

A Criticality Theorem for Fact-Producing Universes

Capacity-Driven Collapse and the Quantum–Classical Boundary

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Abstract

We derive a criticality theorem establishing that a universe capable of supporting both coherent quantum dynamics and stable classical facts must operate near a reversible/irreversible phase boundary. The framework rests on a single structural constraint: the Taylor Limit, which restricts physically admissible observable prediction functionals to those that are analytic, Lipschitz continuous, and effectively finite. From this constraint alone, we derive that collapse (irreversible fact creation) occurs when the distinguishability load $D(\psi)$ of a quantum state exceeds the reversible representational capacity C of the local coherent sector. This yields a deterministic, capacity-driven collapse mechanism requiring no stochastic postulate or observer-dependent projection rule. We translate abstract quantities into experimentally measurable proxies and propose a decisive test using large-molecule matter-wave interferometry. The measurement problem, on this view, reduces to a question of representational capacity—and when capacity is exceeded, collapse is what happens.

Abstract for General Readers

Why do quantum superpositions collapse into definite outcomes?

Quantum mechanics tells us that particles can exist in multiple states simultaneously—a phenomenon called superposition. Yet when we measure a particle, we always get a single definite result. For nearly a century, physicists have treated this "collapse" as a fundamental mystery, adding it to quantum theory as an unexplained rule.

This paper argues that collapse isn't mysterious at all. It's what happens when a system runs out of representational capacity.

Think of a juggler keeping balls in the air. A skilled juggler can handle five balls, maybe seven. But if you keep tossing them more balls, eventually they *must* drop some—not because they choose to, but because they've hit a physical limit. Quantum systems face the same constraint: they can only track so many possibilities at once. When interactions with the environment add possibilities faster than the system can handle, it must "drop" some by collapsing into a definite state.

We prove mathematically that:

1. **There's a capacity limit.** Every quantum system has a maximum "complexity budget" determined by its internal structure.
2. **Collapse is forced, not chosen.** When complexity exceeds capacity, collapse isn't optional—it's the only way for physics to remain well-defined.
3. **The universe must operate in a critical window.** Too little environmental interaction means no stable facts ever form. Too much means quantum effects are instantly destroyed. Only in between can both quantum phenomena and classical reality coexist.
4. **This is testable.** We predict that the threshold for quantum collapse should depend on a molecule's internal complexity, not just its size. This can be checked in existing matter-wave interferometry experiments.

The measurement problem, on this view, was never about physics being incomplete. It was about representations having finite capacity. Collapse is just what happens when the juggler runs out of hands.

Scope and Non-Claims

To prevent misreading, we clarify what this paper does and does not claim:

- **We do not derive the Born rule as a probability postulate.** We take $p = |\psi|^2$ as the standard mapping from amplitudes to probabilities and constrain the resolution scale at which those probabilities become operationally meaningful.
 - **We do not propose a microphysical collapse equation.** We derive collapse as an operational necessity under the Taylor Limit, not as a modification to the Schrödinger equation.
 - **We do not claim the Taylor Limit is derivable from deeper principles here.** We treat it as a minimal operational constraint encoding finite precision and finite resources, consonant with finite-precision and finite-information approaches in the literature, but here elevated to a criterion on admissible observables that drives a collapse threshold.
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1. Introduction and Motivation

1.1 The Measurement Problem Restated

Standard quantum mechanics faces a foundational tension: unitary evolution preserves superposition indefinitely, yet measurements yield definite outcomes. The conventional resolution introduces the Born rule and projection postulate as additional axioms, leaving unexplained why and when collapse occurs.

We propose that collapse is not a fundamental postulate but a consequence of representational capacity limits. The central insight is:

- A quantum state ψ can coherently represent only a finite number of Born-resolvable alternatives.
- Interactions continuously inject new distinguishable alternatives into the system.
- When the number of resolvable alternatives exceeds representational capacity, phase information cannot be tracked and must be discarded.
- That discard *is* measurement.

For the general reader: Imagine a juggler who can keep five balls in the air. If you keep tossing them more balls, eventually they must drop some—not because they choose to, but because they've hit a physical limit. Quantum systems face an analogous constraint: they can only "juggle" so many possibilities at once. When environmental interactions add too many possibilities too fast, the system must "drop" some by collapsing into a definite state. This paper makes that intuition mathematically precise.

1.2 Key Claim

Collapse is deterministic and capacity-driven. The quantum–classical boundary is a critical surface in distinguishability space, not a fundamental divide.

Terminological note: We use "collapse" and "irreversible fact creation" interchangeably throughout this paper, with the latter emphasizing the operational content—the permanent commitment of distinguishability into stable records.

2. Mathematical Framework

For the general reader: This section establishes the precise mathematical language we'll use. The key objects are: (1) a space of possible configurations, (2) quantum states that assign probability amplitudes to those configurations, and (3) a set of rules (the "Taylor Limit") that physical predictions must obey. If you're comfortable with the intuition from Section 1, you can skim the definitions and focus on the boxed interpretations.

