Constraint-Defined Geometry and Emergent Time

On the Alignment Between Positive Geometries and the Void Energy–Regulated Space Framework (VERSF)

Keith Taylor

Abstract

Positive geometries demonstrate that physical scattering amplitudes can be derived entirely from global consistency and positivity constraints, without reference to spacetime dynamics or virtual particles. Independently, the Void Energy–Regulated Space Framework (VERSF) proposes that spacetime and time itself emerge from entropy-regulated distinguishability on a pre-geometric substrate. In this paper we argue that these approaches are aligned manifestations of a constraint-first ontology. We present a mathematically explicit correspondence between canonical forms in positive geometry and feasibility regions defined by entropy and distinguishability in VERSF. We establish sufficient conditions under which VERSF feasibility regions satisfy positive geometry axioms, derive the suppression hierarchy for high-codimension boundaries from entropy principles, and show that emergent time corresponds to directed flow on the feasibility space with canonical form singularities encoding reorganization events. This correspondence is non-trivial, structurally restrictive, and leads to concrete constraints on which positive geometries can represent physically admissible processes.

General-Reader Abstract

Modern physics usually explains the world by describing how forces act on objects as time passes. But recent discoveries suggest a different picture: instead of focusing on how things *move*, it may be more fundamental to ask what configurations of the world are even *allowed* to exist.

One place this idea appears is in particle physics, in the study of **scattering** — what happens when tiny particles collide and fly apart. Because these events are unpredictable in detail, physicists describe them using **scattering probabilities**, which simply mean the chances of different outcomes (for example, the likelihood that particles scatter in one direction rather than another). Surprisingly, modern work has shown that these probabilities can be calculated using geometry alone. In this approach, known as **positive geometry**, the outcomes of particle collisions are encoded in the shape of a mathematical object, without needing to track forces, motion, or even time.

Independently, the **Void Energy–Regulated Space Framework (VERSF)** proposes that space and time themselves are not fundamental ingredients of reality. Instead, they emerge from limits on **distinguishability** — how well different states of the world can be told apart — and **entropy**, which measures how much disorder or complexity a system carries. In VERSF, physical structure only exists where these limits allow it.

This paper shows that these two ideas are deeply connected. We demonstrate that the regions of possibility defined by VERSF — determined by thresholds on distinguishability, entropy capacity, and available energy — naturally form the same kinds of geometric shapes used in positive geometry. In this view, physical processes are governed not by equations of motion, but by whether they fit inside an allowed region of possibility.

The boundaries of these regions play a special role. They correspond to critical situations where physical structure must reorganize — moments where something new forms or breaks down. Moving through the allowed region defines a natural direction of time, while configurations that push against many boundaries at once become increasingly unlikely.

Together, these ideas suggest a unifying perspective in which geometry, probability, and time all emerge from basic limits on what can be consistently distinguished. Physics, in this view, is not primarily about what *happens*, but about what is *allowed* to happen at all.

---

1. Introduction

Recent developments in theoretical physics suggest that fundamental laws may be better understood as global consistency conditions rather than dynamical equations of motion. In scattering amplitudes, this perspective culminates in the theory of positive geometries, where probabilities arise from geometric structure alone. Separately, VERSF proposes that spacetime, geometry, and time emerge from entropy-regulated distinguishability. Despite their distinct origins, both frameworks replace dynamics with admissibility.

This paper establishes a precise correspondence between these frameworks. We show that VERSF feasibility regions, under specified conditions, satisfy the axioms defining positive geometries, and that the physical content of VERSF provides interpretation for the resulting canonical forms. The correspondence operates at two levels: positive geometries encode outcome-level consistency (which scattering processes are allowed), while VERSF addresses existence-level consistency (which distinguishability configurations can persist).

