An RAL Example: Why the Tensor Product Rule Emerges Naturally

This document provides a simple, intuitive example—using Resonant Assembly Language (RAL)—to explain why the tensor product rule arises naturally in quantum mechanics. The goal is not to assume quantum formalism, but to show how it becomes unavoidable once we treat quantum systems as oscillatory assemblies rather than discrete particles.

Important clarification: RAL is not offering an alternative formalism or competing interpretation. The mathematics of quantum mechanics remains unchanged. What RAL provides is an explanatory framework showing *why* the existing formalism has the structure it does.

Table of Contents

Part I: Intuitive Explanation

Part II: Mathematical Support

Part III: Positioning

Appendices

For the General Reader

Before diving into the physics, here's the key idea in plain language:

When physicists combine two quantum systems (like two electrons), they use something called the "tensor product" to describe all possible joint states. In many textbooks, this rule is introduced as a postulate—a starting axiom to be accepted without deeper justification. There are operational reconstructions that motivate it from measurement-level axioms (Hardy, 2001; Chiribella et al., 2011), but these typically don't give a pre-measurement physical picture for why amplitudes must compose this way.

RAL offers a different perspective. Instead of treating quantum particles as tiny billiard balls that happen to obey strange probabilistic rules, RAL treats them as **oscillating patterns**—like vibrations on a drum or waves on water. Before measurement, these oscillations are real and ongoing; measurement is what "freezes" them into definite outcomes.

Once you adopt this view, the tensor product rule stops being mysterious. It's simply saying: *when you have two vibrating systems, every vibration pattern of the first can combine with every vibration pattern of the second*. The mathematics follows from the physics, rather than being imposed on it.

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1. One Quantum System as an Oscillatory Assembly

Consider a single electron spin. In standard quantum mechanics, it has two basis states: spin up $|\uparrow\rangle$ and spin down $|\downarrow\rangle$. In RAL terms, this electron is an assembly that supports two orthogonal oscillatory modes. Before measurement, the system is not in one state or the other—it is actively oscillating across both modes simultaneously.

The Core Assumption

RAL adopts the minimal physical assumption that quantum superpositions correspond to real, unresolved dynamical processes—that is, oscillatory assemblies—rather than merely epistemic catalogues of unknown definite states. This is sometimes called a "realist" or "ontic" interpretation of the wavefunction.

The argument that follows shows that *if this assumption holds*, then the tensor product rule is unavoidable. Readers who maintain that the wavefunction is purely epistemic (a bookkeeping device for observer knowledge) will find the derivation unconvincing—but that disagreement concerns the premise, not the logic.

It is worth noting that the ontic reading is not adopted here merely as a taste preference. Results in the ψ-ontology literature—most famously the Pusey–Barrett–Rudolph theorem (2012)—strongly constrain broad classes of "purely epistemic" wavefunction models under natural independence assumptions. RAL does not depend on PBR, but PBR makes the ontic premise materially less ad hoc than it might otherwise appear.

General reader note: Think of a guitar string that can vibrate in two different ways at once. Before you "measure" it (touch it to stop the vibration), both vibration patterns are genuinely present, superimposed on each other. RAL says quantum systems really do this—they're not secretly in one state with us merely ignorant of which.

The strength of each oscillation corresponds to an amplitude $\alpha \in \mathbb{C}$, and the relative alignment of the oscillations corresponds to phase θ. The general state takes the form:

$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

where $|\alpha|^2 + |\beta|^2 = 1$.

At this stage, nothing is decided. The system is in superposition: a live oscillatory process, not a classical mixture of hidden definite states.

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2. Adding a Second System

Now introduce a second electron spin. It too is an independent assembly with its own two orthogonal oscillatory modes $|\uparrow\rangle$ and $|\downarrow\rangle$. Importantly, neither system has been measured. Both remain fully oscillatory and unresolved.

We denote the first system's state as $|\psi\_A\rangle$ and the second as $|\psi\_B\rangle$:

$|\psi\_A\rangle = \alpha_1|\uparrow\rangle\_A + \beta_1|\downarrow\rangle\_A$

$|\psi\_B\rangle = \alpha_2|\uparrow\rangle\_B + \beta_2|\downarrow\rangle\_B$

The question now becomes: what structure must the joint system possess?