2.1 Configuration Space and States

Let (Λ, d) denote a finite distinguishability metric space with $|\Lambda| = N \geq 3$. Elements $\lambda \in \Lambda$ label distinguishable micro-configurations. The metric $d : \Lambda \times \Lambda \rightarrow \mathbb{R}_{\geq 0}$ quantifies distinguishability between configurations.

Intuition: Think of Λ as the set of all meaningfully different states a system could be in—like the set of all possible positions of a particle that your measuring apparatus could actually distinguish.

Remark on finiteness. The finiteness of Λ is not an assumption of fundamental discreteness but an operational restriction induced by ϵ_0 -resolution distinguishability. Λ represents the Born-resolvable configuration set at scale ϵ_0 ; configurations differing by less than ϵ_0 in all admissible observables are identified. Any underlying continuum is coarse-grained to finite Λ by the resolution threshold. One might ask whether the choice of ϵ_0 determines everything downstream; the answer is that ϵ_0 is not freely chosen but constrained by the Lipschitz condition—it is the scale below which no admissible observable can reliably discriminate states. The precise value of ϵ_0 is system-dependent (set by the best available measurement resolution), but its existence is guaranteed by the requirement that $L_P < \infty$ for all $P \in \mathcal{P}$.

Definition 2.1 (Pure State). A pure state is an amplitude assignment $\psi : \Lambda \rightarrow \mathbb{C}$ satisfying the normalization condition:

$$\sum_{\lambda \in \Lambda} |\psi(\lambda)|^2 = 1$$

The Born probability vector p_ψ is defined by $p_\psi(\lambda) = |\psi(\lambda)|^2$. Interference sensitivity is carried by the relative phases of ψ .

Intuition: A quantum state assigns a complex number (amplitude) to each possible configuration. The squared magnitude gives the probability; the phase (angle) of the complex number determines interference patterns. Two states can have identical probabilities but different phases—and those phases matter for quantum behavior.

2.2 Reversible Dynamics

Let G be the group of isometries of (Λ, d) , i.e., bijections $g : \Lambda \rightarrow \Lambda$ preserving $d(g\lambda, g\mu) = d(\lambda, \mu)$. Reversible dynamics act on amplitudes via the induced unitary representation $U : G \rightarrow U(\mathbb{C}^N)$:

$$(U_g \psi)(\lambda) = \psi(g^{-1}\lambda)$$

Between fact-creation events, $\psi(t)$ evolves under such unitary actions. This ensures that micro-dynamics preserve distinguishability structure.

Intuition: "Reversible dynamics" means evolution that can be undone—no information is lost. Standard quantum evolution (the Schrödinger equation) is reversible. Measurement is not. We're asking: when does nature *have* to switch from reversible to irreversible?

2.3 The Taylor Limit

The Taylor Limit is the central regularity constraint. It restricts physically admissible observable prediction functionals $P : S \rightarrow [0,1]$, where S is the unit sphere in \mathbb{C}^N , to a controlled analytic class.

For the general reader: The Taylor Limit says that physical predictions must be "well-behaved" in three specific ways: smooth (no sudden jumps), stable (small changes in the state produce small changes in predictions), and resource-bounded (any prediction can only depend on finitely many aspects of the state). These aren't arbitrary assumptions—they reflect the fact that real measurements have finite precision and real apparatus has finite complexity.

Definition 2.2 (Taylor-Admissible Functional). Fix a coordinate chart on $S \subset \mathbb{R}^{2N}$. A functional P is Taylor-admissible if it satisfies:

(i) Analyticity. P is real-analytic on S , admitting a convergent Taylor expansion in local coordinates.

Plain language: The prediction varies smoothly—you can approximate it with a polynomial.

(ii) Lipschitz Continuity. There exists $L_P < \infty$ such that for all $\psi, \phi \in S$:

$$|P(\psi) - P(\phi)| \leq L_P \cdot \|\psi - \phi\|_2$$

Plain language: Similar states give similar predictions. There's a maximum "sensitivity" to state changes.

(iii) Effective Finiteness. For every $\varepsilon > 0$, there exists $K = K(P, \varepsilon)$ such that for all ψ there is a subset $S_\varepsilon(\psi) \subset \Lambda$ with $|S_\varepsilon(\psi)| \leq K$ and:

$$\sum_{\lambda \notin S_\varepsilon(\psi)} |\psi(\lambda)|^2 \leq \varepsilon$$

and $P(\psi)$ depends only on ψ restricted to $S_\varepsilon(\psi)$ up to error $\leq \varepsilon$.

Plain language: Any prediction effectively depends on only finitely many configurations. You don't need infinite resources to compute it.

Remark. The Lipschitz condition captures finite measurement resolution. Effective finiteness captures finite physical resources: only finitely many modes can be operationally relevant to any physical prediction. Together, these encode the constraint that physics operates with bounded precision and bounded complexity.

3. Distinguishability Load and Capacity

For the general reader: This section defines the two key quantities: the "load" D (how complex the quantum state is) and the "capacity" C (how much complexity the system can handle). The main theorem will say: when load exceeds capacity, collapse happens.

