Relational Geometry and the Universality of the Two-Planck Scale

Constraint Universality, Saturation as an Attractor, and the Foundations of Emergent Gravity

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Companion Paper: This foundational work accompanies *"Two-Planck Principle: From Quantum Geometry to Emergent Gravity"* (Taylor, 2025), which derives quantitative predictions including the coherence scale $\xi \approx 88$ μm, vacuum energy density $\rho_{vac} \approx 5 \times 10^{-10}$ J/m³, and experimental signatures. The present paper establishes *why* the structural assumptions of that framework are not adjustable parameters but geometric necessities.

Cross-reference note: Section numbers below refer to the companion paper version dated 2025. Key results are also identified by descriptive labels (e.g., "Route A derivation," "CSS postulate," "perihelion calculation") for robustness against future revisions.

Table of Contents

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1. Abstract for General Readers

Imagine trying to measure the length of a table with only one reference point—it's impossible. You need at least two points to define any distance. This paper argues that the same logic applies at the most fundamental level of reality: meaningful space cannot begin with a single point, but requires a *relationship* between two points.

This seemingly simple idea has profound consequences. If space is built from relationships rather than points, then the smallest meaningful unit of space isn't the "Planck length" (the tiniest distance physics allows, about 10^{-35} meters), but rather *twice* that length—the minimum needed for one thing to be related to another.

From this starting point, we show that several features of our universe that have puzzled physicists for decades are not mysterious coincidences but logical necessities:

- **Why does the universe's expansion accelerate at exactly the rate we observe?** Cosmologists attribute this to "dark energy," but our framework shows no mysterious substance is needed. The observed acceleration is simply the natural behavior of structured vacuum at the coherence scale—vacuum energy that saturates a specific gravitational limit, like water filling a container exactly to the brim. Any more and the universe would collapse; any less and space would keep reorganizing without settling down.
- **Why does gravity behave the way it does?** We show that if space emerges from relationships, gravity *must* emerge as the collective effect of these relationships becoming slightly biased near matter.
- **Why is there a special "coherence scale" around 100 micrometers (the width of a human hair)?** This scale emerges as the geometric average between the smallest possible length and the size of the observable universe—a bridge between the quantum world and the cosmos.

Critically, we demonstrate that these results are not the product of clever number-fitting. The key number in our framework—seven independent constraints that must be satisfied for space to form—is *forced* by the geometry of the problem, not chosen to get the right answer. Changing this number doesn't adjust the theory; it replaces it with a completely different one.

This paper establishes the foundations. A companion paper derives the specific predictions that experiments could test.

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2. Technical Abstract

The companion paper (*Two-Planck Principle: From Quantum Geometry to Emergent Gravity*) demonstrated that a relational framework based on the emergence scale $\ell_e = 2\ell_P$ yields:

- A mesoscopic coherence scale $\xi \sim \sqrt{(\ell_p L)} \approx 88$ μm (Route A derivation, Companion §3.2)
- Vacuum energy matching observations to ~20% (CSS + saturation, Companion §3.2)
- Recovery of General Relativity at first post-Newtonian order including Mercury's perihelion precession (foam clock mechanism, Companion §4.9)
- The equation of state $w = -1$ from percolation dynamics (entropy-gradient pinning, Companion §4.11)

However, several foundational questions remained open: whether the Two-Planck scale is uniquely determined or merely a dimensional choice; whether vacuum saturation at the cosmological bound is an assumption or a dynamical attractor; whether the microphysical constraint count underlying dimensional transmutation is robust; and whether the multiple derivation routes employed are genuinely independent.

In this paper we address these questions directly:

1. **Universality of $\ell_e = 2\ell_p$**: We show that the Two-Planck scale arises as the defining length of the minimal binary relational universality class, and that alternative geometric primitives (triangles, simplices) correspond to distinct universality classes rather than tunable variants.
2. **Saturation as attractor**: We demonstrate that cosmological saturation is the unique infrared-stable fixed point of relational coherence under gravitational backreaction, with entropy-gradient pressure providing the restoring force that drives under-saturated states toward saturation.
3. **Constraint completeness**: We provide a structural independence argument for the seven coherence constraints ($K = 7$) governing triangular hinges in simplicial foam, and prove that additional constraints are either gauge-redundant or pertain to higher-order objects.
4. **Route independence**: We establish that Route M (microphysical dimensional transmutation) contains no cosmological input, making its convergence with Route A (infrared consistency) a non-trivial structural success. Small variations in constraint count K or loop content generically push ξ to Planckian or cosmological scales; the mesoscopic window is a narrow region of theory space under discrete combinatorial choices.
5. **Minimal microscopic model**: We introduce an operational model of pre-geometric "stitches" that grounds bias, coherence, and emergent gravity without assuming detailed microphysics.

This work does not introduce new empirical predictions. Instead, it establishes that the assumptions underlying the Two-Planck framework are not adjustable parameters but structural necessities.

Interpretive note: The VERSF framework does not posit "dark energy" as a separate substance or field. The observed cosmological acceleration is explained as the natural behavior of structured vacuum at the coherence scale ξ. What cosmologists parametrize as Λ or ρ_Λ is, in this framework, simply $\hbar c/\xi^4$—the intrinsic energy density of relational geometry, not a mysterious additional component of the universe.

Together with the companion phenomenological paper, this completes a closed, falsifiable argument for emergent geometry, vacuum energy regulation, and gravity as a relational phenomenon.

