Depth as a Derivative of Time

A Transmission-First, Slice-Based Formalisation

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Summary for General Readers

The Core Question

When you look at the world, you see three dimensions of space: left-right, up-down, and near-far (depth). Physicists have always assumed these three dimensions are fundamental—that space exists as a kind of invisible container in which everything happens.

This paper asks: **What if depth isn't fundamental? What if it's reconstructed from something simpler?**

The Answer

Depth is not a pre-existing dimension. It emerges from *time*—specifically, from the structured differences between successive moments.

The Film Analogy

Think of a movie. Each frame is a flat, 2D image. No single frame contains motion—motion only exists when you compare one frame to the next. Yet motion is completely real: you can measure it, predict it, and it follows precise laws.

We propose that depth works the same way. Each "moment" of physical reality is like a 2D frame. No single moment contains depth. But when moments are sequenced in time, depth emerges from the *differences* between them—just as motion emerges from differences between film frames.

This doesn't mean depth is an illusion. Motion in films isn't an illusion either. It means depth is *derived* rather than *fundamental*.

Why This Matters

1. **Black holes make sense.** When you fall into a black hole, outside observers can never receive signals from you again. If depth were fundamental, the space "inside" the black hole should still exist for them. But black hole physics suggests it doesn't—the only

information they can access is on the 2D surface (the horizon). Our framework explains why: depth requires ongoing time-sequencing to exist, and that sequencing stops at the horizon.

2. **The holographic principle is natural.** Physicists have discovered that the information in any region of space can be encoded on its 2D boundary. This is mysterious if 3D space is fundamental. It's obvious if 3D space is reconstructed from 2D structures.
3. **It's testable.** The framework makes specific predictions about correlations in quantum systems that differ from what you'd expect if depth were fundamental. These can be tested in existing quantum simulators.

The One-Sentence Summary

Depth is to space what motion is to film: real, measurable, and law-governed—but emerging from temporal sequencing rather than existing independently.

Abstract (Technical)

This paper develops a rigorous mathematical framework supporting the claim that physical reality is not a pre-existing three-dimensional container, but a system of energy and information transmission. Building on our prior result that renormalisation depth fails all criteria for spatial directions (*Depth Is Not a Direction*), we here establish the positive thesis: apparent three-dimensional depth is reconstructed from temporal sequencing of two-dimensional state manifolds. The minimal consistent substrate is a sequence of 2D states, with depth emerging as a derived coordinate encoding structured inter-slice differences. We establish a no-go theorem eliminating container-space ontology, construct the transmission-first alternative, derive the Depth Reconstruction Theorem with an explicit reconstruction functional, and demonstrate consistency with relativistic causality, horizon physics, and holographic entropy scaling. The framework yields falsifiable predictions distinguishable from container-based theories through precision experiments and correlation analyses. A self-contained mathematical summary is provided in Appendix A.

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Section 1 — Introduction and Relation to Prior Work

1.1 The Central Claim

This paper argues that physical space is not a pre-existing three-dimensional container in which events occur, but an emergent structure reconstructed from temporally ordered information transmission. The apparent depth dimension—the third spatial coordinate we perceive—arises from structured differences between successive two-dimensional state configurations.

Remark (For General Readers): Imagine the universe not as a 3D box that was always there, but as a sequence of 2D "snapshots" that create the appearance of 3D structure when played in order—like how flat film frames create the appearance of motion.

1.2 Connection to Prior Work

In *Depth Is Not a Direction*, we established a negative result: renormalisation depth—the parameter indexing coarse-grained descriptions—fails all four necessary criteria for spatial directions:

- **C1 (Metric Structure)**: Depth lacks intrinsic, scheme-independent distances
- **C2 (Locality)**: Depth does not support tensor factorisation or local couplings
- **C3 (Propagation)**: No trajectories exist through depth; coarse-graining is re-description, not motion
- **C4 (Reversibility)**: Non-injective coarse-graining maps admit no inverses

The No-Go Theorem (Theorem A.4 of that work) established that any parameter governing non-injective coarse-graining cannot constitute a physical spatial direction.

Remark (For General Readers): In the previous paper, we showed what depth ISN'T—it's not a direction you can travel through like left-right or up-down. In this paper, we show what depth IS—something that emerges from time.

The present paper establishes the complementary positive result. Having shown what depth is *not* (a primitive spatial direction), we now propose what depth *is*: a derived coordinate emerging from temporal sequencing of lower-dimensional states.

1.3 Overview of the Argument

Note for readers: The full formal development is summarised in **Appendix A**, which provides the complete axiomatic skeleton, key definitions, and main theorems in compact form. Readers seeking a quick overview of the mathematical structure may wish to consult Appendix A first.

1. **Axioms (Section 2)**: We state explicit axioms governing operational observability, update structure, and ontological minimality.

2. **Mathematical Framework (Section 3)**: We establish precise definitions and typing for all mathematical objects.
3. **No-Go for Container Space (Section 4)**: We establish that pre-existing 3D space is inconsistent with the axioms.
4. **Transmission-First Ontology (Section 5)**: We construct the minimal ontology: temporally sequenced 2D state manifolds.
5. **Depth Reconstruction (Sections 6–7)**: We establish that depth emerges from inter-slice differences via an explicit reconstruction functional.
6. **Metric Emergence (Section 8)**: We establish that the reconstructed depth coordinate admits pseudometric structure.
7. **Uniqueness (Section 9)**: We establish that 2D + sequencing is uniquely minimal via a formal routing requirement.
8. **Applications (Sections 10–12)**: We demonstrate consistency with horizon physics and relativistic causality.
9. **Predictions (Section 13)**: We derive testable empirical signatures.

Section 2 — Axioms and Standing Assumptions

This section states the explicit axioms underlying all subsequent results. These axioms are not hidden assumptions; they are the foundation from which all theorems follow.

Remark (For General Readers): Before proving anything, we need to state our starting assumptions clearly. These are the "rules of the game"—if you accept these reasonable assumptions about physics and measurement, everything else follows logically.

2.1 Operational Axioms

Axiom A1 (Operational Observability)
Only quantities definable via finite measurement procedures are admissible as ontological primitives. Structures that cannot, even in principle, be accessed by any measurement are not fundamental.

Remark (For General Readers): If you can't measure it even in principle, it shouldn't be part of your fundamental description of reality. This is a methodological choice, not a claim about metaphysics.

Axiom A2 (Finite Discrimination)
There exists a minimum discrimination threshold $\varepsilon > 0$ such that state differences below ε are physically meaningless. Formally: if $D(s, s') < \varepsilon$ for distinguishability measure D, then s and s′ are operationally identical.

Remark (For General Readers): There's a limit to how precisely you can distinguish two states. Below that limit, they're effectively the same. This is related to quantum uncertainty and practical measurement limits.

Axiom A3 (Bounded Propagation)
There exists a finite bound $\kappa > 0$ such that structured differences propagate at most distance κ per update, where distance is measured using the intrinsic 2D adjacency metric on each slice (see Definition 3.1). Formally: the influence region $I_n(x)$ of any point x satisfies $I_n(x) \subseteq B\_d\Sigma(x, \kappa)$.

Remark (For General Readers): Nothing travels faster than a certain speed (like the speed of light). Information and influence spread at finite rates.

Axiom A4 (Ontological Minimality)
If two ontologies yield identical operational predictions for all admissible experiments, the ontology with fewer primitive structures is preferred.

Remark (For General Readers): Occam's Razor. Don't assume extra stuff exists if you don't need it to explain observations.

Axiom A5 (Update Objectivity)
The update operator $\mathcal{T}$ is objective: it does not depend on observer choice, though its *representation* may depend on foliation. Different observers using compatible foliations reconstruct equivalent physics.

Remark (For General Readers): The laws of physics are the same for everyone, even if different observers describe them differently.

2.2 Structural Axioms

Axiom A6 (Locality of Interactions)
The update operator $\mathcal{T}_n$ decomposes into local factors:

$$\mathcal{T}_n = \prod_i \mathcal{T}_{n,i}$$

where each $\mathcal{T}_{n,i}$ acts on a region of diameter at most κ (measured in the intrinsic 2D metric).

Remark (For General Readers): Physics is local—things interact with their neighbours, not with distant objects directly.

Axiom A7 (Causal Consistency)
If event E_1 causally influences event E_2, then E_1 occurs on a slice Σ_n with index $n \leq m$, where E_2 occurs on Σ_m. Causal order is consistent with sequence order.

Remark (For General Readers): Causes come before effects. The time-ordering of our slices respects causality.

Axiom A8 (Non-Triviality)
The universe is not trivial: there exist distinct states $s \neq s'$ such that $D(\mathcal{T}_n(s), \mathcal{T}_n(s')) \geq \varepsilon$ for some n.

Remark (For General Readers): Something actually happens. The universe isn't frozen or trivial.

Axiom A9 (Empirical Homogeneity and Isotropy)
On sufficiently large scales, the intrinsic metric $g\Sigma$ is statistically homogeneous and isotropic.

Remark: This axiom reflects an empirically observed property of the physical world, not a logical necessity of the framework. It is included because certain results (notably Theorem 11.2 on Lorentz structure) depend on it. All results depending on A9 are explicitly conditional on its validity in the regime under consideration.

Remark (For General Readers): On large scales, space looks the same everywhere (homogeneous) and in every direction (isotropic). This is observed to be true in our universe. Some of our theorems only hold when this is the case.

2.3 The Routing Requirement

Requirement R (Non-Blocking Parallel Routing)
There exist configurations with sufficiently separated event pairs (a, b) and (c, d) such that signal paths P: $a \to b$ and Q: $c \to d$ can propagate simultaneously without mutual obstruction. Formally: for generic quadruples satisfying $\min(d\Sigma(a,c), d\Sigma(a,d), d\Sigma(b,c), d\Sigma(b,d)) > 2\kappa$, we have $\mathrm{supp}(P) \cap \mathrm{supp}(Q) = \emptyset$.

Remark: R is stated existentially/generically rather than universally. This is sufficient to distinguish dimensional capacities while avoiding pathological cases on compact manifolds with topological constraints.

Remark (For General Readers): Two separate conversations should be able to happen at the same time without interfering. If you're talking to Alice and Bob is talking to Carol, your conversations shouldn't block each other (assuming you're far enough apart).

2.4 Clarification on Primitive Structures

Important: The 2D adjacency metric $g\Sigma$ on each slice is an operational primitive—it encodes "within-slice interaction cost" or neighbourhood structure. **No third spatial coordinate is assumed or smuggled in.** The metric $g\Sigma$ is intrinsic to each 2D slice; it does not reference any embedding space or pre-existing 3D structure.

Remark on the status of $g\Sigma$: A critic might object that we have merely replaced 3D container space with 2D container space plus sequencing—that $g\Sigma$ is doing substantial work (defining distances, neighbourhoods, the propagation bound κ). This is a fair point. The framework does accept $g\Sigma$ as residual primitive spatial structure. However, $g\Sigma$ is less ontologically problematic than a 3D metric for three reasons:

1. **Reduced degrees of freedom**: A 2D metric carries fewer independent components than a 3D metric (one metric function locally vs. three independent spatial directions). The ontological commitment is strictly smaller.

2. **Irreducibility**: $g\Sigma$ encodes the adjacency structure on which Lieb-Robinson-type bounds and locality are enforced—it is the minimal structure required for bounded propagation (A3) and local factorisation (A6). Unlike a 3D container, $g\Sigma$ cannot be further reduced while preserving physical law structure.
3. **Derivability prospect**: Within the broader VERSF programme, $g\Sigma$ itself may be derivable from more fundamental structure (entropy gradients, information density, or distinguishability relations). The present paper takes $g\Sigma$ as given; deriving it is future work. A 3D container, by contrast, has no clear path to derivation from informational principles.

The present framework thus represents genuine ontological progress: from 3D container (assumed, unexplained) to 2D slice metric (minimal, potentially derivable) plus sequencing (operationally defined).

Remark (For General Readers): We're not secretly assuming 3D space and then pretending to derive it. Each 2D slice has its own internal notion of "nearby" and "far apart," but this is purely 2D—there's no hidden third direction.

2.5 Clarification on Time

Sequencing is primitive; metric time is derived.

- The update index $n \in \mathbb{N}$ provides ordering (which update comes before which)
- No external time parameter t is assumed
- **Proper time** τ is an emergent measure defined by the update count of a designated clock subsystem
- The conversion between update counts and SI seconds is empirical, not fundamental

Remark (For General Readers): We don't assume a background clock ticking away. Time IS the sequencing of updates. When we later talk about "seconds" or "proper time," those are derived from counting how many updates a particular physical clock undergoes.

2.6 Summary of Axiom Roles

Axiom	Role	Used In
A1	Grounds ontology in measurement	Lemma 4.1, Theorem 4.4
A2	Defines coarse-graining threshold	Definition 7.2, Theorem 7.9
A3	Bounds information propagation	Theorem 5.5, Theorem 8.7
A4	Eliminates redundant structure	Theorem 4.4, Theorem 9.5
A5	Ensures observer-independence	Theorem 12.2
A6	Ensures locality	Theorem 5.5
A7	Grounds causal structure	Theorem 10.2
A8	Prevents triviality	Proposition 7.6
A9	Empirical symmetry (conditional)	Theorem 11.2

Axiom	Role	Used In
R	Constrains dimensionality	Theorems 9.1–9.5

2.7 Axiom Independence and Non-Engineering

A potential concern in axiomatic approaches is that assumptions may be reverse-engineered to yield a desired conclusion. We address this concern explicitly.

The axioms were not chosen to enforce a two-dimensional substrate or to exclude three-dimensional space. Rather, they encode independently motivated operational commitments that are widely accepted across physics:

- Finite measurability (A1)
- Bounded propagation (A3)
- Locality (A6)
- Causal consistency (A7)
- Ontological minimality (A4)

None of these axioms specifies dimensionality, geometric structure, or the existence of a reconstruction functional.

Indeed, several of the axioms are *weaker* than assumptions commonly made in quantum field theory, general relativity, or information-theoretic reconstructions, where spacetime dimension is typically presupposed rather than derived.

The dimensional conclusions of this paper arise only after combining these operational constraints with explicit topological and informational arguments (Sections 3.5 and 9). They are therefore *consequences* of the axioms, not inputs to them.

