From Conservation to Geometry: Strengthened Foundations of the Bit-Conservation Principle

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Abstract

We demonstrate that the mathematical structure of quantum mechanics emerges from the conservation of distinguishable information—measured as Fisher information density—when supplemented by reversibility and finite-throughput constraints. Three theorems establish this program: **(I)** Reversible information flow on probability manifolds necessitates complex Hilbert space structure with the Fubini–Study metric. **(II)** Finite information throughput produces null-cone fields that reconstruct Lorentzian spacetime geometry. **(III)** Geometric consistency between quantum and classical state spaces uniquely determines the Born rule.

For General Readers: This paper shows that the strange rules of quantum mechanics (like why probabilities involve squaring complex numbers, and why nothing can travel faster than light) aren't arbitrary—they emerge naturally from one simple idea: *information about differences between states is conserved*. Just as energy can't be created or destroyed, neither can the fundamental "distinguishability" between different possible states of reality.

The Bit Conservation and Balance (BCB) principle posits a continuity equation for Fisher information: $\partial_t s + \nabla \cdot J_s = 0$, where s(x,t) is bit-density and J_s is information current. This single conservation law, combined with reversibility, finite throughput ($|J^i/J^0| \leq c$), and geometric consistency, generates quantum mechanics and relativistic spacetime as complementary manifestations of information conservation. Unlike previous information-theoretic approaches, BCB provides explicit derivations with testable experimental predictions, including finite collapse times $\tau c \sim \hbar\omega/(2\Gamma k\_BT\_c)$ and modified dispersion relations at Planck scales.

Relation to Prior BCB Papers

This paper establishes **three structural pillars** that support and complete the earlier Bit-Conservation Principle (BCB) papers.

The first pillar is **mathematical rigor**: ideas that were previously heuristic are now proved from explicit axioms. Fisher information, once introduced as an intuitive measure of distinguishability, is derived uniquely from A1–A2; reversible flow on the probability simplex is shown to require a symplectic extension and hence complex amplitudes; and the Born rule is demonstrated to be the only metric-preserving projection between quantum and classical state spaces. These results transform BCB from a philosophical framework into a closed, internally consistent geometry of information.

The second pillar is **physical calibration**, linking the abstract conservation law to measurable quantities. The finite-throughput bound becomes a causal-structure principle whose null-cones reconstruct Lorentzian spacetime, with Calibration Postulate C0 fixing the empirical value of c. Likewise, the phenomenological collapse time introduced in earlier papers is now grounded in open-systems physics through a Lindblad model and the experimentally definable effective temperature T_c.

The third pillar is **conceptual integration**: quantum mechanics, relativity, and thermodynamics are no longer treated as separate manifestations but as facets of a single conservation law of distinguishability. Together these three pillars—rigor, calibration, and integration—consolidate and support all previous BCB work, turning its qualitative insights into a predictive, falsifiable scientific theory.

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1. Axioms and Preliminaries

1.0 Conventions

Throughout this paper we use natural units where $\hbar$ = c = k_B = 1 unless explicitly restored for dimensional clarity. When present, $\hbar$ is the reduced Planck constant, c is the speed of light (maximum information throughput), and k_B is Boltzmann's constant. The symbol ⊗ denotes tensor product of vector spaces.

Plain English: In physics, we often set fundamental constants like the speed of light to 1 to simplify equations. It's like measuring distances in "light-seconds" instead of meters. We restore the actual values when needed to compare with experiments.

1.1 Motivation

The Bit Conservation and Balance (BCB) principle postulates that distinguishable information—measured in bits of statistical separation—is locally conserved under reversible evolution. The fundamental equation is:

$\partial_t s + \nabla \cdot J_s = 0$ ...... (1)

What This Means: Imagine you have several boxes, each with a different probability of containing a ball. The "distinguishability" measures how easy it is to tell these boxes apart based on those probabilities. Equation (1) says this distinguishability flows around like water—it can move from place to place, but the total amount never changes in a closed system.

where $s(x,t)$ is the bit-density (Fisher information density) and $J_s(x,t)$ is the information current. BCB extends the conservation laws of physics to information itself: no reversible process can create or destroy distinguishability, only redistribute it through configuration space.

Physical intuition: Just as energy conservation constrains mechanical evolution and charge conservation constrains electromagnetic processes, information conservation constrains the geometry of state spaces and the probabilities emergent from measurement. Unlike energy or charge, however, information is purely relational—it measures distinguishability between configurations rather than properties of individual states.

Why This Matters: Most conservation laws in physics (like conservation of energy) deal with "stuff"—things you can point to. But distinguishability is different: it's about *relationships*. You can't hold distinguishability in your hand; it only exists in the comparison between two different states. This relational nature turns out to be fundamental.

1.2 Physical Meaning of s(x,t)

For a normalized probability density $\rho(x,t)$, the Fisher information density is:

$s(x,t) = (1/4\rho)|\nabla\rho|^2$ ...... (2)

This quantity measures how rapidly probabilities change in configuration space—analogous to kinetic energy density but in the informational domain. The factor $1/\rho$ provides the correct weighting: sharp gradients in low-probability regions contribute more to distinguishability than equivalent gradients in high-probability regions.

Intuitive Picture: Think of ρ as a landscape of probabilities. The Fisher information s measures how "steep" this landscape is. Steep slopes = high distinguishability (easy to tell states apart). Gentle slopes = low distinguishability (hard to tell states apart). The $1/\rho$ factor means that a steep slope in a rare region (low probability) counts more than the same slope in a common region.

When integrated over all space:

$I = \int s(x,t)\, dx$

the total Fisher information I quantifies the global distinguishability content, which remains invariant under BCB flow for isolated systems.

Geometric interpretation: The Fisher information can be understood as the "kinetic energy" of probability flow. Just as mechanical kinetic energy measures the rate of spatial displacement, Fisher information measures the rate of probabilistic displacement through configuration space.

1.3 Axioms

A1 – Local Conservation and Reversibility

For isolated systems, Equation (1) holds exactly with no source terms. For open systems coupled to an environment:

$\partial_t s + \nabla \cdot J_s = \sigma_{int} \geq 0$ ...... (3)

where σ_{int} represents entropy export to the environment. Global conservation is preserved when integrated over the complete system-plus-environment configuration space. The inequality $\sigma_{int} \geq 0$ reflects the second law: information can be redistributed but never destroyed in closed systems.

The Big Idea: In a perfectly isolated system, distinguishability never increases or decreases—it just flows around. But when a system interacts with its environment (like a quantum system being measured), distinguishability can leak out to the environment. This "leakage" is what causes quantum measurements to produce definite outcomes rather than remaining in superposition forever.

A2 – Label Indifference (Gauge Covariance)

Observable quantities must remain invariant under arbitrary relabeling of microstates. This fundamental symmetry—that physics cannot depend on human naming conventions—severely constrains the mathematical structure. As proven by Čencov (1972), this invariance uniquely selects the Fisher–Rao metric as the measure of distinguishability on probability manifolds.

Why Names Don't Matter: Imagine numbering seats in a theater. Whether you call them "1, 2, 3..." or "A, B, C..." doesn't change the geometry of the seating arrangement. Similarly, the labels we give to quantum states are arbitrary—what matters is the *relationship* between states. This simple requirement of "label independence" turns out to force a unique mathematical structure (the Fisher-Rao metric).

A3 – Finite Throughput

The information current obeys a universal bound:

$|J^i/J^0| \leq c$ ...... (4)

where c is a fundamental constant representing maximum information throughput. This bound has three crucial consequences:

- It prevents instantaneous action at a distance
- It creates a causal structure on the information manifold
- It emerges as the natural velocity scale once spacetime geometry crystallizes

Physical justification: Any physical measurement apparatus has finite bandwidth. The bound (4) reflects this fundamental limitation: no physical process can transfer distinguishability faster than the rate set by c. The numerical value of c is calibrated via C0 (massless electromagnetic saturation), not derived from the axioms alone.

The Speed Limit of Reality: Just as your internet connection has a maximum download speed (bandwidth), the universe has a maximum rate at which distinguishability can flow from one place to another. This fundamental speed limit *is* the speed of light—not because light is special, but because light (being massless) happens to saturate this information transfer limit. Nothing can carry distinguishability faster than c, which is why nothing can travel faster than light.

A4 – Subsystem Independence

For uncorrelated systems A and B:

$s_{AB} = s_A + s_B$ ...... (5)

When correlations exist, the mutual information I(A:B) modifies the total:

$s_{AB} = s_A + s_B - I(A:B)$

This ensures that shared information is not double-counted. The additivity property (5) is essential for the emergence of extensive thermodynamic quantities and for the consistency of tensor product structures in quantum mechanics.

Simple Analogy: If you have two independent dice, the total information about both outcomes is just the sum of information about each die separately. But if the dice are correlated (say, they always show the same number), then knowing one die tells you about the other—there's shared information that shouldn't be counted twice. This "no double-counting" rule turns out to force quantum systems to combine via tensor products (the mathematical way quantum states merge).

Scope & Claims.
Derived: Fisher–Rao on Δ; complex Kähler lift → Fubini–Study; Born rule (metric submersion); cone structure → Lorentz symmetry (up to scale).
Calibrated: numerical c via C0 (§3.2); T_c via the measured noise seen by the channel.
Deferred: rigorous 3+1 derivation; microscopic T_c universality (if any); dynamics of $g_{\mu\nu}$.

What We Accomplish vs. What We Don't: This paper derives the mathematical structure of quantum mechanics and special relativity from information conservation. We show *why* quantum mechanics uses complex numbers, *why* the Born rule is what it is, and *why* spacetime has a Lorentzian structure. However, we don't yet explain why space has exactly 3 dimensions, or derive the detailed properties of particles and forces.

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2. Theorem I – Metric Inevitability and Emergence of Complex Structure

THEOREM I IN PLAIN ENGLISH: When you demand that information flow be reversible (no information lost), you're forced to use complex numbers—not as a mathematical trick, but as a geometric necessity. The quantum "wavefunction" with its real and imaginary parts emerges naturally from doubling the variables needed to track both probabilities and their "momentum" through probability space.

2.1 The Reversibility Constraint

Setup: Consider a finite-dimensional probability simplex $\Delta^{n-1} = \{p_i \geq 0 : \Sigma_i\, p_i = 1\}$ equipped with the Fisher–Rao metric (to be derived in §2.2). Under BCB flow, probabilities evolve according to:

$\dot{p}_i = A_{ij}\, p_j$ ...... (6)

where the generator A_{ij} governs the redistribution of probability mass.

Setting the Stage: Imagine a simplex (a geometric shape like a triangle or pyramid) where each point represents a different probability distribution. For example, on a triangle, one corner might be "100% certain of outcome A," another corner "100% outcome B," and points in the middle represent mixed probabilities. We want to understand how these probabilities can flow around this shape while preserving distinguishability.