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3. Introduction and Scope

The purpose of this paper is foundational. The companion work (*Two-Planck Principle: From Quantum Geometry to Emergent Gravity*) demonstrated that a relational framework based on the Two-Planck principle leads to:

- A finite vacuum energy density $\rho_{vac} \sim \hbar c/\xi^4$ (vacuum regulation, Companion §2–3)
- A mesoscopic coherence scale $\xi \approx 60$–110 μm via three convergent routes (Route A: §3.2; Route B: §4.5; Route M: §4.8)
- Recovery of gravitational dynamics consistent with General Relativity at weak field, including Mercury's perihelion precession at 43"/century (foam clock mechanism, Companion §4.9)
- A cosmological constant $\Lambda \approx 1.1 \times 10^{-52}$ m^{-2} matching observations to ~20% (CSS derivation, Companion §3.2)
- The equation of state $w = -1$ derived from percolation pinning (entropy-gradient mechanism, Companion §4.11)
- Experimental predictions at the coherence scale including phase signatures (experimental strategy, Companion §5)

That work focused on consequences and testable implications. Here we address a different question: **are the core structural inputs of that framework optional, or are they forced once one adopts a relational view of pre-geometric physics?**

We will show that:

Assumption	Status	This Paper Shows
$\ell_e = 2\ell_p$	Appears arbitrary	Forced by minimal relational geometry
Vacuum saturation (CSS)	Appears postulated	Unique IR attractor with entropy-gradient restoring force
$K = 7$ constraints	Appears fitted	Complete and independent; alternatives change universality class
Route convergence	Could be circular	Routes are genuinely independent; convergence is non-trivial

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4. Universality Classes of Pre-Geometric Structure

For general readers: Before space as we know it exists, *something* must exist that gives rise to space. This section asks: what is the simplest possible "building block" from which space could emerge? The answer determines everything that follows.

4.1 What Is a Universality Class?

Any attempt to define geometry below the Planck scale must first specify what constitutes a **primitive geometric object**. This choice is not numerical but categorical: it defines the *universality class* of the pre-geometric theory.

For general readers: A "universality class" is like a species in biology. All dogs share certain features regardless of breed; all theories in the same universality class share certain features regardless of details. Different universality classes are fundamentally different theories, not adjustable versions of the same theory.

Different choices of primitive objects lead to qualitatively different theories:

Primitive Object	Emergence Scale	Universality Class
Single point	ℓ_p	Not relational (no geometry possible)
Binary relation (edge)	$2\ell_p$	Minimal relational geometry
Triangle	$\sim\sqrt{3}\ \ell_p$	Minimal curvature carrier
Tetrahedron	$\sim\sqrt{6}\ \ell_p$	Minimal volume element

4.2 The Minimal Binary Relation as Geometric Primitive

We adopt the following principle:

Core Principle: Geometry requires relationships between distinct elements. A single point cannot constitute geometry. The minimal geometric entity is therefore an *interval* connecting two points—a binary relation.

This principle reflects deep features of physics:

- **Measurement theory**: All measurements compare at least two entities
- **Information theory**: Information requires distinguishable states, presupposing multiplicity
- **Quantum entanglement**: The fundamental quantum resource is inherently relational
- **Gauge theory**: Physical observables arise from parallel transport *between* points

4.3 Why Not Triangles or Higher Structures?

A natural objection: triangles are the minimal objects that can carry *curvature* (the bending of space that causes gravity). Shouldn't the emergence scale be based on triangles?

The resolution lies in distinguishing two concepts:

Concept	Definition	Minimal Object	Scale
Geometry	Distance, orientation, transport	Binary relation	$2\ell_p$
Curvature	Deviation from flatness around loops	Triangle	$>2\ell_p$

Curvature *presupposes* geometry. You cannot define the "bending" of space without first having space to bend. Triangles require their edges (binary relations) to exist before the triangle can be coherent.

Therefore:

Hierarchy: Binary relations establish *geometry*. Triangles and higher structures carry *curvature* but exist at a higher organizational layer. The geometric emergence scale $\ell_e = 2\ell_p$ precedes and is smaller than any curvature-bearing scale.

4.4 The Factor of Two Is Not Arbitrary

The factor of 2 represents the **minimal multiplicity** required for a relation:

- One cannot have a relation with fewer than two relata
- Larger factors (3, 4, etc.) introduce structure beyond the minimal geometric requirement
- The principle of parsimony dictates starting with the minimal case

Choosing $\ell_e = 3\ell_p$ or $\ell_e = \sqrt{3}\ \ell_p$ would correspond to a different universality class—one based on triangular or higher primitives. This is not a "nearby" theory but a fundamentally different framework with different predictions.

For general readers: Think of it this way: asking "why 2 and not 3?" is like asking "why use atoms instead of molecules as your basic building block in chemistry?" Both are valid choices, but they lead to different theories. We choose the simplest possible building block—a single relationship—which requires exactly two endpoints.

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5. Saturation as an Infrared Fixed Point

For general readers: This section addresses a key question: why does the universe's expansion accelerate at exactly the rate astronomers observe? The VERSF framework doesn't invoke a mysterious "dark energy" substance. Instead, it shows that structured vacuum naturally settles into a specific energy density—pushed toward this exact value from both above and below, like a ball settling into a valley.

A central structural assumption in the Two-Planck framework is that vacuum energy **saturates** the maximal homogeneous energy density consistent with gravitational stability. In the companion paper (Route A derivation, §3.2), this condition was introduced as the **Cosmological Saturation Scenario (CSS)**:

The vacuum state saturates the maximum homogeneous energy density consistent with gravitational stability of the causal patch defined by the future event horizon.

While CSS proved operationally effective—correctly predicting Λ to ~20%—it was introduced as a postulate. Here we show that saturation is not an arbitrary assumption but the **unique infrared-stable fixed point** of relational coherence.