General reader note: Imagine two separate drums, each vibrating independently. We're asking: when we treat them as a single combined system, what vibration patterns must we allow?

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3. What the Joint System Must Support

The key question RAL asks is: *what oscillations must the combined system be able to support?*

Physics gives a clear answer. Every oscillatory mode of the first system must be able to coexist and resonate with every oscillatory mode of the second. To impose any restriction—to say that mode $|\uparrow\rangle_A$ cannot resonate with mode $|\downarrow\rangle_B$, for instance—would be to assert an arbitrary constraint on which combinations nature permits. No physical principle justifies such a restriction at the level of composition.

(Constraints like superselection sectors or identical-particle symmetrisation do restrict which states are physically realisable, but they act as restrictions *within* the composite space rather than replacing the underlying composition rule. For identical particles, the physical state space is the symmetric or antisymmetric subspace of $V_A \otimes V_B$; this constrains admissible states but does not change the underlying composition rule.)

Therefore, the joint system must support four independent joint oscillations:

Joint Mode Configuration
$\uparrow\uparrow\rangle$
$\uparrow\downarrow\rangle$
$\downarrow\uparrow\rangle$
$\downarrow\downarrow\rangle$

This is not a mathematical choice—it is a physical necessity. The dimension of the joint space is $\dim(V_A) \times \dim(V_B) = 2 \times 2 = 4$.

General reader note: If drum A can make sounds X and Y, and drum B can make sounds P and Q, then the combined system must be able to make sounds X+P, X+Q, Y+P, and Y+Q. Forbidding any combination would require some special law—but no such law exists at the fundamental level.

Why Not Direct Sums?

A natural question arises: why does the joint space have dimension $2 \times 2 = 4$ rather than $2 + 2 = 4$? (In this case the numbers coincide, but for larger systems—say $3 \times 3 = 9$ versus $3 + 3 = 6$—they diverge.)

The answer lies in what each operation represents:

- **Direct sum (⊕)** corresponds to *exclusive alternatives*: the system is in subspace A *or* subspace B, but not both. This describes classical uncertainty about which system you have.
- **Tensor product (⊗)** corresponds to *simultaneous coexistence*: both systems exist and oscillate together, unresolved, at the same time.

Since RAL treats superposition as real oscillatory coexistence rather than epistemic uncertainty, the tensor product is the correct composition rule. Direct sums are appropriate for mutually exclusive sectors (including superselection sectors), whereas tensor products describe joint degrees of freedom. The physical situation of two electrons existing simultaneously calls for the latter.

General reader note: Direct sum is like having either a cat or a dog but not knowing which. Tensor product is like having both a cat and a dog at the same time. Quantum systems genuinely coexist before measurement—they don't secretly reduce to one-or-the-other.

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4. Why Amplitudes Multiply

In RAL, amplitudes represent resonance strength. Joint resonance requires simultaneous alignment of oscillations across both subsystems. Because both oscillations must contribute for joint resonance to occur, weakening either one weakens the whole.

Consider system A oscillating in mode $|\uparrow\rangle_A$ with amplitude α_1, and system B oscillating in mode $|\downarrow\rangle_B$ with amplitude β_2. The joint resonance strength for the combined mode $|\uparrow\downarrow\rangle$ must reflect that:

- If $\alpha_1 \to 0$, the joint resonance vanishes (system A contributes nothing to this mode)
- If $\beta_2 \to 0$, the joint resonance vanishes (system B contributes nothing to this mode)
- The joint strength scales linearly with each individual strength

The Bilinearity Assumption

We make this explicit as an assumption:

Independence + linear response: For fixed B, the joint amplitude depends linearly on A's amplitude assignment, and vice versa.

This is the assumption of *bilinearity*. It says that if you double A's amplitude in some mode while holding B fixed, the joint amplitude in the corresponding joint mode also doubles.

Why linear? Because "superposition" is precisely the empirical statement that when a system can realise two oscillatory modes, it can also realise their sum, and interference depends continuously on relative scaling and phase. If two subsystems are independent, scaling the oscillatory contribution of A while holding B fixed should scale the joint contribution proportionally, and vice versa. This is the minimal compositional compatibility between superposition and independence; mathematically, it is bilinearity. Bilinearity is not "Hilbert space by stealth"—it is "superposition behaves like wave addition in each subsystem."

