Dimensional Emergence Calculus: From Informational Ordering to Newtonian Gravity

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Abstract for General Readers

We take for granted that objects move smoothly through three-dimensional space as time flows forward. But why three dimensions? Why does time flow at all? And why does gravity pull things together with a strength that weakens as the square of distance?

This paper proposes that these familiar features of reality are not built into the universe from the start. Instead, they emerge from something simpler: the accumulation of distinctions.

Imagine reality as a process of decisions. Before a quantum measurement, an electron might be "here" or "there"—the distinction hasn't been made yet. When the measurement happens, the universe commits to one option. We call each such commitment a "bit" of realised information. Between these commitments, the universe explores possibilities through "ticks" of logical ordering.

The ratio of ticks to bits—how much exploring happens before each decision—turns out to be crucial. Where this ratio is moderate, things behave normally: time flows, objects move smoothly, and physics works as expected. Where the ratio becomes extreme (near black holes, or at the moment of quantum measurement), ordinary physics breaks down.

The central contribution of this paper is a new mathematical framework called **Dimensional Emergence Calculus (DEC)**. Standard calculus assumes continuous time and space already exist; DEC shows how they can arise from a simpler two-dimensional "ordering space" that tracks exploration and commitment separately. The key mechanism is the "lift"—a mathematical map that converts steps in ordering space into movement through physical space. When this lift has a particular geometric property (non-integrability, or "curl"), it generates forces. Objects don't just move; they accelerate.

We show that when you set up DEC with minimal assumptions—the simplest possible rules consistent with symmetry—inverse-square gravity emerges as a natural limit rather than an independent postulate. The mathematics that Newton invented to describe gravity turns out to be

a special case of DEC, valid only when information is being processed smoothly and continuously.

This matters because it suggests gravity isn't a fundamental force that needs its own explanation. It's a natural consequence of how information organises itself into spatial structure. The paper identifies specific conditions where gravity should deviate from Newton's law—providing concrete predictions that future observations could test.

In essence: DEC reveals how the calculus of motion, three-dimensional space, and gravity may all be what information looks like when there's enough of it, organised in the right way.

Abstract

We develop a differential calculus appropriate to frameworks in which time and space are not fundamental but emerge from deeper informational structures. Starting from the Ticks-Per-Bit (TPB) primitive—a ratio of causal ordering steps to irreversible distinguishability commitments—we construct differential operators that reduce to standard calculus only in appropriate coarse-grained limits. We then extend this to a two-channel ordering space (TPB^2), where reversible exploration and irreversible commitment constitute independent ordering parameters. Overlap and quantum phase are modelled as a U(1) fiber bundle over this base space, with connection curvature encoding interference geometry. The central construction is the Dimensional Emergence Calculus (DEC), which provides a geometric "lift" mapping two-dimensional ordering into three-dimensional spatial displacement. We demonstrate that the non-integrability of this lift—the differentiation curl—generates force-like effects in emergent space. As a concrete application, we show that inverse-square gravitational scaling follows under minimal symmetry assumptions: radial alignment of the lift with a harmonic void field satisfying Laplace's equation. The framework thus provides a falsifiable route from informational foundations toward observable gravitational phenomenology, with explicit conditions under which non-Newtonian corrections would arise.

Scope

This paper is a mathematical construction and consistency demonstration. Except where explicitly stated (e.g., the three assumptions in §10), it does not assert unique microphysical laws, only minimal structures sufficient to recover standard limits. The goal is to establish that a coherent calculus exists in which time, space, and Newtonian gravity emerge from informational ordering—not to claim that nature must implement this particular structure.

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1. Introduction

1.1 The Problem of Assumed Continuity

Standard differential calculus presupposes the existence of a continuous time parameter along which physical quantities evolve smoothly. This assumption, inherited from Newton and Leibniz, has proven extraordinarily successful across classical mechanics, electromagnetism, and general relativity. Yet the assumption itself is never derived—it is simply posited as part of the mathematical infrastructure.

Several lines of contemporary inquiry suggest this assumption may be contingent rather than fundamental. Quantum gravity approaches frequently encounter discreteness at the Planck scale [14, 15]. Information-theoretic reconstructions of quantum mechanics point toward distinguishability as a primitive concept [10, 11, 12]. Black hole thermodynamics implies deep connections between geometry, entropy, and information [6, 20, 21]. These developments invite the question: can we construct a calculus that does not assume continuous time, but instead derives temporal and spatial continuity as emergent phenomena?

1.2 The VERSF Framework

The Void Energy-Regulated Space Framework (VERSF) proposes that spacetime emerges from a more primitive informational substrate characterised by two fundamental fields: the void field φ (representing proximity to the zero-entropy void substrate) and the entropy density field s (representing realised distinguishability). In this framework, what we experience as continuous space and time arises through the accumulation and organisation of distinguishability.

The present paper develops the mathematical machinery required to make this emergence precise. We introduce a calculus based on ordering and commitment rather than space and time, show how standard calculus emerges as a limiting case, and demonstrate that gravitational phenomena arise naturally from the geometry of this deeper structure.

1.3 Overview

Section 2 introduces the primitive measures—ticks and bits—and defines the fundamental TPB ratio. Section 3 constructs differential operators on these measures and establishes the chain rule connecting them. Section 4 shows how ordinary time and the standard derivative d/dt emerge through calibration. Section 5 presents a toy model illustrating the breakdown of standard calculus at crystallisation events. Section 6 develops the covariant formulation suitable for field-theoretic treatment. Section 7 extends to two-channel ordering (TPB^2), distinguishing reversible exploration from irreversible commitment. Section 8 models overlap and phase as a U(1) fiber bundle over TPB^2. Section 9 introduces the Dimensional Emergence Calculus proper, defining the lift from ordering space to physical displacement. Section 10 recovers the Newtonian inverse-square law under explicit assumptions. Section 11 summarises the integrated framework. Section 12 discusses the relationship to classical calculus. Section 13 presents DEC-specific predictions, discusses the deeper significance of treating calculus as a physical phenomenon, and concludes.

1.4 Notation

Symbol	Definition	First appears
T	Tick measure (causal ordering count)	§2
B	Bit measure (irreversible commitments)	§2
τ	TPB ratio: $\Delta T/\Delta B$ (or dT/dB in continuum limit)	§2
T_r	Reversible ordering parameter	§7
T_i	Irreversible ordering parameter	§7
κ	Calibration constant (time per bit)	§4
φ	Void field (proximity to zero-entropy substrate)	§6
s	Entropy density field	§6
A	U(1) connection on TPB^2	§8
F	Curvature two-form of connection A	§8
χ	Overlap-validity function	§8
$\mathbf{e}_r$, $\mathbf{e}_i$	DEC lift vectors	§9
$\mathbf{\Omega}$	Differentiation curl (lift non-integrability)	§9

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2. TPB Primitives and Measures

Along any physical history γ through configuration space, we define two monotone measures that capture distinct aspects of change.

