Ticks-Per-Bit: A Microphysical Foundation for Entropy

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Abstract

Entropy—the quantity that determines why heat flows from hot to cold, why time seems to move forward, and why some processes are irreversible—is traditionally defined by counting microscopic arrangements. But this definition leaves a puzzle: what physical process actually determines which arrangements are "accessible"? We propose an answer grounded in dynamics rather than counting.

We construct a rigorous framework in which entropy emerges from a primitive dynamical quantity: Ticks-Per-Bit (TPB). A tick represents an irreducible micro-event of physical change—the smallest possible "step" a system can take. A bit represents one unit of objectively distinguishable configuration—a measurable difference between states. The central idea is simple: systems that can generate distinguishable states efficiently (few ticks per bit) have high entropy; systems that require many micro-events to produce distinguishable change have low entropy.

We develop explicit axioms grounded in renewal theory, prove the TPB–entropy identity, derive thermodynamic and information-theoretic entropy, establish quantum extensions, and present falsifiable predictions—including a specific linear viscosity–entropy relation in glass-forming liquids that distinguishes TPB from Adam–Gibbs phenomenology. This edition resolves prior issues with ergodicity assumptions, quantum definitions, and partition function derivations.

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1. Introduction

1.1 The Problem: What Does Entropy Actually Measure?

Entropy is one of the most important yet elusive concepts in physics. It governs the direction of time, determines the efficiency limits of engines, underlies the capacity of communication channels, and even appears in the physics of black holes. Yet for all its importance, the standard definition of entropy—$S = k_B \ln \Omega$, where Ω counts accessible microstates—raises an immediate question: what makes a microstate "accessible"?

Consider a gas in a box. Statistical mechanics tells us its entropy depends on how many microscopic arrangements of molecules are compatible with its macroscopic properties (temperature, pressure, volume). But molecules don't simply "exist" in arrangements—they move, collide, and constantly transition between configurations. The standard formula counts arrangements but says nothing about the dynamical process that connects them.

This paper proposes that entropy is fundamentally about dynamics, not static counting. Specifically, entropy measures how efficiently a system's microscopic dynamics can generate distinguishable macroscopic change.

1.2 The Core Idea: Ticks-Per-Bit

Imagine watching a system evolve at the finest possible resolution. Each irreducible step of change—each elementary transition that cannot be subdivided further—we call a **tick**. Now ask: how many ticks does it take, on average, for the system to reach a configuration that is measurably different from where it started? That measurable difference is one **bit** of distinguishability.

The ratio of these quantities defines **Ticks-Per-Bit (TPB)**: the mean number of micro-events required to produce one bit of distinguishable change.

The central claim of this paper is:

Systems with low TPB have high entropy. Systems with high TPB have low entropy.

This becomes clear when TPB is interpreted literally: TPB counts how many microscopic "ticks" the system needs to generate one bit of distinguishable change.

- **A low-TPB system** produces distinguishable changes quickly: each microscopic tick is "effective," and the system moves easily from one microstate to another. Such a system rapidly explores a large region of configuration space, which is precisely the hallmark of high entropy.
- **A high-TPB system** requires many microscopic ticks to produce even a small change. Most ticks are "wasted," in the sense that they fail to generate distinguishable new configurations. The system explores configuration space only slowly and inefficiently, accessing far fewer microstates. This is the hallmark of low entropy.

This matches physical intuition:

- **Hot gas → Low TPB → High entropy:** Molecules collide frequently, and each collision significantly alters the system's microstate. Only a few ticks are needed to produce measurable change, so the system rapidly samples many configurations.
- **Crystal → High TPB → Low entropy:** Atoms vibrate around fixed lattice positions, and most microscopic events do not produce any distinguishable change. Many ticks are required before the system moves into a new configuration.

Thus, TPB provides a mechanistic interpretation of entropy: entropy is high when each tick is dynamically productive, and low when ticks are dynamically inefficient.

1.3 Why This Matters

Statistical mechanics defines entropy as S = k_B ln Ω, where Ω counts accessible microstates. Yet this definition lacks mechanistic grounding—what physical process determines accessibility? We propose that entropy measures the efficiency of microdynamical processes in generating distinguishable configurations.

The central quantity is **Ticks-Per-Bit (TPB)**: the mean number of irreducible transition events required to produce one bit of distinguishable change. Systems with low TPB generate distinguishability efficiently, corresponding to high entropy; systems with high TPB require many transitions per bit, corresponding to low entropy.

This framework provides three principal contributions:

1. **Mechanistic grounding**: Entropy ceases to be a primitive counting exercise and instead emerges from dynamical processes
2. **Unification**: Thermodynamic, information-theoretic, and quantum entropies arise from a single underlying principle
3. **Falsifiability**: The framework generates specific, testable predictions, most notably for glass transition dynamics

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2. Axiomatics of TPB Dynamics

For the general reader: This section builds the mathematical foundation for TPB. We need to precisely define what we mean by "ticks," "bits," and the rules governing how systems transition between configurations. The key insight is that if no timescale is special at the microscopic level, then the statistics of how long it takes to produce distinguishable change must follow a specific mathematical pattern called a "power law."

2.1 Configuration Space Structure

Let $(\mathcal{C}, \Sigma, \mu)$ be a measurable configuration space with σ-algebra Σ and measure μ. We define:

- **Distinguishability boundary** $\partial\mathcal{C}_{\text{dist}} \subset \mathcal{C}$: the set of configurations differing by exactly one bit from a reference state
- **Transition operator** $T: \mathcal{C} \to \mathcal{C}$: a measurable map representing one tick of microdynamical evolution

2.2 Fundamental Axioms

Axiom 1 (Minimal Transition). A tick is the smallest admissible transition such that $\mu(T(A) \triangle A) > 0$ for some measurable A, where $\triangle$ denotes symmetric difference.

Axiom 2 (Distinguishability Quantization). The boundary $\partial\mathcal{C}\_\text{dist}$ partitions $\mathcal{C}$ into equivalence classes of configurations indistinguishable within one bit.

Axiom 3 (Scale-Free Renewal). The first-passage time $\tau(c) = \inf\{n \in \mathbb{N} : T^n(c) \in \partial\mathcal{C}\_\text{dist}\}$ has an asymptotically scale-free distribution:

$$P(\tau > t) \sim C \cdot t^{-\alpha}, \alpha > 0$$

as required by time-scale invariance of the underlying microdynamics.

In plain terms: Axiom 1 says ticks are the smallest possible changes. Axiom 2 says we can meaningfully ask "is this configuration distinguishable from that one?" Axiom 3 says the time to reach a distinguishable configuration follows a specific statistical pattern—one where there's no characteristic timescale built in. This last axiom is crucial: it's what connects microscopic dynamics to the macroscopic concept of entropy.

Standing Assumption (Measure Preservation and Effective Ergodicity). We assume that the transition operator T acts as a measure-preserving map on the accessible region of configuration space, and that on the coarse-grained scales relevant for entropy, the induced dynamics are effectively ergodic: time-averages along typical trajectories coincide with ensemble averages over μ. This is the standard assumption underlying both Boltzmannian and Gibbsian statistical mechanics.

Remark (Non-ergodic and glassy systems). For systems that are strongly non-ergodic on laboratory timescales—most notably structural glasses—the exact assumptions above are violated. In Section 9.5 we therefore switch from "full configuration space" to a coarse-grained basin graph picture: TPB is then defined for transitions between basins of attraction rather than over the entire phase space. The renewal-theoretic language still applies to this coarse-grained dynamics, but the correspondence with the underlying microscopic T is only approximate. All glass predictions in Section 9.5 should thus be understood as TPB-based modeling assumptions, not consequences of strict microscopic ergodicity.

2.3 Derivation of Power-Law Form from Time-Scale Invariance

For the general reader: This section proves something important—we don't have to assume the power-law form; it follows automatically from a deeper principle. If physics at the microscopic level has no "preferred" timescale (no special duration that matters more than others), then mathematics forces the waiting-time distribution to be a power law. This is analogous to how the absence of a preferred direction in empty space forces physical laws to be rotationally symmetric.

The power-law form is not assumed but derived from a deeper principle:

Principle (No Preferred Timescale). There is no intrinsic microscopic timescale for first-passage to one-bit distinguishability.

This means the survival function $F(t) = P(\tau > t)$ must be self-similar under time rescaling:

$F(\lambda t) = g(\lambda) \cdot F(t)$ for all $\lambda > 0$

for some function $g(\lambda)$. Taking logarithms and defining $f(t) = \ln F(t)$:

$f(\lambda t) = \ln g(\lambda) + f(t)$

Let $x = \ln t$ and $h(x) = f(e^x)$. Then:

$h(x + \ln \lambda) = \ln g(\lambda) + h(x)$

This is Cauchy's functional equation. Under mild regularity (continuity or measurability), the only solutions are linear:

$h(x) = -\alpha x + \text{const} \Longrightarrow f(t) = -\alpha \ln t + \text{const}$

Exponentiating:

$F(t) = P(\tau > t) \propto t^{-\alpha}$

Result: Scale-invariant renewal dynamics necessarily produce power-law tails. The exponent α is undetermined by the symmetry alone.

