Area Scaling of TPB-Realized Information and First-Principles Origin of *f*

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Technical Appendix to Companion Paper III: *"Structural Sufficiency, Risk Concentration, and Representation Selection in the One-Fold Framework"*

Abstract

The One-Fold framework derives the internal structure of physical law from five admissibility conditions on a local BCB Hamiltonian acting on $\ell^2(\Lambda) \otimes \mathbb{C}^4$. Companion Paper III establishes that this Hamiltonian class generically produces a unique vacuum (V1), a $3\oplus1$ internal block structure, and a dynamically stable complex excitation representation (GG3). The accompanying Dynamical Forcing Supplement provides the proofs of these results but defers one critical quantity: the vacuum-energy parameter *f*, whose smallness underpins the framework's account of the cosmological constant.

This appendix closes that gap. We show that TPB coarse-graining, combined with locality and finite on-site dimension, induces a realized-information bound that scales with boundary area rather than bulk volume — a holographic area law derived without gravitational input. The ratio of area-bounded realized information to volume-scaled bulk capacity yields $f \sim \ell\_P / R \sim 10^{-61}$–$10^{-62}$, reproducing the observed value of $\Lambda\_obs / \Lambda\_Planck \sim 10^{-122}$ without fine-tuning. The derivation relies only on the admissibility conditions already established in the companion papers, requires no additional postulates, and identifies the horizon-dependence of *f* as a concrete observational prediction.

This document is intended to be read alongside the Dynamical Forcing Supplement and Companion Paper III, completing the quantitative chain from microscopic admissibility to macroscopic cosmological parameters.

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1. Purpose and Context

This appendix establishes that TPB coarse-graining acting on a local BCB Hamiltonian induces an effective realized-information bound that scales with boundary area rather than bulk volume. The argument proceeds in four steps: (i) we define a notion of *realized classical information* appropriate to the BCB/TPB setting, (ii) we bound realized information by quantum mutual information across the region boundary, (iii) we bound that mutual information by a boundary capacity set by locality and finite on-site dimension, and (iv) we combine these bounds to obtain area scaling and extract the vacuum-energy parameter *f* as a scaling relation.

The result supplies a quantitative anchor for the smallness of *f* that was deferred in the Dynamical Forcing Supplement, and establishes holographic information scaling as a structural consequence of the One-Fold framework rather than an additional postulate.

2. Hamiltonian and Coarse-Graining Setup

We work on the global Hilbert space $\ell^2(\Lambda) \otimes \mathbb{C}^4$, where Λ is a *d*-dimensional lattice ($d = 3$ throughout). The Hamiltonian *H* belongs to the admissible class $\mathcal{H}$_BCB defined in the Dynamical Forcing Supplement: it is local with finite interaction range, Hermitian, preserves the primary bit *b* and direction label *d*, and admits a TPB coarse-graining map $\mathcal{R}$_ℓ at scale ℓ. The on-site Hilbert space $\mathbb{C}^4$ carries $\log_2 4 = 2$ bits of raw capacity per site.

Successive applications of $\mathcal{R}$_ℓ produce an effective Hamiltonian *H*_ℓ whose interactions remain local at the coarse scale. We denote by ρ_ℓ the effective state at coarse-graining scale ℓ.

3. Definition of TPB-Realized Information

Definition (Realized Information). A *realized classical bit* in a spatial region *A* at TPB scale ℓ is a two-outcome observable *O* supported in *A* satisfying three conditions:

1. **Local readability.** O is measurable by operations supported entirely within A.
2. **Dynamical stability.** The expectation value $\langle O \rangle$ remains stable under the coarse-grained dynamics generated by H_ℓ for at least $T \gg 1$ TPB steps.
3. **Correlation support.** The distinguishability of the two outcomes of O cannot be removed by any local unitary acting within A alone.

The *total realized information* $N\_\text{real}(A; \ell)$ is the maximum number of independent realized bits that can coexist in region A.

Condition (3) is the key structural requirement. It ensures that realized bits are backed by genuine correlations with the exterior, not by internal microstate labels that are artifacts of a basis choice. Observables satisfying (1) and (2) but failing (3) can be eliminated by local redefinition and therefore do not represent physically accessible classical information.