Why not nonlinear composition? If the joint-amplitude map were nonlinear in either subsystem—for example depending on $|\psi_A(a)|^2\psi_B(b)$ rather than $\psi_A(a)\psi_B(b)$—then "superposition" would cease to behave like wave addition under composition. Concretely, interference visibility would generally depend on absolute intensities rather than relative phase alone, and rescaling or decomposing a preparation into different superposed descriptions would change joint predictions. This would break the operational invariance that experimentally characterises superposition: the statistics depend on the complex amplitudes (including phase) and combine additively under coherent recombination. Bilinearity is therefore not a mathematical convenience but the minimal requirement that composition respect coherent additivity in each subsystem.

Up to an overall scaling and choice of basis, bilinearity picks out the standard product map that becomes the tensor product under linearisation:

$\Psi(\uparrow,\downarrow) = \alpha_1 \cdot \beta_2$

More generally, for configurations $a \in \Lambda_A$ and $b \in \Lambda_B$:

$\Psi(a,b) = \psi_A(a) \cdot \psi_B(b)$

Therefore, the joint-amplitude map is bilinear; the tensor product is the unique linearisation of bilinear composition. This multiplicative structure is exactly what appears mathematically as the tensor product. It is not a quantum oddity—it is how composite wave-like systems compose throughout physics, from classical wave interference to signal processing. For example, in classical wave mechanics the joint amplitude for two independent modes is the product of their complex phasors, and in Fourier analysis separable 2D signals factor as $F(\omega_x, \omega_y) = F_x(\omega_x)F_y(\omega_y)$—the same rank-1 (product) structure that tensor products formalise.

General reader note: For independent systems, the probability of joint outcomes factorises (like coin flips: P(heads AND heads) = ½ × ½). Quantum theory implements this by having amplitudes compose bilinearly (multiplying in the product basis), so that probabilities computed from squared norms also factorise correctly.

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5. Entanglement as a Shared Mode

Not all joint states can be written as products of individual states. Consider:

$|\Phi^+\rangle = (1/\sqrt{2})(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$

This state cannot be factored into $|\psi_A\rangle \otimes |\psi_B\rangle$ for any choice of individual states. Mathematically, no α_1, β_1, α_2, β_2 exist such that:

$$(\alpha_1|\uparrow\rangle + \beta_1|\downarrow\rangle) \otimes (\alpha_2|\uparrow\rangle + \beta_2|\downarrow\rangle) = (1/\sqrt{2})(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

Expanding the left side gives $\alpha_1\alpha_2|\uparrow\uparrow\rangle + \alpha_1\beta_2|\uparrow\downarrow\rangle + \beta_1\alpha_2|\downarrow\uparrow\rangle + \beta_1\beta_2|\downarrow\downarrow\rangle$. Matching coefficients requires $\alpha_1\beta_2 = 0$ and $\beta_1\alpha_2 = 0$, but also $\alpha_1\alpha_2 = \beta_1\beta_2 = 1/\sqrt{2}$. These conditions are mutually incompatible.

In RAL terms, $|\Phi^+\rangle$ represents a global oscillatory mode—a standing wave that belongs to the whole system rather than emerging from independent oscillations in each part. The phase relationships are intrinsically correlated: the two subsystems oscillate in lockstep, and this correlation cannot be decomposed into separate phase evolutions.

Entanglement is therefore not mysterious in RAL. It is the natural result of composite oscillatory assemblies forming resonant modes that span the entire system.

General reader note: Imagine two tuning forks that were coupled while being set into a single standing-wave pattern; once separated, the shared pattern can persist even without continued physical coupling. Entanglement is like this: a vibration pattern that inherently involves both systems at once, with no way to split it into "your vibration" and "my vibration."

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6. Measurement and Collapse

When a measurement occurs, oscillatory energy is irreversibly transferred into a macroscopic record. One mode locks in, superposition ends, and the dynamically active superposition is converted into a stable macroscopic record.

For the entangled state $|\Phi^+\rangle$, measuring system A and finding $|\uparrow\rangle_A$ instantly constrains system B to $|\uparrow\rangle_B$—not because information travels between them, but because the global oscillatory mode $|\Phi^+\rangle$ only supports correlated configurations. The measurement resolves the global mode, and the resolution is inherently joint.