2.4 Absorption of α into the Entropy Scale

The exponent α appears in the entropy formula as:

$S = k_B \ln \Omega \propto k_B \ln(TPB^{-\alpha}) = -\alpha\, k_B \ln TPB = k_{eff} \ln(1/TPB)$

where $k_{eff} := \alpha\, k_B$. At the level of equilibrium thermodynamics, only the product $k_{eff} = \alpha\, k_B$ enters observable quantities (temperature, heat capacities, free energies), not k_B and α separately. In this sense α plays the same role as Boltzmann's constant itself: it fixes the numerical scale that relates entropy to microscopic structure. A world with some renewal exponent $\alpha \neq 1$ is empirically indistinguishable from one with $\alpha = 1$ but a rescaled Boltzmann constant.

Throughout this work we therefore adopt the convention:

$k_{eff} = k_B \Leftrightarrow \alpha = 1$

as a choice of units for entropy. This removes one free parameter from the notation but does not, by itself, constitute a prediction for α; it simply reflects that α and k_B always appear as a product in the TPB–entropy identity.

In particular, throughout this paper we deliberately avoid using α as a separate fit parameter: wherever α appears, it is either absorbed into k_eff or drops out of observable equilibrium relations, ensuring that no spurious predictive content is attributed to its value.

2.5 Universality Argument for $\alpha \simeq 1$ (Conjectural)

While α can always be absorbed into the entropy scale at the level of static thermodynamics, its value still carries dynamical information: it controls the heaviness of renewal tails and the sensitivity of TPB to cutoffs. Here we briefly sketch why $\alpha \simeq 1$ may be dynamically preferred in realistic systems. This section is intentionally more speculative than the rest of the paper.

Aggregated physical systems are built from many subsystems, each with its own local first-passage time τ_i. Coarse-graining over sums, minima, or maxima of such times generically drives the effective renewal statistics toward stable, heavy-tailed forms. In renewal theory, the case $\alpha = 1$ is critical:

- **$\alpha < 1$**: The mean first-passage time diverges strongly; TPB becomes dominated by extremely rare events
- **$\alpha > 1$**: Tails thin out and approach exponential behavior; scale-freeness is lost
- **$\alpha = 1$**: A marginal regime where the mean diverges only logarithmically, preserving scale-free behavior while remaining compatible with finite effective TPB once physical cutoffs are imposed

Conjecture and status. Aggregated, scale-free renewal processes subject to physical UV/IR cutoffs may be attracted to an effective $\alpha \simeq 1$ fixed point. We emphasize that this is **not** used anywhere in the present paper to extract additional predictions: all of our concrete results (the TPB–entropy identity, the information-theoretic derivations, the thermodynamic relations, and the glass predictions) remain valid for arbitrary α once $k_{eff} = \alpha k_B$ is fixed. The $\alpha \simeq 1$ hypothesis should therefore be read as a **prospective universality claim and research direction**, not as a parameter that is being quietly tuned to match data. A quantitative derivation of α from more microscopic dynamics, or a direct extraction of α from measured first-passage tails in specific systems, is left for future work.

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3. The TPB–Entropy Theorem

For the general reader: This is the heart of the paper. We prove that entropy—the quantity physicists have used for 150 years—is mathematically equivalent to (the logarithm of) the inverse of TPB. High entropy means low TPB (efficient generation of distinguishable states); low

entropy means high TPB (sluggish dynamics). This isn't just a reinterpretation; it's a mathematical identity that follows from our axioms.

3.1 Definition of TPB

Definition. For configuration $c \in \mathcal{C}$, the Ticks-Per-Bit is:

$$TPB(c) = \mathbb{E}[\tau(c)]$$

where $\tau(c)$ is the first-passage time to $\partial\mathcal{C}\_dist$ starting from c.

3.2 Accessible Microstate Measure

Lemma. Under scale-free renewal (Axiom 3), the measure of configurations reachable within n ticks satisfies:

$$\mu(T^{-n}(\partial\mathcal{C}\_dist)) \propto n^{-\alpha}$$

Proof. By renewal theory, the expected number of renewals (visits to $\partial\mathcal{C}\_dist$) by time n is:

$$\mathbb{E}[N(n)] \sim n / \mathbb{E}[\tau]$$

The measure of pre-images under T^{-n} equals the density of configurations that reach $\partial\mathcal{C}\_dist$ in exactly n steps. (More precisely: under the invariant measure μ and our ergodicity assumption, $\mu(T^{-n}(\partial\mathcal{C}\_dist))$ equals the probability that a trajectory first hits $\partial\mathcal{C}\_dist$ at or after step n; for a renewal process this is the tail $\Sigma\_\{t\geq n\}\ u(t)$.) For renewal processes with $P(\tau > n) \sim C \cdot n^{-\alpha}$, the renewal density satisfies:

$$u(n) = P(\tau = n) \sim \alpha C \cdot n^{-(\alpha+1)}$$

Integrating over paths reaching $\partial\mathcal{C}\_dist$ at or after time n:

$$\mu(T^{-n}(\partial\mathcal{C}\_dist)) \propto \int_n^\infty u(t)\, dt \propto n^{-\alpha}$$ ■

3.3 Main Theorem

Theorem (TPB–Entropy Identity). Under Axioms 1–3:

$$\Omega(c) \propto TPB(c)^{-\alpha}$$

$$S(c) = k\_eff \ln(1/TPB(c)) + S_0$$

where $k\_eff = \alpha\, k\_B$ and S_0 is a normalization constant.

Proof. The accessible microstate count $\Omega(c)$ is the measure of configurations reachable from c before distinguishability is achieved:

$\Omega(c) = \mu(T^{-TPB(c)}(\partial\mathcal{C}\_dist)) \propto TPB(c)^{-\alpha}$

Taking logarithms:

$S = k\_B \ln \Omega = k\_B \ln(TPB^{-\alpha}) = -\alpha\, k\_B \ln TPB = k\_eff \ln(1/TPB) + S_0$

Since only the product $k\_eff = \alpha\, k\_B$ appears in predictions, we adopt the convention $k\_eff = k\_B$ (equivalently $\alpha = 1$), yielding $S = k\_B \ln(1/TPB)$. ■

3.4 Interpretation

The theorem states that entropy measures inverse tick-efficiency:

- **High entropy (large Ω)**: Few ticks needed per bit → low TPB → each tick is dynamically productive, rapidly generating distinguishable configurations
- **Low entropy (small Ω)**: Many ticks needed per bit → high TPB → most ticks are "wasted," failing to produce distinguishable change

To build further intuition: Consider the difference between shuffling a deck of cards (high entropy process) versus trying to "shuffle" a brick (low entropy). With cards, each manipulation produces a measurably different arrangement—low TPB. With a brick, you can shake it all day and it remains the same brick—astronomically high TPB. The TPB framework says this difference in dynamical productivity is precisely what entropy measures.

3.5 TPB as a Dual to Microstate Counting

For the general reader: This subsection shows that TPB isn't competing with the standard entropy formula—it's the same thing viewed from a different angle. Traditional stat mech counts how many microstates exist; TPB counts how many steps it takes to move between them. Kac's recurrence theorem provides the bridge: the number of microstates equals the average number of steps to return to any given one.

In the microcanonical ensemble, entropy is defined as $S = k\_B \ln \Omega$, where Ω is the number of accessible microstates. Here we show that this is equivalent to the TPB formulation $S = k\_B \ln(1/TPB)$ up to a constant.

Consider a measure-preserving ergodic dynamical system on a finite set of Ω equiprobable microstates. Each microstate corresponds to a cell A of measure $p = 1/\Omega$. By Kac's recurrence lemma, the mean return time to A satisfies:

$\mathbb{E}[\tau\_A] = 1/p = \Omega$

Thus the microstate count Ω is equal to the mean number of ticks required to revisit a given microstate. This reveals that microstate counting already contains a hidden dynamical statement: "If there are Ω equally likely microstates, it takes Ω ticks on average to return to any given one."

In the TPB framework we are not concerned with the recurrence time to a specific microstate, but with the first-passage time to a one-bit distinguishability boundary $\partial\mathcal{C}$_dist. This boundary is a union of many microstates and therefore has measure p_dist that increases with Ω. By the same recurrence logic, the mean first-passage time to this boundary scales as:

$$\text{TPB} \equiv \mathbb{E}[\tau\_\text{dist}] \propto 1/p\_\text{dist}$$

For uniform microcanonical states, p_dist ∝ Ω, so:

$$\Omega \propto 1/\text{TPB}$$

Substituting into Boltzmann's formula:

$$S = k\_B \ln \Omega = k\_B \ln(1/\text{TPB}) + S_0$$

which reproduces the TPB–entropy identity with a constant offset S_0.

The duality: Microstate counting and ticks-per-bit counting are dual descriptions of the same underlying structure:

- **Microstate counting** asks: "How many distinct configurations exist?"
- **TPB counting** asks: "How many ticks does it take to move between them?"

Kac's lemma shows these questions have the same answer (up to constants). The TPB framework thus does not replace statistical mechanics—it reveals its dynamical content.

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4. Information-Theoretic Entropy

For the general reader: Claude Shannon invented information theory in 1948, defining entropy as a measure of uncertainty or "surprise" in a message. Remarkably, his formula looks identical to the one physicists use for thermodynamic entropy. This section shows that the connection isn't a coincidence—both arise from TPB. The "cost" of distinguishing rare symbols (in information terms) equals the number of ticks needed to physically isolate that symbol's corresponding state.

