Structural Closure of the Two-Planck Framework

Independent Verification of K = 7, Minimal Universality-Class Selection, and the CSS Attractor Theorem

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Companion Papers:

- *Two-Planck Principle: From Quantum Geometry to Emergent Gravity* (Taylor, 2025) — Quantitative predictions and experimental signatures
- *Relational Geometry and the Universality of the Two-Planck Scale* (Taylor, 2025) — Foundational arguments and constraint enumeration

This Paper's Role: The companion papers derive quantitative predictions ($\xi \approx 88$ μm, $\Lambda \approx 1.1 \times 10^{-52}$ m^{-2}, $w = -1$) and argue that the structural inputs are forced rather than fitted. This third paper provides independent verification of three critical elements: the coherence constraint count K = 7, the selection of simplicial foam as the minimal universality class, and the Cosmological Saturation Scenario (CSS) as a dynamical attractor under monotone-feedback assumptions. Together, the three papers constitute a structural derivation of the cosmological constant conditional on the empirically measured expansion history H(z).

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1. Abstract for General Readers

Physics has a 120-order-of-magnitude problem. When physicists calculate how much energy empty space should contain, they get an answer that exceeds what astronomers observe by a factor of 10^{120}—a one followed by 120 zeros. This is often called the worst prediction in the history of science.

Our companion papers proposed a solution: space isn't built from points, but from *relationships* between points. The simplest relationship needs two endpoints, so the smallest meaningful unit of space is *twice* the fundamental Planck length. From this single principle, combined with the requirement that space shouldn't collapse into black holes, we derived that empty space has a natural "mesh size" of about 100 micrometers—roughly the width of a human hair. This mesh size determines how much energy empty space contains, and our prediction matches what astronomers observe to within 20%.

But three questions remained:

1. **Why exactly seven constraints?** Our calculation depends on the fact that for a triangular piece of space to be properly formed, seven independent conditions must be satisfied. Why seven and not six or eight?
2. **Why triangles?** We assumed space is built from triangular relationships. But why not some other shape?

3. **Why does space "fill up" to exactly the right energy level?** We assumed empty space contains the maximum energy it can hold without collapsing. But is this an assumption, or does physics force this outcome?

This paper answers all three questions:

- **Seven constraints** is verified through two completely independent mathematical methods—information theory and obstruction theory—both formalizing the same underlying structure in different languages.
- **Triangles** (or equivalent structures) are the minimal building blocks that can carry curvature while allowing the theory to be consistently extrapolated across scales.
- **Maximum energy** is not an assumption but a mathematical inevitability—under very general conditions about how the universe responds to over-filling or under-filling, it naturally evolves toward saturation and stays there.

With these three gaps closed, the derivation of the cosmological constant—the energy density of empty space—is structurally complete. The only observational input is the measured expansion history of the universe, used to determine the horizon scale. Everything else emerges from geometry.

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2. Technical Abstract

The Two-Planck framework derives the cosmological constant $\Lambda \approx 1.1 \times 10^{-52}\ \text{m}^{-2}$ from relational geometry with no fitting to Λ itself. The only empirical input is the measured expansion history $H(z)$, used to evaluate the event horizon scale L_{eh}. Three structural elements required independent verification: the coherence constraint count $K = 7$, the selection of simplicial foam as the pre-geometric universality class, and the Cosmological Saturation Scenario (CSS) as a dynamical attractor.

This paper provides that verification:

Part I establishes $K = 7$ through two independent formalisms:

- Information-theoretic: Triangle coherence requires 7 bits of constraint satisfaction
- Obstruction-theoretic: Descent on the relational sheaf has 7 independent obstruction classes

These methods formalize the same independence decomposition (admissibility/closure/embedding) in non-overlapping mathematical languages; their agreement indicates robustness of that decomposition rather than coincidence.

Part II identifies simplicial foam (or an equivalent 2-complex with triangular minimal loops and hinge-localized curvature) as the *minimal* universality class satisfying four relational axioms: locality, holonomy-based curvature, renormalizability, and UV partiality. Alternative candidates (causal sets, spin networks) fail at least one axiom; equivalent formulations (certain spin foams, Regge calculus) are admitted.

Part III proves the CSS Attractor Theorem under monotone-feedback assumptions: any dynamics satisfying (i) collapse/fragmentation feedback above the gravitational stability bound, and (ii) entropy-gradient pressure below saturation, has a globally attracting fixed point at $x = 1$ (saturation). The specific ODE is the minimal normal form consistent with these monotonicities. As a corollary, the effective equation-of-state parameter $w = -1$ emerges from energy conservation at the attractor.

Together with the companion papers, this completes a structural derivation: $\ell_e = 2\ell_p \rightarrow K = 7 \rightarrow \xi \approx \sqrt{(\ell_p L)} \rightarrow \rho_{va}c \approx \hbar c/\xi^4 \rightarrow \Lambda$. Each arrow represents structural necessity conditional on measured $H(z)$, not parameter adjustment.

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3. Introduction: The Remaining Gaps

The Two-Planck framework, developed across two companion papers, advances a remarkable claim: the cosmological constant—the energy density of empty space driving the universe's accelerated expansion—can be derived from geometric principles without fitting to Λ itself.

The logical chain is:

Relational geometry → Emergence scale $\ell_e = 2\ell_p$
→ Coherence constraints $K = 7$
→ Dimensional transmutation
→ Coherence scale $\xi \approx 88\ \mu m$
→ Vacuum energy $\rho_{va}c \approx \hbar c/\xi^4$
→ Cosmological constant $\Lambda \approx 1.1 \times 10^{-52}\ m^{-2}$

The companion papers established this chain but left three elements as motivated assumptions rather than derived necessities:

Element	Status in Companion Papers	This Paper
K = 7	Enumerated from geometry	Verified by 2 independent methods
Simplicial foam	Assumed universality class	Identified as minimal class from axioms
CSS (saturation)	Postulated	Proven as attractor under monotone feedback

This paper closes these gaps, completing the structural derivation.

Clarification on inputs: The framework requires one empirical input: the measured expansion history $H(z)$, which determines the event horizon scale $L_{eh} \approx 1.6 \times 10^{26}$ m. This is not fitting to Λ—it is using standard cosmological measurements to evaluate an independently derived formula. The distinction matters: fitting would adjust parameters to match Λ; we instead derive Λ from geometric structure and check agreement.

For general readers: Think of this paper as checking our work using different mathematical tools. If you solve a geometry problem using algebra and get the same answer as someone who solved it using trigonometry, that's evidence you're both right. We're doing the mathematical equivalent for our theory of empty space.

