

# Depth Is Not a Direction

## Renormalization, Holography, and the Categorical Error of Geometric Scale

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### Summary for General Readers

**The Core Question:** When physicists study the universe at different scales—from atoms to galaxies—they often draw diagrams showing "scale" as if it were an extra dimension of space, like up-down or left-right. This paper asks: Is scale actually a direction you could travel in, or is it just a useful way of organizing information?

**The Answer:** Scale is not a real spatial direction. It's a bookkeeping tool.

### The Video Game Analogy

Consider a role-playing game (RPG) rendered on a computer. The game world is genuinely three-dimensional. Characters move left–right, forward–back, and up–down. These directions satisfy all the criteria of spatial dimensions: they have distances, neighborhoods, continuous motion, and reversible movement.

Now consider the *rendering resolution* of the game. The same world can be displayed at 4K, 1080p, or 480p. Changing resolution does not move the character anywhere in the game world. No character travels "into" lower resolution. Instead, the same underlying world is re-described with more or fewer pixels. Information is discarded, not relocated.

One can plot resolution as an additional axis in a diagram, or visualize it as "depth" in a rendering pipeline. This is often useful for understanding performance or compression. But resolution is not a spatial direction of the game world. It has no locality, no metric distance within the world, no trajectories, and no reversible motion.

**Renormalization depth plays exactly this role in physics.** It indexes how finely the system is described, not where anything is located. Treating it as a spatial dimension confuses representational structure with physical geometry.

### Why the Confusion Is So Tempting (The VR Extension)

Now imagine the same RPG rendered in virtual reality. The engine dynamically changes resolution depending on where the player looks: nearby objects are sharp, distant objects are

coarse. From the player's perspective, resolution appears to correlate with distance—the game seems to "add depth" through resolution scaling.

But nothing has changed ontologically. Resolution is still not a spatial direction inside the game. It is an encoding strategy used by the engine to organize information efficiently. The apparent geometric role of resolution emerges only under specific rendering regimes and disappears if the rendering strategy changes.

This mirrors what happens in theoretical physics. In certain special regimes, the organization of information across scale admits a geometric interpretation that behaves like an extra spatial dimension. Outside those regimes, the interpretation dissolves. The geometry is emergent, not primitive.

## The One-Sentence Summary

**Renormalization depth is to spacetime what rendering resolution is to a video game: it can be drawn as an axis and behaves geometrically in certain regimes, but nothing in the game world actually moves along it.**

## Why This Matters (And How to Test It)

This isn't just philosophy—it makes predictions. If depth were a real spatial direction, experiments on quantum systems would show certain symmetries (independence between different "depths," reversible information recovery, additive entropy). Our thesis predicts these symmetries will be *violated* in measurable ways. Section 8 develops specific experiments that could test this in tensor network quantum simulators and other systems.

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## Abstract (Technical)

In holographic dualities, tensor-network constructions, and renormalization group analyses, a parameter associated with coarse-graining or scale is routinely represented as an additional spatial dimension. This paper argues that such representations, while computationally powerful, risk a categorical error: conflating descriptive resolution with geometric location. We demonstrate that renormalization depth fails to satisfy the defining criteria for spatial dimensions—it lacks intrinsic metric structure, does not support reversible transport, and indexes irreversible information loss rather than position. We address apparent counterexamples from AdS/CFT, showing that the geometric interpretation of the radial coordinate emerges only in restricted regimes (large- $N$ , semiclassical limits) and represents an organization of boundary data rather than primitive spatial structure. A formal no-go theorem establishes that any parameter governing non-injective coarse-graining maps cannot constitute a physical spatial direction. The structural difference between spatial directions (tensor factorization, reversible transport) and descriptive indices (quotient hierarchies, non-injective maps) generates testable predictions: anomalous correlations in tensor network quantum simulators, structural bounds on recovery fidelity, and characteristic asymmetries in correlation functions. These results clarify the

ontological status of "bulk" reconstructions, support interface-based frameworks for emergent geometry, and sharpen the distinction between mathematical representation and physical reality.

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## Summary of Formal Results

This paper establishes the following theorems and propositions:

| Result  | Statement  |
|---|--|
| <b>Theorem 2.0</b> (Criteria Necessity)         | C1–C4 are necessary conditions for any parameter to be a physical spatial coordinate |
| <b>Theorem A.4</b> (No-Go)                      | Non-injective coarse-graining semigroups cannot define spatial directions            |
| <b>Proposition 3.3</b> (No Metric)              | RG depth lacks intrinsic metric structure due to scheme dependence                   |
| <b>Proposition 3.4</b> (No Locality)            | RG depth lacks locality structure; scales are quotients, not tensor factors          |
| <b>Proposition 3.5</b> (No Propagation)         | RG maps are channels, not automorphisms; no trajectories through depth               |
| <b>Proposition 5.3a</b> (Regime Dependence)     | Bulk geometric interpretation requires semiclassical/code-subspace conditions        |
| <b>Proposition 5.3b</b> (Dictionary Dependence) | Bulk-boundary map is state/subspace-dependent, not universal                         |

### Testable Predictions:

| Prediction                                  | Observable Signature  |
|---|---|
| <b>8.2</b> Independent Manipulation Failure | Operations "at depth $z$ " show anomalous correlations with boundary observables          |
| <b>8.3</b> Structural Recovery Limitation   | States in same equivalence class indistinguishable after recovery, regardless of protocol |
| <b>8.4</b> Correlation Asymmetry            | Depth correlations show channel (not propagator) structure                                |
| <b>8.5</b> State-Dependent Reconstruction   | Bulk operator definitions vary across states/subspaces                                    |
| <b>8.6</b> Entropy Non-Additivity           | $S_{\text{bulk}} \leq S_{\text{boundary}}$ with saturation in holographic systems         |

## 1. Introduction

Physics frequently exploits geometric intuition to organize hierarchical structures. When a system admits descriptions at multiple resolutions, it is tempting to depict these resolutions as positions along an additional axis—to literalize scale as depth. This intuition pervades

contemporary theoretical physics: the holographic correspondence represents conformal field theories as gravitational theories in one higher dimension; tensor networks such as MERA encode entanglement structure through layered geometry; Wilsonian renormalization is visualized as flow through theory space.

**For General Readers:** Physicists often need to describe the same system at different levels of detail. Think of weather: you can describe individual air molecules, or average temperatures across cities, or continental pressure systems. These aren't different places—they're different levels of description. Yet physicists often draw diagrams where "level of detail" appears as an axis, like a new direction in space. This paper examines whether that's just a convenient picture or something deeper.

The utility of these representations is not in question. What concerns us is their interpretation. To depict scale as a spatial direction invites the inference that scale *is* a spatial direction—that objects can move along it, that it possesses intrinsic metric structure, that it participates in geometry on equal footing with the dimensions we directly observe.

This paper argues that such inferences are mistaken. Renormalization depth is not a direction in which anything travels. It is an index labeling equivalence classes of descriptions related by irreversible information loss. The "flow" along depth is not motion but re-description. Treating depth as genuinely spatial introduces conceptual confusion, obscures the true origin of emergent geometry, and reifies what should remain a bookkeeping device.

Our argument proceeds through formal propositions and theorems, not merely conceptual analysis. We prove that the criteria defining spatial directions (metric structure, locality, propagation, reversibility) are *necessary* conditions for any parameter to function as a spatial coordinate, then prove that renormalization depth fails each criterion. We further prove that the geometric interpretation of holographic radial coordinates is regime-dependent and dictionary-dependent, emerging only under restrictive conditions rather than existing as primitive structure.

**On the nature of this contribution.** The mathematical core of the argument is elementary: non-injective maps have no inverses, therefore coarse-graining cannot support the group structure required for spatial translations. The non-trivial content lies elsewhere—in identifying reversible transport as a *necessary* criterion for spatial dimensionality, and in showing that renormalization depth, despite its ubiquitous geometric representations, fails to meet this criterion. The paper's contribution is thus primarily structural and clarificatory: it makes explicit a distinction (between representational convenience and geometric ontology) that is often acknowledged implicitly but rarely formalized.

We proceed as follows. Section 2 establishes criteria for physical spatial directions and proves their necessity. Section 3 proves that renormalization depth fails these criteria. Section 4 directly engages the contrary claim in the literature, identifying the invalid logical step from "geometric representation" to "spatial direction." Section 5 addresses the apparent counterexample from holographic duality with formal propositions on regime and dictionary dependence. Section 6 develops implications for emergent geometry within causal-interface frameworks. Section 7 connects to the broader programme of interface-based emergence. Section 8 derives testable

experimental predictions. Section 9 concludes. Mathematical appendices provide the core no-go theorem, analysis of tensor network geometry, responses to anticipated objections, and a worked quantitative example.

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## 2. Criteria for Physical Spatial Directions

What distinguishes a genuine spatial dimension from a mere parameter? We identify four necessary conditions:

**For General Readers:** Before we can ask whether "scale" is a real direction, we need to define what makes something a direction in the first place. What properties must a direction have to count as part of space? We propose four tests.

**C1. Metric Structure.** A spatial direction admits a well-defined notion of distance. Points along the direction can be separated by measurable intervals, and this separation is observer-independent (up to coordinate transformations preserving the metric).

*In plain terms:* You can measure how far apart two points are, and everyone agrees on the answer (once they account for their reference frames).

**C2. Locality.** A spatial direction supports neighborhoods. Physical interactions respect locality: systems at nearby points can couple directly, while distant points require intermediate connections.

*In plain terms:* Things that are close together can interact directly. Things that are far apart need something in between to carry the interaction—like how sound travels through air molecule by molecule.

**C3. Propagation.** Objects can move through a spatial direction, occupying intermediate positions. Trajectories are continuous paths parametrized by the coordinate.

*In plain terms:* You can actually travel through space, passing through all the points in between where you started and where you end up.

**C4. Reversibility (or Symmetry).** Motion along a spatial direction is, at least in principle, reversible. One can move from point A to point B and return. Spatial translations form a group, not merely a semigroup.

*In plain terms:* If you can go somewhere, you can come back. The laws of physics don't care which direction you're moving.

These criteria are not arbitrary. They capture what it means to say that a direction is "part of space"—that it participates in geometry, supports dynamics, and admits the full structure of spatial reasoning.

## 2.0 Formal Justification: Why These Criteria Are Necessary

We now establish that C1–C4 are not merely intuitive desiderata but *necessary* conditions for any parameter to function as a physical spatial coordinate.

**Definition (Spatial Direction).** A parameter  $u$  is a *physical spatial direction* on a manifold  $M$  if it corresponds to a coordinate on  $M$  such that:

- (A)  $M$  carries a topology inducing neighborhoods along  $u$
- (B)  $M$  carries a metric  $g$  with component  $g_{uu}$  providing operational distances
- (C) Physical transport along  $u$  is represented by a one-parameter group of diffeomorphisms  $\tau_a : M \rightarrow M$  satisfying  $\tau_{\{a+b\}} = \tau_a \circ \tau_b$  and  $\tau_0 = \text{id}$
- (D) Local interactions respect the topology, coupling nearby points along  $u$

**Theorem (Criteria Necessity).** If  $u$  is a coordinate on a physical spatial dimension of  $M$  (i.e., a factor that participates in the physical geometry and kinematics), then:

(C1) There exists a metric component  $g_{uu}$  giving operational distances along  $u$ ;

(C2) The topology induces neighborhoods and local couplings along  $u$ ;

(C3) Continuous paths  $\gamma(t)$  exist with  $\gamma_u$  varying through intermediate values;

(C4) Translations along  $u$  are invertible, hence form a group.