5.1 Gravitational Stability and the Upper Bound

Consider a homogeneous region of characteristic size L containing vacuum energy density ρ. If the total energy exceeds the mass-energy of a black hole of comparable radius, the region collapses. This yields:

$$\rho \lesssim \frac{c^4}{GL^2}$$

For general readers: This equation says: "The energy density of empty space cannot exceed a certain limit that depends on the size of the region. Too much energy in a given volume, and gravity makes it collapse into a black hole."

With the geometric factor $\eta = 3/(8\pi)$ derived from de Sitter geometry:

$$\rho_{max} = \frac{3c^4}{8\pi GL^2}$$

5.2 The Instability of Over-Saturation

If $\rho > \rho_{max}$ at scale L, the causal patch cannot remain homogeneous. Gravitational instability drives:

- Collapse toward black hole formation
- Fragmentation into smaller regions
- Horizon formation isolating the over-dense region

Over-saturation is dynamically unstable. The universe cannot persist in such a state.

5.3 The Instability of Under-Saturation

If $\rho \ll \rho_{max}$, there is no gravitational stopping condition. But this does *not* mean under-saturation is stable. As we now show, under-saturation creates a **restoring force** pushing the system toward saturation.

5.4 Entropy-Gradient Pinning: The Restoring Force

For general readers: This is the key insight. When ordered regions of space try to expand into disordered regions, they encounter resistance—like trying to organize a messy room. This resistance creates pressure that pushes the system toward a specific balance point.

As coherent geometry percolates into incoherent (void) regions, the process encounters resistance:

1. **Coherence requires constraint satisfaction**: At the Two-Planck scale, a geometric triangle is coherent only if $K = 7$ independent constraints are simultaneously satisfied.
2. **Interfaces break constraints**: At the boundary between coherent and incoherent regions, some constraints must be violated.
3. **Broken constraints increase entropy**: Each violated constraint increases the number of accessible microstates, raising entropy.
4. **Entropy gradients resist expansion**: This entropy increase creates an effective "surface tension" opposing further percolation of coherent geometry.

The resulting entropy-gradient pressure is:

$$P_{grad} \sim \frac{\kappa}{L^2} \sim \frac{\hbar c \cdot \xi}{L^2}$$

where $\kappa \sim \hbar c \cdot \xi$ is the gradient stiffness determined by coherence-cell energy.

Crucially: This pressure *increases* as the system moves further below saturation (smaller effective L), driving it back toward the saturation boundary.

5.5 Saturation as the Unique Infrared Fixed Point

Combining the two instabilities:

State	Pressure	Result
Over-saturated ($\rho > \rho_{max}$)	Gravitational collapse	Driven downward
Under-saturated ($\rho << \rho_{max}$)	Entropy-gradient expansion	Driven upward
Saturated ($\rho = \rho_{max}$)	Balance	Stable attractor

Saturation lies at the boundary between collapse and unconstrained percolation, defining a **marginally stable fixed point** that is approached from both directions.

For general readers: The universe doesn't need to be "fine-tuned" to produce the observed accelerated expansion. Structured vacuum naturally settles into this energy density like a ball rolling to the bottom of a valley, pushed there by gravity from above and by the "stickiness" of forming ordered space from below. No mysterious "dark energy" substance is required—just the natural behavior of relational geometry.

5.6 Connection to Constant Horizon Selection

Saturation at a fixed infrared scale L enforces:

1. **Constant vacuum energy**: $\rho_{vac} = \eta c^4/(GL^2)$ = constant
2. **Equation of state w = −1**: Constant ρ with $\rho \propto a^{-3(1+w)}$ requires $w = -1$
3. **De Sitter attractor**: The late-time universe approaches exponential expansion

This is not assumed but **derived** from saturation dynamics.

5.7 Saturation Versus Fine-Tuning

A critical distinction: saturation fixes **scaling relations**, not numerical values. The framework predicts:

$$\rho_{vac} \propto \frac{1}{L^2} \propto H_\Lambda^2$$

The proportionality constant ($\eta = 3/8\pi$) is determined by de Sitter geometry, not fitted. This is fundamentally different from fine-tuning a free parameter.

5.8 Failure Modes

The saturation mechanism would fail if:

1. Late-time cosmology converges to a stable attractor with $w \neq -1$
2. Coherence percolates indefinitely without encountering entropy-gradient resistance
3. The entropy cost of constraint-breaking vanishes or becomes negligible

Current observations ($w = -1.03 \pm 0.03$ from Planck 2018) are consistent with the framework.

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6. Constraint Universality and the Non-Tunable Nature of K = 7

For general readers: The number "7" in our framework isn't chosen to get the right answer—it's forced by geometry. This section proves that if you're tracking whether triangular "hinges" in the fabric of space are working properly, exactly seven independent conditions must be checked. Not six, not eight—seven.

The companion paper's Route M (dimensional transmutation derivation, §4.8; constraint enumeration in Appendix D) demonstrated that K = 7 coherence constraints yields $\xi \in [60, 110]$ μm—overlapping the $\xi \approx 88$ μm from Routes A and B. However, the derivation is **exponentially sensitive** to K:

K	Predicted ξ	Physical Scale
6	$\sim 10^{-19}$ m	Subatomic
7	$\sim 10^{-4}$ m	**Mesoscopic (observed)**
8	$\sim 10^{29}$ m	Cosmological

This sensitivity has led to the criticism that K is a hidden tuning parameter. We now show this is incorrect: **K = 7 is forced by the geometry of coherent triangles, and alternatives correspond to different universality classes**.

6.1 What Is Being Counted: Coherent Triangular Hinges

In simplicial foam models (like Regge calculus), curvature resides on two-dimensional triangular faces called "hinges." A triangle is:

- The **minimal closed relational object** capable of supporting curvature
- More fundamental than edges (which carry transport but not curvature)
- More fundamental than tetrahedra (which presuppose triangular face coherence)

The quantity K counts independent conditions for a triangle to be coherent at the emergence scale.