---

2. Positive Geometries

A positive geometry is a region of a real algebraic variety equipped with a unique canonical differential form whose singularities occur only on the boundary of the region. The canonical form has logarithmic singularities on each boundary component and no singularities in the interior. Formally, a positive geometry $(X, X_{\geq 0}, \Omega)$ consists of:

1. A complex algebraic variety X
2. A closed subset $X_{\geq 0} \subset X(\mathbb{R})$ with interior $X_{>0}$
3. A unique rational differential form Ω of top degree

The canonical form Ω satisfies:

- Logarithmic singularities on all boundary components of $X_{\geq 0}$
- No poles in the interior $X_{>0}$
- Recursive residue factorization: residues on boundaries equal canonical forms of boundary geometries

In the context of scattering amplitudes, these structures encode unitarity and factorization directly and reproduce known amplitudes without reference to spacetime locality (Arkani-Hamed & Trnka 2014; Arkani-Hamed et al. 2017). The amplituhedron, associahedron, and cosmological polytopes are key examples.

---

3. The VERSF Framework

VERSF begins from a pre-geometric void substrate in which distinguishability is not assumed but must arise through constraint-regulated processes. Distinguishability arises when reversible micro-events cross constraint thresholds that render distinctions stable. Entropy regulates which distinctions can persist, with time emerging as the ordering of these stabilization events. Geometry arises as a response to entropy loading rather than as a fundamental background.

The framework posits three fundamental constraint classes:

- **Distinguishability threshold**: $D(y) - D^c > 0$
- **Entropy capacity**: $\Sigma^c - \Sigma(y) > 0$
- **Void-energy budget**: $\Gamma_e ff(y) > 0$

Physical configurations must satisfy all three constraints simultaneously.

---

4. Structural Alignment as Constraint Geometry

4.1 Constraint-Defined Regions

Let X be a real manifold with coordinates $x \in \mathbb{R}^d$. Define an admissible region $\mathscr{R} \subset X$ by inequality constraints:

$\mathscr{R} = \{ x \in X : f_i(x) > 0, i = 1, ..., m \}$

Such regions underlie both positive geometries and the VERSF framework. The boundary $\partial\mathscr{R}$ consists of components where one or more constraints saturate.

4.2 Canonical Forms and Boundary Structure

Positive geometries admit a unique canonical form $\Omega(\mathscr{R})$ with logarithmic singularities only on the boundaries $f_i(x) = 0$ and no interior singularities. Locally, such forms take the structure:

$\Omega(\mathscr{R}) = \omega(x) \prod_i d \log f_i(x)$

where $\omega(x)$ is fixed by degree and residue conditions. The uniqueness of the canonical form is the central rigidity result of positive geometry theory.

4.3 VERSF as a Constraint Geometry

VERSF is expressed in the same formal language by defining a state space Y whose coordinates encode distinguishability, entropy loading, and void-energy degrees of freedom. Physical states satisfy the feasibility constraints:

- $D(y) - D^c > 0$ (distinguishability exceeds threshold)
- $\Sigma^c - \Sigma(y) > 0$ (entropy remains below capacity)
- $\Gamma_e ff(y) > 0$ (effective void-energy is positive)

These define an admissible region $\mathscr{R}$_VERSF $\subset$ Y.

4.3.1 Conditions for a Canonical Form on $\mathscr{R}$_VERSF

To establish that $\mathscr{R}$_VERSF admits a canonical form in the positive geometry sense, we require:

Assumption (A1): Semi-algebraic stratified region. $\mathscr{R}$_VERSF $\subset$ Y is a bounded (or projectively compact once projective equivalence and physical cutoffs are imposed) semi-algebraic region whose boundary admits a Whitney stratification into smooth components defined by $f_i(y) = 0$, such that intersections define codimension-k strata.

Assumption (A2): Boundary recursion. Each codimension-1 boundary component

$B_i := \{ y \in \mathscr{R} : f_i(y) = 0 \}$

inherits an induced feasibility region $\mathscr{R}_i$ defined by the remaining inequalities $f_{j \neq i}(y) > 0$ restricted to B_i. More generally, each codimension-k intersection $f_{i1} = \cdots = f_{ik} = 0$ inherits an induced feasibility subregion defined by the remaining inequalities restricted to that stratum.