3.1 Effective Support

Definition 3.1 (Effective Support). For $\varepsilon \in (0, 1)$, define $K_\varepsilon(\psi)$ as the minimal integer k such that there exists $S \subset \Lambda$ with $|S| = k$ and:

$$\sum_{\lambda \notin S} |\psi(\lambda)|^2 \leq \varepsilon$$

This is the smallest number of configurations needed to capture probability mass $1 - \varepsilon$.

Intuition: If a quantum state is spread across many configurations, K_ε is large. If it's concentrated on just a few, K_ε is small. This measures "how many possibilities are seriously in play."

3.2 Distinguishability Load

Definition 3.2 (Load). Fix a reference resolution $\varepsilon_0 \in (0, 1)$, interpreted as the Born-rule coarse-graining threshold. The distinguishability load is:

$$D(\psi) := \log_2 K_{\{\varepsilon_0\}}(\psi)$$

$D(\psi)$ measures the log-number (in bits) of distinguishable configurations that must be coherently tracked to represent ψ at resolution ε_0 .

Intuition: D counts (in bits) how many "slots" the quantum state is using. A state concentrated on one configuration has $D \approx 0$. A state spread uniformly over 1000 configurations has $D \approx 10$ bits.

Alternative (Entropy Load). For computational convenience, one may use the Shannon entropy:

$$H(p_\psi) = -\sum_\lambda p_\psi(\lambda) \log_2 p_\psi(\lambda)$$

For broadly spread distributions without heavy tails (e.g., near-uniform over the effective support), $H(p_\psi)$ and $\log_2 K_\varepsilon(\psi)$ coincide up to $O(1)$ factors. The effective-support definition is preferred for theoretical precision; the entropy form is often more tractable experimentally.

3.3 Reversible Capacity

Definition 3.3 (Reversible Distinguishability Capacity). Fix the Taylor-admissible class \mathcal{P} of observable functionals and resolution ϵ_0 . The reversible capacity C is:

$$C := \sup \{ D \geq 0 : \text{for all } \psi(t) \text{ evolving under } G\text{-induced unitaries, and all } P \in \mathcal{P}, \text{ the map } t \mapsto P(\psi(t)) \text{ remains Taylor-admissible whenever } D(\psi(t)) \leq D \}$$

Plain language: C is the maximum complexity a quantum state can have while still allowing all physical predictions to remain well-behaved under reversible evolution.

Interpretation. C is the largest log-effective-support load that can be carried while keeping all admissible predictions stable, analytic, and finite-resource-trackable.

Crucial Point. C is a property of the local coherent sector (system plus immediately coupled environment), not a universal constant. It depends on:

- Available degrees of freedom in the local Hilbert space
- Coupling topology between system and environment
- The resolution threshold ϵ_0

For a system with internal Hilbert space dimension d_{int} , we have the scaling:

$$C_{\text{system}} \propto \log(d_{\text{int}})$$

under mild assumptions on the coupling structure.

Intuition: A molecule with more internal vibration modes can "juggle" more quantum possibilities before hitting its limit. Capacity scales with the system's internal complexity.

Constructive Bound on C . The definition of C may appear circular (defined in terms of what it preserves). To ground C physically, we note an upper bound:

$$C \leq \log_2 d_{\text{coh}}(\epsilon_0)$$

where $d_{\text{coh}}(\epsilon_0)$ is the number of mutually distinguishable quantum states that can be coherently maintained at resolution ϵ_0 within the local sector. This is bounded by the effective Hilbert space dimension accessible to the system plus immediate environment, which in turn depends on: (i) the bandwidth of coherent coupling (modes within thermal/dynamical coherence time), (ii) the spatial extent of phase-correlated degrees of freedom, and (iii) the resolution threshold ϵ_0 itself.

In many-body systems, the number of coherently accessible degrees of freedom often scales polynomially or sub-exponentially with the count of active modes within the coherence bandwidth; we write $d_{\text{coh}}(\epsilon_0) \sim N_{\text{active}}^{\{k_{\text{eff}}\}}$ only as a schematic placeholder, not a derived result. A more precise C estimate can be extracted from experimentally measured coherence times, accessible mode bandwidth, and effective local Hilbert dimension inferred from spectroscopy. The precise value is system-dependent, but the existence of a finite bound is guaranteed by finite local resources.

3.4 Distinguishability Influx

Definition 3.4 (Influx via Load Growth). Let $\psi_S(t)$ denote the reduced system state. The distinguishability influx is:

$$\Phi(t) := dD(\psi_S(t))/dt$$

interpreted as an upper Dini derivative when D is not differentiable.

Intuition: Φ measures how fast new "possibilities" are being injected into the system by its environment. High Φ means the environment is rapidly creating new distinguishable alternatives.

Definition 3.5 (Influx via Information Flow). Equivalently, define:

$$\Phi_I(t) := dI(S:E)_t/dt$$

where $I(S:E)$ is the mutual information between system and environment.

Remark on Equivalence. These definitions coincide when environmental correlations faithfully record system alternatives—that is, when each distinguishable system configuration becomes correlated with a distinguishable environmental state. This condition holds in matter-wave interferometry when scattered particles (photons or gas molecules) carry away which-path information without significant recombination or erasure. In such settings, the decoherence rate directly measures Φ . However, in systems with non-Markovian environments or significant back-action, the two definitions may diverge; this would constitute a systematic uncertainty in any experimental test (see Section 6.4).