Remark (For General Readers): We didn't design the axioms to get 2D as an answer. The axioms are standard physics principles (things can be measured, signals have finite speed, causes precede effects). The 2D conclusion comes from working through what these principles actually imply.

Section 3 — Mathematical Framework

This section establishes precise definitions with explicit typing for all mathematical objects.

Remark (For General Readers): This section defines our mathematical vocabulary precisely. You can skim it on first reading and return when specific terms come up later.

3.1 Primitive Structures

Definition 3.1 (State Manifold)
A state manifold is a pair $(\Sigma, g\Sigma)$ where:

- Σ is a compact 2-dimensional smooth manifold (or 2D CW-complex for discrete models)
- $g\Sigma$ is an intrinsic Riemannian metric encoding within-slice adjacency/interaction cost
- $d\Sigma: \Sigma \times \Sigma \to \mathbb{R}_{\geq 0}$ is the induced geodesic distance function

Critical clarification: The metric $g\Sigma$ is an operational primitive reflecting 2D neighbourhood structure. It does not assume or require any embedding in higher-dimensional space. No third spatial coordinate is assumed.

Remark (For General Readers): Think of Σ as a 2D surface (like a sheet or a sphere) where each point represents a possible "location" in that instant. The metric tells you how to measure distances on this surface—purely within the 2D surface, with no reference to any third direction.

Definition 3.2 (Configuration Space)
The configuration space $S(\Sigma)$ is:

- *Classical*: The space of admissible field configurations on Σ, with suitable topology
- *Quantum*: The space $\mathcal{D}(\mathcal{H})$ of density operators on Hilbert space $\mathcal{H}$, i.e., $\mathcal{D}(\mathcal{H}) = \{\rho \in \mathcal{B}(\mathcal{H}) : \rho \geq 0, \mathrm{Tr}(\rho) = 1\}$

Remark (For General Readers): This is the set of all possible "states" the universe could be in at a given moment.

Definition 3.3 (Distinguishability Measure)
A distinguishability measure $D: S \times S \to \mathbb{R}_{\geq 0}$ satisfies:

- $D(s, s') = 0$ if and only if $s = s'$
- $D(s, s') = D(s', s)$
- Triangle inequality: $D(s, s'') \leq D(s, s') + D(s', s'')$

Standard choices:

- *Classical*: Total variation distance $D\_TV(\mu, \nu) = \sup_a |\mu(A) - \nu(A)|$
- *Quantum*: Trace distance $D\_Tr(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1$

Remark (For General Readers): This measures how different two states are. Zero means identical; larger numbers mean more different.

Definition 3.4 (Update Operator)
An update operator $\mathcal{T}_n: S(\Sigma_n) \to S(\Sigma_{n+1})$ is:

- *Classical*: A measurable, continuous map
- *Quantum*: A completely positive trace-preserving (CPTP) map

Remark (For General Readers): This is the "rule" that takes the current state and produces the next state. It encodes all the laws of physics.

Definition 3.5 (Influence Region)
The influence region $I_n(x)$ of a point $x \in \Sigma_n$ is the minimal set such that:

If s_n and s'_n differ only outside $B\_d\Sigma(x, \delta)$ for arbitrarily small $\delta > 0$, then $\mathcal{T}_n(s_n)$ and $\mathcal{T}_n(s'_n)$ can differ only within $I_n(x)$.

Equivalently (in the quantum case, via Lieb-Robinson bounds): $I_n(x)$ is the region where observables can be affected after one update by a perturbation localised at x.

By Axiom A3: $I_n(x) \subseteq B\_d\Sigma(x, \kappa)$.

Remark: This makes Axioms A3 and A6 mathematically precise. The influence region captures the operational content of "bounded propagation."

3.2 Composite Structures

Definition 3.6 (Transmission-First Universe)
A transmission-first universe is a tuple:

$\mathfrak{U} = (\{\Sigma_n\}_{n \in \mathbb{N}}, \{g_n\}, \{S_n\}, \{\mathcal{T}_n\}, \kappa, \varepsilon)$

satisfying Axioms A1–A8.

Definition 3.7 (Signal Path)
A signal path from $a \in \Sigma_n$ to $b \in \Sigma_{n+k}$ is a sequence $P = (p_0, p_1, ..., p_k)$ with:

- $p_0 = a$, $p_k = b$
- $p_{i+1} \in I_{n+i}(p_i)$ for all i

Remark (For General Readers): A signal path is the route that information or influence takes from one event to another across multiple time-steps.

Definition 3.8 (Path Support)
The support of path $P = (p_0, ..., p_k)$ is:

$\mathrm{supp}(P) := \bigcup_{i=0}^{k-1} B\_d\Sigma(p_i, \kappa)$

Remark (For General Readers): The "support" is the region of space that the signal path passes through.

Definition 3.9 (Clock Subsystem)
A clock subsystem is a distinguished degree of freedom whose update count defines proper time:

$\tau := N\_clock \cdot \Delta\tau_0$

where N_clock is the number of updates undergone by the clock and $\Delta\tau_0$ is a calibration constant (determined empirically by comparison with SI standards).

Remark (For General Readers): A clock is just a physical system we use to count updates. "Proper time" is how many updates YOUR clock has experienced.

3.3 Notation Summary

Symbol	Type	Description
Σ, Σ_n	Compact 2-manifold	State manifold at update n
$g\Sigma$	Riemannian metric	Intrinsic 2D metric (no 3rd dimension)
$d\Sigma$	$\Sigma \times \Sigma \to \mathbb{R}_{\geq 0}$	Geodesic distance on Σ
$S(\Sigma)$	Topological space	Configuration space
$\mathcal{D}(\mathcal{H})$	Convex set	Density operators (quantum)
$\mathcal{T}_n$	$S_n \to S_{n+1}$	Update operator
D	$S \times S \to \mathbb{R}_{\geq 0}$	Distinguishability measure
ε	$\mathbb{R}_+$	Discrimination threshold (A2)
κ	$\mathbb{R}_+$	Propagation bound per update (A3)
$I_n(x)$	$\mathcal{P}(\Sigma_{n+1})$	Influence region of x
$R_0 \to N$	$S_0 \to S_n$	Cumulative update
$\mathcal{F}$	$\text{Traj}/\sim \to (\Sigma \times \mathbb{R}_{\geq 0}, g)$	Reconstruction functional
z	$S_0 \times S_0 \to \mathbb{N} \cup \{\infty\}$	Depth function
τ	$\mathbb{R}_{\geq 0}$	Proper time (emergent)
n	$\mathbb{N}$	Update index (primitive)

3.4 Operational Status of the Two-Dimensional Substrate

A natural objection is that if the experienced world is three-dimensional, then a two-dimensional state manifold Σ must be "less real" than 3D space. This is a category error. In the present framework, "real" means ontologically primitive and operationally accessible (Axiom A1), not "what feels volumetric."

Remark (For General Readers): You might think: "But the world LOOKS 3D, so how can 2D be more fundamental?" This section explains why 2D is the real foundation, even though 3D is what we experience.

3.4.1 Operational Reality Criterion

Definition 3.10 (Operational Reality): A structure X is physically real (primitive) if and only if:

1. There exist admissible measurement procedures whose outcomes are functions of X, and
2. Alterations to X induce changes in operational predictions that cannot be reproduced by altering only derived structure.

By this criterion, the 2D slice Σ is real and primitive; depth is real but derived.

3.4.2 Why Σ Is Real

On each update n, the slice Σ_n is the domain on which all primitive observables are defined:

- Event registration occurs at locations $x \in \Sigma_n$
- Local interactions are supported on bounded neighbourhoods B(x, κ) (Axiom A3)
- The update operator factorises into local components acting on regions of Σ_n (Axiom A6)
- Conservation constraints and admissibility conditions operate on the slice configuration $s_n \in S(\Sigma_n)$

In particular, locality and bounded propagation are constraints *on Σ*, not on the reconstructed depth coordinate.

Σ is not an interpretive projection. It is the minimal arena on which physical laws act.

3.4.3 Depth Is Real but Non-Primitive

Depth is operationally real in the sense that observers measure stable 3D distances and navigate volumes. However, depth fails the operational reality criterion for *primitiveness*:

- No depth coordinate exists on any single Σ_n (Lemma 6.1)
- Depth is constructed only from inter-slice structure via $\mathscr{F}$ (Theorem 6.4)
- Different reconstruction conventions yield depth coordinates related by monotone reparametrisation (Theorem 7.8(iii))

Therefore, **depth is real but derived**: it has invariant, measurable consequences, but it is not a primitive structure.

Remark (For General Readers): Depth is like velocity. Velocity is completely real—you can measure it, it has physical effects, cars actually go fast. But velocity doesn't exist in a single instant; it's reconstructed from comparing positions at different times. Depth is the same: real, but reconstructed from comparing slices.

3.4.4 The "Realness" Asymmetry Is Standard in Physics

This asymmetry is not exotic. Many entities are real but derivative:

Entity	Status	Derived From
Temperature	Real but derived	Microstate statistics
Pressure	Real but derived	Momentum flux

Entity	Status	Derived From
Velocity	Real but derived	Ordered position differences
Spacetime curvature	Real but derived	Geodesic deviation, signal timing
Depth	Real but derived	Inter-slice differences

The present framework makes the same move one level deeper: it treats depth the way physics already treats velocity.

3.4.5 Consequence: "Container Space" Is the Wrong Intuition

Because Σ is where causal and conservation constraints are enforced, the natural intuition of "objects moving through a 3D container" is reversed:

- **Wrong intuition**: 3D space exists; 2D slices are projections of it
- **Correct picture**: 2D slices exist; 3D space is reconstructed from their sequence

What exists fundamentally is a sequence of constrained configurations on Σ; the appearance of a third dimension is the stable projection of that sequence.

Conclusion: The 2D substrate is physically real because it carries the primitive event structure, locality constraints, and lawful update dynamics. The third spatial coordinate is physically real only as a reconstruction induced by sequencing. The framework does not deny reality of 3D experience; it denies primitiveness of 3D container space.

Lemma 3.15 (Primitive-Law Locus)
Hypotheses: Axioms A3 (bounded propagation) and A6 (locality) are stated in terms of Σ_n and not in terms of reconstructed coordinates.
Claim: Σ_n is the unique locus where the fundamental laws are applied primitively.

Proof:

1. By A6, $\mathcal{T}_n = \prod_i \mathcal{T}_{n,i}$ where each factor acts on a region of Σ_n.
2. By A3, influence regions $I_n(x)$ are defined relative to the metric dΣ on Σ_n.
3. The depth coordinate z is constructed *from* the action of $\mathcal{T}$ (Definition 7.5), not vice versa.
4. Therefore, the laws (encoded in $\mathcal{T}$) act on Σ and produce z, not on z directly.
5. Σ_n is the primitive arena; z is derived. ■

Remark: This lemma converts the intuitive claim "2D is where the laws act" into a theorem-adjacent form.

On the primitiveness of the intrinsic metric. The present framework treats the intrinsic two-dimensional metric gΣ as primitive. This choice is not meant to suggest that the metric is metaphysically fundamental, but rather that it is the *primitive locus of law application* within the scope of this paper. All constraints on locality, bounded propagation, and update factorisation are stated in terms of $g\Sigma$.

Importantly, the core results of this paper—the no-go theorem for container space and the reconstruction of depth—do not depend on the specific origin or detailed structure of $g\Sigma$, only on its existence as an intrinsic adjacency structure. Any future derivation of $g\Sigma$ from deeper informational or entropic principles would therefore *refine*, but not *undermine*, the conclusions established here.

Furthermore, the conclusions of Sections 4–9 are invariant under any replacement of $g\Sigma$ by a metric-free adjacency structure with bounded degree; no smoothness or continuum properties of $g\Sigma$ are required. The framework is therefore not secretly relying on differential geometry.

3.5 Why Two Dimensions Are Required for Relational Information

The claim that the instantaneous state manifold Σ is two-dimensional is not merely a convenience. It follows from the requirements of relational information itself. Below two dimensions, relational structure collapses; above two dimensions, relational capacity is redundant.

Remark (For General Readers): This section gives a deep reason for "why 2D specifically?"—it's because 2D is the minimum needed for relationships between things to be non-trivial.

3.5.1 What Is Meant by Relational Information

Definition 3.11 (Relational Information): A system carries relational information if it can encode:

1. More than pairwise ordering (i.e., non-linear adjacency)
2. Multiple independent relations simultaneously
3. Distinctions that are not reducible to a single total order

Relational information is the minimal requirement for:

- Locality
- Neighbourhood structure
- Interference
- Contextual dependence

A universe without relational information cannot support physical law beyond trivial sequencing.

Definition 3.11′ (Relational Primitive): A relational primitive is any property of configurations and dynamics that is definable solely from:

1. Within-slice adjacency/interaction structure (the metric neighbourhood relation on Σ_n)
2. Bounded propagation (A3)
3. Causal sequencing (A7)

A property is a relational primitive if and only if it is invariant under any transformation that preserves these three relations.

Assumption (Adjacency-Determined Dynamics): The local factors $\mathcal{T}_{n,i}$ depend only on intrinsic neighbourhood structure on Σ_n (as specified in A3/A6), not on any ambient embedding data.

Remark: This assumption is consistent with the framework as already stated: $g\Sigma$ is intrinsic to each slice, and no embedding space is assumed (§2.4).

Definition 3.11″ (Operationally Closed Dynamics): A dynamics on the accessible state space S_n is *operationally closed* if the update $\mathcal{T}_n$: $S_n \rightarrow S_{n+1}$ is injective on operational equivalence classes (equivalently, reversible on accessible degrees of freedom). In the quantum setting, operational closure is ensured if the accessible evolution is unitary (or isometric with a recoverability map) on the accessible algebra.

Definition 3.11‴ (Reservoir / Hidden Degrees of Freedom): A *reservoir* is any extension $S_n \hookrightarrow \tilde{S}_n$ such that the induced effective evolution on S_n is non-injective (information-losing) because distinct microstates in $\tilde{S}_n$ map to the same operational state in S_{n+1}.

Lemma 3.11A (Operational Irreversibility Requires a Reservoir)
Claim: If the effective update $\mathcal{T}_n$: $S_n \rightarrow S_{n+1}$ is not injective (fails operational closure), then there exists an extended state space $\tilde{S}_n$ and an injective (reversible) micro-update $\tilde{\mathcal{T}}_n$: $\tilde{S}_n \rightarrow \tilde{S}_{n+1}$ such that the effective evolution on S_n is obtained by coarse-graining/restriction from $\tilde{S}_n$.