Definition 2.1 (Tick measure). The tick measure $T: \gamma \rightarrow \mathbb{Z}_{\geq 0}$ is a count of causal ordering steps. Ticks establish the sequence in which events occur—they answer the question "what comes before what?"—without themselves constituting irreversible change. Ticks may be thought of as the minimal units of logical or causal precedence.

Definition 2.2 (Bit measure). The bit measure $B: \gamma \rightarrow \mathbb{Z}_{\geq 0}$ is a count of realised distinguishability, or equivalently, of irreversible commitments. Each increment of B corresponds to a crystallisation event—an irreversible transition from reversible superposition to definite, irretrievable outcome (referred to as "Fold crystallisation" in the broader VERSF framework). Bits answer the question "what has been decided?"

These measures satisfy the following axioms:

Axiom 2.1 (Monotonicity). Both T and B are monotonically non-decreasing along any history γ:

$T(\gamma(s_1)) \leq T(\gamma(s_2))$ and $B(\gamma(s_1)) \leq B(\gamma(s_2))$ for all $s_1 < s_2$

Axiom 2.2 (Ordering precedence). Each bit commitment requires at least one ordering step:

$$\Delta B \geq 1 \Longrightarrow \Delta T \geq \Delta B$$

Motivation (status of Axiom 2.2). Axiom 2.2 is taken as a definitional constraint on what we mean by "ordering": a crystallisation event is an irreversible commitment that must be preceded by at least one ordering step, otherwise commitment would occur without any causal precedence structure. In this sense, T is not merely a counter of events but the minimal carrier of logical precedence. Alternative conventions are possible—for example, allowing $\Delta B > 0$ with $\Delta T = 0$ would correspond to modelling "instantaneous commitment" without antecedent ordering. Such alternatives collapse the distinction between ordering and commitment and are therefore excluded by construction in the present calculus.

Definition 2.3 (TPB ratio). The Ticks-Per-Bit ratio is defined as the finite-difference quantity:

$$\tau := \Delta T / \Delta B \text{ for } \Delta B \geq 1$$

This definition is canonically robust: it requires only that at least one bit has crystallised over the interval in question. When bit formation is sufficiently dense that B can be approximated as continuous, we pass to the continuum limit:

$$\tau = dT / dB$$

The continuum expression should be understood as a Radon-Nikodym derivative of the tick measure with respect to the bit measure [17], valid when many bits crystallise over the scale of interest. Formally, one may treat T and B as nondecreasing measures along histories; the continuum expressions apply in regimes where coarse-graining renders B absolutely continuous with respect to T (and vice versa on the relevant support), so that Radon-Nikodym derivatives exist.

Proposition 2.1 (TPB bounds). By Axiom 2.2, the TPB ratio satisfies $\tau \geq 1$ everywhere. Equality $\tau = 1$ represents maximal commitment efficiency (exactly one tick per bit). The limit $\tau \to \infty$ represents vanishing commitment rate.

The TPB ratio measures experienced time density: how many ordering steps are required to realise one unit of distinguishability. It is not a universal constant but a local field that varies with physical conditions:

- **Low τ** (few ticks per bit): Distinguishability crystallises rapidly. Systems quickly commit to definite outcomes. This characterises high-entropy-production regimes.
- **Moderate τ**: Systems explore configuration space extensively before committing. Reversible dynamics dominate, with occasional crystallisation events.

- **$\tau \to \infty$**: A regime where commitment is arbitrarily suppressed relative to ordering. This limit corresponds to perfect reversibility or, in gravitational contexts, the external description of approach to a horizon where infalling information appears frozen.

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3. TPB Differential Operators

For any observable $Q: \gamma \to \mathbb{R}$ evaluated along a history γ, we define two fundamental derivatives.

Definition 3.1 (Tick-derivative). The tick-derivative of Q is:

$$D\_T Q := dQ / dT$$

This measures the rate of change of Q per ordering step, irrespective of whether distinguishability is being realised. When T is discrete, D_T Q is defined as the forward difference (discrete derivative operator):

$$D\_T Q := Q(T + 1) - Q(T)$$

Definition 3.2 (Bit-derivative). The bit-derivative of Q is:

$$D\_B Q := dQ / dB$$

This measures the rate of change of Q per unit of irreversible commitment. In regimes where B is discrete, D_B Q is meaningful as a distribution (Dirac measure) concentrated on commitment events; the continuum expression applies when bit formation is coarse-grained.

Explicitly, for discrete B with crystallisation events at ticks $\{T_1, T_2, ...\}$:

$$D\_B Q = \Sigma_j \Delta Q_j \cdot \delta(T - T_j)$$

where $\Delta Q_j = Q(T_j^+) - Q(T_j^-)$ is the jump at the j-th crystallisation.

Theorem 3.1 (TPB chain rule). The tick and bit derivatives are related by:

$$D\_T Q = (dB/dT) \cdot D\_B Q = (1/\tau) \cdot D\_B Q$$

Proof. This follows directly from the chain rule for Radon-Nikodym derivatives. In the discrete case, the relationship holds in the distributional sense: the tick-derivative is smooth between crystallisation events and singular at them, with the singularity structure encoded by the concentrated bit-derivative. ■

Corollary 3.1. The tick-derivative is the more fundamental object: it captures change along the causal sequence, while the bit-derivative captures change relative to what has been irreversibly decided. The relationship between them is mediated by the local physics encoded in τ.

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4. Emergence of Ordinary Time and d/dt

To recover standard calculus, we must connect the informational measures T and B to the macroscopic time coordinate t that appears in laboratory physics.

Definition 4.1 (Calibration mapping). The calibration mapping relates bit measure to clock time via:

$$dt = \kappa \cdot dB$$

where $\kappa \in \mathbb{R}_+$ is a calibration constant with dimensions of time per bit (interpreted as $\Delta t = \kappa \, \Delta B$ in the discrete regime, and $dt = \kappa \, dB$ in the coarse-grained limit where B is approximately continuous). This constant maps increments of realised distinguishability to increments of clock time.

The physical interpretation of κ depends on the reference processes used for calibration. In principle, κ could be related to:

1. **Atomic transitions**: The frequency of a caesium hyperfine transition defines the SI second. This implicitly sets a relationship between distinguishability-producing atomic events and clock time.
2. **Planck-scale physics**: If bits correspond to Planck-scale degrees of freedom, κ could be related to the Planck time $t_P \approx 5.4 \times 10^{-44}$ s.
3. **Thermodynamic calibration**: In statistical mechanical terms, κ relates entropy production (in natural units) to elapsed time.

For the purposes of this framework, we treat κ as an empirical constant that encodes how the informational substrate interfaces with macroscopic measurement.