4.1 The Information-Dynamics Correspondence

We now derive (not postulate) that physical tick-costs for symbol resolution are proportional to optimal code lengths.

Theorem (Information-Dynamics Correspondence). For a discrete source emitting symbols s_i with probabilities p_i, the tick-cost τ_i to physically resolve symbol s_i satisfies:

$$\tau_i = L \cdot (-\ln p_i)$$

where L is a system-dependent tick-per-nat conversion factor.

4.2 Derivation from Basin Geometry

Setup: Distinguishing symbol s_i requires driving the system into a basin of configurations corresponding to that symbol. Let:

$$p_i = \mu(\text{basin}_i)$$

be the measure of the basin for symbol i.

Hierarchical partition structure: Physical configuration spaces typically admit hierarchical (multiplicative) partitions. To isolate symbol s_i requires traversing k_i levels of partition, where probability shrinks multiplicatively:

$$p_i \sim r^{k_i} \implies k_i = (-\ln p_i)/(\ln r)$$

The number of transitions to isolate the symbol scales with the number of levels:

$$\tau_i \propto k_i \propto -\ln p_i$$

Result: The logarithmic tick-cost emerges from the geometric property of multiplicative partition refinement in configuration space.

4.3 Derivation from Optimal Distinguishability

Kraft's inequality: If physical dynamics must implement a prefix-free (distinguishable) code in configuration space, then:

$$\Sigma_i \, e^{-\tau_i/L} \leq 1$$

For optimal distinguishability (no wasted microstates), equality holds.

Shannon's source coding theorem: The unique prefix-free code achieving minimal average length has:

$$\tau_i = L(-\ln p_i)$$

Result: Physical distinguishability governed by TPB automatically produces Shannon code lengths under maximal efficiency constraints.

4.4 Derivation from Landauer's Principle

Landauer's bound: Erasing one bit requires energy $k_B T \ln 2$.

Distinguishing rare outcomes: Symbol s_i reduces uncertainty by $-\ln p_i$ nats, requiring proportional erasure of prior possibilities.

Tick-energy correspondence: Let each tick correspond to a fixed micro-amount of dissipated energy ΔE. The energy required to distinguish symbol i is:

$$E_i = k_B T (-\ln p_i) = \tau_i \cdot \Delta E$$

Solving for τ_i:

$$\tau_i = (k_B T / \Delta E)(-\ln p_i)$$

Result: The same functional form emerges directly from thermodynamic irreversibility.

4.5 Shannon Entropy as Mean Tick-Cost

Theorem. Under the Information-Dynamics Correspondence:

$$H = TPB_{eff} / L$$

where H is Shannon entropy and TPB_{eff} is the mean tick-cost.

Proof. The mean tick-cost per symbol is:

$$TPB_{eff} = \Sigma_i p_i \tau_i = \Sigma_i p_i \cdot L \cdot (-\ln p_i) = L \cdot H$$

Rearranging: $H = TPB_{eff} / L$. ■

The effective accessible symbol-space volume per tick is:

$$\Omega_{eff} \propto \exp(H) = \exp(TPB_{eff} / L)$$

Thus Shannon entropy measures the logarithm of typical sequences accessible per unit tick-cost.

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5. Thermodynamics from TPB

For the general reader: Thermodynamics—the science of heat, work, and energy—was developed in the 1800s to understand steam engines. Its key concepts (temperature, heat

capacity, free energy) were later explained by statistical mechanics. Here we show these same concepts emerge naturally from TPB. Temperature, for instance, measures how quickly TPB changes as you add energy to a system.

5.1 Temperature

Given $S(E) = k_B \ln(1/TPB(E))$, temperature emerges from:

$1/T = \partial S/\partial E = -k_B \cdot (1/TPB) \cdot (\partial TPB/\partial E)$

Physical interpretation: Adding energy typically opens new dynamical pathways, reducing TPB. Thus $\partial TPB/\partial E < 0$, ensuring $T > 0$.

5.2 Partition Functions: Corrected Treatment

The standard partition function $Z = \Sigma_j \exp(-\beta E_j)$ counts microstates weighted by Boltzmann factors. We now incorporate tick-dynamics correctly.

Key insight: Dwell times τ_j affect the kinetic accessibility of states but not their equilibrium statistical weight. The Boltzmann distribution emerges from maximizing entropy subject to energy constraints, independent of kinetics.

TPB-Corrected Formulation: Define the dynamically-weighted partition function:

$Z_{dyn} = \Sigma_j [e^{-\beta E_j} / (\tau_j/\tau_0)]$

where τ_0 is a reference timescale. This weights states by both their Boltzmann probability and their dynamical accessibility.

For systems in true equilibrium, detailed balance ensures that dynamical weights cancel in expectation values, recovering standard thermodynamics. We emphasize that in true equilibrium, thermodynamic quantities are governed by the standard partition function Z; the dynamically weighted Z_dyn is introduced only to describe how tick-dynamics modulate the accessibility of states in metastable and driven regimes.

We emphasize that TPB does not modify equilibrium statistical mechanics.

TPB corrections via Z_dyn become significant for:

- Metastable states
- Glassy systems
- Non-equilibrium steady states

Entropy from Z_dyn:

$S = k_B \ln Z_{dyn} + k_B T (\partial \ln Z_{dyn} / \partial T)$

This reduces to standard thermodynamics when $\tau_j = \tau_0$ (uniform tick-costs) and provides corrections otherwise.

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6. Quantum TPB

For the general reader: Quantum mechanics complicates the picture because quantum systems don't have definite configurations until measured. A quantum system can be in a "superposition" of many states simultaneously. How do we define TPB when there's no single trajectory through configuration space? The answer involves "purity"—a measure of how mixed or spread-out a quantum state is. A pure quantum state (like a single photon with definite polarization) has TPB = 1. A maximally mixed state (like unpolarized light) has TPB equal to the number of possible states. Decoherence—the process by which quantum systems become classical—shows up as increasing TPB.

6.1 Definition via Purity

For density operator ρ on Hilbert space $\mathscr{H}$ with $\dim(\mathscr{H}) = d$, define:

- **Purity:** $P = \mathrm{Tr}(\rho^2) = \Sigma_i \lambda_i^2$
- **Effective dimension:** $d_{eff} = 1/P$
- **Quantum TPB:** $TPB_q = d_{eff} = 1/\mathrm{Tr}(\rho^2)$

6.2 Operational Interpretation

In the classical setting, TPB literally counts the expected number of micro-transitions required to reach one-bit distinguishability. For a quantum system described by a mixed state ρ, there is no unique underlying trajectory; instead, distinguishability is mediated by decoherence and measurement.

In that context, we interpret:

$TPB_q = 1/\mathrm{Tr}(\rho^2)$

as an **effective tick-count proxy**: it measures how many orthogonal "branches" are required to support the observed mixed state. Each decoherence event can be viewed as a quantum "tick" that increases the number of effectively distinct branches. More precisely, the rate of purity loss dP/dt tracks an effective tick-rate, though the "tick" picture is necessarily approximate for continuous environmental coupling.

Thus TPB_q tracks how many such branches are needed to represent ρ, even though the underlying unitary evolution does not decompose into literal, discrete ticks in a unique way. The classical "ticks to distinguishability" picture should therefore be read as an intuition pump in the quantum case, not as a microscopic model of wavefunction collapse.

Status of Quantum TPB. In the classical setting, TPB counts literal micro-transitions required to reach one-bit distinguishability. In quantum mechanics, however, there is no underlying trajectory and no unique notion of "micro-events." For this reason the identification $TPB_q = 1/\mathrm{Tr}(\rho^2)$ should be understood as an **operational interpretation** rather than a microscopic derivation. Purity quantifies the number of orthogonal components ("branches") needed to represent the state; decoherence increases this effective branch count; and distinguishability emerges only after decoherence provides classical records.

The TPB_q definition therefore expresses the minimal number of effectively distinguishable sectors required to support ρ, not the number of literal ticks. A fully microscopic derivation of quantum ticks would require a theory of discrete dynamical events in open quantum systems and is left for future work. In this paper we adopt TPB_q as the quantum quantity that plays the same operational role as classical TPB, while explicitly acknowledging that it is interpretive rather than fundamental.

6.3 Properties

- **Pure states** ($\rho = |\psi\rangle\langle\psi|$): $P = 1$, $TPB_q = 1$ (one tick per bit of distinguishability)
- **Maximally mixed** ($\rho = I/d$): $P = 1/d$, $TPB_q = d$ (d ticks per bit)
- **Decoherence:** Increases TPB_q as off-diagonal elements decay

6.4 Entropy Bounds

Theorem. Von Neumann entropy satisfies:

$$\ln(TPB_q) \leq S_{vN}/k_B \leq \ln d$$

where $d = \dim(\mathscr{H})$ is the Hilbert space dimension.

Proof. The lower bound follows from the Rényi-2 entropy:

$$S_2 = -\ln \mathrm{Tr}(\rho^2) = \ln(d_{eff}) = \ln(TPB_q)$$

The standard inequality $S_2 \leq S_{vN}$ gives the lower bound.

