Why the Void Presents a 2D Interface: A First-Principles Derivation

From Gauge Invariance to the Fine-Structure Constant

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Document Context: This is a companion document to Papers I–III on the fine-structure constant. It fills a critical gap in the derivation chain by proving that gauge invariance *requires* electromagnetic coupling to occur on a 2D interface. For the derivation of K=7 and N_loop=14 from 2D geometry, see Paper II. For the dynamical justification of the formula structure, see Paper III.

Abstract for General Readers

What is this about?

There's a number in physics — approximately 1/137 — that determines how strongly light interacts with matter. It's called the fine-structure constant (α). It sets the dimensionless strength of electromagnetic coupling and governs atomic spectra and interaction scales. For nearly 100 years, no one has been able to explain *why* it has this particular value.

This document proves a key step toward that explanation.

The key question answered here:

Our explanation of α depends on electromagnetic forces "seeing" a 2-dimensional surface (like a membrane) rather than the full 3-dimensional volume of space. But why would that be true? Isn't space 3D?

The answer:

It's not an assumption — it's a mathematical requirement. The rules of electromagnetism (specifically, something called "gauge invariance") mathematically *force* electromagnetic measurements to be about surfaces, not volumes. This isn't because space is secretly 2D; it's because of how electromagnetism works.

An analogy:

Imagine measuring how much water flows through a hoop held in a river. You need the hoop (a 1D circle), but the measurement is really about the water passing through the *area inside* the hoop (a 2D surface). You can't even define "flow through the hoop" without implicitly talking about that 2D surface. Electromagnetism works the same way — its measurements are fundamentally about 2D surfaces, even when we write them as line integrals around loops.

Why this matters:

Once we establish that electromagnetic forces operate on a 2D interface, the geometry of that interface (hexagons are most efficient, closure requires 7 constraints, channels come in pairs) mathematically forces the fine-structure constant to be approximately 1/137. The number isn't arbitrary — it's geometry.

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Executive Summary

This document presents a key step in deriving the fine-structure constant $\alpha \approx 1/137$ from first principles. We prove that the interface between a pre-geometric substrate (the "void") and our observable universe must be effectively **two-dimensional** — not as an assumption, but as a mathematical necessity arising from the requirements of gauge invariance.

This result is crucial because once we establish that electromagnetic coupling occurs on a 2D interface, the geometric constraints of that interface force specific numerical values that yield $\alpha^{-1} \approx 137$. Specifically:

- **Hexagonal efficiency** on 2D surfaces → K = 7 closure constraints (derived in Paper II, §3)
- **Interface pairing** → N_{loop} = 14 channels (derived in Paper II, §4)
- **Formula uniqueness** → $\alpha^{-1} = 2^K \times (N+1)/N$ (derived in Paper III, §2; see also Appendix A below)

The present document fills the critical gap: **why does EM coupling access a 2D interface at all?** The answer is that gauge invariance mathematically requires it.

The Problem: Where Does "2D" Come From?

The VERSF framework proposes that the fine-structure constant emerges from the geometry of a pre-geometric substrate. The derivation depends critically on electromagnetic coupling "seeing" a 2D coherence layer rather than the full 3D bulk of space.

A fair criticism is: **why 2D?** If this is merely assumed, it's just trading one unexplained fact ($\alpha \approx 1/137$) for another (the interface is 2D).

We need to **derive** the 2D interface from more fundamental principles.

The Key Insight

The answer comes from gauge invariance — specifically, from the mathematical structure of electromagnetism itself.

Gauge invariance is the principle that certain changes to how we describe electromagnetic fields don't affect any physical predictions. It's not optional; it's the mathematical backbone of electromagnetism and is experimentally verified to extraordinary precision.

When we ask "what can a gauge-invariant electromagnetic measurement actually access?", the answer turns out to be: **the minimal operational content is defined on surfaces, not volumes**.

The Proof

Definitions and Setup

The Void: A pre-geometric substrate with no inherent spatial dimensions. It has only:

- Finite distinguishability capacity (can encode information)
- Relational structure (things can be adjacent or distinct)

Gauge Theory: The mathematical framework describing electromagnetism. The electromagnetic potential A determines the field, but A itself is not directly observable — only certain combinations of A are physical.

Gauge Transformation: A mathematical change to A that doesn't affect any physical observable:

$A \to A + d\lambda$

where λ is an arbitrary function.

For general readers — what is gauge invariance?

Think of it like choosing where to put "zero" on a thermometer. Whether you use Celsius or Fahrenheit, the *difference* in temperature between two objects stays physically meaningful, even though the numbers change. In electromagnetism, there's a similar freedom — you can shift the electromagnetic potential by an arbitrary amount without changing any actual physics. "Gauge invariance" is the requirement that our physics doesn't depend on this arbitrary choice.

Step 1: Relations Require At Least 1D Structure

Even though the void is pre-geometric, it must support *relations* — the ability to distinguish "this" from "that" and to encode adjacency. The minimal mathematical structure for relations is a **graph**:

- **Vertices**: Sites where distinctions are registered
- **Edges**: Relations connecting vertices

This is a 1-dimensional complex (a 1-complex): it has points and lines, but no surfaces yet.

Plain English: Before we have space, we need a network of connections. Think of it like a web of relationships — points connected by threads. This is the bare minimum needed to have any structure at all.

Step 2: Electromagnetic Potential Lives on Edges

In gauge theory, the electromagnetic potential A assigns a **phase** (a number representing rotation around a circle) to each edge of our network.

When we perform a gauge transformation at each vertex using a function $\lambda(v)$, the potential on an edge from vertex u to vertex v transforms as:

$$A_{uv} \to A_{uv} + \lambda(v) - \lambda(u)$$

Now consider a **path** P consisting of several edges, going from vertex *a* to vertex *b*. The total phase accumulated along this path is:

$$\Phi_P = \Sigma \text{ (A over all edges in P)}$$

Under a gauge transformation:

$$\Phi_P \to \Phi_P + \lambda(b) - \lambda(a)$$

The crucial point: This depends on the *endpoints*. Different choices of λ give different values for Φ_P.

Therefore: The phase along an open path is gauge-dependent and cannot be a physical observable.

For general readers — why this matters:

Imagine walking through a city where every intersection has a different "altitude reference point." Your phone's altimeter might say you climbed 50 meters going from A to B, but someone else's phone (using different reference points) might say you climbed 200 meters — for the exact same walk!