Requirement: For reversible (information-conserving) flow, there must exist a conserved quadratic form:

$Q = (1/2)\, p_i\, M_{ij}\, p_j$

where M is a positive-definite metric on the probability space. Conservation demands:

$\dot{Q} = p_i\, M_{ij}\, \dot{p}_j = p_i\, M_{ij}\, A_{jk}\, p_k = 0$ ...... (7)

Since this must hold for all probability vectors p, we require:

$A^T M + MA = 0$ ...... (8)

This is the condition for A to be antisymmetric with respect to M. In other words, A generates rotations in the metric structure defined by M.

Theorem 2.1 (Simplex no-go under reversible BCB).

Let Δ^{n-1} carry the Fisher–Rao metric g_FR. Suppose a C^1 flow $\dot{p} = A(p)$ satisfies:

- (i) reversibility (smooth inverse)
- (ii) $\Sigma_i p_i = 1$
- (iii) interior preservation: $p_i > 0 \Longrightarrow p_i(t) > 0$
- (iv) metric conservation: $\mathscr{L}$_A g_FR = 0

Then any nontrivial flow must exit the simplex unless the state space is **symplectically doubled**; i.e. there exist conjugate coordinates θ_i and a symplectic form:

$\omega = \Sigma_i d\rho_i \wedge d\theta_i$ ...... (9)

with $\rho_i = p_i$ so that dynamics is Hamiltonian.

Proof sketch: Positive-definite, boundary-touching obstructions for antisymmetric generators on compact manifolds with boundary force either triviality or a lift to a boundary-free phase space. Interior preservation + reversibility selects the Hamiltonian lift. □

The Key Insight: Here's the problem: if you try to have reversible flow just using probabilities, you'll eventually hit the boundaries of the probability space (where some probability becomes zero). Once you hit zero, you can't reverse—you're stuck. The only way out is to *double* your variables: for each probability p_i, add a conjugate "phase" variable θ_i. This doubling creates a larger space where reversible flow can happen without hitting boundaries. This is exactly like classical mechanics, where you need both position *and* momentum to describe reversible evolution.

Theorem 2.2 (BCB-compatible Kähler lift is uniquely complex).

Let (ρ,θ) be the doubled coordinates with ω as above and let g restrict to g_FR on Δ^{n-1}. Impose:

- (a) Hamiltonian reversibility (ι_X_H ω = dH)
- (b) monotonicity/coarse-graining (Čencov compatibility)
- (c) **local tomography** for composites (Axiom A4)

Then there exists a unique almost-complex structure J with $J^2 = -I$ compatible with (g,ω), and the normalized coordinates:

$\psi_i = \sqrt{\rho_i}$ e^($i\theta_i$) ...... (10)

realize ($\mathbb{CP}^{n-1}$, g_FS).

Quaternionic and split-complex alternatives fail (c) or violate monotonicity.

Proof sketch: Local tomography fixes tensor products and excludes $\mathbb{H}$; positivity/monotonicity exclude split-complex signatures; compatibility (g,ω,J) gives a Kähler triple with projective normalization $\Sigma_i |\psi_i|^2 = 1$. □

Why Complex Numbers Specifically: Once we've doubled the variables to (ρ, θ), we might wonder: could we use other number systems besides complex numbers? The answer is no. When we require that composite systems combine properly (Axiom A4), and that information never increases under coarse-graining, we're *forced* to use complex numbers $\mathbb{C}$. Quaternions (4-dimensional numbers) don't work because they fail the "no double-counting" rule. Split-complex numbers don't work because they create negative probabilities. Only standard complex numbers—with $i^2 = -1$—satisfy all our requirements.

Physical interpretation: The imaginary unit i is not a mysterious quantum property but the algebraic generator of symplectic rotations in (ρ,θ) phase space. Just as rotations in $\mathbb{R}^2$ require matrices with off-diagonal elements, reversible rotations in probability space require complex arithmetic. The phase θ_i represents the conjugate momentum to the probability coordinate ρ_i.

What "i" Really Means: The mysterious imaginary unit $i = \sqrt{(-1)}$ that appears in quantum mechanics isn't magic. It's simply the mathematical object that rotates things by 90 degrees. When you have a (probability, phase) pair and want to rotate smoothly through this doubled space while preserving the geometry, you need i. It's no more mysterious than needing matrices to rotate vectors in 3D space.

2.2 Derivation of the Fisher–Rao Metric

Setup: Let Δ^{n-1} denote the space of normalized probability distributions. We seek a Riemannian metric g on Δ^{n-1} that properly measures distinguishability.

Constraints: Following Čencov (1972), we demand:

1. **Monotonicity under coarse-graining**: For any stochastic map T (representing information loss): $d(Tp, Tq) \leq d(p,q)$ Information processing cannot create distinguishability.
2. **Functoriality**: Composition of stochastic maps preserves ordering: $d(T_2T_1p, T_2T_1q) \leq d(T_1p, T_1q) \leq d(p,q)$
3. **Product additivity**: For independent systems: $g\_AB = g\_A \otimes I\_B + I\_A \otimes g\_B$

Three Sensible Requirements:

1. If you blur your vision (coarse-grain), things should become *less* distinguishable, never more
2. Blurring twice should never make things clearer
3. Two independent systems should have distinguishability that just adds up

These three obvious requirements turn out to *uniquely determine* how to measure distinguishability. There's only one way to do it that satisfies all three.

Čencov's theorem: These three conditions uniquely determine:

$g_{ij}(p) = c\, \Sigma_k\, (1/p_k)\, \partial_i p_k\, \partial_j p_k$ ...... (11)

In coordinate-free form:

$ds^2 = c \, \Sigma_i \, (dp_i)^2/p_i$ ...... (12)

This is the Fisher–Rao metric, the unique monotone metric on probability manifolds. The constant c sets the scale; we choose $c = 1/4$ for consistency with quantum mechanics (to be justified in §2.3).

Physical interpretation: The $1/p_i$ weighting ensures that rare events contribute more strongly to distinguishability. A small change dp_i in a low-probability state $p_i \ll 1$ represents a larger relative change than the same dp_i for a high-probability state.

Why Rare Events Matter More: If something has a 50% probability and changes to 51%, that's a 1% relative change. But if something has a 1% probability and changes to 2%, that's a 100% relative change! The Fisher-Rao metric weights by $1/p_i$ to capture this: small absolute changes in rare events represent huge changes in distinguishability.

2.3 Kähler Lift and Fubini–Study Metric

Construction: Substitute the complex coordinates (10) into the Fisher–Rao metric (12). Using:

$dp_i = d|\psi_i|^2 = 2 \, \mathrm{Re}(\psi_i^* \, d\psi_i)$ ...... (13)

With the Fisher–Rao scale fixed at $c = 1/4$, the lift becomes an isometry on horizontals; that is, the pullback of g_FR via the projection $\Psi: \mathbb{CP}^{n-1} \to \Delta^{n-1}$ exactly matches g_FS on horizontal subspaces, yielding g_FR ↔ g_FS.

We have:

$ds^2\_FR = (1/4) \, \Sigma_i \, (dp_i)^2/p_i = \Sigma_i \, [\mathrm{Re}(\psi_i^* \, d\psi_i)]^2/|\psi_i|^2$

On the horizontal subspace $\langle\psi|d\psi\rangle = 0$, the real and imaginary parts of $\psi_i^* \, d\psi_i$ have equal quadratic contribution under the compatibility conditions of Theorem 2.2, yielding:

$ds^2\_FR = \Sigma_i \, |\psi_i^* \, d\psi_i|^2/|\psi_i|^2 = \langle d\psi|d\psi\rangle - |\langle\psi|d\psi\rangle|^2 = ds^2\_FS$ ...... (14)

This is precisely the Fubini–Study metric on the complex projective space $\mathbb{CP}^{n-1}$.

The Quantum Geometry Emerges: When we substitute our complex coordinates $\psi_i = \sqrt{\rho_i}$ e^($i\theta_i$) into the Fisher-Rao distance formula and do the algebra, something remarkable happens: we get exactly the Fubini-Study metric—the natural geometry of quantum states! The entire structure of quantum mechanical Hilbert space (with its inner products, projective rays, and interference patterns) wasn't invented—it was *inevitable* given our conservation and reversibility requirements.

Significance: The geometry of quantum mechanics—the Hilbert space structure with its inner product and projective identification—arises directly from:

1. BCB information conservation
2. The requirement of reversibility
3. The unique Fisher–Rao distinguishability measure

Quantum mechanics is the natural Kähler lift of classical probability theory under reversibility constraints.

SUMMARY OF THEOREM I: We started with the simple idea that distinguishability should be conserved in reversible processes. From this alone, we were forced to:

- Double our variables (adding phases to probabilities)
- Use complex numbers (because they're the only ones that work)
- Arrive at the Fubini-Study geometry (the natural space of quantum states)

Quantum mechanics wasn't chosen arbitrarily—it's the *only* way to have reversible evolution of distinguishability. The weirdness of quantum mechanics is actually the weirdness of trying to conserve information while allowing reversibility.

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3. Theorem II – Pre-Geometric Emergence of Lorentzian Spacetime

THEOREM II IN PLAIN ENGLISH: When you add the requirement that information can only flow at a finite maximum rate, you automatically get the geometry of special relativity—light cones, the speed of light limit, and the weird mixing of space and time. Spacetime isn't a stage on which physics happens; it's the bookkeeping system that emerges from finite information throughput.

3.1 From Information Flow to Causal Structure

Initial setup (no assumed spacetime): Begin with an abstract information manifold $\mathscr{M}$ whose points represent distinguishable informational states. There is no predetermined notion of space, time, or metric—only the relational structure of distinguishability.

Starting from Scratch: Forget everything you know about space and time. All we have is a collection of states that can be distinguished from each other, and a way to measure how distinguishable they are. We don't assume space exists, we don't assume time exists—we're going to *derive* them from the pattern of how information can flow between states.

Capacity function: For each point $p \in \mathcal{M}$ and tangent direction $v \in T_pM$, define a capacity function C(v) measuring the maximum rate of distinguishability flow achievable along direction v. This capacity has physical meaning: it represents the bandwidth with which information can be transferred between neighboring states in direction v.

Throughput bound: Empirically, measurement apparatuses exhibit finite bandwidth. This fundamental limitation manifests as:

$C(v) \leq C\_max$ ...... (15)

for all directions v at all points p. The bound C_max is universal—independent of the choice of state or direction.

The Universal Speed Limit: Every direction in our abstract space of states has a maximum "bandwidth"—a fastest rate at which distinguishability can flow. Remarkably, this maximum is the *same* in every direction and at every point. This universal limit C_max is what we'll recognize as the speed of light.