The upper bound $S_{vN} \leq k_B \ln d$ is saturated by the maximally mixed state $\rho = I/d$. Since $d_{eff} = TPB_q \leq d$ always, this completes the bound. ■

Special case: For states with equal nonzero eigenvalues (maximally mixed on their support of dimension $d_{eff} = TPB_q$), we have:

$S_{vN}/k_B = \ln(TPB_q)$

Thus TPB_q directly determines the entropy for this important class of states, which includes thermal states of systems with degenerate energy levels.

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7. Gravitational TPB and Black Hole Entropy

For the general reader: One of the most surprising discoveries in theoretical physics is that black holes have entropy—and enormous amounts of it. A black hole the mass of the Sun has more entropy than all the ordinary matter in the observable universe. Even more surprisingly, this entropy is proportional to the black hole's surface area, not its volume. This section asks: can TPB explain why? The answer involves gravitational time dilation. Near a black hole's horizon, time slows down dramatically, which in TPB terms means the "tick rate" (as seen from far away) becomes extremely high. This concentration of tick-dynamics at the horizon explains why entropy lives on the surface rather than being distributed throughout the volume.

7.1 Background: Standard Horizon Thermodynamics

The thermodynamics of black holes is one of the most robust achievements of semiclassical gravity. Without any reference to TPB, one already knows that:

1. Stationary black holes obey analogues of the four laws of thermodynamics
2. Including quantum fields yields the Hawking temperature $T_H = \kappa/2\pi$ and the Bekenstein–Hawking entropy:

$$S_{BH} = A / 4\ell_P^2$$

where A is the horizon area.

3. Wald's Noether-charge construction shows that, for a wide class of diffeomorphism-invariant Lagrangians, black hole entropy is determined by the gravitational action

In this section TPB does not attempt to re-derive or supersede these results. Instead, we ask a narrower question: given the existence of horizon thermodynamics, can TPB provide a microscopic interpretive layer that explains why entropy localizes on the horizon and scales with area, in terms of distinguishability and tick-efficiency?

7.2 TPB Field Near the Horizon

For a static, spherically symmetric black hole, write the metric in Schwarzschild-like coordinates:

$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2\,d\Omega^2,\ f(r_s) = 0$

Let $\chi^a = (\partial/\partial t)^a$ be the timelike Killing vector outside the horizon. Define the TPB density as seen by an asymptotic observer:

$TPB_\infty(r) \propto 1/|\chi| = 1/\sqrt{f(r)}$

Physical reasoning: Proper time near the horizon relates to coordinate time by $d\tau = \sqrt{f(r)}\,dt$. From infinity, local "ticks" near the horizon are infinitely redshifted—the number of coordinate-time ticks per local physical event diverges as $1/\sqrt{f(r)}$.

(The divergence is frame-dependent: TPB_∞ measures tick-cost in coordinate time seen from infinity. The local tick-efficiency near the horizon remains saturated and determines the area scaling.)

7.3 Distinguishability Current and Horizon Flux

Define a distinguishability current:

$J^a = (1/TPB)\,u^a$

where u^a is the 4-velocity of static observers just outside the horizon. The flux of distinguishability through a surface Σ is:

$\Phi_\Sigma = \int_\Sigma J^a\,d\Sigma_a$

At the horizon:

- u^a becomes null and tangent to the horizon generator
- Ingoing/outgoing directions collapse: one-sided flow aligned with null generators
- The flux per unit area is maximized because transverse directions are fixed by the 2-sphere geometry, and radial distinguishability channels "pile up" against the null surface due to extreme redshift

Key result: The horizon is the unique 2-surface where the distinguishability flux, per unit area and per unit Killing time, is extremal (maximal).

7.4 Area Law from TPB

Impose a UV cutoff at proper length ℓ_{cut} (later identified with $O(\ell_P)$). The number of distinguishable states supported by the horizon flux up to this cutoff determines the entropy:

$S_H \propto A / \ell_{cut}^2$

Derivation:

- Distinguishability is counted per "cell" on the horizon
- Each cell has area ℓ_cut^2 (from UV cutoff + focusing)
- Each cell carries O(1) bits (saturated TPB density at the horizon)
- Total entropy = (number of cells) × (bits per cell) $\propto A/\ell\_cut^2$

This gives area scaling robustly from TPB principles alone.

7.5 Coefficient Matching

TPB fixes the functional dependence $S \propto A$ and provides a conceptual framework for why entropy localizes on a null 2-surface (the maximal distinguishability surface). The proportionality constant is not determined purely from TPB.

Instead, it is fixed by demanding consistency with semiclassical gravity: the same horizon area variation that produces Hawking temperature $T\_H = \kappa/2\pi$ must satisfy the first law:

$dM = T\_H\, dS + \cdots$

This matching yields:

$S = A / 4\ell\_P^2$

which we interpret as fixing the effective horizon cell area:

$\ell\_cut^2 = 4\ell\_P^2$

7.6 Summary

What TPB provides	What GR fixes
Entropy lives on the horizon	—
Entropy scales with area	—
Horizon = maximal distinguishability surface	—
—	Numerical coefficient $A/4\ell\_P^2$
—	Identification $\ell\_cut = 2\ell\_P$

TPB provides a conceptual interpretive layer consistent with semiclassical gravity; semiclassical gravity provides the calibration.

Scope of Gravitational Application. The gravitational results in this section are intended purely as a consistency check, not as new predictions or modifications of semiclassical gravity. TPB offers an interpretive mechanism for why black hole entropy localizes on the horizon and scales with area, but the quantitative results $S = A/4\ell\_P^2$, the Hawking temperature, and the Noether-charge construction remain entirely those of standard GR and quantum field theory in curved spacetime.

This section should therefore be viewed as establishing that TPB is **compatible** with known gravitational thermodynamics and as identifying conceptual parallels (e.g., redshifted tick-rates and horizon-localized distinguishability). Developing a full TPB-based dynamical account of gravitational entropy would require a theory of microscopic ticks in curved spacetime and is beyond the scope of the present work.

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8. Worked Examples

For the general reader: Theory is only as good as its applications. This section works through several systems—starting with a simple toy model that explicitly demonstrates the TPB–entropy identity, followed by classic thermodynamic systems—showing that TPB reproduces the standard entropy formulas in each case. These aren't new predictions; they're consistency checks showing that TPB agrees with established physics.

8.1 Toy Model: Explicit TPB Extraction from Dynamics

This example provides a concrete numerical demonstration that entropy can be computed directly from TPB extracted from dynamics.

System: A 2-bit register with states $(b_1, b_2) \in \{00, 01, 10, 11\}$, giving $\Omega = 4$ microstates.

Dynamics (ticks): At each tick, both bits are re-randomized independently. Each of the 4 configurations is chosen with equal probability 1/4, irrespective of the previous state. This represents maximally mixing dynamics.

Macroscopic observable: We ask only whether the second bit has flipped relative to its initial value. Starting in state 00:

- Macrostate "0": second bit is 0 → states {00, 10}
- Macrostate "1": second bit is 1 → states {01, 11}

The standard macro-entropy of this 1-bit observable is $S_{macro} = k_B \ln 2$.

Step 1: Define the distinguishability boundary. Starting from configuration 00, we define:

$\partial\mathcal{C}_{dist} = \{01, 11\}$

Crossing this boundary means gaining one bit of distinguishable information ("the second bit has flipped"). The probability of being in $\partial\mathcal{C}_{dist}$ at any tick is $p_{dist} = 2/4 = 1/2$.

Step 2: Extract TPB from dynamics. Let τ be the first-passage time to $\partial\mathcal{C}_{dist}$ starting from 00:

$\tau = \inf\{n \geq 1 : \text{state at tick } n \in \{01, 11\}\}$

Since each tick is an independent trial with success probability $p_{dist} = 1/2$, τ is geometrically distributed:

$P(\tau = n) = (1/2)^{(n-1)} \cdot (1/2)$

The mean first-passage time is:

$\mathbb{E}[\tau] = 1/p_{dist} = 2$

So TPB = 2 ticks per bit. On average it takes 2 microscopic ticks to produce 1 bit of distinguishable change.

Step 3: Compute entropy from TPB. Using the TPB–entropy identity $S = k_B \ln(1/TPB) + S_0$ with TPB = 2:

$S_{TPB} = k_B \ln(1/2) + S_0 = k_B(-0.693) + S_0$

Matching to the known macro-entropy $S_{macro} = k_B \ln 2 \approx k_B(0.693)$:

$S_0 = k_B \ln 2 - k_B \ln(1/2) = k_B \ln 4$

Verification (setting $k_B = 1$):

Quantity	Value
TPB	2
ln(1/TPB)	−0.693
S_0	$\ln 4 \approx 1.386$
$S_{TPB} = \ln(1/TPB) + S_0$	$-0.693 + 1.386 = 0.693$
$S_{macro} = \ln 2$	0.693

The TPB-derived entropy exactly matches the standard result.

Interpretation: The additive constant $S_0 = k_B \ln 4$ is the baseline entropy of the underlying 2-bit system (4 microstates). The TPB-dependent part ln(1/TPB) captures how much entropy is associated with the specific coarse-grained question ("has the second bit flipped?"). This demonstrates concretely that the TPB framework extracts the correct entropy from purely dynamical considerations.

8.2 Ideal Gas

Setup: N non-interacting particles in volume V with total energy E.

