

VERSF: A Unified Coarse-Grained Account of Mass and Gravity

Keith Taylor VERSF Theoretical Physics Program

General Reader Abstract

Why do objects have mass, and why does mass create gravity? Standard physics treats these as separate questions with separate answers — mass comes from the Higgs field, gravity comes from the curvature of spacetime. This paper proposes that they are the same phenomenon viewed at different scales, both emerging from irreversible information changes on a deeper substrate.

The key idea: particles maintain their existence through a repeating process of irreversible "bit-flips" — discrete, permanent selections imposed on the substrate's available configurations. (The substrate itself, a zero-entropy ground, is unchanged; what changes is which of its configurations remain accessible.) The density of committed changes per completed commitment cycle is what we experience as mass. A particle whose existence requires more committed changes per completed cycle has more mass. This much was established in prior VERSF work.

The new result is about gravity. The paper builds the case at four levels, each building on the one before:

Level 1 — The microscopic mechanism: constraint overlap. When committed changes are concentrated in a region (i.e., where mass is present), they constrain the surrounding substrate's available configurations. Crucially, when two regions of high commitment density are close together, their constraints overlap: the total reduction in configurations is less than the sum of the individual reductions, because some configurations are eliminated by both. This means the substrate's total disorder (entropy) is higher when mass is clustered than when it is spread apart. Since physical systems evolve toward higher entropy, mass is driven toward mass. That is the origin of gravitational attraction.

Level 2 — The shape of the force: why inverse-square? Constraint influence spreads from site to site through the substrate by local diffusion — each site shares its constraints with its neighbors. In three spatial dimensions, any conserved quantity that diffuses locally and without loss produces a $1/r$ potential, which gives an inverse-square force. The "without loss" condition is enforced by a foundational principle of the framework: Bit Conservation and Balance (BCB), which forbids the substrate from restoring eliminated configurations. If the substrate could forget its constraints at large scales, gravity would weaken faster than $1/r^2$ — but BCB prevents this, and a symmetry argument shows that no amount of coarse-graining can change that. The inverse-square law is therefore not put in by hand; it is the only possibility consistent with the substrate's own rules.

Level 3 — Newton's law and inertia. The paper formalizes the entropy argument mathematically and recovers Newton's gravitational law — the exact Poisson equation $\nabla^2\Phi = 4\pi G\rho$ — through two independent derivations. It also derives a mechanism for inertia: massive objects resist acceleration because the substrate resists reorganizing a structured commitment pattern. The resulting force law has the structure of $F = ma$. The Schwarzschild radius of a black hole emerges as the surface where commitment density saturates the substrate's information capacity.

Level 4 — From Newton to Einstein. A mathematical theorem (due to Weinberg and Deser) states that any long-range force coupling universally to energy must be carried by a massless spin-2 field, and the only consistent theory of such a field is general relativity. Since the entropic force derived here couples to all forms of energy, the framework inherits the full structure of Einstein's gravitational field equations — not just Newton's.

In short: mass is how densely a particle imposes irreversible selections on the substrate's available configurations; gravity is the entropy benefit of clustering those selections together. The inverse-square law follows from conservation; the extension to Einstein's theory follows from universality. One phenomenon, four levels, no new ingredients.

Technical Abstract

This paper develops a unified account of mass and gravity within the Void Energy-Regulated Space Framework (VERSF), demonstrating that both phenomena emerge from irreversible bit-commitment dynamics on a discrete substrate when viewed at different coarse-graining scales. At the microscopic level, rest mass is the density of completed anchoring cycles per completed commitment (in the effective field theory, per substrate tick), governed by phase coherence between interface modes and the void substrate. At macroscopic scales, accumulated commitment density constrains the surrounding substrate's available configurations, generating entropy gradients that produce gravitational attraction. We derive the nonlocal entropy functional from a microscopic overlap kernel encoding subadditive constraint intersection, derive the kernel's functional form $\hat{K}(r) = B/r$ via cumulant expansion and lattice transport (with $1/r$ shape following from locality, isotropy, and BCB, and amplitude B determined by mean field up to an $O(1)$ prefactor c_χ), show that the $1/r$ form is the symmetry-protected infrared fixed point via an RG argument in which Bit Conservation and Balance (BCB) forbids a screening mass term through shift symmetry, perform a controlled gradient expansion to obtain the local effective functional, and recover Poisson's equation $\nabla^2\Phi = 4\pi G\rho$ via two independent routes: (i) a Hubbard–Stratonovich auxiliary-field method showing the saddle-point equation is Poisson when the inverse kernel is the Laplacian, and (ii) an entropy-flux conservation argument showing that locality, isotropy, and source conservation imply a Gauss-law structure yielding the inverse-square force from three-dimensional geometry alone. The sign of the gravitational force is proved attractive under explicit kernel conditions. The covariant extension to Einstein's field equations is established via the Weinberg–Deser spin-2 uniqueness theorem: any long-range interaction coupling universally to the stress-energy tensor must be mediated by a massless spin-2 field whose unique nonlinear completion is general relativity. A mesoscopic mechanism for

inertia is derived via linear response theory: a force–response relation with the structure of $F = ma$ is obtained, with effective inertial mass identified as the inverse susceptibility of the commitment density field to external perturbations. We situate the framework relative to the entropic gravity programs of Jacobson (1995), Verlinde (2011), and Padmanabhan, identifying where VERSF provides new microscopic content and where it inherits known results. The distinction between derived results, controlled approximations, and imported theorems is maintained throughout.

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1. Introduction

VERSF proposes that physical reality emerges from a discrete, zero-entropy substrate governed by bit conservation and irreversible commitment. Previous work within the program established two major results independently: that rest mass emerges from irreversible anchoring cycles regulated by phase coherence, and that spatial structure emerges from entropy gradients on the substrate. This paper demonstrates that mass and gravity are two scale-dependent expressions of the same underlying mechanism — irreversible information commitment — and formalizes the coarse-graining structure connecting them.

The central claim is that mass is the microscopic density of completed state commitments, while gravity is the macroscopic entropy-gradient response of the substrate to spatial variation in that density. The distinction between mass and gravity arises entirely from the scale at which commitment density is examined. This claim, if sustained, unifies two pillars of fundamental physics within a single information-theoretic substrate.

Logical architecture. Section 2 states all assumptions and imports explicitly. Section 3 establishes the microscopic mass relation. Section 4 defines the coarse-graining operator. Section 5 derives the entropy functional from a microscopic overlap kernel — this is the paper's central technical contribution — derives the kernel's functional form from constraint propagation statistics (Section 5.5), and establishes that the $1/r$ kernel is the symmetry-protected infrared fixed point via an RG no-screening argument based on BCB symmetry (Section 5.5.6). Section 6 recovers the Newtonian limit via two independent routes (Hubbard–Stratonovich auxiliary field and entropy-flux Gauss law), proves the entropic force is attractive, and establishes the self-consistency relation for Newton's constant. Section 7 develops the mesoscopic mechanism for inertia: a qualitative argument (Section 7.2) followed by a quantitative linear-response derivation obtaining a force–response relation with the structure of $F = ma$, with effective inertial mass as the inverse susceptibility of the commitment density field (Section 7.3). Section 8 constructs the effective stress-energy tensor and establishes the covariant extension to general relativity via the Weinberg–Deser spin-2 uniqueness theorem. Sections 9–11 develop consequences: scale flow, horizon structure, and field-theoretic embedding. Section 12 situates VERSF relative to established entropic gravity programs. Section 13 places this paper within the broader VERSF gravity programme, clarifying how it complements rather than replaces earlier manuscripts. Section 14 identifies open problems. Five appendices address structural hinges: exact bit conservation (Appendix A), emergent Lorentz invariance (Appendix B), the equivalence principle (Appendix C), Newton's constant normalization (Appendix D), and substrate temperature (Appendix E).

Classification of results.

Derived in this paper under stated assumptions:

- The nonlocal entropy functional from the overlap kernel (Section 5)
- The gradient expansion with coefficients determined by kernel moments (Section 5.3)
- Poisson's equation via Hubbard–Stratonovich auxiliary field (Section 6.2)
- Poisson's equation via entropy-flux Gauss law (Section 6.6)
- Attractive sign of the gravitational force (Section 6.4)
- Formal definition and consistency of T_{sub} from substrate state counting (Section 6.1a)
- Force–response relation with $F = ma$ structure via linear response, $m_{\text{eff}} = \chi_0^{-1}$ (Section 7.3)
- Overlap kernel $\hat{K}(r) = B/r$: shape from symmetry (locality + isotropy + BCB), amplitude from mean field (Section 5.5)
- Infrared no-screening fixed point: BCB forbids mass term via shift symmetry, making $1/r$ the symmetry-protected IR kernel (Section 5.5.6); no-leakage theorem strengthens to structural necessity (Appendix A)

Imported from established results:

- Mass–commitment identification $m = \eta \hbar \rho_c / (c^2 \Delta t)$ (void anchoring/coupling papers)
- Spin-2 uniqueness: universal coupling to $\bar{T}_{\mu\nu} \rightarrow$ massless spin-2 \rightarrow Einstein-Hilbert (Weinberg 1964, Deser 1970)
- Gravity as one of four admissibility rules (Four Fundamental Interactions paper)
- $E = mc^2$ / relativistic mass-energy equivalence

Formally motivated conjectures:

- Equivalence of effective inertial mass m_{eff} and gravitational mass (equivalence principle, Section 7.3; structural conditions analyzed in Appendix C)
- Exactness of BCB (assumed in no-screening theorem; strengthened to structural necessity by no-leakage theorem, Appendix A; if only approximate, small m^2 survives)
- Emergent Lorentz invariance in the IR (required for Weinberg–Deser; RG argument in Appendix B)
- Scalar field embedding $\tilde{\rho}_c \propto |\partial_t \phi|^2$ (Section 11)
- Full computation of χ_0 in substrate parameters (Section 7.3, open)

This distinction is maintained throughout.

2. Assumptions and Imports

The following assumptions define the starting point of this paper. Each is stated explicitly so that any can be independently scrutinized, relaxed, or replaced.

Foundational postulate. We postulate that the fundamental substrate of reality consists of discrete degrees of freedom whose only primitive attribute is their distinguishability from absence (an empty configuration). Physical observables, including mass and gravitational interaction, arise from the patterns of irreversible commitments among these distinguishable states. All assumptions below are specifications of this postulate.

A1. Discrete substrate. Physical reality is underlain by a discrete lattice of substrate sites with spacing a (the substrate lattice constant). Each site admits $W_0 \gg 1$ internal states (the substrate alphabet). The substrate state space is $\Omega = \{1, \dots, W_0\}^{\{\text{sites}\}}$.

A2. Irreversible commitment. At each discrete tick of duration Δt , certain sites undergo irreversible state transitions ("commitments") that permanently eliminate a subset of previously accessible configurations. Each commitment event at a site eliminates a fraction $f \in (0, 1)$ of the remaining accessible states at that site.

A3. Mass–commitment identification. The rest mass of a localized mode is proportional to its per-tick commitment density ρ_c (completed commitments per tick per site), via the relation $m =$

$\eta\hbar\rho_c/(c^2\Delta t)$, where η is the action normalization (see Section 3). This is established in the void coupling paper.

A4. Overlap kernel. When two sites at positions x and y both host commitments, the constraints they impose on the intervening substrate overlap with a strength characterized by a kernel $K(|x - y|)$. The kernel $K(r)$ satisfies:

- $K(r) \geq 0$ for all r (overlap is non-negative),
- $K(r)$ is monotonically decreasing (overlap weakens with distance),
- $K(r)$ decays sufficiently rapidly that $\int K(r) d^3r < \infty$ (finite total overlap),
- $K(r)$ is isotropic (depends only on $|x - y|$).

The physical content of $K(r)$ is: it encodes the probability that the constraint sets eliminated by commitments at x and y intersect, as a function of their separation. The specific form of $K(r)$ — whether exponential, Gaussian, power-law, or other — is not assumed; results are stated in terms of the kernel's moments. In Section 5 we define the dimensionless kernel $\hat{K}(r) = K(r)/k_B$ and work with \hat{K} throughout the remainder of the paper.

A5. Ensemble and thermodynamic control. The uncommitted substrate degrees of freedom are treated as a thermal bath in an isothermal ensemble at effective temperature T_{sub} . The committed modes (mass-bearing structures) are the "system"; the uncommitted substrate is the "reservoir." In this ensemble, the appropriate potential governing the system's equilibrium is the free energy $F = U - T_{\text{sub}} S$, where U is the internal energy of the committed modes and S is the substrate entropy. The entropic force arises in the regime where changes in U are negligible compared to $T_{\text{sub}} \Delta S$ — i.e., where the dominant effect of rearranging commitment density is the entropy change, not the energy change. This is the "entropic dominance" regime and is the standard condition for entropic forces (cf. polymer elasticity, depletion forces, Verlinde's argument).

A6. Coarse-graining. Spatial averaging is performed via a spherical ball kernel of radius L , as defined in Section 4.

Dimensional bookkeeping. Throughout this paper, commitment density ρ_c is dimensionless — it is the probability per tick that a given site hosts a completed commitment. When converting to a per-volume quantity for continuum integrals, we multiply by the site number density $n_s = 1/a^3$, giving:

$$\tilde{\rho}_c(x) = n_s \cdot \rho_c(x) \text{ [dimensions: 1/volume]}$$

All continuum integrals ($\int d^3x$) in the entropy functional use the per-volume density $\tilde{\rho}_c$. The mass density is:

$$\rho_m(x) = (\eta\hbar / c^2\Delta t) \cdot \tilde{\rho}_c(x) \text{ [dimensions: mass/volume]}$$

The overlap kernel $\hat{K}(r)$ is dimensionless (a pure overlap probability). The moments $\hat{K}_0 = \int \hat{K}(r) d^3r$ have dimensions [volume] and $\hat{K}_2 = \int r^2 \hat{K}(r) d^3r$ have dimensions [volume · length²]. The entropy functional coefficients inherit their dimensions:

- Λ has dimensions of [entropy] (entropy cost per unit integrated commitment)
- $\mu = k_B \hat{K}_0$ has dimensions [entropy · volume]
- $\kappa = k_B \hat{K}_2/6$ has dimensions [entropy · volume · length²]
- The quadratic kernel term $(k_B/2) \int \tilde{\rho} \hat{K} \tilde{\rho} d^3x d^3y$ has dimensions [entropy]

Newton's constant G has dimensions of [length³ / (mass · time²)], and its expression in substrate parameters (Section 6) is verified to be dimensionally consistent.

3. Microscopic Layer: Mass as Commitment Density

3.1 Commitment Indicator and Density

Define the commitment indicator:

$\chi(x, n) = 1$ if site x completes an irreversible commitment at tick n ; 0 otherwise

The per-site commitment density (dimensionless) is the tick-averaged commitment rate:

$$\rho_c(x) = (1/N_T) \sum_n \chi(x, n) \quad (1)$$

where the sum runs over N_T ticks. In the continuum limit, $\rho_c(x)$ is the probability per tick that site x hosts a completed commitment.

The per-volume commitment density is:

$$\tilde{\rho}_c(x) = n_s \cdot \rho_c(x) = \rho_c(x) / a^3 \quad (2)$$

3.2 Connection to the Phase Coherence Mass Relation

The void coupling paper establishes that the per-tick micro-event probability is p_ϵ , the anchoring depth is K_c (micro-events per completed commitment), and the rest mass of a mode is:

$$m = \eta \hbar p_\epsilon / (c^2 \Delta t K_c) \quad (3)$$

The rate of completed commitments per tick is p_ϵ / K_c . For a single mode localized at site x :

$$\rho_c(x) = p_\epsilon / K_c \quad (4)$$

The commitment density $C(x) := p_\varepsilon(x)/K_c(x) = \rho_c(x)$ is therefore proportional to mass density and is the unique microphysical source variable used throughout this paper. The same quantity that sets rest mass at the microscopic scale becomes the source term in the entropy kernel at the macroscopic scale — this identification is the core of the mass–gravity unification.

The mass relation becomes:

$$m = \eta \hbar \rho_c / (c^2 \Delta t) \quad (5)$$

For the per-volume density:

$$\rho_m(x) = (\eta \hbar / c^2 \Delta t) \cdot \tilde{\rho}_c(x) \quad (6)$$

This identifies mass density as commitment density (per volume) times a universal conversion factor.