*Proof.* These follow directly from the definition of spatial direction:

- C1 follows from (B): the metric  $g$  provides  $g_{uu}$ .
- C2 follows from (A) and (D): topology provides neighborhoods; physical locality requires local couplings.
- C3 follows from the manifold structure: coordinates on manifolds admit continuous paths.
- C4 follows from (C): the group property  $\tau_{\{a+b\}} = \tau_a \circ \tau_b$  with  $\tau_0 = \text{id}$  implies  $\tau_{\{-a\}} \circ \tau_a = \tau_0 = \text{id}$ , so inverses exist. ■

**Remark.** We do not claim C1–C4 are the *only* possible axioms for spatiality; we claim they are *necessary* for any parameter to be treated as a literal spatial coordinate with transport, locality, and metric meaning. A parameter failing any of these can still be a useful descriptive label—but it is not a spatial direction in the geometric/kinematic sense.

**For General Readers:** We've just proven that our four tests aren't arbitrary—they're the minimum requirements for something to count as a spatial direction. If a parameter fails any of these tests, it might still be useful, but it's not part of space in the way that left-right or up-down are.

## 2.1 Do Ordinary Spatial Dimensions Satisfy These Criteria?

One might object: perhaps ordinary spatial dimensions also fail these criteria at fundamental scales. If space is discrete at the Planck length, does the continuum metric structure of C1 break down? If quantum gravity introduces fundamental limitations on localization, does C2 fail?

**For General Readers:** The Planck length (about  $10^{-35}$  meters) is the scale where quantum mechanics and gravity both become important. Some theories suggest space itself might be "pixelated" at this tiny scale, rather than smooth. Does this undermine our criteria?

This objection conflates two distinct claims. Our thesis is not that spatial dimensions satisfy C1–C4 perfectly at all scales, but that they satisfy these criteria *in principle* and *symmetrically*. Consider the differences:

**Metric structure (C1).** Even if spacetime is discrete at the Planck scale, all spatial directions share the same *structural type*—they are uniformly discrete or continuous, with symmetric discretization scales. Renormalization depth, by contrast, has no intrinsic metric at any scale—the "distance" between resolutions  $z_1$  and  $z_2$  is not a spatial interval but a ratio of cutoffs, and this ratio depends on the renormalization scheme.

**Locality (C2).** Quantum gravity may modify locality at extreme scales, but the modification is again symmetric across spatial directions. Depth has no locality structure at all: there is no sense in which descriptions at nearby resolutions "couple locally" while distant resolutions require mediation.

**Propagation (C3).** Objects propagate through spatial dimensions at all experimentally accessible scales. No object propagates through depth; coarse-graining is not a dynamical process through which systems evolve.

**Reversibility (C4).** This is the decisive asymmetry. Spatial translations are fundamentally reversible: the laws of physics (quantum mechanics, general relativity) are symmetric under spatial reflection and translation. Any irreversibility in spatial motion is emergent (friction, dissipation) rather than constitutive. Renormalization, by contrast, is constitutively irreversible—non-injectivity is built into its definition, not an emergent feature of special conditions.

**For General Readers:** The key point is that even if space gets weird at tiny scales, it gets weird *symmetrically*—all directions are affected equally. But "zooming out" (renormalization) is fundamentally one-way: you lose information and can't get it back. That asymmetry is baked in, not accidental.

The Planck-scale objection thus fails to establish parity between spatial dimensions and depth. Spatial dimensions may have their structure modified at extreme scales, but they retain the symmetry and reversibility that depth intrinsically lacks.

**Remark on Time.** One might object that time satisfies C1–C3 but arguably fails C4, given thermodynamic irreversibility. Yet time differs crucially from renormalization depth: the fundamental laws of physics are time-reversal symmetric, and the arrow of time emerges statistically from boundary conditions rather than from the structure of temporal evolution itself.

The Schrödinger equation, Hamilton's equations, and Einstein's field equations are all time-symmetric. No such symmetry exists for coarse-graining—information loss under renormalization is built into the definition of the operation, not an emergent feature of special conditions.

We acknowledge that time's status remains contested in foundations of physics. Some approaches (e.g., the thermal time hypothesis of Connes-Rovelli) treat entropy increase as more fundamental. However, even on views where time's arrow is emergent, it emerges from *dynamics*—physical evolution constrained by boundary conditions. RG "flow," by contrast, involves no dynamics at all: it is pure re-description, a change in how we represent the same physical situation. This distinction—dynamical emergence vs. representational re-description—is what separates time from depth, regardless of one's view on the arrow of time.

*Scope note:* This paper does not take a position on the deeper ontological status of time; it uses only the structural contrast between dynamical evolution and representational coarse-graining to distinguish time from renormalization depth.

**For General Readers:** "But wait," you might say, "time only goes forward— isn't that also irreversible?" Good question! The fundamental laws of physics actually work the same forwards and backwards in time. The reason we experience time as one-way (eggs break but don't unbreak) comes from statistics and initial conditions, not from the laws themselves. Renormalization is different: the information loss is written into the very definition of the process, not a statistical accident.

### 3. Renormalization Depth Fails All Four Criteria

**For General Readers:** This section shows that "scale" or "zoom level" fails all four tests for being a real spatial direction. The key concept is *renormalization*—the process physicists use to connect descriptions at different scales.

#### 3.1 The Structure of Renormalization

Let  $S$  denote the space of physical states at microscopic resolution (density operators on a Hilbert space  $H$ , or probability measures on phase space). A renormalization scheme defines a family of coarse-graining maps:

$$R_z : S \rightarrow S_z, z \geq 0$$

where  $z$  indexes resolution scale (depth), with  $z = 0$  corresponding to the finest resolution. These maps satisfy the semigroup property:

$$R_{\{z+w\}} = R_z \circ R_w, R_0 = \text{id}$$

Crucially, for  $z > 0$ ,  $R_z$  is generically non-injective: distinct microscopic states  $\rho \neq \sigma$  can satisfy  $R_z(\rho) = R_z(\sigma)$ . This non-injectivity—the hallmark of information loss—is constitutive of coarse-graining.

**Remark on Exact RG.** In exact RG formulations (Polchinski equation, Wetterich effective average action), one tracks the flow of an effective action through theory space. This flow is, in principle, reversible—it is a diffeomorphism on the space of effective actions. Does this undermine our argument? No. The paper's target is the claim that *physical states* move through depth. The exact RG flow preserves information about the functional form of the action at each scale, but this does not imply reversibility on the space of physical configurations—the degrees of freedom integrated out are genuinely eliminated from the state description, even if the flow through theory space is formally invertible. Coarse-grained states lose information about UV configurations regardless of whether we can invert the flow on coupling constants. The distinction between theory space and state space is crucial: reversibility in theory space does not imply reversibility in state space.

**Remark on cMERA.** A related objection targets our use of discrete MERA/isometric coarse-graining: continuous MERA (cMERA) implements a continuous "depth" parameter  $s$  and is generated by continuous unitary evolution. Does continuity restore spatiality? No. cMERA replaces discrete layers with a continuous scale parameter and constructs states via a scale-ordered unitary circuit, but "continuous" does not imply "spatial." Even in cMERA, the depth parameter  $s$  labels resolution/scale, and the map that identifies "the same degree of freedom at different  $s$ " remains dictionary-dependent (a choice of renormalization scheme / representation of field modes). Treating  $s$  as a genuine spatial coordinate would require locality and reversible translations as spatial transport on the underlying physical state space; cMERA instead supplies a continuous re-description/generation structure whose usefulness comes from organizing correlations by scale, not from introducing a new kinematic direction. The continuous limit changes the smoothness of the representation, but not its ontological category: depth remains an index of description, not a place.

**For General Readers:** Imagine you have a detailed photograph (state  $\rho$ ) and a slightly different detailed photograph (state  $\sigma$ ). When you blur both images enough (apply  $R_z$ ), they might become indistinguishable—the same blurry blob. This is "non-injectivity": different starting points can lead to the same endpoint. And once you have the blurry blob, you can't tell which original image it came from. Information has been irreversibly lost.

### 3.2 Failure of C4: Irreversibility

The non-injectivity of  $R_z$  disqualifies any interpretation of depth as a spatial translation coordinate acting on physical states. If  $R_z(\rho) = R_z(\sigma)$  for  $\rho \neq \sigma$ , then no map  $R_{-z}$  can satisfy  $R_{-z} \circ R_z = \text{id}$ , since this would require  $R_{-z}(R_z(\rho))$  to equal both  $\rho$  and  $\sigma$ . Once information is coarse-grained away, it cannot be recovered from the coarse-grained description alone.

This is not a contingent feature of particular renormalization schemes—it is constitutive of what renormalization means. A "reversible coarse-graining" would not be coarse-graining at all; it would be a unitary transformation preserving all information.

**For General Readers:** If two different detailed pictures produce the same blurry picture, there's no way to "un-blur" back to the original. Any "reverse" operation would have to somehow produce two different outputs from the same input—which is impossible for any well-defined process. This isn't a technical limitation; it's logically impossible.

### 3.3 Failure of C1: No Intrinsic Metric

To establish that depth lacks metric structure, we must first specify what "intrinsic metric" means operationally.

**Definition (Intrinsic Metric).** A parameter  $u$  has *intrinsic metric meaning* if there exists an operationally defined distance  $d_u(a,b)$  that is:

1. Determined by physical observables within the theory
2. Invariant under reparameterizations preserving the physical content
3. Corresponds to a measurable geometric quantity (e.g., proper length)

**Proposition (Depth Has No Intrinsic Metric).** For RG depth  $z$ , any "distance"  $|z_1 - z_2|$  depends on the choice of renormalization scheme (cutoff procedure, blocking prescription, renormalization convention) and is therefore not an invariant determined by the physical state/observable content alone. Hence  $z$  does not define an intrinsic metric direction.

*Proof.* Let  $\{R_z\}$  and  $\{R'_{z'}\}$  be two valid renormalization schemes for the same physical system. These schemes are related by a monotone reparameterization  $z' = f(z)$  (and possibly field redefinitions) without changing any physical predictions. The "distance"  $|z_1 - z_2|$  in one scheme maps to  $|f(z_1) - f(z_2)|$  in the other, with no invariant relationship. Since the physical content is unchanged while the "metric" along depth is arbitrary,  $z$  does not carry intrinsic metric structure in the sense required by C1. ■

**For General Readers:** In real space, "10 meters" means something physical—you could lay out measuring sticks, and everyone would agree. But "10 units of zoom" doesn't have the same physical meaning. Different physicists using different "zoom methods" would assign different numbers to the same physical situation, with no way to say who's right. There's no objective "distance" between zoom levels.

### 3.4 Failure of C2: No Locality

Spatial locality requires that a coordinate index independent subsystems that can couple through local interactions. We formalize this requirement and show depth fails it.