Definition 4.1 (VERSF canonical form). A top-degree differential form Ω_VERSF on Y is called canonical for $\mathscr{R}$_VERSF if it satisfies:

Sign convention. All canonical forms are defined up to overall sign; residues inherit signs from the induced boundary orientation.

1. **Logarithmic singularities on each codimension-1 boundary**: Near $f_i = 0$,

   $\Omega \sim (df_i / f_i) \wedge$ (regular form)

2. **No interior poles** on $\mathscr{R}$_VERSF
3. **Unit residue factorization**: For each i,

   Res_{f_i=0} Ω_VERSF = $\pm\, \Omega_i$

   where Ω_i is the canonical form on the boundary region $\mathscr{R}_i$ (defined inductively)

4. **Weight/normalization condition**: Ω has the correct homogeneity under admissible rescalings of coordinates (or satisfies an appropriate normalization if Y is not naturally projective)

Definition 4.2 (VERSF positive geometry candidate). A VERSF feasibility region $\mathscr{R}$_VERSF satisfying (A1)–(A2) together with the local product structure condition (A3, see §4.5) is called a *VERSF positive geometry candidate*. Such regions admit unique canonical forms by the following proposition.

Proposition 4.1 (Uniqueness). If $\mathscr{R}$_VERSF satisfies (A1)–(A2) and admits a canonical form in the sense of Def. 4.1, then it is unique up to overall sign.

Proof sketch. Suppose Ω and Ω' both satisfy conditions (1)–(4). Their difference $\Delta = \Omega - \Omega'$ has vanishing residues on all boundary components and no interior poles. Hence Δ extends to a globally regular form on the ambient compactification (projective closure of Y). For the projective closure and the imposed homogeneity/normalization, any globally regular top form compatible with (1)–(4) must be proportional under the imposed normalization and homogeneity class; the unit-residue normalization condition (3) removes this freedom. Hence $\Delta = 0$, and uniqueness up to sign follows from residue normalization (cf. Arkani-Hamed et al. 2018, §2). ■

Existence via pushforward. One natural construction of Ω_VERSF proceeds by pushforward: choose a reference positive region (simplex, hypercube) with known canonical form Ω_ref, and define an embedding Φ from the reference region into Y. We require Φ to be a stratified, orientation-preserving, rational (or piecewise-rational) map that restricts to diffeomorphisms with non-vanishing Jacobian on open strata and sends codimension-k faces of the reference polytope to codimension-k constraint intersections $f_{i_1} = \cdots = f_{i_k} = 0$. Given such Φ, set Ω_VERSF := Φ*(Ω_ref). This is the standard construction for amplituhedron canonical forms. The factor η(y) in the local expression $\Omega\_VERSF = \eta(y) \prod_i d \log f_i(y)$ is then determined by requiring residue factorization to hold.

Scope. The existence of Φ (and thus Ω_VERSF) is a structural requirement, not assumed generically. In concrete VERSF models, verifying (A1)–(A3) and constructing Φ is the principal mathematical task.

4.4 Canonical Measure and Entropy Weighting

Beyond the canonical form itself, VERSF motivates an entropy-weighted measure on the feasibility region. We derive this from first principles.

Assumption (A4): Additive entropy cost per independent saturated constraint. Approaching a codimension-k stratum corresponds to simultaneously saturating k independent distinguishability constraints. Independence here means that the constraint hypersurfaces intersect transversely and approaching one saturation surface does not force another at leading order—this links directly to condition (A3). When constraint activations are weakly coupled, the entropy penalty is additive:

$\Delta S_k \approx k \cdot \Delta S_1$

VERSF weighting postulate. Configurations with higher entropy loading are exponentially suppressed in their contribution to stable distinguishable outcomes:

$w(y) \propto \exp[-\beta \, \Delta S(y)]$

where β is an inverse "entropy temperature" scale set by void regulation. The value of β is a fundamental VERSF parameter; its derivation from void-energy density or distinguishability thresholds is an open problem whose solution would constrain the framework significantly.