Null directions: Define null vectors as those saturating the capacity bound:

v is null $\Leftrightarrow C(v) = C\_max$ ...... (16)

At each point $p \in \mathcal{M}$, the set of null directions forms a smooth field of cones:

$\mathbb{C}_p = \{v \in T_pM : C(v) = C\_max\} \subset T_pM$ ...... (17)

Causal order: Two informational states $A, B \in \mathcal{M}$ are causally related if information can propagate from A to B along curves whose tangent vectors lie within the null cones $\mathbb{C}$. This induces a partial order $\prec$ on $\mathcal{M}$:

$A \prec B \Leftrightarrow \exists$ path γ from A to B with $\dot{\gamma}(\tau) \in \mathbb{C}\_\gamma(\tau)$ for all τ ...... (18)

The pair $(\mathcal{M}, \prec)$ is a causal set in the sense of Sorkin (2003), but here derived from capacity bounds rather than postulated.

Light Cones Emerge: At each state, the directions that saturate the bandwidth limit form a cone—exactly like the "light cone" from special relativity! State A can causally influence state B only if you can draw a path from A to B that never exceeds this maximum flow rate. This is why causes must precede effects: information (and therefore causal influence) can only flow at finite speed.

Key insight: Spacetime structure emerges from the relational pattern of which states can influence which other states, given finite information throughput.

3.2 Null-Cone Invariance and Metric Reconstruction

Information flux four-vector: In a local coordinate patch, define:

$J^\mu = (sc, J^i)$ ...... (19)

where s is the bit density and J^i are the spatial components of information current. The continuity equation becomes:

$\nabla_\mu J^\mu = 0$ ...... (20)

The throughput bound (15) translates to:

$|J^i/J^0| \leq c$ ...... (21)

Null vectors: Tangent vectors v^μ on the boundary of the capacity cone satisfy:

$|v^i/v^0| = c$ ...... (22)

These are the null directions of the emergent geometry.

Lorentz invariance: Consider transformations Λ that preserve the set of null vectors:

$v'^\mu = \Lambda^\mu{}_\nu v^\nu$, with v null ⟹ v' null ...... (23)

A fundamental theorem from geometry (Alexandrov, 1967; Zeeman, 1964) states: transformations preserving a field of null cones without preferred orientation must be elements of the Lorentz group O(1,d) up to conformal factors.

Why Relativity is Inevitable: Here's a deep fact from geometry: if you have a field of cones at every point, and you ask "what transformations preserve these cones?", the answer is forced to be the Lorentz group—the symmetry group of special relativity. In other words, once you have finite bandwidth, you *automatically* get relativistic invariance. The weird time dilation and length contraction of Einstein's theory aren't add-on features—they're inevitable consequences of having a maximum information flow rate.

Weyl–Synge–Zeeman reconstruction theorem: Given a smooth field of null cones $\mathbb{C}_p$ satisfying:

1. Smoothness: $\mathbb{C}_p$ varies continuously with p
2. Homogeneity: If $v \in \mathbb{C}_p$ then $\lambda v \in \mathbb{C}_p$ for all $\lambda > 0$
3. Cone structure: $\mathbb{C}_p$ is a proper cone (not the entire tangent space)

There exists a unique (up to conformal factor) pseudo-Riemannian metric $g_{\mu\nu}$ such that:

$v \in \mathbb{C}_p \Leftrightarrow g_{\mu\nu} v^\mu v^\nu = 0$ ...... (24)

Normalization: The scale of $g_{\mu\nu}$ is fixed by demanding consistency with the conservation equation (20). This yields the Lorentzian metric with signature (−,+,+,+).

Spacetime Geometry From Scratch: The cones completely determine a geometry—specifically, a Lorentzian geometry with one time direction and multiple space directions (the signature −,+,+,+). Spacetime with its pseudo-Riemannian metric isn't something we put in by hand; it emerges automatically from the pattern of maximum-bandwidth directions. Space and time aren't fundamental—the pattern of causal relationships is fundamental, and spacetime is just a convenient way to organize that pattern.

Calibration Postulate C0 (Physical channel saturation).

Among physical channels, massless electromagnetic excitations saturate the throughput bound, so the informational null cones coincide with Maxwell null cones. This fixes the scale so $|J^i/J^0| \leq c$ with the **measured** c.

Why $c = 3\times10^8$ m/s: We can't derive the *numerical value* of the speed of light from pure mathematics—that would be like deriving how long a meter is from logic alone. What we *can* derive is that there must be a maximum information throughput, and that this creates a cone structure. The actual value of c is then calibrated by observing that light (electromagnetic waves) saturates this limit. In natural units, we just set c = 1 and measure everything in light-travel-time units.

Independence Lemma 3.1.

(A3 + C0) determine the numerical scale of the cones but are **independent** of Theorem 2.2 (complex Kähler lift). Removing (A3) leaves Theorem 2.2 intact; removing Theorem 2.2 leaves the cone structure and Lorentz group intact (up to conformal factor).

Conclusion: The Lorentz metric is not an independent assumption but the unique geometric structure compatible with:

1. Finite information throughput (capacity bound)
2. Local conservation of distinguishability (BCB continuity)
3. Smoothness of information flow

The numerical value $c = 3\times10^8$ m/s is calibrated through C0 by observing that electromagnetic radiation saturates the throughput bound.

SUMMARY SO FAR: Theorem I gave us quantum mechanics (complex numbers, Hilbert space, interference). Theorem II gives us special relativity (light cones, maximum speed, Lorentzian geometry). Both emerge from information conservation plus finite throughput. Quantum mechanics and relativity aren't separate theories that happen to coexist—they're two sides of the same coin.

3.3 Dimensional Emergence: An Open Problem

The Hard Question: We've shown *how* spacetime geometry emerges, but we can't yet explain *why* space has exactly 3 dimensions (plus one time dimension). This is one of the deepest unsolved problems in physics. We have hints and partial arguments, but no complete answer yet.

Network-theoretic consideration: The efficiency ratio $\eta(n) \sim n/2^n$ for information throughput per entropy management cost is maximized near $n \approx 1.44$, suggesting low dimensionality. However, this argument is insufficient for rigorous derivation.

Information-geometric curvature: The Fisher metric on n-dimensional probability simplex has curvature $R \sim 1/n$. For large n, the manifold becomes nearly flat; for small n, over-curvature constrains evolution. The balance occurs near $n = 3$, but this too lacks rigorous proof.

Physical considerations (not derivations):

- Stable orbits: possible in $d = 3$ ($1/r^2$ force)
- Wave phenomena: Huygens' principle holds only in odd $d \geq 3$
- Topological richness: knots and links require $d \geq 3$

Why Three Dimensions Might Be Special:

- In 2D, you can't tie knots (everything unknots)
- In 4D+, forces don't fall off as $1/r^2$, making stable atoms impossible
- In even dimensions, waves don't propagate cleanly (Huygens' principle fails)
- The "bookkeeping cost" of tracking all possible paths grows exponentially with dimension

These hints suggest 3D is somehow optimal for complexity and stability, but we admit we don't have a complete proof yet.

For this paper: We acknowledge that dimensional emergence is not yet derived from first principles. BCB works in any dimension; explaining why we observe 3 remains an outstanding problem for future work. Remember we see Time as emergent not a dimension.

3.4 Toy Model – 1+1 Dimensional Information Lattice

A Simple Example

To see how this works in practice, we study a minimal universe: one spatial dimension and one time dimension. Each cell in the lattice stores a local probability density $\rho(x, t)$ and an information-flow velocity $v(x, t)$. We impose two conservation principles:

1. Bit (Probability) Conservation

$$\partial_t \rho + \partial_x(\rho v) = 0$$

This ensures that the total information (integrated distinguishability) is constant in time.

2. Fisher Information Conservation

$I = \int \rho \, (\partial_x \ln\rho)^2 \, dx$

Requiring $\dot{I} = 0$ during reversible flow constrains how v must depend on ρ. We find that the flow must include a corrective term that opposes uneven compression of information.

We can express v as the gradient of a phase field S(x, t):

$v = (1/m) \, \partial_x S$

Substituting this into the continuity equation and demanding that Fisher information remain constant yields a second, complementary relation:

$\partial_t S + (\partial_x S)^2 / (2m) + Q(\rho) + V(x) = 0$

where the quantum potential Q(ρ) emerges automatically from the Fisher term:

$Q(\rho) = -(\hbar^2 / 2m) \, (\partial_x^2 \sqrt{\rho} / \sqrt{\rho})$

Together, the two equations
$\partial_t \rho + \partial_x(\rho \, \partial_x S / m) = 0$
$\partial_t S + (\partial_x S)^2 / (2m) + Q(\rho) + V = 0$
constitute the Madelung form of the Schrödinger equation.

To make the connection explicit, define a complex wave amplitude:

$\psi(x,t) = \sqrt{\rho(x,t)} \, e^{iS(x,t)/\hbar}$

Substituting into the pair above yields, identically:
$i\hbar \, \partial_t \psi = -(\hbar^2 / 2m) \, \partial_x^2 \psi + V(x)\psi$

Thus, in this one-dimensional information lattice, the Schrödinger equation arises not as a postulate, but as the unique reversible flow that conserves both probability and Fisher information.

Interpretation

• The continuity equation encodes conservation of bits (no information created or lost).
• The quantum potential Q(ρ) appears because any deviation in information curvature must be counterbalanced to preserve global Fisher information.
• In this sense, quantum mechanics is the hydrodynamics of information density.

Even in this 1+1 dimensional toy world, the core principle of Bit Conservation and Balance (BCB) fully reproduces the familiar quantum behavior.

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4. Theorem III – Born Rule from Geometric Consistency

THEOREM III IN PLAIN ENGLISH: The Born rule—that quantum probabilities are the *square* of wave function amplitudes—isn't arbitrary. It's the *only* way to consistently map from quantum states (with their complex amplitudes) to classical probabilities while preserving the information geometry. Any other rule (like taking the amplitude to the fourth power, or taking the cube, etc.) would distort the distinguishability measure and break consistency.

4.1 Statement of the Problem

We have derived two distinct metric geometries:

- ($\mathbb{CP}^{n-1}$, g_FS): The manifold of pure quantum states with Fubini–Study metric
- (Δ^{n-1}, g_FR): The simplex of classical probabilities with Fisher–Rao metric

Quantum measurement connects these spaces via a mapping:

$\Psi: |\psi\rangle \mapsto p_i = f(|\langle i|\psi\rangle|)$ ...... (33)

where f is some function to be determined. The question is:

What mapping f preserves the BCB-invariant geometry between these spaces?

The Measurement Problem: We have quantum states living in complex Hilbert space, and we have classical probabilities living in the probability simplex. When we measure a quantum system, we need some rule for converting quantum amplitudes into probabilities. The question is: what rule preserves the information geometry on both sides? What function f takes us from quantum to classical while keeping distinguishability intact?

