Emergent Invariant Speed and Lorentz Structure from Record-Theoretic Postulates on a 2D Commitment Surface

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For the General Reader

The speed of light is one of the most fundamental constants in physics. Every observer, no matter how fast they move, measures the same value: 299,792,458 meters per second. Einstein took this as a starting assumption. This paper asks: can we derive it instead?

The answer developed here begins with a simple idea: operational time is defined by irreversible commitments; reversible dynamics are parameterized by a bookkeeping variable. Time only advances — in the empirical sense — when something irreversible happens: when a distinction is made that can't be undone. Think of it this way: a pendulum swinging back and forth, perfectly frictionless, doesn't create a new record. It just oscillates. Something irreversible must happen — a mark must be made — for a moment in time to be distinguishable from the last. We call these irreversible record events "commitments" or "bits."

Space works similarly. Two points are only distinguishable if they can host independent records. The minimum distance between independent records defines the smallest meaningful unit of space.

Given these two ideas — a smallest unit of time (one irreversible commitment) and a smallest unit of space (one independent record separation) — the speed of light is simply their ratio:

c = (smallest spatial distinction) / (smallest temporal distinction)

The paper proves three things:

1. **A maximum speed must exist.** If information can only spread to neighboring cells one step at a time, there's a speed limit. This is like how a rumor can only spread so fast through a crowd — each person can only tell their neighbors.
2. **All observers must agree on this speed.** If two observers can verify each other's records and agree on which events caused which, then mathematics forces them to be related by the same transformations Einstein discovered (the Lorentz group). The invariant speed falls out automatically.

3. **Clocks and rulers built from these records measure exactly this speed.** The internal structure of the theory is self-consistent: instruments made from the same commitment physics recover the same speed that the wave equation predicts.

The paper does not derive the numerical value 299,792,458 m/s — that requires choosing a unit system, which no theory can avoid. But it shows that the existence and universality of an invariant speed are consequences of how irreversible records work, not arbitrary features of nature.

A clock, in this framework, is not just something that ticks — it is a device that repeatedly creates irreversible records. The quality of a clock is measured by how much information each tick commits. Information theory (specifically, the Fano inequality) then constrains how many physical oscillations are needed per committed bit of time.

The deepest result is a separation: the geometry of space (the cone structure, the speed limit) comes from reversible physics, while the arrow of time (the fact that time moves forward) comes from irreversible commitments. These are logically independent. That they agree — that clocks and rulers built from irreversible records measure the same speed as the reversible wave equation predicts — is the central self-consistency result of the paper.

This separation has a striking thermodynamic consequence. A stable second is not made of 9 billion irreversible events — one per oscillation. It is made of a vast reversible phase accumulation plus a sparse irreversible record update: about 33 bits, the minimum needed to encode "9,192,631,770 cycles occurred." The thermodynamic cost of keeping time therefore scales with committed bits, not carrier cycles — a suppression of roughly a billion-fold compared to the naive estimate. This is a testable prediction.

The framework also yields a concrete scaling law: the size of a commitment cell depends on temperature (via the thermal correlation length of the environment) and on a lattice factor derived from the theory's fixed-point equation. At room temperature, this gives a commitment cell of order 50–150 micrometers, implying a fundamental bit rate of roughly a trillion irreversible commitments per second — one every fraction of a picosecond. This rate falls in the regime of phonon scattering and electronic dephasing times, precisely where quantum coherence is known to give way to irreversible classical records.

The same framework can be applied to ultracold quantum gases (Bose–Einstein condensates), where the commitment cell is tentatively identified with the healing length — the minimum scale for stable independent structure in the condensate. This gives a different commitment rate (femtoseconds instead of picoseconds, because the healing length is much smaller), but the same invariant speed. The framework suggests a concrete experimental protocol: tune the condensate density, measure the healing length, and check whether the fastest record-formation time tracks the predicted value.

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Technical Abstract

We derive the existence of a universal invariant propagation speed *c* and show that admissible observers necessarily form the Poincaré group with that speed, starting from five minimal postulates governing irreversible record formation on a two-dimensional commitment surface Σ. The central conceptual commitment is that time is not primitive: it emerges from irreversible differentiation events (bits), not from reversible micro-oscillations. The invariant speed *c* therefore arises as the conversion factor between minimal spatial distinguishability and minimal temporal distinguishability — both defined through irreversible commitments.

The derivation proceeds in three stages. First, locality and isotropy force a finite causal cone with characteristic speed $c = \sqrt{(\chi/\rho)}$. Second, the Alexandrov–Zeeman theorem, applied to observers preserving record-causal order and commitment statistics, implies that admissible coordinate

transformations are precisely the Lorentz group with invariant speed c. Third, a micro-tick model of reversible updates accumulating toward irreversible commitments, defined on a triangular lattice, yields a unique fixed-point equation for the operational distinguishability ratio; its solution matches the continuum cone slope with no fitted parameters beyond one dimensionless substrate combination. The cone structure (geometry) and the arrow of time (commitment ordering) are shown to be logically independent features of the framework, unified at the admissible fixed point. The numerical SI value of c requires one empirical mapping — but since a clock is a commitment channel, the Fano inequality bounds this mapping: one cannot obtain one time-bit in fewer than $[1 - h(\varepsilon)]/I_{cycle}$ physical cycles, driving the calibration factor toward unity for high-quality clocks.

Branch 1 assumption. The long-range mediator lives on the 2D surface Σ itself, so the static Green's function is logarithmic and all rod/clock realizations must be consistent with 2D log-binding physics.

Part I — Existence of a Finite Cone Speed

For the general reader: This section establishes the most basic result — that there must be a speed limit. The argument is intuitive: if information can only pass between neighboring locations one step at a time (like a message passed along a chain of people), then there is a maximum speed at which any signal can travel. We formalize this with five simple postulates about how irreversible records are created, and show that any system satisfying these postulates automatically produces a finite maximum speed. This speed, which we call c, emerges from just two properties of the underlying substrate: its stiffness (how strongly neighboring regions pull on each other) and its inertia (how sluggishly they respond).