Theorem 4.1 (Emergence of d/dt). The standard time derivative emerges as a reparameterised tick-derivative:

$$d/dt = (1/\kappa) \cdot d/dB = (\tau/\kappa) \cdot D\_T$$

Proof. From $dt = \kappa \, dB$, we have $d/dt = (1/\kappa) \, d/dB$. Applying the TPB chain rule (Theorem 3.1) in reverse: $d/dB = \tau \cdot d/dT = \tau \cdot D\_T$. Substitution yields the result. ■

Corollary 4.1 (Validity conditions). The emergence of d/dt as a well-defined operator requires:

(i) Bit formation is dense: $|\Delta B| \gg 1$ over the scale of interest

(ii) TPB varies slowly: $|\nabla\tau| \cdot L \ll \tau$ for characteristic length L

(iii) Jumps are sub-resolution: $\max|\Delta Q_j| \ll \langle Q\rangle$

When these conditions fail, the ordinary time derivative d/dt loses meaning, and the more fundamental TPB calculus must be employed.

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5. Breakdown of Standard Calculus: A Crystallisation Model

To illustrate when and why standard calculus fails, consider a system in which an observable Q evolves continuously between discrete crystallisation events but exhibits discontinuous jumps when a bit crystallises.

Model specification. Let bit-formation events occur at ticks $T_1, T_2, T_3, \ldots$ with $\Delta B = 1$ at each event. Define the inter-crystallisation intervals $I_j := (T_j, T_{j+1})$.

Proposition 5.1 (Inter-event dynamics). On each interval I_j:

(i) The bit measure is constant: $dB = 0$

(ii) The tick measure advances: $dT > 0$

(iii) The observable evolves continuously: D_T Q is well-defined and finite

Proposition 5.2 (Crystallisation dynamics). At each crystallisation event T_j:

(i) A unit of distinguishability is committed: $\Delta B = 1$

(ii) The observable exhibits a discrete jump: $\Delta Q_j = Q(T_j^+) - Q(T_j^-)$

(iii) The bit-derivative is concentrated at T_j with weight ΔQ_j (see §3)

Theorem 5.1 (Failure of standard calculus). Under the calibration $dt = \kappa\, dB$:

(i) Between crystallisation events: $dt = 0$, hence dQ/dt is undefined (or infinite)

(ii) At crystallisation events: $dt = \kappa$, $dQ = \Delta Q_j$, hence $dQ/dt = \Delta Q_j/\kappa$ (a delta function in the continuum idealisation)

Proof. Part (i): Since dB = 0 on I_j, we have dt = κ dB = 0. Any nonzero dQ yields dQ/dt = dQ/0, which is undefined. Part (ii): At T_j, ΔB = 1 implies Δt = κ, while ΔQ = ΔQ_j. The ratio is well-defined but concentrated. ■

Corollary 5.1. Standard calculus presupposes continuous bit formation. When distinguishability is realised in discrete events, the continuous time parameter t becomes ill-defined, and dQ/dt fails as a mathematical object.

Remark 5.1 (Hybrid dynamical system). The correct mathematical description in the sparse-bit regime is a hybrid dynamical system [1, 2]: continuous evolution in T governed by D_T Q on each I_j, punctuated by a jump map Φ_j: $Q(T_j^-) \mapsto Q(T_j^+)$ at B events. This is a piecewise-smooth process in T with a jump measure supported on $\{T_j\}$.

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6. Covariant Formulation

To connect TPB calculus to field theory and general-relativistic contexts, we promote ticks and bits from measures along worldlines to spacetime currents.

Definition 6.1 (Tick current). The tick current J^u is a vector field representing the flow of causal ordering through spacetime.

Definition 6.2 (Bit current). The bit current J^{bu} is a vector field representing the flow of realised distinguishability.

In general, both currents satisfy balance laws rather than strict conservation.

Axiom 6.1 (Current balance). The currents satisfy:

$\nabla\_\mu J^u = \Sigma\_T$

$\nabla\_\mu J^{bu} = \Sigma\_B$

where Σ_T and Σ_B are source terms representing tick and bit creation or annihilation. Such sources could arise from measurement events, decoherence processes, or other commitment-generating phenomena.

Definition 6.3 (Closed regime). In the closed/coarse-grained regime analysed in this paper, we assume:

Σ_T = 0, Σ_B = 0

This yields the conservation laws:

$\nabla_\mu J^u = 0$

$\nabla_\mu J^{bu} = 0$

These assert that, in the regime of interest, ordering and distinguishability are neither created nor destroyed but only redistributed through spacetime.

Definition 6.4 (TPB constitutive law). The tick current couples to the VERSF fields via:

$J^u = \mathscr{F}(s, \varphi)\, \nabla^u s$

where s is the entropy density, φ is the void field, and $\mathscr{F}: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a constitutive function. We treat this as the lowest-order, local, entropy-aligned constitutive relation in a hydrodynamic/near-equilibrium gradient expansion; higher-derivative and nonlocal corrections are possible but are neglected here. The sign convention depends on whether J represents ordering flow toward increasing s or its negative; we absorb this choice into the definition of $\mathscr{F}$.

Constraints and status of $\mathscr{F}$. In this paper $\mathscr{F}(s,\varphi)$ is treated as a phenomenological transport function to be constrained by stability, symmetry, and ultimately data. Minimal requirements in the near-equilibrium/local regime include: (i) scalar dependence $\mathscr{F} = \mathscr{F}(s,\varphi)$ to preserve covariance; (ii) locality and regularity (no higher-derivative dependence at leading order); and (iii) sign/positivity chosen so that ordering flow aligns with the entropy-gradient convention adopted. The Newtonian-limit recovery in §10 is driven by the DEC lift structure and harmonic φ assumption; it does not require a specific functional form of $\mathscr{F}$ at leading order, though $\mathscr{F}$ will matter for dynamical coupling and strong-field regimes.

Definition 6.5 (Comoving densities). In a comoving frame with four-velocity u^u, define scalar densities:

$\rho_T := -u_\mu J^u$

$\rho_B := -u_\mu J^{bu}$

These represent the local density of ordering and commitment as experienced by a comoving observer.

Definition 6.6 (Local TPB field). The covariant TPB field is:

$\tau(x) := \rho_T / \rho_B$

This is the spacetime scalar generalisation of ΔT/ΔB, characterising local informational dynamics at each event.

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7. Two-Channel Ordering: The TPB² Base Space

The single TPB ratio captures the relationship between ordering and commitment but does not distinguish between different modes of ordering.

Definition 7.1 (Reversible ordering). The reversible ordering parameter $T_r \in \mathbb{R}_{\geq 0}$ measures exploration of configuration space without commitment. This corresponds to quantum superposition, thermal fluctuation, or any process that could in principle be reversed without thermodynamic cost.

Definition 7.2 (Irreversible ordering). The irreversible ordering parameter $T_i \in \mathbb{R}_{\geq 0}$ measures advancement along the commitment axis. Each increment of T_i corresponds to an approach toward crystallisation—a narrowing of possibilities that cannot be undone.

Remark 7.1. T_i is distinct from the bit measure B: T_i counts ordering steps along the commitment direction, while B counts completed commitments. The relationship is:

$$dB = f(T_i)\, dT_i$$

where $f: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ encodes the commitment rate (a hazard function).