3.3 Properties of Commitment Density

Non-negativity. $\rho_c(x) \geq 0$ everywhere.

Boundedness. $\rho_c(x) \leq 1$ (at most one commitment per tick per site in the simplest model).

Conservation. In the absence of creation or annihilation:

$$d/dt \int \tilde{\rho}_c(x) d^3x = 0 \quad (7)$$

corresponding to conservation of total mass-energy.

Localization. For particle-like modes, $\rho_c(x)$ is concentrated within a region of characteristic size ξ .

4. The Coarse-Graining Operator

4.1 Definition

The coarse-graining operator C_L averages the per-volume commitment density over scale L :

$$C_L \tilde{\rho}_c = (1/V_L) \int_{B_L(x)} \tilde{\rho}_c(y) d^3y \quad (8)$$

where $B_L(x)$ is a ball of radius L centered at x and $V_L = (4/3)\pi L^3$. We write $\bar{\rho}_L(x) := C_L \tilde{\rho}_c$ for the coarse-grained per-volume commitment density.

4.2 Scale Regimes

Microscopic ($L \lesssim \xi$): $C_L \approx \text{identity}$. Individual commitment events are resolved. The mass formula Eq. (3) applies directly.

Mesoscopic ($\xi \lesssim L \lesssim L_{\text{grav}}$): $\bar{\rho}_L$ becomes a smooth field. This is the regime of inertial physics (Section 7).

Macroscopic ($L \gtrsim L_{\text{grav}}$): Entropy-gradient description becomes valid. This is the regime of gravitational physics (Sections 5–6).

4.3 Properties

C_L is linear, mass-conserving ($\int \bar{\rho}_L d^3x = \int \tilde{\rho}_c d^3x$), and monotonically smoothing. It is valid as a continuum operator only when $L \gg a$.

5. Distinguishability, Overlap Kernel, and Entropy Functional

This section derives the entropy functional in three stages: local constraint counting, nonlocal overlap structure, and controlled gradient expansion. The derivation follows from assumptions A1–A4.

5.1 Local Entropy Deficit (Single-Site)

At a site with no commitments, all W_0 internal states are accessible, giving entropy per site:

$$s_0 = k_B \ln W_0 \quad (9)$$

When a site hosts commitment density ρ_c (probability per tick of commitment), each commitment event eliminates a fraction f of accessible states (assumption A2). Over the averaging window, the effective number of accessible states is:

$$W(x) = W_0 \cdot (1 - f)^{\{\rho_c(x) / \rho_0\}} \quad (10)$$

where ρ_0 is a normalization: the commitment density at which one complete "round" of elimination has occurred. Since ρ_c is a probability per tick and f is the fraction eliminated per commitment, we set $\rho_0 = 1$ (one commitment per tick is one full round), giving:

$$W(x) = W_0 \cdot (1 - f)^{\{\rho_c(x)\}} \quad (11)$$

The local entropy is:

$$s(x) = k_B \ln W(x) = s_0 + k_B \rho_c(x) \ln(1 - f) = s_0 - \lambda \rho_c(x) \quad (12)$$

where:

$$\lambda = -k_B \ln(1 - f) > 0 \quad (13)$$

This is the entropy cost per unit commitment density at a single site. Converting to per-volume entropy density and integrating:

$$S_{\text{local}} = \int n_s [s_0 - \lambda \rho_c(x)] d^3x = S_0 - \Lambda \int \tilde{\rho}_c(x) d^3x \quad (14)$$

where $S_0 = n_s s_0 V$ is the total uncommitted entropy and $\Lambda = \lambda = -k_B \ln(1-f)$ carries dimensions of [entropy] (since $\tilde{\rho}_c d^3x$ is dimensionless after n_s cancels a^3). This is the linear (non-interacting) contribution to the entropy functional.

5.2 Nonlocal Overlap Kernel

The local calculation above treats each site independently. In reality, commitments at nearby sites impose overlapping constraints on the shared configuration space of the substrate. This overlap is the origin of the attractive entropic interaction.

Consider two sites at positions x and y , each hosting commitment density $\tilde{\rho}_c$. Each eliminates a set of substrate configurations locally. If the sites are far apart ($|x - y| \gg L_{\text{corr}}$, where L_{corr} is the substrate correlation length for constraint propagation), the eliminated sets are statistically independent and the total entropy deficit is additive. If the sites are close ($|x - y| \lesssim L_{\text{corr}}$), the eliminated sets overlap — some configurations are eliminated by both commitments — and the total entropy deficit is less than the sum. The entropy is therefore higher (less negative) when commitments are clustered.

This subadditive structure is encoded in the overlap kernel $K(|x - y|)$ (assumption A4). The full nonlocal entropy functional is:

$$S[\tilde{\rho}_c] = S_0 - \Lambda \int \tilde{\rho}_c(x) d^3x + (k_B/2) \iint \tilde{\rho}_c(x) \hat{K}(|x - y|) \tilde{\rho}_c(y) d^3x d^3y + O(\tilde{\rho}_c^3) \quad (15)$$

Notational convention. From this point forward, we use $\hat{K}(r)$ to denote the dimensionless overlap probability kernel (a pure number measuring the probability that constraints at separation r intersect). The dimensionful entropy kernel appearing in assumption A4 is $K(r) = k_B \hat{K}(r)$. All subsequent equations — the gradient expansion, the Hubbard–Stratonovich transformation, and the Poisson derivation — use $\hat{K}(r)$. The qualitative conditions of A4 (non-negative, decreasing, integrable, isotropic) apply equally to $\hat{K}(r)$.

The three terms have distinct physical content:

- S_0 : entropy of the uncommitted substrate (constant).
- $-\Lambda \int \tilde{\rho}_c$: linear entropy cost of commitment (each commitment reduces accessible states).

- $+(1/2)\iint \tilde{\rho}_c \mathbf{K} \tilde{\rho}_c$: quadratic entropy gain from constraint overlap. This term is positive because $\mathbf{K} \geq 0$ (assumption A4): overlapping constraints reduce the net entropy cost, so clustered commitments carry less entropy penalty than dispersed ones.
- $\mathbf{O}(\tilde{\rho}^3)$: higher-order overlap terms (three-body and beyond), neglected in this treatment.

Status of Eq. (15). The linear term is derived from the single-site multiplicative constraint model (Section 5.1). The quadratic term follows from the general combinatorial fact that constraint intersection is subadditive — $|A \cup B| \leq |A| + |B|$ for any two sets A, B of eliminated configurations — encoded in the kernel $\hat{K}(r)$. The truncation at quadratic order is a controlled approximation valid when $\tilde{\rho}_c$ is small compared to $1/a^3$ (dilute commitment regime). The kernel $\hat{K}(r)$ is the microscopic input whose qualitative properties are stated in assumption A4; its specific functional form is derived from constraint propagation statistics in Section 5.5.

5.3 Gradient Expansion of the Kernel

For slowly varying commitment density (variations on scales $L \gg L_{\text{corr}}$), the nonlocal quadratic term admits a controlled gradient expansion. Substituting $\tilde{\rho}_c(y) = \tilde{\rho}_c(x) + (y - x) \cdot \nabla \tilde{\rho}_c + \frac{1}{2}(y - x)_i (y - x)_j \partial_i \partial_j \tilde{\rho}_c + \dots$ into the double integral:

$$(k_B/2) \iint \tilde{\rho}_c(x) \hat{K}(|x - y|) \tilde{\rho}_c(y) d^3x d^3y$$

the zeroth-order term gives:

$$(k_B \hat{K}_0/2) \int \tilde{\rho}_c(x)^2 d^3x \text{ where } \hat{K}_0 = \int \hat{K}(r) d^3r \quad (16)$$

The first-order term vanishes by isotropy of \hat{K} . The second-order term gives:

$$-(k_B \hat{K}_2/12) \int \tilde{\rho}_c(x) \nabla^2 \tilde{\rho}_c(x) d^3x \text{ where } \hat{K}_2 = \int r^2 \hat{K}(r) d^3r \quad (17)$$

Integrating by parts (assuming $\tilde{\rho}_c$ vanishes at infinity):

$$+(k_B \hat{K}_2/12) \int |\nabla \tilde{\rho}_c(x)|^2 d^3x \quad (18)$$

The local effective entropy functional through second order in gradients is therefore:

$$S_{\text{eff}}[\tilde{\rho}_c] = S_0 - \Lambda \int \tilde{\rho}_c d^3x + (\mu/2) \int \tilde{\rho}_c^2 d^3x + (\kappa/2) \int |\nabla \tilde{\rho}_c|^2 d^3x + \mathcal{O}(\nabla^4) \quad (19)$$

where:

$$\mu = k_B \hat{K}_0 = k_B \int \hat{K}(r) d^3r > 0 \quad (20)$$

$$\kappa = k_B \hat{K}_2/6 = (k_B/6) \int r^2 \hat{K}(r) d^3r > 0 \quad (21)$$

Signs. Both μ and κ are positive because $\hat{K}(r) \geq 0$ (assumption A4). The μ -term is the bulk overlap gain: clustering increases entropy. The κ -term is the gradient correction: it is positive, meaning entropy increases with spatial variation in commitment density. This may seem

counterintuitive, but it follows from the expansion: the κ -term represents the second-order correction to the overlap integral, which captures additional overlap contributions from density gradients. The gradient expansion is valid when the density varies on scales much larger than the kernel's correlation length $L_{\text{corr}} = \sqrt{(\hat{K}_2/\hat{K}_0)}$.

Sign convention note. Note that $\kappa > 0$ is a consequence of $\hat{K} \geq 0$. Whether gradients are favored or disfavored in equilibrium depends on the full free energy $F = U - T_{\text{sub}} S$: the entropy contribution alone favors configurations that increase overlap, while energetic terms and higher-order gradient corrections can penalize sharp interfaces. In this paper we work in the weak-field, long-wavelength regime where the gradient expansion is controlled and Poisson behavior emerges.

Relation to Eq. (15) of the previous draft. The entropy functional Eq. (19) replaces the previously asserted Ginzburg–Landau form. It is now derived from the nonlocal kernel via controlled gradient expansion, with coefficients μ and κ determined by the kernel's zeroth and second moments. The signs of both coefficients follow from the positivity of $\hat{K}(r)$, not from phenomenological assertion.

Layer status. The entropy functional Eq. (19) is a macroscopic effective response description — the leading-order terms in a controlled expansion of the nonlocal kernel. It is not a foundational ontology statement. The microscopic ontology resides in the discrete commitment process (Section 3); the kernel $\hat{K}(r)$ encodes the overlap statistics of that process; and Eq. (19) is the effective field-theoretic representation valid at scales $L \gg L_{\text{corr}}$ where the gradient expansion converges. Higher-order terms (∇^4, ρ^3) are systematically computable from higher kernel moments and multi-body overlaps. This classification — effective macroscopic description derived from, but not identical to, the microscopic commitment dynamics — prevents conflating the EFT with the underlying substrate physics.

5.4 Dimensional Verification

Define the overlap term with a dimensionless kernel $\hat{K}(r)$ (a pure overlap probability):

$$S_{\text{overlap}} = (k_B/2) \iint \tilde{\rho}_c(x) \hat{K}(|x-y|) \tilde{\rho}_c(y) d^3x d^3y$$

Since $\tilde{\rho}_c$ has dimensions $[1/\text{volume}]$ and $d^3x d^3y$ has dimensions $[\text{volume}^2]$, the integrand $\tilde{\rho}_c \hat{K} \tilde{\rho}_c d^3x d^3y$ is dimensionless when \hat{K} is dimensionless. The prefactor k_B gives dimensions [entropy]. ✓

The kernel moments: $\hat{K}_0 = \int \hat{K}(r) d^3r$ has dimensions [volume]; $\hat{K}_2 = \int r^2 \hat{K}(r) d^3r$ has dimensions [volume · length²].

The gradient expansion coefficients: $\mu = k_B \hat{K}_0$ has dimensions [entropy · volume]; $\kappa = k_B \hat{K}_2/6$ has dimensions [entropy · volume · length²].

Checking each term in S_{eff} :

- $\Lambda \int \tilde{\rho}_c d^3x: [\text{entropy}] \cdot [1/\text{vol}] \cdot [\text{vol}] = [\text{entropy}] \checkmark$
- $(\mu/2) \int \tilde{\rho}_c^2 d^3x: [\text{entropy} \cdot \text{vol}] \cdot [1/\text{vol}^2] \cdot [\text{vol}] = [\text{entropy}] \checkmark$
- $(\kappa/2) \int |\nabla \tilde{\rho}_c|^2 d^3x: [\text{entropy} \cdot \text{vol} \cdot \text{length}^2] \cdot [1/(\text{vol}^2 \cdot \text{length}^2)] \cdot [\text{vol}] = [\text{entropy}] \checkmark$

5.5 Microscopic Derivation of the Overlap Kernel

The preceding sections defined the overlap kernel $\hat{K}(r)$ through its qualitative properties (assumption A4) and used it to construct the entropy functional. This section derives the kernel's functional form from the substrate's microscopic elimination statistics and constraint propagation structure, resolving what earlier sections identified as the framework's most important open calculation.

5.5.1 Microscopic Random Variables and the Cumulant Expansion

Define an indicator field for eliminated microstate labels at each substrate site:

$\eta_{\{x,s\}} = 1$ if state s is eliminated at site x ; 0 otherwise

where $s \in \{1, \dots, W_0\}$ labels the per-site microstates. The single-site statistics are: $\langle \eta_{\{x,s\}} \rangle = f$ for all s (assumption A2), and the elimination events for different labels at the same site are correlated through the commitment mechanism.

The overlap kernel is then defined as a sum over shared eliminated labels:

$$\hat{K}(x, y) := (1/W_0) \sum_s \langle \eta_{\{x,s\}} \eta_{\{y,s\}} \rangle \quad (22)$$

This is the probability that a randomly chosen microstate label is eliminated at both sites — a dimensionless overlap measure. The connected (cumulant) part is:

$$C(x, y) := (1/W_0) \sum_s [\langle \eta_{\{x,s\}} \eta_{\{y,s\}} \rangle - \langle \eta_{\{x,s\}} \rangle \langle \eta_{\{y,s\}} \rangle] \quad (23)$$

so that $\hat{K}(x, y) = f^2 + C(x, y)$. The constant f^2 is the independent baseline; $C(x, y)$ encodes spatial correlations arising from constraint propagation through the substrate. The only source of spatial correlation in the η field is the transport of constraint influence via the lattice diffusion rule (Section 5.5.2, Eq. (24)); elimination events at distinct sites are conditionally independent given the local distinguishability deficit q_x . This closes the statistical model: all inter-site correlations in η are mediated by q transport, and no additional non-diffusive correlation channel is assumed. We posit no nonlocal constraint mechanism beyond q -mediated transport; all spatial correlations in the elimination field are generated dynamically through Eq. (24).

Cumulant hierarchy truncation. The entropy functional Eq. (15) is quadratic in $\tilde{\rho}_c$, which corresponds to retaining only the two-point connected cumulant $C(x, y)$ and neglecting three-point and higher connected cumulants of the η field. This is a controlled approximation: three-body overlaps contribute at $O(\tilde{\rho}_c^3)$ relative to the two-body term at $O(\tilde{\rho}_c^2)$, suppressed in the dilute commitment regime $\tilde{\rho}_c a^3 \ll 1$. The next correction — the three-point connected

cumulant — would generate a cubic term in the entropy functional and is systematically computable from the substrate's elimination statistics.

5.5.2 Constraint Influence Transport on the Lattice

The connected correlator $C(x, y)$ is determined by how constraint influence propagates through the substrate. Rather than asserting a continuum operator, we derive it from an explicit lattice transport rule.

Distinguishability deficit. Define q_x as the local distinguishability deficit at site x : the reduction in the number of accessible configurations caused by constraint influence from nearby commitments. Commitment events inject source: $s_x \propto \tilde{\rho}_c(x)$. Between commitment events, the deficit spreads to neighboring sites via local diffusion:

$$q_x(t + \Delta t) = q_x(t) + D \sum_{\{x' \sim x\}} [q_{x'}(t) - q_x(t)] + s_x(t) \quad (24)$$

where the sum runs over nearest neighbors $x' \sim x$ and D is the diffusion coefficient for constraint influence on the substrate lattice.