For the recorded branch, what remains operationally are classical correlations between outcomes. The tensor product formalism remains mathematically valid, but the physical role of active superposition has ended for that measurement event.

General reader note: This is the phenomenon Einstein famously worried about—but RAL suggests it's not action at all. It's somewhat like discovering that two gloves in separate boxes are a matching pair: the correlation was built into the system from the start, and measurement reveals rather than creates it. However, unlike the glove case, quantum correlations can exceed any classical hidden-variable model (Bell, 1964); the point here is only that correlation does not require superluminal signalling.

Bell correlations in the RAL picture. In RAL terms, an entangled pair is a single global oscillatory mode with joint phase structure; the reduced descriptions of each subsystem are incomplete because they omit that shared phase relation. Local measurements probe the global mode in different local bases, and the joint statistics reflect those pre-existing global constraints. No superluminal signal is required because nothing is transmitted at measurement time: the correlation is a property of the shared mode, and locality is respected in the sense that each outcome is generated by a local interaction with the measuring device. What Bell rules out are local hidden-variable factorisations of the joint probabilities, not the existence of nonseparable global states.

Scope note. The present document uses only a minimal measurement assumption: measurement corresponds to an effectively irreversible coupling to macroscopic degrees of freedom that yields a stable record and ends coherent interference for that event. A full RAL treatment would still owe (i) a dynamical account of how "record formation" suppresses alternative branches/modes, and (ii) an explanation of outcome selection (or why outcome selection is the wrong framing). Those questions are interpretation-sensitive and remain open here.

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7. Conclusion

In RAL terms, the tensor product rule is simply the statement that unresolved oscillatory assemblies must allow all joint oscillations, with amplitudes combining bilinearly. Once this is accepted:

- **Tensor products** are not mysterious mathematical postulates—they are the unique structure preserving superposition and locality
- **Entanglement** is not spooky action—it is the existence of global resonant modes
- **Measurement collapse** is not paradoxical—it is the irreversible resolution of oscillatory possibility into definite record

The formalism of quantum mechanics emerges as physically necessary given the core assumptions of real superposition, bilinear composition, and locality—not arbitrarily imposed.

Proposition: The RAL Tensor Product Rule

Assumptions:

1. **Real superposition** — Pre-measurement states are ontic oscillatory modes, not epistemic uncertainty
2. **Independence** — Subsystems have independent configuration spaces prior to interaction/measurement

3. **Superposition closure** — Admissible amplitude assignments are closed under linear combination
4. **Bilinear compositional response** — Scaling amplitude in either subsystem scales the joint amplitude proportionally
5. **Locality** — Reversible operations on A act as $U_A \otimes I_B$ on the composite

Conclusion: The composite state space is (up to isomorphism) $V_A \otimes V_B$, with product-basis amplitudes $\Psi(a,b) = \psi_A(a)\cdot\psi_B(b)$, and the unique extension of local operations is $U_A \otimes I_B$.

(All uniqueness claims are up to isomorphism and choice of basis/normalisation.)

Mathematical Support for the RAL Tensor Product Rule

This section provides a minimal mathematical formalisation of the RAL intuition presented above. The goal is to demonstrate that the tensor product rule arises inevitably once we impose superposition, locality, and independence on unresolved oscillatory systems.

General reader note: This section is more technical, but the key point is simple: we're showing that the intuitive arguments above translate into rigorous mathematics. If you accept the physical premises, the tensor product is logically forced.

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1. Amplitude Spaces

Let system A have a finite set of distinguishable configurations $\Lambda_A = \{a_1, a_2, \ldots, a_n\}$ and system B have $\Lambda_B = \{b_1, b_2, \ldots, b_m\}$. Pre-measurement states are amplitude assignments:

$\psi_A : \Lambda_A \to \mathbb{C}$

$\psi_B : \Lambda_B \to \mathbb{C}$

The collections of all such assignments form complex vector spaces:

$V_A = \mathbb{C}^{|\Lambda_A|} \cong \mathbb{C}^n$

$V_B = \mathbb{C}^{|\Lambda_B|} \cong \mathbb{C}^m$

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2. Product Resonance and Bilinearity

The joint configuration space is the Cartesian product $\Lambda_A \times \Lambda_B$. RAL requires that simultaneous oscillations combine via a joint-amplitude map:

$\Psi : V_A \times V_B \to V_{AB}$

We impose the **bilinearity assumption**: for fixed ψ_B, the map $\psi_A \mapsto \Psi(\psi_A, \psi_B)$ is linear, and vice versa. This captures the physical requirement that joint resonance strength responds linearly to each subsystem's amplitude. (The physical justification for bilinearity—and why nonlinear alternatives fail—is given in Part I, §4.)