**Definition (Locality Along a Parameter).** A parameter  $u$  supports *locality* if there exists a net of algebras  $\{A([u, u+\epsilon])\}$  indexed by intervals along  $u$  such that:

1. Algebras at disjoint intervals are independent (tensor factor or commuting)
2. Physical interactions can couple neighboring intervals without requiring mediation through distant intervals
3. The decomposition corresponds to distinct physical degrees of freedom

**Proposition (No Locality Along Depth).** There is no natural net  $A([z, z+\varepsilon])$  with independent degrees of freedom at neighboring depths that couple locally. RG depth fails C2.

*Proof.* RG relates descriptions at different depths by functional dependence: coarse-grained variables are derived from (functions of) fine-grained variables via the map  $R_z$ . The relationship between descriptions at depths  $z$  and  $z + \varepsilon$  is not that of independent subsystems coupled by interactions; it is the relationship between a system and its own re-description.

Formally: let  $A_0$  be the UV algebra and  $A_z = R_z(A_0)$  the effective algebra at depth  $z$ . The inclusion  $A_z \subset A_0$  (in the Heisenberg picture) or the quotient structure  $S_z = R_z(S_0)$  (in the Schrödinger picture) is not a tensor factorization. There is no decomposition  $A_0 \cong A_z \otimes A_{\text{complement}}$  with "complement" representing independent degrees of freedom "at depth  $z$ ."

To see why such a decomposition is impossible, suppose it existed. Then every state  $\rho \in S_0$  would decompose as  $\rho = \rho_z \otimes \rho_{\text{comp}}$  (or more generally,  $\rho$  would be determined by its marginals on  $A_z$  and  $A_{\text{complement}}$ ). But this contradicts non-injectivity: if  $\rho \neq \sigma$  satisfy  $R_z(\rho) = R_z(\sigma)$ , they have the same " $A_z$  component" yet differ overall. This difference cannot be captured by an independent "complement" factor, because such a factor would have to record *which* of the infinitely many preimages of  $R_z(\rho)$  we started with—information that is by definition unavailable at depth  $z$ . The "complement" would need to contain information about the coarse-graining equivalence class, which is not a well-defined degree of freedom but merely a label for lost distinctions.

Spatial locality requires decomposition into distinct subsystems indexed by coordinate neighborhoods. RG depth gives a tower of quotients/images (information-losing maps), not a tensor-factorization over  $z$ . "Local couplings in  $z$ " would require coupling a system to its own re-description—a category error. ■

**For General Readers:** In space, if you want to affect something far away, you have to go through everything in between—signals travel, forces propagate, one domino knocks over the next. But zoom levels don't work this way. A description at one scale isn't a separate thing that "interacts" with a description at another scale. They're different views of the same thing—like asking how your face "interacts with" your passport photo.

### 3.5 Failure of C3: No Propagation

Propagation through a spatial coordinate is represented by a specific mathematical structure: invertible maps (automorphisms) on states or observables. We show coarse-graining lacks this structure.

**Definition (Propagation Along a Parameter).** A parameter  $u$  supports *propagation* if physical transport along  $u$  is represented by a one-parameter group of  $*$ -automorphisms  $\{\alpha_u\}$  on the observable algebra (equivalently, unitaries on the Hilbert space), such that states/excitations traverse intermediate values of  $u$  as continuous trajectories.

**Proposition (No Propagation Along Depth).** If propagation along  $u$  requires a one-parameter group of  $*$ -automorphisms  $\alpha_u$  on the observable algebra  $A$ , then any non-injective coarse-graining semigroup  $\{R_z\}$  cannot represent propagation. Hence there is no notion of an excitation "moving through intermediate depths" as a trajectory.

*Proof.* Automorphisms are by definition bijective (invertible). The coarse-graining maps  $R_z$  are generically non-injective (Proposition 3.2). A non-injective map cannot be an automorphism. Therefore  $\{R_z\}$  does not constitute a group of automorphisms, and  $z$  does not support propagation in the required sense.

More concretely: for a state  $\rho$  to "propagate to depth  $z$ ," there would need to be a trajectory  $\rho(t)$  with  $\rho(0) = \rho$  and  $\rho(1) = R_z(\rho)$ , passing through intermediate states at intermediate depths. But the intermediate states  $R_{\{tz\}}(\rho)$  for  $t \in (0,1)$  are not positions along a path—they are increasingly coarse descriptions of the *same* underlying state  $\rho$ . No physical entity traverses this sequence; only the description changes. ■

**For General Readers:** When physicists talk about "renormalization group flow," it sounds like something is moving. But nothing is actually traveling anywhere. It's like saying a book "flows" from detailed to summarized when you write a synopsis. The book doesn't go anywhere—you just have a shorter description of it. There's no trajectory through "synopsis space."

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## 4. Direct Engagement with the "Depth Is Spatial" Claim

**For General Readers:** Before addressing the sophisticated holographic counterexample, we directly confront the widespread claim in physics that renormalization scale "is" a spatial direction. We show exactly where this claim makes an invalid logical step.

A common formulation in the holographic RG and tensor-network holography literatures is that the renormalization scale is (or can be treated as) an additional spatial direction. For example, in influential work on holographic RG, it is stated that "the radial coordinate can be identified with the RG flow parameter." Swingle [6] similarly proposes that one can organize information "in terms of scale" and "define a higher dimensional geometry from this structure."

These statements are widely repeated and are often read—especially by non-specialists—as supporting the stronger ontological claim:

**Contrary view (strong form):** The RG scale/depth is an extra spatial direction—i.e., it supports the same structural properties as ordinary spatial coordinates.

Our position is that this strong form is false. The quoted statements are correct as *representational identifications*—they motivate an extraordinarily useful geometric organization of data—but they do not establish that "depth" is a spatial direction in the physical/kinematic sense.

#### 4.1 Where the Contrary View Makes an Invalid Step

The invalid step is a quiet upgrade:

From: "r labels RG scale / organizes data by scale"

To: "r is a genuine spatial direction of the underlying degrees of freedom"

This upgrade fails because a spatial direction requires the structural package C1–C4 (metric, locality, propagation, invertible translations), whereas RG depth is implemented by coarse-graining maps that are generically non-injective. The representational identification (labeling scale with a coordinate) does not entail the structural identification (possessing the properties of spatial directions).

#### 4.2 Why "Radial = RG Parameter" Does Not Imply Spatiality

To see the mismatch cleanly, separate two notions:

1. **A coordinate in an effective description:** e.g., a bulk radial coordinate  $r$  that is part of a semiclassical spacetime metric
2. **A spatial direction of the underlying state space:** which requires that "motion along the coordinate" be implemented by invertible transport on physical states (C4) and accompanied by locality/propagation structure (C2–C3)

The holographic RG identification in the literature is primarily of type (1). The contrary strong-form claim tacitly assumes it implies type (2). It does not.

Concretely:

- In AdS/CFT,  $r$  is a perfectly good coordinate within the bulk gravitational effective description
- But the *meaning* of that coordinate—what counts as "the same bulk point," what counts as "local," what counts as "nearer/farther"—is tied to a dictionary (reconstruction map) and typically becomes sharp only in semiclassical/code-subspace regimes
- This is fully compatible with the quoted statements, while still contradicting the strong-form claim that depth is spatial "in itself"

#### 4.3 The Decisive Structural Obstruction

The strongest point is independent of AdS/CFT details:

- "Moving in a spatial direction" is represented by a group of invertible transformations (C4)
- RG depth is represented (in state space) by a semigroup of coarse-graining maps  $R_z$  that are generically non-injective
- Non-injective maps cannot admit inverses on physical states

Therefore, depth cannot supply spatial translation structure on the underlying degrees of freedom—no matter how successfully it can be drawn as a dimension.

This is precisely why Swingle's statement—"defining a higher-dimensional geometry from organizing information by scale"—is best read as a claim about *representation* (a geometry constructed from scale-organized correlations), not as a claim that scale is literally a spatial axis of the fundamental ontology.

#### 4.4 What We Agree With

To avoid a strawman: we agree with the reliable core of the holographic RG intuition, namely:

- Scale can be represented geometrically
- Radial flow encodes RG structure
- In special regimes the induced geometry behaves like a spatial direction inside an effective bulk theory

Our disagreement is only with the ontological upgrade:

**Geometrizable  $\neq$  geometric.** A parameter can admit a geometric representation without being a spatial direction of the underlying physics.

This is exactly the categorical distinction the present paper formalizes.

**For General Readers:** Many physics papers say things like "the extra dimension corresponds to energy scale." This is true as a *labeling* convention—it's a useful way to organize information. But labeling something with a coordinate doesn't make it a spatial direction. You could label your mood on a 1-10 scale, but that doesn't mean mood is a direction you can walk in. The physics literature often slides from "useful label" to "actual dimension" without noticing the gap.

## 5. The Holographic Challenge

**For General Readers:** This section addresses the strongest apparent counterexample to our thesis: the famous "holographic principle" from string theory, which seems to treat scale as a genuine extra dimension. We'll explain why this doesn't actually contradict our argument.

### 5.1 The Apparent Counterexample

The AdS/CFT correspondence presents the most serious challenge to our thesis. In this duality, a  $d$ -dimensional conformal field theory on the boundary is equivalent to a  $(d+1)$ -dimensional gravitational theory in the bulk. The additional bulk dimension—the radial coordinate  $r$  in AdS—is commonly identified with RG scale in the boundary theory via the UV/IR connection [2, 3].

**For General Readers:** In 1997, physicist Juan Maldacena proposed something remarkable: certain theories of gravity in a curved space called "Anti-de Sitter space" (AdS) are exactly equivalent to quantum theories living on the boundary of that space. The boundary has one fewer dimension than the bulk. It's as if the information in a 3D room could be completely encoded on its 2D walls—hence "holographic." The extra dimension in the bulk appears to correspond to scale in the boundary theory.

This identification is not merely visual. The bulk possesses genuine metric structure:

$$ds^2 = (L^2/r^2)(dr^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

where  $L$  is the AdS radius and  $\eta_{\mu\nu}$  is the boundary metric. Fields propagate in the bulk; local interactions occur; the radial direction appears to satisfy C1–C4.

Does this not refute our claim?

## 5.2 Resolution: Emergent, Not Primitive Geometry

The resolution lies in recognizing that the bulk geometry is emergent rather than primitive—its geometric interpretation is not intrinsic but arises from specific organizational features of boundary data.

The bulk geometric interpretation—and, in semiclassical/code-subspace regimes, an effective notion of bulk locality and local dynamics—is determined by boundary data [7]. This is the content of holographic reconstruction theorems: bulk fields can be expressed as smeared operators on the boundary (HKLL reconstruction [5]), and bulk geometry emerges from boundary entanglement (Ryu-Takayanagi formula [4], tensor network constructions [6]).

**For General Readers:** Here's the key insight: the "extra dimension" in holography isn't a pre-existing space that was always there. It *emerges* from how information is organized in the boundary theory. Everything about the bulk—its geometry, the distances within it, what happens there—can be reconstructed from boundary data. The bulk is a useful way of *representing* boundary information, not an independent arena where additional things happen.

The radial coordinate  $r$  does not possess geometric properties independently of this encoding. It acquires them through a specific mathematical representation of boundary information. The "metric" on the radial direction is a derived object: a way of organizing boundary correlations that admits a geometric interpretation in appropriate regimes.

