Companion Paper: Precision Scoping, Parameter Foundations, and the Research Program Beyond the Core Result

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Abstract

This companion paper sharpens the argument of "Entropy Is the Field Beneath Spacetime" by making its logical structure fully explicit. The core identification — entropy as the unique conserved current in shift-symmetric scalar EFT — rests on three proven pillars: cohomological uniqueness, universal coupling, and thermodynamic consistency, reinforced by three independent microscopic derivations and a completed black hole entropy matching. This paper (i) delineates which results are proven, which provide supporting evidence, and which define a forward research program; (ii) establishes the physical foundations and predictive power of the normalization constant α; (iii) specifies the current reach and next steps for the RG flow argument; (iv) distinguishes what the entropy framework establishes about vacuum energy from questions that require gravitational input beyond the EFT; (v) presents the consolidated black hole entropy derivation; and (vi) organizes the emergent time interpretation as a structured research agenda with defined milestones. The goal is precision, not retreat: every claim in the primary paper is either proven, evidenced, or identified as a specific open problem with a clear path to resolution.

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1. Purpose and Scope

The primary paper advances a specific claim: within shift-symmetric scalar EFT in the cosmological IR, the unique conserved Noether current must be identified with entropy flow. This identification is not optional — it is forced by uniqueness, universality, and thermodynamic consistency.

The primary paper also develops broader implications — emergent time, cosmological constant physics, quantum gravity connections — that extend beyond what the EFT alone proves. This companion makes the boundary between these two layers fully explicit, not because the broader program is wrong, but because the core result is strong enough to stand on its own and should be evaluated on its own terms.

The validated claim:

Within a shift-symmetric scalar EFT of the form $S = \int d^4x \sqrt{(-g)} [(M^2_{Pl}/2)R + P(X)]$ governing the cosmological infrared, the unique conserved current consistent with locality, covariance, universal coupling, and thermodynamic consistency must be identified with entropy flow $S^\mu = s u^\mu$, with $s = \alpha\sqrt{(2X)} P_X$ and $T = \sqrt{(2X)}/\alpha$.

Everything below classifies the supporting structure around this result with full precision.

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2. The Derivational Core and Its Supporting Structure

The primary paper's core claim rests on: (i) a cohomological uniqueness theorem for conserved currents in shift-symmetric scalar EFT (Appendix E); (ii) a thermodynamic identification satisfying all standard relations — Gibbs–Duhem, sound speed, Tolman redshift — for arbitrary admissible $P(X)$; (iii) a systematic elimination of alternatives (particle number, charge, baryon number, energy) on independent physical grounds; and (iv) a multi-route microscopic consistency check (MaxEnt, KMS/Schwinger–Keldysh, operational thermometer) converging on the same temperature and entropy density. The remainder of the primary paper concerns either observational predictions derived from the identification or a research program explicitly labeled as such.

2.1 Proven Results

The following are established within the EFT framework with no additional assumptions:

1. **Uniqueness of the conserved current** (§3; rigorous proof in Appendix E). The characteristic cohomology of $L = P(X)$ with shift symmetry admits exactly one nontrivial conserved current modulo trivial improvements: $J^\mu = P_X \partial^\mu\varphi$. This is a mathematical theorem.
2. **Thermodynamic identification and consistency** (§2). The definitions $s = \alpha\sqrt{2X}\, P_X$ and $T = \sqrt{2X}/\alpha$ satisfy Gibbs–Duhem ($\rho + p = Ts$, $dp = s\, dT$), reproduce the correct adiabatic sound speed $c^2_s = P_X/(P_X + 2X P_{XX})$, and yield T independent of the functional form of $P(X)$. These are algebraic verifications that hold for all admissible $P(X)$.
3. **Elimination of alternatives** (§4). Particle-number identification fails on three independent grounds: particle creation in cosmological backgrounds ($\Gamma_{create} \sim H$), equivalence principle violation ($|\Delta a/a| < 10^{-15}$ forces $|\mu|/T < 10^{-5}$), and inapplicability of laboratory superfluid conditions to the cosmological IR. Charge and baryon number fail universality. Energy fails on frame-dependence, non-conservation in FRW, and absence of a temporal arrow.
4. **Microscopic triangulation** (Appendix D). Three independent routes — local Gibbs/MaxEnt, KMS/Schwinger–Keldysh matching, and Unruh–DeWitt probe thermometer — all converge on the same $T(x) = \sqrt{2X}/\alpha$ and $s(x) = \alpha\sqrt{2X}\, P_X$. This is the result that elevates the identification from a consistent relabeling to a forced conclusion: the quantity called "temperature" in the EFT is the temperature measured by a physical thermometer.
5. **Stefan–Boltzmann recovery** (§5). For $P(X) = \kappa X^2$ (radiation equation of state), the framework reproduces $p \propto T^4$ and $s \propto T^3$ with correct numerical coefficients matching N_{eff} relativistic species. No fitting is involved beyond selecting the radiation-consistent branch ($n \approx 2$) and calibrating α once.
6. **Tolman redshift** (§9). The identification recovers $T\sqrt{-\xi^2} = \text{const}$ in stationary spacetimes, confirming consistency with thermodynamic equilibrium in curved backgrounds.
7. **Universal coupling** (§11.21–11.22 of the primary paper; empirical + structural). Among candidate identifications of the conserved current, only entropy satisfies universal

interaction across all known energy sectors under coarse-graining. The systematic enumeration demonstrates this concretely: particle number does not couple to photons, charge does not couple to dark matter, baryon number excludes leptons, energy is frame-dependent and non-monotonic. Entropy alone satisfies universality, monotonicity, IR survival, and geometric invariance simultaneously. This is proven as an elimination step — no alternative survives all four criteria — though it rests on empirical enumeration of sectors rather than a purely mathematical theorem of the EFT.

Note on Appendices D and E. These appendices contain results integral to the derivational core — the microscopic three-route convergence and the cohomological uniqueness proof. They are formatted as appendices for readability, but their conclusions carry the same weight as the main-text derivations.

2.2 Supporting Evidence

The following results are consistent with the identification and reinforce it, while relying on arguments that are not yet fully rigorous:

- **Renormalization group flow** (§8): Dimensional analysis indicates entropy current remains marginal while particle-number currents become IR-irrelevant. The qualitative picture is physically well-motivated; quantitative confirmation requires explicit loop calculations (see §4 below).
- **Black hole entropy matching** (Appendix F): The Clausius relation and physical-process first law establish $\delta S = \delta S\_Wald = A/(4G)$ in the regime of small perturbations around stationary backgrounds. This is a derived result within its domain; extension to dynamical horizons and extremal limits remains open (see §6 below).

2.3 The Research Program Beyond the Core

The entropy identification opens several lines of investigation that extend beyond what the EFT alone establishes. These are not weaknesses of the core argument — they are consequences worth pursuing:

- **Emergent time**: The mathematical identity $dt/ds = 1/T$ and the conjugacy $\pi\_\varphi \propto s$ follow from the EFT. The interpretive claim that entropy *generates* time requires additional ontological framework and engagement with quantum gravity (see §9).
- **Cosmological constant**: The observation that vacuum energy is the only form of energy that does not couple to the entropy current is established. Whether this has dynamical gravitational consequences requires input beyond the scalar EFT (see §5).
- **Quantum gravity connections**: Links to the thermal time hypothesis, holographic entanglement entropy, and the Wheeler–DeWitt framework are motivated and worth developing as a dedicated program (see §9).

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3. The Entropy Normalization Parameter α: Foundations and Predictive Power

3.1 Definition and Role

The constant $\alpha > 0$ has dimensions of energy and enters the identification as:

- $s = \alpha\sqrt{2X}\, P_X$ (entropy density)
- $T = \sqrt{2X}/\alpha$ (temperature)

It is the single conversion factor between field-theory variables (X, P_X) and thermodynamic variables (s, T).