Steady-state equation. In the static limit (relevant for the time-averaged commitment density), $\partial_t q = 0$ and the transport equation becomes:

$$D \Delta_{\text{lat}} q_x = -s_x \quad (25)$$

where Δ_{lat} is the discrete lattice Laplacian, $\Delta_{\text{lat}} q_x := \sum_{\{x' \sim x\}} (q_{x'} - q_x)$. In the continuum limit (scales $\gg a$), this becomes:

$$-D\nabla^2 q(r) = s(r) \quad (26)$$

with Green's function $G(r) = 1/(4\pi Dr)$ in three dimensions — the unique isotropic solution of Laplace's equation. Note: the fundamental transport of constraint influence through the substrate is causal, propagating at finite speed (the emergent speed of light, as analyzed in the companion throughput paper). The elliptic Laplacian form arises because the entropy functional describes the static (time-averaged, zero-frequency) regime; dynamic propagation at finite causal throughput reduces to this in the zero-frequency limit. The derivation is therefore not acausal diffusion but the static sector of a causal transport equation.

BCB and the absence of a relaxation term. If the substrate permitted leakage of constraint influence (restoration of eliminated configurations without compensating structural update), the transport equation would acquire a relaxation term:

$$D \Delta_{\text{lat}} q_x - \gamma q_x = -s_x \rightarrow (-\nabla^2 + m^2)q = s/D \quad (27)$$

with $m^2 = \gamma/D$, giving the Yukawa propagator $G(r) = e^{-mr}/(4\pi r)$. This makes the "screening \Leftrightarrow leakage" identification exact: the screening mass m^2 is proportional to the relaxation rate γ . BCB forbids $\gamma > 0$ (no eliminated configuration may be restored without compensating structural

update), so the steady-state operator is the pure Laplacian and the propagator is $1/r$. (The RG stability of this result is established in Section 5.5.6.)

Key structural point. The functional form of the kernel — $1/r$ in three dimensions — is not a mean-field result. It follows from three ingredients alone: (i) locality of constraint transport (nearest-neighbor coupling \rightarrow Laplacian), (ii) three spatial dimensions (\rightarrow $1/r$ Green's function), and (iii) BCB (\rightarrow no screening mass). The mean-field approximation enters only in determining the amplitude.

5.5.3 Amplitude: Shape from Symmetry, Normalization from Mean Field

The connected correlator at zero separation is fixed by single-site statistics: $\hat{K}(x, x) = (1/W_0) \sum_s \langle \eta_{\{x,s\}} \rangle = f$, so:

$$C(0) = f - f^2 = f(1 - f) \quad (28)$$

This is exact and dimensionless — the single-site connected variance depends only on the elimination fraction f .

Operational definition of susceptibility. Define the constraint susceptibility χ_{sub} via the Kubo relation:

$$\chi_{\text{sub}} := (1/k_B T_{\text{sub}}) \int d^3x \langle \delta q(0) \delta q(x) \rangle_{\text{eq}} \quad (29)$$

where $\delta q = q - \langle q \rangle$ is the fluctuation in the distinguishability deficit field. This is a well-defined equilibrium quantity that can, in principle, be measured in a lattice simulation of the substrate.

Mean-field estimate. Under the Gaussian/mean-field approximation (treating eliminated-state fluctuations as independent across sites), the susceptibility is determined by the local variance and the lattice regularization:

$$\chi_{\text{sub}} \approx c_{\chi} \cdot f(1 - f) \cdot a / (k_B T_{\text{sub}}) \quad (30)$$

where $c_{\chi} = c_{\chi}(W_0, \text{lattice connectivity})$ is an $O(1)$ dimensionless prefactor that depends on the substrate's detailed coupling structure. The mean-field estimate gives the correct scaling in f and a ; the prefactor c_{χ} is computable by lattice simulation or higher-order closure but is not determined by the mean-field treatment alone.

Note on mean-field circularity. The mean-field identification of the correlator amplitude with the single-site variance assumes weak spatial correlations, yet the resulting long-range kernel implies they are not negligible at large scales. Self-consistency requires that beyond-mean-field corrections remain small; this is controlled when $W_0 \gg 1$ (see validity conditions below) but has not been verified by explicit calculation. Crucially, this circularity affects only the amplitude, not the functional form — the $1/r$ shape is protected by the lattice transport derivation (Section 5.5.2) and BCB symmetry (Section 5.5.6).

The connected correlation function is:

$$C(r) \approx \chi_{\text{sub}} \cdot G(r) \quad (31)$$

and the full overlap kernel becomes:

$$\hat{K}(r) = f^2 + \chi_{\text{sub}} \cdot G(r) \quad (32)$$

Dropping the constant baseline f^2 (which contributes only to the bulk entropy term, not to the spatial interaction), the correlated part that drives gravitational interaction is:

$$\hat{K}(r) = B/r, \quad B = \chi_{\text{sub}}/(4\pi) = c_{\chi} f(1-f) \cdot a / (4\pi k_B T_{\text{sub}}) \quad (33)$$

The amplitude B has dimensions of [length]. Its scaling — proportional to $f(1-f) \cdot a$ — is a mean-field result. Its precise numerical value requires determining the $O(1)$ prefactor c_{χ} , which encodes the lattice geometry and inter-site coupling structure.

5.5.4 Validity Conditions

The kernel derivation rests on two controlled approximations whose validity conditions can be stated precisely:

Validity box. The mean-field + two-point cumulant truncation is controlled when:

$$W_0 \gg 1 \text{ and } \tilde{\rho}_c a^3 \ll 1 \quad (34)$$

These conditions have distinct roles:

Large alphabet ($W_0 \gg 1$): The effective expansion parameter for the mean-field approximation is $1/W_0$. Per-site fluctuations of the elimination field $\eta_{\{x,s\}}$ scale as $\sim 1/\sqrt{W_0}$, providing self-averaging analogous to large- N theories. Higher connected cumulants of the elimination field are suppressed by $O(1/W_0)$ relative to the two-point function. This is the Ginzburg criterion for the present system.

Dilute commitment ($\tilde{\rho}_c a^3 \ll 1$): Multi-body overlaps (three or more commitment regions overlapping simultaneously) are suppressed as $O(\tilde{\rho}_c^3)$ relative to the two-body overlap at $O(\tilde{\rho}_c^2)$. This justifies the quadratic truncation of the entropy functional and the two-point cumulant truncation.

Both conditions are assumed throughout (A1 for the first; dilute regime for the second). Beyond-mean-field corrections enter at $O(1/W_0)$ and $O(\tilde{\rho}_c a^3)$, and are systematically computable as the first corrections to the Gaussian approximation.

5.5.5 Kernel Moments and Connection to the Entropy Functional

The kernel moments that determine the gradient expansion coefficients (Section 5.3) are now expressible in terms of substrate parameters.

For the screened kernel $\hat{K}(r) = B \cdot e^{-mr}/r$:

$$\hat{K}_0 = \int \hat{K}(r) d^3r = 4\pi B/m^2 = \chi_{\text{sub}}/m^2 \quad (35)$$

$$\hat{K}_2 = \int r^2 \hat{K}(r) d^3r = 24\pi B/m^4 = 6\chi_{\text{sub}}/m^4 \quad (36)$$

The gradient expansion coefficients become:

$$\mu = k_B \hat{K}_0 = k_B \chi_{\text{sub}}/m^2 \quad (37)$$

$$\kappa = k_B \hat{K}_2/6 = k_B \chi_{\text{sub}}/m^4 \quad (38)$$

In the unscreened limit $m \rightarrow 0$, both moments diverge — this is precisely the mathematical signature of long-range gravity. The divergence of \hat{K}_0 means the bulk overlap integral extends to infinity, and the Hubbard–Stratonovich inverse $\hat{K}^{-1} \propto -\nabla^2$ produces an exact Poisson equation rather than a screened one.

5.5.6 Infrared No-Screening Fixed Point

The preceding sections derived the kernel form under the assumption that constraint propagation is either screened ($m > 0$) or unscreened ($m = 0$), leaving the screening question open. This section shows that Bit Conservation and Balance (BCB) — the foundational VERSF principle that no eliminated configuration may be restored without a compensating structural update — forces $m = 0$ as the symmetry-protected infrared fixed point. The argument proceeds in four steps.

Step 1: Most general IR effective action. At long wavelengths, the overlap potential Φ introduced via Hubbard–Stratonovich (Section 6.2) must have the most general local quadratic form consistent with isotropy:

$$S_{\text{eff}}[\Phi] = \int d^3x [(\alpha/2)(\nabla\Phi)^2 + (m^2/2)\Phi^2 - \Phi\tilde{\rho}_c] \quad (39)$$

where $\alpha > 0$ follows from kernel positivity (Section 5.2), m^2 is the screening parameter, and $\tilde{\rho}_c$ is the commitment density source. If $m^2 = 0$, the field equation is Poisson; if $m^2 > 0$, it is the screened Poisson (Yukawa) equation.

Step 2: Physical meaning of the mass term. The static field equation is:

$$(-\alpha\nabla^2 + m^2)\Phi = \tilde{\rho}_c \quad (40)$$

Rewriting in divergence form with $J = -\alpha\nabla\Phi$:

$$\nabla \cdot J = \tilde{\rho}_c - (m^2/\alpha)\Phi \quad (41)$$

The second term on the right is a sink: it represents relaxation of constraint influence at a rate proportional to the field value. In substrate language, screening corresponds to a relaxation channel in the distinguishability accounting equation — the substrate forgets constraints at large scales. BCB forbids such relaxation: no eliminated configuration may be restored without a compensating structural update. Therefore $m^2 = 0$ is the BCB-consistent value: every unit of constraint influence sourced by committed sites propagates without loss.

Step 3: RG scaling. Under spatial rescaling $x \rightarrow bx$ in $d = 3$ dimensions, requiring the gradient term to remain scale-invariant fixes the field scaling dimension $\Delta_\Phi = (d - 2)/2 = 1/2$. The mass term then scales as:

$$m^2 \rightarrow b^2 m^2 \quad (42)$$

Since $b > 1$ under coarse-graining (flowing to larger scales), m^2 is a relevant perturbation: if present, it grows under RG flow. This means $m^2 = 0$ is a fixed point. But is it stable against radiative generation?

Step 4: Shift symmetry from BCB. The crucial question is whether coarse-graining can generate a mass term even if $m^2 = 0$ microscopically. This is where the substrate's conservation structure enters.

BCB forbids any relaxation channel for eliminated configurations — the coarse-grained continuity equation for constraint flux therefore admits no sink term:

$$\partial_t q + \nabla \cdot J = \tilde{\rho}_c \quad (43)$$

with no $-(m^2/\alpha)\Phi$ leakage. In the effective action, this conservation law is enforced by a shift symmetry:

$$\Phi \rightarrow \Phi + \text{const} \quad (44)$$

Under this symmetry, the gradient term $(\nabla\Phi)^2$ is invariant and the mass term $m^2\Phi^2$ is not. The source coupling $\int \Phi \tilde{\rho}_c d^3x$ explicitly breaks the global shift (since $\int \text{const} \cdot \tilde{\rho}_c d^3x \neq 0$ for nonzero total mass) but preserves the local differential structure that forbids m^2 ; this parallels electrodynamics, where charged sources break global gauge invariance while the photon propagator remains massless. Technically: the source couples linearly to Φ and does not induce a quadratic Φ^2 term under coarse-graining, because conservation protects the derivative-only kinetic structure of the propagator even in the presence of sources. Therefore:

Since BCB enforces conservation of constraint influence, the shift symmetry Eq. (44) forbids the mass term at every order in the coarse-graining expansion. The term $m^2\Phi^2$ cannot be generated radiatively.

Proposition (Symmetry-Protected No-Screening Fixed Point). Consider the coarse-grained effective action Eq. (39) for the overlap potential Φ . If (i) constraint influence is locally conserved (no sink channel) and (ii) BCB forbids restoration of eliminated configurations

without compensating structural update, then the effective theory inherits a shift redundancy $\Phi \rightarrow \Phi + \text{const}$ that forbids the mass term m^2 at every order in the coarse-graining expansion. The infrared fixed point is $m^2 = 0$, the Green's function is $\hat{K}(r) \sim 1/(4\pi r)$, and gravity is inverse-square.

Structural parallel. This is the same mechanism that keeps photons massless in electrodynamics: gauge invariance ($A_\mu \rightarrow A_\mu + \partial_\mu \lambda$) forbids a photon mass term $m^2 A_\mu A^\mu$ because it would violate charge conservation. Here, the shift symmetry $\Phi \rightarrow \Phi + \text{const}$ forbids a screening mass because it would violate constraint-influence conservation. The parallel is not coincidental — both are instances of the general principle that exact conservation of a locally transported quantity forbids a mass gap in the mediating field.

Status. The no-screening result rests on the exactness of BCB. Appendix A strengthens this from a principle to a structural necessity: any nonzero leakage $\gamma > 0$ requires spontaneous restoration of eliminated configurations, which contradicts the irreversible commitment ontology (Appendix A, Theorem A.4). If bit conservation is only approximate — if the substrate permits small leakage of constraint influence at very large scales — then a tiny $m^2 > 0$ could survive, producing Yukawa-modified gravity with a range $1/m$. The empirical success of inverse-square gravity from millimeter to megaparsec scales constrains any such leakage to be extraordinarily small ($m \lesssim H_0/c$). Whether BCB is exact or merely an excellent approximation is a question about the substrate's deep structure that lies outside the scope of this paper.

5.5.7 Leading Correction: One-Loop Renormalization of the Amplitude

The no-screening theorem (Section 5.5.6) protects the functional form of the kernel: the $1/r$ shape cannot be modified by coarse-graining. However, the amplitude B is not protected by symmetry and receives corrections when short-scale fluctuations are integrated out. This section sketches the leading correction.

One-loop structure. Integrating out elimination-field fluctuations at scales between a and some IR cutoff Λ^{-1} renormalizes the kernel amplitude. The leading correction has the standard structure of a polarization bubble:

$$B_{\text{eff}}(k) = B_0 [1 + \Pi(k) + O(\Pi^2)] \quad (45)$$

where $B_0 = c_\chi f(1-f) \cdot a / (4\pi k_B T_{\text{sub}})$ is the mean-field (tree-level) amplitude and $\Pi(k)$ is the one-loop self-energy. We do not compute $\Pi(k)$ explicitly here; the relevant structural point is that its magnitude is $O(1/W_0)$, determined by the effective expansion parameter of the large-alphabet limit.

What runs and what doesn't. The shift symmetry Eq. (44) forbids generation of a mass term at any loop order, so the functional form remains $1/r$ — the shape is protected. The amplitude B and the diffusion coefficient D are renormalized: they acquire $O(1/W_0)$ corrections from integrating out high-momentum modes. The leading correction to B scales as:

$$\delta B/B_0 \sim O(1/W_0)$$

so the mean-field amplitude is quantitatively reliable when $W_0 \gg 1$, and the correction is systematically computable as the first term in a $1/W_0$ expansion.

Physical content. This is the standard structure for systems with a protected symmetry: shape protected, normalization renormalized. The precise value of G depends on the renormalized amplitude B_{eff} , which includes $O(1/W_0)$ corrections to the mean-field estimate. Computing these corrections — either analytically via the one-loop self-energy or numerically via lattice simulation — would sharpen the framework's quantitative predictions for G .

5.5.8 Connection to Newton's Constant

Combining the derived kernel amplitude with the self-consistency relation (Section 6.5), Newton's constant becomes:

$$G \propto k_B T_{\text{sub}} \cdot B = k_B T_{\text{sub}} \cdot c_\chi f(1-f) \cdot a / (4\pi k_B T_{\text{sub}}) \propto c_\chi f(1-f) \cdot a \quad (46)$$

modulo geometric factors and the mass-commitment conversion. The strength of gravity is set by the elimination fraction f and the lattice spacing a . Gravity is strongest when $f = 1/2$ (maximum elimination variance) and vanishes in the limits $f \rightarrow 0$ (no commitments) and $f \rightarrow 1$ (all states eliminated). The $O(1)$ prefactor c_χ depends on the lattice connectivity and inter-site coupling structure; if the coupling amplitude scales with W_0 , then W_0 re-enters through c_χ . Determining c_χ precisely requires lattice simulation or higher-order closure.

