Stability of Dimensional Transmutation in Simplicial Foam

Symmetry Constraints, Decorrelation Bounds, and Percolation Control for Route M of the Two-Planck Framework

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Companion Paper Series

This paper provides mathematical consolidation for the microphysical derivation (Route M) within the Two-Planck framework, developed across five companion papers:

1. **Two-Planck Principle: From Quantum Geometry to Emergent Gravity** — Quantitative predictions ($\xi \approx 88$ μm, $\Lambda \approx 1.1 \times 10^{-52}$ m^{-2}, $w = -1$) and experimental signatures
2. **Relational Geometry and the Universality of the Two-Planck Scale** — Foundational arguments for the emergence scale $\ell_e = 2\ell_p$, constraint universality, and the non-tunable nature of $K = 7$
3. **Structural Closure of the Two-Planck Framework** — Independent verification of $K = 7$ through information-theoretic and obstruction-theoretic methods, universality-class selection from axioms, and the CSS attractor theorem under monotone-feedback assumptions
4. **Microphysical Foundations of Route M** — Referee-standard derivations of all Route M parameters: the discrete Bianchi identity establishing $N_{loop} = 14$, the β-function coefficient $b = 0.875$, the bare coupling $g_0^2 = 1/128$, and controlled percolation bounds $p^c \in [0.17, 0.30]$
5. **Epistemic Status of the Two-Planck Derivation** — Critical self-assessment characterising the framework as a conditional but non-retrofitted derivation of Λ, with scheme dependence constrained to O(1) by cross-route agreement

This paper's role. Papers 1–3 establish the framework's predictions, foundations, and structural closure. Paper 4 derives the Route M parameters from simplicial combinatorics but relies on three elements that, while well-motivated, remained at the level of heuristic arguments: (a) the bare coupling $g_0^2 = 2^{-K}$ was justified by maximum-entropy intuition; (b) the multiplicative combination of constraint probabilities assumed strict independence; and (c) the percolation threshold was estimated via Bethe-lattice approximation without rigorous bounds. Paper 5 identified these as the framework's principal technical vulnerabilities.

The present paper addresses all three. It does not introduce new physical assumptions, modify the relational ontology, or extend the phenomenological predictions. Its sole purpose is to convert the three heuristic elements into controlled mathematical results:

- The bare coupling $g_0^2 = 2^{-K}$ is derived as the unique measure invariant under the flip group action on Ω, which acts transitively — replacing maximum-entropy intuition with a one-line symmetry argument (§2)
- The multiplicative structure of the coherence probability is shown to be stable under bounded short-range correlations compatible with simplicial locality, replacing the strict independence assumption with a decorrelation lemma stated in terms of a measurable correlation parameter ε (§3)
- The percolation threshold is bounded from below by spectral methods applied to the proven intra-simplex adjacency graph, and from above by clustering-corrected estimates, replacing heuristic Bethe approximation with controlled graph-theoretic bounds (§4)

Together, these results yield a conditional stability theorem (§6): within the simplicial-foam universality class with K = 7 coherence constraints, the dimensional-transmutation scale is confined to a mesoscopic window centred near 10^{-4} metres, provided constraint correlations satisfy $\varepsilon \leq 10^{-3}$. No cosmological input is used. No parameters are adjusted.

Numerical Convention

Throughout this paper, E denotes the natural-logarithm exponent in the dimensional-transmutation formula:

$\ln(\xi/\ell_e) = E$, so $\xi = \ell_e \cdot e^E$

with $\ell_e = 2\ell_p = 3.232 \times 10^{-35}$ m. To convert to orders of magnitude: $\log_{10}(\xi/\ell_e) = E/\ln(10) = E/2.3026$. All tables in this paper use this convention consistently.

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1. Purpose and Scope

This paper provides a technical consolidation of the microphysical Route M derivation within the Two-Planck framework. It does not introduce new physical assumptions, modify the relational ontology, or extend the phenomenological predictions. Its sole purpose is to formalise and mathematically strengthen three structural elements that underlie the dimensional-transmutation result for the coherence scale ξ.

Specifically, we address: (i) the status of ultraviolet neutrality and the derivation of the bare coupling $g_0^2 = 2^{-K}$; (ii) the additivity of independent coherence constraints under bounded correlation; and (iii) the percolation threshold p^c governing stability of coherent triangular hinges in simplicial foam.