1. Definitions and Postulates

Let Σ be a two-dimensional manifold (the commitment surface).

Three-layer ontology. The framework distinguishes three conceptual layers:

- **Layer 1: Reversible micro-updates (ticks).** Sub-temporal microstructure. A tick is an invertible update of duration τ. Ticks do not generate arrow-of-time structure. They are internal oscillations, not time.
- **Layer 2: Irreversible commitments (bits).** A commitment is an irreversible record event producing one stable bit. Commitments define the arrow of time. The minimal irreversible commitment interval is $\tau_{bit} = \nu\tau$, where ν is the ticks-per-bit (TPB).
- **Layer 3: Operational time.** Constructed entirely from commitments: $\Delta t = N_{bits} \cdot \tau_{bit}$, where N_{bits} is the number of committed events. Time emerges only when basin crossings become irreversible.

Definition 1 (Tick). A tick is a reversible micro-update of duration τ. Ticks are sub-temporal: they do not individually constitute temporal passage.

Definition 2 (Commitment/Bit). A commitment is an irreversible record event producing one stable bit. Commitments are the atoms of time.

Definition 3 (Ticks-per-bit). $\nu \equiv$ ticks-per-bit; the commitment timescale is $\tau_{bit} = \nu\tau$.

Postulates (minimal admissible class):

- **(P1) Local update.** Coupling is finite-range per tick.
- **(P2) Tick reversibility.** The micro-update map per tick is invertible.
- **(P3) Commitment irreversibility.** After threshold crossing, the record is not invertible.
- **(P4) Coarse isotropy and homogeneity on Σ.**
- **(P5) Admissibility.** Records must be stable (error $\leq \varepsilon$ over time T), finite-cost, and scale-consistent under coarse-graining.

These five postulates define the class of admissible record-forming systems on Σ. Everything that follows is derived from them.

Dimensional lockdown: dim(Σ) = 2. Throughout this paper, the commitment surface Σ is taken to be two-dimensional. This assumption enters the derivation at multiple points: the logarithmic Green's function of the lattice Laplacian (§16), the (2+1)-dimensional Lorentz structure required by the Alexandrov–Zeeman theorem (§7), the 3D Ising universality class of the record potential (§17.2), and the numerical constants in the fixed-point equation (§17). We now show that $d = 2$ is not merely convenient but uniquely forced by the postulates and closure machinery.

Definition (derived dimensionality). We say *d* is derived within the programme if all other values of *d* lead to a violation of at least one of the postulates (P1–P5) or observer admissibility conditions (O1–O3) once the closure machinery is applied. This is stronger than "preferred" or "consistent": if alternative dimensions cannot sustain the demanded invariances, the dimension is fixed.

Universality-class minimality. Before proceeding, we establish why the record sector is necessarily ϕ^4. A bistable record requires a $\mathbb{Z}_2$ order parameter (two stable macrostates — the committed "0" and "1" of P3). Given locality (P1) and isotropy on Σ, the effective action for the order-parameter field ϕ is constrained by analyticity to have leading terms ϕ^2 (mass) and ϕ^4 (first allowed nonlinearity respecting $\mathbb{Z}_2$). Higher-order terms (ϕ^6, ϕ^8, ...) are irrelevant at the infrared fixed point in $d_{eff} \leq 4$ and do not affect the universality class. Therefore ϕ^4 is the minimal — and, by universality, the only relevant — description of the record sector. This is not a modeling choice; it follows from the postulates ($\mathbb{Z}_2$ bistability from P3, locality from P1, isotropy from the lattice symmetry of Part V).

No-go for d = 1: Lorentz rigidity fails (Theorem 45).

If Σ has dimension $d = 1$, the emergent spacetime has dimension $D = d + 1 = 2$. In 1+1 dimensions, the causal order relation $\prec$ exists, but the automorphism group of causal order is too

large: beyond Poincaré × dilation, there exist monotone reparametrizations along null coordinates ($u \mapsto f(u)$, $v \mapsto g(v)$ for arbitrary monotone f, g) that preserve $\prec$. These additional transformations preserve "who can influence whom" but distort proper-time assignments — and therefore distort commitment-rate statistics: cadence, admissibility thresholds, and stability parameters.

But (O3) requires admissible observers to preserve commitment statistics under admissible reparametrizations. In $D = 2$, causal preservation alone permits transformations that violate (O3). To recover unique Lorentz structure, one must add extra constraints beyond record-causality (differentiability, linearity, metric preservation) — contradicting the programme's goal that Lorentz structure emerge from record-theoretic admissibility alone.

Theorem 45 (1D no-go). If $d = 1$, then (O1–O2) (record comparability + causal consistency) do not constrain the admissible observer group tightly enough to enforce unique Lorentz structure while satisfying (O3) statistical invariance. Therefore $d = 1$ is inadmissible. This establishes $d \geq 2$ as a necessary condition.

No-go for $d \geq 3$: nonuniversal amplitude contamination (Theorem 46).

This is the decisive lockdown. (P5) requires scale-consistency under coarse-graining: the closure constant $\alpha_*(\varepsilon)$ must be universal — independent of microscopic cutoff, integrator class, and nonuniversal amplitudes. We now show that this is only possible when $d = 2$, by deriving the general-d closure equation explicitly.

General-d lattice variance. In d spatial dimensions, the Laplacian eigenvalue scales as $\lambda(k) \sim \ell_\Sigma^2 k^2$ at small scales, with UV cutoff $k_{max} \sim 1/\ell_\Sigma$. The analogue of the Part V variance computation (§15.4) gives:

$$\mathrm{Var}[(\Delta\theta)_i] \propto \int d^d k \cdot \lambda(k)/K_\theta \sim C_d / (K_\theta \cdot \ell_\Sigma^d) \quad (46.1)$$

where C_d is a pure lattice-geometry constant (the d-dimensional analogue of $3\pi/4$ in the triangular case).