The "true" altitude gain is meaningless unless everyone agrees on a reference. In electromagnetism, the phase accumulated along an open path has the same problem: it depends on arbitrary choices (the gauge), so it can't be a real physical measurement.

Step 3: Gauge Invariance Requires Closure

The only way to construct a gauge-invariant quantity from the potential A on a network is to consider a **closed loop** C that returns to its starting point.

For a closed loop, the start and end vertices are the same point, so:

$\lambda(\text{end}) - \lambda(\text{start}) = 0$

The **holonomy** around a closed loop:

$W(C) = \exp(i \oint_C A)$

is therefore **gauge-invariant**. It gives the same value no matter how we choose to perform gauge transformations.

This is not a choice — it's a mathematical necessity. Gauge-invariant electromagnetic observables *must* be associated with closed loops.

For general readers — the round-trip principle:

Back to our city analogy: even with inconsistent altitude references at every intersection, one thing *is* well-defined: if you walk in a complete loop back to your starting point, the total altitude change must be zero (you ended where you began). Any reference point shifts cancel out perfectly.

In electromagnetism, only measurements taken "around a loop" — returning to the starting point — give unambiguous answers. This is why physicists talk about "Wilson loops": they're the only gauge-invariant way to probe the electromagnetic field.

Step 4: Closed Loops Imply 2D Surfaces (Stokes' Theorem)

Here's where the dimensional jump occurs.

For any closed loop C, there's a fundamental theorem of calculus — **Stokes' theorem** — that says:

$\oint_C A = \int_S F$

where:

- The left side is the line integral of A around the loop C
- The right side is the surface integral of $F = dA$ (the electromagnetic field strength) over any surface S bounded by C

What this means: When we write down a gauge-invariant observable (a loop integral), we are *implicitly* referencing a 2D object — the surface that the loop bounds.

The physical content isn't really "phase around a loop" — it's "flux through the enclosed surface."

Critical point for skeptics: While Wilson loops are formally 1-dimensional observables, their physical interpretation as electromagnetic interaction strength is only defined via their equivalence (by Stokes' theorem) to flux through a bounded surface. Without a 2-chain structure (faces), holonomies remain formally definable but lack operational content, because the experimentally meaningful quantity is flux, and flux is only defined on bounded surfaces via Stokes' theorem.

For general readers — why loops secretly require surfaces:

When you draw a circle on paper, you haven't just drawn a line — you've defined an "inside" and an "outside." The loop creates a 2D region whether you intended to or not.

Stokes' theorem says something profound: measuring "around the loop" is mathematically identical to measuring "through the surface inside." The loop and its interior are two descriptions of the same thing.

For electromagnetism, this means that when physicists write equations about loops, they're *really* talking about surfaces. The 2D structure isn't optional — it's hidden in the mathematics all along.

Step 5: Why 1D Is Fundamentally Insufficient

Could we do gauge-invariant physics with only a 1D structure (a graph with no faces)?

No, for a subtle but important reason.

On a pure 1-complex, there are no 2-chains (faces), hence no intrinsic notion of flux. Any attempt to interpret a Wilson loop as field flux implicitly imports a surface structure not present in the model.

In a pure 1-complex (vertices and edges only, no 2D faces), we can define loops, but we cannot give them operational meaning as flux measurements without reference to surfaces.

Consider a loop C. In 3D space, infinitely many different surfaces could be bounded by C — a flat disk, a curved dome, a wildly wrinkled sheet. The flux through each could be different.

Without 2-cells (faces), the question "how much flux passes through *this* loop?" has no well-defined answer because we haven't specified *which surface* we mean.

Technical statement: The first cohomology H^1 of a 1-complex tells us about loops, but physical flux requires specifying 2-chains. Gauge-invariant physics is underdetermined without 2D structure.

For general readers — the soap film problem:

Hold a wire hoop in a river and ask: "How much water flows through?" The answer depends on what shape of "membrane" you imagine stretched across the hoop — flat, curved, domed?

If you *only* have the hoop (1D), the question is ambiguous. You need to specify a surface (2D) to get an answer.

Electromagnetism has exactly this structure. A loop alone doesn't contain enough information to define the physics. You need the 2D surface it bounds. This is why 1D is fundamentally insufficient — not just inconvenient, but *insufficient* — for gauge-invariant electromagnetism.

Step 6: 2D Is Sufficient

A **2-complex** consists of:

- Vertices (0-cells)
- Edges (1-cells)
- Faces (2-cells)

With faces, we have everything needed for gauge-invariant electromagnetism:

1. **Holonomies** are defined on the boundaries of faces
2. **Flux** through each face is well-defined
3. **Coupling strength** can be computed from 2D combinatorics

Crucially, we do **not** need a full 3D metric or bulk geometry. The electromagnetic coupling can be completely characterized by the 2D face structure.

Connection to lattice gauge theory: In lattice U(1) gauge theory, this is exactly why the action and field strength live on plaquettes (2-cells): without faces, there is no gauge-invariant local curvature. The present argument generalizes this observation to any pre-geometric substrate.

This is exactly what the α derivation does: It counts constraints ($K = 7$) and channels ($N\_loop = 14$) on a 2D coherence layer, with no reference to 3D bulk properties.

For general readers — the screen analogy:

Think of a movie theater. Everything you see happens on the 2D screen. The projector room behind the screen, the volume of the theater — none of it matters for *what appears on screen*.

Electromagnetism is similar. All electromagnetic physics happens on 2D surfaces. The 3D space "behind" those surfaces is irrelevant for electromagnetic interactions. This is why we can calculate the fine-structure constant from 2D geometry alone.

Plain English: A 2D surface — like a membrane or a screen — is enough for electromagnetism to "work." We don't need to know anything about the volume behind the screen. This is why electromagnetic coupling can be computed from 2D geometry alone.

Scope clarification: This document derives the minimal structure required to *define* gauge-invariant coupling; it does not claim that electromagnetic field solutions do not propagate in emergent 3D spacetime once bulk coherence is achieved.

Why does "surface observability" imply the coupling is 2D-computable?

A critic may agree that gauge-invariant observables are naturally expressed as fluxes through surfaces, yet insist that in ordinary QED the field propagates in the 3D bulk and that the relation between different spanning surfaces depends on the 3D embedding. That objection is correct for a pre-existing manifold with a fixed bulk metric. However, the present argument is not a claim about where solutions to Maxwell's equations live once 3D spacetime already exists; it is a claim about what minimal pre-geometric structure is required to define the gauge-invariant coupling data at all.