4. The Criticality Theorem

For the general reader: This is the heart of the paper. We prove that when the complexity of a quantum state (D) exceeds the system's capacity to track it (C), collapse *must* occur—not as a postulate, but as a mathematical necessity. The theorem also shows that fact-producing universes must operate in a "critical window" where both quantum coherence and classical facts can coexist.

4.1 Phase Indistinguishability Beyond Capacity

Before proving the threshold lemma, we establish a key structural result that makes the necessity argument precise.

Proposition 4.1 (Phase Indistinguishability Beyond Effective Finiteness). Let \mathcal{P} be a class of Taylor-admissible functionals satisfying effective finiteness with uniform bound $K_{\max} = \max_P K(P, \epsilon_0)$. Suppose ψ and ϕ are two states with:

(i) Identical Born distributions: $p_\psi = p_\phi$

(ii) Effective support size $K_{\{\varepsilon_0\}}(\psi) = K_{\{\varepsilon_0\}}(\phi) > K_{\max}$

(iii) For every subset $S \subset \Lambda$ with $|S| \leq K_{\max}$, the restrictions $\psi|_S$ and $\phi|_S$ are identical up to an S -dependent global phase, yet ψ and ϕ are not globally phase-equivalent.

Such pairs (ψ, ϕ) exist whenever the phase degrees of freedom exceed the observable phase budget K_{\max} —for example, by assigning phases on $\Lambda \setminus S$ so that all K_{\max} -restricted marginals agree while global phase consistency fails. This is not a pathological construction but a natural consequence of exceeding the observable phase budget.

Then for all $P \in \mathcal{P}$: $|P(\psi) - P(\phi)| \leq 2\varepsilon_0$.

Plain language: If a quantum state is spread over more configurations than any measurement can track, then two states with different phases become indistinguishable. The interference pattern that would reveal their difference simply cannot be detected.

Proof. By effective finiteness, each P depends on ψ only through its restriction to some subset S_P with $|S_P| \leq K_{\max}$, up to error ε_0 . By condition (iii), $\psi|_{S_P}$ and $\phi|_{S_P}$ are phase-equivalent (up to global phase, which P cannot detect). Hence $|P(\psi) - P(\psi|_{S_P})| \leq \varepsilon_0$ and $|P(\phi) - P(\phi|_{S_P})| \leq \varepsilon_0$, with $P(\psi|_{S_P}) = P(\phi|_{S_P})$. By the triangle inequality, $|P(\psi) - P(\phi)| \leq 2\varepsilon_0$. ■

Corollary. When the effective support exceeds K_{\max} , there exist pairs of states with distinct interference signatures that are operationally indistinguishable to all $P \in \mathcal{P}$. Interference at that scale is not merely unmeasured but *undefined* within the admissible observable class.

This proposition makes precise why exceeding capacity forces collapse: the physics cannot track the phase information that would distinguish coherent superposition from mixture.

4.2 Threshold Lemma

Lemma 4.2 (Regularity Breakdown Under Overload). Assume Taylor-admissibility. If on some interval $D(\psi_S(t))$ exceeds C while Φ is bounded away from zero, then there exists $P \in \mathcal{P}$ such that $t \mapsto P(\psi_S(t))$ cannot remain simultaneously analytic, Lipschitz, and effectively finite unless the dynamics performs an irreversible coarse-graining.

Plain language: When load exceeds capacity and keeps growing, something has to give. The nice mathematical properties we required of physical predictions cannot all be maintained. The only way out is collapse.

Proof Sketch. Effective finiteness implies each P can depend on at most K modes at accuracy ε_0 . If $D = \log_2 K_{\{\varepsilon_0\}}(\psi)$ rises above C , then the minimal effective support exceeds the maximal trackable mode budget compatible with Taylor-admissibility across \mathcal{P} .

Maintaining Lipschitz continuity while preserving interference requires phase sensitivity across the entire effective support. This becomes impossible once the support exceeds the trackable budget: the functional would need to track more phase relationships than can be represented within the analyticity constraints. Formally, Proposition 4.1 shows that beyond K_{\max} there exist phase-distinct states that are indistinguishable to all admissible observables, so phase-resolved coherence ceases to be definable within \mathcal{P} unless the dynamics reduces effective support.

Necessity of phase-sensitive observables. One might ask: why must some $P \in \mathcal{P}$ track phase relationships across the full effective support? The answer is that interference *is* phase sensitivity. If no admissible observable were sensitive to relative phases across the support, then superpositions over that support would be empirically indistinguishable from mixtures—interference would be operationally meaningless at that scale. But by assumption, we are in the regime where interference exists ($D < C$ was previously satisfied). Therefore, some $P \in \mathcal{P}$ must have been phase-sensitive. When D crosses C , this P cannot maintain its regularity properties. Restricting \mathcal{P} to exclude all such phase-sensitive functionals would not avoid collapse—it would *be* collapse, merely redescribed as a restriction on observables rather than a projection of states. The physical content is identical.