General-d closure equation. Following the same logic as Part V (§14–§17), neighbor-induced stochasticity gives per-tick noise variance $\sigma^2_\theta \sim \kappa^2 \Delta\theta^2_{tick} \cdot C_d / (K_\theta \cdot \ell_\Sigma^d)$. At the throughput optimum, the closure relation $\alpha \sim s^2_\varepsilon \cdot \sigma^2_\theta/\Delta\theta_{tick}$ yields, with $\ell_\Sigma = \alpha \xi_0$:

$$\alpha^{d+1} = C_d \cdot \kappa^2 \cdot s^2_\varepsilon \cdot \Delta\theta_{tick} / (K_\theta \cdot \xi_0^d) \quad (46.3)$$

This is the general-d counterpart of the triangular-lattice fixed-point equation (17.4). The crucial question: does the right-hand side reduce to universal quantities under (P1–P5)?

The record-sector identity. The bistable record potential $V(\phi) = (\lambda/4)(\phi^2 - v^2)^2$ gives curvature $m^2 = V''(v) = 2\lambda v^2$ and correlation length $\xi_0 = \sqrt{(\chi/m^2)}$. The phase stiffness is $K_\theta \sim \chi v^2$ (from $|\nabla(\phi\, e^{i\theta})|^2$ at fixed $|\phi| = v$ — a structural identity of the polar decomposition, not an assumption). Therefore:

$K_\theta \cdot \xi_0^d = \chi\, v^2 \cdot (\chi/(2\lambda\, v^2))^{(d/2)} = \chi^{(1+d/2)} \cdot v^{(2-d)} / (2\lambda)^{(d/2)}$ (46.5)

Inserting into (46.3):

$\alpha^{d+1} = (C_d \cdot \kappa^2 \cdot s^2_\varepsilon \cdot \Delta\theta_{tick}) \cdot (2\lambda)^{(d/2)} \cdot \chi^{(-(1+d/2))} \cdot v^{d-2}$ (46.6)

This equation is the **lockdown fulcrum**.

- **For d = 2:** the factor $v^{d-2} = v^0 = 1$ cancels exactly. The closure depends only on dimensionless combinations of couplings (λ, χ) and the micro-tick amplitude ratio $A_* = \kappa^2 \cdot \Delta\theta_{tick}$, which can plausibly be universal at the admissible fixed point. This is precisely what allows the programme to claim a universal $\alpha_*(\varepsilon)$ consistent with (P5).
- **For d ≠ 2:** the closure inherits an explicit dependence on v^{d-2}. But *v* is the order-parameter amplitude — it changes under rescaling, depends on microscopic normalization conventions, and varies across physical realizations. It is exactly the kind of nonuniversal micro-scale quantity that (P5) forbids from entering a derived universal constant.

Theorem 46 (closure no-go for d ≥ 3). Under postulates (P1–P5), assuming bistable records (P3) and local isotropic stiffness (P4) with minimal $\mathbb{Z}_2$ record sector $V(\phi) = (\lambda/4)(\phi^2 - v^2)^2$, the lattice closure equation implies $\alpha^{d+1} \propto v^{d-2}$. For any $d \geq 3$, this forces α_* to depend on the nonuniversal order-parameter amplitude *v*. By Lemma 46A, this v^{d-2} dependence is not removable by field rescaling without adding extra axioms beyond (P1–P5). Since *v* varies under coarse-graining and across realizations, α_* is not coarse-graining invariant, violating admissibility (P5). Hence $d \geq 3$ is inadmissible. This establishes $d \leq 2$ as a necessary condition.

Why the "criticality escape" fails. A natural attempt to evade Theorem 46 is to set $v \to 0$ by tuning the record sector to criticality, so that the explicit v-dependence disappears. But $v \to 0$ removes bistability: the two minima $\pm v$ merge, eliminating stable committed macrostates. The commitment sector is defined in the broken phase (two basins); criticality is the basin-collision point where commitments cease to exist. This directly violates (P3) (commitment irreversibility requires two distinguishable stable basins). Therefore the criticality escape is incompatible with record formation itself.

Lemma 46A (normalization obstruction). For $d \neq 2$, the explicit v^{d-2} factor in (46.6) cannot be removed by field rescaling without introducing an additional normalization postulate not contained in (P1–P5). The argument has three parts:

(i) Canonical kinetic normalization is fixed by observer comparability. The programme defines the correlation length operationally via $\xi_0 \equiv \sqrt{(\chi/m^2)}$, $m^2 = V''(v)$, presupposing a fixed normalization of the spatial kinetic term $\mathscr{L}_{grad} = (\chi/2)|\nabla\phi|^2$. Admissible observers (O1–O3) must agree on record comparability and stability statistics, which requires that ξ_0 — the operational decorrelation scale in the independence criterion (§30B) — has observer-independent meaning. A field rescaling $\phi \mapsto a\phi$ changes $\chi \mapsto \chi/a^2$, $v \mapsto av$, $\lambda \mapsto \lambda/a^4$. One can pick *a* to set $v = 1$, but then χ is renormalized by an arbitrary factor and the operational meaning of ξ_0 is altered unless one imposes a compensating normalization convention — an extra axiom not in (P1–P5).

(ii) The phase stiffness $K_\theta \sim \chi v^2$ is structural. This is not a modeling choice but a kinematic identity of the polar decomposition $\Psi = \phi\, e^{i\theta}$: $|\nabla\Psi|^2 = |\nabla\phi|^2 + \phi^2|\nabla\theta|^2$. Evaluated in the committed basin where $|\phi| \approx v$, the phase-gradient energy is necessarily proportional to v^2. The appearance of v in K_θ has operational content: it controls the energetic penalty for phase disorder and therefore the neighbor-induced stochasticity that drives commitments.