Operationally, the electromagnetic coupling strength is not extracted from an arbitrary spanning surface; it is extracted from local, repeatable measurements that compare flux assignments across elementary bounded regions. In ordinary continuum QED, different spanning surfaces for the same loop agree because the bulk already satisfies the relevant closure identities; the point here is that those identities themselves require the 2-cell structure whose minimal presence we are deriving. In discrete gauge theory this is made explicit: the curvature (field strength) is defined on plaquettes (faces), and the coupling constant appears as the weight with which plaquette fluxes contribute to the action and correlation functions. The coupling is fixed by local face-level combinatorics and normalization, while the 3D embedding — when it exists — enters later through how faces are glued into volumes and how waves propagate across large distances.

The key distinction: 3D bulk propagation is a *dynamical* statement about solutions given an emergent bulk, whereas the coupling normalization is a *structural* statement about the minimal gauge-invariant data, which lives on 2-cells. The definition and normalization of electromagnetic coupling is controlled by the 2D face structure of the coherence interface; 3D embedding affects propagation and global consistency, but does not supply additional independent degrees of freedom needed to define the coupling constant at the point of emergence.

Step 7: 3D Requires Additional Emergence

What about 3D structure? Why does gravity "see" the bulk while electromagnetism "sees" the interface?

A 3-complex (adding volumes to our 2-complex) requires something extra: **consistent closure of surfaces around volumes**.

Mathematically, this corresponds to the discrete analogue of Bianchi-type consistency — closure constraints on face-gluing around 3-cells. (In the continuum limit, this becomes the familiar $dF = 0$.)

Enforcing this across a bulk requires **coherent adjacency** — many 2D patches must fit together consistently in three dimensions. This is a **percolation/correlation achievement**: patches must be coherently related across extended regions.

In the VERSF framework, this 3D coherence is achieved at the coherence scale $\xi \approx 88$ μm. Below this scale, only 2D local coupling is well-defined. Above it, bulk geometry emerges.

For general readers — building volumes from surfaces:

Imagine assembling a soccer ball from flat pentagonal and hexagonal patches. Every edge must align perfectly; every corner must close. If the patches don't fit, you get a jumbled mess, not a ball.

3D space is similar: it's "assembled" from 2D surfaces. This assembly requires *coordination* across many surfaces — what physicists call "coherence." At the most fundamental level, surfaces exist first; volumes emerge only when surfaces become coherently organized.

This is why gravity (which needs volumes) is more complex than electromagnetism (which only needs surfaces). Gravity requires an additional achievement — 3D coherence — that electromagnetism doesn't need.

Step 8: Finite Distinguishability Selects Minimal Structure

The void has a finite "budget" for encoding distinctions (finite distinguishability, or FD).

Among structures that support gauge-invariant coupling:

Dimension	Status	Reason
1D	**Insufficient**	Gauge-invariant observables are underdetermined
2D	**Sufficient and minimal**	Flux and holonomy are well-defined
3D	**Requires additional coherence**	Bianchi-type consistency needs bulk correlation

Why should nature select the minimal sufficient structure?

The "minimal structure" principle is not merely aesthetic. It follows from a stability requirement under finite distinguishability (FD): if the substrate can encode only finitely many independent distinctions per coherence domain, then any proposed fundamental structure must satisfy a non-fragility constraint — small fluctuations in encoded distinctions must not destroy the definability of the coupling observables.

A 3D bulk architecture requires additional global consistency (face-gluing closure around 3-cells) beyond what electromagnetism needs to be well-defined. Under FD, that extra requirement consumes distinguishability budget and increases the number of independent constraints that must be simultaneously satisfied. The result is a sharply higher probability of incoherence under generic fluctuations.

Formally, let S_d denote the set of structures of effective dimension d that support gauge-invariant coupling. Define C_d as the minimum number of independent consistency constraints required for stable operation in dimension d. Then under FD, the measure of realizations that satisfy all constraints scales schematically like:

$\mu(\text{stable} \mid S_d) \propto 2^{(-C_d)}$

Here C_d can be interpreted as the number of independent binary consistency checks that must simultaneously be satisfied to realize a stable coupling manifold in dimension d; under FD, each additional independent check suppresses the stable realization measure by a factor ≈ 1/2.

(Up to model-dependent prefactors.) Since $C_3 > C_2$ (bulk closure requires additional coherence conditions), the stable realization measure is exponentially suppressed in 3D relative to 2D at the point of emergence.

Thus, 2D is not merely sufficient; it is the **dominant stable attractor** under finite distinguishability. The framework does not deny that 3D can emerge; it predicts that 3D emergence is a higher-coherence achievement that occurs only once FD resources and correlation length allow the additional closure constraints to be satisfied reliably.

In VERSF terms: 2D coupling is the "first stable fixed point" of gauge-invariant definability; 3D bulk is a subsequent fixed point reached only after coherence percolation.

Information-theoretic extremum principle: Among manifolds capable of supporting gauge-invariant coupling, the 2D complex minimizes distinguishability cost per independent physical degree of freedom.

This is not merely an aesthetic preference for simplicity. It's a variational statement:

- 1D fails the constraint (cannot support the physics)
- 3D exceeds the constraint (requires FD expenditure on bulk coherence that isn't needed for EM coupling)
- 2D saturates the constraint (exactly sufficient, nothing wasted)

By the principle of minimal structure (the same principle that gives hexagonal tiling as the most efficient partition), the **first emergent coupling interface is 2D**.

For general readers — nature's efficiency principle:

If you have limited resources and need to build something functional, you build the simplest thing that works:

- A 1D network isn't enough (can't define electromagnetic physics)
- A 3D volume is more than needed (requires extra "coherence work")
- 2D is just right — exactly sufficient, nothing wasted

This is like asking: "What's the minimum number of legs a table needs to stand?" Three legs are sufficient and minimal. Two legs fail; four legs are more than necessary. Nature, constrained by finite information capacity, selects 2D for electromagnetism the same way — it's the minimal dimension that gets the job done. *(Note: This is a heuristic analogy; the rigorous argument is the information-theoretic extremum principle above.)*

The Theorem

We can now state the result precisely:

Theorem (Minimal Gauge-Invariant Coupling Manifold)

Given:

1. A pre-geometric substrate with finite distinguishability
2. The requirement of gauge invariance for electromagnetic coupling

Then:

- The minimal emergent structure supporting stable gauge-invariant coupling is **2-dimensional**
- This is not an assumption but a **mathematical necessity**

For general readers — why this theorem matters:

This theorem says that 2D surfaces aren't a guess, a simplification, or a modeling choice — they're *required* by electromagnetism. Any universe that has gauge-invariant electromagnetism must have EM interactions happening on 2D structures.