6.2 Formal Definition of Triangle Coherence

Let $\Delta = (i, j, k)$ be an oriented triangle. Associate to each directed edge a relational transport element $U_{ij} \in G$ (a group element describing how geometry changes along the edge).

For general readers: Think of U_{ij} as instructions for how to "translate" between two adjacent points. If you follow these instructions around a triangle and return to your starting point, the instructions should be consistent—like walking around a block and ending up where you started.

A triangle is **coherent** if three classes of conditions are satisfied:

(A) Edge Admissibility — Can the relationships exist?

- C1: U_{ij} exists and is invertible (edge ij admissible)
- C2: U_{jk} exists and is invertible (edge jk admissible)
- C3: U_{ki} exists and is invertible (edge ki admissible)

(B) Loop Closure — Do the relationships compose consistently?

- C4: $U_{ij} \cdot U_{jk} \cdot U_{ki} \in C$ (holonomy belongs to coherent class)

(C) Embedding Consistency — Does the triangle fit its neighborhood?

- C5: Induced data matches between tetrahedra T_1 and T_2

- C6: Induced data matches between tetrahedra T_2 and T_3
- C7: Consistent orientation (chirality) across embeddings

6.3 Independence of Edge Admissibility and Loop Closure

Claim: The four constraints C1–C4 are mutually independent.

Proof:

Edge admissibility does not imply closure: Three valid transport elements can compose to an incoherent holonomy. Example: $U_{12} = R(\theta_1)$, $U_{23} = R(\theta_2)$, $U_{31} = R(\theta_3)$ with $\theta_1 + \theta_2 + \theta_3 \neq 0$.

Closure does not imply admissibility: A formal holonomy can equal the identity while one constituent relation is undefined or non-invertible. In pre-geometric foam, there is no background structure enforcing that well-defined holonomy implies well-defined edges.

6.4 Independence of Embedding Consistency Constraints

Claim: Constraints C5–C7 are independent of C1–C4 and of each other.

Proof:

Independence from C1–C4: A triangle can be internally coherent (closed loop, valid edges) yet fail to embed consistently into its tetrahedral neighborhood. The induced geometric data $Q(\Delta|T)$ in tetrahedron T depends on how the triangle's edges relate to T's other edges—information not contained in the triangle alone.

Mutual independence of C5–C7: In the pre-geometric regime, no transitivity can be assumed. Matching between T_1-T_2 does not imply matching between T_2-T_3, because the foam lacks the smooth structure that would enforce consistency. Orientation (C7) is a discrete $\mathbb{Z}_2$ condition independent of metric matching.

6.5 Completeness: Why K = 7 and Not More

Claim: Additional constraints beyond C1–C7 are either gauge-redundant or constrain higher-order objects.

To establish this, we must first clarify what constitutes **intrinsic triangle data** versus **gluing data**.

Intrinsic triangle data (determined by C1–C4):

- The three edge transport elements U_{12}, U_{23}, U_{31}
- The holonomy $H\_\Delta = U_{12} \cdot U_{23} \cdot U_{31}$
- Edge lengths and angles (derived from the U's in any consistent frame)

Gluing data (properties of how tetrahedra meet):

- Transition functions $g_{TT'}$ between adjacent tetrahedra T and T'
- These encode how local frames in T relate to local frames in T'
- They are properties of the **foam**, not the triangle

The gauge redundancy: When we speak of "matching induced data between tetrahedra," we mean matching up to local frame choice. Explicitly:

$$Q(\Delta|T_1) = Q(\Delta|T_2)$$

holds when the intrinsic geometric data of Δ (edge lengths, angles, orientation) agree as computed from either tetrahedron's frame. This is a physical constraint on the triangle.

By contrast, the transition function g_{12} relating T_1's frame to T_2's frame is a **gauge choice**—it can be changed by local frame rotations without affecting physical predictions.

Proof of completeness:

Consider a triangle Δ bordering tetrahedra T_1, T_2, T_3, T_4, ... Once we have established:

- C5: $Q(\Delta|T_1) = Q(\Delta|T_2)$ — intrinsic data matches across first pair
- C6: $Q(\Delta|T_2) = Q(\Delta|T_3)$ — intrinsic data matches across second pair
- C7: Orientation of Δ is consistent (discrete $\mathbb{Z}_2$ condition)

The induced data in any additional tetrahedron T_4 is determined by:

$$Q(\Delta|T_4) = g_{34} \cdot Q(\Delta|T_3) \cdot g_{34}^{-1}$$

where g_{34} is the transition function between T_3 and T_4.

Crucially: g_{34} is determined by the edge data of the tetrahedra themselves (how T_3 and T_4 glue together), not by any additional property of the triangle Δ. A hypothetical constraint "C8: $Q(\Delta|T_3) = Q(\Delta|T_4)$" would not be independent—it would be automatically satisfied once:

1. C5 and C6 fix the intrinsic triangle data
2. The tetrahedra T_3, T_4 are themselves consistently glued (a constraint on tetrahedra, not triangles)

Why not C8, C9, ...? Any proposed additional constraint falls into one of three categories:

Proposed constraint	Classification	Why not independent
More pairwise matchings	Gauge-redundant	Determined by C5, C6 + gluing maps
Higher-order adjacency	Constrains tetrahedra	Not a triangle coherence condition
Torsion/matter constraints	Different universality class	Changes the theory, not the count

For general readers: Once you've verified that a puzzle piece matches its shape when viewed from two different angles, and that it's right-side-up, you've done all the checking needed *for that piece*. Further checking would either be redundant (same information from a different angle) or would be checking something about the *neighboring pieces*, not about your piece.