What this paper does not assume. To prevent misinterpretation, we state the scope boundaries explicitly:

- No cosmological input is used. The coherence scale ξ is derived solely from microphysical combinatorics and percolation stability within the simplicial-foam universality class.
- No adjustable numerical parameters are introduced. All quantities entering the dimensional-transmutation formula (K, b, g_0^2, p^c bounds) are derived from symmetry, constraint counting, and graph structure.
- No assumption of exact constraint independence is required. Section 3 replaces strict independence with bounded-correlation control compatible with simplicial locality.
- No appeal is made to horizon thermodynamics, cosmological saturation, or infrared gravitational consistency. Those belong to Route A and are not required for the microphysical closure established here.
- No claim is made that the simplicial-foam universality class is uniquely derived from deeper principles. The universality-class selection from axioms A1–A4 is established in Paper 3; the present paper takes it as input.

We treat the simplicial-foam universality class, the Two-Planck emergence scale $\ell_e = 2\ell_p$, the constraint count $K = 7$, the loop channel count $N_{loop} = 14$, and the β-function coefficient $b = 14/16 = 0.875$ as given structural inputs established in companion papers.

2. The Bare Coupling from Flip-Group Transitivity

At the emergence scale ℓ_e, a triangular hinge is coherent if and only if $K = 7$ independent binary constraints are satisfied. Let each constraint be represented by a Bernoulli variable $C_k \in \{0, 1\}$, with $C_k = 1$ indicating satisfaction. The full constraint configuration space is $\Omega = \{0, 1\}^K$, which has $2^K = 128$ elements.

2.1 Two Levels of UV Neutrality

We distinguish two levels of symmetry assumption explicitly to avoid conflation:

Definition 2.1 (Strong UV Neutrality). The UV foam ensemble satisfies strong UV neutrality if the probability measure μ on Ω is invariant under the full flip group $F = (\mathbb{Z}_2)^K$ acting on Ω by independent coordinate flips:

$\mu(A) = \mu(\tau(A))$ for all $\tau \in F$ and all $A \subseteq \Omega$

where each generator τ_k acts by $C_k \mapsto 1 - C_k$, leaving all other components unchanged.

Definition 2.2 (Weak UV Neutrality). The UV foam ensemble satisfies weak UV neutrality if each marginal is unbiased:

$P(C_k = 1) = P(C_k = 0) = 1/2$ for all $k \in \{1, ..., K\}$

Strong UV neutrality implies weak UV neutrality, but the converse is false. One can construct distributions on $\{0, 1\}^K$ with all marginals equal to 1/2 that are not F-invariant — for example, exchangeable correlated distributions arising from random-environment models.

Physical motivation for strong UV neutrality. At the emergence scale $\ell_e = 2\ell_p$, the foam is maximally unstructured. There is no pre-existing geometric background that would favour any particular pattern of constraint satisfaction or violation. Each configuration $C \in \Omega$ corresponds to a specific pattern of "which constraints are met and which are broken," and strong UV neutrality asserts that no such pattern is preferred over any other.

Note that strong UV neutrality does *not* require permutation symmetry (S_K) among constraint labels. The seven constraints have distinct geometric origins, and we make no assumption that they are interchangeable. Strong UV neutrality acts within each configuration, flipping individual bits, without permuting which constraint is which.

2.2 Uniform Invariant Measure Theorem

Theorem 2.3 (Uniform Measure from Flip-Group Transitivity). Let $F = (\mathbb{Z}_2)^K$ act on $\Omega = \{0, 1\}^K$ by independent coordinate flips. If a probability measure μ on Ω is F-invariant — that is, $\mu(\tau(A)) = \mu(A)$ for all $\tau \in F$ and all events $A \subseteq \Omega$ — then μ is the uniform measure:

$\mu(C) = 2^{-K}$ for all $C \in \Omega$

Proof. The action of F on Ω is transitive: for any $C, C' \in \Omega$, define $\tau \in F$ by τ_k = flip if $C_k \neq C'_k$, identity otherwise. Then $\tau(C) = C'$. Applying F-invariance to the singleton event $\{C\}$: $\mu(\{C\}) = \mu(\tau^{-1}(\{C'\})) = \mu(\{C'\})$. Since all 2^K atoms have equal mass, normalisation gives $\mu(C) = 2^{-K}$. ■

Corollary 2.4 (Bare Coupling). Under strong UV neutrality:

$g_0^2 = P(C_1 = 1 \cap \ldots \cap C\_K = 1) = 2^{-K} = 1/128$

for K = 7.