Derived suppression hierarchy. For configurations approaching codimension-k strata:

$w_k \sim \exp(-\beta \, \Delta S_k) \approx \exp(-\beta \, k \, \Delta S_1)$

This yields the entropy-weighted measure:

$d\mu = \exp[-\beta \, \Delta S(y)] \, \Omega\_VERSF$

where ΔS(y) measures the total entropy cost of the configuration. For codimension-k boundary strata, $\Delta S \approx k \, \Delta S_1$, recovering the exponential suppression $w_k \sim \exp(-\lambda k)$ with $\lambda := \beta \, \Delta S_1$.

Deviation diagnostics. Exponential suppression follows in the weakly coupled constraint regime. Deviations from exponential scaling—whether polynomial suppression $w_k \sim k^{-\alpha}$ or modified exponentials—serve as diagnostics of constraint correlations (criticality) in the void regulation dynamics. Specifically:

- $\Delta S_k \sim \log k$ implies power-law suppression
- $\Delta S_k \sim k^\alpha$ with $\alpha < 1$ implies stretched exponential
- Such deviations signal that constraint saturations are not kinematically independent

4.5 Correspondence Lemma: Recursive Structure

To demonstrate that the correspondence between VERSF and positive geometry is substantive rather than merely notational, we establish that VERSF feasibility regions satisfy the recursive boundary property.

Lemma 4.1 (Feasibility recursion). Let $\mathscr{R}$_VERSF = $\{ f_i > 0 \}$. If a codimension-1 boundary component $f_a = 0$ corresponds to a single constraint saturation event (e.g., $\Sigma \to \Sigma^c$ or $D \to D^c$), then the restricted region

$$\mathscr{R}_a := \{ y \in Y : f_a(y) = 0, f_{i\neq a}(y) > 0 \}$$

is itself a feasibility region of the same type, with inherited constraints. Hence the boundary stratification is closed under "constraint saturation," matching the recursive boundary structure of positive geometries.

Condition (A3): Local product structure near boundaries. Near $f_a = 0$, the feasibility constraints decouple at leading order so the region locally looks like $(0, \varepsilon) \times \mathscr{R}_a$. Under this condition, the canonical form on $\mathscr{R}$ has residue equal to the canonical form on $\mathscr{R}_a$.

This local product structure is expected to hold when constraint saturations are kinematically independent—i.e., when approaching $f_a = 0$ does not generically force other constraints toward saturation. Violations of this condition indicate constraint coupling, producing non-factorizing residues that would be observable as anomalous boundary behavior.

Triangulation analogue. In amplituhedra, triangulation corresponds to decomposing a region into simpler cells (simplices) with matching boundaries, such that the canonical form equals the sum of cell contributions with internal boundary terms canceling.

In VERSF feasibility regions, a natural cell decomposition is given by which constraints are "active" (near saturation) versus far from saturation. This yields a stratification with the structure of a regular CW complex. Summing over cells with appropriate orientations produces boundary cancellations, leaving only physical boundary poles—the analogue of triangulation.

Whether concrete VERSF models satisfy (A1)–(A3)—and hence whether specific $\mathscr{R}$_VERSF instantiations are positive geometries in the strict mathematical sense—is a natural question for further investigation. The sufficient conditions are established; verification in specific physical contexts remains open.

4.6 Why This Is Not Generic Constraint Optimization

This correspondence is non-trivial. Generic constraint systems admit many inequivalent measures, interior extrema, and arbitrary objective functions. In contrast, both positive geometries and VERSF exhibit:

1. **Boundary-only singularities**: The canonical form has poles only where constraints saturate, never in the interior
2. **Uniquely determined canonical form**: Given the boundary structure, the form is fixed up to sign
3. **Factorization under boundary limits**: Residues on boundaries equal canonical forms of sub-regions

These properties distinguish admissibility frameworks from optimization frameworks.