To illustrate the distinction, consider an analogy. A temperature field  $T(x,y,z)$  in a room can be visualized as a surface in four-dimensional space  $(x, y, z, T)$ . This surface has geometric

properties—curvature, distances, gradients. But temperature is not thereby a spatial dimension. The "geometry" of the temperature-extended space is a mathematical convenience for visualizing how  $T$  varies, not evidence that temperature participates in physical space.

**For General Readers:** Think of a weather map showing temperature as colors. You could imagine "hot" and "cold" as directions, and draw a surface where height represents temperature. That surface would have geometric properties—slopes, valleys, peaks. But temperature isn't actually a spatial dimension you could walk through. It's just a useful way to visualize data. The "extra dimension" in holography is similar: it's a powerful organizational tool, not literally a new direction in space.

Similarly, the bulk geometry in AdS/CFT represents how boundary data organizes across scales. The radial direction is a visualization of this organization, equipped with induced structure that facilitates calculation. Its geometric meaning is dictionary-dependent and regime-dependent, not ontologically primitive.

### 5.3 The Emergence Asymmetry

AdS/CFT is a duality: in principle, boundary and bulk descriptions encode the same physics, and either can be derived from the other given the appropriate dictionary. Our claim is therefore not that the bulk is "less real" than the boundary in some absolute sense.

Rather, the claim is subtler: the geometric interpretation of the radial coordinate is not an additional ontological dimension appended to boundary spacetime, but an emergent organization of boundary data that becomes effective only in regimes where semiclassical bulk locality exists. These regimes—large- $N$  limits, sparse spectra, code subspaces—are restrictive [7]. In generic boundary states, bulk locality breaks down; the geometric interpretation of  $r$  dissolves.

**For General Readers:** The holographic duality is real and exact—both descriptions are equally valid. But the "extra dimension" interpretation only works under special conditions. In typical quantum states, the nice geometric picture of the bulk falls apart. This tells us that the geometry isn't fundamental—it's a feature that emerges when conditions are right, like how water waves only exist when the water is calm enough.

In this sense, "radial distance" is not primitive structure. It is a derived representation of how boundary correlations and entanglement organize across scale—a representation that becomes sharp and useful only under specific conditions. Outside those conditions, the geometric metaphor loses its grip.

We can make these observations precise:

**Proposition (Regime Dependence of Bulk Geometry).** The geometric interpretation of the radial coordinate  $r$  as "distance into the bulk" is valid only when semiclassical bulk reconstruction applies (large  $N$ , sparse spectrum, appropriate code subspaces). Outside these regimes,  $r$  does not correspond to a stable geometric notion because bulk locality itself is not well-defined.

*Proof sketch.* Semiclassical bulk locality requires that bulk operators can be reconstructed from boundary subregions with appropriate properties (causal wedge reconstruction, entanglement wedge reconstruction). These reconstruction procedures rely on:

1. Large  $N$  to suppress  $1/N$  corrections that would violate bulk locality
2. Restriction to code subspaces where the bulk effective field theory is valid
3. States with geometric duals (not all CFT states have smooth bulk geometries)

For generic CFT states at finite  $N$ , bulk reconstruction fails or becomes state-dependent, and the notion of "position along  $r$ " loses operational meaning. See [7] for detailed conditions. ■

**Proposition (Dictionary Dependence of Radial Coordinate).** The identification of bulk points and operators depends on the choice of code subspace and, in some contexts, on the boundary state itself. Hence the geometric interpretation of  $r$  is not globally fixed across the full CFT Hilbert space.

*Proof sketch.* The HKLL reconstruction [5] and its generalizations express bulk operators as smeared boundary operators. However:

1. The smearing function depends on the background geometry, which can be state-dependent
2. For black hole interiors and other behind-horizon regions, operator reconstruction requires state-dependent maps [12]
3. The same "bulk point" can have multiple inequivalent boundary representations in different subregions (quantum error correction perspective [7, 11])

These features demonstrate that the bulk-boundary dictionary—and hence the meaning of radial position—is subspace/state-dependent rather than universal. ■

**For General Readers:** These propositions establish mathematically what we claimed informally: the "extra dimension" in holography isn't always there. It only makes sense under special conditions. When those conditions aren't met, asking "how far into the bulk?" is like asking "how loud is the color blue?"—the question doesn't have a meaningful answer.

*The VR analogy is apt here:* In a VR game with dynamic resolution scaling, nearby objects are sharp and distant objects are coarse—resolution appears to correlate with spatial depth. But this correlation exists only because of a specific rendering strategy. Change the strategy, and the correlation disappears. The "geometric" role of resolution was always contingent on the rendering regime, never intrinsic to the game world. Similarly, the geometric interpretation of the holographic radial coordinate is contingent on semiclassical conditions, not intrinsic to the boundary theory.

This is consistent with the UV/IR connection emphasized in early holographic work [3]: boundary UV physics maps to bulk IR (near-boundary) physics, and boundary IR maps to bulk deep interior. The radial coordinate *encodes* scale; it does not *constitute* an independent spatial arena.

## 5.4 Information-Theoretic Signature

The entropic structure further supports our interpretation. The Ryu-Takayanagi formula relates boundary entanglement entropy to bulk minimal surfaces [4]:

$$S_A = \text{Area}(\gamma_A) / 4G_N$$

where  $S_A$  is the entanglement entropy of boundary region  $A$  and  $\gamma_A$  is the minimal surface in the bulk homologous to  $A$ . Deeper bulk surfaces (larger  $r$ ) correspond to coarser boundary regions—precisely the signature of renormalization depth. The radial direction organizes entanglement at different scales; it emerges from this organization rather than existing independently of it.

**Remark on Recent Developments.** Several recent results strengthen this picture, each connecting directly to our thesis:

- *ER=EPR* [14]: The conjecture that entanglement between systems is geometrically realized as Einstein-Rosen bridges. This supports our view that entanglement patterns *determine* emergent geometry rather than constituting primitive spatial dimensions—geometry is derived from information-theoretic structure.
- *Complexity=Volume/Action* [15]: Proposals relating computational complexity of boundary states to bulk geometric quantities (volume of maximal slices, gravitational action). This further supports the claim that bulk geometry *encodes* information-theoretic properties of the boundary theory.
- *Islands and replica wormholes* [13]: The demonstration that "bulk" entropy contributions can arise from disconnected boundary regions, with the entanglement wedge determined by extremization. This reinforces the code-subspace and state-dependence themes central to Section 5.3—"where" information is localized in the bulk depends on boundary entanglement structure, not on primitive spatial location.

These developments uniformly support the thesis that bulk geometry represents organization of boundary data rather than primitive spatial structure.

**For General Readers:** There's a beautiful formula connecting the "area" of surfaces in the holographic bulk to the amount of quantum entanglement in the boundary theory. Surfaces deeper in the bulk correspond to entanglement at larger scales. This is exactly what you'd expect if the "depth" direction is really about scale and information organization, not about literal spatial location.

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## 6. Implications for Emergent Geometry

**For General Readers:** If scale isn't a spatial direction, how does space itself arise? This section sketches a picture where space emerges from patterns of correlation and information—and clarifies where scale fits into that picture.

## 6.1 Geometry from Correlations

If depth is not a spatial direction, how does geometry emerge? Within causal-interface frameworks, space arises from correlation structure rather than pre-existing embedding. Physical systems are defined not by positions in a container but by patterns of causal influence and informational correlation.

Consider a network of interacting degrees of freedom. The "distance" between two nodes can be defined operationally: nodes that strongly correlate or rapidly equilibrate are "close"; nodes that weakly correlate or slowly equilibrate are "far." This correlation-based distance can, under appropriate conditions, approximate a metric geometry. Space emerges as the organizational structure that renders correlations geometric.

**For General Readers:** Imagine you know nothing about geography, but you have data on how often people in different cities communicate. Cities with lots of communication between them are probably close together; cities that rarely interact are probably far apart. From the communication patterns alone, you could reconstruct a map. Similarly, space might not be a fundamental container—it might emerge from patterns of interaction and correlation. The "map" is a useful representation of the communication structure.

## 6.2 Persistent Patterns as Physical Objects

In this framework, physical objects are not points traversing a background space. They are temporally persistent patterns of causal activity—stable configurations that maintain coherence across successive updates. We term these patterns "ripples": localized, self-sustaining structures in the correlation network.

Motion, then, is not displacement through a container but re-anchoring of correlations. When the correlations defining a ripple shift relative to other ripples, we observe what appears as motion through space. The underlying process is informational: correlations update, patterns migrate, but no entity travels through a pre-existing spatial medium.

**For General Readers:** Think of a wave in the ocean. The wave moves across the surface, but no water molecule travels from shore to shore—the wave is a pattern that propagates through the water. In emergent geometry frameworks, physical objects might be like waves: persistent patterns in an underlying network of interactions, not things moving through a pre-existing space.

## 6.3 Depth as Descriptive Resolution

Where does depth fit in this picture? Depth indexes how finely we resolve the correlation structure. At fine resolution (small  $z$ ), we track individual degrees of freedom and their detailed correlations. At coarse resolution (large  $z$ ), we aggregate degrees of freedom into collective variables, losing access to fine-grained correlations while preserving macroscopic patterns.

Depth is not a direction within the correlation network—it is not "part of space." It is a parameter describing how much of the network's structure we retain in our description. Moving along depth means changing descriptive resolution, not relocating within the system.

This clarifies why bulk reconstructions work: they are not excavating hidden spatial regions but reorganizing boundary information across resolutions. The "bulk" is the boundary viewed at multiple scales simultaneously, geometrically encoded for calculational convenience.

**For General Readers:** In the picture where space emerges from correlations, scale tells you how much detail you're tracking—are you looking at the whole network or zooming in on individual nodes? It's a property of your description, not a direction within the thing being described. The "depth" direction in holography is like the zoom slider on a map: it changes what you see, but it's not a road you could drive on.

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## 7. Connection to Interface-Based Emergence

The analysis presented here coheres with a broader framework in which space, time, and physical law emerge from information-theoretic structures rather than existing as primitive substrates. Within such frameworks—including interface-based causal models where causality is primary and geometry is derived—physical objects are persistent patterns of causal activity ("ripples") whose apparent spatial relationships emerge from correlation structure.

In this context, the present paper clarifies the status of scale within emergent geometry. Depth is not a fifth (or fourth) dimension waiting to be discovered; it is an axis of description, orthogonal to the spatial directions that emerge from causal correlation. The temptation to geometrize depth reflects a broader tendency to spatialize every parameter—a tendency that obscures rather than illuminates the true structure of physical reality.

**For General Readers:** The take-home message is simple: not everything that looks like a dimension is a dimension. Physicists are very good at drawing pictures where parameters become axes—it's a powerful technique. But we shouldn't mistake the map for the territory. Scale is how we choose to look at things, not a place where things are. Space emerges from patterns of connection; scale describes how finely we examine those patterns.

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## 8. Experimental Signatures and Testable Predictions

**For General Readers:** A good scientific claim should make predictions that can be tested. This section identifies concrete experimental signatures that would distinguish "depth as description" from "depth as genuine spatial direction." If depth were truly spatial, certain experiments would behave one way; if depth is descriptive (as we argue), they should behave differently.