Corollary 2.5 (Exact Independence). The uniform measure on $\{0, 1\}^K$ is a product measure. Strong UV neutrality therefore implies both unbiased marginals and exact independence.

Remark 2.6. The proof requires only the flip group F, not permutation symmetry S_K. This is important because the seven constraints are geometrically distinct, and imposing S_K would be a stronger and less physically motivated assumption. The key property is that F acts transitively on Ω — every configuration can be reached from every other by flipping appropriate bits.

Remark 2.7 (Weak UV neutrality does not determine g_0^2). Under weak UV neutrality alone, the variables need not be independent. Exchangeable distributions with $P(C_k = 1) = 1/2$ but positive pairwise correlations exist and would give $g_0^2 > 2^{-K}$; negative correlations would give $g_0^2 < 2^{-K}$. This is why Section 3 is necessary.

3. Constraint Decorrelation and Additivity Bounds

Section 2 establishes that strong UV neutrality gives $g_0^2 = 2^{-K}$ exactly. In practice, the UV foam may not satisfy full F-invariance — correlations may exist due to shared simplicial data. This section replaces the strong symmetry with weak UV neutrality plus bounded correlations, and quantifies the resulting deviation.

3.1 Setup and Sign Convention

Let $C_1, \ldots, C\_K$ have $P(C_k = 1) = 1/2$ for all k (weak UV neutrality). Define:

$P_{coh} = P(C_1 = 1 \cap \ldots \cap C\_K = 1)$

We parametrise deviations from independence as:

$P_{coh} = 2^{-K} \cdot \exp(+\Delta)$

where Δ is the correlation correction with sign convention:

- **Δ > 0:** positive constraint correlation — constraints tend to succeed together, making coherence more likely. This increases g_0^2.
- **Δ < 0:** negative constraint correlation — constraints tend to anti-correlate, making coherence less likely. This decreases g_0^2.
- **Δ = 0:** exact independence.

The bare coupling under correlations is $g_0^2 = 2^{-K} \cdot \exp(+\Delta)$, giving:

$1/g_0^2 = 2^K \cdot \exp(-\Delta) = 128 \cdot \exp(-\Delta)$

3.2 Effect on the Transmutation Scale

The transmutation exponent is:

$E = (1/2b)(1/g_0^2 - 1/p^c) = (1/2b)(128 \cdot \exp(-\Delta) - 1/p^c)$

Since $\xi = \ell_e \cdot e^E$:

- **$\Delta > 0$** → $\exp(-\Delta) < 1$ → $1/g_0^2$ decreases → E decreases → **ξ decreases**. Physical: coherence is easier, so percolation is reached at a smaller scale.
- **$\Delta < 0$** → $\exp(-\Delta) > 1$ → $1/g_0^2$ increases → E increases → **ξ increases**. Physical: coherence is harder, requiring a larger scale for percolation.

This sign convention is consistent throughout the paper.

3.3 Bounded Correlation Lemma

Definition 3.1 (Pairwise Correlation Bound). Define:

$\varepsilon :=$ max over $i \neq j$ of $|\mathrm{Cov}(C_i, C_j)|$

This is a measurable quantity that can be estimated directly from Monte Carlo simulations of the UV foam ensemble (see §3.6).

Lemma 3.2 (Bounded Correlation). Under weak UV neutrality ($p_k = 1/2$) with pairwise correlation bound ε:

$|\Delta| \leq 2K(K - 1)\varepsilon + O(K^3\varepsilon^2)$

Proof. See Appendix B. ■

Corollary 3.3. For K = 7: $|\Delta| \leq 42\varepsilon + O(\varepsilon^2)$.

This is a conservative envelope bound: bounding a K-way conjunction using only the maximum pairwise covariance is deliberately crude. Sharper control is available under dependency-graph or mixing assumptions (e.g., Dobrushin-type conditions on the constraint factor graph), but the pairwise-ε bound is used here because it is directly measurable in simulations without requiring knowledge of the full correlation structure.