5.5.9 Physical Interpretation and Status

What has been derived. Starting from the substrate's discrete state structure (W_0, f, a) and an explicit lattice transport rule for constraint influence, we have derived:

1. The overlap kernel via a cumulant expansion of the microscopic elimination field $\eta_{\{x,s\}}$ (Section 5.5.1).
2. The Laplacian operator from the lattice diffusion equation for constraint influence, with the screening \Leftrightarrow leakage identification made exact (Section 5.5.2).
3. The functional form $\hat{K}(r) = B/r$ as a consequence of locality + isotropy + BCB, independent of the mean-field approximation (Sections 5.5.2, 5.5.6).
4. The amplitude $B = c_\chi f(1-f) \cdot a / (4\pi k_B T_{\text{sub}})$ under mean-field, with c_χ an $O(1)$ prefactor computable by lattice simulation (Section 5.5.3).
5. The kernel moments \hat{K}_0, \hat{K}_2 that determine the gradient expansion coefficients μ, κ (Section 5.5.5).
6. The no-screening theorem: BCB forbids the mass term m^2 via shift symmetry, making $1/r$ the symmetry-protected IR fixed point (Section 5.5.6); the no-leakage theorem (Appendix A) strengthens this from principle to structural necessity.
7. The one-loop structure: shape protected by shift symmetry, amplitude renormalized at $O(1/W_0)$ (Section 5.5.7).
8. Newton's constant: $G \propto c_\chi f(1-f) \cdot a$, strongest at $f = 1/2$ (Section 5.5.8).

Separation of shape and normalization. A key structural feature of this derivation is that the $1/r$ functional form and the amplitude B are determined by different ingredients. The shape follows from symmetry (locality, isotropy, BCB) and is exact within the framework's postulate structure. The normalization follows from the mean-field estimate and is approximate, subject to $O(1/W_0)$ corrections from the one-loop renormalization. This separation means that the gravitational inverse-square law is a robust structural prediction, while the precise value of G depends on the detailed microphysics encoded in c_χ .

Validity conditions. The derivation is controlled within the validity box: $W_0 \gg 1$ and $\tilde{\rho}_c a^3 \ll 1$ (Section 5.5.4). Higher connected cumulants are suppressed by $O(1/W_0)$; multi-body overlaps are suppressed by $O(\tilde{\rho}_c a^3)$. The no-screening result (Section 5.5.6) is independent of these conditions — it follows from BCB symmetry alone.

Resolution of the screening question. The no-screening fixed-point theorem (Section 5.5.6) resolves the question left open by the kernel derivation. Rather than assuming unscreened propagation (the old A4+), or merely localizing the assumption to "no mass gap in the propagator," we have shown that BCB — the same principle that identifies gravity as the accounting interaction in the Four Fundamental Interactions taxonomy [7] — forbids the mass term via shift symmetry. The $1/r$ kernel is therefore not a parameter choice but a structural consequence of bit conservation.

Relation to assumption A4+. Assumption A4+ (that $\hat{K}(r)$ has a $1/r$ long-range tail) is now derived rather than assumed. It follows from three ingredients already present in the framework: (i) local coupling on the substrate lattice (giving a Laplacian propagator), (ii) three spatial dimensions (giving $1/r$ as the Green's function), and (iii) BCB (forbidding the screening mass via shift symmetry). No new assumptions beyond those already in the VERSF postulate structure are required.

Lattice simulation target. A numerical simulation of the microscopic model — sites with W_0 labels, commitment events eliminating fraction f , constraint influence diffusing with exact accounting — would provide: (i) a direct measurement of $C(r)$ confirming the $1/r$ asymptotic, (ii) the numerical value of c_χ , (iii) verification that screening appears only when leakage ($\gamma > 0$) is introduced, and (iv) the magnitude of beyond-mean-field corrections. Even a toy lattice (small W_0 , simple geometry) would substantially strengthen the quantitative predictions.

6. Entropic Force and the Newtonian Limit

6.1 Thermodynamic Control

Under assumption A5, the uncommitted substrate acts as a thermal reservoir at temperature T_{sub} . The appropriate potential for the committed modes is the free energy:

$$F[\tilde{\rho}_c] = U[\tilde{\rho}_c] - T_{\text{sub}} S[\tilde{\rho}_c] \quad (47)$$

In the entropic dominance regime — where changes in the internal energy U are negligible compared to $T_{\text{sub}} \Delta S$ — the equilibrium condition becomes:

$$\delta F / \delta \tilde{\rho}_{\text{c}} \approx -T_{\text{sub}} \delta S / \delta \tilde{\rho}_{\text{c}} = 0 \quad (48)$$

and forces arise from the entropy gradient:

$$F_{\text{i}} = -\partial F / \partial x_{\text{i}} \approx T_{\text{sub}} \partial S / \partial x_{\text{i}} \quad (49)$$

When is entropic dominance valid? The internal energy U of committed modes is their kinetic and potential energy in the standard sense. For non-relativistic modes at rest, U is dominated by rest-mass energy mc^2 , which is already accounted for in $\tilde{\rho}_{\text{c}}$. The relevant comparison is between the gravitational interaction energy $\Delta U \sim Gm_1m_2/r$ and the entropic contribution $T_{\text{sub}} \Delta S$. Entropic dominance holds when the gravitational field is weak ($\Phi/c^2 \ll 1$), which is precisely the Newtonian regime. This is consistent: the Newtonian limit we derive below is valid in the same regime where the entropic-dominance approximation holds. Operationally, we assume a quasi-static rearrangement of commitment density at fixed total mass-energy, so that the leading change in F arises from the substrate entropy term rather than from changes in kinetic energy or internal binding energy of the committed modes.

Operational definition of T_{sub} . The substrate temperature T_{sub} characterizes the fluctuation spectrum of uncommitted substrate states. Operationally, it is defined as the Lagrange multiplier conjugate to energy in the canonical ensemble of uncommitted substrate configurations:

$$1/T_{\text{sub}} = \partial S_{\text{uncommitted}} / \partial E \big|_{\{N, V\}}$$

where $S_{\text{uncommitted}}$ is the entropy of the uncommitted substrate degrees of freedom and E is the energy exchanged between committed modes and the substrate reservoir. This is the standard thermodynamic definition; no new physical content is introduced. In the self-consistency analysis (Section 6.5), T_{sub} is determined by the constraint $T_{\text{sub}} \mu = 4\pi\eta^2\hbar c$, which fixes it once the overlap kernel moment μ is known.

6.1a Operational Derivation of T_{sub} from Substrate State Counting

To justify the definition of T_{sub} from the substrate's microscopic structure, we treat the uncommitted substrate degrees of freedom in a microcanonical ensemble and show that the standard thermodynamic relation $1/T_{\text{sub}} = \partial S_{\text{uncommitted}} / \partial E$ follows directly from the state counting structure.

Let $\Omega_{\text{uncommitted}}(E)$ be the set of substrate microstates consistent with a fixed energy E and with no commitment events active. The microcanonical uncommitted entropy is:

$$S_{\text{uncommitted}}(E) := k_{\text{B}} \ln |\Omega_{\text{uncommitted}}(E)|$$

where $|\cdot|$ denotes the cardinality of the configuration set. The energy E is the conserved quantity exchanged between the committed modes and the substrate reservoir when commitment events

locally eliminate accessible substrate states. Under standard statistical-mechanical reasoning, the temperature associated with the substrate reservoir is:

$$1/T_{\text{sub}} = \partial S_{\text{uncommitted}} / \partial E \big|_{\{N, V\}}$$

as the reciprocal of the slope of the entropy–energy curve. This definition does not require a physical heat bath external to the substrate; the uncommitted state space itself plays the role of the reservoir.

Connection to substrate parameters. A single commitment event eliminates a fraction f of local accessible states (assumption A2). The uncommitted configuration count decreases multiplicatively with additional energy invested in commitments. For small changes in energy, the dominant change in $S_{\text{uncommitted}}$ arises from the logarithmic state counting of uncommitted substrate sites. Defining:

$$\Delta S_{\text{uncommitted}}(E) := S_{\text{uncommitted}}(E + \delta E) - S_{\text{uncommitted}}(E)$$

and assuming the eliminated state set for δE is small compared to the total uncommitted state count (dilute regime):

$$\Delta S_{\text{uncommitted}} \approx -\lambda \cdot \delta N_{\text{eliminated}}$$

where $\lambda = -k_B \ln(1 - f)$ is the single-site entropy cost (Eq. (13)) and $\delta N_{\text{eliminated}}$ is the number of local states removed per δE . Since the energy cost per eliminated state is substrate-specific and, in the dilute limit, roughly constant, the ratio $\delta N_{\text{eliminated}} / \delta E$ is approximately constant. Taking the derivative in the continuum limit:

$$1/T_{\text{sub}} \approx -\lambda \cdot dN_{\text{eliminated}} / dE$$

so that T_{sub} is positive and proportional to the ratio of energy invested per commitment to the corresponding reduction in uncommitted state count. Inverting:

$$T_{\text{sub}} \approx (-\lambda \cdot dN_{\text{eliminated}} / dE)^{-1} \quad (50)$$

Equation (50) shows that the effective substrate temperature T_{sub} is well-defined in terms of the energy cost of eliminating substrate configurations — it is the energy invested per entropy unit removed. This is an existence and consistency result: it establishes that T_{sub} is a legitimate thermodynamic quantity given the substrate's state-counting structure, and does not require an external heat bath or an ad hoc temperature parameter. However, its numerical value is not predicted by this construction alone; it depends on the detailed microphysics of the substrate's energy-configuration relationship and is constrained only indirectly by the self-consistency relation of Section 6.5. This situation parallels other entropic gravity programs: in Jacobson (1995) and Verlinde (2011), the reservoir temperature is likewise treated as given from horizon thermodynamics rather than computed from microphysics. The definition is consistent with the free energy $F = U - T_{\text{sub}} S$ used throughout Section 6 and justifies entropic forces in the regime where substrate entropy changes dominate the free energy.

6.2 The Auxiliary Overlap Potential (Hubbard–Stratonovich)

Working with the equilibrium weight $\exp[-F/(k_B T_{\text{sub}})]$ and retaining the quadratic overlap contribution to the entropy, the nonlocal kernel term can be decoupled by introducing an auxiliary scalar field $\Phi(x)$ — the overlap potential. This is the standard Hubbard–Stratonovich transformation:

$$\exp[(k_B T_{\text{sub}}/2) \iint \tilde{\rho}_c(x) \hat{K}(|x-y|) \tilde{\rho}_c(y) d^3x d^3y] = \int \mathcal{D}\Phi \exp[-(1/2) \iint \Phi(x) \hat{K}^{-1}(|x-y|) \Phi(y) d^3x d^3y + \int \Phi(x) \tilde{\rho}_c(x) d^3x]$$

up to a $\tilde{\rho}_c$ -independent normalization. Here \hat{K}^{-1} is the inverse kernel defined by $\int \hat{K}^{-1}(|x-z|) \hat{K}(|z-y|) d^3z = \delta^3(x-y)$.

The Poisson structure. The critical step: if the overlap kernel $\hat{K}(r)$ satisfies:

$$\hat{K}^{-1} = -(1/4\pi G_{\text{eff}}) \nabla^2 \quad (51)$$

— i.e., if the inverse kernel is proportional to the negative Laplacian — then the saddle-point equation for Φ (obtained by extremizing the exponent with respect to Φ) is:

$$-(1/4\pi G_{\text{eff}}) \nabla^2 \Phi(x) + \tilde{\rho}_c(x) = 0$$

which gives:

$$\nabla^2 \Phi = 4\pi G_{\text{eff}} \tilde{\rho}_c \quad (52)$$

This is Poisson's equation. The overlap potential Φ is the gravitational potential, and G_{eff} is the effective gravitational coupling.

When does $\hat{K}^{-1} \propto -\nabla^2$ hold? This condition is satisfied when the kernel $\hat{K}(r)$ is the Green's function of the Laplacian — i.e., when $\hat{K}(r) \propto 1/r$ in three dimensions. Physically, this means overlap influence propagates as a $1/r$ potential, which is the natural behavior for a quantity that satisfies an unscreened elliptic equation in three dimensions. If the substrate correlation structure produces a kernel with a finite correlation length L_{corr} (e.g., $\hat{K}(r) \propto e^{-\{r/L_{\text{corr}}\}/r}$), then $\hat{K}^{-1} \propto -\nabla^2 + 1/L_{\text{corr}}^2$, giving a Yukawa-modified Poisson equation. The pure Newtonian limit emerges when $L_{\text{corr}} \rightarrow \infty$ — i.e., when the overlap influence is long-ranged.

Assumption A4+. To recover Newtonian gravity, we therefore strengthen assumption A4 by requiring that the overlap kernel has a $1/r$ tail:

$$\hat{K}(r) \rightarrow (G_{\text{eff}}/k_B T_{\text{sub}}) \cdot (1/r) \text{ as } r \rightarrow \infty \quad (53)$$

Derivation status. This assumption is derived, not postulated. The no-screening theorem (Section 5.5.6) shows that BCB forbids the screening mass via shift symmetry, and the lattice transport derivation (Section 5.5.2) shows that local constraint diffusion with exact accounting

produces the Laplacian operator whose Green's function is $1/r$. We restate A4+ here for clarity and to make the logical dependence of the Newtonian limit explicit.

Empirical content of A4+. Assumption A4+ is the sole dynamical assumption responsible for the long-range Newtonian limit. All other steps in the derivation — the entropy functional, the Hubbard–Stratonovich transformation, the Gauss-law flux argument, the sign of the force — follow from the general kernel conditions A4 and structural symmetries. If the overlap kernel is screened (e.g., Yukawa: $\hat{K}(r) \propto e^{-r/L_{\text{corr}}}/r$), the theory predicts finite-range gravity with a crossover to exponential decay at distances $r \gg L_{\text{corr}}$. The empirical success of inverse-square gravity over scales from millimeters to megaparsecs therefore constrains the asymptotic form of $\hat{K}(r)$: the substrate's constraint-overlap correlation must be unscreened, or screened only at cosmological scales.

Why $1/r$ is generic, not arbitrary. In three spatial dimensions, any conserved scalar flux sourced by localized density and propagating without screening satisfies an elliptic equation whose Green's function decays as $1/r$. Thus, if constraint influence is conserved and unscreened, a $1/r$ kernel is not an arbitrary choice but the generic long-distance solution of the corresponding field equation. Assumption A4+ therefore reduces to two physical conditions: (i) the constraint-overlap influence is conserved in transit (no dissipation), and (ii) it is not screened by a mass gap. Under these conditions, the $1/r$ tail is a mathematical consequence of three-dimensional geometry, not an additional postulate.

Justification of the Laplacian inverse from throughput-limited propagation. The assumption that $\hat{K}^{-1} \propto -\nabla^2$ in the infrared can be physically motivated from the substrate's causal-throughput structure. In the companion analysis of the speed of light as a maximal information-throughput bound, it is shown that constraint influence propagates through the substrate subject to a universal finite transmission capacity, with propagation limited by a maximal causal rate that defines the emergent light cone. In three spatial dimensions, any conserved scalar influence that propagates locally, isotropically, and without screening under such a throughput bound satisfies an elliptic field equation whose Green's function decays as $1/r$. Equivalently, the only rotationally invariant second-order differential operator governing unscreened static propagation consistent with locality and conservation is the Laplacian. Thus, if constraint-overlap influence is conserved and long-ranged, the infrared inverse kernel must reduce to a Laplacian operator, and the overlap kernel itself acquires a $1/r$ tail. The $1/r$ asymptotic form is therefore not imposed arbitrarily but follows from the same causal-throughput structure that determines the universal propagation speed. This provides a physical underpinning for Assumption A4+, linking the gravitational kernel's long-range behavior to the substrate's fundamental information-transmission limit.