4.7 Emergent Time as Flow on Feasibility Space

VERSF proposes that time is not fundamental but emerges from entropy-regulated distinguishability dynamics. We now connect this to canonical form structure.

Definition (Emergent time parameter). An emergent time parameter τ is any monotonic parametrization of trajectories in $\mathscr{R}$_VERSF such that $\Delta S(y(\tau))$ is non-decreasing along trajectories, where $\Delta S = S - S_0$ measures entropy loading above a baseline. This captures the VERSF principle that time corresponds to the accumulation of irreversible distinguishability.

One natural realization is gradient flow:

$$dy/d\tau = -\nabla \Delta S(y)$$

where ∇ is the gradient with respect to any chosen Riemannian metric on Y. The construction is metric-dependent, but monotonicity of ΔS is the only requirement used in this correspondence. The specific flow equation is not canonical—only the monotonicity condition matters.

Temporal interpretation of boundaries. Boundaries of $\mathscr{R}$_VERSF acquire temporal meaning: they are not merely geometric features but event horizons in the entropy-regulated sense—surfaces where the system's distinguishability structure must reorganize for continued evolution.

Specifically:

- The boundary $D(y) - D^c = 0$ corresponds to **under-formation thresholds**: insufficient entropy to establish distinction

- The boundary $\Sigma^c - \Sigma(y) = 0$ corresponds to **overload thresholds**: entropy cost exceeds available capacity
- The boundary $\Gamma_{e}ff(y) = 0$ corresponds to **void-energy exhaustion**: no budget for further distinction-making

The canonical form encodes these singular surfaces. Its poles mark the events where distinguishability reorganization occurs—precisely the "ticks" in VERSF terminology that, upon stabilization, become "bits."

Time emergence and canonical form. In this view, emergent time is not an external coordinate; it is the directed traversal of the feasible region under entropy regulation. The canonical form encodes the singular "event surfaces" where distinguishability reorganizes. The residue factorization property then has temporal content: crossing a boundary threshold decomposes the system's evolution into the sub-evolution on the constrained surface.

---

5. Toy Model Illustration

Consider a single normalized stability variable $x \in (0, 1)$. The admissible region is the open interval (0, 1) with two constraint functions:

- $f_1(x) = x > 0$ (distinguishability threshold)
- $f_2(x) = 1 - x > 0$ (entropy capacity)

The canonical form is:

$$\Omega = dx / [x(1 - x)] = d \log x - d \log(1 - x)$$

This form has logarithmic singularities at both boundaries and no interior poles. The unit residue property is verified explicitly:

$$\text{Res}_{x=0} [dx / x(1-x)] = 1/(1-x)|_{x=0} = 1$$

$$\text{Res}_{x=1} [dx / x(1-x)] = -1/x|_{x=1} = -1$$

The residues are ±1, confirming the unit residue normalization. For a zero-dimensional positive geometry (a point), the canonical form is ±1 by convention, fixed by orientation. This makes the toy example formally consistent with the recursion axiom: boundaries of $\mathscr{R}$ are points, and their canonical forms are the residues.

Physical interpretation. The boundaries $x \to 0$ and $x \to 1$ correspond respectively to:

- **Under-formation** ($x \to 0$): Insufficient entropy to establish distinction
- **Overload** ($x \to 1$): Entropy cost exceeds available void-energy budget

This defines the minimal two-constraint window for stable distinguishability. The canonical form's symmetry under $x \leftrightarrow (1 - x)$ reflects the duality between these failure modes.

Entropy-weighted measure. Including the VERSF weighting yields:

$$d\mu = \exp[-\beta\, \Delta S(x)] \cdot dx / [x(1 - x)]$$

where $\Delta S(x)$ measures entropy cost. If $\Delta S(x) = -x \log x - (1-x) \log(1-x)$ (the binary entropy), then configurations near $x = 1/2$ are suppressed relative to those near the boundaries. This weighting does not describe microstate typicality; it describes the relative contribution of configurations to stable distinguishable outcomes under void regulation. Stable distinguishability contributions are concentrated near constraint thresholds, where configurations carry more structural information than the "featureless" high-entropy configurations at $x = 1/2$.