Proof sketch: This is the standard Stinespring/Naimark-type logic in quantum theory (CPTP maps arise from unitary evolution on a larger space with an environment traced out), and the analogous embedding exists in classical dynamics (non-injective maps arise from projecting an injective map on a larger phase space). The key observation is: irreversibility = projection from a larger reversible system. ■

Theorem 3.11B (Reversible Adjacency-Determined Dynamics Imply 2D Sufficiency)
Hypotheses:

1. Adjacency-determined locality: $\mathcal{T}_n$ depends only on intrinsic neighbourhood structure on Σ_n (A3, A6)
2. Operational closure (reversibility): $\mathcal{T}_n$ is injective on operational states (or unitary/isometric with recovery on the accessible algebra)

Claim: Under (1)–(2), any operational relational primitive (Definition 3.11′) is representationally complete on a two-dimensional substrate: higher-dimensional instantaneous substrates provide no additional necessary degrees of freedom for fundamental dynamics.

Proof:

1. Under operational closure, no information is lost from the accessible structure across updates. Hence there is no need for an auxiliary reservoir to store "missing" relational degrees of freedom.
2. Because the dynamics are adjacency-determined, all causal and interaction structure is fully encoded in intrinsic neighbourhood relations on Σ_n.
3. As shown in Theorem 3.14 (Relational Completeness of 2D), any finite such adjacency-and-update structure admits a representation on a 2D CW-complex preserving locality, bounded propagation, and routing.
4. Therefore, a two-dimensional substrate suffices to realise all primitive relational content of reversible fundamental dynamics. ■

Corollary 3.11C (Apparent Higher-Dimensionality Tracks Coarse-Grained Irreversibility)
If an observer's effective dynamics are non-injective (irreversible) due to coarse-graining over inaccessible degrees of freedom, then additional emergent structure may be required to parameterise the resulting entropy production and information loss. In this case, reconstructed volumetric depth can be interpreted as an *emergent bookkeeping coordinate* for the growth of inaccessible microstructure.

Conversely, at the fundamental reversible level, no such additional bookkeeping dimension is required.

Remark (For General Readers): This says something profound: if the fundamental laws are reversible (as quantum mechanics suggests), then 2D is enough. The apparent third dimension emerges when we coarse-grain—when we lose track of microscopic details. Depth is a way of accounting for what we've lost track of.

3.5.2 Why One Dimension Is Insufficient

In a one-dimensional manifold, all points are totally ordered.

Lemma 3.12 (Relational Collapse in 1D)
Hypotheses: Σ is a connected 1D manifold.
Claim: All relational structure reduces to linear order.

Proof: Let Σ be a connected 1D manifold (interval or circle). For any three distinct points a, b, c $\in \Sigma$, their relations are completely characterised by ordering along the manifold. There exists no notion of "around," "across," or "independent adjacency." Any neighbourhood separates Σ into at most two regions. Thus all relations are reducible to before/after or left/right. ■

Consequences of 1D relational collapse:

- No nontrivial locality beyond adjacency
- No independent routing of influence
- No interference structure

- No contextual relations

This is not merely a geometric limitation; it is an **information-theoretic collapse**. One-dimensional systems cannot encode relational degrees of freedom without embedding in higher dimensions.

Remark (For General Readers): In 1D, everything is just "before" or "after" on a line. You can't have two things be "next to each other but in different directions"—there's only one direction. That's too simple for real physics.

3.5.3 Why Two Dimensions Are Sufficient

Two dimensions are the minimal dimension in which relations are no longer totally ordered.

Lemma 3.13 (Emergence of Non-Linear Relational Structure in 2D)
Hypotheses: Σ is a 2D manifold.
Claim: There exist triples of points whose relations are not reducible to a total order.

Proof: Let Σ be a 2D manifold. For three points a, b, c, their relations require at least two coordinates to specify relative position. There is no global linear order compatible with all neighbourhood relations. Local adjacency graphs admit cycles, crossings, and angular structure. ■

New relational capacities in 2D:

- Independent neighbourhoods
- Angular relations
- Non-blocking paths
- Contextual adjacency
- Interference without annihilation

These are exactly the structures required by Axioms A3 (bounded propagation), A6 (locality), and Requirement R (non-blocking routing).

Remark (For General Readers): In 2D, you can have "north," "south," "east," and "west." Things can be neighbours in multiple independent ways. You can go around obstacles. That's enough for interesting physics.

3.5.4 Why Higher Dimensions Are Not Required

While dimensions greater than two also support relational information, they do not introduce new primitive relational capacity beyond what is already present in 2D.

Theorem 3.14 (Relational Completeness of Two Dimensions)
Hypotheses: Axioms A1–A8. Assume adjacency-determined dynamics. Let the instantaneous substrate be any dimension $d \geq 2$, and let all operational primitives and dynamics be definable

from intrinsic neighbourhood relations, bounded propagation, and sequencing (Definition 3.11′, A3, A7).
Claim: For any operationally realisable (finite-entropy, finite-extent) configuration of events and interactions satisfying A1–A8 and realised on a d-dimensional substrate, there exists a two-dimensional CW-complex $\Sigma^{(2)}$ and an update rule $\mathcal{T}^{(2)}$ such that all relational primitives (in the sense of Definition 3.11′) are preserved. In particular, higher dimensions introduce no new relational primitives beyond those already representable in two dimensions.

Proof:

1. Fix any finite operational instance: a finite set of events and interaction localities at some update n, together with the admissible influence relations implied by A3 and the locality factorisation implied by A6. This operational instance induces a finite adjacency structure: a graph G_n whose vertices are the event-locations and whose edges indicate neighbourhood influence relations relevant to one-step updates.
2. By the hypothesis that relational primitives depend only on adjacency + bounded propagation + sequencing (Definition 3.11′), all relational facts in the instance are functions only of this induced graph structure (and its iteration under $\mathcal{T}_n$).
3. Construct a two-dimensional CW-complex $\Sigma_n^{(2)}$ (the superscript indicating target dimension 2) whose 1-skeleton is exactly G_n. (Every finite graph is the 1-skeleton of some 2D CW-complex; attach 2-cells along cycles as needed to obtain a compact surface or surface-with-boundary. No bounded-degree restriction is required for the embedding—the bounded degree in our setting arises from A3/A6, not from the embedding theorem.)
4. Define $\mathcal{T}_n^{(2)}$ to act locally on the same neighbourhoods (graph balls of radius corresponding to κ) with the same update rule on the corresponding local factors.
5. Because the construction preserves the adjacency relation, the bounded influence relation, and the sequencing index, every relational primitive is preserved by construction.
6. The construction extends coherently across the sequence of updates: for each n, we construct $\Sigma_n^{(2)}$ from G_n, and the update $\mathcal{T}_n^{(2)}: \Sigma_n^{(2)} \rightarrow \Sigma_{n+1}^{(2)}$ is defined by the same local rules as the original $\mathcal{T}_n$, preserving causal sequencing.
7. Therefore, any relational capacity available in dimension $d \geq 2$ is representable in a two-dimensional substrate. Hence, dimensions > 2 do not add new relational primitives in the operational sense used here. ■

Remark on non-triviality: A potential objection is that this theorem is circular—we defined primitives to include only adjacency relations, then proved adjacency is 2D-embeddable. The non-triviality lies in recognising that *physical* locality constraints (A3, A6) naturally yield only adjacency-type relations. The theorem establishes that this physical input has a geometric consequence regarding minimal dimensionality. The definition of "relational primitive" is not chosen to make the theorem true; it is chosen to capture what physical law actually requires.

Remark (Characterisation theorem status): Theorem 3.14 should be understood as a characterisation theorem: "If physics only cares about adjacency-type relations (as axiomatised by A1–A8), then 2D suffices." The physical content lies in justifying the antecedent—that is, in arguing that A1–A8 correctly capture the operational structure of physical law. The theorem then derives the geometric consequence. A concrete example of a property that *would* require higher dimensions is global knot invariants: whether a closed curve is knotted depends on the ambient 3D space, not merely on adjacency relations within the curve. Such properties fail to qualify as relational primitives under Definition 3.11′ because physical dynamics (A3, A6) do not depend on them—the evolution of a local system is determined by its neighbourhood structure, not by global embedding topology.

Remark on finiteness: The theorem explicitly requires finite configurations. This restriction is physically motivated: any operationally realisable experiment involves finitely many events. Infinite graphs can carry genuinely higher-dimensional topological information (e.g., infinite families of disjoint cycles), but such configurations are not operationally accessible under A1.

Remark on geometric properties: Features such as global knotting, chirality, or the existence of three mutually orthogonal directions are geometric properties of embeddings; they are not relational primitives under the present framework unless the update rule $\mathcal{T}$ is explicitly allowed to depend on embedding data. Under adjacency-determined dynamics, such geometric richness is representational rather than operational.

Remark (For General Readers): This theorem says that anything physics needs to do—local interactions, bounded propagation, causal ordering—can be done on a 2D surface. Higher dimensions might give you more geometric options (like three perpendicular directions), but those aren't needed for the fundamental operations of physical law.

Physical significance: This result suggests that three-dimensional space, as experienced, may be a representational structure rather than a fundamental feature of physical law. If the axioms A1–A8 correctly capture the operational constraints on physical systems, then the apparent 3D-ness of space is not primitive but emergent—a consequence of how information is organised and transmitted, not a pre-existing container in which physics occurs. This aligns with holographic intuitions from black hole physics but arrives there from completely different premises: information-locality principles rather than gravitational thermodynamics.

3.5.5 Relational Information vs. Metric Volume

It is important to distinguish relational richness from volumetric intuition.

Aspect	Volumetric Space	Relational Space
Phenomenological feel	"Richer"	Simpler
Informational content	Derived	Primitive
Status	Reconstructed	Fundamental

The two-dimensional substrate is where:

- Adjacency is defined
- Locality is enforced
- Causal influence is constrained
- Conservation laws apply

Depth adds scale and extension, but not new relational primitives.

3.5.6 Consequence: Why Σ Must Be Two-Dimensional

Theorem 3.16 (Minimal Relational Substrate)
Hypotheses: Axioms A1–A8 and Requirement R.
Claim: The instantaneous state manifold Σ must be at least two-dimensional to support non-degenerate relational information, and need not exceed two dimensions to do so.

Proof:

1. One-dimensional substrates collapse relational structure (Lemma 3.12).
2. Two-dimensional substrates support irreducible relational structure (Lemma 3.13).
3. Higher dimensions add no primitive relational capacity under A4 (Theorem 3.14).

Therefore, Σ must be two-dimensional. ■

3.5.7 Interpretation

The two-dimensionality of Σ is not an aesthetic choice. It is the minimal dimensionality required for relations themselves to exist as independent informational facts.

Depth then emerges as a measure of relational persistence across updates, not as an additional relational axis.

One-sentence summary: Two dimensions are required because they are the lowest-dimensional substrate in which relations are not totally ordered; below two dimensions relational information collapses, and above two dimensions it becomes redundant.

Section 4 — No-Go Theorem for Container Space

We establish that pre-existing 3D container space is inconsistent with Axioms A1–A8.

Remark (For General Readers): This section proves that if you accept our axioms (which are quite reasonable), you cannot have a fundamental 3D space that exists independently of measurement and time.

4.1 Container-Space Axioms

The container-space ontology assumes:

Axiom C1: There exists a 3-dimensional manifold M_3 whose points exist independently of physical processes.

Axiom C2: Physical systems occupy locations $x \in M_3$ at each time.

Axiom C3: Time parametrises evolution within M_3.

Remark (For General Readers): This is the intuitive view—space is a 3D box, and stuff moves around inside it over time. We're about to show this view is inconsistent with operational physics.

4.2 Operational Inaccessibility

Lemma 4.1 (Operational Inaccessibility)
Hypotheses: Axioms A1, A3.
Claim: No measurement procedure can access points of M_3 independently of signal transmission and update counting.

Proof:

1. By A1, only quantities accessible via finite measurement are primitive.
2. Any position measurement requires signal exchange: emission at update n_1, propagation, reception at update n_2.
3. By A3, propagation is bounded by κ per update.
4. The inferred distance is: $d = \kappa(n_2 - n_1)/2$ (in units where distance is measured per update).
5. Therefore, spatial location is derived from update counts and the propagation bound, not directly observed.
6. The points of M_3 are not independently accessible. ■

Remark (For General Readers): You can't measure "where something is" without sending and receiving signals. But signals take updates to propagate. So every position measurement is really an update-count measurement in disguise. Space is always inferred from update counting, never measured directly.

Corollary 4.2
Hypotheses: Axioms A1, A3, A4.
Claim: M_3 introduces structure without operational counterpart; by A4, it is ontologically redundant.

4.3 Horizon Inconsistency

Theorem 4.3 (Horizon Inconsistency)
Hypotheses: Axioms C1–C3, A1–A4, A7.
Claim: Container-space ontology predicts physically meaningful volumetric degrees of freedom that are operationally inaccessible at causal horizons.

Proof:

1. Under C1–C3, the interior volume V of region $R \subset M_3$ exists independently of observability.
2. At a causal horizon H, signals from int(R) do not reach exterior observers in finite update count (definition of horizon + A7).
3. By A3, no information from int(R) contributes to exterior state updates.
4. Therefore, int(R) has no operational consequences for exterior observers.
5. Yet C1 asserts int(R) exists with volumetric degrees of freedom.
6. This contradicts A1: the interior introduces structure without measurement access.
7. Empirically, black hole entropy scales as $S \propto$ Area(H), not Volume(int(R)), confirming that interior volume does not carry independent degrees of freedom. ■

Remark (For General Readers): Here's the black hole puzzle. If 3D space is fundamental, then the inside of a black hole should have a huge volume with lots of "degrees of freedom" (ways to arrange stuff). But black hole physics tells us that all the information is on the 2D surface (the event horizon), not in the 3D interior. The container-space view can't explain this; our framework can.

Remark on comparison with standard holography: Conventional physics (Bekenstein-Hawking, AdS/CFT) also arrives at $S \propto$ Area, treating it as a deep constraint on fundamental spacetime. The difference is explanatory: standard holography says "spacetime is fundamental but holographically constrained" (why?), while our framework says "spacetime is derived from 2D structure, so area-scaling is expected" (because information lives on surfaces by construction). Both are consistent with $S \propto$ Area, but our framework explains *why* the constraint holds. Prediction 1 (correlation asymmetry ratio = 2) provides an empirical test distinguishing the two: it follows from our coarse-graining structure but is not predicted by holographic constraints alone.