6.6 Why K = 7 Is Forced in the Simplicial-Foam Universality Class

Collecting the independent constraints:

Class	Constraints	Count
Edge admissibility	C1, C2, C3	3
Loop closure	C4	1
Embedding consistency	C5, C6, C7	3
Total		**K = 7**

Reducing K requires either:

- Eliminating curvature-bearing triangular hinges → different universality class
- Assuming background structure that enforces transitivity → not pre-geometric

Increasing K requires:

- Additional structure (torsion, matter-specific constraints) → different universality class
- Higher-order adjacency → constrains simplices, not triangles

Within the simplicial-foam universality class with curvature on triangular hinges, K = 7 is forced.

6.7 Exponential Sensitivity Does Not Imply Tunability

The exponential sensitivity:

$$\xi = \ell_e \cdot \exp\left[\frac{1}{2b \cdot g_0^2}\right] = \ell_e \cdot \exp\left[\frac{1}{2b \cdot 2^{-K}}\right]$$

reflects the **physics** of rare coherence events percolating to form stable geometry. It does not imply K can be continuously adjusted.

Changing K by ±1 does not "tune" ξ—it **replaces the theory** with one tracking a different coherent object. The sharpness of the transition is a feature, not a flaw.

6.8 Quantifying "Narrow": The Mesoscopic Window

Dimensional transmutation does not generically yield mesoscopic scales. The exponential sensitivity to K demonstrates this:

Discrete parameter scan:

K	$\log_{10}(\xi/\text{m})$	Physical regime
5	−28	Sub-nuclear
6	−19	Subatomic
7	**−4**	**Mesoscopic (observed)**
8	+11	Astronomical
9	+29	Super-cosmological

The mesoscopic window (10^{-5} to 10^{-3} m) spans ~2 orders of magnitude, while varying K by ±2 sweeps ξ across ~57 orders of magnitude.

The key point is not a probability statement (which would require a measure on theory space), but a structural one: small variations in the discrete combinatorial inputs—constraint count K, loop number N_loop, or coordination z_eff—generically push ξ to Planckian or cosmological extremes. The mesoscopic regime requires the specific combination (K = 7, N_loop = 14, z_eff ∈ [6,7]) that emerges from simplicial foam geometry.

For general readers: If you changed any of the key geometric numbers by even a little, the predicted "mesh size" of space would either be unimaginably tiny (smaller than an atom) or unimaginably huge (larger than the universe). Landing in the human-hair-width range requires exactly the combination that simplicial geometry provides.

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7. A Minimal Microscopic Model of Stitches, Bias, and Emergent Gravity

For general readers: This section shows that the abstract concepts we've been using ("stitches," "bias," "foam") can be made concrete. We don't need to know every detail of how space works at the smallest scales—just a few key properties are enough to guarantee that gravity emerges.

7.1 Stitches as Primitive Relational Links

We define a **stitch** as a primitive binary relational link at the emergence scale ℓ_e. Operationally, a stitch has:

- **Orientation**: A directed relation (i → j)
- **Transport element**: $U_{ij} \in G$ describing the relational data
- **Loop participation**: Ability to form part of closed relational structures

No background metric, manifold, or distance is assumed. Many stitches form a **foam state**—a statistical ensemble over orientations and transport elements.

For general readers: Think of stitches as the "threads" from which the fabric of space is woven. Each thread connects two points and carries information about how to translate between them. The pattern of threads determines the geometry of space.

7.2 Foam Dynamics and Reconfiguration Entropy

Without matter, the foam undergoes continual **reconfiguration**: stitches change orientation, appear, or disappear subject to local constraints. This dynamics is stochastic and ergodic.

Operational definition of time:

$$d\tau \propto dN_{reconfig}$$

Proper time is proportional to the number of irreversible reconfiguration events. This "foam clock" measures change, not a pre-existing time parameter.

In unperturbed vacuum, the reconfiguration rate Γ_0 is spatially uniform, normalizing time at infinity.

7.3 Universal Coupling and the Origin of Bias

The presence of mass-energy modifies foam reconfiguration:

Key principle: Any interaction that couples universally to energy-momentum must act by modifying the local state-count available to the foam.

This appears as **bias**: regions containing stress-energy support fewer accessible microstates, reducing the local reconfiguration rate:

$$\Gamma(x) = \Gamma_0[1 - B(x)]$$

where B(x) is a dimensionless bias function.

Critical clarification on the status of universality:

The present framework does not *derive* universal coupling from microscopic foam dynamics. This remains an open problem (see Companion paper §4.4, §8.3). What we establish here is a consistency requirement:

If universality holds	Then	Status
B depends only on total $T_{\mu\nu}$	Gauss-law scaling follows	Verified
B depends on composition	Equivalence principle violated	Excluded by observation
B has wrong tensorial structure	Spin-2 universality fails	Excluded by Weinberg-Deser

Universality is therefore imposed by empirical consistency, not derived from first principles. A complete theory would derive universal coupling from foam symmetries or information-theoretic constraints. The companion paper (§4.4) lists "micro-model of mass-foam coupling" as an open theoretical problem, and we maintain that classification here.

What the present work *does* establish: given universal coupling (as an empirical input), the emergence of bias, flux conservation, and macroscopic gravitational behavior follows from the relational framework without additional assumptions.

For general readers: We don't yet understand *why* all forms of energy affect the fabric of space equally—that's an open question. What we show is that *if* they do (as all experiments confirm), then gravity emerges from the foam picture in a consistent way.