Configuration space: $\mathcal{C} = \{(q_1, ..., q\_N, p_1, ..., p\_N) : \Sigma|p_i|^2/2m = E\}$

Transition operator: T represents molecular collisions redistributing momentum while preserving total energy.

Distinguishability: $\partial\mathcal{C}$_dist consists of configurations differing by one bit in the coarse-grained position-momentum description.

TPB calculation: The number of accessible microstates is:

$\Omega = (V^N / N!) \cdot (2\pi mE)^{(3N/2)} / [h^{(3N)} \Gamma(3N/2)]$

For fixed E and N, $\Omega \propto V^N$. In the TPB framework, increasing V opens more dynamical pathways, so:

$TPB \propto 1/V^N$

Thus:

$S = k\_B \ln(1/TPB) = k\_B \ln V^N + const = Nk\_B \ln V + const$

matching the Sackur-Tetrode equation's volume dependence.

8.3 Quantum Harmonic Oscillator

Setup: 1D oscillator with frequency ω at temperature T.

Density matrix: $\rho = \Sigma_n p_n |n\rangle\langle n|$ with $p_n = (1 - e^{-\beta\hbar\omega}) e^{-n\beta\hbar\omega}$

Purity:

$P = \Sigma_n p_n^2 = (1 - e^{-\beta\hbar\omega})^2 \Sigma_n e^{-2n\beta\hbar\omega} = (1 - e^{-\beta\hbar\omega})/(1 + e^{-\beta\hbar\omega}) = \tanh(\beta\hbar\omega/2)$

Quantum TPB:

$TPB\_q = 1/P = \coth(\beta\hbar\omega/2)$

High-temperature limit ($\beta\hbar\omega \ll 1$):

$TPB\_q \approx 2/(\beta\hbar\omega) = 2k\_B T/(\hbar\omega)$

$S \approx k\_B \ln(TPB\_q) = k\_B \ln(2k\_B T/\hbar\omega)$

matching the classical result $S \propto \ln T$.

8.4 Two-Level System

Setup: States $|0\rangle$, $|1\rangle$ with probabilities p, $1-p$.

Shannon entropy: $H(p) = -p \ln p - (1-p) \ln(1-p)$

Tick-costs: Under the Information-Dynamics Correspondence:

- $\tau_0 = L(-\ln p)$
- $\tau_1 = L(-\ln(1-p))$

Mean TPB:

$$\text{TPB\_eff} = p \cdot L(-\ln p) + (1-p) \cdot L(-\ln(1-p)) = L \cdot H(p)$$

Verification: $H = \text{TPB\_eff}/L$, confirming the Shannon–TPB equivalence.

Quantum case: For $\rho = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$:

$$\text{TPB\_q} = 1/(p^2 + (1-p)^2)$$

At $p = 1/2$: $\text{TPB\_q} = 2$, $S_{vN} = k_B \ln 2$. At $p = 0$ or 1: $\text{TPB\_q} = 1$, $S_{vN} = 0$.

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9. Falsifiable Predictions

For the general reader: A theory that can't be tested isn't science—it's philosophy. This section presents specific, quantitative predictions that distinguish TPB from existing theories. The most developed prediction concerns glass-forming liquids (Section 9.5), where TPB makes a sharp claim: viscosity and configurational entropy should be related by a straight line on a log plot, not the curved relationship predicted by the standard Adam-Gibbs theory. This is a genuine experimental test that could prove or disprove the TPB framework.

9.1 Reaction Kinetics

Prediction: For reactions with multiple pathways, the entropy change ΔS correlates with the ratio of mean dwell times:

$$\Delta S = k_B \ln(\text{TPB\_initial} / \text{TPB\_final}) = k_B \ln(\tau\_\text{final} / \tau\_\text{initial})$$

Test: Compare catalyzed vs. uncatalyzed reactions. Catalysis reduces activation barriers, changing dwell-time ratios. Measure ΔS and τ ratios independently; TPB predicts a linear relationship on a log-log plot.

9.2 Heat Capacity Anomalies

Prediction: Systems with anomalous heat capacity C_V exhibit non-standard TPB temperature dependence:

$C_V = T\,(\partial S/\partial T) = -k_B\,T \cdot (\partial \ln \mathrm{TPB} / \partial T)$

For standard systems, TPB $\propto 1/T$ gives $C_V = k_B$ (equipartition). Anomalies arise when TPB(T) deviates from 1/T.

Test: In glassy systems with sub-logarithmic heat capacity, measure relaxation times $\tau(T)$ and verify that C_V correlates with $-T\,\partial \ln \tau/\partial T$.

9.3 Quantum Decoherence

Prediction: The rate of purity decay equals the rate of TPB_q increase:

$d(\mathrm{TPB}_q)/dt = -(1/P^2)(dP/dt) = (\mathrm{TPB}_q^2/P) \cdot \gamma_{dec}$

where γ_{dec} is the decoherence rate.

Test: In controlled qubit experiments, measure P(t) and verify TPB_q(t) = 1/P(t) tracks decoherence dynamics. Compare sensitivity of TPB_q vs. P as probes of environmental coupling.

9.4 Gravitational Time Dilation

Prediction: If tick-rates vary with gravitational potential as $\mathrm{TPB}(\Phi) \propto \exp(\Phi/c^2)$, then clock rates should satisfy:

$d\tau_{proper}/d\tau_{coordinate} = \sqrt{(1 + 2\Phi/c^2)} \approx 1 + \Phi/c^2$

This reproduces the standard first-order GR redshift relation.

Future directions: A more detailed TPB-based derivation could in principle fix higher-order corrections (for example, relating them to the renewal exponent α or cutoff-dependent effects), offering a possible route to small deviations from GR. We leave the systematic development of gravitational TPB predictions to future work.

9.5 Glass Transition and the Kauzmann Paradox

For the general reader: When you cool a liquid slowly enough, it typically crystallizes—molecules arrange into an orderly lattice. But if you cool certain liquids fast enough, they become "supercooled" and eventually form a glass: a solid that's structurally disordered like a

liquid but mechanically rigid like a crystal. The glass transition is one of the deepest unsolved problems in condensed matter physics.

Two puzzles stand out. First, the viscosity of supercooled liquids increases dramatically—by factors of 10^{14} or more—as they approach the glass transition. Second, the "Kauzmann paradox": if you extrapolate the liquid's entropy to low temperatures, it appears to hit zero at a finite temperature, which would be thermodynamically catastrophic. TPB offers a unified explanation: both viscosity and entropy are controlled by the same underlying dynamics, and the apparent entropy crisis is avoided because systems fall out of equilibrium before reaching the problematic temperature.

This prediction addresses one of the outstanding puzzles in condensed matter physics and provides a quantitative test that distinguishes TPB from existing theories.

9.5.1 Background: The Glass Problem

Supercooled liquids approaching the glass transition exhibit:

- **Super-Arrhenius slowing:** Relaxation time τ increases faster than $\exp(E_a/k_B T)$
- **Kauzmann paradox:** Extrapolated configurational entropy S_{conf} appears to vanish at a finite temperature $T_K > 0$, implying an "entropy crisis"
- **Adam-Gibbs relation:** Empirically, viscosity η and S_{conf} satisfy:

$$\ln \eta = A + B/(T \cdot S_{conf})$$

The Adam-Gibbs relation works phenomenologically but lacks microscopic derivation. The Kauzmann paradox remains unresolved—does S_{conf} really vanish, or does the extrapolation fail?

9.5.2 TPB Analysis

The TPB framework requires careful application to glasses because configurational entropy counts equilibrium basins, while TPB measures dynamical accessibility.

Physical picture: A supercooled liquid has Ω_{conf} distinct configurational basins. The system explores these basins via structural (α) relaxations with timescale τ_α. As temperature decreases:

- Fewer basins remain accessible → S_{conf} decreases
- Transitions between basins slow down → τ_α increases

TPB interpretation: The relaxation time τ_α measures how many ticks are needed to transition between distinguishable configurations. In TPB language:

$$\text{TPB}_{glass} = \tau_\alpha / \tau_0$$

where $\tau_0 \sim 10^{-13}$ s is the microscopic attempt time.

The key relation: Both S_conf and τ_α reflect the same underlying landscape structure—the height and number of barriers between basins. In a broad class of landscape models (including random energy models, trap models, and mean-field p-spin glasses), they are related by:

$$S_{conf} \propto k_B \ln(\tau_{ref} / \tau_\alpha) + S_\infty$$

where τ_ref is a reference timescale and S_∞ is the high-temperature limit.

9.5.3 Core TPB Relation

Within the TPB picture, both viscosity η and configurational entropy S_conf are controlled by the same underlying barrier landscape. In a broad class of simple landscape models, this leads to:

$$S_{conf} = C - (k_B/\gamma) \ln(\tau_\alpha/\tau_0)$$

for some material-dependent exponent $\gamma \geq 1$, where τ_α is the α-relaxation time and τ_0 the microscopic attempt time. Using $\eta \propto \tau_\alpha$ then yields:

$$\ln \eta = A - (\gamma/k_B) S_{conf}$$

Prediction 1 (Shape). The TPB framework predicts that, for each glass-former, a plot of ln η vs S_conf should be approximately linear over the supercooled regime, in contrast to the curved behavior implied by the Adam–Gibbs relation when written in these variables.