The only admissible resolution is projection/coarse-graining into robust records, which reduces effective support and removes inaccessible phase correlations. ■

4.3 Capacity-Driven Collapse

Theorem 4.3 (Capacity-Driven Fact Creation). Under the Taylor Limit and reversible (G -isometric) micro-dynamics:

- (i) If $D(\psi_S) < C$, coherent unitary evolution is admissible.
- (ii) If $D(\psi_S) > C$, analytic regularity fails and irreversible coarse-graining (fact creation) is forced.

Extended coexistence of coherence and stable classical facts is possible only near $D(\psi_S) \approx C$.

The bottom line: Below capacity, quantum mechanics works normally. Above capacity, collapse is forced. The quantum-classical boundary is where load meets capacity.

Why collapse is forced, not merely permitted. One might ask: when D exceeds C , why not allow predictions to simply become undefined or unstable, rather than forcing collapse? The answer is that a universe with undefined or unstable predictions is not fact-producing. Persistent records require stable correlations between system and environment; science requires reproducible measurement outcomes; observers require reliable information storage. A regime where predictions fluctuate or fail to exist cannot support any of these. Since we are characterizing *fact-producing* universes (the only kind that can contain observers asking such questions), the alternative of "undefined predictions" is not physically admissible—it would constitute a failure of the universe to produce facts, which is precisely the subcritical regime $\kappa <$

κ_1 . Thus, when $D > C$, the only resolution compatible with continued fact-production is collapse into a reduced state with $D < C$.

What this argument establishes and what it leaves open. The above shows that superposition must give way to mixture (operationally indistinguishable alternatives), and that mixture must resolve to definite facts for the universe to remain fact-producing. In operational terms, the forced transition is first to a classical mixture over robust records; the emergence of a single realized outcome from that mixture is handled by standard probabilistic selection governed by the Born rule. What this argument does *not* derive is *which* outcome occurs or with *what* probability. The Born rule $p = |\psi|^2$ governs outcome selection; we take this as the standard probability mapping and do not claim to derive it here. Our contribution is identifying *when* collapse occurs ($D > C$), not the mechanism by which one outcome is selected from the resulting mixture. The selection mechanism remains an open question, though we note it must reproduce Born statistics to match observation.

Corollary 4.4 (Collapse Criterion). The operational boundary condition is:

$$D(\psi_S) \approx C \text{ (load form)}$$

or equivalently:

$$\Phi \cdot \tau \approx C \text{ (integrated rate form)}$$

where τ is the interaction time.

4.4 The Criticality Parameter

Definition 4.5. Define the criticality parameter:

$$\kappa := (\Phi \cdot \tau) / C$$

This dimensionless ratio compares distinguishability injection to representational capacity.

Intuition: κ is like a "stress ratio" for the quantum system. $\kappa < 1$ means the system can handle the incoming complexity; $\kappa > 1$ means it's overwhelmed and must collapse.

Theorem 4.6 (Criticality Theorem for Fact-Producing Universes). There exist constants $0 < \kappa_1 < \kappa_2$ such that:

(i) Subcritical regime ($\kappa \leq \kappa_1$): Distinguishability influx is insufficient to overcome analytic regularity. Correlations remain reversible and cannot be stabilized into records. The universe fails to be fact-producing.

(ii) Supercritical regime ($\kappa \geq \kappa_2$): Distinguishability influx exceeds reversible capacity. Analytic tracking of alternatives fails, forcing immediate coarse-graining and projection-like behavior. Coherence is rapidly destroyed.

(iii) Critical window ($\kappa_1 < \kappa < \kappa_2$): Both coherent dynamics and stable fact creation can coexist. This is the only regime supporting the physics we observe.

Theorem Summary. A fact-producing universe must operate in a critical window: too little environmental coupling prevents stable records; too much destroys coherence. The quantum–classical boundary is this window's edge.

For the general reader: This theorem explains why we see the world we do. If the universe operated in the subcritical regime, nothing would ever become definite—no measurements, no records, no facts. If it operated in the supercritical regime, quantum effects would be instantly destroyed and we'd never see interference, entanglement, or quantum computing. The universe we inhabit—where both quantum phenomena and classical facts exist—can only occur in the critical window between these extremes.

Remark on κ_1, κ_2 . The existence of a critical window is the theorem's core claim; the precise numerical values of κ_1 and κ_2 are system-dependent and not claimed here. What follows from the Taylor Limit structure alone is:

- $\kappa_1 > 0$: Record formation requires nonzero distinguishability influx (a universe with $\Phi = 0$ produces no facts).
- $\kappa_2 < \infty$: Coherence requires bounded influx (unbounded Φ destroys all superposition).
- $\kappa_1 < \kappa_2$: The subcritical and supercritical failure modes are distinct, so an intermediate window must exist.

Order-of-magnitude estimates for laboratory systems interacting with thermal environments suggest $\kappa_1 \sim O(0.1)$ and $\kappa_2 \sim O(10)$, with the observable quantum–classical boundary near $\kappa \approx 1$. These estimates are placeholders; deriving precise bounds requires specifying the coupling topology, environmental spectrum, and resolution threshold ε_0 for each physical system. The theoretical content lies in the existence and structure of the window, not in particular numerical values.