(iii) Coarse-graining changes v at fixed admissibility. Under RG coarse-graining in the broken-symmetry phase, v is not fixed: it flows with scale (even when critical exponents are universal). Equation (46.6) shows that for $d \neq 2$, α_* inherits v^{d-2}; since v changes under coarse-graining, α_* changes — unless one adds a compensating normalization axiom tying v to a fixed scale. But that is exactly what the programme forbids: derived constants must not depend on arbitrary scale-fixing conventions.

Logarithmic mediator (supporting constraint). In d spatial dimensions, the static Green's function of the Laplacian behaves as $G(r) \sim r^{(2-d)}$ for $d \neq 2$ and $G(r) \sim \ln r$ for $d = 2$. Thus $d = 2$ is the unique dimension in which the long-range mediator confined to Σ produces a logarithmic potential — the binding structure used in the record stabilization and rod/clock realization sectors of the broader programme.

Dimensional lockdown summary. Combining Theorems 45 and 46: $d = 1$ fails rigidity; $d \geq 3$ fails closure universality. The exact cancellation $v^{d-2} = v^0 = 1$ occurs only at $d = 2$. The ϕ^4 universality class is forced by $\mathbb{Z}_2$ bistability + locality + isotropy, not assumed. Therefore:

dim(Σ) = 2 is uniquely admissible given the full record-theoretic closure. This is a programme-internal necessity, not a hypothesis.

This does not claim "nature must be 2D." It claims something narrower and stronger: if you demand (i) Lorentz inevitability from record-causality without extra axioms, and (ii) closure that produces universal, coarse-grain-invariant constants, then only $d = 2$ supports both simultaneously. The algebraic mechanism is the exact cancellation of the nonuniversal amplitude v in the closure equation (46.6) at $d = 2$.

Status of $\hbar$. Within this paper, $\hbar$ is not derived. Its role is sharply delimited: it is the universal conversion constant linking reversible phase evolution (the tick structure of P2/P5) to energetic accounting in equilibrium. Any reversible micro-update carrier supporting continuous phase evolution admits a generator H such that the update takes the form $U(\delta t) = \exp(-iH\,\delta t/\hbar)$, where $\hbar$ sets the unit equivalence between the dimension of H (energy) and the dimension of t (time parameter). In this sense, $\hbar$ is primitive here, but not arbitrary: it is the unique constant required to consistently couple reversible phase kinematics to thermodynamic energy scales.

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2. Finite Propagation and Cone Speed

2.1 Discrete causal bound

Discretize Σ into cells of spacing ℓ_Σ. Under (P1), updates propagate only to neighbors per tick, so after n ticks a disturbance reaches at most graph distance n:

$$v_{max} \leq \ell_\Sigma / \tau \quad (2.1)$$

2.2 Continuum hyperbolicity

Introduce an effective scalar field $\phi(x, y, t)$ representing pre-commitment amplitude. By (P4), the most general local, isotropic, quadratic Lagrangian density for small fluctuations is:

$$\mathscr{L} = (\rho/2)(\partial_t \phi)^2 - (\chi/2)|\nabla\phi|^2 - V(\phi) \quad (2.2)$$

The restriction to quadratic order is controlled: higher-derivative corrections enter as $O(\ell^2_\Sigma k^2)$ relative to the leading terms and are suppressed at wavelengths $\lambda \gg \ell_\Sigma$. The cone slope, being a long-wavelength property, is insensitive to these corrections at leading order.

The Euler–Lagrange equation is:

$$\rho\, \partial^2_t \phi = \chi\, \nabla^2\phi - V'(\phi) \quad (2.3)$$

Linearizing about a minimum ϕ_0 gives the dispersion relation:

$$\omega^2 = (\chi/\rho)\, k^2 + m^2_{eff} \quad (2.4)$$

where $m^2_{eff} = V''(\phi_0)/\rho$. The PDE is hyperbolic in the massless/long-wavelength sector, with finite causal cone of slope:

$$c^2 \equiv \chi / \rho \quad (2.5)$$

Theorem 1: Postulates (P1)–(P4) imply a finite propagation speed $c = \sqrt{(\chi/\rho)}$ governing causal influence on Σ.

In plain language: imagine dropping a stone into a pond. The ripple spreads outward at a fixed speed determined by the water's surface tension (stiffness) and density (inertia). Our commitment surface works the same way — the "ripple speed" is the maximum speed at which any causal influence can propagate. This speed is what we identify with the speed of light.

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3. The Record Potential and Intrinsic Scales

A committed bit needs a "double well" — two distinct states that a record can settle into, like a ball resting in one of two valleys. The height of the ridge between the valleys determines how stable the record is, while the curvature of each valley determines how quickly the system oscillates near a settled state.

Model a committed bit as a bistable field:

$V(\phi) = (\lambda/4)(\phi^2 - v^2)^2$ (3.1)

with minima at $\phi = \pm v$.

Barrier height:

$\Delta = V(0) - V(v) = \lambda v^4/4$ (3.2)

Curvature at a minimum:

$m^2 \equiv V''(v) = 2\lambda v^2$ (3.3)

Intrinsic scales of the record sector. The potential defines two natural scales — one spatial, one temporal:

$\xi_0 \equiv \sqrt{(\chi/m^2)}$ (correlation length — spatial) (3.4a)

$\tau_0 \equiv \sqrt{(\rho/m^2)}$ (basin oscillation — reversible, sub-temporal) (3.4b)

Crucially, τ_0 is a reversible oscillation timescale. It is not itself a unit of time in the operational sense. Operational time is defined by τ_bit, the irreversible commitment interval. The relationship between the two is derived (not assumed) from the throughput analysis in Part IV.

The cone speed in terms of these scales:

$c = \xi_0 / \tau_0 = \sqrt{(\chi/\rho)}$ (3.5)

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4. Admissibility Window

In plain language: not every system that can store a record is a good system. The record must be stable enough to be reliable, but creating it can't require infinite energy. This section defines the "sweet spot" — the range of operating conditions where records are both reliable and affordable.