3.4 Numerical Impact on ξ

The following table shows the effect of correlation strength on the transmutation scale, using b = 0.875, $p^c = 0.20$ (the intra-simplex bound; the foam-level bound at d* = 7 gives $p^c = 0.167$, which would widen the ξ range slightly), and $\ell_e = 3.232 \times 10^{-35}$ m:

ε	$\lvert\Delta\rvert$ bound	$1/g_0^2$ range	E range	ξ range
10^{-2}	≤ 0.42	[84, 195]	[45, 109]	[$\sim10^{-15}$ m, $\sim10^{13}$ m]
10^{-3}	≤ 0.042	[123, 133]	[67, 73]	[5 μm, 2.5 mm]
5×10^{-4}	≤ 0.021	[125, 131]	[69, 72]	[24 μm, 510 μm]
3×10^{-4}	≤ 0.013	[126, 130]	[69, 71]	[43 μm, 273 μm]
10^{-4}	≤ 0.0042	[127, 129]	[70.0, 70.6]	[80 μm, 147 μm]

Worked example. For $\varepsilon = 10^{-3}$, $|\Delta| \leq 0.042$. At the lower extreme ($\Delta = +0.042$, positive correlation): $1/g_0^2 = 128 \times \exp(-0.042) = 128 \times 0.959 = 122.7$. $E = (122.7 - 5.00)/1.75 = 67.3$. $\log_{10}(\xi/\ell_e) = 67.3/2.303 = 29.22$. $\xi = 3.232 \times 10^{-35} \times 10^{29.22} = 5.4 \times 10^{-6}$ m = 5.4 μm. At the upper extreme ($\Delta = -0.042$): $1/g_0^2 = 128 \times \exp(+0.042) = 128 \times 1.043 = 133.5$. $E = (133.5 - 5.00)/1.75 = 73.4$. $\xi = 3.232 \times 10^{-35} \times 10^{31.87} = 2.5 \times 10^{-3}$ m = 2.5 mm.

Critical observation. The bound $\varepsilon \leq 10^{-2}$ is insufficient to confine ξ to the mesoscopic window — the resulting range spans subatomic to astronomical scales. Mesoscopic confinement requires $\varepsilon \leq 10^{-3}$ or tighter. This is a genuine constraint on the framework, not previously identified.

3.5 Physical Estimate of ε from Simplicial Locality

The seven constraints act on partially overlapping simplicial data:

- C_1–C_3 (edge admissibility): Each acts on a distinct edge. No two share simplicial support, so $\mathrm{Cov}(C_i, C_j) = 0$ for pairs within this group.
- C_4 (loop closure): Functionally depends on edge transports certified by C_1–C_3. In a disordered ensemble, the correlation is suppressed by group averaging: for a gauge group of effective order $|G|$, the correlation is $O(1/\sqrt{|G|})$.
- C_5–C_6 (embedding consistency): Depend on how the triangle embeds into adjacent tetrahedra. Correlation with C_1–C_4 arises through shared boundary edges and is bounded by the same locality argument.
- C_7 (orientation): A discrete $\mathbb{Z}_2$ condition topologically independent of metric data.

If edge-transport variables are approximately Haar-mixed at the emergence scale ℓ_e, correlations induced by functional dependence (e.g., loop closure on transports) are suppressed by group averaging over the gauge group manifold. For non-abelian gauge groups relevant to gravity (SU(2) or larger), this suppression is strong: correlation coefficients between constraint channels scale inversely with the effective group volume. The required regime $\varepsilon \leq 10^{-3}$ is therefore physically plausible, but should be verified numerically. We adopt $\varepsilon \leq 10^{-3}$ as the physically motivated bound, yielding $\xi \in$ [5 μm, 2.5 mm].

3.6 Dependency-Graph Perspective

The constraint correlations can be formalised via a dependency graph D on {1, ..., K}, where C_i and C_j are connected if they share simplicial support. The maximum degree of D is $\Delta_D \leq 5$. Under a Dobrushin-type condition on D — if the total influence of all neighbours on any single constraint is bounded by $\alpha < 1$ — standard correlation-decay results guarantee $\varepsilon \leq \alpha$ per connected pair, providing a structural basis for the bounded-correlation assumption.

3.7 Making ε Testable

The correlation parameter ε is directly measurable in dynamical triangulation simulations:

1. Generate an ensemble of simplicial foams at the emergence scale
2. For each triangle, evaluate all $K = 7$ constraint satisfaction indicators
3. Compute $\mathrm{Cov}(C_i, C_j)$ for all pairs (i, j)
4. Report $\varepsilon = \max$ over $i \neq j$ of $|\mathrm{Cov}(C_i, C_j)|$

The stability theorem (§6) is explicitly conditional on $\varepsilon \leq 10^{-3}$ and on a bounded foam degree cap d^*. Numerical measurement of ε and d^* are concrete, achievable tests of the framework — both measurable in the same dynamical triangulation simulations.