6.3 Newton's Constant in Substrate Parameters

The saddle-point equation Eq. (52) uses the per-volume commitment density $\tilde{\rho}_c$. Converting to mass density via $\rho_m = (\eta\hbar/c^2\Delta t) \tilde{\rho}_c$:

$$\nabla^2\Phi = 4\pi G_{\text{eff}}(c^2\Delta t/\eta\hbar) \rho_m \quad (54)$$

Matching to the standard Poisson equation $\nabla^2\Phi = 4\pi G\rho_m$ identifies:

$$G = G_{\text{eff}} \cdot c^2\Delta t/(\eta\hbar) \quad (55)$$

Under the Planck-tick calibration $\Delta t = t_P = \sqrt{(\hbar G/c^5)}$:

$$G = G_{\text{eff}} \cdot c^2 t_P / (\eta\hbar) = G_{\text{eff}} \cdot c^2 \sqrt{(\hbar G/c^5)} / (\eta\hbar) = G_{\text{eff}} \sqrt{(G/(\hbar c^5))} \cdot c^2 / \eta$$

Squaring both sides and solving:

$$G = G_{\text{eff}}^2 / (\eta^2\hbar c) \quad (56)$$

This is a self-consistency relation between G , the kernel-determined coupling G_{eff} , and substrate parameters. It constrains G_{eff} rather than predicting G from first principles.

What this means. The framework does not derive Newton's constant from first principles alone, but it constrains it tightly. The entropy functional, combined with the Hubbard–Stratonovich auxiliary field, produces Poisson's equation with a coupling constant that is consistent with the observed G provided the overlap kernel's long-range behavior satisfies Eq. (53) with the correct amplitude. The kernel derivation (Section 5.5) separates shape from normalization: the $1/r$ form follows from symmetry (locality + isotropy + BCB), while the amplitude $B = c_\chi f(1-f) \cdot a/(4\pi k_{\text{BT_sub}})$ is a mean-field result with $O(1)$ prefactor c_χ computable by lattice simulation. Extracting the precise numerical value of G requires determining c_χ .

6.4 Sign of the Force: Rigorous Proof of Attraction

Theorem. Under assumption A4 ($\hat{K}(r) \geq 0$, decreasing), the entropic force between two localized commitment distributions is attractive.

Proof. Consider two non-overlapping commitment distributions $\tilde{\rho}_1(x)$ and $\tilde{\rho}_2(x)$ separated by distance R , with $\tilde{\rho}_2(x) = \tilde{\rho}_2^{\wedge}\{(0)\}(x - R\hat{e})$ for some unit vector \hat{e} . The entropy gain from their mutual overlap is:

$$\Delta S(R) = (k_B/2) \iint \tilde{\rho}_1(x) \hat{K}(|x - y|) \tilde{\rho}_2(y) d^3x d^3y \quad (57)$$

(The factor $1/2$ is absent for the cross-term; the expression above gives the interaction contribution.) Since both $\tilde{\rho}_1, \tilde{\rho}_2 \geq 0$ and $\hat{K} \geq 0$, we have $\Delta S(R) \geq 0$ for all R .

The force on distribution 2 along \hat{e} is:

$$F_R = T_{\text{sub}} d\Delta S/dR \quad (58)$$

Computing:

$$d\Delta S/dR = k_B \iint \tilde{\rho}_1(x) [d\hat{K}(|x - y|)/dR] \tilde{\rho}_2^{\wedge}\{(0)\}(y - R\hat{e}) d^3x d^3y$$

Since increasing R increases $|x - y|$ for the typical pair, and \hat{K} is monotonically decreasing (assumption A4):

$$d\hat{K}/d|x-y| < 0 \implies d\Delta S/dR < 0 \quad (59)$$

Therefore:

$$F_R = T_{\text{sub}} d\Delta S/dR < 0 \quad (60)$$

The force is negative (attractive): it points in the direction of decreasing R , toward the other mass. ■

Physical content. Bringing commitments closer increases their constraint overlap, which increases the substrate entropy (reduces the net entropy cost). The second law drives the system toward higher entropy — toward smaller separation. This is gravitational attraction.

6.5 Self-Consistency Relation

From the kernel's long-range behavior Eq. (53) and the Poisson equation Eq. (52), the self-consistency condition is:

$$G_{\text{eff}} = k_B T_{\text{sub}} \cdot (\text{long-range amplitude of } \hat{K}) \quad (61)$$

Combined with Eq. (56):

$$G = (k_B T_{\text{sub}})^2 \cdot (\hat{K} \text{ amplitude})^2 / (\eta^2 \hbar c) \quad (62)$$

This constrains the product $T_{\text{sub}} \times (\hat{K} \text{ amplitude})$ given the observed G . It is a consistency relation, not a prediction: the framework is compatible with the observed value of G for a specific combination of substrate parameters, but does not compute that combination from first principles.

6.6 Second Derivation: Entropy Flux and the Gauss Law

The Hubbard–Stratonovich route (Section 6.2) derives Poisson's equation from a specific kernel condition ($\hat{K}^{-1} \propto -\nabla^2$). This section provides an independent derivation from symmetry principles alone, requiring no assumption about the kernel's functional form. The two derivations arrive at the same result via different logic; their agreement strengthens the conclusion.

Entropy-loading flux. Each committed site imposes constraints on the surrounding substrate's available configurations, reducing the number of accessible states in its neighborhood. This constraint influence propagates outward through the substrate. Define the entropy-loading flux $J_{\Sigma}^{\rightarrow}(x)$ as the rate at which entropy deficit (constraint influence) flows through a surface element at position x . The flux measures how much "ordering influence" passes through a given point per unit time.

Three symmetry principles. The entropy-loading flux satisfies three conditions that follow from the substrate's structure:

Locality: The flux at x depends only on the local commitment density and its immediate neighborhood. There is no action at a distance — constraint influence propagates through the substrate, site by site.

Isotropy: On a substrate with no preferred direction (assumption A4, isotropy of K), the flux from a point source is radially symmetric: J_{Σ} depends only on the radial distance and direction from the source.

Source conservation: Every committed site is a source of entropy deficit. The total flux through any closed surface equals the total entropy-loading rate enclosed:

$$\oint_{\partial V} J_{\Sigma} \cdot dA^{\vec{}} = \kappa_{\Sigma} \int_V \tilde{\rho}_c(x) d^3x \quad (63)$$

where κ_{Σ} is the entropy-loading rate per unit commitment density (a substrate-determined constant). This is the statement that constraint influence is neither created nor destroyed in transit — it originates at committed sites and propagates outward conservatively.

Gauss law structure. By the divergence theorem, Eq. (63) is equivalent to:

$$\nabla \cdot J_{\Sigma}(x) = \kappa_{\Sigma} \tilde{\rho}_c(x) \quad (64)$$

This is a Gauss law: the divergence of the entropy-loading flux is proportional to the commitment density. It follows from locality, isotropy, and source conservation alone — no kernel shape is assumed.

From Gauss law to Poisson. For a spherically symmetric point source of total commitment $\tilde{\rho}_{\text{total}} = \int \tilde{\rho}_c d^3x$, isotropy gives $J_{\Sigma} = J_{\Sigma}(r) \hat{r}$, and the Gauss law yields:

$$4\pi r^2 J_{\Sigma}(r) = \kappa_{\Sigma} \tilde{\rho}_{\text{total}}$$

$$J_{\Sigma}(r) = \kappa_{\Sigma} \tilde{\rho}_{\text{total}} / (4\pi r^2) \quad (65)$$

The entropy-loading flux falls off as $1/r^2$ — the inverse-square law follows from three-dimensional geometry and flux conservation, with no dynamical input.

Assuming the flux field is curl-free in the static regime (no vortical constraint transport), $\nabla \times J_{\Sigma} = 0$, we may write $J_{\Sigma} = -\nabla\Phi$, defining the gravitational potential Φ . Static equilibrium implies $\partial J_{\Sigma} / \partial t = 0$ and no rotational constraint transport, justifying the irrotationality condition. Then:

$$\nabla^2\Phi = -\nabla \cdot J_{\Sigma} = -\kappa_{\Sigma} \tilde{\rho}_c \quad (66)$$

Identifying $-\kappa_{\Sigma} = 4\pi G_{\text{eff}}$ recovers the Poisson equation Eq. (52). The sign is fixed by the convention that Φ is negative near sources (attractive potential) and J^{Σ} points outward (entropy deficit flows away from sources).

What this derivation adds. The Gauss-law route derives Poisson's equation from symmetry (isotropy + locality + conservation) rather than from a specific kernel assumption. It shows that the $1/r^2$ force law is a geometric consequence of three-dimensional flux conservation, not an artifact of choosing $\hat{K}(r) \propto 1/r$. The Hubbard–Stratonovich route (Section 6.2) shows that the same result follows from the entropy functional's saddle point when the kernel has the correct structure. The two routes are complementary: the flux argument shows that $1/r^2$ is inevitable given the symmetries; the kernel argument shows how it connects to the microscopic overlap statistics.

Consistency check. The two derivations agree if and only if the overlap kernel $\hat{K}(r)$ that satisfies $\hat{K}^{-1} \propto -\nabla^2$ (Section 6.2) is the same kernel whose moments produce the entropy-loading rate κ_{Σ} (this section). This is indeed the case: for $\hat{K}(r) \propto 1/r$, the Hubbard–Stratonovich saddle point and the Gauss-law flux both yield $\nabla^2\Phi = 4\pi G_{\text{eff}} \tilde{\rho}_c$. The two derivations are independent routes to the same equation, confirming its robustness.

7. Mesoscopic Layer: Inertia as Commitment Stability

Classification: Section 7.1 presents the physical picture. Section 7.2 gives a qualitative plausibility argument. Section 7.3 provides a quantitative linear-response derivation that obtains a force–response relation with the structure of $F = ma$, with the effective inertial mass identified as the inverse susceptibility of the commitment density field. Proving exact equivalence between this effective mass and the gravitational mass (the equivalence principle) remains open.

7.1 The Physical Picture

Between the microscopic scale (individual commitments define mass) and the macroscopic scale (entropy gradients define gravity), there is a mesoscopic question: why do massive objects resist acceleration?

The VERSF answer, at the level of physical argument, is that a spatially structured commitment distribution requires coordinated reorganization of micro-events to alter, and this coordination cost manifests as inertial resistance.

7.2 Plausibility Argument

Consider a mode with equilibrium commitment density $\tilde{\rho}_c^{\{0\}}(x)$ at rest. An applied force attempts to shift the pattern by displacement $\delta x(t)$. For the pattern to shift:

1. The phase coherence structure generating the commitments must re-establish at new positions — requiring at least ξ/c ticks (coherent domain size divided by maximum signal speed).
2. Partial commitments at old sites are lost; K_c new micro-events must accumulate at new positions.
3. The coarse-grained entropy structure readjusts to the new density profile.

The reorganization timescale is bounded below by the flip period $T_{\text{bit}} = \Delta t K_c / p_\epsilon$. An acceleration a requires continuous reorganization at rate $d\delta x/dt = at$, and the entropy cost of this reorganization is proportional to $\int |\partial \tilde{\rho}_c / \partial t| d^3x \propto |a| \int |\nabla \tilde{\rho}_c^{\{(0)\}}| d^3x$. Since both this entropy cost and the mass are proportional to the integrated commitment density, the resulting resistive force is proportional to $m \cdot a$ — the correct structure for inertia.

7.3 Inertia from Commitment Flux Linear Response

The plausibility argument of Section 7.2 can be elevated to a quantitative mechanism using linear response theory applied to the commitment density field.

Dynamical equation. Consider the coarse-grained per-volume commitment density field $\tilde{\rho}_c(x, t)$ evolving in time. We postulate a continuity equation expressing conservation of commitment density:

$$\partial \tilde{\rho}_c(x, t) / \partial t + \nabla \cdot J(x, t) = 0 \quad (67)$$

where $J(x, t)$ is the commitment flux vector field. Changes in local density arise solely from net flux divergence.

Constitutive relation. To close the system, we introduce a constitutive relation for the commitment flux in terms of an effective commitment potential $\Phi_c(x, t)$:

$$J(x, t) = -D \nabla \Phi_c(x, t) \quad (68)$$

where $D > 0$ is a substrate-dependent diffusivity coefficient and Φ_c encodes the local resistance of the substrate to spatial reorganization of commitment density. This is analogous to Fick's law and is justified in the near-equilibrium regime, where the flux responds linearly to small gradients of the potential.

Linear response to applied force. An external applied force $F_{\text{ext}}(x, t)$ attempting to accelerate a localized commitment pattern perturbs the equilibrium distribution $\tilde{\rho}_c^{\{(0)\}}(x)$. In linear response theory, the induced deviation $\delta \tilde{\rho}_c(x, t) := \tilde{\rho}_c(x, t) - \tilde{\rho}_c^{\{(0)\}}(x)$ is proportional to the applied force:

$$\delta \tilde{\rho}_c(x, t) = -\chi(x) \cdot F_{\text{ext}}(x, t) \quad (69)$$

where $\chi(x)$ is the susceptibility kernel relating perturbations in commitment density to applied forcing. For modes localized on spatial scales much smaller than the coarse-graining scale L , $\chi(x)$ may be approximated as spatially homogeneous, $\chi(x) \approx \chi_0$.

Momentum and acceleration. The momentum associated with a localized commitment distribution is defined analogously to a fluid momentum density:

$$P(x, t) := \rho_m(x) v(x, t)$$

where $\rho_m(x) = (\eta\hbar/c^2\Delta t) \tilde{\rho}_c(x)$ is the mass density (Eq. (6)) and $v(x, t)$ is the mode's velocity field. Under a small acceleration $a(x, t)$, the rate of change of momentum density is:

$$\partial P(x, t)/\partial t = \rho_m(x) a(x, t) \quad (70)$$

Kubo formula for the susceptibility. The susceptibility χ_0 is related to equilibrium fluctuations via the Kubo formula:

$$\chi_0 = (1/k_B T_{\text{sub}}) \int_0^\infty dt' \langle \delta\tilde{\rho}_c(0) \delta\tilde{\rho}_c(t') \rangle_{\text{eq}} \quad (71)$$

where $\langle \cdot \rangle_{\text{eq}}$ denotes an equilibrium ensemble average. In the regime where fluctuations of $\tilde{\rho}_c$ are stationary and ergodic, χ_0 is finite and positive. Since $\rho_m \propto \tilde{\rho}_c$, perturbations in ρ_m respond proportionally to the applied force with a coefficient determined by $\chi_0 k_B T_{\text{sub}}$.

Force–response relation. Equation (69) gives the local density response; to obtain the total inertial response, we integrate over the mode's spatial support volume $V_{\text{mode}} = \int d^3x$ (the region where $\tilde{\rho}_c^{\{0\}}$ is appreciable). The total commitment number perturbation is:

$$\delta N_c = \int \delta\tilde{\rho}_c(x, t) d^3x = -\chi_0 V_{\text{mode}} F_{\text{ext}} \quad (72)$$

Identifying the effective inertial mass as:

$$m_{\text{eff}} \equiv (\chi_0 V_{\text{mode}})^{-1} \quad (73)$$

which has dimensions [force/acceleration] = [mass] (since $[\chi_0] = [1/(\text{volume} \cdot \text{force})]$ and $[V_{\text{mode}}] = [\text{volume}]$). Under a spatially uniform applied acceleration a :

$$m_{\text{eff}} \cdot a = F_{\text{ext}} \quad (74)$$

We obtain a linear force–response relation whose structure mirrors $F = ma$, with the effective inertial mass given by the inverse of the integrated susceptibility. Because m_{eff} is proportional to the equilibrium commitment density $\tilde{\rho}_c^{\{0\}}$ integrated over V_{mode} — which is exactly the total commitment number, and hence proportional to the rest mass of the mode — the resulting response coefficient is consistent with the identification of inertial and gravitational mass. However, proving the exact equivalence $m_{\text{eff}} = m_{\text{grav}}$ (the equivalence principle) would require showing that the susceptibility coefficient $\chi_0 V_{\text{mode}}$ is related to the gravitational

coupling by a symmetry or dynamical identity — not merely by dimensional consistency. This remains an open problem.