4. Realized Information Is Bounded by Mutual Information

Lemma 1 (Realized Bits ≤ Mutual Information) [Sketch]. Let A be a spatial region and A^c its complement. Then

$$N\_\text{real}(A; \ell) \leq I\_\ell(A : A^c) / \ln 2,$$

where $I\_\ell(A : A^c) = S(\rho\_A) + S(\rho\_A^c) - S(\rho\_AA^c)$ is the quantum mutual information computed from the TPB-coarse-grained state ρ_ℓ.

Argument. Each realized bit constitutes a one-bit classical channel readable from within A. By condition (3), the distinguishability of its two outcomes requires correlations across the A–A^c boundary: a realized bit uncorrelated with A^c defines a purely internal degree of freedom, and the partial trace over A^c during coarse-graining mixes such degrees of freedom within O(1) TPB steps, violating stability condition (2). Each independent bit that is both locally readable and stabilized by boundary correlations consumes at least ln 2 of mutual information. Since mutual information is additive over independent channels, the bound follows. Purely internal microstate entropy — entropy of ρ_A not backed by correlations with A^c — does not contribute to N_real.

5. Boundary Capacity Bound on Mutual Information

Lemma 2 (Boundary Capacity Bound) [Sketch]. For any Hamiltonian in $\mathcal{H}\_\text{BCB}$ with interaction range r, the mutual information across the A–A^c boundary satisfies

$$I\_\ell(A : A^c) \leq c(\ell) \cdot |\partial A|,$$

where $|\partial A|$ is the boundary area measured in units of $\ell\text{^}(d-1)$, and $c(\ell)$ is a scale-dependent constant of order O(1) per boundary block.

Argument. Locality of H implies a Lieb–Robinson bound on information propagation: correlations between A and A^c are mediated exclusively by degrees of freedom within a boundary layer of thickness O(r). Under TPB coarse-graining at scale $\ell \geq r$, this layer is captured by boundary blocks. The number of such blocks scales as $|\partial A| / \ell^{(d-1)}$. Each block has finite-dimensional Hilbert space (dim $\mathbb{C}^4$ per constituent site), so the mutual information contributed per block is bounded above by $2 \cdot \log_2 4 = 4$ bits. Summing over boundary blocks yields the bound with $c(\ell)$ determined by the on-site dimension and effective interaction strength at scale ℓ.

The O(1) character of $c(\ell)$ is not incidental: it is enforced by the finiteness of the on-site Hilbert space $\mathbb{C}^4$. No matter how strongly boundary blocks interact, the mutual information per block pair cannot exceed $2 \cdot \log \dim(\mathbb{C}^4) = 4$ bits. This prevents $c(\ell)$ from growing under coarse-graining and is essential for the numerical estimate in §7.

6. The Area Law for TPB-Realized Information

Theorem (Area Scaling of Realized Information) [Sketch]. For any region A in a state produced by TPB-coarse-grained evolution of an admissible Hamiltonian,

$$N_{\text{real}}(A; \ell) \leq [\, c(\ell) / \ln 2 \,] \cdot |\partial A|.$$

Proof. Direct composition of Lemma 1 and Lemma 2. ■

The bound has two immediate consequences. First, as region size increases at fixed lattice scale, the realized-information *density* (per unit volume) vanishes: $N_{\text{real}} / V \sim |\partial A| / V \sim 1/R$, where R is the linear size of A. Second, the bound is *universal* within $\mathcal{H}_{\text{BCB}}$ — it depends on the admissibility conditions and on-site dimension, not on specific Hamiltonian parameters.

7. First-Principles Scaling of the Vacuum-Energy Parameter *f*

Within the One-Fold framework, the vacuum-energy parameter f is defined as the ratio of realized information to total fold capacity in a region of linear size R:

$$f \equiv N_{\text{real}} / N_{\text{capacity}}.$$

Bulk capacity. Each Planck-scale site contributes $\log_2 4 = 2$ bits of raw fold capacity. For a three-dimensional region of linear size R, the total number of sites scales as $(R / \ell_P)^3$, giving

$$N_{\text{capacity}} \sim (R / \ell_P)^3.$$

Realized information. By the Area Scaling Theorem, with $c(\ell) \sim O(1)$ per boundary Planck cell (enforced by the four-dimensional on-site space, as established in §5),

$$N_{\text{real}} \sim (R / \ell_P)^2.$$

The scaling relation. Taking the ratio,

$f \sim (R / \ell\_P)^2 / (R / \ell\_P)^3 = \ell\_P / R$.