4. Controlled Percolation Bounds on the Triangle Adjacency Graph

4.1 Definition and Intra-Simplex Combinatorics

Let $G\triangle$ be the graph whose nodes are triangular hinges (2-simplices) in the simplicial foam, with two nodes adjacent iff the corresponding triangles share a common edge (1-simplex).

Within a single 4-simplex: $N\triangle = C(5,3) = 10$ triangular faces, each with 3 edges, each edge shared with exactly 2 other triangles. Intra-simplex coordination: $z_{intra} = 3 \times 2 = 6$.

4.2 Identification as Johnson Graph

The intra-simplex triangle adjacency graph is the Johnson graph $J(5, 3)$: vertices are the 3-element subsets of a 5-element set, with adjacency defined by intersection in exactly 2 elements.

Proposition 4.1. $J(5, 3)$ is 6-regular with eigenvalues 6 (multiplicity 1), 1 (multiplicity 4), and −2 (multiplicity 5).

Proof. See Appendix C. Verification: $6(1) + 1(4) + (-2)(5) = 0 = \mathrm{Tr}(A)$. ✓

4.3 Rigorous Lower Bound

For site percolation on a locally finite graph G with maximum degree d_{max}, a rigorous universal lower bound is (Grimmett, 1999; Hamilton & Pryadko, 2014, who cite it as the existing bound their Hashimoto result improves upon):

$p^c \geq 1/(d_{max} - 1)$

(See Hamilton & Pryadko, 2014, who discuss this as the "existing" $(d_{max} - 1)^{-1}$ bound that their Hashimoto spectral-radius result $\rho(H)^{-1}$ improves upon; see also Grimmett, 1999.)

For locally finite infinite graphs (and in particular for quasi-transitive graphs), this bound follows from comparison with the Galton–Watson branching process on the universal cover; it is achieved with equality on the d-regular tree.

Application to the intra-simplex graph. For J(5, 3), every vertex has degree exactly 6. Applied to this subgraph alone: $p_c \geq 1/5 = 0.20$.

Degree-cap condition (required for a foam-level bound). The theorem $p_c \geq 1/(d_{max} - 1)$ applies to the full triangle-adjacency graph $G_\triangle$ with its global maximum degree $d_{max}(G_\triangle)$. While the intra-simplex subgraph has $d_{max} = 6$, cross-simplex gluing can increase triangle degree in the full foam. Therefore a theorem-grade numerical floor for the foam requires an additional condition: that the foam ensemble satisfies a bounded local-degree cap $d_{max}(G_\triangle) \leq d^*$.

This condition is directly testable in simulations and is treated as part of the Route M stability hypothesis, alongside the correlation bound $\varepsilon \leq 10^{-3}$.

*Physical estimate of d.** In the simplicial-foam universality class, each triangle has 3 edges available for cross-simplex adjacency. The analysis in §4.4 estimates $z_{eff} \in [6, 7]$, suggesting that gluing typically adds at most ~1 adjacency per triangle on average. We take $d^* \approx 7$ as a physically motivated working hypothesis for the degree envelope in the Route M universality class, to be verified numerically. The theorem-grade statement is conditional: $p_c \geq 1/(d^* - 1)$ once d^* is empirically established. At $d^* = 7$:

*$p_c \geq 1/(d - 1) = 1/6 \approx 0.167$ (at $d = 7$)***

If future simulations establish $d^* = 6$ (no cross-simplex degree inflation), the bound tightens to $p_c \geq 1/5 = 0.20$.

Remark (Spectral tightening via non-backtracking walks). Hamilton & Pryadko (2014) construct a strictly tighter lower bound for site percolation on infinite quasi-transitive graphs in terms of the spectral radius of the Hashimoto (non-backtracking) matrix H: $p_c \geq 1/\rho(H) > 1/\rho(A)$. This provides a route to a genuinely controlled spectral lower bound that could improve upon the maximum-degree result. Computing $\rho(H)$ for the full foam graph is identified as future work.

4.4 Conservative Cap via Clustering-Corrected Branching Estimate

Clustering raises p_c by reducing effective branching. On a graph with coordination z and clustering C, the effective independent branching number is approximately $z(1 - C)$, giving the heuristic estimate:

$p_c \approx 1/(z(1 - C) - 1)$

Derivation. If a fraction C of neighbour pairs are directly connected, approximately Cz of the z neighbours are reachable from other neighbours without passing through the central node. The Bethe criterion then becomes $p_c \cdot (z(1 - C) - 1) \approx 1$.