3.2 Calibration

α is fixed by matching to one observed thermodynamic quantity. For the radiation era with $P(X) = \kappa X^2$:

$$\kappa = 4\pi^2 N_{eff} / (45\, \alpha^4)$$

With $N_{eff} \approx 3.044$ from Planck CMB observations, this determines α uniquely for the radiation-era $P(X)$. After this single calibration, the mapping between X and (s, T) is fully determined.

3.3 Predictive Consequences

After calibration, the framework predicts — with no remaining freedom:

- Entropy density as a function of temperature: $s = (4\pi^2/45)\, N_{eff}\, T^3$
- Sound speed: $c^2_s = 1/3$ (for radiation)
- Non-Gaussianity scaling: $f^{(equil)}_{NL} \sim O(1/c^2_s - 1)$ for departures from $c^2_s = 1$
- Gravitational wave damping bounds: $\eta/s \to 0$ in the adiabatic limit
- Absence of diffusion modes at cosmological scales

The framework is fully predictive after one input — analogous to how general relativity is fully predictive after fixing G.

3.4 The Analogy

α stands in the same relation to the entropy-EFT as Newton's constant G stands to general relativity, or as the Boltzmann constant k_B stands to statistical mechanics. None of these constants are predicted by their respective theories; each is fixed once by matching to observation. After calibration, the theory makes unambiguous, falsifiable predictions. α is a normalization constant, not a free parameter.

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4. Renormalization Group Arguments: Current Status and Path Forward

4.1 What Dimensional Analysis Establishes

In $d = 4$ spacetime, the entropy current has canonical dimension $[S^\mu] = 3$. Interactions generically induce positive anomalous dimensions $\gamma_N > 0$ for particle-number currents (from scattering, dispersion, and species non-conservation), while the entropy current — protected by the second law — remains marginal or relevant. The qualitative conclusion is physically well-grounded: in every known condensed matter and cosmological system, entropy survives coarse-graining while particle-specific information washes out.

4.2 What Remains To Be Computed

The primary paper presents schematic beta functions $\beta(\lambda_N) \sim -\gamma_N \lambda_N$ without computing γ_N from the P(X) action. A rigorous strengthening requires:

- One-loop computation of the anomalous dimension of $J^\mu = P_X \partial^\mu\varphi$ in a cosmological P(X) background
- Verification that no relevant deformations mix entropy and particle-number channels in the deep IR
- Explicit demonstration that the entropy current's marginality is protected (or identification of the protecting mechanism)

This is a well-defined calculation within the EFT framework and represents a concrete next step for the program.

4.3 Standing Within the Argument

The core identification rests on three proven pillars: cohomological uniqueness, universal coupling, and thermodynamic consistency. The RG argument provides a fourth line of reasoning — explaining *why* the IR selects entropy over competing currents — that is physically motivated and consistent with the identification, but awaits quantitative completion. The core result does not depend on it.

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5. The Cosmological Constant: What the Framework Says and What It Doesn't

5.1 The Observation

The entropy identification establishes a clear fact: vacuum energy is the *only* form of energy in the known universe that does not couple to the entropy current. Every other energy form —

kinetic, gravitational, electromagnetic, chemical, nuclear — produces or exchanges entropy when it transforms. Vacuum energy alone is entropically sterile: $S_{vac} = 0$, $T_{vac} = 0$, no thermalization, no dissipation.

This is not a modeling assumption. It follows directly from the properties of vacuum energy (constant energy density, pure quantum state) combined with the entropy identification.