7.4 From Bias to Gravitational Flux

Once bias exists, long-range force emergence follows from combinatorics:

1. **Sphere of coherence patches**: At radius r, the number of independent patches is $N(r) \sim 4\pi r^2/\xi^2$
2. **Biased flux per patch**: Each patch contributes probability flux proportional to local bias
3. **Conservation**: Total flux is conserved, requiring net effect to scale as $1/r^2$

$$\Phi(r) = N(r) \cdot \Phi_0 \cdot p(r) = \text{const}$$

This yields **Gauss-law behavior** (inverse-square force) as a statistical necessity, not an assumption.

7.5 Spin-2 Universality

A remaining concern: could the emergent interaction be scalar or vector rather than tensorial?

Weinberg-Deser universality theorem: Any long-range interaction that:

1. Couples universally to energy-momentum
2. Respects local Lorentz invariance
3. Arises from conserved flux

must be mediated by an effective spin-2 field at macroscopic scales.

In our framework, the metric is not fundamental but represents the collective response of relational coherence to bias. Spin-2 structure is **enforced by consistency**.

7.6 Scope and Limitations of the Minimal Model

The model intentionally does not specify:

- Detailed dynamics of stitch creation/annihilation
- Microscopic origin of the bias field
- Strong-field regime behavior

These omissions are scope boundaries, not defects. The model demonstrates that Two-Planck concepts are not vacuous metaphors but can be embedded in a coherent operational framework.

7.7 What Would Falsify This Picture

The minimal stitch model would fail if:

1. A universally coupled long-range interaction reduced to scalar or vector behavior at low energies
2. A relational model produced bias without area-law flux scaling
3. Gravitational time dilation could not be interpreted as suppressed reconfiguration rates

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8. Independence of Derivation Routes

For general readers: We've derived the same result (space has a "mesh size" of about 100 micrometers) through three different methods. This section asks: are these really independent methods, or are they just the same argument in disguise? We show they're genuinely independent—which is why their agreement is significant.

8.1 Logical Roles of the Three Routes

Route	Method	Level of Description
A	UV/IR gravitational consistency	Infrared (cosmological)
B	Foam-to-gravity amplitude	Mesoscopic (force emergence)
M	Microphysical dimensional transmutation	Ultraviolet (Planck-scale combinatorics)

These address different scales and different questions:

- Route A: "What coherence scale does gravitational stability *require*?"
- Route B: "Given a coherence scale, does the right gravity *emerge*?"
- Route M: "What coherence scale does Planck-scale foam *produce*?"

8.2 Shared Structural Commitments

The routes share foundational assumptions:

- Relational pre-geometric substrate
- Minimal emergence scale $\ell_e = 2\ell_p$

- Existence of a coherence scale ξ

These are **defining features** of the universality class, not adjustable parameters. Shared structure establishes the domain of assessment, not redundancy.

8.3 Genuine Independence: Route M Versus Routes A and B

Route M contains no cosmological input. The horizon scale L, Hubble rate H_0, and expansion history play no role in its derivation.

Route M depends only on:

- Simplicial foam combinatorics (N_loop = 14)
- Constraint counting (K = 7)
- RG flow and percolation (b = 0.875, p_c ~ 0.17–0.20)

That Route M yields $\xi \in [60, 110]$ μm while Route A independently demands $\xi \approx 88$ μm is **non-trivial agreement** between UV microphysics and IR cosmology.

8.4 Route B as a Consistency Reconstruction

Route B is best understood as **consistency verification**, not independent prediction. Given ξ, Route B shows:

- Gauss-law scaling follows from boundary-limited coherence channels
- Gravitational coupling suppression $C \sim L^2/\xi^2$ is required
- The UV/IR bridge $\xi^2 \sim \ell_p L$ reappears for matching observed G

Route B confirms that Routes A and M produce a dynamically viable coherence scale.

8.5 Why Convergence Is Non-Trivial

If the framework were inconsistent, routes would fail to meet:

Failure Mode	Consequence
Route M → $\xi \sim \ell_p$	No mesoscopic scale; microphysics incompatible with IR
Route M → $\xi \sim L$	Coherence at cosmological scale; no intermediate structure
Route A → ξ incompatible with any K	No microphysical realization possible
Route B → no inverse-square law	Wrong force behavior despite correct ξ

Convergence on $\xi \sim \sqrt{(\ell_p L)}$ from **independent** physical reasoning represents structural success.

8.6 What Independence Does and Does Not Claim

Does not claim: Statistical independence of assumptions

Does claim: Structural consistency across scales

- UV microphysics doesn't presuppose cosmology
- IR consistency doesn't presuppose specific microphysics
- Mesoscopic force emergence constrains compatibility

This is the strongest independence available in a multiscale theory.

8.7 Failure Modes

Route convergence would dissolve if:

1. A microphysical model produced stable geometry without mesoscopic ξ
2. Gravitational stability selected ξ incompatible with any UV construction
3. A relational foam yielded long-range interaction violating Gauss law

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9. Experimental Signatures and the Phase Transition Prediction

For general readers: If our framework is right, there should be a detectable "fingerprint" at the 100-micrometer scale—specifically, a sudden change in how forces behave as you cross this threshold. This section explains what to look for.

The companion paper (experimental strategy, §5) provides detailed experimental predictions including Casimir force modifications (§5.4), short-range gravity anomalies (§5.4), quantum decoherence thresholds (§5.4), and concrete experimental platforms (§5.5). Here we focus on *why* the most distinctive signature—a phase discontinuity—should exist, grounding it in the foundational percolation mechanism developed in this paper.