Numerical estimate. For the cosmological horizon $R \approx 4.4 \times 10^{26}$ m and the Planck length $\ell\_P \approx 1.6 \times 10^{-35}$ m,

$f \sim (1.6 \times 10^{-35}) / (4.4 \times 10^{26}) \sim 10^{-61} - 10^{-62}$.

This falls within the range required to reproduce the observed cosmological constant via the vacuum-energy relation $\Lambda = C \cdot f^2 \cdot \Lambda\_\text{Planck}$, yielding $\Lambda\_\text{obs} / \Lambda\_\text{Planck} \sim 10^{-122}$ without fine-tuning.

Status of the estimate. This is an order-of-magnitude scaling result, not a precision prediction. The O(1) prefactors — set by $c(\ell)$, the exact definition of fold capacity, and geometric factors — are uncontrolled at this level of argument. What is derived from structure is the *exponent*: the 10^{-61} scaling follows from locality, finite on-site dimension, three spatial dimensions, and TPB coarse-graining, without inserting Λ_obs by hand. Note that the dimensional dependence is a feature: the area-to-volume ratio ℓ_P / R is specific to $d = 3$ spatial dimensions, tying the cosmological constant scaling to the same spatial dimensionality derived in the Dynamical Forcing Supplement.

8. Scale Dependence and the Role of *R*

The scaling $f \sim \ell\_P / R$ raises a natural question: if R is identified with the cosmological horizon, and the horizon evolves in time, does f inherit a cosmological time dependence?

Within the One-Fold framework, the answer is structurally constrained. The horizon radius $R(t)$ sets the largest scale at which TPB coarse-graining produces a stable effective description. As R grows, the ratio of boundary to bulk degrees of freedom decreases, and f decreases accordingly. This implies a *slowly decreasing effective cosmological constant*, consistent with the general behaviour of late-time de Sitter expansion in which the horizon asymptotes to a constant value.

Whether this time dependence produces observable deviations from a strict cosmological constant is a quantitative question that depends on the rate of horizon evolution relative to TPB coarse-graining timescales. This constitutes a concrete observational target: precision measurements of the dark energy equation of state parameter w could in principle distinguish a strictly constant Λ from the slow logarithmic drift predicted by $f(t) \sim \ell\_P / R(t)$. We flag this as an open problem for future work.

9. Connection to Holographic Bounds

The area scaling derived here is structurally analogous to — but logically independent of — the holographic entropy bounds of 't Hooft, Susskind, and Bousso. In the standard holographic programme, area scaling of entropy is motivated by black hole thermodynamics and the covariant entropy bound. Here, it arises from a different starting point: locality of the Hamiltonian, finite on-site dimension, and TPB coarse-graining.

The two routes converge on the same conclusion (information content bounded by area), which constitutes a non-trivial consistency check on the One-Fold framework. However, the present derivation is more restrictive in one respect — it bounds *realized* information rather than total entropy — and more general in another — it does not require a gravitational or black hole context. Whether the two bounds can be shown to be equivalent under appropriate conditions is an open question.

10. Summary

The central results of this appendix are:

1. **Realized information**, defined by local readability, dynamical stability, and correlation support, is bounded above by the quantum mutual information across the region boundary (Lemma 1).
2. **Locality and finite on-site dimension** bound this mutual information by boundary area with an O(1) coefficient per Planck cell (Lemma 2).
3. **Area scaling** of realized information follows by composition (Theorem).
4. **The vacuum-energy parameter** $f \sim \ell\_P / R \sim 10^{-61} - 10^{-62}$ emerges as a scaling relation from the area-to-volume ratio, reproducing the observed cosmological constant without fine-tuning.
5. **Scale dependence** of f on the horizon radius constitutes a concrete observational prediction distinguishable in principle from a strict cosmological constant.

The remaining open problems are: (a) an explicit RG calculation (e.g. tensor-network renormalization) confirming the stability of $c(\ell) \sim O(1)$ under iterated coarse-graining, and (b) a controlled computation of the O(1) prefactors to sharpen the estimate from order-of-magnitude to quantitative prediction.