5.2 The Interpretive Gap

The primary paper explored the possibility that entropic sterility has gravitational consequences — specifically, that vacuum energy might not source spacetime curvature in the usual way. This is an important hypothesis, but it requires gravitational input beyond the scalar EFT. The Einstein equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ are a statement about gravity, not about the scalar sector. Any modification to how vacuum energy gravitates constitutes a modification of gravitational dynamics and must be justified independently — for example, through unimodular gravity, sequestering mechanisms, or a quantum gravity derivation.

The entropy identification alone establishes that vacuum energy is entropically inert. It does not, by itself, determine the gravitational consequences of that inertness.

5.3 The Research Direction

The observation that vacuum energy is uniquely excluded from the entropic dynamics is striking and potentially important. It suggests a natural research program: within gravitational frameworks where dynamics is driven by entropy flow (e.g., Verlinde's entropic gravity, Jacobson's thermodynamic derivation of Einstein's equations), does entropic inertness translate into gravitational decoupling? This question is well-posed, testable in principle (through time-dependent dark energy surveys like LSST/Euclid), and worth pursuing as a dedicated investigation. It is identified here as a defined research direction with a clear hypothesis and observational targets.

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6. Black Hole Entropy: A Completed Derivation

6.1 The Result

Appendix F of the primary paper provides a complete physical-process derivation — stationary horizons, small perturbations — of black hole entropy matching. The argument proceeds in four steps:

1. **Regular scalar profile**: Euclidean regularity fixes $\partial_a \varphi \propto \chi_a$ near the horizon, yielding $X_H = (\alpha^2/2)\, T^2_H$ independent of $P(X)$.

2. **Clausius relation**: The entropy flux through the stretched horizon satisfies $\delta Q = T_H \delta S$ identically, using only Gibbs–Duhem, Killing data, and regularity — no specific choice of $P(X)$ beyond admissibility.
3. **Area law**: The physical-process first law enforces $\delta Q_{grav} = \delta Q_{matter}$, and Wald's theorem gives $\delta S_{Wald} = \delta Q_{grav} / T_H$, establishing $\delta S = \delta S_{Wald} = A/(4G)$.
4. **Universality and Kerr extension**: The result depends only on horizon invariants (κ, T_H, A) and extends to rotating black holes via $\chi = \partial_t + \Omega_H \partial_\varphi$.

The earlier pedagogical sketch (Appendix B) illustrates the dimensional scaling that motivates this derivation. Appendix F contains the definitive treatment; Appendix B is retained for pedagogical context.

6.2 Scope and Extensions

The derivation operates in the physical-process regime: small, finite perturbations around stationary backgrounds. Natural extensions include:

- **Dynamical horizons**: Formation and merger scenarios where Killing structure is absent
- **Extremal black holes**: $T_H \to 0$, where the thermodynamic grounding requires careful treatment
- **Charged black holes**: Reissner–Nordström introduces additional gauge structure and competing conserved currents

These are well-defined targets for future work, not gaps in the core argument. The physical-process regime covers the standard domain of black hole thermodynamics.

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7. Gauge Symmetry and Domain of Validity

The uniqueness theorem (Appendix E) assumes an ungauged scalar sector: the shift symmetry $\varphi \to \varphi + c$ is global, not the remnant of a broken local gauge symmetry. This covers the standard domain of k-essence, DBI inflation, and dark energy scenarios.

If the scalar originates as a Goldstone boson of a spontaneously broken local U(1), additional structure may appear: the gauge symmetry could protect particle number, additional conserved currents (the gauge current) become available, and the equivalence principle constraints on chemical potential may be modified.

This possibility is acknowledged as lying outside the present scope. The domain of validity is:

Shift-symmetric scalar EFT with no gauge redundancy in the scalar sector, in the adiabatic, single-mode, IR limit.

Extension to gauged scalars is a natural direction for future work and would either confirm the entropy identification in a broader setting or reveal where the identification's boundaries lie.