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4. Part I: Independent Reconstructions of the K = 7 Decomposition

4.1 Why K = 7 Matters

The coherence constraint count K is the most sensitive parameter in the framework. The coherence scale ξ depends on K exponentially:

$$\xi = \ell_e \cdot \exp[1/(2b \cdot 2^{-K})]$$

where $b \approx 0.875$ is the β-function coefficient. This extreme sensitivity means:

K	Predicted ξ	Physical regime
5	$\sim 10^{-28}$ m	Sub-nuclear
6	$\sim 10^{-19}$ m	Subatomic
7	**$\sim 10^{-4}$ m**	**Mesoscopic (observed)**
8	$\sim 10^{11}$ m	Astronomical
9	$\sim 10^{29}$ m	Super-cosmological

For general readers: The number 7 isn't chosen to get the right answer—it's counted from the geometry of triangular relationships. But because small changes in this count produce enormous changes in our prediction, we need to be absolutely certain the count is correct. If K were 6, our predicted mesh size would be smaller than an atom. If K were 8, it would be larger than the Earth's orbit. Only K = 7 gives the human-hair scale that matches observation.

The companion paper (Appendix D) derived K = 7 by explicitly enumerating seven constraints C1–C7 for triangle coherence in simplicial foam. Here we verify this count through two independent mathematical frameworks that formalize the same underlying independence structure in different languages.

4.2 Route α: Information-Theoretic Derivation

Core principle: In a pre-geometric regime without background structure, triangle coherence is an informational event requiring exclusion of all independent failure modes.

Setup: Let Δ = (i, j, k) be an oriented triangle with relational transport elements U_{ij}, U_{jk}, U_{ki}. Define the coherence event C_Δ as the conjunction:

$C\_\Delta = (E_1 \wedge E_2 \wedge E_3) \wedge L \wedge (M_1 \wedge M_2 \wedge O)$

where:

- $\mathbf{E_1, E_2, E_3}$: Edge admissibility events (each edge relation exists and is invertible)
- **L**: Loop closure event (holonomy $H\_\Delta = U_{ij} \cdot U_{jk} \cdot U_{ki}$ belongs to coherent class)
- $\mathbf{M_1, M_2}$: Embedding matching events (triangle data consistent across tetrahedra)
- **O**: Orientation consistency event (chirality preserved, $\mathbb{Z}_2$-valued)

Independence argument: In the absence of background metric or transitivity:

- Edge admissibility E_i does not imply E_j for $i \neq j$ (different relational degrees of freedom)
- Loop closure L is not implied by edge admissibility (valid edges can compose incoherently)
- Embedding matches M_1, M_2 are not implied by L (internal consistency ≠ external consistency)
- Orientation O is a discrete condition independent of metric matching

Information cost: Each independent binary constraint contributes 1 bit of information that must be specified for coherence. The total information cost is:

$I(C\_\Delta) = 3 + 1 + 2 + 1 = 7$ bits

Theorem α: *The minimal information required to specify triangle coherence in a pre-geometric relational foam is 7 bits.*

Proof: See Appendix B.1 for the detailed independence argument establishing that no constraint is implied by the others in the pre-geometric setting. ■

Lemma (Lower Bound on Coherence Encoding): *Any coherence predicate C_Δ satisfying:*

1. *Independent edge existence (no edge implied by others)*
2. *Loop-level holonomy non-reducibility (holonomy not determined by edge data alone)*
3. *Embedding consistency across ≥2 adjacent 3-simplices*
4. *Independent $\mathbb{Z}_2$ orientation consistency*

requires at least 7 independent binary degrees of freedom.

Proof sketch: Assume a coherence encoding using ≤6 bits. By the pigeonhole principle, at least one of the four constraint categories must share encoding with another (i.e., two logically distinct constraints map to the same bit pattern).

Consider the possible overlaps:

- **Edge sharing:** If E_1 and E_2 share encoding, then configurations with edge (12) present but edge (23) absent are indistinguishable from configurations with both present. But these have different coherence status (first fails, second may succeed). Contradiction.
- **Holonomy-edge sharing:** If L shares encoding with any E_i, then a triangle with valid edges but incoherent holonomy ($H\_\Delta \notin \mathcal{C}$) would be indistinguishable from one with a missing edge. But these fail for different reasons and require different remediation. Contradiction.
- **Embedding sharing:** If M_1 and M_2 share encoding, consider a triangle coherent in the T_1-T_2 overlap but incoherent in the T_2-T_3 overlap. This configuration cannot be distinguished from full coherence. But the triangle is not globally embeddable. Contradiction.
- **Orientation-metric sharing:** If O shares encoding with M_1 or M_2, consider a triangle with consistent metric matching but flipped orientation. This is geometrically distinct (produces a reflection rather than a rotation) but would be encoded identically. Contradiction.

All possible ≤6-bit encodings produce distinguishability failures. Therefore $K \geq 7$. Combined with Theorem α ($K \leq 7$ by explicit construction), we have $K = 7$ exactly. ■

For general readers: Information theory tells us that specifying any yes/no choice requires 1 "bit" of information (like a coin flip: heads or tails). For a triangle to be properly formed in our framework, seven independent yes/no questions must all be answered "yes." The lemma above proves you can't cheat by combining questions—if you try to use only 6 bits, you inevitably confuse situations that should be distinguished. Seven is both necessary and sufficient.

4.3 Route β: Obstruction-Theoretic Verification

Core principle: Triangle coherence is a descent problem—local relational data must glue consistently into a global geometric object. Each independent way gluing can fail defines an obstruction class.

Mathematical framework: Consider the presheaf of relational data on the simplicial complex. Local sections (relational transport elements on edges) must descend to a global section (coherent triangle). Obstructions to descent live in cohomology groups.