The predictions in this section are *operational signatures* in engineered renormalization and holographic-code platforms (e.g., MERA-like circuits, holographic tensor networks), designed to distinguish "depth-as-dynamical-dimension" interpretations from "depth-as-descriptive-index" interpretations. They are not claims of new fundamental interactions, but rather diagnostics that can be implemented in quantum simulators to test whether the mathematical structure of coarse-graining behaves like spatial transport or like information-discarding re-description.

The preceding analysis has been structural: we proved that renormalization depth lacks the formal properties of spatial directions. These operational tests translate that structural distinction into observable differences.

## 8.1 The Core Empirical Distinction

The central difference between spatial directions and descriptive indices is:

| Property                 | Spatial Direction  | Descriptive Index (Depth)   |
|--------------------------|--|---|
| Subsystem structure      | Tensor factorization: $H = H_A \otimes H_B$                                      | Quotient/image hierarchy: $S_z = R_z(S_0)$                          |
| Independent manipulation | Degrees of freedom at different positions can be prepared/measured independently | "Degrees of freedom at different depths" are functionally dependent |
| Correlation structure    | Limited by locality and propagation  | Constrained by isometry/channel structure                           |
| Information recovery     | Limited only by noise/decoherence  | Bounded by non-injectivity (kernel dimension)                       |

These differences generate testable predictions.

## 8.2 Prediction 1: Independent Manipulation Test

**Statement.** If depth were a genuine spatial direction, degrees of freedom at "neighboring depths"  $z$  and  $z + \epsilon$  would be independent—like particles at positions  $x$  and  $x + \epsilon$  in ordinary space. They could be prepared, manipulated, and measured independently.

If depth is a descriptive index (hierarchy of quotients), then "degrees of freedom at depth  $z$ " are functionally dependent on boundary degrees of freedom. They cannot be independently manipulated: any operation targeting "depth  $z$ " necessarily disturbs the "depth 0" (fine-grained) description.

**Prediction (Independent Manipulation Failure).** In engineered holographic systems—cold atom arrays implementing MERA-like tensor networks, photonic circuits realizing entanglement renormalization, or other analog quantum simulators—attempts to independently prepare or manipulate states "at different depths" will fail in a characteristic way.

Specifically: let  $O_0$  be an operator on the fine-grained (boundary) Hilbert space, and let  $O_z$  be an operator on the effective (coarse-grained) degrees of freedom at depth  $z$ . If depth were spatial,

$[O_0, O_z]$  could be made arbitrarily small by appropriate choice of operators (locality along depth). If depth is descriptive:

$[O_0, O_z] \neq 0$  generically, with the commutator structure determined by the coarse-graining map  $R_z$ .

More concretely: prepare a state  $|\psi\rangle$  at the boundary. Apply an operation  $U_z$  intended to act "only at depth  $z$ " (on coarse-grained variables). Measure a fine-grained observable  $O_0$ . The outcome statistics will show correlations with  $U_z$  that are *absent* when  $U_x$  acts along a genuine spatial direction  $x$ .

### Experimental Protocol.

1. Implement a MERA-like tensor network in a controllable quantum system (trapped ions, superconducting qubits, photonic circuits)
2. Prepare a known boundary state  $|\psi\rangle$
3. Apply unitaries  $U_z$  that act on effective degrees of freedom at layer  $z$
4. Measure boundary observables
5. Compare correlation structure to the same protocol with operations along spatial directions

**Observable Signature.** Operations "along depth" produce correlations with boundary observables that violate the independence expected for spatial directions. The violation pattern matches the isometry structure  $V^\dagger V = I$ ,  $VV^\dagger \neq I$ .

A fully worked quantitative example demonstrating this prediction in a minimal two-qubit model is provided in Appendix D, including explicit commutator calculations and predicted measurement outcomes ( $\langle Z_1 \rangle = \cos \theta$  for depth rotation angle  $\theta$ ).

## 8.3 Prediction 2: Recovery Fidelity Bound

**Statement.** For transport along a genuine spatial direction, information fidelity is limited only by noise and decoherence in the physical channel. For non-injective coarse-graining, there exists a fundamental, protocol-independent limitation on recovery that is structural rather than merely practical.

**Prediction (Structural Recovery Limitation).** Non-injective coarse-graining implies an irreducible ambiguity: there exist states  $\rho \neq \sigma$  such that  $R_z(\rho) = R_z(\sigma)$ . For any recovery map  $G$ :

$$G(R_z(\rho)) = G(R_z(\sigma))$$

since  $G$  receives identical inputs. Therefore uniform recovery of arbitrary fine-grained states is impossible without either (a) restricting to a subspace where  $R_z$  is injective, or (b) providing side information not contained in  $R_z(\rho)$ .

This is not a quantitative bound but a qualitative distinction: the recovery limitation is *structural* (from non-injectivity) rather than *practical* (from noise or finite resources). Optimal recovery protocols (e.g., Petz recovery map) can achieve the best possible fidelity given additional assumptions (reference states, subspace restrictions), but they cannot overcome the fundamental ambiguity when  $R_z$  is genuinely non-injective.

**Contrast with Spatial Transport.** For information transport along a genuine spatial direction  $x$ , no such structural limitation exists. In the ideal (noiseless) limit, transport is unitary and  $F \rightarrow 1$ . Any fidelity loss is due to decoherence, not to the structure of spatial translation itself.

### Experimental Protocol.

1. Prepare an ensemble of states  $\{\rho_i\}$  including pairs designed to be mapped to identical coarse-grained descriptions
2. Apply a known coarse-graining operation  $R_z$
3. Attempt recovery using various protocols (Petz map, variational recovery, etc.)
4. Verify that states with  $R_z(\rho) = R_z(\sigma)$  cannot be distinguished after recovery, regardless of protocol sophistication

**Observable Signature.** Recovery attempts for states in the same coarse-graining equivalence class converge to the same output, revealing the structural (not noise-induced) nature of the information loss. This contrasts with spatial channels, where improved isolation yields improved fidelity without bound.

## 8.4 Prediction 3: Correlation Asymmetry in Tensor Networks

**Statement.** In a MERA tensor network, equal-time correlations between operators at the same boundary position but "different depths" should be structurally different from correlations between operators at different spatial positions.

**Prediction (Correlation Asymmetry).** Let  $O_A$  and  $O_B$  be operators at boundary positions A and B separated by distance  $r$ . Let  $O_z$  and  $O_{z'}$  be operators representing the same boundary region at depths  $z$  and  $z'$ . Then:

Spatial correlations:  $\langle O_A O_B \rangle_c \sim r^{-2\Delta}$  (power law from conformal structure)

Depth correlations:  $\langle O_z O_{z'} \rangle$  constrained by isometry relations, not spatial locality

Specifically, if  $V_z : H_{z-1} \rightarrow H_z$  is the isometry implementing the  $z$ -th coarse-graining layer:

$$\langle O_z O_{z'} \rangle = \langle \psi | V_z^\dagger \dots V_{z'}^\dagger O_{z'} V_{z'} \dots O_z V_z | \psi \rangle$$

This has the structure of a *channel* (completely positive map), not a *propagator* (unitary evolution). The correlation function is constrained by  $V^\dagger V = I$  but not by  $VV^\dagger = I$ .

**Observable Signature.** If depth were spatial, "depth correlations" would show the same functional form as spatial correlations (modified by any metric along depth). Instead, depth correlations should show:

- Asymmetry between "ascending" and "descending" correlations
- Dependence on isometry structure rather than metric structure
- Saturation behavior reflecting the finite-dimensional effective Hilbert space at large  $z$

A fully worked quantitative example is provided in Appendix D.7, demonstrating that a depth round-trip changes operator moments ( $\langle O^2 \rangle = 2$  but  $\langle (\text{POP})^2 \rangle = 1$ ) and correlators ( $\langle OO \rangle = 2$  but  $\langle O'O \rangle = 1$ )—signatures impossible for genuine spatial translation.

### Experimental Protocol.

1. Implement a MERA tensor network in a controllable system
2. Measure two-point functions  $\langle O_A O_B \rangle$  for varying spatial separation
3. Measure "depth correlations"  $\langle O_z O_{z'} \rangle$  for varying depth separation
4. Compare functional forms and identify structural differences

## 8.5 Prediction 4: State-Dependent Operator Reconstruction

**Statement.** If the bulk-boundary dictionary is state/subspace-dependent (Propositions 4.3a–b), then "bulk" operators should show reconstruction ambiguities absent for operators along genuine spatial directions.

**Prediction (Reconstruction Ambiguity).** For operators  $O_x$  along a spatial direction  $x$ , the operator itself is state-independent: only the expectation value  $\langle \psi | O_x | \psi \rangle$  depends on the state  $\psi$ .

For "bulk" operators  $O_r$  associated with holographic depth:

The *definition* of  $O_r$  may depend on the state or code subspace.

Concretely: there exist states  $\psi, \phi$  and a "bulk operator"  $O_r$  such that the boundary representation of  $O_r$  differs:

$$O_r^{\{(\psi)\}} \neq O_r^{\{(\phi)\}} \text{ as boundary operators}$$

even though both correctly compute  $\langle \psi | O_r | \psi \rangle$  and  $\langle \phi | O_r | \phi \rangle$  respectively.

**Observable Signature.** Attempts to define a state-independent bulk operator algebra will encounter obstructions that are absent for spatial operator algebras. Specifically:

- Commutation relations  $[O_r, O_{r'}]$  may be state-dependent
- The operator product expansion may require state-dependent structure constants
- Tomographic reconstruction of "bulk operators" from boundary measurements will show ambiguities

## 8.6 Prediction 5: Entropy Non-Additivity

**Statement.** In theories with genuine extra dimensions (Kaluza-Klein, Randall-Sundrum), modes along the extra dimension carry independent entropy. If holographic depth is descriptive, "bulk" configurations should not contribute entropy beyond the boundary.

**Prediction (Entropy Bound).** In any holographic system:

$$S_{\text{bulk}} \leq S_{\text{boundary}} \text{ (saturation when bulk is fully encoded)}$$

"Bulk excitations" are reorganizations of boundary data at different scales, not independent degrees of freedom. The covariant entropy bound should be saturated, not merely respected.

**Contrast with Extra Dimensions.** In Kaluza-Klein compactification, KK modes contribute *additional* entropy:  $S_{\text{total}} = S_{\text{4D}} + S_{\text{KK}}$ . In holography, if our analysis is correct, the "bulk entropy" is a re-counting of boundary entropy, not an addition.

**Observable Signature (Cosmological).** In holographic cosmology scenarios, the entropy content of primordial perturbations should be fully accountable from boundary data. Any apparent "bulk contribution" should be rewritable as boundary entropy at a different scale. This constrains inflationary models in holographic cosmology.