The bilinear composition corresponding to independent joint configurations is:

$\Psi(a,b) = \psi_A(a) \cdot \psi_B(b)$

This defines *separable* or *product* states:

$|\psi_A\rangle \otimes |\psi_B\rangle$

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3. Superposition Closure

Physical admissibility requires closure under superposition. If Ψ_1 and Ψ_2 are both valid joint states, then so is any linear combination:

$\Psi = c_1\Psi_1 + c_2\Psi_2$, for $c_1, c_2 \in \mathbb{C}$

Hence all linear combinations of product resonances must be included. The smallest vector space containing all product states is the algebraic tensor product:

$V_A \otimes V_B$

This space has dimension $\dim(V_A) \times \dim(V_B) = n \times m$, with basis elements $|a_i\rangle \otimes |b_j\rangle$.

Key theorem: The tensor product $V_A \otimes V_B$ is the unique (up to isomorphism) vector space that linearises bilinear maps from $V_A \times V_B$. This is the *universal property* of tensor products.

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4. Why Tensor Products, Not Direct Sums

It is worth pausing to address why the composition rule is $\otimes$ (tensor product) rather than $\oplus$ (direct sum).

The direct sum $V_A \oplus V_B$ has dimension $n + m$, not $n \times m$. It represents *mutually exclusive alternatives*: the system is described by a state in V_A or a state in V_B, but not both simultaneously. This is appropriate for classical ignorance—when you have one of two possible systems but don't know which.

The tensor product $V_A \otimes V_B$, by contrast, represents *simultaneous coexistence*. Both systems exist; both are oscillating; neither has been resolved. Every mode of A can resonate with every mode of B.

Given the RAL assumption that pre-measurement superpositions are real oscillatory processes (not epistemic uncertainty), simultaneous coexistence is the correct physical picture. Hence:

Real coexistence $\Longrightarrow$ Tensor product, not direct sum

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5. Locality

Local reversible operations must act independently on each subsystem. For a unitary operation U_A acting only on system A:

$$(U_A \otimes I_B)(|\psi_A\rangle \otimes |\psi_B\rangle) = (U_A|\psi_A\rangle) \otimes |\psi_B\rangle$$

The universal property of the tensor product guarantees that this extension to the full joint space is unique and well-defined. Any bilinear map from $V_A \times V_B$ factors uniquely through $V_A \otimes V_B$:

For $f : V_A \times V_B \to W$ bilinear, $\exists!\ \tilde{f} : V_A \otimes V_B \to W$ linear

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6. Inner Product and Probability

With probabilities given by squared norms (the Born rule), independence of measurements on separate systems requires factorisation of inner products:

$$\langle \psi_A \otimes \psi_B \mid \varphi_A \otimes \varphi_B \rangle = \langle \psi_A|\varphi_A\rangle \cdot \langle \psi_B|\varphi_B\rangle$$

This extends by linearity to all states in $V_A \otimes V_B$. For infinite-dimensional systems, completing the algebraic tensor product under the induced norm yields the Hilbert space tensor product:

$$\mathscr{H}_A \otimes \mathscr{H}_B = \mathrm{cl}(V_A \otimes V_B)$$

where cl(·) denotes norm completion. The completion introduces limits of Cauchy sequences, which is where infinite-dimensional subtleties reside (see Reed & Simon, 1980, Vol. I, Ch. II). For finite-dimensional systems (such as spin-½ particles), the algebraic and Hilbert tensor products coincide.

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7. Entanglement

States in V_A ⊗ V_B that cannot be written as a single product |ψ_A⟩ ⊗ |ψ_B⟩ correspond to global resonant modes. These are *entangled* states.