This is as certain as saying "triangles have three sides." It's not about our universe specifically; it's about what gauge invariance mathematically requires.

Key Lemma

The following lemma captures the operational content of the proof:

Lemma (Operational Gauge Observables, U(1))

In abelian U(1) gauge theory, any gauge-invariant electromagnetic observable that distinguishes physical configurations (i.e., is not pure gauge) can be represented as flux of F = dA over a bounded 2-chain.

Proof sketch: Gauge invariance removes open-path phases; nontrivial observables reduce to loop holonomies (Step 3). By Stokes, holonomy is equivalent to flux through a spanning surface; the latter requires a 2-chain structure (Steps 4-5). ■

Note on "operational": Here "operational" means that two distinct field configurations can be distinguished by the observable under allowed gauge transformations. An observable that gives identical values for gauge-inequivalent configurations is not operationally meaningful.

Note on non-abelian theories: This lemma is specific to abelian U(1) gauge theory. For non-abelian gauge theories (SU(2), SU(3), etc.), the situation is more complex: Wilson loops themselves are not gauge-invariant — only their traces are. However, the fundamental point remains: gauge-invariant observables in non-abelian theories are still constructed from holonomies around loops, and their physical content still involves surface-like structures (the loops bound surfaces on which the curvature is integrated, albeit in a path-ordered fashion). The same minimal-structure moral persists — curvature is a 2-form and local gauge-invariant content is naturally face-based — though the non-abelian case requires path ordering and additional algebraic care.

This lemma is the fulcrum of the entire argument. It says that **operationally meaningful electromagnetism is intrinsically 2-dimensional** — not as an approximation or convenience, but as a consequence of U(1) gauge structure.

For general readers — the lemma in plain terms:

This lemma is the heart of everything. It says: "If you can actually *measure* an electromagnetic quantity in the real world, that measurement is necessarily about flux through a surface."

Not "often about surfaces" or "conveniently described by surfaces" — but *necessarily, always, inescapably* about surfaces. Every real electromagnetic measurement is a 2D measurement, whether the experimenter realizes it or not.

Consequences for the Fine-Structure Constant

This theorem is the keystone of the α derivation:

The logical chain:

1. **Gauge invariance** → requires closed loops (this document, Steps 2-3)
2. **Closed loops** → reference bounded surfaces via Stokes (this document, Step 4)
3. **Bounded surfaces** → 2D structure is necessary and sufficient (this document, Steps 5-6)
4. **Minimal 2D structure** → hexagonal efficiency applies (Paper II, §3; Hales 2001)
5. **Hexagonal + closure** → K = 7 constraints (Paper II, §3.2)
6. **Interface pairing** → N_loop = 14 channels (Paper II, §4)
7. **Combinatorics** → $\alpha^{-1} = 2^7 \times (15/14) =$ **137.143** (Paper III, §2; Appendix A below)

The observed value is $\alpha^{-1} = 137.036$, a match to **0.08%** with no adjustable parameters.

On the formula structure: The specific form $\alpha^{-1} = 2^K \times (N_{loop} + 1)/N_{loop}$ is not chosen but forced by structural requirements:

- The factor 2^K arises from K independent binary constraints (each P = ½ by entropy maximization; Paper III, §2.1)
- The correction (N_loop + 1)/N_loop arises from democratic channel contributions plus global closure (Paper III, §2.3)
- **No alternative functional combination of K and N_loop satisfies symmetry, extensivity, and independence simultaneously** (proven two ways in Appendix A)

Appendix A provides two independent uniqueness proofs:

- **Proof I** assumes the correction form and derives uniqueness
- **Proof II** (stronger) *derives* the Möbius form (N+1)/N from countability, coarse-graining consistency, and single-offset emergence

This is why the derivation produces a unique numerical prediction rather than a family of possibilities.

For general readers — how we get 137:

Here's the punchline:

1. Electromagnetism must operate on 2D surfaces (proven above)
2. What's the most efficient way to tile a 2D surface? Hexagons (mathematically proven — it's why honeycombs are hexagonal)
3. Hexagonal tiling + closure requirements = exactly 7 independent constraints
4. Pairing up information channels = 14 channels
5. Now do the math:
    - $2^7 = 128$ (the base factor from 7 binary constraints)
    - $128 \times (15/14) = 137.14$ (after the channel correction)

The measured value is $\alpha^{-1} = 137.04$. We got within 0.08% using nothing but geometry — no adjustable parameters, no fitting to data.

On the residual discrepancy (~0.08%)

The prediction $\alpha^{-1} = 2^7 \times (15/14) = 137.142857...$ should be interpreted as a **zeroth-order structural value**: the value obtained when (i) the interface is treated as perfectly democratic across all N channels, and (ii) the closure constraints are taken as exactly binary and exactly independent at the interface scale.

In a complete theory, small corrections are expected once one includes:

- Weak channel non-democracy due to finite correlation length across the interface
- Renormalization of the effective coupling between the interface scale and the laboratory scale
- Curvature/defect corrections to ideal hexagonal closure (e.g., rare disclinations required by global topology)

VERSF therefore treats the 0.08% gap not as an arbitrary mismatch but as a **finite-coherence correction**:

$\alpha^{-1} = 2^K \times (N+1)/N \times (1 - \varepsilon_{corr})$, where $\varepsilon_{corr} \sim 10^{-3}$

The correction ε_{corr} encodes the leading deviation from perfect channel democracy and perfect closure. Importantly, the sign is physically interpretable: any leakage of coherence or partial channel suppression reduces effective coupling normalization, lowering α^{-1} toward the measured value.

The leading-order derivation is parameter-free; the residual magnitude is of the order expected for a finite-correlation interface rather than an exact mathematical identity. Paper III treats this correction as a dynamical fixed-point perturbation.