With zeff ∈ [6, 7] and Ceff ∈ [0.3, 0.6] (cross-simplex dilution): the most relevant estimate at zeff ≈ 6.5, Ceff ≈ 0.4 gives $p^c \approx 1/2.9 \approx 0.34$. We adopt $p^c \leq 0.30$ as a conservative cap, noting this is an anchored estimate rather than a rigorous bound.

4.5 Working Interval and Effect on ξ

*Working interval (conditional on d): $p^c \in [1/(d-1), 0.30]$***

The lower endpoint is a theorem-level bound from the maximum-degree result, conditional on the foam degree cap d*. At d* = 7 (the physically motivated estimate): $p^c \geq 1/6 \approx 0.167$. If d* = 6 (no cross-simplex degree inflation): $p^c \geq 1/5 = 0.20$. The upper endpoint (0.30) is a conservative estimate from clustering-corrected branching — physically anchored but not a theorem-level bound. Numerical determination of both d* and p^c by Monte Carlo simulation would sharpen the interval.

Effect on ξ (at Δ = 0, b = 0.875, $g_0^2 = 1/128$, $\ell_e = 3.232 \times 10^{-35}$ m):

p^c	$1/p^c$	$128 - 1/p^c$	E	$\log_{10}(\xi/\ell_e)$	ξ	d*
1/6 (≈ 0.1667)	6.00	122.00	69.71	30.28	61 μm	7
1/5 (= 0.20)	5.00	123.00	70.29	30.52	109 μm	6
0.25	4.00	124.00	70.86	30.77	190 μm	—
0.30	3.33	124.67	71.24	30.93	275 μm	—

Rows $p^c = 0.25$ and 0.30 illustrate the effect of the upper-end working cap; they are not tied to a particular d* value.

Worked example ($p^c = 0.20$). E = (128 − 5.00)/1.75 = 123.00/1.75 = 70.29. $\log_{10}(\xi/\ell_e)$ = 70.29/2.3026 = 30.52. $\xi = 3.232 \times 10^{-35} \times 10^{30.52} = 3.232 \times 10^{-35} \times 3.31 \times 10^{30} = 1.07 \times 10^{-4}$ m ≈ 107 μm. ✓

5. Stability Under Bounded Perturbations

5.1 Sensitivity Analysis

To Δ (constraint correlations, $\varepsilon \leq 10^{-3}$, $|\Delta| \leq 0.042$): $|\delta E| \leq (128 \times 0.042)/1.75 \approx 3.1$. Shifts ξ by factor $e^{3.1} \approx 22$ (~1.3 orders of magnitude).

To b (loop channel count, δb = ±0.1): $|\delta E_b| = (0.1/2 \times 0.875^2) \times 122.4 \approx 8.0$. Shifts ξ by factor $e^8 \approx 3000$ (~3.5 orders of magnitude). However, $N_{loop} = 14$ is geometrically fixed.

To p^c (percolation threshold, $\delta p^c = \pm 0.05$, evaluated at $p^c = 0.20$): $|\delta E_p| = (1/1.75)(0.05/0.04)$ ≈ 0.71. Shifts ξ by factor $e^{0.71} \approx 2.0$ (~0.3 orders of magnitude).

Combined ($\varepsilon \leq 10^{-3}$, $\delta b/b \leq 0.15$, $\delta p^c = \pm 0.05$): The conservative envelope (summing absolute values of all individual sensitivities) gives $|\delta E| \leq 12$, keeping $E \in [58, 82]$ and ξ within $[10^{-8}$ m, 10^2 m$]$. The physically motivated range ($\varepsilon \leq 5 \times 10^{-4}$, nominal b and p^c) gives $E \in [69, 72]$, $\xi \in [24\ \mu\text{m}, 510\ \mu\text{m}]$.

5.2 The Discrete Sensitivity Gap

K	Exponent (central)	ξ
6	~33	$\sim 10^{-20}$ m
7	~70	$\sim 10^{-4}$ m
8	~143	$\sim 10^{27}$ m

Changing K by ±1 shifts E by ~73 — far larger than all continuous perturbations combined. The mesoscopic result is pinned by the discrete constraint count K = 7, which is enumerated from simplex geometry and cannot be continuously varied.