Formally, a pure state |Ψ⟩ ∈ V_A ⊗ V_B is entangled if and only if its Schmidt rank exceeds 1. Any joint state admits a Schmidt decomposition (Nielsen & Chuang, 2010, §2.5):

$|\Psi\rangle = \Sigma_k \lambda_k |a_k\rangle \otimes |b_k\rangle$

where $\lambda_k > 0$ are the Schmidt coefficients and r is the Schmidt rank. Product states have r = 1; entangled states have r ≥ 2.

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8. Summary

The tensor product rule follows directly from four physical requirements, given the foundational assumption of real superposition:

Requirement	Mathematical Consequence
Bilinear composition	Joint amplitudes multiply; tensor product as unique linearisation
Superposition closure	Linear combinations of product states → full tensor space
Locality	Independent operations → universal property satisfied
Probability consistency	Factorised inner product → Hilbert tensor product

The tensor product rule is therefore *physically forced given real superposition, bilinear composition, locality, and independence*—not mathematically assumed as a bare axiom.

Positioning Against Standard Accounts of the Tensor Product Rule

To clarify what is new in the RAL account, it is useful to contrast it explicitly with how the tensor product rule is usually introduced in quantum mechanics.

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1. Standard Textbook Account: Postulate

In most quantum mechanics textbooks (Griffiths, 2018; Sakurai & Napolitano, 2017; Cohen-Tannoudji et al., 1977), the tensor product rule is introduced as a basic postulate: when two systems are combined, their joint state space is the tensor product of their individual Hilbert spaces.

$\mathscr{H}\_AB = \mathscr{H}\_A \otimes \mathscr{H}\_B$

No physical derivation is offered; the rule is presented as part of the formal machinery that must be accepted to proceed. This approach is operationally effective but foundationally silent. It explains *how* to calculate, but not *why* nature requires this specific composition rule over any alternative.

General reader note: Imagine learning arithmetic by being told "multiplication exists" without any explanation of what it means or why it works. You could still do calculations, but you wouldn't understand why the rules are what they are.

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2. Mathematical Account: Vector Space Composition

A more sophisticated presentation, common in mathematical physics texts (Isham, 1995; Hall, 2013), motivates the tensor product by appealing to linear algebra: the tensor product is described as the universal bilinear construction—the natural way to combine vector spaces while preserving linearity and independence.

While mathematically correct, this argument assumes that Hilbert spaces are fundamental objects of nature rather than derived structures. From a foundational perspective, this explanation is circular: it presupposes the very framework whose physical origin is under examination.

The question "why tensor products?" becomes "why Hilbert spaces?"—which remains unanswered.

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3. Operational and Information-Theoretic Accounts

In quantum information theory and axiomatic reconstruction programmes (Hardy, 2001; Chiribella et al., 2011; Masanes & Müller, 2011; Barrett, 2007), the tensor product is often justified indirectly through operational constraints:

- **No-signalling**: Local operations cannot transmit information
- **Local tomography**: Joint states are determined by local measurements
- **Bell inequality violations**: Certain correlations exceed classical bounds (Bell, 1964; Aspect et al., 1982)

These approaches successfully derive or constrain the tensor product structure from operational axioms. However, they do not provide a pre-measurement physical picture for *why* amplitudes compose bilinearly. Composition is characterised by what measurements reveal, rather than derived from the physics of superposition prior to measurement.

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4. Historical Precedent: Wave Mechanics

It should be acknowledged that the oscillatory or wave-mechanical view of superposition is not unique to RAL. Schrödinger (1926) himself emphasised that quantum states represent genuine wave phenomena, not merely probability catalogues. De Broglie's pilot wave theory (de Broglie, 1927; Bohm, 1952) and modern approaches like stochastic electrodynamics (de la Peña & Cetto, 1996) share this intuition.

What RAL contributes is not a new physical picture but a *systematic derivation*: showing precisely how the tensor structure follows from treating superposition as real oscillatory process. RAL organises and extends the wave-mechanical intuition into a rigorous explanatory framework.

General reader note: Schrödinger always believed quantum waves were physically real, not just mathematical tools. RAL builds on this idea and shows that if you take it seriously, much of quantum formalism follows automatically.

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5. The RAL Account: Pre-Measurement Necessity

The RAL account differs from all of the above in its explanatory target. It does not begin with Hilbert space, nor with operational constraints on measurement outcomes. Instead, it starts from the physical assumption that unmeasured quantum systems behave as oscillatory assemblies supporting superposition.