5. Physical Interpretation

For the general reader: This section translates the abstract mathematics into concrete physical quantities that experimentalists can measure.

5.1 Translation to Physical Proxies

| Abstract Quantity | Physical Proxy | Everyday Analogy |
|-------------------------------------|--|---|
| $D(\psi)$ — distinguishability load | Effective Hilbert-space dimension at Born resolution; approximately the Shannon entropy of the Born distribution | Number of balls the juggler is trying to keep in the air |
| C — reversible capacity | Maximum coherent mode budget of system + local environment; bounded by available degrees of freedom and coupling topology | Maximum balls the juggler can handle |
| Φ — distinguishability influx | Rate of entanglement/mutual information flow to environment; experimentally approximated by decoherence rate, scattering rate, or amplification gain | Rate at which new balls are being tossed to the juggler |
| κ — criticality parameter | Ratio of environmental injection rate to coherent processing capacity | Stress level: incoming balls per unit time divided by juggling capacity |

5.2 Physical Examples

Large molecules: Decohere when environmental scattering injects distinguishable paths faster than internal coherence can track. A molecule in vacuum can maintain quantum superposition of paths; add gas molecules that scatter off it, and the "which-path" information leaks to the environment, driving D above C .

Measurement devices: Engineered so amplification gain guarantees $\Phi \gg C$, ensuring rapid fact creation. A Geiger counter, for instance, amplifies a single ionization into an avalanche of $\sim 10^8$ electrons—deliberately pushing the system deep into the supercritical regime so that a definite "click" is guaranteed.

Mesoscopic systems: Remain quantum because $\Phi \approx C$ for experimentally accessible times; they operate in the critical window. This explains why systems like superconducting qubits can maintain coherence for microseconds—they're carefully engineered to sit just below the collapse threshold.

5.3 What This Framework Explains

1. **No stochastic collapse postulate.** Collapse is deterministic capacity saturation—not a roll of cosmic dice, but an inevitable consequence of exceeding representational limits.
2. **No observer dependence.** The threshold $D \approx C$ is objective and physical. Collapse doesn't require consciousness or observation—it requires only that the system's complexity exceed its tracking capacity.
3. **Born rule integration.** The Born rule, measurement threshold, and quantum–classical boundary share a single scale ϵ_0 . Born probabilities remain the standard mapping from amplitudes to operational frequencies; the Taylor Limit selects the resolution scale ϵ_0 at which those probabilities are physically discriminable and stable. Below ϵ_0 , probability differences cannot be reliably resolved by any admissible observable. The appearance of

the same ϵ_0 in the capacity definition and the collapse threshold is not a coincidence but a consequence of both being set by the regularity requirements on physical predictions.

4. **Criticality is structural.** The critical window is not fine-tuned but follows from the requirement that physics be both coherent and fact-producing. We don't need to explain why the universe is "tuned" to allow both quantum and classical behavior—the theorem shows this is the only regime where observers can exist to ask the question.
 5. **Graceful failure.** Quantum mechanics fails at the capacity limit in a predictable, continuous way—not mysteriously. There's no "quantum magic" that suddenly switches off; there's a smooth transition as load approaches and exceeds capacity.
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6. Experimental Test

For the general reader: A theory is only as good as its testable predictions. This section proposes a specific experiment that could distinguish our framework from standard quantum mechanics. The key prediction: the threshold for losing quantum interference should depend on a system's *internal complexity*, not just its size or mass.

6.1 Core Prediction

The framework predicts that loss of quantum interference occurs when the number of Born-resolvable alternatives injected into the system exceeds the reversible representational capacity C .

Key distinction from standard decoherence:

- Standard decoherence predicts monotonic visibility loss driven by environmental coupling strength, with visibility declining continuously as coupling increases.
- Capacity-driven criticality predicts a crossover that is systematically sharper across molecular families, and—more importantly—exhibits a universal curve collapse when plotted against $\Phi\tau/C$.

Analogy: Standard decoherence is like turning up background noise—signal quality degrades smoothly. Capacity-driven collapse exhibits a sharper crossover structure, and crucially, data from different systems should collapse onto one curve when properly normalized.

6.2 Proposed Experiment: Large-Molecule Matter-Wave Interferometry

System. Use a matter-wave interferometer with progressively larger molecules (e.g., oligoporphyrins, nanoclusters, or tailored macromolecules), extending experiments by Arndt and collaborators.

Control parameters:

- Molecular complexity (internal degrees of freedom, determining C)

- Environmental scattering rate (background gas pressure, determining Φ)
- Interferometer path separation and interaction time τ

6.3 Experimental Protocol

1. Prepare molecular beams spanning a range of internal complexity (vibrational mode count, conformational degrees of freedom), controlling for mass and scattering cross-section as much as feasible—or explicitly normalizing for standard decoherence parameters.
2. For each molecular species, vary environmental coupling (gas pressure or photon scattering) and measure interference visibility V .
3. Extract the critical scattering rate or pressure at which V collapses.
4. Compare scaling of the critical point with molecular internal complexity, after factoring out mass and cross-section dependence predicted by standard decoherence.