Status. This linear response derivation elevates the sketch of Section 7.2 to a quantitative mechanism: the effective inertial mass $m_{\text{eff}} = (\chi_0 V_{\text{mode}})^{-1}$ is the resistance coefficient of the commitment density field to perturbations, with correct dimensions of [mass] arising from integration over the mode's spatial support. Inertia in VERSF is the substrate's dynamical resistance to reorganization under external forcing, following directly from the commitment density's fluctuation–response relation combined with the conservation and constitutive laws postulated above. The derivation establishes the structural result that inertia has the correct form (linear in acceleration, proportional to total commitment number); a full computation of χ_0 in terms of substrate parameters ($W_0, f, a, \Delta t$) and a proof that m_{eff} equals the gravitational mass remain open calculations. Appendix C analyzes the structural conditions under which the equivalence principle holds: if χ_0 depends only on local substrate parameters and not on particle identity, exact equivalence follows.

8. Stress-Energy and the Covariant Extension

8.1 Effective Stress-Energy Tensor

For modes with four-velocity u^μ , the effective stress-energy tensor is:

$$T^{\mu\nu}_{\text{eff}}(x) = \rho_m(x) u^\mu u^\nu \quad (75)$$

In the rest frame, $T^{00}_{\text{eff}} = \rho_m c^2$ (energy density), and all other components vanish. This is the standard perfect-fluid stress-energy for pressureless dust, consistent with the mass relation Eq. (6).

8.2 Covariant Completion via Spin-2 Uniqueness

The Newtonian limit (Section 6) establishes that the entropic force couples universally to commitment density $\tilde{\rho}_c$, which is proportional to T^{00} (the energy density component of the stress-energy tensor) in the rest frame. This universal coupling to stress-energy has a powerful consequence that elevates the covariant extension from conjecture to mathematical inevitability.

The Weinberg–Deser theorem. A classical result in theoretical physics, established independently by Weinberg (1964) [15] and Deser (1970) [16], states:

Any consistent, Lorentz-invariant, long-range interaction that couples universally to the stress-energy tensor $T_{\mu\nu}$ must be mediated by a massless spin-2 field. The unique nonlinear self-consistent completion of this field's dynamics is the Einstein-Hilbert action — i.e., general relativity.

The theorem proceeds in three steps:

1. **Spin from coupling structure.** A field that couples to the symmetric tensor $T_{\mu\nu}$ and produces a long-range ($1/r$) potential must have spin 2. Spin 0 couples to the trace T ; spin 1 couples to a conserved current J^μ ; only spin 2 couples to the full symmetric $T_{\mu\nu}$. Since the VERSF entropic force couples to $\tilde{\rho}_c \propto T^{00}$ in the rest frame and — assuming emergent Lorentz invariance in the IR continuum limit (a standard IR requirement in the VERSF programme) — must transform covariantly, the mediating field is necessarily spin-2.
2. **Masslessness from range.** The $1/r$ potential derived in Section 6 corresponds to a massless mediator. A massive spin-2 field would produce a Yukawa potential e^{-mr}/r with finite range. The unscreened Newtonian limit (assumption A4+) requires the mediator to be massless.
3. **Uniqueness of nonlinear completion.** A massless spin-2 field coupling to $T_{\mu\nu}$ must couple to its own stress-energy (since the field itself carries energy-momentum). Self-consistent incorporation of this self-coupling, order by order, uniquely produces the Einstein-Hilbert action. There is no consistent alternative: any attempt to truncate the self-coupling at finite order produces pathologies (ghosts, acausality, or loss of gauge invariance). The full nonlinear theory is general relativity.

Application to VERSF. The entropic force derived in Section 6 satisfies all three premises of the theorem:

- It couples to commitment density, which is proportional to T^{00} and generalizes covariantly to $T_{\mu\nu}$ (Section 8.1).
- It is long-ranged ($1/r$ potential from the unscreened overlap kernel or Gauss-law flux).
- The substrate dynamics are Lorentz-covariant in the emergent continuum limit (the substrate clock and lattice structure produce Lorentz symmetry at scales $L \gg a$, which is a foundational result of the VERSF program; the conditions for this emergence are analyzed in Appendix B).

The Weinberg–Deser theorem then guarantees that the covariant completion of the VERSF gravitational sector is general relativity — not as a conjecture, but as a mathematical consequence of universal coupling, long range, and Lorentz invariance.

What VERSF adds to the theorem. The Weinberg–Deser theorem tells us the form of the covariant theory given universal coupling to $T_{\mu\nu}$. It does not tell us why the coupling is universal. VERSF provides the answer: the coupling is universal because all mass arises from the same substrate mechanism (commitment density), and the gravitational interaction is the entropy response to that density. Universality of gravity is a consequence of the universality of mass origin.

Gravity as one of four admissibility rules. Within the VERSF framework, gravity is identified with the "accounting rule" — Bit Conservation and Balance (BCB) — for irreversible commitment in distinguishable information. The Four Fundamental Interactions paper [7] shows that this accounting constraint is one of exactly four irreducible admissibility rules that can govern information without violating finite distinguishability, irreversible commitment, or global consistency. This situates the entropic origin of gravity derived here within a broader taxonomy

that also includes electromagnetism (flow rule), the weak interaction (unlocking rule), and the strong interaction (locking rule), accounting for why no additional fundamental interaction is observed.

Caveats. The Weinberg–Deser theorem applies in the continuum limit where Lorentz invariance holds. At scales approaching the substrate lattice spacing a , Lorentz invariance breaks down, and the spin-2 uniqueness argument no longer applies. Crucially, the emergence of Lorentz invariance from a discrete substrate is itself a non-trivial requirement that is assumed here rather than derived. This is a recognized open challenge for all discrete-substrate gravity programs: while many lattice and condensed-matter–inspired models produce emergent Lorentz symmetry in the IR under appropriate conditions, no general proof exists that an arbitrary discrete substrate will do so. In the VERSF programme, the causal-throughput analysis of the speed of light provides a physical mechanism (finite maximal information transmission rate) that is consistent with emergent Lorentz invariance, but a rigorous derivation from the substrate's dynamics remains open. The theory of gravity at sub-Planckian scales — where the discrete structure becomes relevant and Lorentz invariance breaks down — is an open problem and a potential source of observable departures from general relativity.

Relation to the Weinberg–Witten theorem. A further constraint on emergent gravity programs comes from the Weinberg–Witten theorem [18], which states that massless particles with spin $j > 1$ cannot carry a Lorentz-covariant, conserved stress–energy tensor within a conventional relativistic quantum field theory. This is often interpreted as ruling out composite or emergent gravitons in theories where the graviton is built from more fundamental Lorentz-covariant degrees of freedom. The VERSF framework sidesteps this constraint for a specific structural reason: it is not a conventional QFT with fundamental Lorentz-covariant fields. Lorentz invariance is emergent in the IR continuum limit, arising from the substrate's finite causal-throughput structure rather than being imposed as a fundamental symmetry. The overlap potential Φ introduced via Hubbard–Stratonovich (Section 6.2) is an auxiliary field of the coarse-grained entropy functional, not a fundamental propagating degree of freedom of the substrate. The Weinberg–Witten theorem's premises — fundamental Lorentz covariance and a well-defined stress–energy tensor carried by the massless spin-2 particle within a standard QFT Hilbert space — are therefore not satisfied in the substrate-level description. The theorem remains important as a consistency check: it confirms that the gravitational sector cannot be formulated as a conventional QFT of composite gravitons, which is consistent with VERSF's claim that gravity emerges from coarse-grained entropy dynamics on a pre-geometric substrate rather than from field-theoretic composition within an existing spacetime.

9. Scale Flow and Fixed-Point Structure

9.1 Scale-Dependent Density

The coarse-grained density at scale L defines a family of fields:

$$\bar{\rho}_L(x) := C_L \tilde{\rho}_c \quad (76)$$

The flow under scale change satisfies:

$$\partial \bar{\rho}_L / \partial (\ln L) = B_L[\bar{\rho}_L] \quad (77)$$

where B_L encodes the difference between the density on the boundary of the averaging ball and the bulk average.

9.2 Properties

Smoothing. High-frequency spatial fluctuations are progressively eliminated with increasing L .

Conservation. $\int \bar{\rho}_L d^3x$ is independent of L .

Monotone variance decrease. $\text{Var}[\bar{\rho}_L]$ decreases monotonically with L .

9.3 Fixed Points

Homogeneous ($L \rightarrow \infty$): $\bar{\rho}_L \rightarrow \text{const}$. Cosmological regime.

Microscopic ($L \rightarrow 0$): $\bar{\rho}_L \rightarrow \tilde{\rho}_c$. Particle-physics regime.

Mass corresponds to the microscopic fixed-point data; gravity corresponds to the entropy functional's response to structure at finite L between the fixed points.

9.4 Temporal Fixed Points

A mode with stable high coherence R produces time-independent $\tilde{\rho}_c$ — a temporal fixed point corresponding to a stable particle. Modes with fluctuating coherence (as analyzed in the void coupling paper's stability analysis) produce time-dependent $\tilde{\rho}_c$ — unstable particles whose mass fluctuates and which eventually decay.

10. Schwarzschild Structure and Horizon Phenomenology

10.1 Spherically Symmetric Solution

For a spherically symmetric source with total mass M , the Poisson equation (Section 6) gives:

$$\Phi(r) = -GM/r \text{ for } r \gg L_{\text{corr}} \quad (78)$$

At $r \lesssim L_{\text{corr}}$, the finite range of the kernel regulates the $1/r$ divergence, producing a smooth core.

10.2 Horizon Scale via Areal Flux Saturation

The Schwarzschild radius emerges from the entropy-flux formalism (Section 6.6) through a saturation argument. The entropy-loading flux through a sphere of radius r enclosing mass M is, from Eq. (65):

$$|J_{\Sigma}(r)| = \kappa_{\Sigma} \tilde{\rho}_{\text{total}} / (4\pi r^2)$$

The flux per unit area — the areal entropy-loading intensity — is:

$$\sigma_{\Sigma}(r) = \kappa_{\Sigma} \tilde{\rho}_{\text{total}} / (4\pi r^2) \quad (79)$$

As r decreases, the areal intensity grows without bound in the continuum theory. However, the discrete substrate imposes a finite areal capacity: each substrate cell of area a^2 can process at most one unit of entropy-loading flux per tick. The maximum areal flux is therefore:

$$\sigma_{\text{max}} = 1 / (a^2 \Delta t) \quad (80)$$

The critical radius at which the areal flux saturates the substrate's capacity is determined by $\sigma_{\Sigma}(r_{\text{crit}}) = \sigma_{\text{max}}$:

$$\kappa_{\Sigma} \tilde{\rho}_{\text{total}} / (4\pi r_{\text{crit}}^2) = 1 / (a^2 \Delta t)$$

Using $\tilde{\rho}_{\text{total}} = Mc^2\Delta t/(\eta\hbar)$ (from Eq. (6) integrated) and identifying the proportionality constants via the Newtonian calibration (Section 6.3):

$$r_{\text{crit}} = 2GM/c^2 \quad (81)$$

This is the Schwarzschild radius. It appears not as a coordinate singularity of a metric but as the surface at which the substrate's finite information-processing capacity is exhausted by the entropy-loading flux. Inside this radius, the substrate cannot propagate constraint influence fast enough to maintain the equilibrium entropy structure — the entropy-gradient description breaks down.

Independence of this derivation. This derivation does not depend on the density-based local saturation argument $\tilde{\rho}_c \sim 1/a^3$; it follows purely from global flux conservation (Eq. (63)) and the substrate's finite areal information-processing capacity. The emergence of r_s from areal flux saturation does not require curvature input; it follows from capacity and conservation alone. The density saturation $\tilde{\rho}_c(r_s) \sim 1/a^3$ can be recovered as a consequence — it is the volume-interior view of the same threshold that the flux argument identifies at the surface. The flux route is primary because it relies only on the Gauss-law structure (already derived in Section 6.6) and the substrate's finite capacity, with no additional assumptions about the density profile.

Status: consistency check, not independent prediction. It should be noted that this derivation uses the Newtonian potential $\Phi(r) = -GM/r$, which already embeds the gravitational constant G calibrated in Section 6.3. The result $r_s = 2GM/c^2$ therefore does not constitute an independent prediction from new substrate variables alone — it is a consistency check confirming that the framework's flux-saturation mechanism reproduces the known Schwarzschild radius once G is

fixed. This situation parallels other entropic gravity programs (Jacobson 1995, Verlinde 2011), where horizon relations are recovered as kinematic consequences of the formalism rather than dynamical predictions. What the derivation does establish is that the VERSF framework naturally produces a horizon-like structure at the correct scale, using only the substrate's finite capacity and the Gauss-law structure derived in Section 6.6 — no additional geometric input from general relativity is required.

10.3 Singularity Regulation

The discrete substrate imposes an upper bound on commitment density: $\tilde{\rho}_c \leq 1/a^3$. This prevents the classical singularity at $r = 0$. Whether the resulting structure reproduces the Bekenstein-Hawking entropy $S = A/(4l_P^2)$ depends on the relationship between substrate parameters and the Planck length — an open computation.

11. Scalar Field Embedding

This section presents one possible effective parametrization of commitment density in field-theoretic language. It is an effective representation (the field-theoretic VERSF layer), not a claim of fundamental scalar mediation or a statement about the substrate's ontology. Its purpose is to provide a translation layer between substrate-level and EFT-level descriptions.

11.1 Identification

For a mode described by a scalar field $\phi(x, t)$, the commitment density can be identified with the field's irreversible update rate:

$$\tilde{\rho}_c(x) \propto n_s \langle |\partial_t \phi|^2 \rangle_\tau \quad (82)$$

where $\langle \cdot \rangle_\tau$ denotes time-averaging over an anchoring cycle. The physical content: $\partial_t \phi$ measures the rate of phase change, and its square gives the intensity of irreversible updates.

11.2 What This Buys

This identification connects VERSF to standard EFT language: the commitment density maps to a canonical kinetic term, the entropy functional maps to an effective action, and the gravitational coupling emerges from the overlap kernel's role in the effective action. It provides a translation layer between the substrate-level description and the field-theory description, allowing results from either framework to inform the other.

11.3 Limitations

The identification $\tilde{\rho}_c \propto |\partial_t \phi|^2$ has the structure of a Lagrangian density, while $\tilde{\rho}_c$ simultaneously serves as an entropy source. These roles are compatible — the field's kinetic

energy measures its irreversible update rate, which generates entropy cost — but the dual interpretation has not been derived from first principles. The embedding is an effective parametrization at the field-theoretic layer of the VERSF hierarchy, not a fundamental claim about the substrate. In particular, φ is a collective variable summarizing commitment dynamics at mesoscopic scales — not a fundamental field of the substrate.

12. Relation to Entropic Gravity Programs

12.1 Jacobson (1995)

Jacobson derived Einstein's field equations from the Clausius relation $\delta Q = T\delta S$ applied to local Rindler horizons, using Bekenstein-Hawking area-entropy proportionality.

Agreement: Both frameworks derive gravity from entropy relations and treat gravity as emergent.

Difference: Jacobson uses area entropy on causal horizons; VERSF uses volume entropy from commitment density. Jacobson's result is kinematic (follows from entropy-area regardless of microphysics); VERSF's is dynamical (depends on the commitment process).

VERSF adds: A candidate microphysics for the entropy — constrained configuration space of committed substrate cells.

12.2 Verlinde (2011)

Verlinde derived Newton's law from the holographic principle, equipartition on holographic screens, and Bekenstein's entropy-displacement relation.

Agreement: Both derive gravity as an entropic force. The logical structure (entropy \rightarrow force \rightarrow gravity) is identical.

Difference: Verlinde uses holographic screens and surface entropy; VERSF uses bulk substrate and volume entropy. Verlinde requires the holographic principle as input.

VERSF adds: A connection to specific microscopic processes (bit-commitment) that generate the entropy, and a unified origin for mass and gravity from the same commitment density.

12.3 Padmanabhan

Padmanabhan derives field equations from maximizing entropy on null surfaces plus matter entropy.

Agreement: The overarching philosophy — gravity as emergent thermodynamics of microscopic information — is shared.

Difference: Padmanabhan uses null surfaces and Noether charge; VERSF uses a discrete lattice and bulk commitment density. The mathematical frameworks differ substantially.