4.4 No-Go Theorem

Theorem 4.4 (No-Go for Container Space)
Hypotheses: Axioms A1–A4, A7.
Claim: No ontology satisfying these axioms can include a pre-existing 3D container manifold M_3 as primitive structure.

Proof: Combine Lemma 4.1 (operational inaccessibility), Corollary 4.2 (redundancy), and Theorem 4.3 (horizon inconsistency). Container space is: (a) operationally inaccessible (violates A1) (b) redundant (violates A4) (c) inconsistent with horizon physics Therefore, M_3 cannot be primitive under the stated axioms. ■

Remark (For General Readers): The 3D container view fails three tests: you can't measure it directly, it adds unnecessary complexity, and it can't explain black holes. It's out.

4.5 Additional Arguments Against Container Space

The no-go theorem above (Theorem 4.4) establishes that container space is incompatible with our operational axioms. Here we present eight additional arguments showing that container space faces tensions across multiple domains of physics.

Important framing: The arguments in this subsection are consistency constraints and cross-domain tensions rather than independent no-go theorems. They are included to demonstrate that the derivative picture aligns naturally with multiple known physical limits—information bounds, relativistic structure, quantum mechanics, and cosmology.

Ranking by rigor:

- *Established constraints* (derived from known physics): §4.5.1 (information bounds), §4.5.2 (simultaneity), §4.5.4 (entanglement), §4.5.5 (speed of light)
- *Suggestive connections* (plausible but requiring further development): §4.5.3 (vacuum energy), §4.5.6 (initial conditions), §4.5.7 (problem of now)
- *Speculative interpretation* (philosophical rather than physical): §4.5.8 (factuality problem)

The core argument of this paper does not depend on the speculative arguments; they are included for completeness and to indicate directions for future work.

4.5.1 The Information-Theoretic Argument

The Problem: A 3D container, taken literally, would require specifying the state at every point in the volume.

Consider naive voxel degree-of-freedom counting: even discretised at the Planck scale ($\ell_P \approx 1.6 \times 10^{-35}$ m), a single cubic metre contains approximately:

$N_{voxels} \approx (1 \text{ m} / \ell_P)^3 \approx 10^{105}$ voxels

If each voxel carried independent degrees of freedom, the information cost would scale as **volume**.

But the Bekenstein-Hawking bound tells us that the maximum information in any region scales with **surface area**, not volume:

$S_{max} = A / (4\ell_P^2)$

For a cubic metre, this gives roughly 10^{70} bits—vastly less than 10^{105}.

The Consequence: If all voxels in a 3D container were independent, the naive DOF count would be incompatible with holographic bounds. Therefore, the bulk degrees of freedom must be massively constrained or redundant.

The Resolution: If you tried to populate a 3D bulk with independent states, you would either:

1. Violate information bounds (impossible)
2. Force most of the volume into correlated/constrained states (making the independent bulk picture misleading)
3. Accept that the "container" is mostly redundant specification—structure without independent content

Option 3 is essentially the derivative-depth picture: the independent degrees of freedom live on surfaces, and volumetric appearance is reconstructed.

Conclusion: The derivative picture is *compatible* with finite information bounds. Container models can also be made compatible by imposing sufficient constraints, but then the "container" is no longer doing independent work—it becomes a redundant scaffold.

Remark (For General Readers): There's a maximum amount of information that can fit in any region of space, and that maximum is proportional to the surface area, not the volume. A naive 3D container with independent voxels would need more information to describe than physics allows. The math forces either massive redundancy or a surface-based picture.

4.5.2 The Simultaneity Problem

The Problem: A 3D container implies a privileged "now" slice—all spatial points existing simultaneously.

But special relativity forbids observer-independent simultaneity. Different reference frames disagree about which events are concurrent. A primitive 3D container requires picking a preferred foliation of spacetime, which relativity explicitly rules out.

The Resolution: The derivative picture sidesteps this entirely. The 2D substrate and its temporal sequencing can be local to each causal patch. There is no global "now" that needs to be metaphysically privileged—different observers simply use different compatible foliations (Section 12).

Remark (For General Readers): If 3D space exists "all at once," we have to say which events happen "at the same time." But Einstein showed there's no objective answer to that question—it depends on how fast you're moving. The 2D picture avoids this problem.

4.5.3 The Vacuum Energy Problem

The Problem: If space is a container, every point in the volume should contribute vacuum energy (zero-point fluctuations of quantum fields).

The naive QFT calculation gives a cosmological constant roughly 10^{120} times larger than observed—the worst prediction in the history of physics.

A Possible Resolution: If space is derivative, there is no empty container sitting there accumulating vacuum energy. Only the 2D surfaces that are actually instantiated as operational degrees of freedom would contribute. The bulk that "isn't really there" as primitive structure would not gravitate in the same way.

Remark: This framework suggests a route to reinterpreting vacuum energy contributions, because only operationally instantiated degrees of freedom contribute. A full treatment of the cosmological constant problem is beyond the scope of this paper; we flag this as suggestive, not demonstrated. The derivative picture removes one potential source of overcounting, but whether this is sufficient to resolve the vacuum energy problem requires detailed calculation.

Remark (For General Readers): Empty space should have energy. If you add up all the energy from every point in a 3D container, you get a number that's 10^{120} times too big. If the bulk isn't primitive structure, this overcounting may be avoided—but proving this rigorously is future work.

4.5.4 Entanglement and Non-Locality

The Problem: A container picture says spatially separated points are fundamentally distinct locations.

But entangled particles maintain correlations that do not respect this separation—correlations that hold regardless of distance and cannot be explained by local hidden variables (Bell's theorem).

A Reframing: In the derivative picture, what we call "spatial separation" is a projection artifact of the reconstruction. Entangled states might be adjacent or connected in the underlying 2D structure but project into apparent 3D distance. The "non-locality" of entanglement becomes a statement about the reconstruction map, not about faster-than-light influences.

Important clarification: This does not reinstate local hidden variables; Bell's theorem still holds. Rather, it changes the meaning of "spatial separation" in the effective description. The correlations are not explained by hidden variables but by the structure of the underlying 2D states before reconstruction.

Remark (For General Readers): Entangled particles stay correlated no matter how far apart they are in 3D. If 3D distance is fundamental, this is deeply mysterious. If 3D distance is reconstructed from something simpler, the "distance" between entangled particles may not mean what we thought.

4.5.5 The Speed of Light Problem

The Problem: In a container theory, you must impose a speed limit as an external constraint. Nothing about the container itself explains why information cannot propagate arbitrarily fast.

The Resolution: In the derivative picture, the speed of light emerges naturally as a bound on how much successive 2D states can differ (Axiom A3). If each update can only change local regions by a finite amount κ, then c falls out of the sequencing rules ($c = \kappa/\Delta\tau_0$) rather than being imposed on top.

Remark (For General Readers): Why can't anything go faster than light? The container view just has to assert this as a rule. The 2D-sequence view derives it: if each update can only change things by a limited amount, there's automatically a speed limit.

4.5.6 The Initial Conditions Problem

The Problem: A 3D container at the Big Bang requires specifying initial conditions across the entire spatial volume simultaneously—an enormous amount of coordinated information that must be given all at once.

The Resolution: The derivative picture requires only a 2D initial state. Complexity builds iteratively through temporal sequencing. The information cost of "starting the universe" drops by a dimensional factor.

Remark (For General Readers): To start a 3D universe, you need to specify everything everywhere at once. To start a 2D-sequence universe, you just need one 2D sheet. Much simpler.

4.5.7 The Problem of Now

The Problem: Container theories treat time as just another dimension—a "block universe" where past, present, and future all equally exist. But this makes the felt reality of "now" a complete mystery. Why is there a present moment that advances?

The Resolution: The derivative picture makes "now" fundamental. It is the current 2D configuration. Time is not a direction you can look along; it is the process that generates the next state. The present is privileged because it is where the update is happening.

Remark (For General Readers): In the block universe view, all of history exists frozen in 4D. But then why does it feel like time passes? In our view, "now" is real—it's the current slice, and time is the process of generating the next one.

4.5.8 The Factuality Problem

The Problem: A fact requires commitment—a definite outcome that excludes alternatives and remains stable for comparison. Where do facts live in a 3D container?

The Comparison Problem: Facts are relational ("A is greater than B"). In a 3D bulk, information is distributed throughout the volume. To compare, you must transport information across space. During transport, states evolve, decohere, entangle with the environment. The more volume to traverse, the harder it is to execute a clean comparison. And what counts as "at the same time" for the comparison? The bulk provides no answer.

The Landing Problem: This is the measurement problem in disguise. A quantum superposition must become a definite outcome. Where does it land? In a 3D container, the boundary between system and apparatus is arbitrary. You can always expand the system to include the apparatus, and then the combined system is in superposition. Every point is interior to some larger region. The bulk has no edge where definiteness occurs.

The Distinguishability Problem: Facts require distinguishable states, but information bounds (Section 4.5.1) concentrate distinguishability at surfaces. The bulk interior would be epistemically empty—indistinguishable states about which no facts can form.

The Resolution: On a 2D surface with temporal sequencing:

- All information coexists on the same sheet—comparison is local
- Each temporal update is a commitment event—the moment when the configuration stabilises
- The surface IS the locus of fact formation—definiteness is built into the structure
- Persistence comes from sequential commitment—each update references the previous

Facts form because the 2D configuration must be definite to serve as input to the next update.

Speculative interpretation: This structure is suggestive of a resolution to the measurement problem: if measurement is identified with temporal updating, then definiteness emerges from the sequential structure itself. However, this is a philosophical interpretation rather than a demonstrated solution. A rigorous connection to collapse models, decoherence, or quantum foundations would require further development. We flag this as a potentially fruitful direction, not a claim of completion.

Remark (For General Readers): For something to be a fact, it has to be definite and comparable to other facts. In a 3D bulk, there's no natural place where definiteness happens. On a 2D surface that updates sequentially, every update IS the moment when things become definite. Whether this fully solves the measurement problem is an open question, but the structure is suggestive.

4.5.9 Summary: The Container Is Problematic

Problem	Container Theory	Derivative Picture
Information bounds	Naive DOF count incompatible	Naturally satisfied
Simultaneity	Requires privileged foliation	Foliation-independent
Vacuum energy	Overcounting of DOF	Reduced DOF count
Entanglement	Fundamentally non-local	Locality is emergent
Speed of light	Externally imposed	Derived from updates
Initial conditions	Requires 3D specification	Only 2D needed
The present moment	Mysterious	Fundamental
Fact formation	No natural locus	Built into structure

Conclusion: Under Axioms A1–A8 and Requirement R, the derivative picture is the unique minimal option that avoids these tensions. This is a theorem-level conditional statement, not a claim of metaphysical monopoly: *given these operational axioms, the derivative picture is the unique minimal structure*.

Section 5 — Formal Transmission-First Ontology

Having eliminated container space, we construct the minimal ontology satisfying A1–A8.

Remark (For General Readers): Having ruled out the "3D container" view, we now build the alternative: a sequence of 2D states evolving in time, from which 3D appearance emerges.

5.1 The Minimal Structure

Definition 5.1 (Transmission-First Universe)
A transmission-first universe $\mathfrak{U}$ is a tuple satisfying Definition 3.6 and Axioms A1–A8.

Proposition 5.2
Hypotheses: Definition 5.1.
Claim: A transmission-first universe $\mathfrak{U}$ satisfies Axioms A1–A8.

Proof: By construction. A2 holds by inclusion of ε. A3 holds by inclusion of κ. A6 holds by requiring local decomposition of $\mathcal{T}$. A7 holds because sequence index provides causal ordering. A8 holds generically. A1, A4, A5 are satisfied because we include only operationally necessary, observer-independent structure. ■

5.2 Time as Sequencing

Definition 5.3 (Time)
Time is the ordering relation induced by the index n on $\{\Sigma_n\}$. The passage of time is the application of update operators $\mathcal{T}_n$.

Lemma 5.4 (No Timeless Change)
Hypotheses: Definition 5.1.
Claim: There is no notion of change within a single Σ_n.

Proof: Σ_n encodes a complete instantaneous configuration. Change requires comparison of configurations at distinct indices. Without applying $\mathcal{T}_n$, no new index is reached. ■

Remark (For General Readers): A single "frame" of reality is frozen. Change only happens when you go from one frame to the next. Time IS the sequencing of frames.

5.3 Clarification: Time as Primitive Sequencing

We accept that time is primitive in this framework—but *only as sequencing*, not as a geometric dimension or external parameter.

What time IS in this framework:

- An ordering of resolved state updates (the index n)
- The index labelling which configuration is "current"
- Primitive in the sense that sequencing is not derived from anything more fundamental

What time is NOT:

- A geometric dimension with metric structure (metric time is derived)
- A background parameter external to the physics
- A container in which updates "happen"

Proper time is emergent: $\tau = N\_clock \cdot \Delta\tau_0$ where N_clock is the update count of a clock subsystem and $\Delta\tau_0$ is a calibration constant.

Remark (For General Readers): Yes, time is fundamental here. But it's not a "dimension" like length or width. It's the process of updating—the fact that one state follows another. "Seconds" and "proper time" are derived from counting updates on a particular clock.

Why Sequencing Is an Acceptable Primitive

Why accept temporal sequencing but not spatial extension as primitive?

The distinction is operational rather than metaphysical. **Temporal sequencing is epistemically irreducible**: every physical measurement, comparison, or inference presupposes a before/after ordering. Even the act of distinguishing two outcomes requires an ordering relation—one outcome must be registered prior to another.

Spatial extension, by contrast, is never accessed directly. All spatial information is inferred via signal exchange involving emission, propagation, and reception. As shown in Lemma 4.1, spatial distances reduce operationally to update counting combined with a propagation bound. Spatial extension is therefore always *mediated* by sequencing.

Sequencing is thus epistemically prior: it is required to define measurement itself. **Spatial extension is epistemically derivative**: it is reconstructed from how distinguishable differences propagate across sequences.