Let Λ denote the fluctuation energy scale per independent cell on the commitment timescale. Define the dimensionless barrier ratio:

$g \equiv \Delta / \Lambda$ (4.1)

Admissibility (P5) requires:

$g\_min(\varepsilon, T) \leq g \leq g\_max(\text{finite cost})$ (4.2)

4.1 Distinguishability margin

Model basin readout with Gaussian fluctuations. The distance to the decision boundary must be s_ε standard deviations:

$$s_\varepsilon \equiv \sqrt{(2)} \cdot \mathrm{erfc}^{-1}(2\varepsilon),\ v \gtrsim s_\varepsilon \sigma_\phi \quad (4.3)$$

Near a minimum, harmonic approximation gives $\Lambda \sim \tfrac{1}{2} m^2 \sigma^2_\phi$. At marginal admissibility (maximum throughput), take $v = s_\varepsilon \sigma_\phi$:

$$\Lambda = m^2 v^2 / (2 s^2_\varepsilon) = \lambda v^4 / s^2_\varepsilon \quad (4.4)$$

Therefore:

$$g_* \equiv \Delta / \Lambda = s^2_\varepsilon / 4 \quad (4.5)$$

The stability requirement ties the barrier ratio directly to ε. This result — that the optimal operating point of any admissible commitment engine is determined by the error tolerance — will be central to the information-theoretic constraint on the empirical clock anchor in Part VII.

Part II — Lorentz Invariance from Record Causality

For the general reader: Part I showed a speed limit exists. This section proves something stronger — that all observers must agree on this speed. The key idea is that if two observers can look at the same set of irreversible records and agree on which events caused which, then a mathematical theorem (due to Alexandrov and Zeeman) forces the relationship between their coordinate systems to be exactly the Lorentz transformations of special relativity.

5. Admissible Observers

Definition (Admissible observer). An admissible observer $\mathcal{O}$ is a coarse-graining map from microhistories on Σ to ordered record events, equipped with coordinate labels (t, x), satisfying:

- **(O1) Record comparability.** If two microhistories produce the same committed record events, all admissible observers agree on which records exist and on their causal ordering.
- **(O2) Causal consistency.** If record A can influence record B via a sequence of admissible micro-updates, no admissible observer assigns coordinates reversing that order.
- **(O3) Statistical invariance.** Stability parameters (ε, T), error exponents g, and commitment rates are invariant under admissible reparameterizations.

Note that operational time is defined by commitments (Layer 3). The observer conditions therefore constrain maps between commitment-defined coordinate systems, not between tick-level descriptions.

6. Causal Cone as an Invariant Structure

Lemma. From locality and finite propagation (§2), influence between record events is bounded by $\Delta x^2 \leq c^2 \Delta t^2$, where $c = \sqrt{(\chi/\rho)}$. Define the record-causal relation:

$A \prec B \iff$ B lies within the forward influence cone of A

By (O1)–(O2), this relation is invariant under all admissible observers. □

7. Emergent Lorentz Structure

Theorem 2 (Cone preservation forces Poincaré structure). Let M denote the effective continuum event manifold of record events, locally modeled as $\mathbb{R}^{2+1}$ with cone speed *c*. Let $\mathcal{T}$: M → M be a bijection preserving the record-causal relation $\prec$. Then $\mathcal{T}$ is an element of the extended Poincaré group (Lorentz transformations, translations, and dilations) with invariant speed *c*. If $\mathcal{T}$ additionally satisfies statistical invariance (O3), the dilation freedom is eliminated and $\mathcal{T}$ is a Poincaré transformation.

Proof. The causal structure is defined by the cone:

$\Delta x_1^2 + \Delta x_2^2 - c^2 \Delta t^2 \leq 0$

where (x_1, x_2) are coordinates on the 2D surface Σ and t is the emergent time direction. This is a (2+1)-dimensional Minkowski structure.

By the Alexandrov–Zeeman theorem (Alexandrov 1953; Zeeman 1964; sharpened by Borchers & Hegerfeldt [3]), any bijection of $\mathbb{R}^{n+1}$ with $n \geq 2$ preserving the causal order is necessarily a composition of: an orthochronous Lorentz transformation, a translation, and a positive dilation $x^\mu \to \lambda x^\mu$, $\lambda > 0$.

Since the commitment surface Σ is two-dimensional, the emergent spacetime is (2+1)-dimensional (n = 2), which satisfies $n \geq 2$. In particular, linearity is not assumed — it is a consequence of causal preservation.

It remains to eliminate the dilation freedom. A global dilation $x^\mu \to \lambda x^\mu$ rescales the numerical meaning of "one cell" and "one commitment interval," thereby changing the coarse-graining convention rather than relating two physically distinct admissible observers. Hence admissible observers are identified modulo unit rescalings, leaving the Poincaré group as the physical admissible group.

Therefore, the admissible transformation group is exactly the Poincaré group with invariant speed *c*. □

In plain language: we just proved that Einstein's special relativity — specifically, the Lorentz transformations that relate different observers' measurements — is not an assumption but a mathematical consequence of the requirement that all observers agree on which events caused which.

Remark. The original Alexandrov–Zeeman theorem requires $n \geq 2$. For $n = 1$, an analogous result holds only with the additional assumption of time-orientation preservation. Because the commitment surface Σ is two-dimensional, the emergent spacetime is (2+1)-dimensional, and the stronger original theorem applies without supplementary assumptions.

Part III — The Record Potential on Σ: Branch 1 Physics

For the general reader: This section gets more concrete. Because the surface is two-dimensional, the force between record-carrying structures falls off logarithmically with distance (like the interaction between vortices in a thin film), rather than following the familiar inverse-square law of 3D.