Definition 7.3 (TPB² base space). The TPB² base space is the two-dimensional manifold:

$$M := \{(T_r, T_i) \in \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}\}$$

equipped with the natural smooth structure inherited from $\mathbb{R}^2$.

Proposition 7.1 (Minimality). TPB² is the minimal extension that separates reversible exploration from irreversible approach-to-commitment. Two channels are the minimal structure that permits independent control of:

(i) Reversible overlap/phase geometry

(ii) Irreversible commitment dynamics

without conflating them into a single parameter. Additional channels could capture finer distinctions but are not required at this resolution.

Proposition 7.2 (Physical correspondence). The two-channel structure corresponds to distinct aspects of quantum evolution:

(i) Unitary evolution (Schrödinger equation): advances T_r while leaving T_i unchanged

(ii) Measurement/decoherence: advances T_i (and eventually B) while collapsing explored configuration space

8. Overlap as a U(1) Fiber Bundle over TPB^2

Quantum systems in superposition exhibit overlap between configuration branches, manifesting as interference. We model this as a U(1) fiber bundle over TPB^2.

8.1 The Bundle Structure

Definition 8.1 (Phase bundle). The phase bundle is a principal U(1) bundle [3, 4, 5]:

$\pi: P \to M$

where $M \equiv TPB^2$ is the base space and each fiber $\pi^{-1}(p) \cong U(1) \cong S^1$ represents the continuous phase degree of freedom associated with reversible overlap.

Proposition 8.1 (Separation of structure). The bundle construction separates:

(i) Ordering information: position (T_r, T_i) on the base space M

(ii) Phase information: position $\theta \in [0, 2\pi)$ on the fiber U(1)

This reflects the physical distinction: ordering determines what possibilities exist, while phase determines how they interfere.

8.2 Connection and Parallel Transport

Definition 8.2 (Phase connection). A U(1) connection on P is a 1-form on M:

$A = A_r(T_r, T_i)\, dT_r + A_i(T_r, T_i)\, dT_i$

where $A_r, A_i: M \to \mathbb{R}$ are smooth functions encoding phase advancement per unit of reversible and irreversible ordering, respectively.

Definition 8.3 (Parallel transport). For a path $\gamma: [0,1] \to M$, parallel transport induces a phase shift:

$\theta(\gamma) = \theta_0 + \int_\gamma A = \theta_0 + \int_0^1 [A_r(\gamma(s))\, \dot{\gamma}_r(s) + A_i(\gamma(s))\, \dot{\gamma}_i(s)]\, ds$

This captures phase accumulation along a history parameterised by ordering measures.

8.3 Curvature and Interference

Definition 8.4 (Curvature). The curvature two-form is:

$F := dA = (\partial A_i/\partial T_r - \partial A_r/\partial T_i)\, dT_r \wedge dT_i$

Theorem 8.1 (Path dependence). For two paths γ_1, γ_2: $[0,1] \rightarrow M$ with the same endpoints:

$\theta(\gamma_1) - \theta(\gamma_2) = \int_\Sigma F$

where Σ is any surface bounded by $\gamma_1 - \gamma_2$.

Proof. Stokes' theorem. ■

Corollary 8.1. Nonzero curvature $F \neq 0$ implies path-dependent phase evolution, yielding observable interference effects. Vanishing curvature $F = 0$ implies phase depends only on endpoints (flat overlap geometry).

8.4 The Overlap-Validity Function

Definition 8.5 (Overlap validity). The overlap-validity function $\chi: M \rightarrow [0,1]$ tracks the degree to which superposition remains intact:

- $\chi \approx 1$: Fully reversible regime; U(1) fiber description applies
- $\chi \rightarrow 0$: Approaching crystallisation; phase becoming undefined
- $\chi = 0$: Post-commitment; definite branch with no remaining overlap

Axiom 8.1 (Monotone decoherence). We impose:

$\partial\chi/\partial T_i \leq 0$

This captures the monotone loss of coherence under irreversible ordering: advancement toward commitment diminishes superposition.

8.5 Complex Amplitudes and the Born Rule

Proposition 8.2 (Geometric interpretation of amplitudes). Complex quantum amplitudes decompose as:

$\psi = |\psi| \cdot e^{i\theta}$

where:

- $|\psi|$: Magnitude governed by TPB/BCB constraints on distinguishability distribution
- θ: Phase; position on the U(1) fiber

Complex numbers are thus reinterpreted geometrically as the minimal structure to represent overlap over TPB^2 ordering space.

Born weighting and first-passage realisation (model statement). We adopt $|\psi|^2$ as the operational probability weight because it is a measure consistent with stability under coarse-graining and consistent composition in standard quantum practice; in the present paper we treat this as a structural requirement on any admissible probability assignment and defer a full derivation to future work.

We provide a concrete dynamical realisation compatible with TPB commitment. Consider competing branches j with complex amplitudes ψ_j. Model crystallisation as a first-passage (race) process [23] in which each branch produces a record at a stochastic hazard rate λ_j satisfying $\lambda_j \propto |\psi_j|^2$. If the record time in branch j is exponentially distributed, $T_j \sim \mathrm{Exp}(\lambda_j)$, then the probability that branch j wins the race is:

$$P(j \text{ wins}) = \lambda_j / \Sigma_k \lambda_k = |\psi_j|^2 / \Sigma_k |\psi_k|^2$$

This yields the Born frequencies as an emergent consequence of first-passage record formation. We emphasise that this is a minimal mechanism-level model (not a unique derivation): it shows that TPB-style crystallisation dynamics can implement $|\psi|^2$ weighting through irreversible record competition, while the deeper origin of the $\lambda_j \propto |\psi_j|^2$ scaling can be anchored either in an entropy/composability argument or in a more microscopic TPB hazard derivation.

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9. Dimensional Emergence Calculus (DEC): The Lift

We now arrive at the central construction: the Dimensional Emergence Calculus, which maps TPB^2 into physical displacement.

9.1 The Differentiation Lift

Definition 9.1 (Lift vectors). The lift map is specified by a pair of vector fields on physical 3-space, parameterised by TPB^2 coordinates:

$\mathbf{e}_r: M \to \mathbb{R}^3$ (reversible lift vector)

$\mathbf{e}_i: M \to \mathbb{R}^3$ (irreversible lift vector)

Definition 9.2 (Emergent displacement). The displacement generated by an infinitesimal step in TPB^2 is:

$$d\mathbf{x} = \mathbf{e}_r\, dT_r + \mathbf{e}_i\, dT_i$$

This is the DEC analogue of $dx = v\,dt$ in classical mechanics, with the "velocity" replaced by two lift vectors and "time" replaced by the two-dimensional ordering space.

9.2 Physical Interpretation

Proposition 9.1 (Lift semantics).

(i) $\mathbf{e}_r$ (reversible lift): Displacement generated by exploratory ordering. This corresponds to motion that does not commit distinguishability—fluctuations, quantum spreading, reversible dynamics.