Treating sequencing as primitive while treating spatial extension as derived is therefore not an asymmetry of convenience. It reflects the asymmetric roles these notions play in operational physics: **sequencing is required to form facts, while spatial extension is reconstructed from them.**

Remark on operationalism: This argument grounds ontology in what is epistemically accessible. A hard-nosed realist might reject this as anthropocentric. We acknowledge: this framework IS operationalist in its foundations (Axiom A1). We regard this as a feature, not a bug. Operationalism provides a principled criterion for distinguishing primitive from derived structure. Those who reject operationalism may reject A1 and thereby the framework's conclusions—but they must then provide an alternative criterion for ontological primitiveness that doesn't reduce to "what feels intuitive."

5.4 Emergent Causal Cones

Theorem 5.5 (Causal Cone Emergence)
Hypotheses: Axioms A3, A6, A7; transmission-first universe $\mathfrak{U}$.
Claim: Influence from point $x \in \Sigma_0$ after N updates is contained in:

$$C_n(x) = \{y \in \Sigma_n : d\Sigma(y, x) \leq N\kappa\}$$

Proof: By induction on N.

- Base (N = 1): By A3, $I_0(x) \subseteq B\_d\Sigma(x, \kappa)$. ✓
- Inductive step: Assume $C_n(x) \subseteq B\_d\Sigma(x, N\kappa)$. For $y \in C_{n+1}(x)$, there exists $z \in C_n(x)$ with $y \in I_n(z)$. By A3, $d\Sigma(y, z) \leq \kappa$. By triangle inequality:

$$d\Sigma(y, x) \leq d\Sigma(y, z) + d\Sigma(z, x) \leq \kappa + N\kappa = (N+1)\kappa$$ ✓ ■

Remark (For General Readers): This proves that "light cones" emerge naturally. If information can only travel distance κ per update, then after N updates it can only have reached distance Nκ. That's exactly what we mean by a light cone—the region that could possibly be affected by an event.

This is the emergent light cone. The invariant speed c is defined by relating update counts to proper time (Section 8).

5.5 Why This Is Not Block Spacetime

A potential misreading of this framework is that it describes "block spacetime" with a preferred foliation. We explicitly deny this.

In block spacetime models:

- All slices Σ_n coexist timelessly
- Past and future are equally real
- Time is an illusion; only the 4D block exists

In the present framework:

- Only one Σ_n is physically realised per update

- Past slices have been updated *from*; future slices have not yet been generated
- Time is real as sequencing; the sequence has a "current" index
- The framework is *presentist* about update realisation, not eternalist

Remark (For General Readers): This is NOT the view that "all of time exists at once" (block universe). We're saying there's a real "now" that advances. Past states have been processed; future states don't exist yet. Time is the process of generating new states from old ones.

Section 6 — Depth Reconstruction Theorem

We establish that depth emerges from inter-slice differences.

Remark (For General Readers): Now we get to the key result. How does 3D depth emerge from 2D slices? The answer: from the differences between successive slices.

6.1 No Intrinsic Depth

Lemma 6.1 (No Intrinsic Depth)
Hypotheses: (Σ_n, g_n) is a 2D Riemannian manifold.
Claim: Σ_n contains no intrinsic third coordinate.

Proof: By definition, $\dim(\Sigma_n) = 2$. Any coordinate system has exactly two independent coordinates. A third cannot be defined intrinsically. ■

Remark (For General Readers): A single 2D slice is just 2D—you can't find a third direction within it. This is obvious but important.

6.2 Inter-Slice Differences

Definition 6.2 (Inter-Slice Difference)
For observable $f: S \to \mathbb{R}$, the inter-slice difference is:

$$\Delta_n f := f \circ \mathcal{T}_n - f : S_n \to \mathbb{R}$$

Lemma 6.3
Claim: The sequence $\{\Delta_n f\}_n$ encodes feature evolution under updates, constrained by $\mathcal{T}$.

Remark (For General Readers): The "inter-slice difference" measures how much a feature changes from one frame to the next. These differences are what we'll use to reconstruct depth.

6.3 Depth Reconstruction Theorem

Theorem 6.4 (Depth Reconstruction)
Hypotheses: Axioms A1–A8; transmission-first universe $\mathfrak{U}$.
Claim:
(i) No depth coordinate exists on any single Σ_n.
(ii) A depth coordinate z emerges from inter-slice differences via reconstruction functional $\mathscr{F}$.
(iii) Observers sharing update sequence $\{\mathcal{T}_n\}$ agree on reconstructed depth.

Proof: (i) Lemma 6.1. (ii) Theorem 7.8 constructs $\mathscr{F}$ explicitly. (iii) By A5, $\mathcal{T}$ is observer-independent. The reconstruction depends only on $\{\mathcal{T}_n\}$ and ε (both objective). ■

Remark (For General Readers): This is the main theorem. Part (i) says each frame is flat. Part (ii) says depth emerges from comparing frames. Part (iii) says everyone agrees on the result. Depth is real but derived.

Section 7 — The Reconstruction Functional: Explicit Construction

We construct $\mathscr{F}$ explicitly as a well-defined mapping between equivalence classes.

Remark (For General Readers): We now give the precise mathematical recipe for extracting depth from slice differences.

7.1 Cumulative Update

Definition 7.1 (Cumulative Update)
The cumulative update from slice 0 to N is:

$$R_{0\to n} := \mathcal{T}_{n-1} \circ \cdots \circ \mathcal{T}_0 : S_0 \to S_n$$

Type: $R_{0\to n}: S_0 \to S_n$

Remark (For General Readers): This is what you get by applying the update rule N times in a row. It takes you from the initial state to the state N steps later.

7.2 Equivalence Under Coarse-Graining

Definition 7.2 (Preliminary ε/2-Neighbourhood Relation)
States $s, s' \in S_0$ are N-proximate, written $s \approx_n s'$, if:

$$D(R_{0\to n}(s), R_{0\to n}(s')) < \varepsilon/2$$

where D is the distinguishability measure and ε the threshold from A2.

Remark on the ε/2 threshold: The choice of ε/2 (rather than ε or ε/3) is conventional. Any fixed fraction $\alpha \cdot \varepsilon$ with $0 < \alpha < 1$ would work; different choices yield depth functions related by monotone transformation (Theorem 7.9(iii)). We use ε/2 for concreteness and because it allows clean triangle-inequality arguments. The physical content is insensitive to this choice.

Definition 7.3 (N-Equivalence)
States $s, s' \in S_0$ are N-equivalent, written $s \sim_n s'$, if there exists a finite chain $s = s_0, s_1, ..., s_k = s'$ such that $s_i \approx_n s_{i+1}$ for all $i \in \{0, ..., k-1\}$.

That is, $\sim_n$ is the transitive closure of $\approx_n$.

Remark (For General Readers): Two starting states are "N-equivalent" if you can connect them by a chain of states that are each pairwise close after N updates. This ensures the equivalence relation is well-behaved mathematically.

Lemma 7.4 (Equivalence Relation)
Hypotheses: A2; D symmetric.
Claim: $\sim_n$ is an equivalence relation.

Proof:

- Reflexivity: $s \approx_n s$ since $D(R(s), R(s)) = 0 < \varepsilon/2$. Hence $s \sim_n s$ via the trivial chain. ✓
- Symmetry: If $s \sim_n s'$ via chain $(s_0, ..., s_k)$, then $s' \sim_n s$ via reversed chain $(s_k, ..., s_0)$, since D is symmetric. ✓
- Transitivity: If $s \sim_n s'$ via chain $(s_0, ..., s_k)$ and $s' \sim_n s''$ via chain $(s'_0, ..., s'_m)$, concatenate to get $s \sim_n s''$ via $(s_0, ..., s_k = s'_0, ..., s'_m)$. ✓ ■

Remark on chain length: For physically relevant state spaces satisfying reasonable regularity conditions (e.g., compactness and finite operational entropy), the length of chains required to establish N-equivalence is bounded. In particular, the maximum chain length is bounded above by the diameter of the state space under D divided by ε/2. Pathological cases requiring arbitrarily long chains correspond to non-physical, non-regular state spaces and are excluded by the operational assumptions of the framework.

Technical note: For infinite-dimensional quantum state spaces (e.g., continuous-variable systems), the finiteness of chain lengths requires an additional regularity assumption: that the accessible state space S_0 has finite diameter under D, or equivalently that the operationally accessible configurations span a finite-entropy subspace. This is implicit in A2 (finite resolution) and the physical requirement that accessible experimental configurations involve finite energy and entropy. Formally, one may add to A2: "The state space S(Σ) has finite diameter under D."

7.3 The Depth Function

Definition 7.5 (Distinguishability Depth)
For $s \neq s'$ in S_0:

$z(s, s') := \min\{N \in \mathbb{N} : s \sim_n s'\}$

with $z(s, s') = \infty$ if no such N exists.

Type: $z: S_0 \times S_0 \to \mathbb{N} \cup \{\infty\}$

Remark (For General Readers): The "depth" between two states is how many updates it takes before they become indistinguishable. If two features are very similar (shallow difference), they'll merge quickly. If they're very different (deep difference), it takes longer. This is how we measure depth.

Proposition 7.6
Hypotheses: A2, A3, A8.
Claim: $z(s, s')$ is well-defined and finite for generic distinct states.

Proof: By A3, information spreads boundedly. By A2, distinctions below ε vanish. By A8, non-trivial dynamics exist. For typical coarse-graining, local features eventually become indistinguishable. ■

7.4 The Reconstruction Functional

Definition 7.7 (Trajectory)
A trajectory is $\tau = (s_0, s_1, \ldots)$ with $s_{n+1} = \mathcal{T}_n(s_n)$. Let $\mathrm{Traj}(\mathfrak{U})$ denote trajectory space.

Definition 7.8 (Reconstruction Functional)
The reconstruction functional is:

$$\mathscr{F}: \mathrm{Traj}(\mathfrak{U})/\sim \; \to (\Sigma_0 \times \mathbb{R}_{\geq 0}, g\_eff)$$

defined by $\mathscr{F}([\tau]) = (x_0, z(\tau))$, where:

- $[\tau]$ is the equivalence class under $\sim$
- $x_0 \in \Sigma_0$ is the spatial location on the initial slice
- $z(\tau)$ is the distinguishability depth
- g_eff is the effective metric (Section 8)

Type: $\mathscr{F}: \mathrm{Traj}(\mathfrak{U})/\sim \; \to (\Sigma_0 \times \mathbb{R}_{\geq 0}, g\_eff)$

Remark (For General Readers): The reconstruction functional $\mathscr{F}$ takes the history of a feature and outputs its position in 3D space: where it is on the 2D surface (x_0) and how "deep" it is (z). This is how 3D coordinates emerge from 2D + time.

Theorem 7.9 (Existence and Well-Definedness)
Hypotheses: Axioms A1–A8.
Claim: $\mathscr{F}$ exists, is well-defined, and satisfies:
(i) $\mathscr{F}$ is invariant under admissible coarse-graining reparametrisation

(ii) $\mathscr{F}$ preserves causal ordering
(iii) $\mathscr{F}$ is unique up to monotone reparametrisation of z

Proof:

- Existence: Traj($\mathfrak{U}$) ≠ ∅ by A8; ~ well-defined by Lemma 7.4; z well-defined by Proposition 7.6.
- (i): z depends on when equivalence is achieved, invariant under index relabelling.
- (ii): By A7, causal order = sequence order, preserved by $\mathscr{F}$.
- (iii): Any other admissible $\mathscr{F}$ yields z′ = f(z) for monotone f, since both track the same equivalence structure. ■

Section 8 — Metric Emergence Theorem

We establish that the depth coordinate admits ultrametric-type structure and combine with the 2D slice metric to form an effective spacetime.

Remark (For General Readers): Having defined depth, we now show it behaves like a real spatial coordinate—it has distances, satisfies geometric rules, and combines with the 2D surface to give a proper 3D geometry.

8.1 Ultrametric Structure from Depth

Definition 8.1 (Physical Depth Coordinate)
The physical depth coordinate Z is defined by:

$Z := \kappa \cdot z$

where z is the dimensionless distinguishability depth (Definition 7.5) and κ is the propagation bound (A3). Z has units of length.

Definition 8.2 (Rescaled Depth for Boundedness)
For applications requiring a bounded depth measure:

$\tilde{Z}(s, s') := \kappa \cdot (1 - \exp(-z(s,s')/\xi))$

for characteristic scale ξ > 0, with the convention lim_{z→∞} exp(−z/ξ) = 0.

Remark on Z vs $\tilde{Z}$: The unbounded coordinate Z = κz is used throughout the main text to represent physical depth. The bounded form $\tilde{Z}$ is useful for applications involving compact representations (e.g., numerical simulations or visualisation) and does not alter any structural results.

Theorem 8.3 (Ultrametric-Type Depth Inequality)
Hypotheses: Definition 7.5; D satisfies triangle inequality; ∼ defined via transitive closure of ε/2-proximity (Definition 7.3); coarse-graining is monotone contractive ($D(\mathcal{T}(s), \mathcal{T}(s')) \leq D(s, s')$).
Claim: The depth function z satisfies:
(i) $z(s, s) = 0$
(ii) $z(s, s') = z(s', s)$
(iii) $z(s, s'') \leq \max(z(s, s'), z(s', s''))$ (ultrametric inequality)

Remark on contractivity: The monotone contractivity hypothesis ($D(\mathcal{T}(s), \mathcal{T}(s')) \leq D(s, s')$) is automatically satisfied in the quantum setting when D is trace distance and $\mathcal{T}$ is CPTP, since trace distance is contractive under completely positive trace-preserving maps. For classical coarse-graining, contractivity follows from the definition of coarse-graining as a projection to equivalence classes.

Proof: (i) $s \sim_0 s$ via trivial chain, so $z(s, s) = 0$. ✓ (ii) $\sim_n$ is symmetric (Lemma 7.4), so z is symmetric. ✓ (iii) Let $N = \max(z(s, s'), z(s', s''))$. Then $s \sim_n s'$ and $s' \sim_n s''$. By transitivity of $\sim_n$, we have $s \sim_n s''$. Therefore $z(s, s'') \leq N = \max(z(s, s'), z(s', s''))$. ✓ ■

Remark: The ultrametric inequality (iii) is stronger than the standard triangle inequality. It arises naturally from coarse-graining/hierarchical structures and is characteristic of tree-like spaces. This is consistent with the MERA tensor network realisation (Appendix B).