Prediction 2 (Slope Universality vs. γ). The simplest TPB hypothesis is $\gamma = 1$, which would imply a universal slope of $-1/k_B$ across all materials. We treat $\gamma = 1$ as a concrete, falsifiable conjecture. If experiments instead find material-dependent, but still roughly constant slopes $-\gamma/k_B$, this would falsify the $\gamma = 1$ universality while leaving the structural TPB claim (linear relation between ln η and S_conf) intact.

Falsification criterion: A clear and systematic breakdown of linearity in ln η vs S_conf would be a genuine falsification of the TPB glass picture itself.

9.5.4 Comparison of Predictions

Model	Viscosity relation	ln η vs S_conf
Arrhenius	$\ln \eta = A + E_a/k_B T$	No direct relation
Adam-Gibbs	$\ln \eta = A + B/(T \cdot S_{conf})$	Curved (1/T factor)
TPB	$\ln \eta = A - (\gamma/k_B) S_{conf}$	Linear, slope $-\gamma/k_B$

The critical difference: Adam-Gibbs predicts curvature in a ln η vs S_conf plot (due to the temperature dependence), while TPB predicts a straight line. The conjectural case $\gamma = 1$ would give a universal slope $-1/k_B$ across all materials; if γ proves material-dependent, slopes would vary but linearity would persist.

9.5.5 Kauzmann Paradox: Weak vs. Strong Form

In the TPB framework, the "strong" Kauzmann paradox—an actual thermodynamic crisis at finite temperature where S_conf = 0—is never realized.

The paradox: Extrapolating S_conf(T) to low temperature suggests S_conf → 0 at a finite temperature T_K > 0. This "entropy crisis" would imply a thermodynamic phase transition, but no such transition is observed—the system simply falls out of equilibrium at T_g > T_K.

TPB reinterpretation: From the relation $S_{conf} = C - (k_B/\gamma) \ln(\tau_\alpha/\tau_0)$, we have S_conf → 0 when:

$$\ln(\tau_\alpha/\tau_0) \to \gamma C/k_B$$

$$\tau_\alpha \to \tau_0 \exp(\gamma C/k_B) = \tau_{max}$$

This defines a maximum relaxation time τ_max. But crucially, as τ_α → τ_max, the system falls out of equilibrium—it can no longer explore configuration space on experimental timescales.

Physical interpretation: The "entropy crisis" never occurs because:

1. S_conf → 0 requires τ_α → τ_max (astronomically long)
2. Long before this, at T_g, the system falls out of equilibrium
3. Below T_g, S_conf is no longer a well-defined equilibrium quantity

In that sense TPB formalizes a viewpoint already common in the glass literature: T_K marks the breakdown of naive extrapolation of equilibrium S_conf, not a true thermodynamic phase transition. What TPB adds is an explicit dynamical ceiling (τ_max) and a direct link between that ceiling and the entropy budget through γ.

9.5.6 Quantitative Test Protocol

Data required:

- Relaxation time τ_α(T) from dielectric spectroscopy or mechanical relaxation
- Configurational entropy S_conf(T) from calorimetry (S_conf = S_liquid − S_crystal)
- Viscosity η(T)

Test 1 (Primary): Plot ln η vs S_conf/k_B.

- TPB predicts: straight line (slope −γ/k_B). The γ = 1 conjecture predicts slope = −1/k_B.
- Adam-Gibbs predicts: curved (due to implicit T dependence)

Test 2 (Universality): Compare slope across different glass-formers.

- The γ = 1 conjecture predicts: universal slope = −1/k_B for all materials

- If slopes vary but remain constant within each material: indicates material-dependent $\gamma \neq 1$, but TPB structure (linearity) survives
- If slopes vary unsystematically: potential challenge to TPB framework

Test 3 (Kauzmann): Examine S_conf(T) as $T \to T_g$.

- TPB predicts: S_conf remains positive, system falls out of equilibrium
- Check: Does extrapolated T_K coincide with $\tau_\alpha \to \tau_{max}$?

9.5.7 Candidate Systems

Systematic analysis across fragility classes would test whether the TPB relation $\ln \tau \propto -S_{conf}$ holds universally or with material-dependent corrections:

- **Ortho-terphenyl (OTP):** Well-characterized fragile glass-former with extensive τ_α and S_conf data
- **Glycerol:** Intermediate fragility, good calorimetric data
- **SiO_2:** Strong glass-former, different universality class

9.5.8 Significance

If the TPB viscosity relation holds:

1. **Explains Adam-Gibbs:** The empirical Adam-Gibbs relation would be an approximation to the more fundamental TPB relation, valid when $T \cdot S_{conf}$ varies slowly
2. **Reinterprets Kauzmann:** The apparent entropy crisis is replaced by a divergence of relaxation time that is cut off by loss of equilibrium—S_conf is bounded below by dynamical constraints
3. **Unifies glass physics:** Connects thermodynamic (S_conf) and dynamic (τ_α) anomalies through a single principle
4. **Validates TPB:** Provides a quantitative, falsifiable test in a well-studied system

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10. Discussion

For the general reader: This section steps back to assess what TPB accomplishes, where it fits among existing theories, what its limitations are, and what questions remain open. The key message: TPB doesn't replace statistical mechanics—it provides a deeper foundation for it, explaining why the traditional formulas work and generating new testable predictions.

10.1 Relationship to Existing Frameworks

The TPB framework is compatible with but conceptually distinct from:

- **Boltzmann:** TPB provides a dynamical mechanism for microstate counting
- **Gibbs:** Ensemble averages correspond to TPB averages over renewal processes
- **Shannon:** Information entropy emerges from physical tick-costs
- **Landauer:** Bit erasure costs arise from TPB dynamics

10.2 Limitations and Scope

Several important limitations should be acknowledged:

1. **Quantum interpretation:** The classical "ticks to distinguishability" picture becomes metaphorical in quantum systems. $TPB_q = 1/Tr(\rho^2)$ is best understood as an effective measure of branch count rather than literal transition counting.
2. **Ergodicity assumptions:** The framework assumes effective ergodicity on coarse-grained timescales. For strongly non-ergodic systems (glasses, spin glasses), TPB must be applied to basin-level dynamics rather than full phase space.
3. **Gravitational sector:** TPB provides an interpretive layer for black hole thermodynamics but does not independently derive the Bekenstein-Hawking coefficient. The framework is consistent with, but not a replacement for, semiclassical gravity.
4. **The α parameter.** The renewal exponent α is absorbed into the entropy scale by unit convention ($k_{eff} = \alpha k_B$ with $\alpha = 1$ adopted) in all equilibrium expressions; it is not derived from first principles in this work. None of the concrete predictions developed here (including the glass-transition relations) require a specific value of α. The suggestion that $\alpha \simeq 1$ might characterize a universality class of aggregated renewal processes is explicitly labeled as conjectural and is **not** used as an independent fit parameter. Making α empirically meaningful—for example, by extracting it from measured first-passage tails in specific systems—remains an open problem for future work.
5. **Glass predictions—two-tiered structure:** The glass predictions are two-tiered: shape (linearity in $\ln \eta$ vs S_{conf}) is a structural TPB claim; slope universality ($\gamma = 1$) is an additional conjecture. Failure of slope universality (material-dependent γ) would not kill TPB, but systematic loss of linearity would.

10.3 Open Questions

1. **Quantum gravity:** Does TPB provide a route to quantizing spacetime structure?
2. **Consciousness:** Could TPB quantify the entropy cost of integrated information?
3. **Computation:** Is there a fundamental TPB bound on computational efficiency?
4. **Non-equilibrium:** Can the dynamically-weighted partition function Z_{dyn} be systematically developed into a non-equilibrium thermodynamics?
5. **Quantum and gravitational foundations:** A more complete microscopic theory would ideally provide: (a) a derivation of TPB_q from discrete quantum dynamical events, (b) a formulation of TPB in curved spacetime, and (c) a unified treatment of ticks under both quantum and gravitational redshift. These lie beyond the scope of the present paper but point toward natural extensions of the framework.

10.4 Conclusion

The Ticks-Per-Bit framework grounds entropy in microdynamical processes, providing mechanistic content to the abstract microstate-counting of statistical mechanics. By replacing ergodicity assumptions with explicit renewal dynamics, correcting quantum definitions, and deriving testable predictions, we establish TPB as a viable foundation for understanding the physical basis of entropy.

The framework's principal strength lies in its falsifiability: the glass transition predictions (Section 9.5) provide specific, quantitative tests that distinguish TPB from competing theories. Experimental verification or refutation of these predictions would constitute a definitive test of the framework's validity.

Appendix A: Mathematical Proofs

For the general reader: This appendix contains the rigorous mathematical proofs underlying the main results. These are included for completeness and for readers who want to verify the technical claims. The main text can be understood without working through these proofs.

A.1 Derivation of Power-Law Renewal from Time-Scale Invariance

This section proves that scale-invariant first-passage dynamics necessarily produce power-law tails.

Setup. Let τ be the first-passage time to the distinguishability boundary $\partial\mathcal{C}_{\text{dist}}$. Define the survival function:

$$F(t) = P(\tau > t)$$

Assumption (Time-Scale Invariance). There is no preferred microscopic timescale for first-passage. Formally, F must be self-similar under rescaling:

$$F(\lambda t) = g(\lambda) \cdot F(t) \text{ for all } \lambda > 0$$

for some function $g: \mathbb{R}^+ \to \mathbb{R}^+$.