In the complete program, ε_{corr} is not a tunable fit parameter; it is determined by (i) correlation-length-limited channel democracy on the interface, (ii) defect/disclination density implied by global topology, and (iii) the renormalization flow from the interface scale to laboratory scale.

Why Gravity Is Different

The derivation also explains why gravity accesses 3D bulk statistics rather than the 2D interface:

Interaction	What It Couples To	Geometric Access	Relevant Structure
Electromagnetism	Phase coherence	2D interface	Surfaces, holonomies
Gravity	Adjacency/embedding	3D bulk	Volumes, percolation

Why phase is intrinsically 2D, while embedding is intrinsically 3D

The distinction here is not post-hoc. In gauge theory, the physically meaningful content is encoded in holonomy — the parallel transport of phase around loops — which is naturally detected by flux through surfaces. This is why, even in conventional formulations, field strength is a two-form and couples to 2-dimensional integration domains.

By contrast, gravitational structure concerns mutual embedding and closure: whether collections of faces can be glued into consistent volumes and whether curvature constraints close around 3-cells. In discrete gravity (e.g., Regge calculus), curvature is concentrated on 2-dimensional hinges, but the consistency of those hinges as a geometry requires a surrounding 3D assembly; the deficit angles have meaning only relative to a 3D simplicial neighborhood.

This perspective also resonates with holographic scaling intuitions: gauge-invariant informational content is naturally bounded by surfaces (area-like bookkeeping), while the emergence of a stable bulk geometry requires extended correlation and consistency across many surface patches.

In VERSF terms: electromagnetism is the first "surface-closed" interaction (holonomy/flux closure), whereas gravity is the first "volume-closed" interaction (face-gluing closure around 3-cells). The difference is geometric-operational: phase coherence is testable on surfaces; embedding coherence is testable only once volumes exist.

Electromagnetism is about phase relationships, which are captured by surface flux.

Gravity is about how regions fit together in space, which requires bulk coherence.

This isn't an ad hoc distinction — it follows from what each interaction mathematically *is*.

For general readers — why gravity and electromagnetism are different:

Electromagnetism asks: "What's the phase relationship between point A and point B?" — a question answered by looking at the surface between them.

Gravity asks: "How does this region of space connect to its neighbors?" — a question that requires knowing about the full 3D volume.

It's like the difference between:

- "What color is this wall?" (a surface question)
- "What shape is this room?" (a volume question)

Different questions need different geometric information. Electromagnetism needs surfaces; gravity needs volumes. The math isn't arbitrary — it follows from what each force actually *does*.

What Remains Postulated vs. Derived

After this proof, here is the status of assumptions in the α derivation:

Element	Status	Source
Pre-geometric void	**Postulated**	VERSF foundation
Finite distinguishability	**Postulated**	VERSF foundation
Gauge invariance	**Orthodox physics**	Experimentally established
2D interface	**DERIVED**	This document
Hexagonal efficiency	**Derived**	Paper II, §3; Hales (2001)
K = 7	**Derived**	Paper II, §3.2
N_loop = 14	**Derived**	Paper II, §4
Formula structure	**Derived**	Paper III, §2; Appendix A
$\alpha^{-1} \approx 137$	**DERIVED**	Combined result

The only genuine postulates are:

1. The existence of a pre-geometric substrate (the void)
2. That it has finite capacity for distinctions
3. Standard gauge theory (which is just orthodox physics)

Everything else follows.

For general readers — what we're NOT assuming:

Look at what we didn't have to assume:

- We don't assume the universe is 2D — we *derive* that EM sees 2D
- We don't assume hexagons — we *derive* them from efficiency
- We don't assume the numbers 7 or 14 — we *derive* them from geometry
- We don't assume the formula — we *derive* it from symmetry

The only things we take for granted are: (1) there's some fundamental substrate, (2) it has limited information capacity, and (3) electromagnetism works the way experiments show. Everything else — including the value 137 — is *calculated*, not assumed.

This is what makes this different from numerology. Numerology fits numbers after the fact. This framework predicts 137.14 before looking at the answer.

Summary for General Readers

The Big Picture

For nearly a century, physicists have wondered why the fine-structure constant has the value it does (approximately 1/137). It determines the strength of electromagnetic interactions and appears to be arbitrary — a number we must measure but cannot explain.

This document establishes a key step in explaining that number.

The key insight: The minimal operational content of electromagnetism is defined on 2D surfaces, not 3D volumes. This isn't because the universe is secretly 2D, but because gauge symmetry — the mathematical backbone of electromagnetism — restricts what can be physically measured to surface-based quantities (flux).

Once we know that electromagnetic coupling occurs on a 2D interface, the geometry of that interface forces the fine-structure constant to be approximately 1/137:

- Hexagons are the most efficient way to tile a 2D surface (see Paper II)
- This tiling requires 7 closure constraints (see Paper II)
- Channels come in pairs, giving 14 (see Paper II)
- The formula $2^7 \times (15/14) = 137.14$ is uniquely forced (see Paper III and Appendix A)

The bottom line: $\alpha \approx 1/137$ isn't arbitrary. It's the mathematical consequence of:

- How gauge invariance works (this document)
- The geometry of 2D surfaces (Paper II)
- The combinatorics of closure and channels (Papers II and III)

What This Means

If this derivation is correct:

1. **The fine-structure constant is not a free parameter** — it's derivable from geometry and gauge theory
2. **The "hierarchy" of dimensions makes sense** — 2D interfaces emerge before 3D bulk, which explains why electromagnetic and gravitational constants have different origins
3. **The 0.08% match is not numerology** — it's a parameter-free prediction from first principles
4. **Physics may have fewer arbitrary inputs than we thought** — if α can be derived, perhaps other "fundamental constants" can too

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Technical Notes

Relation to Standard Gauge Theory

This work does not contradict standard gauge theory; it derives its minimal structural prerequisites.

The argument presented here is fully compatible with — indeed, implicit in — the standard formulation of U(1) gauge theory. We are not proposing exotic physics or modifications to electromagnetism. Rather, we are asking a foundational question that standard treatments typically bypass:

What is the minimal geometric structure required to support gauge-invariant electromagnetic coupling?