## 8.7 Summary of Predictions

| Prediction                       | Tests                                  | Key Observable   |
|----------------------------------|--|--|
| Independent Manipulation Failure | Tensor network quantum simulators      | Correlations between "depth" operations and boundary observables |
| Structural Recovery Bound        | Coarse-graining experiments            | Fidelity saturation below 1, independent of noise                |
| Correlation Asymmetry            | MERA implementations                   | Different functional forms for spatial vs. depth correlations    |
| State-Dependent Reconstruction   | Holographic simulations                | Ambiguities in bulk operator definition                          |
| Entropy Non-Additivity           | Black hole / cosmological observations | Saturation of covariant entropy bounds                           |

**Feasibility Assessment.** Predictions vary in near-term testability:

- *Near-term* (existing platforms): **8.2** and **8.4** can be tested in current tensor network quantum simulators (trapped ions, superconducting qubits, photonic circuits implementing MERA). These are the most immediately actionable predictions.
- *Medium-term* (requires dedicated experiments): **8.3** requires preparing states known to be in the same coarse-graining equivalence class and verifying recovery indistinguishability; **8.5** requires platforms capable of simulating holographic codes with sufficient fidelity.

- *Long-term* (observational/cosmological): **8.6** involves black hole information physics or cosmological observations that may require advances in gravitational wave astronomy or precision cosmology.

We recommend experimentalists focus initially on **8.2** (independent manipulation) and **8.4** (correlation asymmetry), as these directly probe the quotient-vs-tensor-factor distinction using existing technology.

**For General Readers:** These predictions translate the abstract claim "depth isn't spatial" into concrete experimental tests. The key idea: if depth were really a spatial direction, experiments would show certain symmetries (independence, reversibility, additive entropy). Our thesis predicts these symmetries will be *violated* in specific, measurable ways. This makes the claim scientific rather than merely philosophical.

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## 9. Conclusion

Renormalization depth is not a dimension of space. It is a monotonic index of information loss—a label for equivalence classes of descriptions related by coarse-graining. The criteria that define spatial directions (metric structure, locality, propagation, reversibility) are necessary conditions for any parameter to function as a spatial coordinate, and renormalization depth fails each criterion. Apparent counterexamples from holographic duality dissolve upon recognizing that bulk geometry emerges from boundary data in restricted regimes rather than existing as primitive spatial structure.

This conclusion is not merely philosophical. It has methodological implications: theories of emergent space should not treat depth as an additional direction to be navigated, but as a descriptive parameter encoding resolution. Geometry emerges from correlations across scales, not from motion through scales. The bulk is a way of seeing the boundary, not a place beyond it.

More importantly, the thesis generates testable predictions. The structural difference between spatial directions (tensor factorization, independent degrees of freedom, reversible transport) and descriptive indices (quotient hierarchies, functional dependence, non-injective maps) leads to observable signatures in correlation structure, recovery fidelity, and operator reconstruction. These predictions can be tested in tensor network quantum simulators and may have implications for holographic cosmology.

Depth tells us not where things are, but how much we have chosen to forget.

Or, in the language of analogy: *Renormalization depth is to spacetime what rendering resolution is to a video game—it can be drawn as an axis and behaves geometrically in certain regimes, but nothing in the game world actually moves along it.*

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# Appendix A: No-Go Theorem for Depth as Spatial Direction

**For General Readers:** This appendix provides the rigorous mathematical proof that scale cannot be a spatial direction. The key insight is simple: if the "zoom out" process loses information (as it must), then there's no way to "zoom back in" and recover what was lost. This makes scale fundamentally different from genuine spatial directions, where you can always return to where you started.

## A.1 Definitions

Let  $S$  denote a space of physical states (e.g., density operators on a Hilbert space, or probability distributions on phase space).

**Definition (Coarse-graining map).** A map  $R: S \rightarrow S$  is a coarse-graining if it is non-injective: there exist distinct states  $\rho \neq \sigma$  such that  $R(\rho) = R(\sigma)$ .

Non-injectivity captures the essential feature of coarse-graining: information loss. Distinct microscopic configurations become indistinguishable at coarser resolution.

**Remark.** Non-injectivity is implied by—but not equivalent to—contractivity under information measures. A map can be strictly contractive (reducing distinguishability) while remaining injective on the full state space. Our theorem requires only non-injectivity, the weaker and more directly relevant condition.

## A.2 Renormalization as Non-Injective Semigroup

A renormalization flow is a one-parameter family  $\{R_z\}_{z \geq 0}$  satisfying:

- **Semigroup:**  $R_{z+w} = R_z \circ R_w$  for all  $z, w \geq 0$
- **Identity:**  $R_0 = \text{id}$
- **Non-injectivity:** For some  $z > 0$ ,  $R_z$  is non-injective

The non-injectivity condition captures that coarse-graining discards information: multiple microscopic states map to the same effective description.

## A.3 Definition of Spatial Direction

A parameter  $u$  constitutes a physical spatial direction only if translations  $T_u$  along  $u$  form a group:

- **Closure:**  $T_u \circ T_v = T_{u+v}$  for all  $u, v \in \mathbb{R}$
- **Identity:**  $T_0 = \text{id}$

- **Inverses:** For each  $T_u$ , there exists  $T_{-u}$  such that  $T_u \circ T_{-u} = \text{id}$

The existence of inverses is essential: one can move along a spatial direction and return.

## A.4 No-Go Theorem

**Theorem.** Let  $\{R_z\}_{z \geq 0}$  be a renormalization flow satisfying the semigroup property such that  $R_z$  is non-injective for some  $z > 0$ . Then  $z$  cannot be a physical spatial direction.

**Proof.** Suppose for contradiction that  $z$  is a spatial direction. Then there exist inverse maps  $R_{-z}$  for all  $z > 0$  satisfying  $R_{-z} \circ R_z = \text{id}$ .

By assumption,  $R_z$  is non-injective for some  $z > 0$ . Thus there exist distinct states  $\rho \neq \sigma$  with  $R_z(\rho) = R_z(\sigma)$ . Applying the supposed inverse:

$$\rho = R_{-z}(R_z(\rho)) = R_{-z}(R_z(\sigma)) = \sigma$$

This contradicts  $\rho \neq \sigma$ . Therefore no inverse  $R_{-z}$  exists for non-injective  $R_z$ , and  $z$  does not satisfy the group property required for spatial directions. ■

**For General Readers:** The proof is straightforward: if "zooming out" makes two different things look identical, then "zooming back in" would have to produce two different outputs from the same input. But that's impossible—any well-defined operation gives one output for each input. So there can be no true "reverse" of coarse-graining, which means scale can't be a real spatial direction.

## A.5 On Approximate Recovery

A spatial interpretation of depth would require at minimum a uniform approximate inverse—a recovery procedure that approximately restores pre-coarse-grained states. Non-injectivity rules this out in the strongest possible way.

If  $R(\rho) = R(\sigma)$  for  $\rho \neq \sigma$ , then any recovery procedure  $G$  receives identical input on both states. Since  $G$  is a function, it cannot output distinct approximations to  $\rho$  and  $\sigma$  simultaneously:  $G(R(\rho)) = G(R(\sigma))$ , regardless of how  $G$  is constructed. No deterministic recovery map can approximate both  $\rho$  and  $\sigma$  from their common image.

At best, recovery can be state-restricted (e.g., to a code subspace where  $R$  remains injective) or can succeed given additional side information not contained in  $R(\rho)$ . This is consistent with the data-processing principle for relative entropy under CPTP maps [9]; see also the textbook discussion in [8].

**Remark (Code-Subspace Recovery Does Not Restore Spatiality).** In AdS/CFT, quantum error correction shows that certain coarse-grainings become approximately reversible on a chosen code subspace, enabling bulk reconstruction [11]. This does not promote  $z$  to a spatial direction

in our sense: reversibility obtained by restricting the domain is not a global translation symmetry, and the decoding map is defined relative to a subspace (often tied to a semiclassical regime or state class) rather than as an intrinsic geometric degree of freedom. A spatial translation dimension requires invertible action on the full state space (or at least on an open neighborhood structure compatible with locality), whereas code-subspace recovery is a conditional decoding map that depends on restricting the domain rather than providing a global inverse.

## A.6 Corollary: Bulk Motion is Re-description

Any statement of the form "an excitation moves deeper into the bulk" must be interpreted as: the fine-grained description  $\rho$  is replaced by the coarse-grained description  $R_z(\rho)$ . No ontological motion occurs. The excitation does not traverse an additional spatial dimension; its representation changes.

The bulk radial coordinate in holographic duality, insofar as it corresponds to RG depth, inherits this interpretation. "Motion" along the radial direction is reorganization of boundary data, not transport through space.

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# Appendix B: Relation to Tensor Network Geometry

**For General Readers:** This appendix examines "tensor networks"—a computational tool that makes the "depth as geometry" intuition maximally concrete. We show that even here, where the geometric picture is most compelling, depth remains a representational device rather than genuine space.

Tensor networks based on entanglement renormalization (e.g., MERA) [10] provide explicit constructions where RG depth appears geometric [6]. Because these constructions make the depth-as-geometry intuition maximally concrete, they deserve careful analysis.

## B.1 The MERA Construction

MERA represents the ground state of a many-body system as a hierarchical network of tensors. Starting from a lattice of physical degrees of freedom (the "boundary"), the network proceeds through layers:

1. **Disentangler**s remove short-range entanglement between neighboring sites
2. **Isometries** coarse-grain pairs of sites into single effective sites

Each layer reduces the number of degrees of freedom by a factor (typically 2), producing a sequence:

$$N \rightarrow N/2 \rightarrow N/4 \rightarrow \dots \rightarrow 1$$

The network naturally forms a tree-like structure, and when drawn in two dimensions, it resembles a discrete version of AdS space. The "radial" direction through the layers corresponds to RG depth.

**For General Readers:** MERA is like a compression algorithm for quantum states. Imagine you have a row of 1000 quantum bits. MERA compresses them: first to 500, then to 250, then to 125, and so on, until you have just one "super-bit" that captures the essential large-scale properties. When you draw this process as a diagram, it looks like a tree—or, if you squint, like a funnel-shaped extra dimension of space.

## B.2 Why the Geometry Is Representational

Despite its geometric appearance, the MERA network does not constitute an additional spatial dimension. The key observations:

**Information flow is unidirectional.** Tensors map from fine to coarse layers. The isometries  $V: \mathbb{C}^d \otimes \mathbb{C}^d \rightarrow \mathbb{C}^d$  satisfy  $V^\dagger V = I$  but  $VV^\dagger \neq I$  (they are isometries, not unitaries). Information about the boundary state flows upward through the network; it cannot flow back down. This is precisely the non-injectivity of coarse-graining.

*In plain terms:* Information only flows one way through the network—from detailed to compressed. You can't run the compression backwards to recover the original.

**The "metric" is inherited from entanglement.** The graph distance through the MERA network correlates with entanglement structure: the minimal path connecting two boundary regions passes through layers proportional to  $\log(\text{separation})$ . This is why MERA reproduces the Ryu-Takayanagi formula. But this "distance" is not primitive—it is a representation of how entanglement organizes across scales.

*In plain terms:* The "distance" through the network reflects how entangled different regions are—it's measuring information relationships, not spatial separation.

**No propagation through layers.** Physical excitations do not travel through the network. A local perturbation at the boundary does not "move upward" through successive layers as a dynamical process. Rather, the network encodes how the perturbation's description changes under coarse-graining.

*In plain terms:* If you poke the system at the boundary, nothing literally travels up through the network layers. The layers just show how the "poke" looks at different levels of description.