Obstruction classification:

(i) 0-cochain obstructions (edge level):

Each edge (ij) carries a transport element U_{ij}. The obstruction to existence/invertibility is a 0-cochain condition. Three edges yield three independent obstructions:

$\mathcal{O}^0 = \{o_{12}, o_{23}, o_{31}\}$ $|\mathcal{O}^0| = 3$

(ii) 1-cocycle obstruction (loop level):

Given valid edge data, the holonomy $H\_\Delta = U_{ij} \cdot U_{jk} \cdot U_{ki}$ defines a 1-cocycle. The obstruction to coherence is:

$\mathcal{O}^1 = \{[H\_\Delta] \in H^1(\partial\Delta, G)\}$ $|\mathcal{O}^1| = 1$

(iii) Descent obstructions (embedding level):

Even with coherent holonomy, the triangle may fail to embed consistently into its tetrahedral neighborhood. The Čech cohomology of the nerve yields:

- Two independent metric matching obstructions (from pairs of adjacent tetrahedra)
- One orientation obstruction (discrete $\mathbb{Z}_2$ cohomology)

$\mathcal{O}^{desc} = \{m_{12}, m_{23}, \omega\}$ $|\mathcal{O}^{desc}| = 3$

Total obstruction count:

$|\mathcal{O}| = |\mathcal{O}^0| + |\mathcal{O}^1| + |\mathcal{O}^{desc}| = 3 + 1 + 3 = 7$

Theorem β: *The descent of relational data to a coherent triangle has exactly 7 independent obstruction classes.*

Proof: The independence follows from the distinct cohomological degrees and the absence of connecting homomorphisms between obstruction types in the pre-geometric setting. See Appendix B.2 for details. ■

Remark (Explicit Descent Setup). To make the obstruction-theoretic argument concrete, consider a triangle Δ shared by tetrahedra T_1, T_2, T_3. Define:

- **Cover:** $\mathcal{U} = \{T_1, T_2, T_3\}$
- **Overlaps:** $T_i \cap T_j = \Delta$ for adjacent pairs; triple overlap $T_1 \cap T_2 \cap T_3 = \Delta$
- **Coefficient group:** G = relational transport group (e.g., SO(3) for rotational holonomy)

The obstruction decomposition follows directly:

- **0-cochains** (edge data): Live on vertices of the nerve; 3 edges → 3 independent sections
- **1-cocycle** (holonomy): Lives on the unique 1-cycle ∂Δ; coherence requires $[H\_\Delta] = 0$ in $H^1(\partial\Delta; G)$
- **Čech 1-cocycles** (metric matching): On overlaps $T_i \cap T_j$; 2 independent matching conditions for the minimal nerve
- $\mathbb{Z}_2$ **0-cocycle** (orientation): Discrete condition on the nerve; 1 independent obstruction

The stated count $|\mathcal{O}| = 3 + 1 + 2 + 1 = 7$ follows from this explicit cover structure.

For general readers: When you try to assemble a jigsaw puzzle, each piece must fit its neighbors in specific ways. If any fit fails, the puzzle doesn't work. For our triangular pieces of space, there are exactly seven independent ways the fitting can fail. All seven must succeed for the triangle to be properly formed.

4.4 Convergence and Robustness

Observation (Decomposition Robustness): *Routes α and β formalize the same independence decomposition—admissibility (3), closure (1), embedding (3)—in non-overlapping mathematical languages. Their agreement to K = 7 indicates robustness of that decomposition rather than coincidence.*

Route	Framework	Decomposition	Count
Original	Geometric enumeration	C1-C3, C4, C5-C7	7
α	Information theory	$E_1E_2E_3$, L, M_1M_2O	7
β	Obstruction theory	$\mathcal{O}^0$, $\mathcal{O}^1$, $\mathcal{O}^{desc}$	7

The three-part structure (admissibility/closure/embedding) appears in every formalization. This is the signature of a genuine structural decomposition, not an artifact of any particular mathematical language.

For general readers: We've now counted the constraints three different ways using completely different mathematical tools, and every method gives the answer 7 with the same internal structure (3 + 1 + 3). This isn't about probability—it's about the same underlying reality being described in different languages, like describing a table as "four-legged" in English, French, and Chinese.

4.5 Numerical Verification Strategy

Beyond analytical methods, K = 7 admits numerical verification through lattice simulations.

Protocol:

1. Generate random 4D simplicial complexes (dynamical triangulations)
2. Assign each triangle coherence probability $p = 2^{-K}$
3. Declare triangle coherent iff K independent Bernoulli trials all succeed
4. Perform iterative coarse-graining
5. Measure scale ξ^* at which spanning coherent cluster emerges

Predictions:

- K = 6: Percolation at microscopic scales ($\xi^* \sim \ell_p$)
- K = 7: Percolation at mesoscopic scales ($\xi^* \sim 10^{-4}$ m in appropriate units)
- $K \geq 8$: No percolation until cosmological scales

Robustness criterion: If $\xi^*/\ell_p \sim 10^{31}$ emerges consistently for K = 7 across different random ensembles, topologies, and coarse-graining schemes, this constitutes numerical confirmation independent of analytical arguments.

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5. Part II: Minimal Universality-Class Selection

5.1 What Is a Universality Class?

For general readers: A universality class is like a species in biology. All dogs share certain features (four legs, fur, bark) regardless of breed. Similarly, all theories in the same universality class share certain mathematical features regardless of details. The question is: which "species" of pre-geometric theory correctly describes our universe?

In physics, a universality class is defined by the fundamental degrees of freedom and symmetries of a theory. Theories in the same class share:

- The same symmetry structure
- The same relevant operators under renormalization
- The same critical exponents and scaling relations

Different pre-geometric proposals correspond to different universality classes:

Proposal	Fundamental object	Universality class
Simplicial foam	Triangulated manifolds	Regge-like
Causal sets	Partial orders	Poset-based
Spin networks	Graphs with group labels	Spin-foam
Tensor models	Higher-rank tensors	Tensorial

The Two-Planck framework requires simplicial foam or an equivalent structure. We now derive this as the *minimal* choice satisfying necessary physical axioms.

5.2 The Relational Axioms

We posit four axioms that any viable pre-geometric theory must satisfy:

Axiom A1 (Relational Locality): *Geometric data is encoded in local relational structures. There exists a notion of "neighboring" relations, and physical observables depend only on finite neighborhoods.*

For general readers: Space should be built from local relationships, not global ones. What happens "here" shouldn't directly depend on what happens in a distant galaxy.

Axiom A2 (Holonomy-Based Curvature): *Curvature—the bending of space that causes gravity—is detected via holonomy (parallel transport) around closed loops. Curvature localizes on codimension-2 substructures.*

For general readers: Gravity comes from the bending of space. To detect bending, you need to walk in a small loop and see if you come back rotated. This requires closed paths, which means loops, which means at minimum triangles.

Axiom A3 (Renormalizability): *The theory admits a controlled coarse-graining procedure. Physics at large scales emerges systematically from small-scale dynamics.*

For general readers: We should be able to "zoom out" and still have a sensible theory. The rules at microscopic scales should lead to consistent rules at macroscopic scales.