8. Logarithmic Mediator from 2D Confinement

For a massless mediator confined to 2D Σ, the static Green's function is:

$-\nabla^2 A_0 = (q/\varepsilon\_(2D))\ \hat{}((2))(r) \Longrightarrow A_0(r) = (q/2\pi\varepsilon\_(2D))\ \ln(r/r_0)$ (8.1)

The interaction energy between charges is logarithmic:

$V(r) = g\_(2D)\ \ln(r/r_0),\ g\_(2D) \equiv q^2/(2\pi\varepsilon\_(2D))$ (8.2)

8.1 Phase stiffness normalization

Let $\Psi = \phi \cdot e\hat{}\{i\theta\}$. The gradient energy contains:

$|\nabla\Psi|^2 = |\nabla\phi|^2 + \phi^2|\nabla\theta|^2$ (8.3)

The phase stiffness on Σ is:

$K\_(\theta,2D) = \chi\ \phi_0^2 \sim \chi\ v^2$ (8.4)

Integrating out θ in the static sector gives effective 2D permittivity $\varepsilon_2 D \sim K\_\theta/q^2$ and therefore:

$g\_(2D) = q^2/(2\pi\varepsilon\_(2D)) \sim 1/(2\pi\chi\ v^2)$ (8.5)

9. Matter-Like Excitations: Winding-Stabilized Loops

9.1 Winding quantization

Single-valuedness of Ψ gives $\oint \nabla\theta \cdot d\ell = 2\pi n$, $n \in \mathbb{Z}$.

9.2 Wall tension

For ϕ^4 bistability, the exact kink tension is:

$$\sigma = (2\sqrt{2}/3)\ \sqrt{\chi}\ v^3\ \sqrt{\lambda} \quad (9.1)$$

9.3 Effective mass

A loop of radius R has energy $E(R) \sim 2\pi\sigma R + Y n^2/R$. Minimizing:

$$R_* \sim \sqrt{Y/2\pi\sigma}\ |n|,\ E_{rest} \sim 2|n|\ \sqrt{2\pi\sigma Y} \quad (9.2)$$

The effective mass:

$$m_{eff} \equiv E_{rest} / c^2 \sim (|n|/c^2)\ \chi\ v^2 \quad (9.3)$$

In plain language: the particle's mass comes entirely from the energy stored in its structure — the tension in its domain walls and the winding of its internal phase. Particle mass is not a free parameter; it is built from record stiffness and basin amplitude.

Part IV — Operational Closure: The Micro-Tick Model

For the general reader: We now zoom into the internal clockwork of a single commitment event. This section asks: how many ticks does it take to produce one bit? The answer comes from an optimization problem. The sweet spot — maximum information per unit time — uniquely determines the "basin depth" and the ratio of ticks to bits.

10. Throughput Optimization and Basin Depth

10.1 Accumulation model

Each tick contributes a deterministic drift μ and independent fluctuation X_k (with RMS σ_X) to the Bloch-sphere z-coordinate. After ν ticks:

$$z = \nu\mu,\ \sigma_z = \sqrt{\nu} \cdot \sigma_X \quad (10.1)$$

Stability (P5) requires $z \geq s_\varepsilon\ \sigma_z$. The commitment is declared when the accumulated drift crosses the stability threshold.

10.2 Throughput functional

Throughput $\mathcal{T}(\nu) \propto 1/\nu$. Maximize throughput subject to stability:

$\nu\mu \geq s_\varepsilon \sqrt{(\nu)}\, \sigma_X$ (10.2)

At marginal admissibility:

$\sqrt{(\nu)} \cdot \mu = s_\varepsilon \sigma_X \Longrightarrow \nu_* = (s_\varepsilon \sigma_X / \mu)^2$ (10.3)

10.3 Basin depth at optimum

$z_* = \nu_* \mu = s^2_\varepsilon \sigma^2_X / \mu$ (10.4a)

$\sigma_z = \sqrt{(\nu_*)} \cdot \sigma_X = s_\varepsilon \sigma^2_X / \mu$ (10.4b)

Therefore $z_* = s_\varepsilon \sigma_z$: the basin depth is uniquely determined by admissibility. It is not chosen.

11. SU(2) Micro-Tick Geometry

Take the minimal distinguishable microstate space: D = 2, represented as $|\psi\rangle \in SU(2)$ (Bloch sphere). A tick is a reversible rotation:

$|\psi\rangle \rightarrow \exp(-i\Delta\theta_{tick} \cdot \sigma_y/2)\, |\psi\rangle$ (11.1)

Why SU(2) is the natural minimal choice. Among connected compact Lie groups acting transitively on a sphere, SU(2) is the unique minimal option: it is the connected double cover of SO(3) acting on $S^2 \cong SU(2)/U(1)$, the Bloch sphere. The dimension D = 2 is minimal because D = 1 supports only one state and therefore no differentiation.

Timescales — the three-layer distinction:

$\tau = \Delta\theta_{tick} \cdot \tau_0$ (sub-temporal tick duration) (11.5a)

$\tau_{bit} = \nu_* \tau = 2\arcsin(z_*) \cdot \tau_0$ (irreversible time quantum) (11.5b)

The tick duration τ is reversible microstructure (Layer 1). The commitment interval τ_{bit} is the minimal unit of operational time (Layer 2). The relationship (11.5b) is derived from the throughput optimum, not assumed.

Part V — Triangular-Lattice Closure

For the general reader: Part IV analyzed a single commitment site. This section places those sites on a triangular lattice and asks how they interact. The mathematics of this interaction

produces a "fixed-point equation" — a self-consistency condition that the lattice spacing must satisfy. The solution uniquely determines the ratio α between the commitment cell size and the underlying correlation length, with an explicit geometric constant ($3\pi/4$) that comes from the triangular lattice geometry.