Alternative framing: Compare molecular families with similar cross-sections but differing internal state densities (e.g., rigid vs. floppy molecules of comparable size). The capacity framework predicts residual dependence on internal complexity beyond what standard decoherence parameters account for.

Systematic uncertainties. Two potential sources of systematic error deserve attention: (i) The equivalence between load-growth and mutual-information definitions of Φ assumes faithful environmental recording (see Section 3.4). In matter-wave interferometry with gas scattering, this assumption is well-justified since scattered molecules carry away which-path information irreversibly. However, in setups with significant photon reabsorption or non-Markovian effects, the measured decoherence rate may not accurately reflect the true distinguishability influx. (ii) Estimating C from internal mode counts requires assumptions about which modes are coherently coupled; spectroscopic validation of the effective mode budget would strengthen any claimed C value.

6.4 Discriminating Predictions

Addressing apparent thresholds in standard decoherence. One might object that standard decoherence already produces sharp-looking thresholds in practice, due to detection limits, postselection, or noise floors. This is true but does not undermine our discriminant. The capacity-driven prediction is not merely that thresholds exist, but that:

1. The threshold location should shift systematically with internal complexity at fixed mass and geometry (or after normalizing for standard decoherence parameters).
2. Visibility curves from different molecular species should collapse onto a universal function when plotted against $\Phi\tau/C$.

Standard decoherence does not predict this collapse under capacity-normalized scaling. The scaling test, not the mere existence of a threshold, is the discriminant.

Standard decoherence predicts:

- Critical scattering rate depends primarily on mass, cross-section, and coupling strength
- Internal degrees of freedom act only indirectly via heating or effective noise
- Visibility decays monotonically with environmental coupling

Capacity-driven criticality predicts:

- Critical point shifts systematically with internal complexity, even at fixed mass and geometry
- Systems with larger internal Hilbert-space budgets (larger C) tolerate higher environmental coupling before collapse
- Visibility remains high until a sharp threshold, then drops rapidly
- Curves for different molecular complexities collapse when plotted against $\Phi \cdot \tau / C$

6.5 Observable Signature

The distinguishing signature is a nonlinear, threshold-like dependence of interference visibility on environmental coupling, with the threshold location scaling with internal representational capacity.

Predicted scaling law:

$$V = f(\Phi \cdot \tau / C)$$

where f is a universal function with $f(x) \approx 1$ for $x \ll 1$ and $f(x) \rightarrow 0$ rapidly for $x > 1$.

What to look for: Plot interference visibility against the "stress parameter" $\Phi \tau / C$. If this framework is correct, data from molecules of wildly different sizes and structures should all fall on the same curve. That universal collapse is the smoking gun.

Graphical test: Plot visibility V against the scaled variable $\Phi \cdot \tau / C$. Data from molecules of different internal complexity should collapse onto a single curve.

6.6 Why This Test Is Decisive

This experiment directly probes whether collapse is driven by environmental noise alone (standard decoherence) or by saturation of reversible representational capacity (this framework).

A positive result would:

- Provide the first empirical support for a capacity-based collapse mechanism
- Explain why mesoscopic quantum systems exist at all
- Establish criticality as a physical, measurable boundary
- Vindicate the Taylor Limit as a fundamental constraint on physical predictions

7. Discussion

7.1 Relation to Other Approaches

Decoherence theory: Our framework is compatible with decoherence but adds a threshold mechanism. Decoherence describes how environmental entanglement suppresses interference; we explain *when* this suppression becomes irreversible. Standard decoherence is the "how"; capacity saturation is the "when."

Objective collapse models (GRW, Penrose): These introduce stochastic or gravitational collapse triggers. Our approach derives collapse from information-theoretic capacity limits without new physical postulates. Where GRW adds randomness and Penrose adds gravity, we add only the requirement that physical predictions be well-behaved.

QBism and relational interpretations: These locate collapse in the observer's knowledge update. Our framework locates it in objective capacity saturation, independent of observers. The threshold $D \approx C$ exists whether or not anyone is watching.

7.2 The Status of the Taylor Limit

The Taylor Limit functions as the foundational constraint in this framework, but one may ask: why does it hold? Several perspectives are available:

(a) Brute physical fact. The Taylor Limit may simply characterize our universe's physics—an empirical regularity with no deeper explanation, akin to the dimensionality of space or the existence of conservation laws.

(b) Consequence of deeper principles. The three Taylor conditions (analyticity, Lipschitz continuity, effective finiteness) may follow from more fundamental constraints. Information-theoretic approaches suggest that finite channel capacity implies effective finiteness; thermodynamic considerations suggest that finite free energy implies bounded sensitivity (Lipschitz); and the requirement that predictions be computable or simulable may enforce analyticity. These connections remain to be rigorously established.

(c) Observer selection. Only universes with Taylor-admissible physics may produce stable records, and hence observers capable of formulating physical theories. On this view, we observe the Taylor Limit because we could not exist in a universe without it—an anthropic constraint on admissible physics.

Importantly, the criticality theorem itself is non-anthropic; observer selection is discussed only as one possible explanation for *why* the Taylor Limit holds, not as part of the derivation. The theorem follows from the Taylor Limit alone, regardless of its ultimate justification.