(ii) $\mathbf{e}_i$ (irreversible lift): Displacement generated by commitment ordering. This corresponds to motion driven by approach to crystallisation. In gravitational contexts, $\mathbf{e}_i$ aligns with the direction of "falling."

9.3 The Differentiation Curl

Definition 9.3 (Differentiation curl). The non-integrability of the lift is captured by:

$$\boldsymbol{\Omega} := \partial\mathbf{e}_i/\partial T_r - \partial\mathbf{e}_r/\partial T_i \in \mathbb{R}^3$$

This 3-vector measures the failure of the lift to define a consistent coordinate transformation from TPB^2 to physical space.

Definition 9.4 (Lift 1-form). Define the $\mathbb{R}^3$-valued 1-form on TPB^2:

$$\alpha := \mathbf{e}_r\, dT_r + \mathbf{e}_i\, dT_i$$

Its exterior derivative is:

$$d\alpha = (\partial\mathbf{e}_i/\partial T_r - \partial\mathbf{e}_r/\partial T_i)\, dT_r \wedge dT_i = \boldsymbol{\Omega}\, dT_r \wedge dT_i$$

Theorem 9.1 (Force generation). When $\boldsymbol{\Omega} \neq 0$:

(i) Displacement depends on the path through TPB^2, not just endpoints

(ii) For a closed loop $\gamma = \partial\Sigma$ bounding a region Σ in TPB^2, the net displacement is:

$$\oint_{\partial\Sigma} \alpha = \iint_\Sigma d\alpha = \iint_\Sigma \boldsymbol{\Omega}\, dT_r \wedge dT_i$$

(iii) Force-like effects emerge in physical space

Proof. Part (ii) is Stokes' theorem for $\mathbb{R}^3$-valued forms. The path dependence in (i) is the contrapositive: if displacement were path-independent, then $\oint\alpha = 0$ for all loops, implying $d\alpha = 0$

and hence **Ω** = 0. Part (iii) follows from interpreting path-dependent displacement as work done by an effective force field. ■

Corollary 9.1. The differentiation curl **Ω** is the DEC mechanism for generating forces from geometry, analogous to spacetime curvature in general relativity.

9.4 VERSF Field Determination

Definition 9.5 (Field-lift coupling). The lift vectors are determined by the VERSF fields (φ, s):

$\mathbf{e}_i = a(\varphi, s)\,\hat{\mathbf{n}}$

$\mathbf{e}_r = b(\varphi, s)\,\hat{\mathbf{m}}$

where:

- a, b: $\mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are scalar lift-strength functions
- $\hat{\mathbf{n}}, \hat{\mathbf{m}} \in S^2$ are unit vectors encoding lift direction

Axiom 9.1 (Canonical alignment). In the simplest realisation:

(i) $\mathbf{e}_i \parallel \nabla\varphi$: Commitment moves toward lower void field

(ii) $\mathbf{e}_r \perp \nabla\varphi$: Exploration is tangent to void-field isosurfaces

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10. The Newtonian Limit

We demonstrate that DEC recovers Newtonian gravity under explicit assumptions.

10.1 Assumptions

The derivation rests on three explicit assumptions:

Assumption 1 (Radial alignment). The irreversible lift vector aligns with the radial void gradient:

$\mathbf{e}_i \parallel \nabla\varphi = \varphi'(r)\,\hat{\mathbf{r}}$

Assumption 2 (Linear coupling). The lift strength depends linearly on the void field:

$a(\varphi) = k\varphi,\ k \in \mathbb{R}_+$

Assumption 3 (Harmonic void field). The void field satisfies Laplace's equation in vacuum:

$$\nabla^2\varphi = 0$$

These represent the simplest choices consistent with spherical symmetry and standard field-theoretic structure.

10.2 Setup: Spherically Symmetric Void Field

Consider a static, spherically symmetric void field $\varphi(r)$ centred on a mass M, where $r = \|\mathbf{x}\|$. Define:

- Radial unit vector: $\hat{\mathbf{r}} = \mathbf{x}/r$
- Void gradient: $\nabla\varphi(r) = \varphi'(r)\,\hat{\mathbf{r}}$

In the spherically symmetric Newtonian configuration, $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ and $\hat{\mathbf{m}}$ spans the transverse tangent plane (any orthonormal basis of vectors perpendicular to $\hat{\mathbf{r}}$).

10.2a Matter Sourcing of the Void Field (Required Extension)

The vacuum condition $\nabla^2\varphi = 0$ used here is sufficient to recover inverse-square scaling outside sources, but a complete gravitational theory requires a source equation relating φ to matter/energy distribution. The natural analogue is a Poisson-type relation, schematically $\nabla^2\varphi = S(\rho, ...)$, where S encodes how matter couples to the void field in the effective limit. Determining S (and its relativistic extension) is beyond the scope of this paper and is deferred to future work; the present result should be read as a vacuum-limit consistency demonstration: if φ is harmonic outside sources and the DEC lift aligns radially, inverse-square scaling follows.

10.3 Lift Configuration

Under Assumption 1:

$$\mathbf{e}_i = a(\varphi)\,\hat{\mathbf{r}}$$

where $a(\varphi)$ has dimensions [length · (TPB² ordering)$^{-1}$]. The reversible lift spans transverse directions:

$$\mathbf{e}_r \perp \hat{\mathbf{r}}$$

10.4 Newtonian Regime

Definition 10.1 (Newtonian regime). The Newtonian regime is characterised by:

(i) $dT_i \gg dT_r$ over macroscopic scales (commitment dominates exploration)

(ii) Motion is predominantly radial: $d\mathbf{x} \approx \mathbf{e}_i\, dT_i$

In this regime, physical radial velocity is proportional to the lift strength $a(\varphi)$.

10.5 Emergent Acceleration

Definition 10.2 (Effective acceleration). In the coarse-grained Newtonian regime, the emergent acceleration is defined as the second derivative of radial coordinate with respect to calibrated time t:

$$a\_eff := d^2r/dt^2$$

Theorem 10.1 (Acceleration from lift variation). The effective acceleration scale is:

$$a\_eff(r) \propto \|\partial \mathbf{e}_i/\partial r\| = |d[a(\varphi(r))]/dr|$$

Proof sketch. From $d\mathbf{x} \approx \mathbf{e}_i\, dT_i$, the radial velocity is $dr/dT_i = a(\varphi(r))$. The acceleration in T_i is:

$$d^2r/dT_i^2 = (da/dr)(dr/dT_i) = a'(r) \cdot a(r)$$

Converting to t via $dt = \kappa\, dB$ and $dB = f\, dT_i$, the Jacobians dT_i/dB and dB/dt are absorbed into the proportionality constant. Up to multiplicative factors absorbed into calibration—where a(r) varies slowly in the near-Newtonian regime—the acceleration scale tracks $|da/dr|$. ■

Role of the differentiation curl in the Newtonian regime. In full DEC, force-like effects are associated with lift non-integrability $\mathbf{\Omega} = \partial \mathbf{e}_i/\partial T_r - \partial \mathbf{e}_r/\partial T_i$, which measures path dependence in TPB^2. The Newtonian recovery in §10 is obtained in a restricted regime where (i) motion is near-radial and (ii) ordering is dominated by the commitment channel $dT_i \gg dT_r$, so the dynamics effectively project onto a one-dimensional slice in which $\partial \mathbf{e}_r/\partial T_i$ and transverse path effects are suppressed. In this limit, the dominant "force proxy" reduces to the spatial variation of the irreversible lift $\partial \mathbf{e}_i/\partial r$. Away from this regime—when reversible–irreversible coupling is significant or when paths explore nontrivial loops in TPB^2—the full $\mathbf{\Omega}$ term contributes and can generate additional non-Newtonian accelerations.