The answer — 2D cell structure (faces/plaquettes) — is already embedded in:

- **Lattice gauge theory**: The Wilson action is defined on plaquettes (2-cells), not on edges alone. This is not a discretization artifact; it reflects the fact that gauge-invariant curvature requires faces.
- **Topological field theory**: In TQFT, observables are associated with manifolds of specific dimension. For abelian gauge theory, the natural observables (holonomies, fluxes) are intrinsically 2D in character.
- **Differential geometry**: The curvature 2-form F lives in $\Omega^2(M)$, not $\Omega^1(M)$. The "2" is not accidental.

What this document adds is the explicit recognition that this 2D structure is not just convenient but *necessary* for operational gauge-invariant physics — and that this necessity has consequences for coupling constants when combined with efficiency principles on the emergent interface.

The VERSF framework provides a context (pre-geometric substrate, finite distinguishability) in which this observation becomes predictive. But the core gauge-theoretic argument (Steps 1–6) stands independently of VERSF and should be uncontroversial to anyone familiar with lattice gauge theory or geometric quantization.

On the "Wilson Loops Are 1D" Objection

A sophisticated critic might argue: "Wilson loops are 1-dimensional observables. Your argument that 2D is required is overstated."

The response: This confuses the *mathematical definition* of a Wilson loop (a 1D path integral) with its *physical/operational content*.

In ordinary abelian gauge theory with F = dA well-defined, the holonomy around a loop is determined by the loop alone. However, the *operational meaning* of that holonomy — what it tells us about the physical field — comes from its equivalence (via Stokes) to flux through a bounded surface.

On a pure 1-complex (vertices and edges only), you can formally define holonomies, but you cannot give them operational content as flux measurements. The surface structure is what makes the observable physically meaningful, not just mathematically well-defined.

Gauge theory textbooks often suppress this distinction because they work in contexts where the surface structure is already present. But at the foundational level — asking what minimal structure is required to support gauge-invariant physics — the 2D structure is essential.

On the EM/Gravity Dimensional Split

The claim that EM accesses 2D interfaces while gravity accesses 3D bulk is not ad hoc. It follows from what each interaction *couples to*:

Interaction	Couples To	Mathematical Object	Minimal Dimension
Electromagnetism	Phase	Holonomy / Flux	2D (surfaces)
Gravity	Adjacency	Embedding coherence	3D (volumes)

This aligns with:

- **Regge calculus**: curvature lives on 2D hinges, but deficit angles require 3D embedding
- **Spin foam models**: faces carry group representations (2D), but amplitudes require consistent 3D assembly
- **Entropic gravity ideas**: entropy is associated with surfaces, but gravitational dynamics requires bulk

The VERSF framework captures this structure without borrowing the specific machinery of these programs.

On the Cohomological Perspective

For readers familiar with algebraic topology:

The argument can be phrased in terms of de Rham cohomology. Gauge potentials are elements of $\Omega^1(M)$ (1-forms). Gauge transformations are exact forms $d\Omega^0(M)$. Physical observables live in the first de Rham cohomology $H^1\_dR(M)$.

For a manifold with nontrivial H^1, the cohomology classes are detected by integration over 1-cycles — but the physical *interpretation* (electromagnetic flux) requires Stokes' theorem, which references 2-chains.

The key point is that while loops detect cohomology, flux is defined on surfaces. The physical content of H^1 electromagnetic observables is intrinsically 2-dimensional.

Connection to lattice gauge theory: This is precisely why lattice gauge theory places the action on plaquettes (2-cells) rather than on edges alone. The Wilson action $S = \Sigma\_p (1 - Re[U\_p])$

sums over plaquettes because gauge-invariant curvature requires a 2-cell to be well-defined. Edges carry the connection (group elements U_e), but the physics — the curvature that enters the action — lives on faces. The lattice formulation makes explicit what the continuum theory encodes implicitly: electromagnetic dynamics is fundamentally about 2D structures.

On Null Surfaces

The light-cone gauge analysis in Paper I provides an independent route to the same conclusion, reinforcing the 2D claim from a different direction.

The argument from Paper I: In light-cone gauge ($A^+ = 0$), the electromagnetic field on a null surface (x^+ = const) reduces to exactly two physical degrees of freedom — the transverse polarizations. The longitudinal and timelike components are eliminated by gauge fixing and constraints. What remains lives on the 2D transverse plane.

Why this supports the present argument:

1. **Dimensional reduction on null surfaces**: The physical content of the EM field, when restricted to a null surface, is intrinsically 2D. This is not a coordinate artifact — it reflects the fact that light-like propagation "sees" a transverse screen.
2. **Impedance interpretation**: Paper I derives $\alpha = Z_0/(2R_K)$ as an impedance mismatch between quantum channels (~26 kΩ) and the vacuum load (~188 Ω per polarization). The factor of 2 comes precisely from the two transverse modes — i.e., the 2D structure of the null surface.
3. **Operational equivalence**: Measuring electromagnetic coupling (e.g., photon absorption by graphene, vacuum admittance) always involves transverse modes on an effective screen. The 2D interface is where operational physics happens.

Connection to the gauge-invariance argument: The gauge-invariance argument (Steps 1–6) shows that 2D is *necessary* for operational observables. The null-surface argument shows that physical EM degrees of freedom naturally *reduce* to 2D on propagation boundaries. These are two faces of the same coin: gauge theory requires surfaces; dynamics delivers surfaces.

The consistency between the gauge-invariance argument (this document), the null-surface analysis (Paper I), and the statistical geometry argument (Paper II) provides strong mutual reinforcement for the 2D interface claim.

On the Emergence Order

The claim that "2D emerges before 3D" should be understood carefully:

We are not claiming that the universe went through a 2D phase historically. Rather, in the logical/constitutive order of emergence from a pre-geometric substrate:

1. Relations (0D points, 1D edges) come first
2. Gauge-invariant coupling requires 2D structure

3. Bulk 3D geometry requires coherent assembly of 2D patches

This is an order of *logical priority*, not temporal sequence (especially since time itself is emergent in VERSF).

Appendix A: Uniqueness of the Formula Structure

This appendix proves that the formula $\alpha^{-1} = 2^K \times (N+1)/N$ is the *unique* form consistent with the structural requirements, not merely one possibility among many. We provide two independent proofs: the first assumes the correction form directly; the second derives it from weaker "physics-y" axioms.

A.1 The Claim

Uniqueness Claim: Given only the integers K (closure constraints) and $N \equiv N_{loop}$ (independent loop channels), there is no other combination consistent with the structural requirements (symmetry, independence/composition, and channel democracy) that can define the coupling strength — except the form $2^K \times (N+1)/N$ (up to trivial reparameterizations).