We do not commit to a resolution here. The Taylor Limit may be fundamental, derived, or selected; what matters for the present argument is that it holds and yields testable consequences. Investigating its origins is a task for future work.

7.3 Open Questions

1. **Explicit computation of κ_1, κ_2** for specific physical systems (e.g., spin chains, harmonic oscillators coupled to thermal baths).
2. **Connection to gravity:** Does gravitational interaction provide a natural capacity bound, as Penrose suggests? Could spacetime curvature limit local representational capacity?
3. **Cosmological implications:** What does criticality imply for the early universe and the emergence of classical spacetime? Was there a phase transition as the universe cooled into the critical window?
4. **Biological systems:** Do living systems exploit the critical window for quantum coherence in functional processes? Photosynthesis and bird navigation show hints of quantum effects—is biology optimized to operate near $\kappa \approx 1$?

7.4 Conclusion

We have established that a fact-producing universe must operate near a reversible/irreversible phase boundary. The quantum–classical transition is not a fundamental mystery but a capacity-driven threshold: when distinguishability load exceeds representational capacity, coherent superposition gives way to definite fact.

This framework:

- Derives collapse from the Taylor Limit alone
- Predicts a measurable criticality threshold
- Unifies the Born rule, measurement, and classicality under a single scale
- Offers decisive experimental tests

The measurement problem, on this view, was never a problem of physics but of representation—and representations have finite capacity.

Final thought for the general reader: For nearly a century, physicists have struggled to explain why quantum superpositions collapse into definite outcomes. We've shown that collapse doesn't require modifying the unitary equations of quantum mechanics; it follows once we impose the minimal operational constraint that physical predictions must remain stable and finite-resource. Just as a computer runs out of memory, a quantum system runs out of "coherence budget." When it does, it must make a choice. That choice is measurement. That's all measurement ever was.

Appendix A: Notation Summary

| Symbol | Definition |
|---|--|
| (Λ, d) | Finite distinguishability metric space |
| $N = \Lambda $ | Number of micro-configurations |
| $\psi : \Lambda \rightarrow \mathbb{C}$ | Pure state (amplitude assignment) |

| Symbol | Definition |
|--|---|
| $p_\psi(\lambda) = \psi(\lambda) ^2$ | Born probability |
| G | Isometry group of (Λ, d) |
| U_g | Unitary induced by isometry g |
| ε_0 | Born-rule resolution threshold |
| $K_\varepsilon(\psi)$ | Effective support at resolution ε |
| $D(\psi) = \log_2 K_{\{\varepsilon_0\}}(\psi)$ | Distinguishability load (bits) |
| C | Reversible capacity |
| $\Phi = dD/dt$ | Distinguishability influx |
| $\kappa = \Phi\tau/C$ | Criticality parameter |
| \mathcal{P} | Class of Taylor-admissible functionals |

Appendix B: Proof Details for Lemma 4.2

Note on proof methodology. The following is a *regularity argument*: it shows that maintaining Taylor-admissibility under the stated conditions is impossible, forcing a transition to a coarse-grained description. This is not a functional-analytic existence proof but a structural necessity argument. The conclusion—that collapse must occur—follows from the incompatibility of the regularity conditions with the state's complexity, not from constructing an explicit collapse mechanism.

Full Proof of Regularity Breakdown.

Let \mathcal{P} be the class of Taylor-admissible functionals and suppose $D(\psi_S(t)) > C$ on an interval $[t_0, t_1]$ with $\Phi(t) \geq \Phi_{\min} > 0$.

By Definition 3.3, C is the supremum over loads D such that all $P \in \mathcal{P}$ remain Taylor-admissible under unitary evolution. Since $D(\psi_S) > C$, there exists $P^* \in \mathcal{P}$ such that at least one of the three Taylor conditions fails for $t \mapsto P^*(\psi_S(t))$.

Case 1: Analyticity failure. If $P^*(\psi_S(t))$ develops a singularity, the Taylor expansion diverges. This signals a breakdown of stable phase-resolved predictions; operationally, this is indistinguishable from decoherence at resolution ε_0 .

Case 2: Lipschitz failure. If $|P^*(\psi) - P^*(\phi)|/\|\psi - \phi\| \rightarrow \infty$, infinitesimal state changes produce finite prediction changes. Operationally, this is indistinguishable from outcome selection: infinitesimal perturbations amplify into macroscopically different records, yielding effectively discrete outcomes at resolution ε_0 .

Case 3: Effective finiteness failure. If the minimal $K(P^*, \varepsilon)$ exceeds all bounds, P^* depends on arbitrarily many modes. This cannot be realized with finite physical resources; the system must project onto a tractable subspace.

In all cases, restoring Taylor-admissibility requires reducing $D(\psi_S)$ below C . The only dynamically available mechanism is irreversible coarse-graining: projecting onto robust record states that reduce effective support while discarding inaccessible phase correlations.

Since $\Phi > 0$, new distinguishability is continuously injected. Without coarse-graining, D would continue to grow, perpetuating the violation. Thus coarse-graining is not merely permitted but *forced* by the dynamics. ■

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