10.6 Inverse-Square Scaling

Under Assumption 2:

$$a(\varphi) = k\varphi \Longrightarrow a\_eff(r) \propto k|\varphi'(r)|$$

Under Assumption 3, we use Laplace's equation. We emphasise that Laplace's equation is not derived here, but adopted as the simplest vacuum condition consistent with locality and spherical symmetry. The spherically symmetric solution is:

$$\varphi(r) = -GM\_\varphi/r + \varphi\_\infty$$

where M_φ is the "void charge" of the central mass ("void charge" here denotes the coupling strength of matter to the void field, not a new conserved quantity) and φ_∞ is set to zero by reference choice. Formally, M_φ is defined as the coefficient of the 1/r term in the exterior solution for φ, and is determined by the matter–void coupling in the sourced field equation (see §10.2a).

Theorem 10.2 (Inverse-square law). Under Assumptions 1-3:

$$a_{eff}(r) \propto 1/r^2$$

Proof. From $\varphi(r) = -GM_\varphi/r$:

$$\varphi'(r) = GM_\varphi/r^2$$

Therefore $a_{eff}(r) \propto |\varphi'(r)| = GM_\varphi/r^2$. ■

10.7 Calibration to Newton's Constant

Proposition 10.1. Matching to observed gravity $g = GM/r^2$ requires:

$$M_\varphi \propto M$$

with proportionality fixed by calibration to G. In the simplest identification, $M_\varphi = M$ and the coupling k absorbs remaining calibration factors.

10.8 Conditions for Non-Newtonian Corrections

Theorem 10.3 (Correction conditions). Departures from inverse-square gravity arise when:

(i) **Non-harmonic void field**: $\nabla^2\varphi \neq 0$ implies $\varphi(r) \neq -GM_\varphi/r$, hence $\varphi'(r) \neq 1/r^2$

(ii) **Nonlinear lift coupling**: $a(\varphi) = k\varphi + k_2\varphi^2 + \cdots$ introduces higher-order terms:

$$a_{eff} \propto |k\varphi' + 2k_2\varphi\varphi' + \cdots|$$

(iii) **Entropy modulation**: $a(\varphi,s) = k(\varphi)\cdot f(s)$ implies entropy gradients modulate gravitational strength (potentially producing MOND-like phenomenology [19] in low-acceleration regimes)

(iv) **TPB² curvature**: Significant $\partial \mathbf{e}_r/\partial T_i$ contributions add force-like terms from reversible-irreversible coupling

These conditions provide falsifiable predictions.

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11. Integration: The Complete Framework

The constructions combine into a unified architecture:

Layer	Object	Role
TPB² base	$M = (T_r, T_i)$	Two-channel ordering manifold
U(1) fiber	$\theta \in S^1$	Phase/overlap geometry
Connection	$A = A_r\, dT_r + A_i\, dT_i$	Governs phase evolution
Curvature	$F = dA$	Encodes interference geometry
Overlap validity	$\chi: M \to [0,1]$	Tracks superposition status
DEC lift	$(\mathbf{e}_r, \mathbf{e}_i): M \to \mathbb{R}^3 \times \mathbb{R}^3$	Maps ordering to displacement
Diff. curl	$\boldsymbol{\Omega} = \partial\{T_r\}\boldsymbol{e}_i - \partial\{T_i\}\mathbf{e}_r$	Generates force-like effects
VERSF fields	(φ, s)	Determine lift and connection

Theorem 11.1 (Coarse-grained limits). In the regime where bit formation is dense and TPB varies slowly:

(i) d/dt emerges as reparameterised tick-derivative (§4)

(ii) Classical mechanics emerges from differentiation curl (§9)

(iii) Newtonian gravity emerges from harmonic void field (§10)

Outside this regime, the full TPB²/DEC structure is required.

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12. Relationship to Classical Calculus

12.1 What TPB²-DEC Adds to Classical Calculus

Classical calculus provides $dx = v\, dt$, turning an abstract parameter t into spatial displacement. However, it does not explain why space or time exist, why motion is continuous, or what t represents physically—these are assumed infrastructure.

TPB²-DEC operates at a deeper level, describing how continuous 3D motion arises from a 2D informational substrate via:

$$d\mathbf{x} = \mathbf{e}_r\, dT_r + \mathbf{e}_i\, dT_i$$

The ordering variables (T_r, T_i) are not spatiotemporal but informational. Continuous motion emerges through integration, with the lift vectors encoding how ordering steps generate physical displacement.

12.2 The Role of Curvature

Just as spacetime curvature in GR manifests as gravity, lift non-integrability **Ω** manifests as force-like effects. Geometry explains why motion deviates from straight-line behaviour.

12.3 The Hierarchy

1. **TPB²-DEC**: Ordering-based calculus with emergent dimensionality
2. **Standard calculus**: Coarse-grained limit with continuous time
3. **Newtonian mechanics**: Forces from spatial potential variation
4. **General relativity**: Spacetime curvature effects

Each emerges from the one below through limiting procedures.

12.4 Summary

Standard calculus: language of motion within space.

TPB²-DEC: language of how space-like continuity emerges from informational ordering.

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13. Discussion and Conclusions

13.1 Summary of Results

1. **TPB primitives** (§2): Ticks and bits as fundamental measures; $\tau = \Delta T/\Delta B$ characterises local dynamics with continuum limit dT/dB when bits are dense.
2. **TPB operators** (§3): D_T and D_B as fundamental derivatives with chain rule relating them through τ.
3. **Time emergence** (§4): $d/dt = (\tau/\kappa)D\_T$ under validity conditions.
4. **Calculus breakdown** (§5): Hybrid dynamical system in sparse-bit regime.
5. **Covariant formulation** (§6): Balance laws for currents; constitutive relation as lowest-order gradient expansion.
6. **TPB² structure** (§7): Minimal two-channel separation of reversible/irreversible ordering.
7. **U(1) bundle** (§8): Phase geometry with curvature encoding interference; overlap-validity tracking decoherence.
8. **DEC lift** (§9): Mapping from TPB² to $\mathbb{R}^3$; differentiation curl generating forces.

9. **Newtonian limit** (§10): $1/r^2$ from three explicit assumptions; conditions for corrections identified.