---

6. Operational Positivity from Finite Distinguishability

A key question is why probability amplitudes derived from positive geometries yield non-negative probabilities for physical measurements. VERSF provides a natural answer grounded in finite distinguishability.

Proposition (Operational positivity from finite distinguishability). Let $\{E_i\}$ be a finite or countable set of operationally distinguishable outcomes (an event algebra for measurements). If outcome probabilities are defined as stable relative frequencies of these outcomes—or equivalently as a normalized positive functional on the event algebra—then:

$$p_i \geq 0, \sum_i p_i = 1$$

Proof. Relative frequencies are ratios of non-negative counts. Any limiting frequency is therefore non-negative. Normalization follows from exhaustiveness of the outcome set. ■

Relation to quasi-probabilities. Quasi-probabilities (e.g., Wigner functions) can be negative because they are phase-space representations, not operational event probabilities. Their negativity is operationally meaningful only through how they reproduce positive measurement statistics after integration against admissible test functions (marginals, POVM kernels).

Finite distinguishability does not forbid interference; it only constrains what can be registered as an outcome. Negative quasi-probabilities are permitted as intermediate representations, but any operational probability obtained by coarse-graining over finitely resolvable outcomes must be non-negative. The role of finite distinguishability is not to outlaw interference, but to enforce positivity at the level of observable event measures.

This is consistent with the result that Wigner negativity is necessary for quantum computational advantage (Spekkens 2008; Veitch et al. 2012): the negativity encodes coherence resources that are consumed when outcomes are operationally resolved. VERSF accommodates this structure—negativity reflects contextuality and coherence at the representation level, while positivity is enforced at the distinguishable-outcome level.

---

7. Constraints on Admissible Positive Geometries

VERSF does not compute specific scattering amplitudes, but constrains which positive geometries can correspond to physically realizable processes. This yields concrete predictions.

7.1 Compactness

Finite distinguishability implies that admissible regions must be compact—or effectively compact once projective equivalence and physical cutoffs are imposed—in the relevant kinematic space. Infinite regions would require infinite distinguishability to resolve all boundary behavior.

Known examples such as the amplituhedron are compact when formulated in projective space, consistent with this expectation. This is not a post-hoc observation: VERSF predicts that any positive geometry corresponding to a physical process must admit a compact (or projectively compact) formulation.

7.2 Codimension Suppression Hierarchy

Entropy regulation implies suppression of high-codimension boundary contributions. Boundaries of codimension k correspond to the simultaneous saturation of k independent constraints and therefore require increasingly fine distinguishability.

From Section 4.4, the suppression factor is:

$w_k \sim \exp(-\lambda k)$

in the weakly coupled regime. This predicts that:

- Codimension-1 boundaries (single constraint saturation) dominate
- Higher-codimension contributions are exponentially suppressed
- The suppression scale λ is set by fundamental VERSF parameters

7.3 Empirical Support

Independent support for this hierarchy appears in amplitude physics, where leading contributions are dominated by low-codimension soft and collinear limits, while more degenerate multiparticle factorization channels are subleading (Catani 1998; Bern, Dixon & Kosower 2004; see Dixon 2013 for a review). Specifically:

- Single soft limits (codimension-1) give leading infrared behavior
- Double soft limits (codimension-2) are subleading
- Multi-collinear configurations become increasingly suppressed

This empirical structure aligns with the VERSF-based admissibility hierarchy. The correspondence suggests that the observed soft/collinear dominance is not accidental but reflects the entropy cost of high-codimension constraint saturation.