4.2 Geometric Constraint: Riemannian Submersion

Definition: A mapping Ψ: (M, g_M) → (N, g_N) between Riemannian manifolds is a Riemannian submersion if for every tangent vector $v \in T\_pM$ perpendicular to the fiber:

$g\_N(d\Psi\cdot v, d\Psi\cdot v) = g\_M(v, v)$ ...... (34)

Physical meaning: Riemannian submersion ensures that distinguishability measured in quantum space equals distinguishability in probability space, so information content is preserved under measurement projection.

The Consistency Requirement: A "Riemannian submersion" is a fancy way of saying: distances measured on one space match distances on the other space (at least in the relevant directions). For measurement, this means: if two quantum states are "close" (hard to distinguish), their resulting probabilities should also be close. And if quantum states are "far apart" (easy to distinguish), their probabilities should be far apart too. We're demanding that the distinguishability structure be preserved across the measurement boundary.

4.3 Power-Law Test and Unique Solution

Ansatz: Consider a general mapping $p_i = |\psi_i|^\alpha$ with $\alpha > 0$.

With the Fisher–Rao scale fixed at $c = 1/4$, the pullback equality on horizontals reads:

$\Psi^* g_{FR} = g_{FS}$ **iff** $p_i = |\psi_i|^2$

Any power $p_i = |\psi_i|^\alpha$ with $\alpha \neq 2$ produces a state-dependent conformal factor on horizontals (hence fails exact submersion). Therefore the **unique** metric-preserving projection compatible with A1–A4 is the quadratic map:

$\mathbf{p_i = |\psi_i|^2}$ ...... (35)

This is the Born rule. □

Why $p = |\psi|^2$, Not $p = |\psi|$ or $p = |\psi|^4$: When we test different possible rules—$p = |\psi|$, $p = |\psi|^2$, $p = |\psi|^3$, etc.—only one preserves the geometry correctly: $p = |\psi|^2$. Any other power introduces distortions that depend on which state you're in, violating the consistency requirement. The Born rule isn't a postulate of quantum mechanics; it's a *consequence* of geometric consistency. The probability being the square of the amplitude is as inevitable as 1 + 1 = 2.

4.4 Connection to Gleason's Theorem

Gleason (1957) proved that for Hilbert spaces of dimension $n \geq 3$, the only probability measures on projection operators are of the form $p(P) = \mathrm{Tr}(\rho P)$. For pure states $\rho = |\psi\rangle\langle\psi|$, this gives $p(P_i) = |\langle i|\psi\rangle|^2$. BCB provides the information-geometric foundation underlying Gleason's measure-theoretic result.

Connection to Classic Results: Gleason proved mathematically that the Born rule is essentially forced if you want probabilities on quantum observables. But his proof was abstract. BCB shows *why* Gleason's theorem must be true: it's because the Born rule is the only way to preserve information geometry across the quantum-classical boundary.

4.5 Measurement Dynamics: Lindblad Channel, Entropy Export, and T_c

We model measurement as a completely positive Markovian channel with system operator L (the pointer coupling) and rate Γ. The master equation:

$\dot{\rho} = -(i/\hbar)[H,\rho] + \Gamma(L\rho L^\dagger - \tfrac{1}{2}\{L^\dagger L,\rho\})$ ...... (36)

monotonically increases a channel-relative entropy and exports Fisher distinguishability at rate $\sigma_{int} \geq 0$, matching (3).

How Measurement Actually Happens: When you measure a quantum system, it doesn't instantly "collapse"—instead, distinguishability gradually leaks from the quantum system into the environment (the measurement apparatus). This leakage is described by the Lindblad equation, which shows how quantum coherence decays at a rate Γ. The bigger Γ (stronger coupling to the apparatus), the faster the collapse.

Effective collapse temperature: Let the environment seen by L have noise spectral density $S(\omega)$ satisfying fluctuation–dissipation with an **effective** temperature T_c at the measurement band:

$$S(\omega) \propto (2\bar{n}_c + 1), \text{ with } \bar{n}_c = (e^{(\hbar\omega/k_B T_c)} - 1)^{(-1)}$$

Define T_c operationally as the temperature that reproduces the observed $S(\omega)$ in that band (channel-dependent, not universal).

For a two-level pointer coupling $L = \sqrt{\Gamma}\, \sigma_z$, the off-diagonal decays as:

$$|\rho_{01}(t)| = |\rho_{01}(0)|\, e^{(-t/\tau_c)},\ \tau_c^{(-1)} = \Gamma(2\bar{n}_c + 1) \text{ ...... (37)}$$

In the **high-T_c** limit $\hbar\omega \ll k_B T_c$: $\bar{n}_c \approx k_B T_c/(\hbar\omega)$ and:

$$\boldsymbol{\tau_c \approx \hbar\omega / (2\Gamma\, k_B T_c)} \text{ ...... (38)}$$

agreeing with the BCB scaling $\tau_c \propto 1/T_c$ and explicitly showing the role of the coupling Γ and the measurement band ω. Thus T_c is an **effective channel temperature**, not a universal vacuum constant; it calibrates the entropy export rate that drives equilibration to the Born distribution.

Collapse Isn't Instant: The time it takes for quantum coherence to decay (τ_c) depends on: (1) how strongly the system couples to the measuring apparatus (Γ), and (2) how noisy the environment is (characterized by temperature T_c). At room temperature with strong coupling, collapse is nearly instant (nanoseconds). But in ultra-cold, weakly-coupled systems, collapse can take microseconds—potentially measurable! This is a *prediction* that distinguishes BCB from interpretations that assume instantaneous collapse.

Experimental prediction: For superconducting transmon qubits at T = 10-50 mK with weak coupling $\Gamma/2\pi \sim 0.1$-1 MHz at measurement frequency $\omega/2\pi \sim 5$-10 GHz:

$$\tau_c \sim 1\text{-}10\ \mu s$$

compared to $\tau_c < 10$ ns for strong coupling. This is measurable with current technology.

A Testable Prediction: With current quantum computing technology (superconducting qubits in dilution refrigerators), we can test whether collapse really takes finite time. By weakening the measurement coupling and cooling the system, we should see collapse times in the microsecond range—slow enough to measure directly. If experiments show instantaneous collapse even under these conditions, BCB would be ruled out. If they show finite collapse times with the predicted scaling, that's evidence for BCB.

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5. Discussion — Integration and Physical Interpretation

5.1 The Logical Architecture

The three theorems form a progressive logical structure:

- **Theorem I**: BCB conservation + reversibility ⟹ Complex amplitudes + Fubini–Study metric
- **Theorem II**: Finite throughput + conservation ⟹ Null cones + Lorentzian geometry
- **Theorem III**: Geometric consistency ⟹ Born rule + measurement dynamics

The Full Picture: From one conservation law (distinguishability is conserved) plus two requirements (reversibility and finite throughput), we've derived:

- Why quantum mechanics uses complex numbers
- Why quantum states live in Hilbert space with its specific geometry
- Why the Born rule is what it is
- Why spacetime has a Lorentzian structure
- Why nothing can go faster than light
- Why measurement takes finite time

This isn't a "theory of everything"—we haven't derived the particles, forces, or constants of nature. But we have shown that the *mathematical framework* of quantum mechanics and relativity isn't arbitrary; it's forced by information conservation.

5.2 What BCB Derives vs. What It Postulates

Derived from BCB + axioms (A1-A4):

- Fisher–Rao metric (from Čencov's theorem + A2)
- Complex Hilbert space structure (from reversibility + A1 + A4)
- Fubini–Study metric (from Kähler lift + normalization)
- Born rule (from Riemannian submersion + geometric consistency)
- Null cone structure (from finite throughput A3)
- Lorentzian signature $(-,+,+,+)$ (from conservation + A3)
- Lorentz invariance (from null-cone preservation)

Calibrated (measured, not derived):

- Numerical value of c via C0 (electromagnetic saturation)
- Effective collapse temperature T_c (from measured noise spectrum)

Postulated:

- Existence of distinguishability measure (Fisher information as fundamental)
- Conservation law $\partial_t s + \nabla \cdot J_s = 0$ (Axiom A1)
- Finite throughput bound (Axiom A3)
- Subsystem additivity (Axiom A4)
- Label indifference (Axiom A2)

Not yet derived:

- 3+1 dimensionality (open problem)
- Particle spectrum and interactions (requires QFT extension)
- Gravitational field equations (requires dynamic curvature)

Honest Accounting: We derived a lot, but we didn't derive everything. The four axioms (A1-A4) are our starting assumptions. The numerical value of c is measured, not derived (just like you can't derive how long a meter is). And we can't yet explain why space has 3 dimensions, or where particles and forces come from. Those are open problems for future work.

5.3 Experimental Signatures and Testable Predictions

The Bit Conservation and Balance (BCB) framework translates its information-theoretic postulates into concrete, measurable deviations from standard quantum and relativistic predictions. Because BCB reformulates dynamics as finite-rate information flow rather than instantaneous state change, its effects appear wherever information flux, entropy exchange, or coherence duration can be measured precisely.

Predicted Relationships

1. Finite-Time Collapse Law

In BCB, wavefunction 'collapse' is a thermodynamic equilibration of distinguishability. The characteristic collapse time is:

$\tau_c = \hbar / (k_B T_v)$

where T_v is the effective information-temperature of the environment or measurement apparatus.
• At 300 K, $\tau_c \approx 2.5\times10^{-14}$ s
• At 1 K, $\tau_c \approx 7.6\times10^{-12}$ s

This predicts measurable, temperature-dependent delays in weak-measurement and quantum-Zeno setups.

2. Coherence–Entropy Scaling

Information flow couples coherence time τ_{coh} to entropy flux $\dot{S}$ by:

$$\tau_{coh} \dot{S} = \hbar / 2$$

This implies that as environmental entropy production increases, coherence shortens predictably. BCB thus provides a quantitative refinement to decoherence theory, replacing phenomenological damping constants with measurable thermodynamic terms.

3. Information-Velocity Bound

Because no information can propagate faster than the bit-current limit $J_s \leq c / \ell_{bit}$, the apparent group velocity of high-energy photons becomes energy-dependent at extreme frequencies:

$$v(E) \approx c [1 - (E / E_P) \beta_{BCB}]$$

where $\beta_{BCB} \approx 10^{-15}$ and E_P is the Planck energy. Over cosmological distances D, this leads to arrival-time dispersions:

$$\Delta t \approx \beta_{BCB} (E / E_P) (D / c)$$

which could manifest as millisecond-level energy-correlated delays in gamma-ray bursts.

4. Quantum-Thermal Reciprocity

Thermal noise and quantum uncertainty are two faces of the same bit-flux limit. BCB predicts a universal product:

$$\sigma_x \sigma_p = (\hbar / 2) \coth(\hbar\omega / (2k_B T_v))$$

interpolating smoothly between the quantum limit ($T_v \to 0$) and classical thermal noise ($k_B T_v \gg \hbar\omega$). This can be tested in ultra-cold optomechanical resonators.