A.2 Proof I: Direct Axiomatization

Let F(K,N) denote the derived dimensionless coupling inverse (i.e., $F = \alpha^{-1}$).

Axiom A1 — Domain/discreteness $K \in \mathbb{N}$ is the count of independent closure constraints; $N \in \mathbb{N}$ is the count of indistinguishable loop channels. F is defined on $\mathbb{N} \times \mathbb{N}$ and depends only on these integers.

Axiom A2 — Constraint independence (additive composition) If you combine two independent closure sets, constraints add while the channel architecture N remains unchanged. The effect on inverse coupling must multiply:

$$F(K_1 + K_2, N) = F(K_1, N) \times F(K_2, N)$$

This is the standard "independent factors multiply" principle.

Axiom A3 — Channel democracy (permutation symmetry) All loop channels are physically indistinguishable up to relabeling. F may depend on N only through the count, not through any structure like "channel 3 is special."

Axiom A4 — Minimality in N: "no structure, no correction" If there are infinitely many channels, finite-channel discreteness should disappear:

$\lim(N\to\infty) F(K,N)/F(K,\infty) = 1$

Axiom A5 — Minimal correction form (single-hub offset) The finite-N correction arises from one extra equivalence class interacting democratically with N identical channels:

$C(N) = 1 + c/N$ for some constant integer c

Axiom A6 — Normalization of the hub (one extra state) The hub contributes exactly one additional effective degree among the N channels:

$c = 1$, so $C(N) = (N+1)/N$

Why exactly one hub class (why c = 1, not 0 or 2)?

Within the interface construction (Paper II), N counts the number of democratic loop channels available for phase transport across the coherence layer. Closure requires one additional global equivalence class: a single non-local "closure bookkeeping" degree that enforces consistency of loop composition across the interface.

This object is not another channel; it is the unique global constraint that ties the otherwise independent channels into a coherent gauge structure. Equivalently: the hub is the minimal global closure functional that lifts local face holonomies into a consistent global gauge structure; introducing more than one independent such functional adds unmotivated global degrees of freedom.

- If **c = 0**, there is no global closure bookkeeping and loop composition remains underconstrained (the interface is not a stable coupling manifold)
- If **c ≥ 2**, one has introduced additional non-democratic structure — multiple privileged global classes — which would (i) violate channel democracy and (ii) imply extra independent closure mechanisms not present in the minimal hexagonal-hub interface

Thus the minimal consistent gauge-closure architecture contains exactly one such global class, yielding the unique offset $N \to N + 1$.

Theorem (Proof I): Under axioms A1–A6, the only possible form of F(K,N) is:

$F(K,N) = 2^K \times (N+1)/N$

Proof:

Step 1 — A2 forces exponential dependence on K

Fix N. Define $f\_N(K) := F(K,N)$. Axiom A2 says:

$f\_N(K_1 + K_2) = f\_N(K_1) \times f\_N(K_2)$

On ℕ, the only positive solutions are $f_N(K) = [f_N(1)]^K$. Therefore:

$$F(K,N) = A(N)^K$$

for some positive function $A(N) = F(1,N)$.

This rules out any formula like $K \cdot g(N)$, K^2, $\log K$, K + stuff, etc.

Step 2 — Binary constraint meaning fixes the base to 2

Each independent closure constraint is a binary admissibility requirement (pass/fail). Independent binary constraints multiply equivalence classes by factor 2 each. Therefore:

$$F(K+1, N) = 2 \times F(K,N)$$

Could $A(N)$ depend on N? If so, adding a closure constraint would have different effects depending on channel count, contradicting "independent closure constraint." So $A(N) = 2$, giving:

$$F(K,N) = 2^K \times C(N)$$

where $C(N)$ captures only the finite-channel correction.

Step 3 — A4–A6 force $C(N) = (N+1)/N$

From A4: $C(N) \to 1$ as $N \to \infty$ From A5: $C(N) = 1 + c/N$ (minimal first-order correction) From A6: $c = 1$

Therefore $C(N) = (N+1)/N$, and:

$$\mathbf{F(K,N) = 2^K \times (N+1)/N} \blacksquare$$

A.3 Proof II: Democracy + Coarse-Graining Consistency + Countability ⇒ Möbius Form

This proof is stronger: it *derives* the $(N+1)/N$ form from weaker axioms rather than assuming it.

As established in Proof I, constraint independence forces $F(K,N) = 2^K \times C(N)$, so all remaining freedom is in the finite-channel correction $C(N)$. We now prove $C(N)$ is uniquely forced to be $(N+1)/N$.

Axiom B1 — Channel democracy (exchangeability) All N channels are indistinguishable: any physical quantity may depend on the channels only through the cardinality N. So $C(N)$ is a function of N alone.

Axiom B2 — Coarse-graining consistency (replication invariance) If we replicate the interface m times (creating m identical non-interacting copies), the effective "finite-N correction" per copy should not change; i.e., the correction is an intensive structural factor, not extensive with replication.

This implies C(N) depends only on the per-copy channel count, not on total channels across disjoint replicas. Equivalently: C cannot encode any additional scale besides the integer N.

Axiom B3 — Countability (no hidden continuous structure) The correction C(N) comes from combinatorics of a democratic discrete interface, not from a continuous parameter. Therefore C(N) must be representable as a ratio of polynomial counts in N with integer coefficients:

$C(N) = p(N)/q(N)$, where $p, q \in \mathbb{Z}[N]$

This is the "physics" version of: if the correction is purely combinatorial, it must literally be a ratio of counts.

Axiom B4 — Vanishing correction at large N Finite-channel effects vanish as $N \to \infty$:

$\lim(N\to\infty)\ C(N) = 1$

Axiom B5 — Single-offset emergence The finite-N correction is produced by exactly one additional interface equivalence class beyond the N democratic loop channels (the "hub/closure" class), not by a whole tower of new structures.

This is weaker than assuming the explicit algebraic form: it only says the difference between numerator and denominator counts is of order one, not order N or N^2.

Theorem (Möbius Uniqueness): Under B1–B5,

$C(N) = (N + c)/N$ for some integer constant c

and single-offset emergence forces $c = 1$, hence $C(N) = (N+1)/N$.

Proof:

Step 1 — Rationality + large-N limit forces equal degrees

Write $C(N) = p(N)/q(N)$ with integer polynomials.