## B.3 Worked Example: Correlation Functions

Consider two boundary operators  $O_A$  and  $O_B$  separated by distance  $r$  on a 1D critical system. Their connected correlation function scales as:

$$\langle O_A O_B \rangle_c \sim r^{-2\Delta}$$

In MERA, this correlation is computed by tracing paths through the network. The dominant contribution comes from paths that ascend from  $A$  to some layer  $z^*$ , then descend to  $B$ , where  $z^* \sim \log(r)$ . The "meeting point" in the network—sometimes called the *causal cone apex* or *causal wedge* in the MERA literature—corresponds to the scale at which  $A$  and  $B$  first become part of the same coarse-grained block.

Crucially,  $z^*$  is not a location where some physical process occurs—it is the resolution scale at which  $A$  and  $B$  become indistinguishable as separate regions. The network geometry encodes this scale-dependence of distinguishability. The apparent "bulk point" at depth  $z^*$  is a representation of information-theoretic structure, not a place in an additional spatial dimension. The causal cone structure of MERA has motivated much of the tensor network/holography connection [6], but the geometric interpretation remains representational.

**For General Readers:** Imagine two cities,  $A$  and  $B$ . At street-level detail, they're clearly separate. Zoom out to a state map—still separate. Zoom out to a continental map—they might merge into one dot. The "scale at which they merge" isn't a place between the cities; it's a property of how we're looking at them. MERA works the same way: the "depth" where two points meet isn't a location in space—it's the scale at which they become indistinguishable.

## B.4 The Holographic Correspondence

Swingle's observation [6] that MERA geometry resembles AdS is not coincidental—both encode the same information-theoretic structure. The AdS radial coordinate and MERA depth both represent the scale-organization of boundary entanglement. The success of tensor networks in approximating holographic systems demonstrates that bulk geometry *is* a representation of boundary entanglement structure, supporting our thesis that the radial direction is derived rather than primitive.

The lesson of MERA is that geometric appearance does not imply geometric ontology. A network can look spatial while encoding something entirely different: the hierarchical structure of information under coarse-graining.

**For General Readers:** MERA provides the clearest lesson of this paper: something can look exactly like a spatial dimension—complete with distances, paths, and geometric structure—while actually being something completely different: a representation of how information organizes across scales. The map is not the territory. The diagram is not the dimension.

# Appendix C: Anticipated Objections and Responses

**For General Readers:** This appendix addresses questions that expert reviewers are likely to raise. It's structured as a FAQ to make the paper's position clear on potential points of confusion.

## C.1 On Terminology and Significance

**Q1. "Everyone in holography calls the RG scale an extra dimension. Why are you making a fuss about terminology?"**

Because the terminology is doing conceptual work it cannot support. Our claim is not that geometric representations of scale are useless—on the contrary, they are extraordinarily effective—but that interpreting scale literally as a spatial direction introduces a category error. The paper isolates precisely which structural properties distinguish spatial dimensions from descriptive parameters and shows that renormalization depth does not possess them. Clarifying this distinction avoids conflating representational convenience with ontology.

**Q2. "Isn't this just semantics? You're arguing about words, not physics."**

No. The distinction has structural consequences. Treating depth as a spatial direction invites assumptions about locality, propagation, and transport that are false for coarse-graining maps and lead to conceptual confusion in discussions of emergent geometry, bulk reconstruction, and information flow. The no-go theorem shows this is not merely linguistic: non-injective coarse-graining cannot support the group structure required for spatial translation.

## C.2 On the Criteria C1–C4

**Q3. "Your criteria C1–C4 seem arbitrary. Why should those define a spatial dimension?"**

They are not arbitrary; they are necessary. Any parameter that functions as a physical spatial direction must support metric distances, locality, propagation through intermediate positions, and reversible translations. These are minimal structural features of geometry used throughout classical mechanics, quantum field theory, and general relativity. A parameter failing these criteria may still be a useful label, but it does not participate in geometry as a spatial coordinate. (See Theorem 2.0 for the formal proof of necessity.)

**Q4. "Time also fails reversibility (C4). Why doesn't your argument rule out time as a dimension?"**

Time differs crucially from renormalization depth. The fundamental dynamical laws are time-reversal symmetric; irreversibility arises statistically from boundary conditions, not from the

structure of time evolution itself. Renormalization depth, by contrast, is irreversible by definition: information loss is built into the map. Thus the analogy does not hold.

## C.3 On Quantum Error Correction and Code Subspaces

**Q5. "Within a quantum error-correcting code subspace, bulk reconstruction is reversible. Doesn't that give depth the reversibility C4 requires?"**

No. Reversibility obtained by restricting the domain does not constitute a spatial translation symmetry. A spatial direction requires invertible action on the full physical state space (or at least on open neighborhoods compatible with locality). Code-subspace recovery is a conditional decoding map defined relative to a chosen subspace and often to a specific state class. This does not promote renormalization depth to a global geometric coordinate. (See Remark in A.5.)

**Q6. "But within the code subspace, bulk locality and geometry behave exactly like real space. Why isn't that enough?"**

Because effective behavior within a restricted regime does not define ontology. Many effective parameters behave geometrically in limited contexts without being spatial dimensions (e.g., temperature gradients, order parameters, collective coordinates). The paper's claim is precisely that the geometric interpretation of depth is regime-dependent, not primitive.

Moreover, there is a crucial structural difference: genuine spatial directions exhibit *uniform locality* across different states and subspaces. The spatial structure of a system does not depend on which state we prepare—left-right is left-right regardless of whether we're studying a ground state or an excited state. Bulk reconstruction, by contrast, is subspace-dependent: the dictionary mapping boundary operators to bulk operators changes depending on which code subspace we work in, and for some states (e.g., black hole interiors), the reconstruction becomes state-dependent [12]. This subspace/state dependence is incompatible with the uniform character of spatial directions.

One might object that general relativity makes geometry dynamical, so spatial structure is "state-dependent" even in ordinary physics. But this state-dependence is smooth and governed by Einstein's equations: small changes in the matter configuration produce small changes in the metric. The *existence* of spatial directions is universal; only their precise metric structure varies continuously with the state. Bulk reconstruction exhibits a qualitatively different dependence: the very definition of bulk operators changes discretely across code subspaces, not as a smooth function of state governed by dynamical equations.

## C.4 On the Status of Bulk Spacetime

**Q7. "Your argument seems to deny the reality of bulk spacetime in AdS/CFT. Are you claiming the bulk isn't real?"**

No. The paper makes no claim about ontological "reality" in a metaphysical sense. It claims that the bulk geometric description is emergent and encoded, not an additional primitive spatial arena appended to the boundary theory. This is fully consistent with AdS/CFT as a duality and with the modern quantum-error-correction interpretation of bulk reconstruction.

**Q8. "If bulk geometry is emergent, emergent from what exactly?"**

From boundary correlation and entanglement structure organized across scale. The radial coordinate encodes how boundary degrees of freedom at different resolutions relate to one another. In regimes where semiclassical bulk locality exists, this organization admits an effective geometric description; outside those regimes, it does not.

## **C.5 On Metric, Locality, and Propagation**

**Q9. "You say depth has no metric. But in AdS there is a perfectly good metric involving  $dr^2$ ."**

That metric is derived, not intrinsic. Its geometric meaning depends on the holographic dictionary and holds only in semiclassical regimes. The coordinate  $r$  does not define an invariant metric structure independent of the boundary theory and the reconstruction map. This is why different dictionaries, states, or regimes can alter or destroy the geometric interpretation. (See Propositions in Section 5.3.)

**Q10. "Why can't renormalization depth support locality along the depth direction?"**

Because different depths are not independent subsystems. Coarse-grained descriptions are functionally dependent on fine-grained ones; they do not coexist as neighboring degrees of freedom that can interact locally. There is no natural tensor-factorization or net of local algebras indexed by depth, only a hierarchy of quotients. (See Proposition 3.4.)

**Q11. "Isn't RG flow literally a flow? Why deny propagation?"**

"Flow" is metaphorical. RG evolution is a sequence of re-descriptions of the same system, not the motion of states or excitations through an additional coordinate. There is no automorphism group generating trajectories along depth—only a semigroup of information-discarding maps. (See Proposition 3.5.)

**Q12. "Couldn't one define an approximate notion of motion or locality along depth?"**

Approximate notions can exist in restricted regimes, just as approximate invertibility exists in code subspaces. But approximation does not establish a spatial dimension. A spatial direction requires uniform structural properties, not regime-dependent or state-dependent behavior.

## **C.6 On Non-Injectivity**

**Q13. "Your argument depends heavily on non-injectivity. Isn't that too strong an assumption?"**

Non-injectivity is not an additional assumption; it is the defining feature of coarse-graining. If a map were injective, it would not discard information and would not be a renormalization in the usual sense. The no-go theorem isolates this structural fact and shows its geometric consequences.

## **C.7 On Practical Implications**

**Q14. "What does this actually buy us? Does it change predictions?"**

The paper is not aimed at new empirical predictions but at conceptual clarity. It prevents the misapplication of geometric intuition where it does not apply and sharpens the distinction between representational tools and physical structure. This has downstream implications for how we interpret bulk reconstruction, locality, and information flow in emergent spacetime frameworks.

**Q15. "Isn't this just philosophy of physics?"**

No. The paper uses explicit structural criteria, formal properties of coarse-graining maps, and reconstruction results from quantum gravity. While the conclusions are conceptual, the arguments are mathematical and structural rather than interpretive or metaphysical.

**Q16. "Why hasn't this been stated clearly before if it's so important?"**

Because the geometric representation of scale has been enormously successful, and its utility often obscures its limitations. The distinction between representation and ontology is widely understood implicitly but rarely made explicit. This paper's contribution is to formalize that distinction and make its consequences precise.

**Q17. "Are you saying we should stop drawing bulk diagrams?"**

No. Bulk diagrams are powerful and indispensable. The claim is that they should be understood as representations of how boundary data organizes across scale, not as literal pictures of motion through an additional spatial dimension.

**Q18. "What about continuous MERA (cMERA)? Doesn't continuity restore spatiality?"**

No. cMERA replaces discrete layers with a continuous scale parameter  $s$  generated by continuous unitary evolution, but "continuous" does not imply "spatial." The depth parameter  $s$  still labels resolution/scale, and identifying "the same degree of freedom at different  $s$ " remains dictionary-dependent. Treating  $s$  as spatial would require locality and reversible translations on the physical state space; cMERA provides continuous re-description whose utility comes from organizing correlations by scale, not from introducing a new kinematic direction. Smoothness of representation  $\neq$  ontological category change.

## C.8 Summary

### Q18. "Summarize your position in one sentence."

Renormalization depth can be drawn as if it were a spatial dimension, but it cannot be one, because it lacks the structural features—metric, locality, propagation, and reversible translation—that define space.

*Or, by analogy:* Renormalization depth is to spacetime what rendering resolution is to a video game: it can be drawn as an axis and behaves geometrically in certain regimes, but nothing in the game world actually moves along it.

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## Appendix D: Worked Examples— Quantitative Tests of Predictions 8.2 and 8.4

This appendix provides minimal, fully explicit calculations demonstrating Prediction 8.2 (Independent Manipulation Failure) and Prediction 8.4 (Correlation Asymmetry) in a toy MERA-like model.

### D.1 Setup: One Isometry Layer

Consider a 2-qubit "boundary" Hilbert space  $H_{UV} \cong \mathbb{C}^2 \otimes \mathbb{C}^2$  with computational basis  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ .

Define a coarse-graining isometry  $V : H_{UV} \rightarrow H_{IR}$  mapping 2 qubits to 1 qubit:

$$V|00\rangle = |0\rangle, V|11\rangle = |1\rangle, V|01\rangle = V|10\rangle = 0$$

In matrix form (rows =  $|0\rangle, |1\rangle$ ; columns =  $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ ):

$$V = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This satisfies:

- $VV^\dagger = I_{IR}$  (isometry property)
- $V^\dagger V = P_{\text{code}}$ , where  $P_{\text{code}} = |00\rangle\langle 00| + |11\rangle\langle 11|$  is the projector onto the "code subspace"

**Interpretation:** This is the minimal MERA structure: coarse degrees of freedom are an isometric image of UV degrees of freedom, not an independent tensor factor.

## D.2 Depth Operation and Its Boundary Avatar

Let  $X$  be the Pauli- $X$  operator acting on the IR qubit. A unitary "operation at depth" is:

$$U_{\text{IR}}(\theta) = \exp(-i\theta X/2)$$

Its induced boundary operation is:

$$U_{\text{UV}}(\theta) = V^\dagger U_{\text{IR}}(\theta) V$$

which acts only within the code subspace. The lifted boundary generator is:

$$X_{\text{code}} = V^\dagger X V = |00\rangle\langle 11| + |11\rangle\langle 00|$$

## D.3 Demonstrating Prediction 8.2: Commutator Calculation

Pick a fine-grained boundary observable: Pauli  $Z$  on the first UV qubit:

$$Z_1 = Z \otimes I = \text{diag}(1, 1, -1, -1)$$

**Claim:** The commutator  $[Z_1, X_{\text{code}}] \neq 0$ .

**Proof:** Using  $Z_1|00\rangle = +|00\rangle$  and  $Z_1|11\rangle = -|11\rangle$ :

$$Z_1|00\rangle\langle 11| = +|00\rangle\langle 11| \quad |00\rangle\langle 11|Z_1 = -|00\rangle\langle 11| \quad Z_1|11\rangle\langle 00| = -|11\rangle\langle 00| \quad |11\rangle\langle 00|Z_1 = +|11\rangle\langle 00|$$

Therefore:

$$[Z_1, X_{\text{code}}] = 2(|00\rangle\langle 11| - |11\rangle\langle 00|) \neq 0$$

**Meaning:** An operation "purely at depth" (generated by  $X$  in the IR) does not commute with UV-local observables. Operations at depth necessarily disturb boundary measurements. This is exactly Prediction 8.2.

## D.4 Quantitative Effect: Expectation Value Shift

Take a UV state inside the code subspace:  $|\psi_0\rangle = |00\rangle$ .

Apply the depth unitary (lifted to the boundary):

$$|\psi(\theta)\rangle = U_{\text{UV}}(\theta)|00\rangle = \cos(\theta/2)|00\rangle - i \sin(\theta/2)|11\rangle$$

Compute the boundary expectation of  $Z_1$ :

$$\langle Z_1 \rangle_\theta = \cos^2(\theta/2)(+1) + \sin^2(\theta/2)(-1) = \cos \theta$$

**Concrete values:**

| $\theta$ | $\langle Z_1 \rangle$ |
|----------|-----------------------|
| 0        | +1                    |
| $\pi/2$  | 0                     |
| $\pi$    | -1                    |

A "depth-only" operation flips a UV-local expectation value continuously and predictably. This is the quantitative signature of Prediction 8.2.

## D.5 Connection to Prediction 8.4 (Correlation Asymmetry)

The isometry asymmetry is visible directly:

- $VV^\dagger = I$  (complete on IR)
- $V^\dagger V = P \neq I$  (projector on UV)

Therefore:

- **Ascending** (UV  $\rightarrow$  IR):  $\rho_{\text{IR}} = V \rho_{\text{UV}} V^\dagger$  is a CPTP channel
- **Descending** (IR  $\rightarrow$  UV):  $\rho_{\text{UV}} \sim V^\dagger \rho_{\text{IR}} V$  is an embedding into a subspace, not an inverse

This is precisely the structural difference claimed in Prediction 8.4: depth correlations are constrained by isometries (channels), not by automorphisms (propagators). The asymmetry between ascending and descending is built into the mathematics.

## D.6 Experimental Implementation

This toy model can be directly implemented in:

- **Two-qubit systems:** Trapped ion pairs, superconducting qubit pairs, photonic polarization qubits
- **Measurement protocol:**
  1. Prepare  $|00\rangle$
  2. Apply  $V$  (projective preparation into code subspace)
  3. Apply  $U_{\text{IR}}(\theta)$  at the "coarse" level
  4. Measure  $Z_1$  on the first qubit
  5. Verify  $\langle Z_1 \rangle = \cos \theta$

The predicted  $\cos \theta$  dependence is a direct, falsifiable signature distinguishing depth (quotient/isometry structure) from a genuine spatial direction (tensor factor structure, where the commutator could vanish).

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## D.7 Worked Example for Prediction 8.4: Channel vs. Propagator

This section demonstrates Prediction 8.4 (Correlation Asymmetry) using the same MERA isometry framework.

### D.7.1 The Depth Maps: Ascending vs. Descending

Using our isometry  $w : H_{IR} \rightarrow H_{UV}$  (where  $w|0\rangle = |00\rangle$ ,  $w|1\rangle = |11\rangle$ ), define:

**Ascending** ( $UV \rightarrow IR$ ):  $A(O_{UV}) = w^\dagger O_{UV} w$

**Descending** ( $IR \rightarrow UV$ ):  $D(o_{IR}) = w o_{IR} w^\dagger$

Now compute the two compositions:

**(i)  $IR \rightarrow UV \rightarrow IR$  is identity:**

$$(A \circ D)(o) = w^\dagger (w o w^\dagger) w = (w^\dagger w) o (w^\dagger w) = o$$

Going down then up returns the same IR operator.

**(ii)  $UV \rightarrow IR \rightarrow UV$  is a projection channel:**

$$(D \circ A)(O) = w (w^\dagger O w) w^\dagger = (w w^\dagger) O (w w^\dagger) = P_{\text{code}} O P_{\text{code}}$$

Going up then down returns not  $O$  but the projected operator POP.

**This is the channel-vs-propagator distinction:**

- **Propagator** (unitary translation): would give identity both ways
- **Depth** (isometric coarse structure): gives identity one way, projection the other

### D.7.2 Quantitative Correlation Asymmetry

**Setup:** Take the IR state  $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ . The corresponding UV state is:

$$|\Psi\rangle = w|+\rangle = (|00\rangle + |11\rangle)/\sqrt{2} \equiv |\Phi^+\rangle \text{ (Bell state)}$$

**Choose a UV operator with both code and non-code components:**

$$O = Z_1 + X_1$$

where  $Z_1 = Z \otimes I$  and  $X_1 = X \otimes I$  act on the first UV qubit.

- $Z_1$  preserves the code subspace  $\text{span}\{|00\rangle, |11\rangle\}$
- $X_1$  takes code states out:  $|00\rangle \mapsto |10\rangle, |11\rangle \mapsto |01\rangle$

**Compute the depth-projected operator:**

$$O' = (D \circ A)(O) = P_{\text{code}} O P_{\text{code}}$$

Since  $P X_1 P = 0$  ( $X_1$  exits the code) and  $P Z_1 P = Z_1 P$ :

$$O' = P(Z_1 + X_1)P = P Z_1 P = Z_1 P_{\text{code}}$$

### D.7.3 Signature 1: Operator Moments Change Under Depth Round-Trip

**Compute  $\langle O^2 \rangle$ :**

Using Pauli algebra ( $Z_1^2 = X_1^2 = I$ , and  $Z_1 X_1 + X_1 Z_1 = 0$ ):

$$O^2 = (Z_1 + X_1)^2 = Z_1^2 + X_1^2 + (Z_1 X_1 + X_1 Z_1) = 2I$$

$$\text{Therefore: } \langle \Psi | O^2 | \Psi \rangle = 2$$

**Compute  $\langle (O')^2 \rangle$ :**

Since  $|\Psi\rangle$  lies in the code subspace ( $P|\Psi\rangle = |\Psi\rangle$ ) and  $Z_1^2 = I$ :

$$(O')^2 |\Psi\rangle = (Z_1 P Z_1 P) |\Psi\rangle = Z_1^2 |\Psi\rangle = |\Psi\rangle$$

$$\text{Therefore: } \langle \Psi | (O')^2 | \Psi \rangle = 1$$

**Quantity Value**

$$\langle O^2 \rangle \quad 2$$

$$\langle (O')^2 \rangle \quad 1$$

**Interpretation:** A "round trip in depth" changes the operator's second moment. A genuine spatial translation (unitary propagator) would preserve operator norms under conjugation. Depth doesn't.

### D.7.4 Signature 2: Mixed-Depth Correlator Differs from Boundary Correlator

### Boundary correlator:

$$C_{UV} = \langle \Psi | O O | \Psi \rangle = \langle O^2 \rangle = 2$$

### Depth-projected correlator:

$$C_{\text{depth}} = \langle \Psi | O' O | \Psi \rangle = \langle \Psi | Z_1 (Z_1 + X_1) | \Psi \rangle = \langle \Psi | Z_1^2 | \Psi \rangle + \langle \Psi | Z_1 X_1 | \Psi \rangle$$

The first term is 1. The second term is 0 for  $|\Phi^+\rangle$  (it maps code  $\leftrightarrow$  non-code with zero overlap).

Therefore:  $C_{\text{depth}} = 1$

| Correlator  | Value |
|---|-------|
| $\langle O O \rangle$ (both at boundary)            | 2     |
| $\langle O' O \rangle$ (one "lifted" through depth) | 1     |

**Interpretation:** If depth were a spatial coordinate with unitary translation, "moving one insertion along depth" would not halve the correlator. The asymmetry is structural, not noise-induced.

### D.7.5 Summary: What This Proves

In this explicit MERA isometry model, the depth round-trip ( $D \circ A$ ) acts as a projection channel  $O \mapsto \text{POP}$ , not a unitary conjugation. Consequently:

1. **Moments are not preserved:**  $\langle O^2 \rangle = 2$  but  $\langle (\text{POP})^2 \rangle = 1$
2. **Correlators are asymmetric:**  $\langle OO \rangle = 2$  but  $\langle O'O \rangle = 1$

This quantitative asymmetry is impossible for a genuine spatial dimension, where translations act invertibly (group structure) and preserve operator norms under conjugation. **Depth correlations exhibit channel structure rather than propagator structure**, exactly as claimed in Prediction 8.4.

### D.7.6 Experimental Protocol for Prediction 8.4

1. Prepare the Bell state  $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$
2. Measure  $\langle O^2 \rangle$  for  $O = Z_1 + X_1$  (should give 2)
3. Implement the projection channel  $P(\cdot)P$  via post-selection or active projection
4. Measure  $\langle (O')^2 \rangle$  where  $O' = \text{POP}$  (should give 1)
5. Verify the 2:1 ratio as signature of channel (not propagator) structure

The factor-of-2 difference is a clean, noise-robust signature that can be verified in any two-qubit platform.

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