Axiom A4 (UV Partiality): *At the emergence scale, relations are partial—not all compositions are defined, and coherence is a rare event requiring constraint satisfaction.*

For general readers: At the smallest scales, space is "foamy" and unreliable. Relationships can fail to connect properly. This is what makes coherence rare and valuable.

5.3 Selection Theorem: Simplicial Foam as Minimal Class

Theorem (Minimal Universality Class): *Simplicial foam (or an equivalent 2-complex with triangular minimal loops and hinge-localized curvature) is the minimal universality class satisfying Axioms A1–A4.*

Proof:

(1) A1 + A2 → Local loops with holonomy

Relational locality (A1) requires finite local structures. Holonomy-based curvature (A2) requires closed loops. The minimal closed loop in a relational structure is a triangle (3 edges forming a cycle). Holonomy is defined on these minimal loops.

(2) A2 → Codimension-2 curvature localization

In d dimensions, curvature localizes on $(d-2)$-dimensional substructures. For $d = 4$, this means 2-dimensional surfaces—i.e., triangular faces in a simplicial decomposition.

(3) A3 → Simplicial blocking

Renormalizability requires a coarse-graining scheme. For structures satisfying (1) and (2), the natural coarse-graining is simplicial blocking: combining adjacent simplices into larger simplices while preserving triangulation structure.

(4) A4 → Probabilistic coherence on simplices

UV partiality means coherence probability $p < 1$ for minimal structures. Combined with (1), this means triangular coherence is probabilistic, governed by constraint satisfaction (K independent conditions).

(5) Minimality

Any structure satisfying A1–A4 must have triangular loops (from 1), curvature on 2-faces (from 2), simplicial-type blocking (from 3), and constraint-governed coherence (from 4). Simplicial foam is the minimal structure with these properties—no additional structure (group labels, causal relations, tensor indices) is required.

Structures with additional features (spin foams with representation labels, tensor models with higher symmetries) contain simplicial foam as a subsector but are not minimal. ■

Note on "minimal": We do not claim simplicial foam is the *unique* universality class satisfying A1–A4. Equivalent formulations exist (see §5.6). The claim is that it is *minimal*: any structure satisfying these axioms contains simplicial foam or an equivalent as its core geometric sector.

5.4 Why Not Causal Sets?

Causal set theory posits that spacetime is fundamentally a partially ordered set (poset) of events, with the order relation encoding causal structure.

Failure mode: Causal sets fail **Axiom A2**.

In a causal set:

- There is no intrinsic notion of "closed loop" (the partial order is acyclic)
- Curvature must be extracted via non-local dimension estimators (Myrheim-Meyer)
- There are no codimension-2 substructures on which curvature localizes

For general readers: Causal sets encode the "before and after" relationships between events, but not the "beside" relationships. You can't walk in a loop in a causal set because time only flows forward. Without loops, you can't detect curvature the way our framework requires.

Causal sets constitute a different universality class, potentially valid but with different phenomenology. They are not excluded as physical theories—they simply don't satisfy A2 and thus yield different predictions.

5.5 Why Not Spin Networks?

Spin networks (from loop quantum gravity) are graphs with edges labeled by group representations, encoding quantum geometry.

Failure mode: Spin networks fail **Axiom A2** in its strong form, and fail **minimality**.

In spin networks:

- Holonomy is defined on edges (Wilson lines)
- Curvature requires passing to the dual 2-complex (spin foam)
- The curvature-bearing structures are not native to the spin network itself
- Group representation labels add structure beyond minimal relational geometry

For general readers: Spin networks encode geometry on their edges and vertices, but to get curvature, you need to construct a separate dual structure. It's like having a road map where the roads carry information, but to find out if the terrain is hilly, you need to build a completely separate elevation map.

Spin foams (the dynamics of spin networks) are closer to simplicial foam. The dual 2-complex of a spin foam can be equivalent to simplicial foam for appropriate choices—see §5.6.

5.6 Equivalent Formulations

The minimality theorem does not exclude equivalent formulations:

Formulation	Relationship to simplicial foam
Regge calculus	Discretized GR on simplicial manifolds; equivalent
Spin foams (BF-type)	Dual 2-complexes can match simplicial structure
Dynamical triangulations	Specific ensemble over simplicial foams; equivalent
Piecewise-linear manifolds	Different language, same objects

Criterion for equivalence: A pre-geometric structure is equivalent to simplicial foam if:

1. Its minimal curvature-bearing objects are 2-cells (triangular or equivalent)
2. Curvature localizes on these 2-cells as deficit angles or holonomies
3. Coarse-graining preserves the 2-cell structure

The Two-Planck framework applies to any structure in this equivalence class.

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6. Part III: CSS as Attractor Under Monotone-Feedback Assumptions

6.1 The Saturation Hypothesis

The Cosmological Saturation Scenario (CSS) states:

The vacuum state saturates the maximum homogeneous energy density consistent with gravitational stability of the causal patch defined by the future event horizon.

Mathematically:

$\rho_{vac} = \rho_{max}(L) = \eta c^4/(GL^2)$

where L is the event horizon scale and $\eta = 3/(8\pi)$ is derived from de Sitter geometry.

In the companion papers, CSS was introduced as a postulate. Here we prove it is the unique stable attractor under general monotone-feedback assumptions—*not* by assuming a specific dynamical equation, but by deriving that any dynamics with the required monotonicity properties converges to saturation.

For general readers: We claimed that empty space contains the maximum energy it can hold without collapsing. But why should the universe "choose" this maximum? This section proves that the universe doesn't choose—it's forced there by very general physical principles about how over-filled and under-filled states behave.

6.2 Dynamical Variables and Phase Space

Define two dynamical variables:

Coherence order parameter:

$p(t) \in [0, 1]$

The probability that a minimal triangle remains coherent under coarse-graining.

- $p = 0$: Fully incoherent (void-dominated)
- $p = p_s \approx 0.17$–0.20: Percolation threshold
- $p = 1$: Fully coherent (classical geometry)

Saturation ratio:

$x(t) = \rho/\rho_{max}(L) \in [0, \infty)$

The vacuum energy density relative to the gravitational stability bound.

- $x < 1$: Under-saturated
- $x = 1$: Saturated (CSS)
- $x > 1$: Over-saturated (unstable)

The phase space is $\mathbb{R}^+ \times [0,1]$ with coordinates (x, p).

6.3 The Two Monotone-Feedback Channels

The key insight is that the saturation dynamics are governed by two universal feedback mechanisms:

Channel I: Gravitational collapse/fragmentation (x > 1)

When $\rho > \rho_{max}(L)$, the energy density exceeds the gravitational stability bound. This triggers:

- Black hole formation in over-dense regions
- Fragmentation of coherent geometry
- Net reduction of homogeneous energy density

This feedback is *monotonically restoring*: the further above saturation, the stronger the collapse tendency.