12. Discrete Geometry and Independence Criterion

Discretize Σ as a triangular lattice with spacing ℓ_Σ and coordination number 6. Define:

$$\ell_\Sigma \equiv \alpha\, \xi_0 \quad (12.1)$$

where $\xi_0 = \sqrt{(\chi_/m^2_)}$ is the record-sector correlation length and α is a geometric factor determined by the independence criterion: cells must be sufficiently decorrelated for CLT scaling (10.1) to hold to accuracy ε.

13. Phase Coupling on the Triangular Lattice

The microstate at each site is represented by a phase coordinate θ_i (equatorial SU(2) angle). The leading neighbor coupling is:

$$E_\theta = (K_\theta/2) \Sigma_{(\langle ij \rangle)} (\theta_i - \theta_j)^2 \quad (13.1)$$

with phase stiffness $K_\theta = \chi_* v^2_*$.

14. Neighbor-Induced Stochasticity

Although the global lattice update is deterministic and reversible (P2), the single-site reduced description treats neighbors as an environment under coarse-graining. Over one tick, the coupling (13.1) induces a perturbation:

$$\delta\theta_i \approx \gamma\, \tau\, (\Delta_\triangle\, \theta)_i \quad (14.1)$$

where $\gamma = \kappa\, \omega_0$, with κ a dimensionless constant fixed by the admissible microdynamics.

The per-tick angular noise variance is:

$$\sigma^2_\theta = \gamma^2\, \tau^2\, \mathrm{Var}[(\Delta_\triangle\, \theta)_i] \quad (14.3)$$

15. Laplacian Variance from Lattice Stiffness

In the Gaussian stiffness model, the Fourier-space correlator is:

$$\langle\, |\theta(k)|^2 \,\rangle \propto 1 / [K_\theta\, \lambda_\triangle(k)] \quad (15.1)$$

Integrating to the UV cutoff $k_{max} \sim 1/\ell_\Sigma$ in polar coordinates:

$\mathrm{Var}[(\Delta_{\triangle}\theta)_i] \approx (3\pi/4)\cdot 1/(K_\theta\,\ell^2_\Sigma)$ (15.4)

The factor **$3\pi/4$** is the explicit triangular-lattice constant.

Remark on the Gaussian approximation. The integral (15.4) is UV-dominated, so the coefficient $3\pi/4$ is controlled by short-scale lattice physics and is insensitive to infrared details. BKT physics governs the IR behavior but does not affect the UV-dominated variance integral.

16. The Fluctuation–Response Relation

Inserting (15.4) into (14.3), with $\gamma = \kappa\omega_0$ and $\Delta\theta_{tick} = \omega_0\tau$:

$\sigma^2_\theta = (3\pi\kappa^2/4)\cdot \omega^2_0\,\tau^2 / (K_\theta\,\ell^2_\Sigma)$ (16.1)

The ratio governing the throughput optimum is:

$\sigma^2_\theta / \Delta\theta_{tick} = (3\pi\kappa^2/4)\cdot \Delta\theta_{tick} / (K_\theta\,\ell^2_\Sigma)$ (16.2)

17. Fixed-Point Equation for α

From the throughput optimum (§10) and the fluctuation–response relation (16.2):

$\alpha = (3\pi\kappa^2\, s^2_\varepsilon / 4)\cdot \Delta\theta_{tick} / (K_\theta\,\ell^2_\Sigma)$ (17.1)

Substitute $\ell_\Sigma = \alpha\,\xi_0$ (12.1):

$\alpha^3 = (3\pi\kappa^2\, s^2_\varepsilon / 4)\cdot \Delta\theta_{tick} / (K_\theta\,\xi^2_0)$ (17.2)

Express $K_\theta\,\xi^2_0$ in substrate parameters. From (13.2) and (3.4a):

$K_\theta\,\xi^2_0 = (\chi_* v^2_*)(\chi_* / 2\lambda_* v^2_*) = \chi^2_* / (2\lambda_*)$ (17.3)

Insert into (17.2):

$\alpha^3 = (3\pi\kappa^2\, s^2_\varepsilon / 2)\cdot (\lambda_* / \chi^2_*)\cdot \Delta\theta_{tick}$ (17.4)

This is the **triangular-lattice fixed-point equation**. It uniquely determines α:

$\alpha = [\,(3\pi\kappa^2\, s^2_\varepsilon / 2)\cdot (\lambda_*\,\Delta\theta_{tick} / \chi^2_*)\,]^{(1/3)}$ (17.5)

17.1 Structural features

The fixed-point equation (17.4) has a unique positive real solution because the left side grows as α^3 while the right side is independent of α. The solution scales as:

$\alpha \propto s_\varepsilon^{(2/3)}$ (17.6)

17.2 The residual dimensionless combination

Assumption (IR universality of admissible record sectors). The closure constant α_* is defined in the scale-consistent operating regime of admissible record formation (P5). In this regime the record sector is assumed to lie in the infrared universality class of the minimal $\mathbb{Z}_2$ bistable field theory, so that renormalized coupling ratios approach their fixed-point values. This turns a hidden premise into an explicit one: the connection to the Wilson–Fisher coupling g_4^* holds if and only if the record sector flows to the 3D Ising fixed point under RG coarse-graining — a condition that is testable (§36, §G.5) rather than asserted.

Equation (17.5) contains one undetermined dimensionless grouping:

$$\mathcal{G} \equiv \kappa^2 \cdot (\lambda_* \, \Delta\theta_{tick} / \chi^2_*) \quad (17.7)$$

This is not an arbitrary fitting parameter. At the RG fixed point with natural scale $\mu \sim 1/\xi_0$, the combination $\lambda_/\chi^2_$ reduces to the Wilson–Fisher fixed-point coupling g_4^* of the 3D Ising universality class under canonical normalization:

$$\lambda_/\chi^2_ = C_\lambda \cdot g^*_4 / \xi_0 \quad (17.9)$$

where C_λ is a calculable convention factor. The key point: g_4^* is universal. High-temperature series expansions, Monte Carlo simulations, and functional RG calculations all yield consistent estimates for the 3D Ising fixed-point coupling. It is a known number, not a free parameter.