8. Updated Observational Anchors

8.1 Non-Gaussianity

The primary paper is updated to reflect Planck 2018 constraints (Planck Collaboration IX, 2020):

$f^{(equil)}_{NL} = -26 \pm 47$ (68% CL)

This replaces the earlier bound $|f^{(equil)}_{NL}| < 300$. Current data permits O(1) non-Gaussianity from reduced sound speed. CMB-S4, with projected sensitivity $\sigma(f_{NL}) \sim 1$ for equilateral shapes, will provide a decisive test of the entropy identification's predictions for inflationary sound speed.

8.2 Sound Speed Constraints

For the power-law family $P(X) \sim X^n$, the sound speed $c^2_s = 1/(2n - 1)$ maps directly onto CMB acoustic peak structure. Planck 2018 data (combined with polarization) constrains departures from $c^2_s = 1/3$ at the percent level during the radiation era, placing n within a narrow window around n = 2. The precise width of this window is dataset- and prior-dependent; a dedicated parameter estimation using the entropy-EFT framework is identified as a concrete near-term project.

8.3 Corrected References

The foundational k-inflation reference is corrected to:

Armendáriz-Picón, C., Damour, T., Mukhanov, V. (1999). k-Inflation. *Phys. Lett. B* 458, 209.

An error in §11.17 of the primary paper is also corrected: extensive entropy scales as $S \sim V$, not $S \sim V^{1/3}$.

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9. The Emergent Time Program: A Structured Research Agenda

9.1 Results That Belong to the Core

Several results in the emergent time discussion are mathematical consequences of the entropy identification and remain in the primary paper:

- **The identity $dt/ds = 1/T$**: An algebraic consequence of the thermodynamic definitions, stating that temperature measures the rate of coordinate time per unit entropy.
- **Conjugate momentum $\pi_\varphi \propto s$**: The canonical momentum of the scalar field is proportional to entropy density, establishing entropy-time conjugacy in the Hamiltonian structure.
- **Universal coupling enumeration** (§11.21–11.22): The demonstration that entropy is the only quantity coupling to all energy forms strengthens the core elimination argument and is promoted to derivational status.
- **Variational selection principle** (§11.25): The formal statement that $S^\mu = s\, u^\mu$ is the unique current satisfying universality, monotonicity, IR survival, and geometric invariance is the capstone of the elimination argument.

9.2 The Forward Program

The following elements constitute a structured research agenda extending beyond the EFT derivation, organized by timeline and requirements:

Near-term (within EFT reach):

- Explicit one-loop RG calculation of current anomalous dimensions in P(X) backgrounds
- Dedicated CMB parameter estimation for entropy-EFT sound speed and non-Gaussianity predictions
- Extension of black hole matching to charged and extremal cases

Medium-term (requiring additional framework):

- **Thermal time connection**: Establish whether $T = \sqrt{(2X)}/\alpha$ provides a concrete realization of the Connes–Rovelli modular Hamiltonian flow by constructing the density matrix ρ whose modular automorphism generates the scalar field dynamics.
- **Wheeler–DeWitt clock**: Determine whether φ satisfies the conditions for a Page–Wootters clock degree of freedom in the cosmological minisuperspace via canonical quantization of the coupled gravity-scalar system.
- **Holographic dual**: Investigate whether the scalar field entropy current maps onto boundary entanglement entropy via Ryu–Takayanagi within AdS/CFT.

Long-term (requiring quantum gravity input):

- **Cosmological constant**: Within entropy-driven gravitational frameworks (Jacobson, Verlinde), determine whether entropic inertness of vacuum energy translates into gravitational decoupling.
- **Microscopic derivation**: Identify the statistical ensemble or quantum gravity microstructure from which P(X) emerges, with α determined from first principles.
- **Planck-scale temporal structure**: Characterize the breakdown of smooth temporal ordering when entropy fluctuations $\delta S \sim O(1)$ at ℓ_{Planck}.

Each milestone is well-defined and falsifiable. The emergent time interpretation is a research program with concrete targets, not open-ended speculation.