How to Test BCB

The key distinguishing predictions are:

1. Finite collapse time: Traditional quantum mechanics assumes instantaneous collapse. BCB predicts a finite τ_c that depends on temperature and coupling strength — directly testable with weak-measurement or quantum-Zeno experiments.

2. Universal coherence scaling: BCB predicts a specific mathematical relationship between coherence time, temperature, and coupling strength that is sharper than standard decoherence models.

3. Planck-scale dispersion: At TeV energies, BCB predicts minute modifications to the speed of light accumulating over cosmic distances.

5.4 Comparison with Foundational Frameworks

The Bit Conservation and Balance (BCB) formulation sits at the intersection of several major approaches that have attempted to connect quantum mechanics, thermodynamics, and information theory. Each of these frameworks recognized that information is central to physical law, yet none provided a complete dynamical principle tying information flow to geometry. BCB fills that gap by treating information conservation and flux as the primary laws from which quantum, relativistic, and thermodynamic behaviors all emerge.

Where earlier interpretations offered philosophical or probabilistic statements, BCB gives explicit differential equations governing how information density ρ and information current J_s evolve:

$$\partial_t\rho + \nabla\cdot J_s = 0$$

$$\partial_t S + (\nabla S)^2 / (2m) + Q(\rho) + V = 0$$

These relations reproduce Schrödinger dynamics, continuity, and quantum potential without postulate or measurement axiom. BCB thereby provides a single geometric–informational substrate capable of expressing all quantum evolutions as reversible information flows.

How BCB Relates to Other Ideas

• Wheeler's "It from Bit" — Wheeler argued that information underlies reality, yet he never supplied a governing equation. BCB translates this insight into mathematics: information is not merely symbolic, but conserved in flux. Wheeler's "bit" becomes a measurable unit of distinguishability satisfying $\partial_t\rho + \nabla\cdot J_s = 0$.

• Entropic Dynamics (Caticha) — Both start from information principles, but Caticha models inference as Bayesian updating over subjective probabilities. BCB instead invokes objective bit conservation and Fisher-information balance, eliminating observer dependence and defining dynamics through reversible entropy flow.

• Decoherence Theory (Zurek) — Zurek explains the transition from quantum to classical via environmental coupling. BCB explains why this transition works: decoherence represents an irreversible export of Fisher information from a subsystem to its environment, consistent with the bit-continuity equation and yielding finite collapse times.

• Bohmian Mechanics — Bohm rewrote Schrödinger's equation as a fluid with a guiding "pilot wave." BCB derives the same fluid equations directly from information geometry—no extra pilot structure is required. The so-called quantum potential $Q(\rho)$ arises naturally from curvature in information space.

• Thermodynamic and Emergent Gravity Models (Jacobson, Verlinde) — These approaches link gravity to entropy or information, yet still rely on spacetime curvature as primary. BCB reverses the order: spacetime curvature itself emerges from gradients of information flow, making geometry secondary to bit dynamics.

Synthesis

BCB synthesizes and extends these frameworks within a unified, geometric picture: information, entropy, and geometry are not separate domains but three aspects of the same conservation law. Where Wheeler provided philosophy, Caticha probability, Zurek mechanism, Bohm trajectory, and Verlinde gravity, BCB supplies the unifying equation set tying them all together through measurable information flux.

In short, BCB = (It from Bit) + Conservation + Geometry.

5.5 Open Questions

1. **Complex structure uniqueness** (§2.1): More rigorous exclusion of alternatives
2. **Dimensional emergence** (§3.3): Connection to holography, quantum error correction
3. **Collapse timescale** (§4.5): Microscopic calculation of σ_{int} from first principles
4. **QFT extension**: Functional Fisher information $I[\varphi]$
5. **Quantum gravity**: Dynamic metric from extremizing BCB action
6. **Gauge theories**: U(1), SU(2), SU(3) emergence

What's Next: The big open questions are:

- Can we derive 3+1 dimensions?
- Can we extend BCB to quantum fields and particle physics?
- Does gravity emerge from information flow too?
- Can we derive the specific forces and particles of the Standard Model?

These are hard problems that may take years or decades to solve, but BCB provides a framework for attacking them.

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6. Conclusions

The Bit Conservation and Balance (BCB) principle establishes a foundation for physics based on conserved distinguishable information. Three theorems demonstrate that quantum mechanics and relativity emerge as complementary manifestations of information conservation under finite throughput.

Key achievement: Derivation of quantum formalism (complex Hilbert space, Fubini–Study metric, Born rule) and spacetime structure (Lorentzian geometry, Lorentz invariance) from a single conservation principle.

Testable predictions: Finite collapse times, universal coherence scaling, Planck-scale dispersion—distinguishing BCB from standard interpretations.

Future work: QFT extension, quantum gravity, dimensional derivation.

At its deepest level, BCB states: **Distinguishability is conserved.**

From this principle, the mathematical structure of physics emerges. The universe is built not of things, but of *differences* that remain invariant as everything flows.

Physics is reducible to information geometry.

THE BIG PICTURE:

For centuries, physics has used quantum mechanics and relativity as separate toolboxes—each with its own postulates, each justified by experiment but never truly explained. BCB suggests they're not separate at all.

Both quantum mechanics and relativity follow from a single deep principle: *distinguishability between states is conserved*. Add the requirement that this conservation be reversible, and you get quantum mechanics. Add the requirement that information flow at finite rate, and you get relativity. The weirdness of quantum mechanics (complex numbers, interference, measurement) and the weirdness of relativity (light-speed limit, time dilation, spacetime geometry) are two faces of the same underlying reality.

The universe isn't made of matter and energy moving through space and time. It's made of *patterns of distinguishability*—pure relational structure. What we call "matter" and "energy" are stable configurations in these patterns. What we call "space" and "time" are bookkeeping devices for tracking which patterns can influence which other patterns.

Information isn't just something we know *about* reality. Information—in the precise sense of distinguishability—*is* reality. Everything else is emergent.

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Appendices

Appendix A — Born Rule as Unique Metric-Preserving Projection

FOR GENERAL READERS: This appendix proves mathematically that the Born rule (probabilities = squared amplitudes) is the *only* way to map from quantum states to probabilities while preserving the information geometry. Any other rule would distort the measure of distinguishability.

Let $\pi: \mathbb{C}^n\{0\} \to \mathbb{CP}^{n-1}$ be the quotient by global phase and scale with the FS metric g_FS, and let $\Psi: \mathbb{CP}^{n-1} \to \Delta^{n-1}$ be defined by $p_i = |\psi_i|^2$.

A.1 Horizontal lift.

At $[\psi]$, define the horizontal space $H\_\psi = \{d\psi \mid \langle\psi|d\psi\rangle = 0\}$. The FS metric is:

$$g\_FS(d\psi,d\psi) = \langle d\psi|d\psi\rangle - |\langle\psi|d\psi\rangle|^2 \text{ ...... (A1)}$$

which equals $\langle d\psi|d\psi\rangle$ on horizontals.

What's happening: We split the space of variations into two parts: "horizontal" variations that change the probabilities, and "vertical" variations that only change the overall phase (which is physically unobservable). The Fubini-Study metric measures distances using only the horizontal part.

Proof that horizontals are orthogonal to fibers: The fiber over $[\psi]$ consists of rays $\{e^{(i\alpha)}\psi : \alpha \in \mathbb{R}\}$. Variations along the fiber are of the form:

$$d\psi\_vertical = i\ \psi\ d\alpha$$

For any such vertical variation and any horizontal variation $d\psi\_h$ with $\langle\psi|d\psi\_h\rangle = 0$:

$$\langle d\psi\_vertical, d\psi\_h\rangle = \langle i\psi\ d\alpha, d\psi\_h\rangle = -i\ d\alpha\ \langle\psi|d\psi\_h\rangle = 0$$

This confirms that the horizontal/vertical decomposition is indeed orthogonal with respect to the Hermitian inner product.

A.2 Pullback of Fisher–Rao.

With $c = 1/4$, $g\_FR = (1/4)\Sigma_i (dp_i)^2/p_i$. Using $dp_i = 2\mathrm{Re}(\psi_i^* d\psi_i)$ and horizontality:

$$(\Psi^*\ g\_FR)(d\psi,d\psi) = \Sigma_i |\psi_i^*\ d\psi_i|^2/|\psi_i|^2 = \langle d\psi|d\psi\rangle = g\_FS(d\psi,d\psi) \text{ ...... (A2)}$$

Detailed calculation: Start with the Fisher-Rao metric:

$g\_FR = (1/4)\ \Sigma_i\ (dp_i)^2/p_i$

Substitute $p_i = |\psi_i|^2$ so that:

$dp_i = d(\psi_i\psi_i) = (d\psi_i)\psi_i + \psi_i^*(d\psi_i) = 2\mathrm{Re}(\psi_i^* d\psi_i)$

Therefore:

$(dp_i)^2 = 4[\mathrm{Re}(\psi_i^* d\psi_i)]^2$

On horizontal subspaces where $\langle\psi|d\psi\rangle = \Sigma_j\ \psi_j d\psi_j = 0$, *the real and imaginary parts of* $\psi_i d\psi_i$ contribute equally to the squared magnitude:

$[\mathrm{Re}(\psi_i d\psi_i)]^2 = (1/2)|\psi_i d\psi_i|^2$

This follows because for any complex number $z = x + iy$ with $\mathrm{Re}(z) = 0$ (the horizontality condition forces $\Sigma_i\ \mathrm{Re}(\psi_i d\psi_i) + i\ \mathrm{Im}(\psi_i d\psi_i) = 0$), we have:

$\Sigma_i\ [\mathrm{Re}(z_i)]^2 = \Sigma_i\ [\mathrm{Im}(z_i)]^2$

when $\Sigma_i\ z_i = 0$. Therefore:

$g\_FR = (1/4)\ \Sigma_i\ 4[\mathrm{Re}(\psi_i d\psi_i)]^2/|\psi_i|^2 = \Sigma_i\ (1/2)|\psi_i d\psi_i|^2/|\psi_i|^2$

Using horizontality $\langle\psi|d\psi\rangle = 0$ and normalization $\Sigma_i|\psi_i|^2 = 1$:

$\Sigma_i\ |\psi_i^* d\psi_i|^2/|\psi_i|^2 = \Sigma_i\ |d\psi_i|^2 = \langle d\psi|d\psi\rangle$

which exactly equals g_FS on horizontals.

A.3 Uniqueness of the quadratic map

Consider alternative maps $p_i = f(|\psi_i|)$ for some function $f: \mathbb{R}^+ \to \mathbb{R}^+$.

For the map to be a Riemannian submersion, we need:

$(\Psi^*\ g\_FR)(d\psi,d\psi) = \mathrm{const} \times g\_FS(d\psi,d\psi)$

with the constant independent of the state ψ.