12.4 Distinctive VERSF Contribution

The specific contribution is the unification of mass and gravity in one substrate quantity: commitment density is mass at short distances and gravity at long distances, connected by coarse-graining. In all prior entropic gravity programs, mass is an input. In VERSF, mass is derived from the same substrate that generates gravity.

12.5 Known Challenges for Entropic/Emergent Gravity Programs

Intellectual honesty requires acknowledging that entropic and emergent gravity programs — including this one — face recognized conceptual and technical challenges:

Conservative forces from entropy gradients. Deriving conservative gravitational behavior from entropy gradients without hidden assumptions can be problematic. Visser (2011) and others have argued that representing Newtonian gravity as a thermodynamic entropic force requires careful construction of entropy and temperature functions; naive implementations can produce inconsistencies with conservative potentials or violate energy conservation.

Temperature as input vs. output. In all existing entropic gravity derivations — Jacobson's, Verlinde's, and ours — the reservoir temperature is ultimately constrained by self-consistency rather than predicted from first principles. Section 6.1a establishes that T_{sub} is well-defined; it does not predict its numerical value independently of the self-consistency relation. This is an honest limitation shared across the field.

Emergent Lorentz invariance. The spin-2 uniqueness argument (Section 8.2) requires Lorentz invariance in the IR. For discrete substrate theories, the emergence of this symmetry is a non-trivial requirement that is typically assumed rather than derived. While the VERSF causal-throughput analysis provides a physical mechanism consistent with emergent Lorentz invariance, a rigorous derivation from substrate dynamics remains open.

Equivalence principle. The linear-response derivation of inertia (Section 7.3) produces a force–response relation with the structure of $F = ma$, but proving that the effective inertial mass exactly equals the gravitational mass requires either a symmetry argument or a detailed computation of χ_0 that has not yet been performed.

Horizon relations as kinematic vs. dynamical. The Schwarzschild radius derivation (Section 10.2) recovers $r_s = 2GM/c^2$ using the already-calibrated G . This is a consistency check, not a dynamical prediction — a status shared with horizon derivations in Jacobson and Verlinde.

VERSF addresses some of these challenges better than prior programs (particularly by deriving mass and the kernel form from substrate microphysics) while inheriting others. The remaining open challenges are catalogued in Section 14.

13. Relationship to Prior VERSF Gravity Papers

This paper should be read as a synthesis and bridge paper within the VERSF gravity programme. It does not replace, contradict, or "reset" the earlier gravity manuscripts; it consolidates their strongest elements into a single coarse-grained mechanism linking mass microphysics to Newtonian and relativistic gravity through one shared substrate variable: commitment density.

13.1 What This Paper Is — and Is Not

What it is: a unification of mass origin and gravity origin in one scale-linked account. It provides a single pipeline:

- *Microscopic:* mass as commitment/anchoring completion density
- *Macroscopic:* gravity as entropy response to the spatial organization of that density
- *Covariant completion:* GR as the unique IR theory given universal coupling (spin-2 inevitability)

What it is not: a repudiation of earlier VERSF gravity lenses (entropy field language, critical back-pressure, distinguishability conservation). Those lenses remain valid — but are now placed into a clearer hierarchy.

13.2 How It Complements Each Earlier Gravity Strand

(A) BCB / Distinguishability Conservation papers [3] — "Why Poisson is inevitable."

Earlier BCB papers framed gravity as the equilibrium pattern of a conserved "difference" or distinguishability flow, yielding Poisson structure as the static limit of a continuity law.

Complement, not contradiction: This paper preserves that core result but micro-grounds the source term: instead of treating "distinguishability" as abstract, it identifies the physical driver of distinguishability loading with irreversible commitment density. The Gauss-law derivation in this paper (Section 6.6) is the same structural result as BCB, now expressed in the commitment language and paired with an independent Hubbard–Stratonovich derivation. In short:

- BCB gave the conservation-law inevitability of Poisson.
- This paper supplies a concrete substrate quantity that plays the role of the source across scales.

(B) Gravity as Critical Entropic Back-Pressure [4] — "Why horizons and saturation exist."

The critical-capacity papers emphasize that gravity and horizons arise when a finite-capacity substrate approaches saturation: proper-time suppression, horizon formation as areal capacity exhaustion, and smooth regulation near "black hole" regimes.

Complement, not contradiction: This paper inherits the same saturation intuition but places it after the Newtonian derivation, as the high-loading regime of the same underlying entropy mechanism. Put simply:

- The present paper establishes the weak-field entropic response from overlap statistics and flux structure.
- The critical back-pressure paper describes the nonlinear response / saturation regime when the same substrate cannot continue to accommodate constraint influence.

So the relationship is: linear response here; nonlinear saturation there — two regimes of the same capacity-limited substrate.

(C) Gravity-as-Entropy-Gradient / Entropy-field (ϕ, χ) papers [6 and related field-theoretic VERSF notes] — "Effective fields as calculational tools."

Those papers use bulk scalar fields and Green's function solutions to represent how "entropy influence" propagates and how timing effects appear in clock networks.

Complement, not contradiction: This paper clarifies the status of those fields: the "entropy field" language is an effective parametrization of the coarse-grained entropy functional and its kernel, not a separate ontology. Where the earlier papers sometimes read like they postulate a field, this paper derives the need for an effective field via:

- the overlap kernel formulation, and
- the Hubbard–Stratonovich transformation, which naturally introduces an auxiliary potential.

So the earlier field language remains valid as the EFT representation of the kernel-based entropy structure developed here.

(D) Why Emergent Gravity Must Be Spin-2 [5] — "Why GR is the unique covariant completion."

That paper closes the "tensor structure gap," showing that universal coupling and consistency force spin-2 and Einstein–Hilbert in the IR.

Complement, not contradiction: This paper supplies the missing upstream input that makes the spin-2 theorem applicable in the VERSF context: it shows how a universal long-range interaction arises from commitment density through entropy maximization and flux structure. Then the spin-2 paper provides the downstream closure:

- This paper: why you get a long-range universal interaction from the substrate.

- Spin-2 paper: why the only consistent covariant IR completion is GR.

Together they turn "covariant extension" from conjecture into inevitability under stated IR assumptions.

(E) Void Anchoring / Mass Scale papers [1, 2] — "Where the mass source comes from."

Those papers derive the mass scale as a function of anchoring parameters (p_{ϵ}/K_c) and show consistency with rest energy, redshift structure, and (conditionally) inertial equivalence.

Complement, not contradiction: This paper is the gravity-side completion of the anchoring work. It takes the mass result as an imported microphysical identification and shows that the same quantity $\rho_c \sim p_{\epsilon}/K_c$ is also the entropy-loading source that generates gravitational attraction after coarse-graining. So:

- Void anchoring explains why matter has a mass scale at all.
- This paper explains why that mass scale must generate gravitational attraction through the substrate's entropy structure.

13.3 The Programme-Level Hierarchy

A clean way to view the collection is as a layered stack:

Layer	Paper(s)	Role
Foundational constraint	BCB / distinguishability [3]	Conservation + admissibility → Poisson/geometry in equilibrium
Microphysical mass	Void anchoring / coupling [1, 2]	Stable modes → commitment density → rest mass
Macroscopic response (linear)	This paper	Overlap kernel + flux conservation → Newtonian gravity
Macroscopic response (nonlinear)	Critical back-pressure [4]	Capacity saturation → horizons and regulation
Covariant IR completion	Spin-2 uniqueness [5]	Universal coupling + long range → GR forced
Effective-field phenomenology	Entropy/clock fields [6 and related notes]	EFT variables for Green's functions, clock tests, cosmology

This hierarchy makes the earlier papers mutually reinforcing rather than competing: each paper occupies a distinct explanatory layer.

13.4 What Is Newly Added in This Paper (the "Delta")

This paper's primary contribution relative to the earlier gravity papers is:

- an explicit overlap kernel micro-to-macro bridge (subadditive constraint intersection → nonlocal entropy functional),
- a controlled gradient expansion linking kernel moments to EFT coefficients,
- two independent routes to Poisson (Hubbard–Stratonovich + Gauss law),
- a rigorous attraction proof under explicit kernel conditions,
- and a clean connection from the resulting universal long-range coupling to the spin-2 inevitability closure.

So rather than contradicting earlier work, it consolidates and upgrades the logical spine: it makes clear which pieces are structural inevitabilities, which are effective parametrizations, and which remain open microphysical calculations.

13.5 Why Apparent Differences Across Papers Are Not Contradictions

Some earlier manuscripts talk in terms of entropy fields, back-pressure and proper-time suppression, distinguishability flow, commitment density, and horizons as capacity saturation. These are not competing theories; they are different coordinate systems on the same research programme:

- *Flow language* emphasizes conservation structure (Gauss law, Poisson).
- *Field language* emphasizes calculational convenience (Green's functions, propagation).
- *Capacity language* emphasizes nonlinear response and horizon physics.
- *Commitment language* emphasizes microphysical sourcing (mass origin).

This paper's role is to state the dictionary between them clearly and show how the same source variable can be tracked through all layers.

13.6 Summary: "One Programme, Many Lenses"

Taken together, the VERSF gravity papers form a coherent progression:

- **BCB** tells you the shape gravity must take in equilibrium (Poisson/geometry).
- **Void anchoring** tells you what matter is at the substrate level (commitment density).
- **This paper** tells you why matter's substrate definition must produce Newtonian attraction (overlap entropy + Gauss law) and how that implies the IR covariant completion.
- **Spin-2 inevitability** tells you why the completion is uniquely GR.
- **Critical back-pressure** tells you what happens when the response saturates (horizons, regulation).
- **Entropy/clock field papers** provide the effective field toolkit for predictions.

That is complementarity by design: different papers illuminate different layers, but they all point back to the same conserved substrate story.

14. Limitations and Open Questions

14.1 Overlap kernel — beyond mean field. Section 5.5 derives the overlap kernel $\hat{K}(r)$ via cumulant expansion and lattice transport, with the $1/r$ shape following from symmetry (locality + isotropy + BCB) and the amplitude $B = c_\chi f(1-f) \cdot a / (4\pi k_B T_{\text{sub}})$ from mean-field. Appendix D summarizes the predictive program for Newton's constant. The remaining open problems are: (i) computing the $O(1)$ prefactor c_χ (via lattice simulation or higher-order closure), (ii) computing the one-loop correction $\Pi(k)$ to quantify beyond-mean-field renormalization of B (Section 5.5.7), (iii) establishing the precise W_0 -dependence of c_χ , and (iv) computing the numerical prefactors relating B to G via the mass-commitment conversion. A lattice simulation of the microscopic model — sites with W_0 labels, elimination at fraction f , constraint diffusion with exact accounting — could settle all four questions and would provide the first quantitative test of the mean-field amplitude.

14.2 Exactness of BCB. The no-screening theorem (Section 5.5.6) and the no-leakage theorem (Appendix A) together show that $1/r$ is the symmetry-protected IR fixed point provided BCB is exact — and that any violation of BCB requires spontaneous restoration of eliminated configurations, contradicting the irreversibility postulate. Whether bit conservation is a fundamental property of the substrate or merely an excellent approximation is the key remaining structural question for the gravitational sector. If BCB is only approximate, a tiny screening mass $m^2 > 0$ survives, producing Yukawa-modified gravity with range $1/m$. The empirical success of inverse-square gravity constrains any such leakage to $m \lesssim H_0/c$. A lattice simulation testing whether the discrete substrate's constraint propagation respects exact bit conservation would settle this question directly.

14.3 Substrate temperature. T_{sub} is constrained by the self-consistency relation Eq. (62) but not derived from fluctuation statistics. Appendix E establishes its thermodynamic definition ($1/T_{\text{sub}} = \partial S_{\text{uncommitted}} / \partial E$) and clarifies that T_{sub} is determined by the substrate's energy–configuration map but not predicted without specifying the detailed microphysics. An operational derivation connecting T_{sub} to the variance of uncommitted substrate configurations — or equivalently, a microscopic Hamiltonian for the substrate — would remove T_{sub} from the input parameters and close the G normalization.

14.4 Sub-Planckian regime. The Weinberg–Deser theorem (Section 8.2) guarantees the covariant extension to general relativity in the continuum limit where Lorentz invariance holds. Appendix B analyzes the conditions for emergent Lorentz invariance via RG suppression of irrelevant operators. At scales approaching the substrate lattice spacing a , Lorentz invariance breaks down and the spin-2 uniqueness argument no longer applies. The form of the gravitational interaction at sub-Planckian scales — where the discrete substrate structure becomes relevant — is an open problem and a potential source of observable departures from GR (see Appendix B.4 for falsifiable consequences).

14.5 Inertia and the equivalence principle. The linear-response framework (Section 7.3) establishes a force–response relation with the structure of $F = ma$, with effective inertial mass $m_{\text{eff}} = \chi_0^{-1}$. Two open calculations remain: (i) computing χ_0 from substrate parameters ($W_0, f, a, \Delta t$), and (ii) proving that m_{eff} equals the gravitational mass — i.e., that the equivalence principle holds within the framework. Appendix C identifies the structural condition: exact equivalence holds if χ_0 depends only on local substrate parameters (f, W_0, a) and not on particle

identity (the "susceptibility universality" condition). Current Eötvös-type experiments constrain violations to $\Delta m/m \lesssim 10^{-15}$.

14.6 Higher-order terms. The entropy functional is truncated at quadratic order in $\tilde{\rho}_c$. The cubic and higher terms (three-body overlaps) could produce corrections to Newtonian gravity at high densities.

14.7 Bekenstein bound. Whether the commitment-density saturation at the horizon reproduces $S = A/(4l_P^2)$ depends on substrate parameters. This is checkable once $\hat{K}(r)$ is determined.

14.8 Dark energy. The homogeneous fixed point of the scale flow produces a constant entropy density that could source a cosmological constant. Whether the magnitude matches observations is unknown.

14.9 Connection to void coupling coherence. The phase coherence formalism determines $\rho_c = p_\varepsilon/K_c$ for individual modes. The present paper treats $\tilde{\rho}_c$ as a classical field. A complete treatment should derive multi-mode commitment density from interacting coherence dynamics.

Conjecture: coherence collapse and symmetry breaking. The synchronization transition in the void coupling paper — the threshold below which $R = 0$ and no mass emerges — has a parallel here: below the critical coherence, commitment density vanishes and the entropy-gradient gravitational structure disappears. If particle families correspond to distinct synchronization phases, the gravitational sector would exhibit family-dependent structure. This is entirely untested.

15. Conclusion

Mass and gravity are unified in VERSF as scale-separated manifestations of irreversible information commitment.

The derivation chain:

```

Irreversible anchoring micro-events (A2)
↓
Commitment density  $\rho_c = n_s \cdot p_\varepsilon / K_c$  (Section 3)
↓
Constraint overlap → Nonlocal entropy kernel  $\hat{K}(r)$  (Section 5)
↓ (constraint propagation + mean-field)
Derived kernel:  $\hat{K}(r) = B/r$ ,  $B = c_\chi f(1-f) \cdot a / (4\pi k_{BT\_sub})$  (Section 5.5)
↓ (BCB → shift symmetry → no mass term)
No-screening fixed point:  $m^2 = 0$  symmetry-protected in IR (Section 5.5.6)
↓ (irreversibility → no restoration → no leakage)
No-leakage theorem:  $\gamma = 0$  structurally necessary (Appendix A)
↓ (one-loop: shape protected, amplitude renormalized at  $O(1/W_0)$ )
(Section 5.5.7)
↓ (gradient expansion)

```

Local effective entropy functional $S_{\text{eff}}[\tilde{\rho}_c]$ (Section 5.3)

↓

├─ T_{sub} from substrate state counting (Section 6.1a)
 ↓ (two independent routes to Poisson)

├─ Hubbard–Stratonovich $\rightarrow K^{-1} \propto -\nabla^2 \rightarrow$ Poisson (Section 6.2)
 └─ Entropy flux \rightarrow Gauss law $\rightarrow 1/r^2 \rightarrow$ Poisson (Section 6.6)

↓

Newtonian gravity: $\nabla^2\Phi = 4\pi G\rho_m$, $G \propto c_\chi f(1-f) \cdot a$ (Section 6.3)

↓

├─ Inertia: linear response $\rightarrow m_{\text{eff}} = (\chi_0 V_{\text{mode}})^{-1} \rightarrow F = m_{\text{eff}} \cdot a$
 (Section 7.3)

↓

Universal coupling to $T_{\mu\nu}$ (Section 8.1)
 ↓ (Weinberg–Deser theorem)

Massless spin-2 \rightarrow Einstein–Hilbert \rightarrow General relativity (Section 8.2)
 ↓ (four admissibility rules)

Gravity = BCB (accounting rule) – one of exactly four interactions (Section 8.2)

Each arrow represents either a derivation under stated assumptions, a controlled approximation, or an imported theorem — and each is labeled as such throughout the paper.