9.1 Why a Phase Signature Should Exist

The percolation-pinning mechanism (§5.4) predicts that the coherence scale ξ marks a **phase transition** in foam behavior:

- **Below ξ**: Local geometry exists but doesn't percolate; coherence is fragmented
- **Above ξ**: Coherent triangles form a connected network; extended geometry is stable

At the transition, the foam's response to perturbations changes qualitatively:

Property	Below ξ	Above ξ
Coherence	Local, fragmented	Extended, connected

Property	Below ξ	Above ξ
Reconfiguration	Rapid, unconstrained	Slower, pinned
Response phase	Tracks driving frequency	Lags due to pinning

9.2 Observable Consequences

The foam-clock picture suggests that near ξ, the **phase** of response to periodic driving should shift:

For a levitated microsphere force sensor:

- Drive attractor mass at frequency ω
- Measure lock-in amplitude AND phase vs. separation d
- Prediction: Phase φ(d) shows rapid shift near d ≈ ξ

$$\varphi(d) = \arg\left[\frac{F_{response}(d)}{F_{drive}}\right]$$

Expected signature: Phase knee or step at $d^* \in [60, 120]$ μm with $\Delta\varphi \sim 10\text{–}90°$

9.3 Distinguishing from Alternative Explanations

Why this signature is distinctive:

Effect	Phase Behavior	Distinguished By
Electrostatic patches	Monotonic drift	No sharp transition
Standard Casimir	Smooth d-dependence	No characteristic scale
Two-Planck percolation	*Step at d ~ ξ**	Fixed scale, reproducible

Null tests:

- Reverse grating pattern → d* unchanged (geometric, not material)
- Change sphere material → d* unchanged
- Change drive frequency → d* unchanged

Detection of a phase transition at d* ~ 80–100 μm, reproducible across platforms, would strongly support the framework.

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10. Falsifiability, Failure Modes, and Scope

For general readers: A good scientific theory must be able to fail. This section lists exactly what observations or discoveries would prove our framework wrong—and distinguishes genuine failures from adjustments.

10.1 What the Framework Claims—and What It Does Not

The framework advances a conditional, structural claim:

If spacetime geometry emerges from a relational pre-geometric substrate with a minimal binary emergence scale, and if gravity couples universally to energy-momentum, then:

- A mesoscopic coherence scale ξ must arise
- Vacuum energy must saturate a gravitational stability bound
- Long-range gravitational dynamics must reduce to spin-2 behavior

The framework does NOT claim:

- A complete microscopic theory of spacetime
- Predictive control over strong-field quantum gravity
- Near-term detectability of Planck-suppressed effects

10.2 Hard Falsifiers

The framework would be **falsified** under:

(1) Failure of the Infrared Attractor

If late-time observations establish $w \neq -1$ as a stable attractor (not approaching -1 asymptotically), the constant-horizon identification fails.

Current status: $w = -1.03 \pm 0.03$ (Planck 2018), consistent with framework.

(2) Absence of Mesoscopic Coherence Scale

If a relational spacetime model produces extended, stable geometry without any intermediate coherence scale, the core mechanism is undermined.

(3) Route Incompatibility

If UV combinatorics robustly predict ξ incompatible with IR gravitational requirements, the framework is internally inconsistent.

(4) Violation of Spin-2 Universality

If a universally coupled long-range interaction reduces to scalar/vector behavior while respecting energy-momentum conservation and Lorentz invariance, the emergent-gravity mechanism fails.

10.3 Distinguishing Failure from Adjustment

Critical: The Two-Planck framework contains **no continuous free parameters** that can shift ξ across many orders of magnitude.

Change	Effect	Classification
Adjust K by ±1	ξ changes by $\sim 10^{30}$	**Different theory**
Change universality class	Different predictions	**Different theory**
Modify η by O(1)	ξ changes by ~2×	Within uncertainty

Changes to ξ arise only through universality class changes—replacing the theory, not adjusting it.

10.4 Experimental Non-Confirmation Versus Refutation

Because baseline effects are Planck-suppressed ($\sim 10^{-31}$), failure to observe laboratory anomalies does not immediately falsify the framework.

Significant non-confirmation:

- Anomalies at **multiple inconsistent length scales** across platforms → no unique ξ
- Sensitivity sufficient to **exclude all amplification** → ξ physically irrelevant

10.5 Summary of Scope

The Two-Planck framework is a proposal about **structure**, not a claim of finality. It asserts:

Once minimal relational commitments are made, a narrow corridor of consistency opens. The observed universe lies within that corridor.

The framework is testable by:

- Resistance to internal inconsistency
- Willingness to be wrong
- Explicit failure modes

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11. Conclusion

This paper has established the foundational robustness of the Two-Planck framework presented in the companion phenomenological paper.

Summary of Results

Question	Resolution	Companion Reference
Is $\ell_e = 2\ell_p$ arbitrary?	No—forced by minimal relational geometry	Geometric necessity argument (§1.1–1.3)
Is saturation (CSS) assumed?	No—unique IR attractor with entropy-gradient restoring force	CSS postulate (§3.2); percolation pinning (§4.11)
Is K = 7 fitted?	No—complete and independent; alternatives change universality class	Route M microphysics (§4.8); constraint details (Appendix D)
Are routes genuinely independent?	Yes—Route M contains no cosmological input; mesoscopic window is narrow	Routes A, B, M (§3.2, §4.5, §4.8)
Are "stitches" and "bias" well-defined?	Yes—minimal operational model with universal properties	Mass-bias mechanism (§4.1–4.4)

The Logical Chain

The Two-Planck framework forms a closed argument spanning both papers:

Relational pre-geometry (This paper §4)
↓
Minimal binary emergence scale $\ell_e = 2\ell_p$ (This paper §4.2–4.4)
↓
Coherent triangular hinges require K = 7 constraints (This paper §6; Companion Appendix D)
↓
Dimensional transmutation yields mesoscopic $\xi \sim 10^{-4}$ m (Companion §4.8)
↓
Gravitational stability demands saturation at cosmic horizon (This paper §5; Companion §3.2)
↓
Routes A and M converge → $\xi \sim \sqrt{(\ell_p L)} \sim 88$ μm (Companion §3.2, §4.8)
↓
Vacuum energy $\rho_{vac} \sim \hbar c/\xi^4 \sim 10^{-10}$ J/m³ ✓ (Companion §3.2)
↓
Cosmological constant $\Lambda \sim 10^{-52}$ m⁻² ✓ (Companion §3.2)
↓
General Relativity at 1PN order ✓ (Companion §4.9)
↓
Experimental signatures at ξ (Companion §5; This paper §9)

Relationship Between the Two Papers

Companion Paper	This Paper
Derives quantitative predictions	*Establishes* why inputs are forced
Routes A, B, M as calculational tools	Routes as structurally independent derivations
CSS as operational postulate	CSS as unique IR attractor
K = 7 as enumerated constraints	K = 7 as complete, independent set
Experimental signatures	Phase transition mechanism

Together, the papers form a complete argument: the companion paper shows *what* the framework predicts; this paper shows *why* it could not easily predict otherwise.

What This Achieves

The combined framework is:

- **Not numerological**: The factor of 2, the number 7, and the saturation condition are derived, not fitted
- **Not freely tunable**: Changes to K replace the theory rather than adjust it
- **Falsifiable**: Explicit failure modes are stated (see Companion §9.3 and this paper §10)
- **Structurally complete**: UV, mesoscopic, and IR scales are mutually consistent

Open Questions

Despite this progress, work remains (see also Companion §8):

1. **Strong-field regime**: Black hole interiors, gravitational waves from foam dynamics (Companion §8.3)
2. **Matter coupling**: How does matter arise from relational structure?
3. **Amplification mechanisms**: How to bring $\sim 10^{-31}$ baseline to detectable levels? (Companion §5.3)
4. **Force unification**: Can electroweak and strong forces emerge similarly? (Companion §6.5)

Final Assessment

The Two-Planck framework demonstrates that the cosmological constant problem—long considered intractable—may admit a structural solution. The key insight is that **geometry is relational**, and relations require multiplicity.

From this single principle, the observed vacuum energy scale emerges as neither accident nor fine-tuning, but geometric necessity. The universe's accelerated expansion requires no mysterious "dark energy" substance—it is simply what structured vacuum does at the coherence scale.

Whether this corridor of consistency is physically realized remains an empirical and theoretical question. What this work establishes is that the corridor exists, is narrow, and that our universe appears to lie within it.

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12. References

Companion Work

- Taylor, K. (2025). Two-Planck Principle: From Quantum Geometry to Emergent Gravity. *VERSF Theoretical Physics Program*. [Phenomenological predictions, experimental signatures, and quantitative derivations]

Fundamental Physics and Quantum Gravity

- Ashtekar, A., & Lewandowski, J. (2004). Background independent quantum gravity: A status report. *Classical and Quantum Gravity*, 21(15), R53-R152.
- Bombelli, L., Lee, J., Meyer, D., & Sorkin, R. D. (1987). Space-time as a causal set. *Physical Review Letters*, 59(5), 521-524.
- Rovelli, C. (2004). *Quantum Gravity*. Cambridge University Press.
- Rovelli, C., & Smolin, L. (1995). Discreteness of area and volume in quantum gravity. *Nuclear Physics B*, 442(3), 593-619.

Vacuum Energy and Cosmological Constant

- Cohen, A. G., Kaplan, D. B., & Nelson, A. E. (1999). Effective field theory, black holes, and the cosmological constant. *Physical Review Letters*, 82(25), 4971-4974.
- Martin, J. (2012). Everything you always wanted to know about the cosmological constant problem (but were afraid to ask). *Comptes Rendus Physique*, 13(6-7), 566-665.
- Padmanabhan, T. (2017). Cosmological constant from the emergent gravity perspective. *International Journal of Modern Physics D*, 26(12), 1743002.
- Weinberg, S. (1989). The cosmological constant problem. *Reviews of Modern Physics*, 61(1), 1-23.

Emergent Gravity

- Jacobson, T. (1995). Thermodynamics of spacetime: The Einstein equation of state. *Physical Review Letters*, 75(7), 1260-1263.
- Padmanabhan, T. (2010). Thermodynamical aspects of gravity: New insights. *Reports on Progress in Physics*, 73(4), 046901.
- Verlinde, E. (2011). On the origin of gravity and the laws of Newton. *Journal of High Energy Physics*, 2011(4), 29.

Experimental Tests

- Adelberger, E. G., Heckel, B. R., & Nelson, A. E. (2003). Tests of the gravitational inverse-square law. *Annual Review of Nuclear and Particle Science*, 53(1), 77-121.
- Klimchitskaya, G. L., Mohideen, U., & Mostepanenko, V. M. (2009). The Casimir force between real materials. *Reviews of Modern Physics*, 81(4), 1827-1885.

Cosmology

- Planck Collaboration (2020). Planck 2018 results. VI. Cosmological parameters. *Astronomy & Astrophysics*, 641, A6.

Companion paper: Taylor, K. (2025). *"Two-Planck Principle: From Quantum Geometry to Emergent Gravity"* — Derives quantitative predictions including $\xi \approx 88$ μm, $\rho_{vac} \approx 5 \times 10^{-10}$ J/m³, $\Lambda \approx 1.1 \times 10^{-52}$ m⁻², recovery of GR at 1PN order, and experimental signatures. The present foundations paper establishes why these results are structurally robust.