From this starting point, the tensor product emerges as the unique way to combine unresolved oscillatory systems while preserving:

1. **All joint oscillations**: No mode combinations arbitrarily forbidden
2. **Bilinear composition**: Joint amplitude responds linearly to each subsystem
3. **Superposition**: Closure under linear combination
4. **Locality**: Independent operations on subsystems
5. **Probability**: Factorised inner products for independent measurements

The rule governs how *possibilities* combine before any measurement occurs, not how *outcomes* correlate afterwards.

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6. Summary of Differences

Account	Status of Tensor Product Rule	Explanatory Basis
Textbook	Postulated	None (axiom)
Mathematical	Justified	Abstract vector space properties
Operational	Derived/constrained	Measurement-level axioms
RAL	**Derived**	**Pre-measurement oscillatory physics**

This positioning clarifies that RAL is not offering an alternative formalism, but a deeper explanation of why the existing formalism has the structure it does. The mathematics of quantum mechanics remains entirely unchanged; what changes is our understanding of where that mathematics comes from.

RAL and operational reconstructions are complementary rather than competing: operational approaches characterise quantum structure from measurement statistics; RAL explains why a world with real superpositions would necessarily have that structure.

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7. Implications

If the RAL account is correct, several consequences follow:

1. **The tensor product rule is not contingent**: Given real superposition and bilinearity, it could not have been otherwise
2. **Entanglement is demystified**: It reflects global oscillatory modes, not action at a distance
3. **Quantum mechanics is less arbitrary**: Its structure is physically forced, not axiomatically chosen
4. **Foundational debates shift**: From "which interpretation?" to "what is oscillating?"

The last question—what physical substrate supports these oscillations—remains open and connects RAL to deeper questions about the nature of quantum fields and spacetime itself.

Pre-emptive Reviewer Q&A (Objections & Replies)

Q1. "Aren't you just assuming Hilbert space by assuming bilinearity?"

A. No. The bilinearity requirement is not adopted as abstract linear algebra; it's the minimal compatibility condition between (i) real superposition (coherent additivity of oscillatory modes) and (ii) independence (scaling one subsystem's oscillatory contribution while holding the other fixed should scale the joint contribution proportionally). If composition were nonlinear, joint predictions would depend on arbitrary decompositions of a preparation into superposed components, and interference visibility would generically depend on intensity rather than relative phase—contradicting the operational invariances that define coherent superposition.

Q2. "Why must the joint configuration space be the Cartesian product $\Lambda_A \times \Lambda_B$?"

A. Because we are modelling two simultaneously existing subsystems with independent configuration degrees of freedom prior to measurement. The Cartesian product is the minimal joint label set that allows each distinguishable configuration of A to co-occur with each distinguishable configuration of B. Restrictions like superselection or particle indistinguishability constrain admissible states within the composite space (e.g., to symmetric/antisymmetric subspaces), but they do not replace the underlying composition rule.

Q3. "There are many bilinear maps $V_A \times V_B \to V_{AB}$. Why is $\Psi(a,b) = \psi_A(a)\psi_B(b)$ special?"

A. We are not claiming mathematical uniqueness of bilinear maps in the abstract. We pick out the bilinear map that realises independent joint configurations in the product basis—i.e., the map that assigns the joint amplitude for (a,b) as the simultaneous contribution of the amplitude for a and the amplitude for b. The universal property then guarantees that this bilinear composition linearises uniquely into the tensor product structure (up to isomorphism and choice of basis/normalisation).

Q4. "Operational reconstructions already 'derive' the tensor product rule—what's new?"

A. Operational reconstructions derive/constrain composition from measurement statistics and information-theoretic axioms. RAL targets a different explanatory layer: a pre-measurement physical picture for why amplitudes compose bilinearly when superposition is treated as a real oscillatory process. The approaches are complementary: operational frameworks tell you what must be true of observed statistics; RAL explains why a world with real superpositions naturally realises that structure.

Q5. "Your argument depends on a 'real superposition' premise. Isn't that just an interpretation choice?"

A. It is a premise, explicitly stated. The note is conditional: if pre-measurement superpositions correspond to real unresolved dynamics, then the tensor product rule follows. That premise is not arbitrary: ψ-ontology results (e.g., PBR under natural independence assumptions) strongly constrain broad classes of purely epistemic wavefunction models. RAL does not rely on PBR, but it makes the ontic premise materially less ad hoc.