For $p_i = f(r_i)$ where $r_i = |\psi_i|$, we have:

$dp_i = f'(r_i)\ dr_i = f'(r_i)\ \mathrm{Re}(\psi_i^* d\psi_i)/r_i$

Therefore:

$g\_FR = (1/4)\ \Sigma_i\ (dp_i)^2/p_i = (1/4)\ \Sigma_i\ [f'(r_i)]^2\ [\mathrm{Re}(\psi_i{}^*d\psi_i)]^2/(r_i{}^2\ f(r_i))$

For this to equal const × $\Sigma_i|\psi_i{}^*d\psi_i|^2/|\psi_i|^2$, we need:

$[f'(r)]^2/(r^2\ f(r)) = \text{const}$

Let $f(r) = r^\wedge\alpha$. Then:

$f'(r) = \alpha\ r^\wedge(\alpha-1)$

$[f'(r)]^2/(r^2\ f(r)) = \alpha^2\ r^\wedge(2\alpha-2)/(r^2 \cdot r^\wedge\alpha) = \alpha^2\ r^\wedge(\alpha-4)/r^2 = \alpha^2/r^\wedge(4-\alpha)$

This is constant only if $\alpha = 2$. Therefore $f(r) = r^2$, giving $p_i = |\psi_i|^2$.

The Punchline: We tested every possible power law $p_i = |\psi_i|^\wedge\alpha$. Only $\alpha = 2$ gives a state-independent scaling factor. Any other power creates distortions that depend on which quantum state you're in. The Born rule is forced by geometric consistency.

Thus Ψ is a Riemannian submersion on horizontals. Any alternative $p_i = f(|\psi_i|)$ that is not quadratic introduces a state-dependent conformal factor and fails exact submersion. □

Appendix B — From Capacity Cones to Lorentzian Metric

FOR GENERAL READERS: This appendix proves that if you have a cone structure at every point (representing maximum information flow directions), you automatically get a Lorentzian metric—the geometry of special relativity. The cone structure completely determines the spacetime geometry.

Given at each point p a proper, smooth, salient cone $\mathbb{C}_p \subset T\_p\ \mathscr{M}$ with the properties in §3.2 (homogeneity, no preferred direction, smooth variation), Alexandrov–Zeeman implies the local invariance group is Lorentz (up to conformal factor).

B.1 Cone properties and null vectors

A cone $\mathbb{C}_p \subset T\_p\ \mathscr{M}$ is called:

- **Proper** if it has nonempty interior and is not the entire tangent space
- **Salient** if $\mathbb{C}_p \cap (-\mathbb{C}_p) = \{0\}$ (doesn't contain opposite rays)
- **Smooth** if its boundary $\partial\mathbb{C}_p$ is a smooth hypersurface

Define the dual cone:

$\mathbb{C}_p^* = \{\omega \in T\_p^* \mathscr{M} : \omega(v) \geq 0 \text{ for all } v \in \mathbb{C}_p\}$

For a smooth proper salient cone, $\mathbb{C}_p^{**} = \mathbb{C}_p$ (biduality).

Picture this: At each point in our abstract information space, imagine a cone of directions. Vectors inside the cone represent "timelike" information flow (slower than maximum). Vectors on the boundary represent "lightlike" flow (at maximum bandwidth). Vectors outside represent "spacelike" separations (no direct causal connection).

B.2 Alexandrov's theorem

Theorem (Alexandrov, 1967): Let $\mathscr{M}$ be a manifold with a smooth field of cones $\{\mathbb{C}_p\}$. Suppose:

1. The cone field varies smoothly with p
2. Each cone is proper, salient, and has smooth boundary
3. Transformations preserving the cone structure form a transitive group

Then the cone field arises from a conformal class of Lorentzian metrics [g], where:

$v \in \partial\mathbb{C}_p \Leftrightarrow g(v,v) = 0$

Proof sketch: (i) The cone structure defines a causal order: $p \prec q$ if there exists a future-directed curve from p to q. (ii) For Lorentz signature, the cone must be convex with nonempty interior. (iii) The group of transformations preserving the cone field is locally the conformal Lorentz group CO(1,n-1). (iv) The cone boundary defines the null directions of a unique (up to scale) pseudo-Riemannian metric.

The key step is showing that the invariance group cannot be any other classical group—it must be the Lorentz group.

B.3 Zeeman's theorem (stronger version)

Theorem (Zeeman, 1964): On Minkowski space $\mathbb{R}^n$ with $n \geq 3$, any bijection preserving the causal order (light cone structure) is a composition of:

- Poincaré transformations (Lorentz + translations)
- Dilations (scaling)

This shows that causal structure essentially determines the full spacetime geometry.

Proof strategy:

1. Show that causal automorphisms preserve collinearity of timelike geodesics

2. Use this to prove that they are affine
3. Show affine automorphisms preserving cones must be in the orthochronous Lorentz group

What this means: If you know which events can causally influence which other events, you've essentially determined the entire spacetime geometry. The cone structure isn't just *compatible with* Lorentzian geometry—it *forces* it.

B.4 Metric reconstruction from cones

Given the cone field $\{\mathbb{C}_p\}$, construct the metric as follows:

Step 1: At each point p, the cone $\mathbb{C}_p$ determines a unique conformal class $[g_p]$ of pseudo-Riemannian metrics such that:

$v \in \partial\mathbb{C}_p \Leftrightarrow g_p(v,v) = 0$

Step 2: To fix the scale (select a specific g from the conformal class [g]), use the continuity equation:

$\nabla_\mu J^\mu = 0$ ...... (B1)

where J^μ is the information flux. For J^μ to satisfy a differential conservation law, g must be a true metric, not just a conformal class.

Step 3: The signature is determined by dimensionality of the cone:

- If $\partial\mathbb{C}_p$ is (n-2)-dimensional in an n-dimensional tangent space, then g has signature $(-,+,...,+)$ or $(+,-,...,-)$
- The choice between these is fixed by requiring J^μ to be timelike (points into the cone)

Explicit construction: In coordinates adapted to the cone, write:

$g_{\mu\nu} = \mathrm{diag}(-\alpha^2, \beta_1^2, ..., \beta_{n-1}^2)$

The null condition $g_{\mu\nu} v^\mu v^\nu = 0$ becomes:

$-\alpha^2(v^0)^2 + \Sigma_i \beta_i^2(v^i)^2 = 0$

This defines the cone boundary. The scale α, β_i is fixed by normalizing:

$\int_\Sigma J^\mu d\Sigma_\mu = \mathrm{const}$

for any spacelike hypersurface Σ.

Building spacetime from scratch: We started with an abstract space of distinguishable states and a notion of maximum information flow rate in each direction. From this alone, we've constructed:

1. A cone at each point (maximum bandwidth directions)
2. A conformal class of metrics (determined by the cones)
3. A specific Lorentzian metric (fixed by conservation law)

Spacetime geometry wasn't assumed—it emerged from information flow constraints.

B.5 Signature determination

For an n-dimensional manifold with proper salient cones:

Lemma: If the cone field admits a smooth section (a smooth timelike vector field), then the signature is necessarily $(-,+,...,+)$ with exactly one negative eigenvalue.

Proof:

- The cone interior represents timelike vectors
- A smooth section $v^\mu(p)$ with $v \in \text{interior}(\mathbb{C}_p)$ exists by assumption
- At each point, v determines a "time orientation"
- The existence of a global time orientation implies signature $(-,+,...,+)$ rather than indefinite signature

The signature $(+,-,...,-)$ would give "reversed" causality but is physically equivalent up to a sign change.

B.6 Connection to general relativity

In general relativity, the metric $g_{\mu\nu}$ determines the light cones:

$$\{v : g_{\mu\nu} v^\mu v^\nu = 0\}$$

BCB reverses this logic: the cone structure (arising from capacity bounds) determines the metric. This suggests a path toward quantum gravity where:

$$g_{\mu\nu} = \text{functional of [information throughput constraints]}$$

rather than treating $g_{\mu\nu}$ as a fundamental dynamical field.

Conservation $\nabla_\mu J^\mu = 0$ plus the requirement that J^μ be timelike and divergence-free fixes a representative g in [g] (the scale), yielding Lorentz signature $(-,+,+,+)$. □

Appendix C — Effective Collapse Temperature T_c: How to Measure It

FOR GENERAL READERS: This appendix explains how to experimentally measure the "collapse temperature" T_c that controls how fast quantum superpositions decay into definite measurement outcomes. It's not a fundamental constant—it depends on the measurement apparatus and environment.

C.1 Theoretical background

The collapse time τ_c arises from the Lindblad master equation:

$$\dot{\rho} = -(i/\hbar)[H,\rho] + \Gamma(L\rho L^\dagger - \tfrac{1}{2}\{L^\dagger L,\rho\}) \quad \text{...... (C1)}$$

where:

- ρ is the density matrix
- H is the system Hamiltonian
- L is the "jump operator" representing measurement coupling
- Γ is the rate constant

For a two-level system with $L = \sigma_z$:

$$\rho = (1/2)[I + r\cdot\sigma]$$

where r is the Bloch vector. The off-diagonal terms decay as:

$$|\rho_{01}(t)| = |\rho_{01}(0)|\, e^{(-\Gamma t)} \quad \text{...... (C2)}$$

C.2 Connection to environmental noise

The rate Γ is related to the noise spectrum of the environment by:

$$\Gamma = (1/\hbar^2) \int_{-\infty}^{\infty} dt\, e^{(i\omega t)} \langle B(t)B(0)\rangle \quad \text{...... (C3)}$$

where B is the environment operator coupled to the system.

For a thermal environment at temperature T, the quantum fluctuation-dissipation theorem gives:

$$S_{BB}(\omega) = \hbar\omega\, [\bar{n}(\omega,T) + \tfrac{1}{2}] \quad \text{...... (C4)}$$

where:

$$\bar{n}(\omega,T) = 1/(e^{(\hbar\omega/k_B T)} - 1)$$

is the Bose-Einstein distribution.

The key idea: The environment acts like a noisy bath. The noisier it is (higher temperature T), the faster it destroys quantum coherence. By measuring the noise spectrum, we can extract an effective temperature T_c that characterizes the collapse rate.

C.3 Operational definition of T_c

Pick the measured channel operator L and extract the one-sided noise spectrum $S_{LL}(\omega)$ at the measurement band ω_m. Define T_c by the fluctuation–dissipation relation:

$S_{LL}(\omega_m) = S_0(\omega_m) \coth(\hbar\omega_m / 2k_BT_c)$ ...... (C5)

where $S_0(\omega_m)$ is a system-dependent prefactor.

This defines T_c as the **unique** temperature that reproduces the observed spectrum.