For $x > 1$: $dx/dt < 0$, with $|dx/dt|$ increasing in $(x - 1)$

Channel II: Entropy-gradient pressure (x < 1)

When $\rho < \rho_{max}(L)$, the coherent geometry is under-saturated. Constraint-breaking entropy at coherence boundaries creates:

- Surface tension opposing further dilution
- Effective negative pressure driving expansion
- Net increase toward saturation

This feedback is *monotonically driving*: the further below saturation, the stronger the pressure toward filling.

For $x < 1$: $dx/dt > 0$, with $|dx/dt|$ increasing in $(1 - x)$

Critical observation: Both channels point toward $x = 1$. This is not a choice—it follows from the physics of gravitational stability and statistical mechanics.

Remark (Necessity of Monotone Feedback). The sign structure of dx/dt is not an optional assumption within the Two-Planck ontology—it is forced by consistency:

Channel I (x > 1): If over-saturated regions did not fragment or collapse, homogeneous energy density would exceed the gravitational stability bound indefinitely. This contradicts the Cohen–Kaplan–Nelson constraint: a region of size L cannot support energy density $\rho > \eta c^4/(GL^2)$ without forming horizons that fragment the homogeneous patch. Therefore $dx/dt < 0$ for $x > 1$.

Channel II ($x < 1$): If under-saturated coherent regions did not experience entropy-gradient pressure, coherence boundaries would expand indefinitely into the void. But constraint-breaking at boundaries costs entropy $\Delta S = \bar{n} \cdot k_B \ln 2$ per boundary triangle (§6.4). This creates a surface tension $\sigma \sim \hbar c/\xi^3$ that resists dilution. The resulting pressure drives the system toward saturation. Therefore $dx/dt > 0$ for $x < 1$.

The monotonicity requirements are thus consequences of gravitational stability and statistical mechanics, not independent assumptions.

6.4 Derivation of the Entropy Functional

For general readers: Entropy measures disorder. When ordered regions of space (coherent geometry) try to expand into disordered regions (void), they encounter resistance—like trying to organize a messy room. This section derives exactly how much resistance, starting from basic counting of microscopic configurations.

Step 1: Constraint-breaking entropy

At the coherence boundary, $K = 7$ constraints govern each triangle. When a coherent region borders an incoherent one, some constraints must be violated.

Let n be the number of violated constraints at a boundary triangle. Each violation increases the number of accessible microstates by factor 2, so the entropy increase is:

$$\Delta S = n \cdot k_B \ln 2$$

Step 2: Boundary energy cost

Each violated constraint costs energy of order:

$$\varepsilon_c \approx \hbar c/\xi$$

This is the characteristic energy of a coherence cell.

Step 3: Surface tension

The boundary between coherent and incoherent regions has thickness $\sim\xi$ and carries energy density:

$$\sigma = \bar{n} \cdot \varepsilon_c/\xi^2 = \bar{n} \cdot \hbar c/\xi^3$$

where $\bar{n} \leq K = 7$ is the average number of broken constraints per boundary triangle.

Step 4: Entropy functional

The total foam entropy takes the Landau-Ginzburg form:

$$S[p] = S_0 - \int d^3x \, [\, V(p) + (\kappa/2)|\nabla p|^2 \,]$$

where:

- $V(p)$ is the local potential encoding coherence energetics
- κ is the gradient stiffness from constraint-breaking
- ∇p represents coherence gradients at boundaries

Step 5: Fixing the stiffness κ

Matching the surface tension $\sigma \approx \sqrt{(\kappa \cdot \Delta V)}$ with $\Delta V \approx \hbar c/\xi^4$:

$$\kappa \approx \hbar c \cdot \xi$$

This is determined by microphysics—no new parameter is introduced.

6.5 The Gradient Flow Equations as Minimal Normal Form

Key principle: We do not *assume* a specific ODE. Instead, we derive that the *minimal normal form* consistent with the two monotone-feedback channels takes a specific structure.

Monotonicity requirements:

From Channel I ($x > 1$): dx/dt must be negative and monotonically decreasing in $(x - 1)$.

From Channel II ($x < 1$): dx/dt must be positive and monotonically increasing in $(1 - x)$.

Continuity requirement: dx/dt must be continuous at $x = 1$, with $dx/dt|_{x=1} = 0$ (fixed point).

Minimal normal form:

The simplest (lowest-order polynomial) function satisfying these requirements is:

$$dx/dt = -a(x - 1) - b(x - 1)^3$$

where:

- The linear term $-a(x - 1)$ provides the leading-order monotone feedback
- The cubic term $-b(x - 1)^3$ ensures monotonicity is maintained for large deviations
- $a, b > 0$ are determined by microphysics (collapse rate, entropy gradient strength)

Why this form is not arbitrary:

Any dynamics satisfying the monotonicity requirements can be written as:

$dx/dt = -(x - 1) \cdot f(x)$

where $f(x) > 0$ for all x. The minimal normal form corresponds to $f(x) = a + b(x - 1)^2$.

Higher-order terms (quintic, etc.) do not change the attractor structure—they only modify approach rates. The qualitative behavior is determined by the monotonicity requirements, not the specific polynomial order.

Coherence dynamics:

Similarly, the order parameter p evolves by gradient descent on the entropy functional:

$dp/dt = -\Gamma_p \cdot [\ \partial V/\partial p - \kappa \nabla^2 p\ ]$

where $\Gamma_p > 0$ is a kinetic coefficient. At the percolation threshold $p = ps$, $V(p)$ has a minimum.

6.6 General Attractor Theorem

Theorem (CSS Attractor under Monotone Feedback): *Let the saturation dynamics satisfy:*

1. *$dx/dt < 0$ for $x > 1$, with $|dx/dt|$ monotonically increasing in $(x - 1)$*
2. *$dx/dt > 0$ for $x < 1$, with $|dx/dt|$ monotonically increasing in $(1 - x)$*
3. *dx/dt is continuous with $dx/dt|_{x=1} = 0$*

Then $x = 1$ is the unique globally asymptotically stable fixed point.

Proof:

(1) Existence of Lyapunov function:

Define:

$\mathscr{L}(x) = \frac{1}{2}(x - 1)^2$

This satisfies:

- $\mathscr{L}(1) = 0$
- $\mathscr{L}(x) > 0$ for $x \neq 1$

(2) Lyapunov derivative:

$d\mathscr{L}/dt = (x - 1) \cdot dx/dt$

For $x > 1$: $(x - 1) > 0$ and $dx/dt < 0$, so $d\mathscr{L}/dt < 0$.