Q6. "How does RAL account for Bell-inequality violations without signalling?"

A. In RAL terms, entangled systems are single global oscillatory modes with joint phase structure. Local descriptions omit the shared phase relation, so classical factorisations of joint probabilities fail. Local measurements probe the same global mode in different local bases; the joint statistics reflect pre-existing global constraints, not signals sent at measurement time. Bell rules out local hidden-variable factorisations, not nonseparable global states.

Q7. "What about identical particles, fermions/bosons, and symmetrisation?"

A. Symmetrisation constrains the physical state space to the symmetric/antisymmetric subspace of $V_A \otimes V_B$. This restricts admissible states but does not replace the composition rule; it is a constraint within the composite space.

Q8. "What does RAL add about measurement? Why does one outcome occur?"

A. This note adopts only a minimal measurement assumption: measurement corresponds to an effectively irreversible coupling that produces a stable macroscopic record and ends coherent interference for that event. A full RAL account would still owe a dynamical model of record formation and an account of outcome selection (or why that framing is wrong). Those issues are interpretation-sensitive and are treated as open here.

Q9. "Does the argument generalise beyond finite-dimensional spins?"

A. Yes at the structural level. The algebraic tensor product is defined for general vector spaces; for infinite-dimensional Hilbert spaces one takes the appropriate completion under the induced norm to obtain $\mathscr{H}_A \otimes \mathscr{H}_B$. The note focuses on finite-dimensional spins only to keep the intuition transparent.

Q10. "Is this just 'wave mechanics' restated?"

A. RAL's novelty is not the claim that quantum states are wave-like; that has historical precedent. The novelty is the systematic derivation: from real superposition + independence + locality + bilinear response to the tensor product via the universal property, with explicit pre-emption of standard objections (direct sum confusion, superselection/indistinguishability, Bell/no-signalling).

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Glossary for General Readers

Amplitude: A complex number describing the "strength" of a quantum oscillation. Its squared magnitude gives probability.

Basis states: The fundamental, distinguishable configurations a system can be found in upon measurement (like spin up vs. spin down).

Bell inequality / Bell correlations: Mathematical constraints that any local hidden-variable theory must satisfy. Quantum mechanics predicts—and experiments confirm—that entangled particles violate these inequalities, demonstrating that quantum correlations cannot be explained by pre-existing local properties. This rules out certain classical explanations but does not imply faster-than-light signalling.

Bilinear: A function of two variables that is linear in each variable separately. If you double one input while holding the other fixed, the output doubles.

Complex number ($\mathbb{C}$): Numbers of the form a + bi, where i = $\sqrt{(-1)}$. They encode both magnitude and phase.

Direct sum ($\oplus$): A way of combining vector spaces representing *exclusive alternatives*—one or the other, but not both.

Entanglement: A quantum state of multiple systems that cannot be described as independent states of each part. The systems share a global oscillatory mode.

Epistemic: Relating to knowledge or belief. An epistemic interpretation treats the wavefunction as representing what we know, not what exists.

Hilbert space: The mathematical space containing all possible quantum states of a system. A complete vector space with an inner product.

Inner product ⟨ψ|φ⟩: A way of measuring "overlap" between two quantum states. Used to calculate probabilities.

Measurement: An irreversible interaction that transfers quantum information into a macroscopic record, resolving superposition.

Ontic: Relating to reality or existence. An ontic interpretation treats the wavefunction as representing something physically real.

Phase: The timing or alignment of an oscillation. Two waves can interfere constructively or destructively depending on their relative phase.

Schmidt decomposition: A way of writing any joint quantum state as a sum of product terms, revealing how entangled it is.

Superposition: The simultaneous existence of multiple oscillatory modes before measurement. Not a mixture, but genuine coexistence.

Superselection rule: A constraint prohibiting superpositions between certain states (e.g., different electric charges). Acts within the tensor product structure, not as a replacement for it.

Tensor product (⊗): The mathematical operation combining two vector spaces representing *simultaneous coexistence*. Dimension multiplies: $n \times m$.

Unitary operation: A reversible quantum transformation that preserves probabilities. Rotations in Hilbert space.

Universal property: A mathematical characterisation stating that the tensor product is the unique space through which all bilinear maps factor.