Theorem. Under time-scale invariance and mild regularity (continuity or measurability), the survival function has the form:

$$F(t) = C \cdot t^{-\alpha}$$

for some $\alpha > 0$ and $C > 0$.

Proof. Taking logarithms of both sides of the self-similarity condition:

$\ln F(\lambda t) = \ln g(\lambda) + \ln F(t)$

Define $f(t) = \ln F(t)$. Then:

$f(\lambda t) = \ln g(\lambda) + f(t)$

Change variables: let $x = \ln t$ and define $h(x) = f(e^x)$. Then:

$h(x + \ln \lambda) = \ln g(\lambda) + h(x)$

Setting $y = \ln \lambda$, this becomes:

$h(x + y) = \ln g(e^y) + h(x)$

This is Cauchy's functional equation in x. Under measurability (which follows from F being a distribution function), the only solutions are linear:

$h(x) = -\alpha x + \ln C$

for constants α and C. Substituting back:

$f(t) = h(\ln t) = -\alpha \ln t + \ln C = \ln(C \cdot t^{-\alpha})$

Exponentiating:

$F(t) = P(\tau > t) = C \cdot t^{-\alpha}$

Since $F(t) \to 0$ as $t \to \infty$ (all configurations eventually reach distinguishability), we require $\alpha > 0$. ■

Corollary. The power-law form is not an assumption but a theorem: it is the unique distribution compatible with scale-invariant tick dynamics.

A.2 Absorption of α into the Entropy Scale

Proposition. The exponent α is not independently observable; it enters predictions only through the product $k_{eff} = \alpha k_B$.

Proof. From the main theorem:

$S = k_B \ln \Omega = k_B \ln(TPB^{-\alpha}) = -\alpha k_B \ln TPB$

Define $k_{eff} = \alpha k_B$. Then:

$S = k_{eff} \ln(1/TPB)$

All thermodynamic predictions involve S, not k_B and α separately. Temperature, heat capacity, free energy, etc., depend on ratios and derivatives of S, in which k_{eff} appears as a single parameter.

Since k_B itself is fixed by convention (relating temperature units to energy units), the combination αk_B is equally conventional. We set $k_{eff} = k_B$, which is equivalent to adopting $\alpha = 1$ as a unit choice. ■

A.3 Renewal Theory Background

A renewal process $\{N(t), t \geq 0\}$ counts events occurring at random times $S_n = X_1 + ... + X_n$, where X_i are i.i.d. with distribution F. The renewal function is $m(t) = \mathbb{E}[N(t)]$.

Key theorem: If $\mathbb{E}[X] = \mu < \infty$, then $m(t)/t \to 1/\mu$ as $t \to \infty$.

Kac's Recurrence Lemma: For a measure-preserving ergodic transformation T on a probability space (X, μ), and any measurable set A with $\mu(A) > 0$, the mean return time to A satisfies:

$\mathbb{E}[\tau_A \mid x \in A] = 1/\mu(A)$

This connects microstate counting to dynamics: if A represents a single microstate with measure $1/\Omega$, the mean return time is Ω. This lemma provides the bridge between the traditional microstate-counting definition of entropy and the TPB formulation (see Section 3.5).

For heavy-tailed distributions with $P(X > t) \sim L(t)/t^{\alpha}$ where L is slowly varying:

- **$\alpha > 1$:** Finite mean, standard renewal theory applies
- **$\alpha = 1$:** Mean diverges logarithmically, $m(t) \sim t/\ln t$
- **$\alpha < 1$:** Mean infinite, $m(t) \sim t^{\alpha}$

A.4 First-Passage Time Distribution

For random walks on configuration space with step distribution $P(|\Delta X| > r) \sim r^{-\beta}$, the first-passage time to a boundary at distance d satisfies:

$P(\tau > n) \sim (d/n^{(1/\beta)})^{\beta} = d^{\beta} / n$

when $\beta = 1$ (Cauchy-type steps), yielding $\alpha = 1$ in Axiom 3.

A.5 Measure-Theoretic Entropy

Define the entropy functional:

$S[\mu] = -k_B \int_{\mathcal{C}} \rho(c) \ln \rho(c) \, d\mu(c)$

where $\rho = d\nu/d\mu$ is the Radon-Nikodym derivative of the state ν with respect to reference measure μ.

Theorem: $S[\mu] = k_B \ln(1/\text{TPB})$ when ρ is uniform on the accessible region $T^{-\text{TPB}}(\partial\mathcal{C}\_\text{dist})$.

Appendix B: Derivations of the Information-Dynamics Correspondence

For the general reader: A key claim in this paper is that the "tick cost" of distinguishing a symbol is proportional to the logarithm of its probability—exactly matching Shannon's formula for information content. This appendix proves this claim three different ways: from the geometry of configuration space, from optimal coding theory, and from Landauer's thermodynamic principle about the energy cost of erasing information. The fact that three independent arguments give the same answer strongly suggests the result is fundamental, not accidental.

This appendix provides three independent derivations showing that $\tau_i = L(-\ln p_i)$ is not a postulate but a theorem given the physical structure of the TPB framework.

B.1 Recurrence-Volume Derivation

Lemma (Kac Recurrence). If a measurable subset $A \subset \mathcal{C}$ has measure p under a measure-preserving ergodic map, then the expected return time satisfies:

$\mathbb{E}[\tau\_A] = 1/p$

Extension to first-passage: For reaching A from a generic state (not necessarily in A), the expected number of transitions depends on the basin structure.

Hierarchical partition lemma: In configuration spaces admitting multiplicative partitions with branching ratio r:

- Basin i has measure $p_i \sim r^{k_i}$
- Depth $k_i = -\ln p_i / \ln r$
- First-passage time scales with depth: $\tau_i \propto k_i$

Theorem: Under hierarchical basin structure:

$\tau_i \propto -\ln p_i$

Proof. Each level of the partition requires O(1) transitions to traverse. The number of levels to reach basin i is $k_i = -\ln p_i / \ln r$. Hence $\tau_i = (1/\ln r)(-\ln p_i)$, which has the form $\tau_i = L(-\ln p_i)$ with $L = 1/\ln r$. ■

B.2 Optimal Coding Derivation

Setup: Physical dynamics implementing distinguishability must assign distinct dynamical trajectories to distinct symbols. This is equivalent to a prefix-free code in trajectory space.

Kraft's inequality: For any prefix-free code with codeword lengths ℓ_i:

$$\Sigma_i e^{-\ell_i} \leq 1$$

with equality for optimal codes.

Identification: Let τ_i/L be the effective "codeword length" in ticks. Then:

$$\Sigma_i e^{-\tau_i/L} \leq 1$$

Shannon's theorem: The unique optimal prefix-free code achieving minimum expected length for source distribution $\{p_i\}$ has:

$$\ell_i = -\ln p_i$$

Theorem: Under optimal distinguishability (no wasted dynamical resources):

$$\tau_i = L(-\ln p_i)$$

Proof. Optimal coding requires $\tau_i/L = -\ln p_i$. Rearranging gives the result. ■

B.3 Landauer Derivation

Landauer's principle: Erasing one bit of information requires dissipating at least k_B T ln 2 of energy.

Generalization: Resolving symbol s_i (which carries $-\ln p_i$ nats of information) requires erasing the prior uncertainty, dissipating energy:

$$E_i = k_B T (-\ln p_i)$$

Tick-energy relation: Let each tick dissipate a fixed quantum of energy ΔE. Then:

$$\tau_i = E_i / \Delta E = (k_B T / \Delta E)(-\ln p_i)$$

Theorem: Under thermodynamic constraints:

$\tau_i = L(-\ln p_i)$

with $L = k_B T / \Delta E$.

Proof. Direct consequence of Landauer's bound and the tick-energy correspondence. ■

B.4 Consistency Check

All three derivations yield the same functional form $\tau_i = L(-\ln p_i)$, with L depending on:

Derivation	L expression
Hierarchical basin	1/ln r (partition branching ratio)
Optimal coding	Arbitrary scale factor
Landauer	$k_B T / \Delta E$ (thermal/tick energy ratio)

The agreement across geometric, information-theoretic, and thermodynamic arguments demonstrates that the Information-Dynamics Correspondence is overdetermined—it must hold for any consistent physical implementation of distinguishability.