For $x < 1$: $(x - 1) < 0$ and $dx/dt > 0$, so $d\mathscr{L}/dt < 0$.

For $x = 1$: $d\mathscr{L}/dt = 0$.

(3) Conclusion:

By Lyapunov's theorem, $x = 1$ is globally asymptotically stable. The monotonicity requirements ensure no other fixed points exist. ■

Corollary: *The specific polynomial form $dx/dt = -a(x - 1) - b(x - 1)^3$ is not essential. Any dynamics satisfying the monotone-feedback conditions converges to saturation.*

For general readers: We've proven that the universe *must* settle into the saturated state—not because we assumed a particular equation, but because we showed that *any* reasonable law of physics (one where over-filled states collapse and under-filled states expand) leads to the same conclusion. It's like proving that all balls roll to the bottom of valleys, without having to specify the exact shape of any particular valley.

6.7 Derivation of w = −1

Corollary: *At the CSS attractor, the effective equation-of-state parameter of the late-time accelerating component is $w = -1$.*

Proof:

At the attractor:

- $x = 1$ (saturation)
- $L = L_{eh} \to$ const (de Sitter horizon)
- $\rho = \rho_{max}(L) = \eta c^4/(GL^2) =$ const

For a perfect fluid with equation of state $P = w\rho c^2$:

Energy conservation in FRW cosmology:

$$\dot{\rho} + 3H(\rho + P/c^2) = 0$$

At the attractor, $\dot{\rho} = 0$, so:

$$3H(\rho + P/c^2) = 0$$

Since $H \neq 0$ (de Sitter expansion):

$$\rho + P/c^2 = 0 \Longrightarrow P = -\rho c^2 \Longrightarrow w = -1$$

■

Terminology note: We refer to $w = -1$ as the "effective equation-of-state of the late-time accelerating component" rather than "dark energy equation of state." In the ΛCDM parameterization, this component is often called dark energy; in the Two-Planck framework, it is geometric vacuum energy. The phenomenology ($w = -1$) is identical.

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7. Part IV: Synthesis and Structural Closure

7.1 The Complete Derivation Chain

We can now trace the complete derivation of Λ with all gaps closed:

STEP 1: Relational Geometry
├── Geometry requires relations between elements
├── Minimal relation has 2 endpoints
└── Emergence scale: $\ell_e = 2\ell_p$
[Philosophical commitment, well-motivated]

STEP 2: Minimal Universality Class (This paper, Part II)
├── Axioms A1–A4 (relational, local, holonomy, renormalizable, partial)
├── Selection theorem → Simplicial foam (or equivalent)
└── Curvature on triangular hinges
[Derived as minimal class from axioms]

STEP 3: Constraint Count (This paper, Part I)
├── Route α: Information theory → $K = 7$
├── Route β: Obstruction theory → $K = 7$
└── Robust decomposition: $3 + 1 + 3 = 7$
[Verified by independent methods, same structure]

STEP 4: Dimensional Transmutation (Companion paper, Route M)
├── β-function: $b = 14/16 = 0.875$
├── Bare coupling: $g_0^2 = 2^{-7} = 1/128$
├── Percolation threshold: $p_s \in [0.17, 0.20]$
└── Coherence scale: $\xi \in [60, 110]$ μm
[Computed from foam combinatorics]

STEP 5: UV/IR Consistency (Companion paper, Route A)
├── Gravitational stability bound: $\rho \leq \eta c^4/(GL^2)$
├── De Sitter geometry: $\eta = 3/(8\pi)$
├── Horizon scale: L_{eh} from measured $H(z)$
└── UV/IR bridge: $\xi = \eta^{-1/4}\sqrt{(\ell_p L)} \approx 88$ μm
[Geometric consistency + empirical $H(z)$]

STEP 6: CSS Attractor (This paper, Part III)
├── Two monotone-feedback channels identified
├── Minimal normal form derived (not assumed)
├── General attractor theorem: $x = 1$ is globally stable
└── Corollary: $w = -1$
[Proven under monotone-feedback assumptions]

STEP 7: Output
├── Vacuum energy: $\rho_{vac} = \hbar c/\xi^4 \approx 5 \times 10^{-10}$ J/m³
├── Cosmological constant: $\Lambda = 8\pi G\rho_{vac}/c^4 \approx 1.1 \times 10^{-52}$ m⁻²
└── Matches observation to ~20%

Every step is either a well-motivated principle, a derived consequence, or a verified necessity. No fitting to Λ is performed.

7.2 What "Structural Derivation" Means

Clarification: "Structural derivation" does not mean "assumption-free." The framework rests on foundational commitments:

Commitment	Status
Geometry is relational	Philosophical principle
Minimal relation has 2 endpoints	Definitional
Axioms A1–A4	Physical principles
Monotone feedback (collapse above, expansion below)	Physical requirement
Entropy maximization governs dynamics	Statistical mechanics

Empirical input: The measured expansion history H(z) is used to evaluate L_{eh}. This is not fitting to Λ—it is using standard cosmological data to evaluate an independently derived formula.

What "structural derivation conditional on H(z)" means is:

1. **No continuous tuning:** K = 7 is an integer from counting, not a fitted real number
2. **No fitting to Λ:** The observed Λ is an output, not an input
3. **No adjustable coefficients:** $\eta = 3/(8\pi)$ is derived, b = 0.875 from loop counting
4. **One empirical input:** H(z), used only to evaluate L_{eh}

The framework could be wrong if the foundational commitments are wrong. But it cannot be "adjusted" to fit different values of Λ—it either works or it doesn't.

7.3 Comparison with Other Approaches

Approach	Fitted to Λ?	Derives w?	Derives ξ?	Lab predictions?
ΛCDM	Yes (by definition)	No (assumed)	N/A	No
Weinberg anthropic	No	No	N/A	No
Padmanabhan CosMIn	Partial	Partial	No	No
String landscape	Somewhere in 10^{500}	Varies	No	No
Two-Planck	**No**	**Yes**	**Yes**	**Yes**

The Two-Planck framework is distinguished by:

1. Deriving (not assuming) $w = -1$
2. Predicting a specific coherence scale $\xi \sim 100$ μm
3. Making laboratory-testable predictions
4. Using H(z) as input, not Λ

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8. Falsifiability and Experimental Tests

A derivation is only as good as its testability. The framework makes specific predictions:

Prediction 1: Coherence scale

$\xi \in [60, 110]$ μm

Testable via Casimir measurements, short-range gravity tests, and quantum decoherence experiments at this scale.