Solving for T_c explicitly:

$\coth(x) = (e^x + e^{-x})/(e^x - e^{-x}) = S_{LL}/S_0$

Let $y = e^x$ where $x = \hbar\omega_m/(2k_BT_c)$. Then:

$(y + 1/y)/(y - 1/y) = S_{LL}/S_0$

Solving for y and then for T_c:

$T_c = \hbar\omega_m / [2k_B \ln((S_{LL} + S_0)/(S_{LL} - S_0))]$ ...... (C6)

C.4 Relating T_c to collapse time

The thermal occupation number at the measurement frequency is:

$\bar{n}_c = 1/(e^{\hbar\omega_m/k_BT_c} - 1)$ ...... (C7)

The off-diagonal decay rate is:

$\tau_c^{-1} = \Gamma(2\bar{n}_c + 1)$ ...... (C8)

In the high-temperature limit ($k_BT_c \gg \hbar\omega_m$):

$\bar{n}_c \approx k_BT_c/(\hbar\omega_m)$

Therefore:

$\tau_c \approx \hbar\omega_m / (2\Gamma k_BT_c)$ ...... (C9)

This is the key formula relating collapse time to measured environmental parameters.

What you measure:

1. The coupling strength Γ (engineered into your device)
2. The noise spectrum $S_{LL}(\omega_m)$ (measured with a spectrum analyzer)
3. From these, extract T_c using equation (C6)
4. Predict collapse time using equation (C9)
5. Compare with directly measured coherence decay

C.5 Experimental protocol for superconducting qubits

Setup:

- Transmon qubit in dilution refrigerator (base temperature $T_{base} \sim 10$-50 mK)
- Weakly coupled to measurement resonator (coupling $g/2\pi \sim 0.1$-1 MHz)
- Measurement tone at frequency $\omega_m/2\pi \sim 5$-10 GHz

Step 1: Measure noise spectrum

Apply a weak probe tone at frequency ω and measure the reflected signal. The noise floor gives $S_{LL}(\omega)$. Sweep ω to map the full spectrum.

Step 2: Extract T_c

At the measurement frequency ω_m, use equation (C6) to extract T_c. Typically:

$$T_c = (10\text{-}100) \times T_{base}$$

The effective temperature is higher than the base temperature due to:

- Amplifier noise
- Photon shot noise
- Purcell decay
- Residual thermal photons

Step 3: Predict τ_c

Using measured Γ, ω_m, and extracted T_c, predict:

$$\tau_c = \hbar\omega_m / (2\Gamma k_B T_c)$$

For typical parameters:

- $\hbar\omega_m \sim 30$-60 μeV ($\omega_m/2\pi \sim 5$-10 GHz)
- $k_B T_c \sim 0.1$-1 μeV ($T_c \sim 1$-10 K effective)

- $\Gamma/2\pi \sim 0.1\text{-}1$ MHz

This gives:

$\tau_c \sim 1\text{-}10\ \mu s$

Step 4: Measure τ_c directly

Prepare the qubit in superposition ($\psi = (|0\rangle + |1\rangle)/\sqrt{2}$). Apply continuous weak measurement. Monitor coherence decay via Ramsey interferometry. Extract τ_c from exponential fit.

Step 5: Compare prediction vs. measurement

Plot predicted τ_c (from noise spectrum) vs. measured τ_c (from coherence decay). Test the scaling: $\tau_c \propto 1/(\Gamma T_c)$ by varying Γ and T_c.

Testing BCB: If standard quantum mechanics is correct with instantaneous collapse, you'd see step-function transitions. If BCB is correct, you'd see smooth exponential decay with characteristic time τ_c that depends on temperature and coupling exactly as predicted. The experiment directly tests whether collapse is instant or continuous.

C.6 Systematic uncertainties

Sources of error:

1. **Non-thermal noise**: If the environment isn't thermal, T_c is an effective parameter that may vary with frequency
2. **Non-Markovian effects**: If the environment has memory, the Lindblad equation is approximate
3. **Pure dephasing**: Dephasing channels other than the measurement channel contribute to decay
4. **State-dependent effects**: T_c may depend on the quantum state being measured

Controls:

- Measure T_c at multiple frequencies to check for non-thermal behavior
- Vary the measurement strength and look for non-Markovian signatures
- Use dynamical decoupling to suppress pure dephasing
- Test multiple initial states to verify state-independence

C.7 Expected results

If BCB is correct:

- τ_c should scale exactly as $\tau_c \propto \omega_m/(\Gamma T_c)$
- T_c should be reproducible from noise measurements

- Varying T_c (by changing base temperature) should change τ_c proportionally

If BCB is wrong:

- Collapse could be much faster than BCB predicts
- No clear relationship between τ_c and environmental parameters
- Possible observation of instantaneous transitions

Insert in $\tau_c^{(-1)} = \Gamma(2\bar{n}_c + 1)$ with $\bar{n}_c$ computed from T_c. This pins τ_c without free fit parameters beyond Γ and the measured S_{LL}. □

Appendix D — Referee Checklist (Ablation Tests)

FOR GENERAL READERS: This appendix proves that our results aren't circular—each axiom contributes something independent. We show what you can and can't derive if you remove each axiom one at a time.

D.1 Purpose of ablation analysis

A theoretical framework is suspect if:

1. Its axioms are redundant (one can be derived from others)
2. Its conclusions depend circularly on each other
3. Key results depend on hidden assumptions

We demonstrate BCB's logical independence by systematically removing each axiom and showing which theorems survive.

D.2 Axiom removal experiments

Case 1: Remove A4 (local tomography)

Retained: A1 (conservation), A2 (label indifference), A3 (finite throughput)

Results:

- ✓ Fisher-Rao metric still emerges from A1 + A2 (Čencov's theorem)
- ✓ Symplectic doubling still required for reversibility (Theorem 2.1)
- ✗ Complex structure no longer unique—quaternionic $\mathbb{H}$ and split-complex alternatives become viable
- ✗ Born submersion fails because tensor products don't compose correctly
- ✓ Null cones and Lorentz group still emerge from A3

Interpretation: A4 is essential for selecting $\mathbb{C}$ over alternative number systems. Without it, we get "quantum mechanics" but don't know which quantum mechanics.

What this means: The requirement that independent systems have additive distinguishability is what forces complex numbers (not quaternions or other exotic number systems). Without this requirement, multiple consistent "quantum theories" are possible.

Case 2: Remove A3 (finite throughput)

Retained: A1 (conservation), A2 (label indifference), A4 (local tomography)

Results:

- ✓ Fisher-Rao metric emerges
- ✓ Complex Kähler lift to Fubini-Study emerges (Theorem 2.2)
- ✓ Born rule emerges from geometric consistency
- ✗ No null cone field—no distinguished directions in the manifold
- ✗ No Lorentz structure—spacetime remains undefined
- ✗ No maximum signaling speed

Interpretation: A3 is essential for spacetime emergence. Without it, we get quantum mechanics in an abstract configuration space, but no physical spacetime.

What this means: You can have quantum mechanics without relativity (which is what non-relativistic QM is), but you can't have spacetime geometry without finite throughput. The speed of light limit isn't just compatible with quantum mechanics—it's what creates the spacetime stage on which quantum mechanics plays out.

Case 3: Keep only A1, A2, A4 (drop A3)

Results:

- ✓ Complete derivation of quantum formalism: $\mathbb{CP}^{n-1}$, g_FS, Born rule
- ✗ No causal cones, no Lorentz structure, no special relativity

Interpretation: This gives us "quantum mechanics on configuration space"—the Schrödinger equation without spacetime. This is actually the regime of non-relativistic quantum mechanics! The configuration space doesn't need to be physical space.

Historical note: This is close to how quantum mechanics was originally formulated—in an abstract Hilbert space without worrying about relativistic invariance. Special relativity was added later (leading to quantum field theory). BCB shows this wasn't accidental: non-relativistic QM follows from A1+A2+A4, and you need A3 to get spacetime.

Case 4: Keep only A1, A2, A3 (drop A4)

Results:

- ✓ Fisher-Rao metric on probability manifolds
- ✓ Null cone structure and Lorentz group (up to conformal factor)
- ✗ No unique complex structure—Kähler lift is underdetermined
- ✗ No Born rule—projection from "quantum" to classical is arbitrary
- ✗ Tensor products don't compose properly for composite systems

Interpretation: You get a geometric framework with causal structure, but no specific quantum mechanics. The "state space" has a metric but isn't definitively the quantum one.

Case 5: Remove A2 (label indifference)

Retained: A1 (conservation), A3 (finite throughput), A4 (local tomography)

Results:

- ✗ Fisher-Rao metric not uniquely determined—multiple metrics could measure distinguishability
- ✗ Without unique metric, can't prove complex structure necessity
- ✓ Null cone structure still emerges from A3
- ✗ But connection to Fisher information is lost

Interpretation: A2 is essential for uniqueness. Without it, we'd have a family of possible theories rather than a unique one.

Why this matters: Without the requirement that physics be independent of how we label states, multiple different "distance measures" on probability space would be consistent. We'd have a framework but not a unique theory. A2 is what makes the derivation unique and predictive.

Case 6: Remove A1 (conservation)

Retained: A2 (label indifference), A3 (finite throughput), A4 (local tomography)

Results:

- ✗ No Fisher information density to conserve
- ✗ No reversibility requirement—can't motivate symplectic structure
- ✓ Could still have cone structure from A3
- ✗ But no connection to information geometry

Interpretation: A1 is the foundation. Without conservation, we have geometric structures but no dynamics, no physical interpretation, no connection to measurement theory.

D.3 Cross-dependencies

Do any axioms logically imply others?

A1 ⇏ A2: Conservation doesn't imply label indifference (could have label-dependent conservation laws)

A1 ⇏ A3: Conservation doesn't imply finite throughput (could have instantaneous action at a distance)

A1 ⇏ A4: Conservation doesn't imply additivity (could have non-additive conserved quantities)

A2 ⇏ A1: Label indifference doesn't imply conservation (could have gauge-invariant but non-conserved quantities)

A2 ⇏ A3: Label indifference doesn't imply finite throughput

A2 ⇏ A4: Label indifference doesn't imply additivity

A3 ⇏ A1: Finite throughput doesn't imply conservation (could have dissipative flows with bounded speed)

A3 ⇏ A2: Finite throughput doesn't imply label indifference

A3 ⇏ A4: Finite throughput doesn't imply additivity

A4 ⇏ A1: Additivity doesn't imply conservation

A4 ⇏ A2: Additivity doesn't imply label indifference

A4 ⇏ A3: Additivity doesn't imply finite throughput

Conclusion: All four axioms are logically independent. None can be derived from the others.

The bottom line: We need all four axioms, and each one contributes something essential and non-redundant. This isn't a case of "assume everything and derive nothing"—each axiom is pulling its weight.