At the microscopic scale, commitment density $\tilde{\rho}_c = n_s p_\varepsilon / K_c$ determines rest mass via $\rho_m = (\eta\hbar/c^2\Delta t)\tilde{\rho}_c$. At the macroscopic scale, the nonlocal entropy functional — derived from the subadditive overlap of substrate constraints via the kernel $\hat{K}(r)$ — generates gravitational attraction through entropy maximization. The covariant extension to full general relativity follows from the Weinberg–Deser spin-2 uniqueness theorem, given that the entropic force couples universally to the stress-energy tensor.

Explicit classification of results.

Derived in this paper:

- Entropy functional from overlap kernel with coefficients $\mu = K_0$, $\kappa = K_2/6$ (Section 5)
- Overlap kernel via cumulant expansion and lattice transport (Section 5.5.1–5.5.3)
- Shape/normalization separation: $1/r$ from symmetry, amplitude from mean field (Section 5.5.2–5.5.3)
- Infrared no-screening fixed point: BCB forbids mass term via shift symmetry (Section 5.5.6); no-leakage theorem (Appendix A)
- One-loop structure: shape protected, amplitude renormalized at $O(1/W_0)$ (Section 5.5.7)
- Newton's constant $G \propto c_\chi f(1-f) \cdot a$ in terms of substrate parameters (Section 5.5.8)
- Poisson equation from Hubbard–Stratonovich saddle point (Section 6.2)
- Poisson equation from entropy-flux Gauss law (Section 6.6)
- Attractive sign of gravitational force under kernel conditions (Section 6.4)
- Self-consistency relation constraining G in terms of substrate parameters (Section 6.5)
- Formal definition and consistency of T_{sub} from substrate state counting (Section 6.1a)
- Force–response relation with $F = ma$ structure via linear response, $m_{\text{eff}} = (\chi_0 V_{\text{mode}})^{-1}$ (Section 7.3)
- Schwarzschild radius as consistency check from flux saturation (Section 10.2)

Imported from established physics:

- Spin-2 uniqueness theorem: Weinberg (1964), Deser (1970) (Section 8.2)
- Weinberg–Witten theorem: constraint on emergent gravity programs (Section 8.2)
- Four fundamental interactions as admissibility rules (Section 8.2)
- Mass–commitment identification: void anchoring/coupling papers (Section 3)
- Hubbard–Stratonovich transformation: Hubbard (1959), Stratonovich (1957)
- Mass-energy equivalence $E = mc^2$

Formally motivated conjectures:

- Exactness of BCB (assumed in no-screening theorem; strengthened to structural necessity by no-leakage theorem, Appendix A; if only approximate, small m^2 survives)
- Equivalence of effective inertial mass m_{eff} and gravitational mass (equivalence principle, Section 7.3; structural conditions in Appendix C)
- Emergent Lorentz invariance in the IR (required for Weinberg–Deser; RG argument in Appendix B)
- G normalization: c_χ computable but not yet computed (Appendix D)
- T_{sub} : thermodynamically defined but not predicted without substrate Hamiltonian (Appendix E)
- Scalar field embedding $\tilde{\rho}_c \propto |\partial_t \phi|^2$ (Section 11)
- Full computation of χ_0 in substrate parameters (Section 7.3, open)

What is open: The mean-field kernel derivation (Section 5.5) provides the functional form and substrate-parameter dependence. The no-screening theorem (Section 5.5.6) and the no-leakage theorem (Appendix A) establish that $1/r$ is the symmetry-protected IR fixed point under BCB — with BCB itself shown to be structurally necessary given the irreversibility postulate. The remaining frontier is computing beyond-mean-field corrections and extracting precise numerical prefactors that would make the framework's predictions for G quantitatively testable.

Appendix A: Structural Necessity of Exact Bit Conservation (No-Leakage Theorem)

A.1 Statement of the Issue

Section 5.5.6 establishes that screening corresponds to the presence of a relaxation term $D\nabla^2 q - \gamma q = -s$ with screening mass $m^2 = \gamma/D$. The argument given there asserts that BCB forbids $\gamma > 0$, thereby protecting the massless $1/r$ kernel. This appendix strengthens that result by demonstrating that any nonzero leakage term γ is incompatible with the irreversible commitment ontology defined in Sections 2–4: any relaxation channel that restores eliminated configurations without compensating structural update violates irreversibility and therefore contradicts the foundational substrate postulate.

A.2 Irreversible Commitment and State Elimination

From Section 5.1, each commitment eliminates a fraction f of accessible substrate configurations. Let $\Omega_x(t)$ denote the set of accessible microstates at site x . Irreversible commitment implies:

$$\Omega_x(t + \Delta t) \subseteq \Omega_x(t) \text{ (A1)}$$

i.e., the accessible set shrinks monotonically under commitment. This monotonic inclusion is the operational definition of irreversibility in the framework. No allowed operation enlarges Ω_x unless accompanied by a compensating commitment elsewhere (global distinguishability balance).

A.3 What Screening Physically Implies

A relaxation term $-\gamma q$ in the transport equation corresponds to spontaneous decay of distinguishability deficit. Operationally this means: previously eliminated configurations become re-accessible, constraint influence fades without compensating structural update, and the eliminated configuration set is partially restored. That is equivalent to $\Omega_x(t + \Delta t) \supset \Omega_x(t)$ without corresponding compensating elimination elsewhere. This directly violates irreversibility of commitment, conservation of distinguishability accounting, and the global entropy bookkeeping that underlies the entropy functional. Screening therefore corresponds to spontaneous restoration of eliminated states — which is disallowed by the substrate postulate.

A.4 No-Leakage Theorem

Theorem. Let commitment events be defined as irreversible eliminations of accessible substrate configurations. Assume: (i) eliminated configurations cannot be restored without compensating commitment elsewhere (BCB); (ii) constraint influence is transported locally; (iii) no external reservoir of distinguishability exists. Then the coarse-grained transport equation for distinguishability deficit cannot contain a relaxation term of the form $-\gamma q$.

Proof sketch. A relaxation term implies exponential decay of constraint influence: $q(t) \sim e^{-\gamma t}$, which requires restoration of eliminated configurations to reduce deficit. But restoration without compensating elimination contradicts (i). Since transport is local and there is no external distinguishability source (iii), no allowable mechanism exists to generate such decay. Therefore $\gamma = 0$. ■

A.5 Shift Symmetry as a Consequence

In Section 5.5.6 the shift symmetry $\Phi \rightarrow \Phi + \text{const}$ was used to forbid the mass term. This symmetry is not an independent assumption — it is the macroscopic reflection of the microscopic monotonicity of configuration elimination. Because eliminated states cannot be restored, only differences in potential (gradients) have physical meaning; absolute potential offset corresponds to uniform distinguishability baseline, which has no physical effect.

Therefore: the gradient term is allowed, the mass term is forbidden, and the shift symmetry emerges from irreversible elimination rather than being imposed.

A.6 Stability Under Coarse-Graining

One might worry that coarse-graining could generate an effective γ term even if microscopic leakage is absent. However, the shift symmetry derived above is exact under BCB. Radiative generation of $m^2\Phi^2$ would break this symmetry, so such terms cannot appear at any scale. This parallels gauge symmetry protection of photon masslessness — but here the protecting symmetry arises from irreversibility, not gauge redundancy.

A.7 Empirical Implication

If BCB were only approximate, one would expect $\nabla^2\Phi - m^2\Phi = 4\pi G\rho$ with extremely small m . Current observations constrain $m \lesssim H_0/c$. The empirical success of inverse-square gravity over scales from millimeters to megaparsecs is therefore direct support for effective BCB exactness.

A.8 Structural Conclusion

The infinite-range nature of gravity is not an arbitrary kernel assumption. It follows from: irreversible elimination of configurations, no spontaneous restoration, local transport, and absence of an external distinguishability reservoir. Screening would require violation of the commitment ontology. Within the VERSF substrate model, strict infinite range is structurally necessary unless the foundational irreversibility postulate is relaxed.

Appendix B: Conditions for Emergent Lorentz Invariance

B.1 The Structural Role of Lorentz Invariance

Section 8 invokes the Weinberg–Deser theorem to establish that universal long-range coupling implies a massless spin-2 field whose nonlinear completion is general relativity. That theorem requires Lorentz invariance in the infrared. Since the VERSF substrate is discrete and possesses a fundamental lattice spacing a , Lorentz invariance cannot be fundamental. It must emerge in the coarse-grained limit $L \gg a$. This appendix clarifies the conditions under which emergent Lorentz invariance is expected and the consequences if it is only approximate.

B.2 Necessary Conditions

Emergent Lorentz symmetry requires: (i) isotropic lattice connectivity, (ii) finite maximal signal speed (causal throughput bound), (iii) linear dispersion at low momentum, and (iv) absence of preferred-frame operators in the IR effective action.

Condition (ii) is already present: the throughput-limited propagation analysis establishes a universal maximal transmission speed c . Condition (iii) follows if the linearized transport equation for small perturbations takes the form $\partial_t^2 \psi = c^2 \nabla^2 \psi + O(a^2 \nabla^4)$, which yields relativistic dispersion at leading order.

B.3 RG Argument for IR Restoration

Under coarse-graining, higher-order lattice anisotropies are suppressed by powers of a/L , and Lorentz-violating operators scale as irrelevant perturbations if their mass dimension exceeds 4 in the IR effective field theory. Thus, if the leading IR kinetic term is quadratic and isotropic, Lorentz invariance emerges as an accidental symmetry at long distances. This mechanism parallels emergent Lorentz symmetry in graphene, relativistic phonon modes in critical systems, and Wilsonian RG restoration of isotropy in lattice field theories.

B.4 Falsifiable Consequence

If Lorentz invariance is only approximate, modified dispersion relations appear at high energies, preferred-frame effects arise, and the gravitational sector acquires additional propagating modes. These effects are tightly constrained by current observations (gamma-ray burst timing, gravitational wave speed measurements, Michelson–Morley-type experiments). Lorentz invariance in VERSF is therefore not assumed as exact — it is an IR property contingent on operator suppression, with violations bounded empirically.

B.5 Status

The derivation of exact Lorentz invariance from substrate microdynamics remains open. However, the IR restoration of Lorentz symmetry is structurally plausible under RG suppression of irrelevant operators. The GR completion (Section 8) is therefore conditional on emergent Lorentz invariance in the infrared — a condition that is generically expected but not yet proven from the substrate rules.

Appendix C: Structural Basis of the Equivalence Principle

C.1 Statement of the Issue

Section 7 derives a force–response relation with structure $F = m_{\text{eff}} \cdot a$, where effective inertial mass m_{eff} is the inverse susceptibility of the commitment density field. But equality between m_{eff} and the gravitational mass m_{grav} — which sources entropy gradients in Section 6 — is not formally proven. This appendix clarifies when and why equality is expected.

C.2 Structural Identity

Both masses derive from the same underlying quantity: the commitment density $\tilde{\rho}_c = n_s p_\epsilon / K_c$. Gravitational mass sources entropy gradients (Section 6). Inertial mass measures the susceptibility of commitment density to external perturbation (Section 7.3). If the susceptibility χ_0 scales inversely with commitment density — $\chi_0 \propto 1/\tilde{\rho}_c$ — then $m_{\text{eff}} \propto \tilde{\rho}_c$, which matches the gravitational mass scaling exactly.

C.3 Universality Condition

Exact equivalence requires: (i) homogeneous scaling of susceptibility across all modes, and (ii) no composition-dependent corrections to χ_0 . If χ_0 depends only on local substrate parameters (f , W_0 , a) and not on particle identity, then inertial and gravitational mass coincide for all objects regardless of composition.

C.4 Empirical Constraint

Eötvös-type experiments constrain violations of the equivalence principle to $\Delta m/m \lesssim 10^{-15}$. VERSF therefore requires universality of χ_0 at this precision — any composition-dependent correction to the susceptibility would produce detectable equivalence principle violations.

C.5 Status

The equivalence principle is structurally expected if susceptibility derives solely from commitment density and does not depend on the internal structure of the committed mode. A full microscopic computation of χ_0 from substrate parameters — verifying its independence from particle identity — would elevate this from structural plausibility to derived result.

Appendix D: Normalization of Newton's Constant

D.1 Separation of Shape and Amplitude

Section 5.5 establishes $\hat{K}(r) = B/r$ with $B = c_\chi f(1-f) \cdot a / (4\pi k_B T_{\text{sub}})$. The $1/r$ shape is symmetry-protected (Sections 5.5.2, 5.5.6, Appendix A). The amplitude B determines Newton's constant G .

D.2 Expression for G

From Section 6: $G \propto c_\chi f(1-f) \cdot a$. The strength of gravity depends on: the lattice spacing a , the elimination fraction f (maximized at $f = 1/2$), and the renormalization prefactor c_χ encoding lattice geometry and inter-site coupling structure.

D.3 Beyond Mean Field

One-loop corrections give $B_{\text{eff}} = B_0[1 + O(1/W_0)]$ (Section 5.5.7). The shape is fixed by symmetry; the amplitude is renormalized. The leading correction scales as $1/W_0$ and is systematically computable.

D.4 Predictive Program

A lattice simulation of the microscopic model — W_0 states per site, elimination at fraction f , constraint diffusion with exact accounting — would compute c_χ numerically and yield a quantitative prediction for G in terms of substrate parameters. This is the most direct path to a testable numerical prediction from the framework.

D.5 Status

The amplitude of G is not derived from first principles alone. It is constrained to the form $G \propto c_\chi f(1-f) \cdot a$ with $O(1)$ prefactor c_χ computable by lattice simulation or higher-order closure. Determining c_χ is the primary quantitative open problem in the gravitational sector.

Appendix E: Substrate Temperature and Thermodynamic Consistency

E.1 Role of T_{sub}

The effective substrate temperature T_{sub} appears in: the free energy $F = U - T_{\text{sub}} S$ (Section 6.1), the kernel amplitude B (Section 5.5.3), and the self-consistency relation for G (Section 6.5, Eq. (62)).

E.2 Thermodynamic Definition

From Section 6.1a, Eq. (50):

$$1/T_{\text{sub}} = \partial S_{\text{uncommitted}}/\partial E \quad (\text{E1})$$

This is a structural identity from state counting: T_{sub} is the energy invested per unit of entropy removed when configurations are eliminated. It is a legitimate thermodynamic quantity given the substrate's state-counting structure.

E.3 Why T_{sub} Is Not Arbitrary

T_{sub} is: (i) the energy cost per eliminated configuration, (ii) a property of substrate microphysics, and (iii) constrained by the entropy–energy relation of the uncommitted substrate modes. It is not a free parameter — it is determined by the substrate's energy landscape — but its value is not predicted by the framework without specifying the detailed energy–configuration map.

E.4 Why It Is Not Yet Predicted

The detailed microphysics of the substrate's energy–configuration relationship is not specified within the scope of this paper. T_{sub} therefore remains an input parameter constrained by the self-consistency relation Eq. (62): $G = (k_B T_{\text{sub}})^2 \cdot (\hat{K} \text{ amplitude})^2 / (\eta^2 \hbar c)$. This parallels other entropic gravity programs: in Jacobson (1995) and Verlinde (2011), the reservoir temperature is likewise treated as given rather than computed from microphysics.

E.5 Future Direction

A microscopic Hamiltonian for the substrate would allow: explicit calculation of $dN_{\text{eliminated}}/dE$, independent determination of T_{sub} , and full closure of the G normalization. This would remove T_{sub} from the input parameters and make the framework's gravitational predictions fully self-contained.

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