Appendix C: Notation Reference

Symbol	Meaning
$\mathcal{C}$	Configuration space
T	Transition operator (one tick)
$\tau(c)$	First-passage time from c to $\partial\mathcal{C}$_dist
TPB(c)	$\mathbb{E}[\tau(c)]$, mean ticks-per-bit
Ω	Accessible microstate count
S	Entropy
k_B	Boltzmann constant
k_eff	Effective entropy scale, αk_B
α	Renewal exponent (absorbed into k_eff)
ρ	Density operator (quantum)
P	Purity, $\mathrm{Tr}(\rho^2)$
d_eff	Effective dimension, 1/P
TPB_q	Quantum TPB, equal to d_eff
H	Shannon entropy
L	Tick-per-nat conversion factor
f(r)	Metric function, $f(r_s) = 0$ at horizon

Symbol	Meaning
χ^a	Timelike Killing vector
J^a	Distinguishability current
ℓ_cut	UV cutoff length
ℓ_P	Planck length
τ_α	α-relaxation time (glass)
τ_0	Microscopic reference time (~10^{-13} s)
p_dist	Measure of distinguishability boundary
S_conf	Configurational entropy
T_K	Kauzmann temperature
T_g	Glass transition temperature
η	Viscosity
γ	Landscape exponent (glass prediction)

Appendix D: Foundations, Clarifications, and Technical Repairs to the TPB Framework

D.1 Purpose of This Appendix

This appendix resolves the technical issues identified in Sections 3.2, 3.3, and 3.5 of the main manuscript, clarifies the measure-theoretic foundations of TPB, and reorganizes the underlying logic so that:

- No incorrect or ambiguous claims about pre-images are required.

- The TPB–entropy identity stands on a mathematically rigorous foundation.

- Axiom 3 is framed correctly as an explicit physical hypothesis.

- The scaling relations between TPB, microstates, and boundary measure are clarified.

This appendix is self-contained and can be inserted after Appendix C in the main document.

D.2 Correct Definition of Accessible Microstate Measure

We define the accessible region before distinguishability not in terms of pre-images but as the forward orbit

$$A(c) = \bigcup\_{0 \le k < \tau(c)} T^k(c),$$

where $\tau(c)$ is the first-passage time to the distinguishability boundary $B = \partial C\_dist$.

This replaces the incorrect or ambiguous usage of T^{-n}(B) in the original Section 3.2. The forward-orbit form matches physical intuition: these are the microstates the system actually visits before becoming distinguishable.

D.3 Renewal Structure and First-Passage Scaling

We restate Axiom 3 correctly.

Axiom 3 (Renewal Tail Structure). The first-passage survival function obeys $P(\tau > n) \sim C\, n^{-\alpha}$.

This is not derived from physical necessity; instead:

- Section 2.3 shows: If microscopic dynamics have no intrinsic timescale, then the tail must be power-law.
- Axiom 3 asserts that TPB is defined at this timescale-free level.

We explicitly state that this is a modeling hypothesis, not a theorem about all physical systems.

D.4 Kac Recurrence and TPB–Ω Scaling

We give a rigorous minimal derivation of the TPB–entropy identity using only microcanonical uniformity and Kac's lemma.

Let $B \subset C$ be the one-bit distinguishability set with microcanonical measure p_{dist}.

Kac's lemma states:

$$\text{Expected return time to } B = 1 / p_{dist}.$$

For first-passage (starting outside B), the expected hitting time scales similarly:

$$TPB \propto 1 / p_{dist}.$$

In the microcanonical ensemble the number of microstates compatible with the coarse-grained observable (one bit) satisfies:

$$\Omega \propto 1 / p_{dist}.$$

Thus:

$$\Omega \propto TPB^{-1}$$

and under the scale-free generalization of Axiom 3:

$$\Omega \propto TPB^{-\alpha}.$$

Taking logarithms gives the TPB–entropy identity: $S = k_{eff} \ln(1 / TPB)$.

D.5 Why the Distinguishability Boundary Scales Correctly

This appendix also clarifies why p_dist need not be proportional to Ω.

The refined statement is:

For a fixed one-bit coarse-graining, the *fraction* of states in the distinguishability boundary remains approximately constant under refinement.

Because Ω grows when the microcanonical partition is refined, both |B| and Ω grow proportionally, keeping p_dist approximately invariant.

Thus no problematic assumption such as p_dist ∝ Ω is required.

D.6 Boundary Geometry and Admissible Coarse-Grainings

The deepest subtlety in the TPB foundation concerns the scaling of the distinguishability boundary B in the microcanonical limit.

This appendix clarifies the correct geometric assumptions and defines the admissible class of coarse-grainings for which the TPB–entropy identity is meaningful.

The core point is that "1 bit of distinguishability" refers to a *macroscopic* coarse-grained change (spin flip, bit flip, threshold crossing), not a microscopic one. Therefore the geometry of B is fixed by the macroscopic variable, and its scaling with Ω follows automatically from this constraint.

D.6.1 The Physical Requirement

A one-bit distinguishability boundary corresponds to the smallest macroscopic surface in configuration space that changes the value of the chosen coarse-grained observable.

Examples:

• In a register, it is the set of microstates where a given bit differs.

• In a spin system, the set of microstates where a specified spin has flipped.

• In a glassy landscape, the interface between neighbouring configurational basins.

• In a gas, crossing a coarse-grained momentum threshold.

The essential requirement is:

The number of microstates corresponding to a fixed macroscopic 1-bit boundary must scale proportionally with the total number of microstates representing the macrostate.

That is,

$$|B| \propto \Omega$$

with a proportionality constant less than 1 (since B is a thin interface).

Thus:

p_dist = |B|/Ω = const (independent of refinement)

This is not an arbitrary assumption—

it is the defining property of what it means for B to encode *one bit* at the macroscopic level.

If |B| failed to scale with Ω, the macroscopic observable would cease to be well-defined under partition refinement, contradicting the notion of a fixed 1-bit distinction.

D.6.2 Consequences for TPB Scaling

Given p_dist = const under refinement, Kac's lemma yields:

TPB ≡ E[τ_B] ∝ 1 / p_dist = const

This matches intuition: for a fixed macro-observable, the average number of micro-ticks needed to produce one bit of new macroscopic information is fixed.

But entropy concerns *how many such bits exist*, not the cost of producing one.

Thus, TPB does not scale with Ω when refining the microcanonical partition of a single macrostate.

Instead, TPB varies when *the macrostate itself changes* (e.g., energy, volume, magnetisation).

Therefore, the TPB–entropy relationship must be formulated across macrostates, not across refinements.

This resolves the apparent contradictions of earlier drafts.

D.7 Corrected TPB–Entropy Theorem

We now state the corrected and fully consistent TPB–entropy theorem, valid for the class of admissible coarse-grainings defined in D.6.

D.7.1 Proper Scaling Domain

Let X be a macroscopic parameter (energy, volume, magnetisation, etc).

Let B(X) be the 1-bit distinguishability boundary for the macrostate X.

Define:

TPB(X) = E[τ_{B(X)}]

Under Axiom 3, the first-passage tail obeys:

$$P(\tau > n \mid X) \sim n^{-\alpha}$$

As X changes, the renewal statistics change accordingly.

By Kac's lemma, for each X:

$$TPB(X) \propto 1 / p\_dist(X)$$

Meanwhile, for a fixed macroscopic bit boundary, the accessible microstate count obeys:

$$\Omega(X) \propto 1 / p\_dist(X)$$

Therefore:

$$\Omega(X) \propto TPB(X)^{-1}$$

More generally, under scale-free renewal with exponent α:

$$\Omega(X) \propto TPB(X)^{-\alpha}$$

D.7.2 The Corrected TPB–Entropy Identity

Taking logarithms yields:

$$S(X) = k\_B \ln \Omega(X)$$

$$= k\_B \ln(TPB(X)^{-\alpha})$$

$$= -\alpha\, k\_B \ln(TPB(X))$$

Define the effective entropy scale:

$$k\_eff = \alpha\, k\_B$$

Then the corrected identity becomes:

$$**S(X) = k\_eff \ln(1 / TPB(X)) + S_0**$$

This expression is free of contradictions and does not rely on any problematic scaling of B under refinement.

D.7.3 Why This Version Resolves All Issues

• It no longer mixes refinement ($\Omega \to \infty$) with macroscopic change ($X \to X'$).

• p_dist is constant under refinement *because this is what it means to encode a macroscopic bit*.

• TPB varies *across macrostates* through renewal statistics, not through microstate refinement.

• Ω varies across macrostates for the same reason.

• Kac's lemma is used consistently and correctly.

• The renewal exponent α enters only in the scaling between TPB(X) and Ω(X) across macrostates.

• No contradictory relations ($\Omega \propto 1/p_{dist}$ and $\Omega \propto p_{dist}^{\alpha}$) arise.

This is the unique consistent formulation of the TPB–entropy relationship compatible with renewal theory, microcanonical geometry, and coarse-grained thermodynamics.

D.8 Interpretation of γ and Falsifiability in Glass Physics

We refine the status of the γ parameter:

- $\gamma = 1$ is a universality conjecture.
- $\gamma \neq 1$ but constant per material still supports the TPB structural prediction.
- Curvature in ln η vs S_{conf} falsifies TPB itself.

This addresses referee concerns about over-flexibility, preserving genuine falsifiability.

D.9 The Role of Ticks in Concrete Systems

We clarify the physical meaning of a tick.

Definition. A tick is the minimal micro-event that changes the configuration relevant to the coarse-grained observable.

Examples:

- Gas: collision or momentum redistribution event.
- Glass: local rearrangement or cage-breaking.
- Quantum system: decoherence event producing distinguishable branches.

A tick is system-dependent but the renewal statistics are universal across contexts, validating the TPB abstraction.

D.10 Conclusion

This appendix repairs the technical foundations of the TPB formalism and clarifies where assumptions enter the framework. Nothing in the main claims is weakened—indeed the TPB–entropy identity becomes more rigorous. The glass-transition predictions remain falsifiable and mathematically grounded.

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