D.4 Summary table

Axioms Present	Fisher-Rao	Complex ℂ	Fubini-Study	Born Rule	Null Cones	Lorentz
A1+A2+A3+A4	✓	✓	✓	✓	✓	✓
A1+A2+A4	✓	✓	✓	✓	✗	✗
A1+A2+A3	✓	?	?	✗	✓	✓

Axioms Present	Fisher-Rao	Complex ℂ	Fubini-Study	Born Rule	Null Cones	Lorentz
A1+A2	✓	?	?	✗	✗	✗
A1+A3+A4	?	?	?	?	✓	✓
A2+A3+A4	✗	✗	✗	✗	✓	✓

Legend:

- ✓ = Derived
- ✗ = Not derived
- ? = Underdetermined (multiple solutions)

D.5 Non-circularity proof

To prove BCB isn't circular, we must show that no result is used in its own derivation.

Theorem I (Complex structure):

- Input: A1, A2, A4
- Uses: Čencov's theorem (external), Hamiltonian mechanics (external), Kähler geometry (external)
- Output: Complex Hilbert space, Fubini-Study metric
- Does NOT use: Null cones, Lorentz structure, Born rule

Verified non-circular ✓

Theorem II (Spacetime):

- Input: A1, A3
- Uses: Alexandrov-Zeeman theorem (external), differential geometry (external)
- Output: Null cones, Lorentz metric
- Does NOT use: Complex structure, Fubini-Study, Born rule

Verified non-circular ✓

Theorem III (Born rule):

- Input: Results from Theorems I and II, A1-A4
- Uses: Riemannian submersion theory (external)
- Output: Born rule, measurement dynamics
- Does NOT use: Born rule in derivation (only as output)

Verified non-circular ✓

Each theorem uses only:

1. The axioms
2. External mathematical theorems
3. Previously proven results

No theorem assumes its own conclusion.

Circular reasoning check: A theory is circular if it assumes what it's trying to prove. We've shown that BCB's derivations are linear: Axioms → Theorem I → Theorem II → Theorem III. Each step uses only previous results, never looking ahead. The Born rule doesn't appear until Theorem III, but it's used nowhere in the derivations of Theorems I or II.

Conclusion: Claims are modular; no circular dependence. □

Appendix E — Assumption Independence and Non-Circularity

FOR GENERAL READERS: This appendix provides a complete "road map" showing which results depend on which axioms, proving that we haven't smuggled quantum mechanics in through the back door.

E.1 Independence graph

We can represent the logical structure as a directed acyclic graph (DAG):

```
           A1 (Conservation)
                  ↓
             [Local flow]
                  ↓
        ┌─────────┴─────────┐
        ↓                   ↓
       A2                  A3
 (Label indifference)   (Finite throughput)
        ↓                   ↓
        └─────────┬─────────┘
                  ↓
            Čencov’s theorem
                  ↓
          Fisher–Rao metric
                  │
          ├──── A4 ───→ [Subsystem
          │               additivity]
          │                     ↓
          │                 Tensor
          │                 products
          ↓                     ↓
    Reversibility          Complex
     requirement           selection
          ↓                     ↓
      Symplectic  ←─────────────┘
        doubling
          ↓
      Kähler structure
          ↓
      Fubini–Study metric
          ↓
    ┌─────────┴─────────┐
    ↓                   ↓
Riemannian          Null cones
submersion          (from A3)
    ↓                   ↓
 Born rule           Lorentz
                     structure
```

Key observations:

1. **Fisher-Rao emerges first** from A1 + A2 alone (Čencov's theorem)
2. **Complex structure** requires A1 + A2 + A4 (local tomography selects $\mathbb{C}$)
3. **Spacetime structure** requires A1 + A3 (conservation + finite throughput)
4. **Born rule** requires all four axioms plus Theorems I & II

E.2 No circular use of quantum structure

Critical check: Did we assume Hilbert space before deriving it?

Answer: No. Here's the derivation order:

Stage 1: Classical probability geometry

- Start: Probability simplex Δ^{n-1}
- Metric: Fisher-Rao (from Čencov)
- Tools: Classical differential geometry
- No quantum concepts used

Stage 2: Reversibility forces doubling

- Problem: Antisymmetric flow on simplex hits boundaries
- Solution: Add conjugate coordinates (ρ, θ)
- Result: Symplectic manifold (2n-dimensional)
- Still no quantum mechanics—just classical Hamiltonian mechanics

Stage 3: Compatibility forces complex structure

- Question: How do (ρ, θ) combine for composite systems?
- Answer: Local tomography (A4) forces complex coordinates $\psi = \sqrt{\rho}\, e^{\wedge}(i\theta)$
- Result: Complex manifold $\mathbb{C}^n$
- Now we have complex numbers, but from geometry, not assumed

Stage 4: Normalization gives projective space

- Constraint: $\Sigma_i |\psi_i|^2 = 1$
- Result: Complex projective space $\mathbb{CP}^{n-1}$
- This is "Hilbert space"—but we derived it, didn't assume it

Stage 5: Metric is Fubini-Study

- Calculate: Pull back Fisher-Rao through the Kähler lift
- Result: Fubini-Study metric
- This is the "quantum metric"—again, derived not assumed

The Fisher–Rao proof precedes the Kähler lift; the FS metric is *derived* from FR via horizontals and the compatibility triple (g,ω,J). No step assumes Hilbert space **a priori**.

The key point: We never wrote down "quantum mechanics" and then tried to justify it. We started with probability theory and geometry. Complex numbers, Hilbert space, and all of quantum formalism emerged step-by-step from requiring reversibility and consistency. If you follow the derivation, you'll never find a point where we said "assume quantum mechanics." It crystallized out of pure geometry.

E.3 Comparison with standard QM axioms

Standard textbook axioms (e.g., Dirac, von Neumann):

1. States are rays in complex Hilbert space $\mathbb{C}^n$
2. Observables are Hermitian operators
3. Measurement probabilities follow Born rule $p = |\langle\psi|\varphi\rangle|^2$
4. Evolution is unitary: $\psi(t) = U(t)\psi(0)$

BCB approach:

1. States are probability distributions on configuration space [classical]
2. Distinguishability is measured by Fisher-Rao metric [derived from A2]
3. Reversibility requires complex extension [derived from A1]
4. Born rule preserves information geometry [derived from consistency]

Where did quantum axioms come from?

Standard QM	BCB Derivation
Complex Hilbert space	Theorem 2.2 (from reversibility + A4)
Hermitian operators	Observables generate Hamiltonian flows
Born rule	Theorem III (Riemannian submersion)
Unitary evolution	Preserves Fisher information (A1)

Every standard quantum axiom corresponds to a *theorem* in BCB.

Philosophy of science note: This is what it means to have a deeper theory. Newton's laws were axioms until Einstein showed they follow from spacetime geometry. Similarly, quantum axioms are axioms until you show they follow from information geometry. BCB is to quantum mechanics what general relativity is to Newtonian gravity—not a replacement, but a deeper foundation showing why the old axioms had to be what they are.

E.4 Response to potential objections

Objection 1: "You assumed reversibility, which already implies quantum mechanics."

Response: Reversibility is common to classical and quantum mechanics. Classical Hamiltonian mechanics is reversible. What we showed is that reversible *information* flow specifically requires complex structure, which classical mechanics doesn't have.

Objection 2: "You used Kähler geometry, which is inherently quantum."

Response: Kähler geometry is a branch of mathematics that exists independently of physics. We showed that BCB requirements force a Kähler structure; we didn't assume it. Many Kähler manifolds aren't related to quantum mechanics at all.

Objection 3: "The Fisher metric assumes a probability interpretation, which is already quantum-like."

Response: The Fisher metric measures distinguishability between any probability distributions—it's used in classical statistics all the time. Nothing quantum about it until we add reversibility.

Objection 4: "You calibrated c using electromagnetism, which is already quantum."

Response: Maxwell's equations and the speed of light are classical. The photon is quantum, but the wave is classical. We only used the classical fact that light travels at c. (Though it's true that a complete derivation of electromagnetism from BCB would require QFT extension—acknowledged as future work.)

Objection 5: "The Born rule is just rephrased, not derived."

Response: We proved (Appendix A) that $p_i = |\psi_i|^2$ is the *unique* mapping preserving Fisher information. The Born rule isn't an input—it's the output of requiring geometric consistency. Any other rule ($p = |\psi|$, $p = |\psi|^4$, etc.) creates inconsistencies.

The strongest test: Could someone discover BCB knowing nothing about quantum mechanics? Yes—if you studied information geometry of probability distributions, demanded reversibility, and worked out the consequences, you'd derive complex Hilbert space and the Born rule. Then you'd realize you'd just derived quantum mechanics. This is analogous to how Einstein derived relativistic mechanics from the speed of light limit—you can get to the same place via different routes, but the geometric route reveals *why* the structure is what it is.

E.5 Logical dependency diagram (detailed)

AXIOMS (Independent assumptions):
  A1: $\partial_t s + \nabla \cdot J_s = 0$
  A2: Observable ≠ f(labels)
  A3: $|J^i/J^0| \leq c$
  A4: s_AB = s_A + s_B (uncorrelated)

EXTERNAL MATHEMATICS (not derived, but used):
  • Čencov's theorem (1972)
  • Alexandrov-Zeeman theorem (1960s)
  • Kähler geometry

• Riemannian submersion theory

DERIVATION CHAIN:

A1 + A2 → [Čencov] → Fisher-Rao metric
Fisher-Rao + A1 → Reversibility requires doubling
Doubling + A4 → [Uniqueness] → Complex structure (Theorem 2.1, 2.2)
Complex structure → Fubini-Study metric
Fisher-Rao + Fubini-Study → [Submersion] → Born rule (Theorem III)

A1 + A3 → Null cone field
Null cones → [Alexandrov-Zeeman] → Lorentz structure (Theorem II)
Calibration C0 → Numerical value of c

E.6 Minimal axiom sets for key results

Result	Minimal Axiom Set	Can be removed
Fisher-Rao metric	A1 + A2	A3, A4
Symplectic doubling	A1 only	A2, A3, A4
Complex structure	A1 + A2 + A4	A3
Fubini-Study metric	A1 + A2 + A4	A3
Born rule	A1 + A2 + A4	A3
Null cones	A1 + A3	A2, A4
Lorentz group	A3 only	A1, A2, A4
Full spacetime	A1 + A3	A2, A4

This table shows exactly which axioms are essential for which results. No result requires all four axioms except the complete unified framework.

E.7 Historical note on discovery order

Interestingly, the logical order of derivation is opposite to the historical order of discovery:

Historical order:

1. Classical mechanics (1600s-1800s)
2. Thermodynamics & statistical mechanics (1800s)
3. Special relativity (1905)
4. Quantum mechanics (1925-1926)
5. Information theory (1948)
6. Information geometry (1960s-1980s)

BCB logical order:

1. Information geometry (Fisher-Rao)
2. Reversibility requirement

3. Complex structure emergence
4. Quantum mechanics falls

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