Prediction 2: Effective equation of state

$w = -1$ exactly (asymptotically)

Current constraint: $w = -1.03 \pm 0.03$ (Planck 2018). Future surveys (Euclid, LSST, Roman) will tighten this to $\sigma\_w \sim 0.01$.

Prediction 3: Horizon-coherence identity

$\xi^4 = \ell_p^2 L_{eh}^2/\eta$

If cosmological measurements update L_{eh}, the predicted ξ shifts accordingly. This is a consistency test, not a free parameter.

Hard falsifiers:

Observation	Consequence
$w \neq -1$ at late times (stable, >5σ)	CSS attractor fails
Anomalies at inconsistent scales	No unique ξ; framework fails
$K \neq 7$ by numerical simulation	Constraint counting fails
Monotone feedback violated	Attractor theorem fails

The framework is falsifiable. It makes predictions that can be wrong.

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9. Conclusion

This paper has closed the three remaining structural gaps in the Two-Planck derivation of the cosmological constant:

Gap	Resolution
K = 7 verification	Two independent methods recover same 3+1+3 decomposition
Universality class	Simplicial foam (or equivalent) is minimal class from A1–A4
CSS as attractor	Proven under general monotone-feedback assumptions

The complete derivation chain is now:

Relational geometry —(minimal)→ $\ell_e = 2\ell_p$

—(A1–A4)→ Simplicial foam (minimal)

—(counting)→ K = 7

—(RG flow)→ $\xi \approx 88$ μm

—(saturation)→ $\rho_{vac} \approx \hbar c/\xi^4$

—(attractor)→ $\Lambda \approx 1.1 \times 10^{-52}$ m^{-2}, w = −1

Each arrow represents structural necessity conditional on measured H(z), not parameter adjustment. The observed cosmological constant emerges from geometry.

For general readers: We started with a simple idea—space is made of relationships, and the simplest relationship needs two endpoints. We followed this idea through counting exercises, stability proofs, and mathematical consistency checks. At the end, we got a number: the energy density of empty space. And that number matches what astronomers observe, without us ever adjusting anything to make it fit.

This doesn't prove we're right. Nature might work differently. But it does show that the accelerating expansion of the universe—one of the deepest puzzles in physics—might not require exotic new substances or fine-tuned parameters. It might just be geometry.

The theory is testable. Experiments at the 100-micrometer scale can confirm or refute it. The answer lies in laboratories, not just equations.

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Appendix A: Homological Perspective on Constraint Counting

This appendix provides a supporting analogy between constraint counting and homological algebra. It is not claimed as a rigorous independent derivation but as a conceptual bridge for readers familiar with algebraic topology.

A.1 The Constraint Complex (Heuristic)

The K = 7 constraints can be organized into a chain-complex-like structure:

$C_0 \xrightarrow{\partial_1} C_1 \xrightarrow{\partial_2} C_2$

where:

- C_0 represents edge-level constraints (dimension 3)
- C_1 represents loop and embedding constraints (dimension 4)
- C_2 represents gauge redundancies (dimension 0 for a single triangle)

A.2 Analogy with Euler Characteristic

In a genuine chain complex, the Euler characteristic $\chi = \Sigma_i (-1)^i \dim(C_i)$ counts the alternating sum of dimensions.

For the constraint "complex," a similar counting gives:

"χ" = 3 − 4 + 0 + (homology contribution)

The fact that K = 7 = 3 + 4 suggests that the "homology" (truly independent constraints not killed by boundary maps) contributes the full count—i.e., the constraints are independent rather than redundant.

A.3 Limitations

This analogy should not be over-interpreted:

- The boundary maps ∂_1, ∂_2 are not rigorously defined
- The "complex" structure is heuristic
- The Euler characteristic interpretation is suggestive, not proven

The rigorous verification of K = 7 comes from Routes α and β (information theory and obstruction theory), which do not rely on this homological framing.

Appendix B: Detailed Proofs

B.1 Proof of Theorem α (Information-Theoretic K = 7)

Claim: The minimal information required to specify triangle coherence is 7 bits.

Detailed proof:

Consider a triangle $\Delta = (i, j, k)$ in pre-geometric foam. We establish the independence of each constraint:

(1) Edge admissibility (3 bits):

In pre-geometric foam, there is no background manifold guaranteeing that relations exist. Each edge (ij), (jk), (ki) requires:

- Existence of transport element U_{ij}
- Invertibility: U_{ij}^{-1} exists

These are independent for each edge because:

- No transitivity: existence of U_{ij} and U_{jk} does not imply existence of U_{ki}
- No background: there is no manifold structure enforcing relation existence

Information: 3 binary choices = 3 bits.

(2) Loop closure (1 bit):

Given valid edges, the holonomy $H\_\Delta = U_{ij} \cdot U_{jk} \cdot U_{ki}$ may or may not lie in the coherent class $\mathcal{C} \subseteq G$.

This is independent of edge admissibility:

- Valid edges can compose to incoherent holonomy (example: rotations with angles summing to nonzero)
- Coherent holonomy cannot be deduced from edge validity alone

Information: 1 binary choice = 1 bit.

(3) Embedding matching (2 bits):

The triangle Δ is shared by multiple tetrahedra. For the first pair (T_1, T_2):

- Induced geometric data $Q(\Delta|T_1)$ must match $Q(\Delta|T_2)$

For the second pair (T_2, T_3):

- Induced geometric data $Q(\Delta|T_2)$ must match $Q(\Delta|T_3)$

These are independent because:

- No transitivity: matching across T_1-T_2 does not imply matching across T_2-T_3
- Additional tetrahedra T_4, T_5, ... are gauge-redundant (determined by previous matches plus gluing data)

Information: 2 binary choices = 2 bits.

(4) Orientation (1 bit):

The triangle has a discrete $\mathbb{Z}_2$ orientation (chirality). This must be consistent across all embeddings.

This is independent of metric matching:

- Orientation is topological (discrete)
- Metric matching is geometric (continuous)

Information: 1 binary choice = 1 bit.

Total: 3 + 1 + 2 + 1 = 7 bits. ■

B.2 Proof of Theorem β (Obstruction-Theoretic K = 7)

Claim: The descent of relational data to a coherent triangle has exactly 7 independent obstruction classes.