13.2 Predictions and Falsifiability

The DEC framework makes predictions that are distinct from those of general relativity, quantum field theory, causal set theory, and entropic gravity. The key distinguishing feature is that DEC ties force-like behaviour to path dependence in ordering space (TPB^2), not just to metric potentials or local field values. The combination of (i) two-channel ordering as a base space, (ii) phase as a U(1) fiber over that base rather than over spacetime, and (iii) force as non-integrability of a lift from ordering space into $\mathbb{R}^3$ produces "ordering-history" effects that are not automatic in other frameworks.

13.2.1 Path-dependent phases from ordering history

In standard quantum mechanics, phase differences depend on action along spacetime paths (and gauge fields), but the "clock" is coordinate time. In DEC, phase lives on a U(1) fiber over (T_r, T_i), so phase shifts can depend on how much reversible exploration versus irreversible commitment occurred along the way—even for paths with the same spacetime endpoints.

Distinctive signature: Interference fringes that shift when one changes the decoherence/commitment profile of one interferometer arm (altering T_i history) while keeping classical path length and elapsed time the same. This is not a standard decoherence effect (which destroys interference); it is a phase shift that preserves coherence but modifies the fringe pattern.

13.2.2 Forces from reversible–irreversible coupling (the **Ω** term)

In the Newtonian limit (§10), the differentiation curl $\mathbf{\Omega} = \partial \mathbf{e}_i/\partial T_r - \partial \mathbf{e}_r/\partial T_i$ was suppressed by the $dT_i \gg dT_r$ projection. In general, however, nonzero $\mathbf{\Omega}$ creates force-like effects that are distinct from "gravity from a potential" or "curvature from stress-energy." These are forces arising from ordering non-integrability.

Distinctive signature: Small additional accelerations that correlate with changes in local commitment dynamics (decoherence rate, entropy-production environment), not only with mass distribution. In other words: identical mass geometry but different "commitment profile" yields slightly different dynamics. This is testable in principle by comparing free-fall trajectories in environments with different decoherence rates.

13.2.3 Environment-linked deviations from the equivalence principle

General relativity's universality of free fall (independence of composition and environment) is extremely strict. DEC can reproduce it in the coarse-grained limit, but it provides a clear mechanism for controlled violations when:

- The lift depends on both φ and s (entropy field), so that $a(\varphi,s) \neq a(\varphi)$ alone

- **Ω** effects become appreciable (significant reversible–irreversible coupling)
- τ varies rapidly (sparse-bit, near-horizon, or strong-gradient regimes)

Distinctive signature: A small, regime-limited equivalence principle violation that tracks entropy gradients or decoherence rate, not composition per se. This is not a claim that EP is violated generally; it is a candidate falsifiable corner-case in specific high-gradient or low-commitment-density environments.

13.2.4 Calculus breakdown criterion

Many frameworks invoke discreteness at the Planck scale, but DEC provides a sharp operational criterion for when standard calculus fails:

Standard d/dt calculus is valid only when: (i) bit formation is dense, (ii) τ varies slowly, and (iii) jumps are sub-resolution.

This yields specific breakdown predictions in "commitment-sparse" conditions.

Distinctive signature: Measurable non-smooth (jump-like) behaviour in effective dynamics near crystallisation events, where the correct description is a hybrid dynamical system (continuous in T, discontinuous in B). Other frameworks mention collapse or decoherence, but DEC ties it to a replacement of calculus itself—the ordinary differential equation description fails and must be replaced by the TPB formalism.

13.2.5 Strong-field structure: multiple channels for non-Newtonian corrections

In GR, deviations from Newtonian gravity arise from metric curvature. In MOND, they arise from modified dynamics at low acceleration. In entropic gravity, they arise from modified information-geometry relations. DEC can produce modified gravity through at least three distinct mechanisms:

(i) φ deviates from harmonic (non-Laplacian void field in strong-field or boundary regions)

(ii) a(φ,s) becomes nonlinear (higher-order lift coupling)

(iii) **Ω** becomes non-negligible (reversible–irreversible mixing)

Distinctive signature: A family of deviations where the shape of corrections is linked to ordering geometry (TPB^2 coupling structure), not just to acceleration scale or metric potentials. The different mechanisms produce different correction profiles, potentially distinguishable through precision gravitational measurements.

13.2.6 Horizon behaviour

As $\tau \to \infty$ near horizons, the relationship between ordering and commitment breaks down in a specific way: ordering continues indefinitely while commitment freezes from the external perspective. This suggests regimes where the GR continuum description may be insufficient.

Distinctive signature: Modified near-horizon physics where the $\tau \to \infty$ limit produces specific predictions for information flow and effective dynamics that differ from standard GR horizon thermodynamics.

Summary: What makes these predictions DEC-specific

Some other frameworks can mimic individual effects above. What is distinctive about DEC is the combination:

- Two-channel ordering (T_r, T_i) as a base space
- Phase as a U(1) fiber over that base (not over spacetime)
- Force as non-integrability of a lift from ordering space into $\mathbb{R}^3$

This trio produces ordering-history effects—dependence on the path through (T_r, T_i), not just through spacetime—that are not automatic in GR, QFT, causal sets, or entropic gravity.

13.3 Relationship to Other Approaches

Shared features with:

- Information-theoretic QM reconstructions [10, 11, 12, 13] (distinguishability primitive)
- Causal set theory [14, 15, 16] (ordering over geometry)
- Entropic gravity [6, 7, 8, 9] (thermodynamic-geometric connection)
- Shape dynamics [22] (3D over 4D primacy)

Distinguished by: explicit two-channel structure and geometric lift mechanism.

13.4 Open Questions

1. VERSF field dynamics: equations for φ and s, matter coupling
2. Source terms: physical interpretation of non-zero Σ_T, Σ_B
3. QFT extension: infinite degrees of freedom
4. Cosmology: large-scale structure, early universe, dark sector
5. Experimental signatures: laboratory tests of discrete-commitment structure

Strong-field regime ($r \to 0$). The present Newtonian recovery concerns the exterior/vacuum limit and does not address the inner/strong-field behaviour as $r \to 0$. In DEC terms, the strong-field regime is precisely where (i) the harmonic approximation for φ may fail, (ii) entropy dependence in the lift $a(\varphi,s)$ may become dominant, and (iii) TPB^2 non-integrability $\mathbf{\Omega}$ may no longer be suppressed by the $dT_i \gg dT_r$ projection. Whether this structure regularises the classical singularity or reproduces it depends on the sourced field equation for φ and the high-gradient behaviour of the lift functions. We note that Penrose [18] has proposed gravity-induced state reduction as a mechanism linking gravitational physics to quantum measurement—a connection that resonates with the TPB crystallisation dynamics developed here; detailed comparison is deferred to future work.

13.5 The Deeper Significance: Calculus as Physical Phenomenon

The core contribution of DEC is that it treats calculus itself as a physical phenomenon rather than assumed mathematical infrastructure.

Standard physics uses differential calculus to describe how things change—dx/dt, $\partial\psi/\partial t$, and so on. But this presupposes that continuous time and space already exist. Newton and Leibniz gave us the language of motion; they did not explain why that language applies to reality.