Remark (Physical interpretation): The ultrametric inequality means depth behaves hierarchically rather than additively. If states A and B diverged at update N_1 (deep difference) and B and C diverged at update $N_2 > N_1$ (shallow difference), then $z(A,C) = z(A,B)$—the deeper divergence dominates. This is characteristic of tree-like or genealogical structures, consistent with MERA's hierarchical coarse-graining. Physically, it means that once two configurations have diverged deeply in the coarse-graining hierarchy, subsequent shallow divergences do not increase the effective depth between them.

Remark (For General Readers): These properties mean depth behaves like a legitimate distance measure: the depth from A to A is zero, the depth from A to B equals the depth from B to A, and depth satisfies a strong form of "the shortest path" rule.

8.2 Effective Spacetime Metric

Definition 8.4 (Proper Time)
Proper time τ for an observer is defined by:

$$\tau := N \cdot \Delta\tau_0$$

where N is the update count experienced by the observer's clock subsystem and $\Delta\tau_0$ is a calibration constant (with units of time per update, determined empirically).

Definition 8.5 (Invariant Speed)
The invariant speed c is defined by:

$c := \kappa / \Delta\tau_0$

This converts the propagation bound κ (length per update) to speed (length per time). Note: c has units [length/time].

Definition 8.6 (Effective Spacetime Metric)
The effective (2+1+1)-dimensional spacetime metric (i.e., 2D surface + 1D emergent depth + 1D time) is:

$$ds^2 = -c^2\, d\tau^2 + g_{a\beta}(x)\, dx^a\, dx^b + dZ^2$$

where:

- τ is proper time (Definition 8.4)
- $g_{a\beta}$ is the intrinsic 2D metric on Σ_0 (with signature +,+)
- $Z = \kappa z$ is the physical depth coordinate (Definition 8.1)
- c is the invariant speed (Definition 8.5)

Remark on dimensional consistency: All terms have units of [length²]:

- $c^2 d\tau^2 = (\kappa/\Delta\tau_0)^2 \cdot (N\Delta\tau_0)^2 = \kappa^2 N^2$ [length²] ✓
- $g_{a\beta}\, dx^a\, dx^b$ [length²] ✓
- $dZ^2 = (\kappa\, dz)^2$ [length²] ✓

Remark (For General Readers): This is the formula for spacetime intervals in our emergent space. The first part ($-c^2\, d\tau^2$) is the time contribution. The second part ($g_{a\beta}\, dx^a\, dx^b$) is distance on the 2D surface. The third part (dZ^2) is distance in the depth direction. Together, they give a full spacetime geometry.

Theorem 8.7 (Metric Emergence)
Hypotheses: Axioms A1–A8; transmission-first universe 𝔘; Definitions 8.4–8.6.
Claim: The effective metric g_eff is:
(i) Well-defined on $\Sigma_0 \times \mathbb{R}_{\geq 0} \times \mathbb{R}$
(ii) Compatible with causal cones: null geodesics ($ds^2 = 0$) satisfy

$$g_{a\beta}\, (dx^a/d\tau)(dx^b/d\tau) + (dZ/d\tau)^2 = c^2$$

(iii) Reduces to $g_{a\beta}$ on constant-Z, constant-τ slices

Proof: (i) $g_{a\beta}$ exists by Definition 3.1; $\kappa > 0$ by A3; z well-defined by Theorem 7.9; τ well-defined by Definition 8.4; $c > 0$ by Definition 8.5; $Z = \kappa z$ is well-defined. (ii) Setting $ds^2 = 0$: $-c^2\, d\tau^2 + g_{a\beta}\, dx^a\, dx^b + dZ^2 = 0$ $g_{a\beta}\, dx^a\, dx^b + dZ^2 = c^2\, d\tau^2$

Dividing by $d\tau^2$: $g_{a\beta}\, (dx^a/d\tau)(dx^b/d\tau) + (dZ/d\tau)^2 = c^2$

This is the null cone condition. For motion purely along the surface ($dZ = 0$): $|dx/d\tau| = c$. For motion purely in depth ($dx = 0$): $|dZ/d\tau| = c$. Both directions have the same limiting speed c. ✓
(iii) At fixed Z and τ, $ds^2 = g_{a\beta}\, dx^a\, dx^b$ by definition. ✓ ■

Corollary 8.8
Claim: Observers embedded in 𝔘 experience effective (3+1)-dimensional spacetime despite fundamental 2D slices.

Remark (For General Readers): The punchline: even though reality is fundamentally 2D slices evolving via updates, anyone living inside it would experience a fully functional (3+1)D spacetime with proper distances, geometry, and light cones. The 3D world we perceive is real—it's just not fundamental.

Section 9 — Minimality and Uniqueness Theorem

We establish that 2D + sequencing is uniquely minimal using the routing requirement R. This section provides the topological/graph-theoretic argument; for the complementary information-theoretic argument, see Section 3.5.

Remark (For General Readers): Why specifically 2D? Why not 1D or 4D? This section proves that 2D is the unique "Goldilocks" dimension—1D is too small to support complex physics, and 3D+ is redundant. We give two independent arguments: one based on signal routing (here), one based on relational information (Section 3.5).

9.1 Dimensional Failure in 1D

Theorem 9.1 (1D Fails R)
Hypotheses: Σ is a 1-dimensional manifold (interval or circle).
Claim: Requirement R cannot be satisfied.

Proof:

1. Let $\Sigma \cong [0, 1]$ or S^1 with induced linear/circular ordering.
2. Consider any four points with ordering $a < c < d < b$ (in the linear case) or arranged so that paths $a \to b$ and $c \to d$ must cross (in the circular case).
3. Any path from a to b must traverse the interval $[c, d]$ (1D topology forces this).
4. Any path from c to d is contained in $[c, d]$.
5. Therefore $\mathrm{supp}(P) \cap \mathrm{supp}(Q) \supseteq [c, d] \neq \emptyset$ for all choices of paths.
6. No separated quadruple admits disjoint routing.
7. R fails. ■

Remark (For General Readers): In 1D (a line), if Alice wants to send a message to Bob, and Carol is between them, the message MUST pass through Carol's location. There's no way *around. This means signals can't travel independently—they block each other. That's too restrictive for real physics.*

Corollary 9.2: 1D substrates cannot support complex causal structure, interference, or isotropy without embedding in higher dimensions.

9.2 Dimensional Sufficiency in 2D

Lemma 9.3 (Local Planar Routing)
Hypotheses: Σ is a 2-dimensional manifold; consider a disk-like patch $D \subset \Sigma$.
Claim: For four points $a, b, c, d \in D$ with min pairwise distance $> 4\kappa$, there exist paths $P: a \to b$ and $Q: c \to d$ with $\text{supp}(P) \cap \text{supp}(Q) = \emptyset$.

Proof (constructive):

1. Place a, b, c, d in the disk D with pairwise distances $> 4\kappa$.
2. Connect a to b via a narrow corridor of width $< \kappa$, staying $> 2\kappa$ away from both c and d.
3. Connect c to d via a narrow corridor of width $< \kappa$, staying $> 2\kappa$ away from the first corridor.
4. In 2D, such corridors can be routed around each other (one can go "north" while the other goes "south").
5. The κ-neighbourhoods of the two corridors are disjoint because the corridors themselves are separated by $> 2\kappa$.
6. Therefore $\text{supp}(P) \cap \text{supp}(Q) = \emptyset$. ✓ ■

Remark: This constructive argument does not require heavy machinery. It relies only on the fact that in 2D, you can "go around" obstacles, which is the intuitive content of planar topology. For more general results on disjoint paths in planar graphs, see Robertson-Seymour [11], but the local disk argument suffices for our purposes.

Theorem 9.4 (2D Satisfies R)
Hypotheses: Σ is a 2-dimensional manifold; we consider configurations on locally planar patches.
Claim: Requirement R is satisfied.

Proof:

1. R requires only that *some* sufficiently separated quadruples admit disjoint routing.
2. By Lemma 9.3, any disk-like patch with four points at distance $> 4\kappa$ admits such routing.
3. Such configurations exist generically on any 2-manifold.
4. R is satisfied. ✓ ■

Remark: For manifolds of higher genus (e.g., torus), global topological constraints may obstruct some routings. However, R is an existential/generic requirement, not a universal one, so it holds even on higher-genus surfaces for sufficiently localised patches.

Remark (For General Readers): In 2D (a surface), you can go AROUND obstacles. If Carol is between Alice and Bob, Alice can route her message around Carol. Two separate conversations can happen without interference. This is enough for complex physics.

9.3 Dimensional Redundancy in 3D

Theorem 9.5 (3D is Redundant)
Hypotheses: Axioms A1–A4.
Claim: A 3D fundamental manifold M_3 satisfies R but introduces operationally inaccessible structure.

Proof:

- R satisfied: 3D provides strictly more routing freedom than 2D (additional dimension for path separation).
- Redundancy: By Theorem 4.4, M_3 as primitive violates A1 and A4.
- The third dimension, if experienced, must be derived (= reconstructed depth), not primitive. ■

Remark (For General Readers): 3D also allows routing around obstacles, but it adds extra structure we can't independently measure (as shown in Section 4). Since 2D is enough, 3D is overkill.

9.4 Uniqueness Theorem

Theorem 9.6 (Uniqueness of 2D + Sequencing)
Hypotheses: Axioms A1–A8, Requirement R.
Claim: The minimal instantaneous state manifold is 2-dimensional, and apparent 3D geometry emerges via $\mathscr{F}$.

Proof:

1. $\dim(\Sigma) \geq 2$: 1D fails R (Theorem 9.1).
2. $\dim(\Sigma) \leq 2$: 3D violates A4 (Theorem 9.5).
3. Therefore $\dim(\Sigma) = 2$.
4. Effective 3D geometry emerges via $\mathscr{F}$ (Theorem 8.7).
5. This is the unique minimal structure. ■

Remark (For General Readers): The conclusion: exactly 2D is required. Less is not enough; more is wasteful. The third dimension we perceive must emerge from time-sequencing, because it can't be fundamental.

Section 10 — Horizons, Black Holes, and Dimensional Collapse

Remark (For General Readers): This section explains how our framework naturally accounts for the weird physics of black holes—something the "3D container" view struggles with.

10.1 Horizons as Update Boundaries

Definition 10.1 (Causal Horizon)
A causal horizon H relative to observer O is a surface such that for $x \in \text{int}(H)$, no signal path from x reaches O in finite update count.

Remark (For General Readers): A horizon is a boundary beyond which you can never receive information, no matter how long you wait. The edge of a black hole is the classic example.

Theorem 10.2 (Depth Collapse)
Hypotheses: Axioms A1–A8; Definition 10.1.
Claim: For x beyond H relative to O, z(x) is undefined for O.

Proof:

1. Depth requires inter-slice differences accessible to O.
2. By definition of H, no update from x reaches O.
3. No $\Delta_n f$ from x contributes to O's reconstruction.
4. Hence z(x) is undefined for O. ■

Remark (For General Readers): If you can't receive any signals from a region, you can't reconstruct its depth. Depth REQUIRES ongoing time-sequencing to exist. At a horizon, sequencing (from your perspective) stops—so depth stops too. This explains why black hole interiors are "dimensionally reduced" for outside observers.

10.2 Holographic Entropy

Theorem 10.3 (Area Scaling)
Hypotheses: Axioms A1–A8; region R bounded by horizon H.
Claim: Information accessible to external observers about R scales as Area(H).

Proof:

1. By Theorem 10.2, int(R) contributes nothing to external reconstruction.
2. Accessible information is encoded on H (last accessible surface).
3. H is 2-dimensional.
4. Information $\propto$ Area(H), matching Bekenstein-Hawking. ■

Remark (For General Readers): This is why black hole entropy is proportional to surface area, not volume. In our framework, this is obvious: the interior doesn't contribute anything because depth collapses at the horizon. All the information is on the 2D boundary.

Section 11 — Relativistic Causality and Emergent Light Cones

Remark (For General Readers): This section shows that Einstein's special relativity—light cones, time dilation, and all that—emerges naturally from our framework without being put in by hand.

11.1 Invariant Speed

Definition 11.1: The invariant speed $c := \kappa/\Delta\tau_0$ (Definition 8.5).

11.2 Lorentz Structure

Scope note: The following theorem is conditional on Axiom A9 (empirical homogeneity and isotropy). It applies in regimes where these symmetries hold (as they do to high precision on large cosmological and laboratory scales).

Theorem 11.2 (Lorentz Structure in Homogeneous and Isotropic Regimes)
Hypotheses: Axioms A3, A7, A9; uniform κ across Σ; smooth reparametrisations in the continuum limit.
Claim: The symmetry group preserving the emergent causal cone structure is locally isomorphic to the Lorentz group SO(2,1) (or the Poincaré group including translations). If scale invariance is also present, the conformal extension SO(3,2) is admitted.

Proof sketch:

1. The causal cone from Theorem 5.5 and Theorem 8.7 defines a null surface $ds^2 = 0$.
2. Under A9 (homogeneity and isotropy), the metric takes the standard form $ds^2 = -c^2 d\tau^2 + dx^2 + dz^2$.
3. Transformations preserving $ds^2 = 0$ while respecting linearity and smoothness are precisely Lorentz transformations.
4. The group structure follows from composition.
5. Conformal transformations (preserving null cones up to scale) extend this to the conformal group if scale invariance holds. ■

Status of this proof: This is a proof sketch, not a complete derivation. The key steps are standard (the symmetry group preserving a null cone in a homogeneous isotropic space is Lorentz), and can be found rigorously in standard references on the axiomatic foundations of special relativity (e.g., Zeeman's theorem characterising the Lorentz group as the automorphism group of the causal structure). A complete treatment would verify that our specific causal structure (derived from bounded propagation on 2D + depth) satisfies the hypotheses of such theorems. We flag this as a theorem-in-outline; the full derivation is mechanical but lengthy.

Technical note: Without A9, larger or smaller symmetry groups may arise. The Lorentz group arises specifically in the empirically relevant regime where homogeneity and isotropy hold. In more general settings, the emergent symmetry group may be the conformal group, an anisotropic subgroup, or a discrete approximation.

What this derivation achieves: Standard physics also derives Lorentz symmetry from isotropy via symmetry arguments. The gain here is structural: we show that Lorentz structure emerges from the *combination* of bounded propagation (A3), causal ordering (A7), and empirical isotropy (A9) applied to a transmission-first ontology. The framework explains *why* isotropy leads to Lorentz structure (because the null cone is determined by the propagation bound κ and proper time τ, both derived from update structure), rather than imposing Lorentz structure directly on a pre-existing spacetime manifold.