6. Route M Microphysical Closure Theorem

Theorem 6.1 (Route M Microphysical Closure). Consider a 4D simplicial-foam universality class with: (i) emergence scale $\ell_e = 2\ell_p$; (ii) K = 7 triangle-coherence constraints; (iii) weak UV neutrality: $P(C_k = 1) = 1/2$ for all k; (iv) bounded correlations: $\varepsilon :=$ max over $i \neq j$ of $|\mathrm{Cov}(C_i, C_j)| \leq 10^{-3}$; (v) bounded triangle-adjacency degree: $d\max(G\triangle) \leq d^*$ for a measurable degree cap d^*, with working interval $p^c \in [1/(d^* - 1), 0.30]$. At $d^* = 7$: $p^c \in [0.167, 0.30]$.

Then $\ln(\xi/\ell_e) = (1/2b)(1/g_0^2 - 1/p^c)$ with $b = 14/16$, $g_0^2 = 2^{-K} \cdot \exp(+\Delta)$, $|\Delta| \leq 42\varepsilon$, yields:

$\xi \in [O(10^{-6}\ \text{m}), O(10^{-3}\ \text{m})]$

with the central value near 10^{-4} m, independent of cosmological input.

Corollary 6.2 (Exponential Rigidity). Bounded perturbations at $\varepsilon \leq 10^{-3}$ shift E by at most ~12. Replacing $K \to K \pm 1$ shifts E by ~73. The mesoscopic result is pinned by discrete combinatorics.

Corollary 6.3 (Universality-Class Dependence). The mesoscopic scale is rigidly tied to K = 7 and 4D loop combinatorics. Modifying the universality class replaces the theory rather than deforming the prediction.

7. Comparison with Paper 4

Element	Paper 4	This Paper	Change
Bare coupling justification	Maximum-entropy heuristic	Flip-group transitivity theorem	Logical error corrected

Element	Paper 4	This Paper	Change
Independence assumption	Assumed	Bounded-correlation lemma with measurable ε	Controlled
Required ε bound	Not specified	$\varepsilon \leq 10^{-3}$ (explicit, testable)	New constraint identified
Sign convention for Δ	Inconsistent	$\Delta > 0$ = positive correlation = ξ decreases	Corrected throughout
Percolation lower bound	Bethe: $p_c \geq 0.167$	Maximum-degree theorem: $p_c \geq 1/(d^* - 1)$; at $d^* = 7$: $p_c \geq 0.167$	Theorem-grade, conditional on d*
Central ξ ($d^* = 7$)	75 μm	60–109 μm ($p_c \in [0.167, 0.20]$)	Depends on foam degree cap
All table entries	Minor inconsistencies	Verified from single convention	Numerically consistent

New finding: The honest sensitivity analysis reveals that $\varepsilon \leq 10^{-2}$ is insufficient for mesoscopic confinement. The framework requires $\varepsilon \leq 10^{-3}$, physically motivated by gauge-group averaging but awaiting numerical confirmation. This was obscured in the previous version by a sign-convention error in the correlation analysis.

8. Conclusion

1. Symmetry theorem for g_0^2 (§2). The bare coupling $g_0^2 = 2^{-K} = 1/128$ is the unique measure invariant under the flip group $(\mathbb{Z}_2)^K$ acting transitively on $\{0, 1\}^K$. The proof is one line. No permutation symmetry is needed. This corrects a logical error in the previous version.

2. Bounded correlation lemma (§3). Relaxing from strong to weak UV neutrality, mesoscopic confinement of ξ requires $\varepsilon \leq 10^{-3}$ — a specific, measurable, testable condition. All signs and numerical propagation are verified consistently.

3. Controlled percolation bounds (§4). $p_c \geq 1/(d^* - 1)$ from the maximum-degree theorem, conditional on a testable foam degree cap d*. At $d^* = 7$ (physically motivated): $p_c \geq 1/6 \approx 0.167$. The upper cap $p_c \leq 0.30$ is a conservative clustering estimate pending simulation. The Hashimoto non-backtracking bound is identified as a route to further tightening.

Next steps: (1) Monte Carlo measurement of ε and d* in dynamical triangulation simulations, testing whether the constraint correlation and degree-cap conditions are satisfied. (2) Numerical determination of p_c for triangle percolation on 4D simplicial foams, converting the working interval to a precise value.

9. References

Companion Papers

- Taylor, K. (2025). Two-Planck Principle: From Quantum Geometry to Emergent Gravity. VERSF Theoretical Physics Program.
- Taylor, K. (2025). Relational Geometry and the Universality of the Two-Planck Scale. VERSF Theoretical Physics Program.
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Appendix A: Proof of the Uniform Invariant Measure Theorem

Theorem 2.3. If μ on $\Omega = \{0, 1\}^K$ is F-invariant ($\mu(\tau(A)) = \mu(A)$ for all $\tau \in F$, all $A \subseteq \Omega$), then $\mu(C) = 2^{-K}$ for all C.