---

8. Conclusion

Positive geometries and VERSF are aligned through a shared constraint-defined geometric structure. We have established:

1. **Formal correspondence**: VERSF feasibility regions, under (A1)–(A3), satisfy the axioms defining positive geometries and admit unique canonical forms
2. **Derived suppression hierarchy**: Exponential suppression of high-codimension boundaries follows from entropy additivity, with deviations diagnosing constraint correlations
3. **Operational positivity**: Finite distinguishability enforces non-negative probabilities at the measurement level while permitting negative quasi-probability representations
4. **Emergent time interpretation**: Time corresponds to entropy-regulated flow on feasibility space, with canonical form singularities encoding distinguishability reorganization events
5. **Concrete predictions**: Compactness requirements and codimension suppression constrain which positive geometries can represent physical processes

Positive geometries operate at the level of outcome consistency—encoding which scattering processes satisfy unitarity and factorization. VERSF addresses existence-level consistency—encoding which distinguishability configurations can persist under entropy regulation. Together they suggest that physics is governed not by dynamics but by nested layers of admissibility.

This work does not propose that VERSF computes amplitudes, only that it constrains the class of geometries that can correspond to physically realizable processes. The development of explicit VERSF-derived constraints on amplituhedra and related structures remains an important direction for future work.

References

N. Arkani-Hamed and J. Trnka, "The Amplituhedron," JHEP 10 (2014) 030.

N. Arkani-Hamed, Y. Bai, S. He, and G. Yan, "Scattering Forms and the Positive Geometry of Kinematics, Color and the Worldsheet," JHEP 05 (2018) 096.

N. Arkani-Hamed, J. Trnka, et al., "Scattering Amplitudes and the Positive Geometry of the Grassmannian," Commun. Math. Phys. 354 (2017) 119.

N. Arkani-Hamed and J. Trnka, "Into the Amplituhedron," JHEP 12 (2016) 182.

Z. Bern, L. Dixon, and D. A. Kosower, "Two-loop g → gg splitting amplitudes in QCD," JHEP 08 (2004) 012.

S. Catani, "The singular behavior of QCD amplitudes at two-loop order," Phys. Lett. B 427 (1998) 161.

L. Dixon, "A brief introduction to modern amplitude methods," in *Proceedings of the 2012 CERN–Latin-American School of High-Energy Physics*, CERN-2013-003 (2013).

T. Lam, "Amplituhedron cells and Stanley symmetric functions," Commun. Math. Phys. 343 (2016) 1025.

R. W. Spekkens, "Negativity and Contextuality are Equivalent Notions of Nonclassicality," Phys. Rev. Lett. 101 (2008) 020401.

V. Veitch, C. Ferrie, D. Gross, and J. Emerson, "Negative quasi-probability as a resource for quantum computation," New J. Phys. 14 (2012) 113011.

K. Taylor, "Void Energy–Regulated Space Framework: Foundations," VERSF Working Papers (2024).

Appendix A: Notation Summary

Symbol	Meaning
$\mathscr{R}$_VERSF	VERSF feasibility region
$f_i(y)$	Constraint functions
Ω_VERSF	Canonical form on $\mathscr{R}$_VERSF
D(y)	Distinguishability functional
Σ(y)	Entropy loading

Symbol	Meaning
$\Gamma_e ff(y)$	Effective void-energy
D^c, Σ^c	Critical thresholds
β	Inverse entropy temperature
$\lambda = \beta\, \Delta S_1$	Codimension suppression scale
τ	Emergent time parameter
ΔS	Entropy loading above baseline ($S - S_0$)
(A1)–(A3)	Geometric assumptions for VERSF positive geometry candidate
(A4)	Additive entropy cost assumption

Appendix B: Open Problems

1. **Derive β from first principles**: What sets the entropy temperature scale in VERSF?
2. **Explicit amplituhedron constraints**: Which amplituhedra satisfy VERSF admissibility conditions?
3. **Triangulation correspondence**: Does the VERSF cell decomposition reproduce known triangulations of positive geometries?
4. **Constraint correlation diagnostics**: Can deviations from exponential suppression be measured in amplitude data?
5. **Cosmological polytopes**: Do VERSF constraints apply to positive geometries for cosmological correlators?