Remark (For General Readers): The "Lorentz group" is the mathematical structure behind special relativity—it describes how space and time mix when you change reference frames. We get it naturally from our axioms (with the additional empirical assumption that space looks the same everywhere and in every direction), without assuming relativity in advance.

11.3 Time Dilation

Proposition 11.3 (Time Dilation)
Hypotheses: A3, A6.
Claim: Systems expending update capacity internally experience fewer external updates per unit proper time.

Argument: Total update capacity per unit proper time is bounded by A3. A system that expends update capacity on internal processes (internal dynamics, computation, interaction) has correspondingly less capacity available for external evolution relative to its environment. External observers, whose clocks are not expending capacity internally, count more updates while the internal clock counts fewer. This is operationally equivalent to time dilation.

Remark on scope: This proposition establishes qualitative consistency with relativistic time dilation. A full derivation of the Lorentz factor $\gamma = (1 - v^2/c^2)^{(-1/2)}$ from update-budget considerations requires specifying how velocity relates to update expenditure, which involves the detailed kinematics of motion in the emergent spacetime. This quantitative connection is developed in companion work within the VERSF programme and lies beyond the present paper's scope.

Remark (For General Readers): Time dilation (moving clocks run slow) emerges naturally. If a system uses its "update budget" on internal processes, it has less for external evolution. From outside, it appears to age more slowly.

Section 12 — Observer Dependence and Foliation Freedom

Remark (For General Readers): Different observers can "slice" time differently (relativity of simultaneity). This section shows our framework handles that correctly.

12.1 Compatible Foliations

Definition 12.1: A foliation $\{\Sigma_n\}$ is compatible if: (i) Each Σ_n is spacelike (ii) Sequence respects causal ordering (A7) (iii) $\mathcal{T}_n$ maps consistently between slices

Theorem 12.2 (Foliation Equivalence)
Hypotheses: A5, A7; compatible foliations; homogeneity and isotropy in the relevant patch.
Claim: All compatible foliations yield equivalent effective geometries up to Lorentz-like diffeomorphism.

Proof: By A5, $\mathcal{T}$ is observer-independent. Different foliations correspond to different inertial frames. Reconstructions are related by the transformations of Theorem 11.2. ■

Remark (For General Readers): Different observers can disagree on which events are "simultaneous," but they all reconstruct the same physics. This is exactly what relativity says, and our framework reproduces it.

Section 13 — Testable Predictions and Empirical Signatures

Remark (For General Readers): A good scientific framework makes predictions that can be tested. Here are ours.

13.1 Classification

We distinguish predictions by:

- **Status**: Derived (follows from axioms) vs. Conjectural (requires additional assumptions)
- **Testability**: Near-term, medium-term, or long-term

13.2 Prediction 1: Correlation Asymmetry

Statement: Depth correlations exhibit channel structure, not propagator structure.

Quantitative Signature:

⟨OO⟩_boundary / ⟨O′O⟩_depth = 2

where O′ = POP is the depth-projected operator.

Status: Derived (from isometry structure $V^\dagger V = I$, $VV^\dagger \neq I$).

Testability: Near-term. Implementable in MERA tensor network simulators (using trapped ions, superconducting qubits, or photonic circuits).

Generality of the ratio = 2: This prediction assumes coarse-graining via isometries ($V^\dagger V = I$, $VV^\dagger \neq I$), as in MERA-type tensor networks. The ratio = 2 follows from the rank deficiency of $VV^\dagger$ relative to $V^\dagger V$. Different tensor network architectures (e.g., TTN, PEPS) may yield different ratios.

Robust form: For any coarse-graining with $\text{rank}(VV^\dagger) = \text{rank}(V^\dagger V)/k$, the ratio is k. The qualitative prediction (ratio > 1) holds universally for information-losing coarse-graining. **Ratio = 1 would falsify the framework's core mechanism**—it would indicate that no information is being lost under coarse-graining, contradicting the fundamental structure of depth emergence.

Interpretation of Non-Detection:

- If ratio = $k \neq 2$ (e.g., ratio = 1.5 or 3): The specific binary isometry structure is incorrect, but the framework's core mechanism (information-losing coarse-graining) is preserved. This would indicate a different coarse-graining architecture (non-binary branching, different rank deficiency).
- If ratio = 1: The coarse-graining mechanism central to the framework is falsified—no information is being lost under coarse-graining, contradicting depth emergence.
- The headline prediction is ratio > 1 (asymmetry exists); ratio = 2 is the specific value for binary MERA.

Remark (For General Readers): In quantum simulators that implement our kind of structure, we predict a specific ratio (2:1) between certain measurements. This can be tested with existing technology. If we see the ratio, the framework is supported. If not, we learn something important about coarse-graining.

13.3 Prediction 2: Horizon Dimensional Reduction

Statement: Effective dimensionality reduces near horizons.

Form: $D_{eff}(d) \to 2$ as proper distance $d \to 0$.

Status: Derived (Theorem 10.2).

Testability: Medium-term. Requires near-horizon observations or analog systems.

Remark (For General Readers): Near black hole horizons, space should effectively become 2D (not 3D). Future observations of black hole physics could test this.

13.4 Prediction 3: Entropy Non-Additivity

Statement: $S_{bulk} \leq S_{boundary}$ with saturation.

Status: Derived (Theorem 10.3).

Testability: Long-term. Black hole observations, holographic cosmology constraints.

Remark (For General Readers): The information "inside" a region can never exceed the information on its boundary. This is already believed to be true (holographic principle), but our framework explains WHY.

13.5 Prediction 4: Sequential Anisotropy

Statement: Residual anisotropy at order $(\ell/L)^\alpha$, $\alpha \geq 2$.

Status: Conjectural. Scale ℓ and exponent α not determined by axioms.

Testability: Long-term. Precision cavity experiments.

Interpretation: Absence of effect constrains ℓ_{update} to below experimental sensitivity.

Remark (For General Readers): If space emerges from 2D structures, there might be tiny directional asymmetries at very small scales. Current experiments haven't seen this, which tells us the underlying scale is very small (probably near the Planck length, ~10^{-35} meters).

13.6 Prediction 5: Energy-Time Coupling

Statement: Time dilation correlates with internal reconfiguration.

Status: Conjectural. Must respect Einstein Equivalence Principle violation bounds (fractional anomalous redshift < 10^{-5} from precision clock comparisons).

Testability: Medium-term. Precision clocks with varying complexity.

Remark (For General Readers): Systems that are internally more complex might experience time slightly differently. This is speculative and must be consistent with existing precision tests.

13.7 Summary Table

#	Prediction	Status	Testability	Key Observable
1	Correlation Asymmetry	Derived	Near-term	Ratio = 2
2	Dimensional Reduction	Derived	Medium-term	D_{eff} profile
3	Entropy Non-Additivity	Derived	Long-term	$S_{bulk} \leq S_{boundary}$
4	Sequential Anisotropy	Conjectural	Long-term	$\delta c/c$ scaling

#	Prediction	Status	Testability	Key Observable
5	Energy-Time Coupling	Conjectural	Medium-term	Clock correlations

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Section 14 — Relation to Existing Physical Theories

Remark (For General Readers): How does this framework relate to physics you may have heard of?

14.1 General Relativity

GR treats 4D spacetime as primitive. This framework derives (3+1)D geometry from 2D + sequencing. Both agree on causal structure and horizon physics. Curvature ↔ nonuniform update capacity.

Remark (For General Readers): We don't contradict Einstein. We provide a deeper explanation for why his equations work.

14.2 Quantum Field Theory

QFT assumes fields on spacetime. Here, fields = inter-slice correlation patterns. Vacuum = update equilibrium. Renormalisation = update coarse-graining.

Remark (For General Readers): Quantum fields are reinterpreted as patterns in how successive slices relate to each other.

14.3 Holography / AdS-CFT

Holography: bulk = boundary encoding. Here: expected, since bulk depth is reconstructed. This framework provides the mechanism.

Remark (For General Readers): The holographic principle says 3D information is encoded on 2D surfaces. Our framework explains why: because 3D is BUILT from 2D.

14.4 Quantum Gravity Approaches

Approach	Shared Features	Differences
Causal Sets	Causal ordering primitive	We derive dimensionality
CDT	Discrete building blocks	We don't assume target dimension
LQG	Geometry discrete	We make geometry emergent, not quantised

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Section 15 — Scope and Limitations

15.1 What This Paper Does NOT Claim

To prevent misreading, we explicitly state what this paper does *not* claim:

This paper does NOT claim:

❌ That 3D space is an illusion or "not real"
❌ That General Relativity or QFT are false
❌ That spacetime emerges from consciousness
❌ That this is a Theory of Everything
❌ That we have derived all constants of nature
❌ That the framework is complete

This paper DOES claim:

✓ That 3D space is not *primitive*—it is reconstructed
✓ That depth emerges from temporal sequencing
✓ That a transmission-first ontology is *required* under Axioms A1–A8
✓ That specific, falsifiable predictions follow
✓ That this complements (not replaces) established physics

Remark (For General Readers): We're not saying space is fake or that Einstein was wrong. We're saying space is REAL but DERIVED—like how motion in a film is real but derived from the sequence of frames.

15.2 Axiom Sensitivity and Scope

The conclusions of this paper are not hard-coded by the axioms themselves. In particular:

- **Axioms A1–A8 do not fix the dimensionality** of the state manifold
- **They do not require reconstruction** of a depth coordinate
- **They do not preclude higher-dimensional substrates** a priori

The uniqueness of a two-dimensional instantaneous substrate follows only after imposing additional, independently motivated requirements: non-blocking parallel routing (R) and ontological minimality (A4). **Relaxing either of these admits alternative structures.**

The results should therefore be read as conditional: *given* these operational commitments, the derivative picture is the unique minimal option. Rejecting the conclusion requires rejecting or modifying specific axioms, not disputing hidden assumptions.

Why Stronger or Weaker Axioms Were Not Used

One could adopt *stronger* axioms that trivially enforce the desired conclusion—for example, assuming holography, boundary encoding, or pre-existing renormalisation structure. We deliberately do not do so.

Conversely, one could *weaken* the axioms by allowing unbounded propagation, non-local update rules, or unobservable bulk structure. Such frameworks evade the present conclusions but do so at the cost of violating empirical constraints (e.g., finite signal speed), operational observability, or ontological economy.

The axioms chosen here represent a minimal set that is:

- **Strong enough** to support physical law
- **Weak enough** to allow nontrivial derivation

The conclusions are therefore not engineered but earned.

15.3 Open Questions

1. **Quantum completion**: How does $\mathcal{T}$ relate to unitary evolution?

Remark on quantum completion: In the quantum setting, the update operator $\mathcal{T}$ may be realised in two complementary ways. At the microscopic level, $\mathcal{T}$ can be taken as unitary (or isometric) evolution on an enlarged Hilbert space, consistent with reversibility. Irreversibility and entropy increase then arise effectively through coarse-graining, restriction to subsystems, or the discarding of inaccessible degrees of freedom—a mechanism familiar from quantum information theory and consistent with CPTP dynamics.

In this sense, $\mathcal{T}$ need not violate unitarity at the fundamental level to generate an arrow of time at the effective level. The detailed relationship between update sequencing, entropy flow, and emergent thermodynamic irreversibility is developed elsewhere in the VERSF programme and lies beyond the scope of the present paper.

Explicit scope limitation: The present framework is developed at the *effective/classical level*. Key questions requiring further development for full quantum completion include:

- **Arrow of time**: If $\mathcal{T}$ is fundamentally unitary, the arrow of time emerges from coarse-graining asymmetry, not from $\mathcal{T}$ itself. This is consistent with standard quantum statistical mechanics.
- **Reconstruction and superposition**: How does the reconstruction functional $\mathscr{F}$ interact with quantum superposition? For superposed states, the depth $z(s, s')$ should be understood as operating on the density matrix representation (trace distance), not on individual superposition branches.
- **Depth of superposed states**: For pure states $|\psi\rangle$ and $|\varphi\rangle$, depth would be defined via the induced density matrices; $z(|\psi\rangle\langle\psi|, |\varphi\rangle\langle\varphi|)$ is well-defined via trace distance distinguishability.

These questions mark the boundary of the present treatment. Full quantum completion is future work.

Remark on entropy and irreversibility: Although the update operator $\mathcal{T}$ is not assumed to be irreversible at the fundamental level, effective irreversibility arises naturally from update sequencing combined with coarse-graining and restriction to subsystems. The thermodynamic arrow of time is not imposed but arises from the asymmetry between primitive sequencing (which defines the direction of update application) and derived reconstruction (which inherits this directionality). This connection between sequencing and the second law is a feature of the framework, not an additional assumption.

2. **Cosmological regime**: What is sequencing like near the Big Bang?
3. **Symmetry class**: What determines α in anisotropy predictions?
4. **Consciousness**: If observers are embedded, what determines their foliation?

15.4 Relation to Metaphysical Positions

The framework is compatible with multiple metaphysical stances:

- **Scientific realism**: Space exists but is emergent, not primitive
- **Structural realism**: Only relational structure is fundamental
- **Operationalism**: Only measurable quantities matter

It is incompatible with:

- **Naive realism about 3D space**: Container space as primitive

15.5 What Would Falsify This Framework?

To preempt the criticism that the framework is unfalsifiable, we state explicit falsification conditions in one place:

1. Correlation Asymmetry (Prediction 1)

- *Falsified if*: The ratio ⟨OO⟩_boundary / ⟨O′O⟩_depth = 1 (no asymmetry) in MERA-type tensor network implementations
- *Interpretation*: The coarse-graining structure assumed (isometric with $V^\dagger V = I$, $VV^\dagger \neq I$) is incorrect
- *Severity*: Would require revising the specific reconstruction mechanism, though not necessarily the framework's ontology

2. Horizon Dimensional Reduction (Prediction 2)

- *Falsified if*: Near-horizon physics shows dimensional enhancement ($D_{eff} > 3$) rather than reduction ($D_{eff} \to 2$)
- *Interpretation*: The depth-collapse mechanism at horizons is wrong

- *Severity*: Would challenge the core claim that depth is derivative of temporal sequencing

3. Entanglement and Reconstruction (Prediction 4)

- *Falsified if*: Entanglement correlations systematically violate the reconstruction-map interpretation—i.e., if "spatial separation" cannot be consistently understood as a derived property
- *Interpretation*: The "distance is derived" picture is incorrect
- *Severity*: Would undermine the framework's treatment of non-locality

4. Holographic Entropy Scaling (Prediction 3)

- *Falsified if*: Physical systems are discovered where $S_{bulk} > S_{boundary}$ without saturation
- *Interpretation*: Information can be encoded in ways that exceed surface capacity
- *Severity*: Would contradict the claim that bulk degrees of freedom are not independent

General Falsification Condition: If any physical phenomenon is discovered that *requires* treating three-dimensional space as primitive—i.e., that cannot be reconstructed from 2D sequential structure—the framework would be falsified. The burden is to exhibit such a phenomenon and demonstrate that reconstruction is impossible, not merely inconvenient.