Proof. *Step 1 (Transitivity):* For any $C, C' \in \Omega$, the flip τ defined by τ_k = flip iff $C_k \neq C'_k$ satisfies $\tau \in F$ and $\tau(C) = C'$. *Step 2 (Equal atoms):* Applying F-invariance to singleton events: $\mu(\{C\}) = \mu(\{C'\})$ for all C, C′. *Step 3 (Normalisation):* $|\Omega| = 2^K$, so $\mu(C) = 2^{-K}$. ■

Remark. The previous version attempted: S_K-invariance + unbiased marginals ⇒ product measure. This is false (counterexample: draw p ~ Uniform[0,1], set C_k = Bernoulli(p) independently — marginals are 1/2, distribution is exchangeable, but variables are positively correlated). The corrected proof uses only F-transitivity.

Appendix B: Derivation of the Bounded Correlation Lemma

Lemma 3.2. Under weak UV neutrality with $|\mathrm{Cov}(C_i, C_j)| \leq \varepsilon$:

$$|\Delta| \leq 2K(K - 1)\varepsilon + O(K^3\varepsilon^2)$$

Proof. Write $P_{coh} = \prod_k p_k \cdot (1 + R)$ where R encodes correlations. Then $\Delta = \ln(1 + R)$. For K = 2: $R_2 = \mathrm{Cov}(C_1, C_2)/(p_1p_2)$. For general K: $|R| \leq \sum$ over i<j of $|\mathrm{Cov}(C_i, C_j)|/(p_ip_j) + O(K^3\varepsilon^2) \leq K(K-1)/2 \cdot \varepsilon/(1/4) + O(K^3\varepsilon^2) = 2K(K-1)\varepsilon + O(K^3\varepsilon^2)$. For small |R|: $|\Delta| = |\ln(1+R)| \leq |R|/(1-|R|) \approx |R|$. ■

Appendix C: Spectral Properties of J(5, 3)

The intra-simplex triangle adjacency graph is J(5, 3): vertices are 3-element subsets of {0,1,2,3,4}, adjacent iff intersection has size 2.

J(5, 3) is 6-regular. Eigenvalues of J(n, k): $\lambda_i = (k-i)(n-k-i) - i$ for i = 0, ..., min(k, n−k).

For J(5, 3): $\lambda_0 = 3\cdot2 - 0 = 6$; $\lambda_1 = 2\cdot1 - 1 = 1$; $\lambda_2 = 1\cdot0 - 2 = -2$. Multiplicities: 1, 4, 5.

Verification: 6(1) + 1(4) + (−2)(5) = 0 = Tr(A). ✓ $\mathrm{Tr}(A^2) = 36 + 4 + 20 = 60 = 2 \times 30$ edges. ✓

Spectral radius $\lambda_1 = 6$. Spectral gap: 6 − 1 = 5 (strong connectivity). The spectral data confirm 6-regularity of the intra-simplex graph (dmax = 6 within a single 4-simplex). The maximum-degree percolation bound $p_c \geq 1/(d_{max} - 1)$ applied to the full foam graph requires knowledge of $d_{max}(G\triangle)$, which depends on gluing topology (see §4.3).

Appendix D: Glossary for General Readers

Bare coupling (g_0^2): Probability of a minimal triangle being coherent at the emergence scale.

Correlation parameter (ε): Maximum absolute pairwise covariance between constraint variables. Measurable in simulations.

*Degree cap (d):** The maximum vertex degree in the full foam triangle-adjacency graph G△. Determines the percolation lower bound via $p^c \geq 1/(d^* - 1)$. Measurable in simulations; physically estimated at $d^* = 7$.

Flip group: $(\mathbb{Z}_2)^K$ acting on $\{0,1\}^K$ by flipping individual coordinates. Acts transitively: any configuration reaches any other.

Johnson graph J(n,k): Graph on k-element subsets of an n-set, adjacent iff subsets differ by one element. J(5,3) is the intra-simplex triangle adjacency graph.

Strong UV neutrality: Full F-invariance of the foam measure. Implies uniform distribution and exact independence.

Weak UV neutrality: Each constraint marginal is unbiased ($P(C_k = 1) = 1/2$), but correlations are permitted.

Transitive action: Every element maps to every other via some group element. The key property enabling the one-line proof.