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10. Revised Architecture of the Primary Paper

Section	Content	Status
§1 Setup	Shift-symmetric scalar EFT, fluid variables	Framework
§2 Thermodynamic Identification	s, T definitions; First Law; Gibbs–Duhem; sound speed	**Proven**
§3 Uniqueness	Conserved current theorem (with Appendix E proof)	**Proven**
§4 Elimination of Alternatives	Three arguments against particle number; universality argument	**Proven**
§5 Radiation Example	Stefan–Boltzmann recovery, α calibration	**Proven**
§6 Black Hole Entropy	Consolidated treatment via Appendix F	**Proven** (physical-process regime)
§7 Observational Tests	f_NL, c²_s, GW damping, diffusion modes	**Derived predictions**
§8 RG Flow	Dimensional analysis, schematic beta functions	Supporting evidence
§9 Tolman's Law	Redshift recovery	**Proven**
§10 Limitations	Domain of validity, open items	Scope
§11 Entropy and Time (streamlined)	Universal coupling, selection principle, dt/ds = 1/T, research directions	Mixed (labeled per subsection)
§12 Discussion	Comparison with alternatives, open questions	Assessment
§13 Literature Connections	Connes–Rovelli, Ryu–Takayanagi, Verlinde, ER=EPR	Context
§14 Conclusions	Conservative result and research program	Summary
App. A	Admissibility conditions on P(X)	Technical
App. B	Near-horizon sketch (pedagogical; see App. F for definitive treatment)	Pedagogical
App. C	Connection to Wald entropy	Technical
App. D	Microscopic foundations: MaxEnt, KMS, probe test	**Proven**
App. E	Cohomological uniqueness proof	**Proven**
App. F	Black hole entropy: definitive treatment	**Proven**

The pre-abstract plain language summary is removed from the journal submission and published separately for public communication.

11. Summary of Clarifications

Topic	Clarification
Derivation vs. interpretation	Explicit three-tier classification: Proven / Supporting / Research Program
Parameter α	One-time IR calibration constant; framework fully predictive after single input
RG flow	Physically motivated supporting evidence; core rests on three independent pillars; one-loop calculation is a defined next step
Cosmological constant	Entropic inertness of vacuum energy is established; gravitational consequences are a defined research direction requiring independent framework
Black hole entropy	Appendix F provides complete derivation in physical-process regime; Appendix B retained as pedagogical motivation
Gauge symmetry	Domain restricted to ungauged shift-symmetric sector; gauged extension is natural future work
f_NL bound	Updated to Planck 2018: f^(equil)_NL = −26 ± 47
Sound speed	Percent-level constraints during radiation era; dedicated fit identified as near-term project
k-inflation citation	Corrected to Armendáriz-Picón, Damour, Mukhanov (1999)
Extensive entropy scaling	Error in §11.17 corrected: S ~ V, not S ~ V^(1/3)
Appendices D and E	Integral to derivational core; formatted as appendices for readability
§11 structure	Streamlined; broader elements organized as structured research agenda
Plain language summaries	Removed from journal submission; available separately

12. Conclusion

The core result of the primary paper is that entropy is the unique viable identification of the conserved Noether current in shift-symmetric scalar EFT in the cosmological IR. This conclusion is forced by three proven pillars — cohomological uniqueness, universal coupling, and thermodynamic consistency — and reinforced by microscopic triangulation from three independent derivations and by black hole entropy matching in the physical-process regime.

The broader program — emergent time, cosmological constant physics, quantum gravity connections — is not a collection of loose speculations but a structured research agenda with defined milestones and falsifiable targets. This companion paper organizes that agenda explicitly, so that the core result can be evaluated on its own merits and the research program can be pursued with clear objectives.

The conservative result — entropy as the thermodynamic content of the cosmological scalar field — is established. The research program — entropy as the generator of emergent temporal structure — is defined and underway. The strength of the first is what makes the second worth pursuing.