Remark: The framework is falsifiable because it makes specific, quantitative predictions about correlation ratios, dimensional behaviour, and entropy scaling. These are not post-hoc adjustable parameters but derived consequences of the axioms.

Section 16 — Conclusion

16.1 Summary of Results

Theorem	Statement	Section
3.12	Relational collapse in 1D	§3.5
3.16	2D is minimal relational substrate	§3.5
3.15	Σ is unique primitive-law locus	§3.4
4.4	No-Go for container space	§4
—	Eight additional problems with container	§4.5
7.9	Reconstruction functional exists	§7
8.3	Ultrametric depth inequality	§8
8.7	Effective spacetime metric emerges	§8
9.6	2D + sequencing uniquely minimal	§9
10.2	Depth collapses at horizons	§10

Theorem	Statement	Section
11.2	Lorentz structure emerges (with conditions)	§11

16.2 The Two-Paper Programme

- *Depth Is Not a Direction*: Established that renormalisation depth fails spatial criteria (negative result)
- *This paper*: Established that depth emerges from temporal sequencing (positive result)

Together: **Space is not given but generated.**

16.3 The Central Result

What this paper establishes:

Given operational observability, finite discrimination, bounded propagation, and causal consistency (Axioms A1–A8), three-dimensional container space cannot be primitive. Under these same axioms, a temporally sequenced two-dimensional substrate is the unique minimal structure from which observed spatial depth can be reconstructed.

This is a theorem-level conditional claim, not a metaphysical assertion.

The framework does not assume that space is two-dimensional. It shows that if space is treated as primitive and three-dimensional, then one must abandon at least one of the operational commitments that modern physics already relies on.

The burden is therefore on those who reject the conclusion: **Which axiom would they drop? And are they willing to pay that price?**

- Dropping A1 (operational observability) abandons the link between theory and measurement
- Dropping A3 (bounded propagation) violates empirical constraints on signal speed
- Dropping A4 (ontological minimality) permits arbitrary unobservable structure
- Dropping A6 (locality) contradicts the structure of known physics
- Dropping A7 (causal consistency) undermines the foundation of dynamics

The derivative picture is not a speculation. Under standard operational commitments, it is the unique minimal option.

Acknowledgment: A critic might respond: "I don't drop any axiom; I reject that operationalist axioms are the right starting point for fundamental ontology." This is a coherent position. The framework is conditional on accepting operationalist foundations, as defended in Section 5.3. Those who reject operationalism in favour of a priori ontological commitments may reach different conclusions. We maintain that operationalism provides the most principled criterion for distinguishing primitive from derived structure, but acknowledge this is itself a foundational choice.

16.4 Closing Remark

Depth does not exist on any instantaneous state. It emerges from structured temporal differences. Where time sequences updates, depth appears. Where updates halt, depth cannot exist.

The universe does not contain space as a stage; it generates spatial appearance through the unfolding of information in time.

Remark (For General Readers): The final message is simple. Each moment of reality is 2D. The 3D world you perceive emerges from the sequence of these moments—from the differences between them, accumulated over time. Space isn't the stage; it's part of the performance.

Appendix A: Mathematical Core (Self-Contained Summary)

A.1 Axioms

Axiom	Statement
A1	Only measurement-accessible quantities are primitive
A2	$\exists\varepsilon > 0$: $D(s,s') < \varepsilon \Longrightarrow s \equiv s'$ operationally
A3	$\exists\kappa > 0$: $I_n(x) \subseteq B\_d\Sigma(x, \kappa)$ [2D metric, no 3rd dim assumed]
A4	Prefer ontologies with fewer unobservable primitives
A5	$\mathcal{T}$ is observer-independent (up to foliation)
A6	$\mathcal{T}_n = \prod_i \mathcal{T}_{n,i}$ with local support
A7	Causal order ⊆ sequence order
A8	Non-trivial dynamics exist
A9	$g\Sigma$ is statistically homogeneous and isotropic (empirical, conditional)
R	Disjoint routing possible for generic separated pairs

A.2 Key Definitions

- **State manifold**: $(\Sigma, g\Sigma)$ compact 2D Riemannian (intrinsic; no embedding assumed)
- **Influence region**: $I_n(x)$ = minimal set affected by perturbation at x after one update (Def 3.5)
- **Operational reality**: X is primitive iff measurements depend on X and changes to X cannot be reproduced by derived structure alone (Def 3.10)
- **Relational information**: Non-linear adjacency, multiple independent relations, non-total-order structure (Def 3.11)

- **Relational primitive**: Property definable from adjacency, bounded propagation, and sequencing only (Def 3.11′)
- **Operationally closed**: $\mathcal{T}_n$ injective on operational equivalence classes (reversible) (Def 3.11″)
- **Reservoir**: Extension $\tilde{S}_n$ where effective evolution on S_n is non-injective (Def 3.11‴)
- **Update**: $\mathcal{T}_n$: $S_n \to S_{n+1}$ (CPTP)
- **Cumulative**: $R_0 \to_n = \mathcal{T}_{n-1} \circ \cdots \circ \mathcal{T}_0$
- **ε/2-proximity**: $s \approx_n s' \Leftrightarrow D(R_0 \to_n(s), R_0 \to_n(s')) < \varepsilon/2$ (Def 7.2)
- **N-Equivalence**: $s \sim_n s' \Leftrightarrow$ transitive closure of $\approx_n$ (Def 7.3)
- **Depth**: $z(s,s') = \min\{N : s \sim_n s'\}$ (Def 7.5)
- **Physical depth**: $Z = \kappa z$ [length] (Def 8.1)
- **Proper time**: $\tau = N\_clock \cdot \Delta\tau_0$ (emergent) (Def 8.4)
- **Invariant speed**: $c = \kappa/\Delta\tau_0$ (Def 8.5)
- **Reconstruction**: $\mathscr{F}$: Traj/∼ $\to (\Sigma \times \mathbb{R}_{\geq 0}, g\_eff)$ (Def 7.8)

A.3 Main Theorems

Theorem	Hypotheses	Conclusion
3.11A	$\mathcal{T}_n$ non-injective	Requires reservoir extension
3.11B	Adjacency-determined + reversible	2D sufficiency for relational primitives
3.12	Σ connected 1D	Relational structure collapses to linear order
3.13	Σ is 2D	Non-linear relational structure emerges
3.14	Thm 3.14	Relational completeness in 2D
3.15	A3, A6	Σ_n is unique primitive-law locus
3.16	A1–A8, R	dim(Σ) = 2 is the minimal relational substrate
4.4	A1–A4, A7	No primitive 3D container
7.4	A2, D symmetric	$\sim_n$ is equivalence relation
7.9	A1–A8	$\mathscr{F}$ exists and is well-defined
8.3	Def 7.5, monotone contraction	z satisfies ultrametric inequality
8.7	A1–A8, Defs 8.4–8.6	Effective metric $ds^2 = -c^2 d\tau^2 + g_{a\beta} dx^a dx^b + dZ^2$
9.6	A1–A8, R	dim(Σ) = 2 uniquely minimal
10.2	A1–A8	z undefined beyond horizons
11.2	A3, A7, uniformity, isotropy, homogeneity	Causal symmetries ≅ Lorentz (locally)

A.4 Eight Problems with Container Space (Section 4.5)

Problem	Container Issue	Derivative Resolution
Information bounds	Naive DOF count incompatible with holographic bounds	Only surfaces carry independent DOF

Problem	Container Issue	Derivative Resolution
Simultaneity	Requires privileged foliation	Foliation-independent
Vacuum energy	Overcounting of DOF	Reduced DOF count
Entanglement	Non-local correlations unexplained	Locality is emergent; "distance" is reconstructed
Speed of light	Must be imposed externally	Derived from $c = \kappa/\Delta\tau_0$
Initial conditions	Requires 3D specification	Only 2D needed
Problem of Now	Block universe has no present	Now = current slice
Factuality	No locus for definiteness	Updates ARE commitment events

A.5 Key Proof: Why Exactly 2D (Theorem 3.16 + 9.6)

Information-theoretic argument (Section 3.5):

1. 1D collapses relational information to total order (Lemma 3.12)
2. 2D supports irreducible relational structure (Lemma 3.13)
3. Higher dimensions add no new relational primitives (Theorem 3.14)

Routing argument (Section 9):

1. 1D fails R: paths must overlap in 1D topology (Thm 9.1)
2. 3D violates A4: unobservable structure (Thm 9.5)
3. 2D satisfies R: local disk routing works (Lemma 9.3, Thm 9.4)

Combined: Under A1–A8 and R, $\dim(\Sigma) = 2$ is the unique minimal choice satisfying both relational requirements AND routing requirements. ■

A.6 Why Sequencing Is Primitive but Space Is Derived (Section 5.3)

Epistemic asymmetry:

- Temporal sequencing is epistemically irreducible: measurement itself presupposes before/after ordering
- Spatial extension is epistemically derivative: all spatial information is inferred via signal exchange across multiple updates (Lemma 4.1)

This asymmetry is not a choice but reflects the operational structure of measurement.

A.7 One-Sentence Summary

Two dimensions are required because they are the lowest-dimensional substrate in which relations are not totally ordered; below two dimensions relational information collapses, and above two dimensions it becomes redundant.

Appendix B: Connection to Tensor Network Geometry

Remark (For General Readers): Tensor networks are computational tools physicists use to study quantum systems. They provide a concrete example of how our framework works in practice.

MERA (Multi-scale Entanglement Renormalisation Ansatz) provides explicit realisation:

- Layers = updates
- Isometries V implement $\mathcal{T}$
- Depth $z \sim \log(\text{separation})$
- Ryu-Takayanagi: $S_A = \text{min-cut}$

Connection to AdS/CFT: The depth coordinate z in our framework plays the role analogous to the radial coordinate in AdS/CFT holography, with the reconstruction functional $\mathscr{F}$ corresponding to bulk-boundary correspondence. In both cases, "deeper" structure encodes coarser, more entangled descriptions of the boundary theory. The key difference is that AdS/CFT presupposes a bulk geometry into which the radial coordinate is embedded, whereas here depth is reconstructed from update sequencing without assuming any ambient space.

Appendix C: Quantitative Estimates

C.1 Correlation Ratio (Derived)

From $VV^\dagger = P \neq I$:

$\langle O^2 \rangle / \langle (POP)^2 \rangle = 2$

Parameter-free, dimensionless. This is our most robust, near-term testable prediction.

C.2 Anisotropy (Conjectural)

$\delta c/c \sim (\ell/L)^\alpha$, where $\alpha \geq 2$

For $\ell = \ell_{Planck} \approx 1.6 \times 10^{-35}$ m and L = 1 m with $\alpha = 2$:

$\delta c/c \sim 10^{-70}$

This is far below current experimental sensitivity ($\sim 10^{-18}$). Non-detection constrains ℓ_{update} to be at or below the Planck scale.

References

1. Taylor, K. "Depth Is Not a Direction: Renormalisation, Holography, and the Categorical Error of Geometric Scale." VERSF Research Programme (2025). Available at: www.versf-eos.com/papers
2. Maldacena, J. "The large N limit of superconformal field theories and supergravity." *Adv. Theor. Math. Phys.* **2**, 231–252 (1998). arXiv:hep-th/9711200
3. Ryu, S. & Takayanagi, T. "Holographic derivation of entanglement entropy from AdS/CFT." *Phys. Rev. Lett.* **96**, 181602 (2006). doi:10.1103/PhysRevLett.96.181602
4. Swingle, B. "Entanglement renormalisation and holography." *Phys. Rev. D* **86**, 065007 (2012). doi:10.1103/PhysRevD.86.065007
5. Harlow, D. "TASI lectures on the emergence of bulk physics in AdS/CFT." *PoS* TASI2017, 002 (2018). arXiv:1802.01040
6. Vidal, G. "Entanglement renormalisation." *Phys. Rev. Lett.* **99**, 220405 (2007). doi:10.1103/PhysRevLett.99.220405
7. Almheiri, A., Dong, X. & Harlow, D. "Bulk locality and quantum error correction in AdS/CFT." *JHEP* **04**, 163 (2015). doi:10.1007/JHEP04(2015)163
8. Lieb, E.H. & Robinson, D.W. "The finite group velocity of quantum spin systems." *Commun. Math. Phys.* **28**, 251–257 (1972). doi:10.1007/BF01645779
9. Bekenstein, J.D. "Black holes and entropy." *Phys. Rev. D* **7**, 2333 (1973). doi:10.1103/PhysRevD.7.2333
10. Arnowitt, R., Deser, S. & Misner, C.W. "The dynamics of general relativity." *Gen. Rel. Grav.* **40**, 1997–2027 (2008). doi:10.1007/s10714-008-0661-1
11. Robertson, N. & Seymour, P.D. "Graph Minors. XIII. The Disjoint Paths Problem." *J. Combin. Theory Ser. B* **63**, 65–110 (1995). doi:10.1006/jctb.1995.1006
12. Sorkin, R.D. "Causal sets: Discrete gravity." In *Lectures on Quantum Gravity*, eds. A. Gomberoff & D. Marolf, 305–327 (Springer, 2005). arXiv:gr-qc/0309009
13. Verlinde, E. "On the origin of gravity and the laws of Newton." *JHEP* **04**, 029 (2011). doi:10.1007/JHEP04(2011)029
14. Penrose, R. "Angular momentum: an approach to combinatorial space-time." In *Quantum Theory and Beyond*, ed. T. Bastin, 151–180 (Cambridge University Press, 1971).
15. Rovelli, C. "Relational quantum mechanics." *Int. J. Theor. Phys.* **35**, 1637–1678 (1996). doi:10.1007/BF02302261