DEC operates one level deeper. It asks: what has to be true for continuous change to be possible at all? And it answers: there must be enough distinguishability, accumulated densely enough, with slowly varying commitment dynamics. When those conditions hold, standard calculus emerges (Theorem 4.1, Corollary 4.1). When they fail, you need the more primitive TPB formalism.

A genuine level-shift. General relativity made spacetime dynamic—geometry responds to matter. But spacetime remains fundamental infrastructure. DEC makes spacetime derived—it emerges from the organisation of ordering and commitment. The continuous manifold we integrate over is not given; it is built.

The lift mechanism. This is where emergence becomes concrete. You have a 2D informational space (reversible exploration T_r, irreversible commitment T_i). You have a map—the lift—from that space into physical displacement. The geometry of that map, specifically whether it is integrable, determines whether forces exist. Non-integrability of the lift is force, in the same sense that curvature of spacetime is gravity in GR. But now the base manifold is informational, not spatiotemporal.

This architecture delivers three things simultaneously:

1. Unification of scale. The same framework describes both the quantum regime (where phase geometry over TPB^2 matters, where crystallisation events are discrete) and the classical regime (where coarse-graining yields standard calculus). The quantum-to-classical transition is not a mystery requiring interpretation—it is the validity conditions of §4 being satisfied or violated.

2. Falsifiability with teeth. Because standard physics is derived as a limit, we know exactly where it should break down: sparse commitment, rapid τ variation, significant reversible–irreversible coupling. These are not vague gestures toward "Planck-scale effects"—they are operational criteria (§5, §13.2.4) that specify when the hybrid dynamical system description must replace ordinary differential equations.

3. Gravity without gravity. The framework does not postulate a gravitational force or field as a primitive. It postulates informational ordering, a lift map, and minimal symmetry assumptions (§10.1). Inverse-square attraction falls out as a theorem (Theorem 10.2). This suggests gravity is not a thing to be explained but a pattern that emerges when information organises itself spatially through the void field.

The deepest implication. The reason physics is describable by differential equations may not be that reality is fundamentally continuous. It may be that we only perceive the regimes where distinguishability is dense enough to make continuity a good approximation. DEC provides the mathematical framework for asking—and potentially answering—what lies beneath.

13.6 Conclusion

Dimensional Emergence Calculus provides a mathematical framework in which continuous three-dimensional space, ordinary differential calculus, and Newtonian gravity all emerge from a two-dimensional informational ordering substrate. The key constructions are:

- **TPB primitives** that separate ordering (ticks) from commitment (bits)
- **Two-channel extension** (TPB^2) distinguishing reversible exploration from irreversible approach-to-commitment
- **U(1) phase geometry** over ordering space rather than spacetime
- **The differentiation lift** mapping ordering increments to physical displacement
- **Non-integrability (Ω)** as the mechanism generating force-like effects

The recovery of inverse-square gravity under three explicit assumptions (Theorem 10.2) demonstrates that the framework makes contact with observable physics. The identification of six categories of DEC-specific predictions (§13.2) provides falsifiable criteria distinguishing this approach from GR, entropic gravity, and causal set theory.

The framework suggests that the applicability of differential calculus to physics is not a brute fact about reality but an emergent feature of regimes where distinguishability is dense enough, commitment dynamics vary slowly enough, and discrete jumps are fine-grained enough to be invisible. Where these conditions fail, the TPB formalism provides the replacement description.

Whether nature actually implements this structure remains to be determined by experiment. What the present work establishes is that such an implementation is mathematically coherent and empirically viable—a proof of concept for treating the infrastructure of calculus as itself a physical phenomenon requiring explanation..

Appendix A – Validity Regime of the Newtonian Limit

We clarify the conditions under which the proportionality used in Theorem 10.1 is valid. The conversion from irreversible ordering T_i to calibrated time t proceeds through Jacobians dT_i/dB and $dB/dt = 1/\kappa$. These factors may be absorbed into calibration provided they vary slowly over the spatial scale L of interest.

Define the small parameters:

$\varepsilon_a := |(1/a)(da/dr)|\ L \ll 1$
$\varepsilon\_\tau := |\nabla\tau|\ L / \tau \ll 1$

When both conditions hold, lift strength variation and TPB gradients are subleading, and the effective acceleration scale satisfies:

$a\_eff(r) \propto |da/dr| + O(\varepsilon_a, \varepsilon\_\tau)$

This links Theorem 10.1 directly to Corollary 4.1 and specifies the regime of Newtonian validity.

Appendix B – Constraints on the Constitutive Function $\mathscr{F}(s,\varphi)$

The constitutive relation $J^u = \mathscr{F}(s,\varphi)\ \nabla^u s$ is treated as the lowest-order, local, covariant term in a gradient expansion.

Minimal constraints include:

1. Positivity/Stability:
$\mathscr{F}(s,\varphi)\ \partial^u s\ J^u \geq 0$

2. Monotonicity in entropy:
$\partial\mathscr{F}/\partial s \geq 0$

3. Locality and covariance:
$\mathscr{F}$ depends only on scalar fields s and φ at leading order.

These conditions restrict admissible forms of $\mathscr{F}$ without fixing its detailed functional structure.

Appendix C – Status of the Born Rule Scaling

The first-passage race model introduced in §8.5 is an existence proof, not a uniqueness derivation. It demonstrates that irreversible TPB-style crystallisation dynamics can implement $|\psi|^2$ weighting.

Quadratic weighting is singled out by:
- Additivity under branch composition
- Stability under coarse-graining
- Normalisation under repeated branching

No claim of uniqueness is made. A full microscopic derivation of hazard scaling $\lambda_j \propto |\psi_j|^2$ is deferred.

Appendix D – Relativistic Extension of the DEC Lift

The DEC lift is formulated in emergent 3-space appropriate to dense-commitment rest frames. A relativistic extension may be obtained by promoting the lift to a spacetime-valued one-form:

$dX^u = E^u{}_r\, dT_r + E^u{}_i\, dT_i$

Lorentz covariance emerges in the dense-bit limit when causal constraints are imposed on $E^u{}_i$. This extension is deferred to future work.

Appendix E – Strong-Field and r → 0 Behaviour

As $r \to 0$, the projection $dT_i \gg dT_r$ fails. Reversible ordering becomes significant, and differentiation curl Ω is no longer suppressed.

This suggests a natural mechanism for modifying strong-field behaviour, potentially avoiding singular acceleration by redistributing ordering between channels.

Appendix F – Measure-Theoretic and Dimensional Clarifications

The derivative D_B Q is understood as a Radon–Nikodym derivative of a jump measure with respect to the bit measure. Delta functions $\delta(T - T_j)$ are distributions acting under integration over T.

Dimensional summary:
T, T_r, T_i, B: dimensionless
κ: time
$\mathbf{e}_r$, $\mathbf{e}_i$: length
φ: potential-like scalar
ℓ(φ): length per ordering unit

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