Since $\lim(N\to\infty)\ C(N) = 1$, we must have deg(p) = deg(q), and the leading coefficients of p and q are equal.

So we can write: $p(N) = aN^d + \cdots$, $q(N) = aN^d + \cdots$

Thus $C(N) = 1 + O(1/N)$ automatically.

Step 2 — Single-offset emergence kills higher-order structure

B5 says the correction comes from one extra equivalence class relative to the N democratic channels.

The difference between numerator-count and denominator-count must be constant order, not growing like N, N^2, etc.

The simplest way a ratio of counts tends to 1 while differing by only O(1) is:

- Denominator counts "how many democratic channels": proportional to N
- Numerator counts "democratic channels + one extra class": proportional to N + c

Any higher-degree polynomial (N^2, N^3, …) would correspond to emergent structure scaling like areas, volumes, higher adjacency constraints — violating "single-offset" minimality.

Therefore the only allowed degree is $d = 1$:

$p(N) = aN + b$, $q(N) = aN + d$

Step 3 — Large-N limit pins the leading coefficients

From Step 1, leading coefficients match automatically for degree 1: both must be $a \neq 0$.

So: $C(N) = (aN + b)/(aN + d)$

Step 4 — Democracy + "no extra scale" pins the denominator shift to zero

The denominator represents the actual number of democratic channels available for loop routing, which is exactly N.

If the denominator were $N + d$ with $d \neq 0$, then two interfaces with different true channel counts could produce the same predicted correction by reparameterizing N, making N non-identifiable from observation. But N is defined as an observable combinatorial count (Paper II, §4), so it must enter without hidden offsets.

Equivalently: a shift $d \neq 0$ would imply the effective channel count differs from the measured channel count, hiding non-democratic structure inside the "channel count" definition itself.

So consistency of interpretation forces $q(N) \propto N$, hence $d = 0$:

$C(N) = (aN + b)/(aN) = 1 + (b/a) \times (1/N)$

Since p, q count discrete classes, b/a must be an integer c. Therefore:

$C(N) = (N + c)/N$

Step 5 — Single extra class implies c = 1

B5 says exactly one additional equivalence class beyond the N democratic loop channels contributes to the coupling observable. That is precisely c = 1:

C(N) = (N + 1)/N ■

Corollary: Since F(K,N) = 2^K × C(N), we obtain uniquely:

F(K,N) = 2^K × (N+1)/N

A.4 What These Proofs Rule Out

Proposed Form	Violation
F = K·N or F = K/N	Violates A2 (not multiplicative)
F = 2^K × (N+a)/(N+b) with b≠0	Violates "denominator = N" (Step 4 of Proof II)
F = 2^K × (1 + 1/N + 1/N²)	Violates single-offset (not Möbius form)
F = 2^K × exp(1/N)	Violates countability B3 (not a ratio of counts)
F = 3^K × (N+1)/N	Violates binary constraint meaning
F = 2^K × (N+2)/N	Violates single extra class (c ≠ 1)

A.5 Where Skeptics Can Push (and How to Defend)

The uniqueness hinges on two "meaning axioms":

1. Binary closure constraints → base 2

Defense:

- Closure is admissible/non-admissible (pass/fail)
- Finite distinguishability operates as bit-commitments
- K is literally a count of binary closures (Paper II, §3.2)

2. Single hub offset → (N+1)/N

Defense:

- The interface construction has exactly one hub class coupling democratically to N symmetric channels (Paper II, §4.1)
- The independence between K and N means the only cross-term permitted is the minimal democratic offset
- Proof II derives this from countability + coarse-graining consistency, not by assumption

A.6 Why Proof II Is Stronger

Proof I assumes A5–A6 (the correction has form 1 + c/N with c = 1).

Proof II derives the Möbius form (N+c)/N from:

- Countability (ratio of integer polynomials)
- Large-N limit (equal degrees)
- Single-offset (degree 1)
- Democracy (denominator = N exactly)

Then c = 1 follows from "one extra class."

A skeptic can no longer say "you just chose (N+1)/N" — because Proof II shows it's the *only* rational correction compatible with democracy, countability, and single-offset emergence.

References

This Paper Series:

1. **Paper I**: "The Fine-Structure Constant as Electromagnetic Impedance Mismatch" — Derives $\alpha = Z_0/(2R\_K)$ from vacuum/quantum impedance ratio; proves factor of 2 from transverse polarizations
2. **Paper II**: "The Fine-Structure Constant from Vacuum Geometry" — Derives K=7 from hexagonal efficiency and closure; derives N_loop=14 from interface pairing
3. **Paper III**: "Dynamical Completion of the Fine-Structure Constant" — Shows A1-A4 are dynamical fixed points; proves formula uniqueness

External References:

4. Hales, T. C. "The Honeycomb Conjecture." *Discrete & Computational Geometry* 25, 1–22 (2001) — Proves hexagonal tiling minimizes perimeter/area ratio
5. CODATA 2022: $\alpha^{-1} = 137.035999177(21)$

Related VERSF Programme Documents:

6. "Two-Planck Scales from Entropy-Momentum Foundations" — Establishes $\xi \approx 88$ μm coherence scale
7. "Void Energy-Regulated Space Framework: Foundations" — Core VERSF postulates (finite distinguishability, pre-geometric void)

Conclusion

The 2D interface between the void and observable physics is not assumed — it is **derived** from gauge invariance.

This completes the logical chain from first principles to $\alpha \approx 1/137$:

Gauge Invariance → 2D Interface → Hexagonal Geometry → K=7, N_loop=14 → α^{-1} = 137.14

The fine-structure constant, far from being an arbitrary parameter, emerges as a mathematical necessity from the structure of gauge theory and the geometry of emergent space.

For general readers — the final word:

For a century, physicists have treated 1/137 as a brute fact — a number that simply *is*, with no explanation possible. This document shows otherwise.

The number isn't arbitrary. It's what you get when you combine:

- The mathematics of electromagnetism (gauge invariance)
- The fact that loops imply surfaces (Stokes' theorem)
- The geometry of efficient 2D tiling (hexagons)
- Simple counting (7 constraints, 14 channels)

The universe isn't "fine-tuned" with arbitrary constants. It's *mathematically constrained*. The value 1/137 is as inevitable as the fact that hexagons tile a plane more efficiently than squares.

That's a fundamentally different way to understand physical reality.