Detailed proof:

(1) Edge obstructions ($\mathcal{O}^0$):

Each edge (ij) requires a transport element $U_{ij} \in G$. The obstruction to existence is:

- $o_{ij} \in H^0(\{i,j\}; G)$ (0-cochain with values in G)

For three edges, we have three independent 0-cochains. These are independent because:

- Different edges correspond to different 0-simplices in the nerve
- No coboundary relation connects them at this level

Count: $|\mathcal{O}^0| = 3$.

(2) Loop obstruction ($\mathcal{O}^1$):

Given valid edges, the composition H_Δ = $U_{ij} \cdot U_{jk} \cdot U_{ki}$ defines a 1-cocycle on the triangle boundary.

The obstruction to coherence is the cohomology class:

- $[H\_\Delta] \in H^1(\partial\Delta; G)$

This is a single obstruction (the triangle has one independent 1-cycle).

Count: $|\mathcal{O}^1| = 1$.

(3) Descent obstructions ($\mathcal{O}^{desc}$):

The triangle Δ is shared by multiple tetrahedra. Consistent embedding requires:

Metric matching: For adjacent tetrahedra T_1, T_2 sharing Δ, the induced metrics must agree:

- $Q(\Delta|T_1) = Q(\Delta|T_2)$

In the nerve of the tetrahedral cover, this is a Čech 1-cocycle condition. For a generic foam, there are 2 independent matching conditions (determined by the minimal cycle structure of the dual graph).

Orientation: The triangle orientation must be consistently induced from all surrounding tetrahedra. This is a $\mathbb{Z}_2$-valued 0-cocycle condition on the nerve.

Count: $|\mathcal{O}^{desc}| = 2 + 1 = 3$.

(4) Independence:

The three types of obstructions live in different cohomological degrees or different coefficient groups:

- $\mathcal{O}^0$: 0-cochains with G coefficients
- $\mathcal{O}^1$: 1-cocycles with G coefficients
- $\mathcal{O}^{desc}$ (metric): Čech 1-cocycles
- $\mathcal{O}^{desc}$ (orientation): $\mathbb{Z}_2$ 0-cocycles

There are no connecting homomorphisms between these in the pre-geometric setting (no background differential structure to induce relations).

Total: $|\mathcal{O}| = 3 + 1 + 3 = 7$. ■

B.3 Proof of Minimal Universality Class Theorem

Claim: Simplicial foam (or equivalent) is the minimal universality class satisfying A1–A4.

Proof: See main text (§5.3). The proof establishes that each axiom constrains the structure, and the intersection of these constraints yields simplicial foam as minimal. ■

B.4 Proof of General Attractor Theorem

Claim: Under monotone-feedback conditions, $x = 1$ is the unique globally asymptotically stable fixed point.

Detailed proof:

(1) Setup:

Let $x(t) \in \mathbb{R}^+$ satisfy dynamics with:

- $dx/dt < 0$ for $x > 1$, with $|dx/dt|$ increasing in $(x - 1)$
- $dx/dt > 0$ for $x < 1$, with $|dx/dt|$ increasing in $(1 - x)$
- dx/dt continuous with $dx/dt|_{x=1} = 0$

(2) Fixed point:

By continuity and sign conditions, $x = 1$ is the unique fixed point where $dx/dt = 0$.

(3) Lyapunov function:

Define $\mathscr{L}(x) = ½(x - 1)^2$.

- $\mathscr{L}(1) = 0$ ✓
- $\mathscr{L}(x) > 0$ for $x \neq 1$ ✓

(4) Lyapunov derivative:

$d\mathscr{L}/dt = (x - 1) \cdot dx/dt$

For $x > 1$: $(x - 1) > 0$ and $dx/dt < 0 \Longrightarrow d\mathscr{L}/dt < 0$

For $x < 1$: $(x - 1) < 0$ and $dx/dt > 0 \Longrightarrow d\mathscr{L}/dt < 0$

For $x = 1$: $d\mathscr{L}/dt = 0$

(5) Global asymptotic stability:

By Lyapunov's theorem, since $d\mathscr{L}/dt < 0$ for all $x \neq 1$ and $d\mathscr{L}/dt = 0$ only at $x = 1$, the fixed point $x = 1$ is globally asymptotically stable.

(6) Uniqueness:

Any other fixed point $x^* \neq 1$ would require $dx/dt|_{x=x^*} = 0$. But by the sign conditions:

- $x^* > 1 \Longrightarrow dx/dt < 0 \neq 0$
- $x^* < 1 \Longrightarrow dx/dt > 0 \neq 0$

Therefore no other fixed points exist. ■

Appendix C: Glossary for General Readers

Attractor: A state toward which a system naturally evolves. Like the bottom of a valley for a rolling ball.

Coherence: The property of being well-formed and self-consistent. A coherent triangle has all its relationships properly aligned.

Constraint: A condition that must be satisfied. Like "the angles of a triangle must sum to 180°."

Cosmological constant (Λ): The energy density of empty space, causing the universe's expansion to accelerate. Often parameterized in the ΛCDM model.

Coarse-graining: The process of "zooming out"—ignoring small-scale details to focus on large-scale behavior.

Descent problem: In mathematics, the challenge of assembling local data into a consistent global structure. Like assembling puzzle pieces into a complete picture.

Effective equation of state (w): The ratio of pressure to energy density for a cosmological component. $w = -1$ corresponds to a cosmological constant.

Entropy: A measure of disorder or the number of possible microscopic arrangements. Higher entropy = more disorder.

Holonomy: The rotation accumulated when parallel-transporting a vector around a closed loop. Measures curvature.

Lyapunov function: A mathematical tool for proving stability. If you can find one, you've proven the system settles to a fixed state.

Monotone feedback: A response that always pushes in the same direction and gets stronger the further you are from equilibrium.

Obstruction: Something that prevents a mathematical construction from working. Like a puzzle piece that doesn't fit.

Order parameter: A quantity that distinguishes different phases of a system. Like magnetization distinguishing magnetic from non-magnetic states.

Percolation: The formation of a connected path through a random medium. Like water finding a path through coffee grounds.

Planck length (ℓ_p): The smallest meaningful length in physics, about 10^{-35} meters.

Relational: Defined in terms of relationships between things, rather than intrinsic properties of things.

Saturation: The state of being at maximum capacity. Like a sponge that can't absorb more water.

Simplicial foam: A model of space built from triangles, tetrahedra, and their higher-dimensional analogues.

Universality class: A category of theories that share the same fundamental mathematical structure.

UV/IR: Ultraviolet/Infrared. UV = small scales (high energy). IR = large scales (low energy).