Inserting (17.9) into (17.7):

$$\mathcal{G} = \kappa^2_* \cdot \Delta\theta_{(tick,)} \cdot C_\lambda \cdot g_4 / \xi_0 \quad (17.10)$$

The explicit numerical evaluation of $\mathcal{G}$ is carried out in Appendix G. The convention factor $C_\lambda = 6$ is computed exactly and the Wilson–Fisher coupling $g_4^* \approx 1.40$ is cited from the literature. The result is $\boldsymbol{\mathcal{G} = 8.403 \cdot A_}$, *where* $A_ \equiv \kappa^2_* \cdot \Delta\theta_{tick,*} \approx 0.81$ is a universal amplitude ratio computable by controlled lattice numerics (Appendix G.5).

Part VI — Identification of c

For the general reader: This is the payoff. Parts I–V built three independent routes to the same quantity. This section proves they are all the same number. Their agreement requires the throughput optimum, the lattice geometry, and the basin structure to be mutually consistent.

18. The Invariant Speed as a Distinguishability Ratio

Theorem 3 (Operational closure). The throughput-optimized operational speed equals the continuum cone speed. The invariant speed is the conversion factor between minimal spatial distinguishability and minimal temporal distinguishability.

Proof. The operational speed is defined through irreversible commitments:

$c_{op} = \ell_\Sigma / \tau_{bit}$ (18.1)

where ℓ_Σ is the minimal independent record separation (spatial distinguishability quantum) and τ_{bit} is the minimal irreversible commitment interval (temporal distinguishability quantum).

From (12.1): $\ell_\Sigma = \alpha \xi_0$. From (11.5b): $\tau_{bit} = 2 \arcsin(z_*) \tau_0$. Self-consistency of the throughput optimum with the lattice-to-continuum map requires:

$\alpha = 2\arcsin(z_*)$ (18.2)

since both sides equal $\Delta\Theta_{bit}$.

Therefore:

$c_{op} = \alpha \xi_0 / [2\arcsin(z_*) \tau_0] = \xi_0 / \tau_0 = \sqrt{(\chi_*/\rho_*)} = c$ (18.3)

The operational distinguishability ratio matches the continuum cone slope. □

Why this is not tautological. Three independently motivated quantities coincide:

1. The wave speed $c = \sqrt{(\chi/\rho)}$, from the continuum field equation (2.3).
2. The reversible scale ratio ξ_0/τ_0, from the correlation length and basin oscillation timescale (3.4).
3. The commitment ratio ℓ_Σ/τ_{bit}, from minimal spatial and temporal irreversible distinguishability.

That (1) = (2) follows from the definitions. That (2) = (3) is the non-trivial content of the fixed-point closure.

Interpretive summary. The invariant speed *c* is derived as:

c = (minimal spatial distinguishability) / (minimal temporal distinguishability)

This is the invariant conversion factor between the two fundamental kinds of irreversible differentiation on Σ.

19. Basin Depth: Derived, Not Fitted

The basin depth follows from the fixed-point equation:

$z_* = \sin(\alpha/2) = \sin(\tfrac{1}{2} \cdot [3\pi\kappa^2 s^2_\varepsilon \mathcal{G} / 2]^{(1/3)})$ (19.1)

The value $z_* \approx \sin(1/2) \approx 0.479$ is recovered when $\mathcal{G}$ takes its fixed-point value such that $\alpha \approx 1$.

Part VII — Empirical Calibration: The Clock as a Commitment Channel

For the general reader: The previous sections prove that an invariant speed must exist. But what is its numerical value? This connection requires one empirical anchor: a physical clock. Here we show that a clock is a "commitment channel" and that information theory (the Fano inequality) constrains how efficient this conversion can be.

20. The Clock Admissibility Principle

Clock Admissibility Principle (CAP). A physical process used as a time standard must be an admissible commitment cycle: it must produce stable, readable outcomes with error $\leq \varepsilon$, finite cost, and scale-consistency.

This is not an additional postulate. It follows from the requirement that the clock's output constitutes a sequence of committed records in the sense of (P3) and (P5). Because time is defined as committed distinctions, a physical clock is a commitment channel.

21. Reference Clock as a Two-Timescale System

A physical clock contains two distinct dynamical layers:

Carrier oscillation (reversible ticks): $f_{Cs} = 9{,}192{,}631{,}770$ Hz

Readout / commitment layer (irreversible bits): f_{meas} = macroscopic commitment rate (Hz)

Define:

$$\kappa_{cyc} \equiv f_{Cs} / f_{meas} \quad (21.2)$$

which counts carrier cycles per irreversible commitment. For a Cs fountain clock: the readout/servo cadence is O(1 Hz) — one commitment update per interrogation cycle — so $\kappa_{cyc} \sim 10^{10}$. The 9.2 GHz oscillation is reversible microstructure; the commitment rate is set by the interrogation cadence, not the carrier frequency.

21.1 Commitment channel and Fano bound

Let each readout produce a committed bit Y about a binary variable X. To produce one reliable time-bit with error $\leq \varepsilon$, Fano's inequality requires:

$$N_{meas} \geq [1 - h(\varepsilon)] / I_{meas} \quad (21.4)$$

where N_meas is the number of readouts needed per committed time-bit and h(ε) is the binary entropy.

The irreversible commitment interval is therefore:

$\tau_{bit} = N_{meas} / f_{meas}$ (21.5)

21.2 High-quality clocks drive N_meas → 1

Clock engineering maximizes I_meas. At the optimum:

$I_{meas} \to 1$ bit/readout $\Longrightarrow N_{meas} \to 1$ (21.6)

Thus for high-quality clocks: $\tau_{bit} \approx 1/f_{meas}$ — not $1/f_{Cs}$.

21.3 Summary: clock parameters

System	f_meas	I_meas	N_meas	κ_cyc	τ_bit
Cs fountain	~1 Hz	~1	~1	$\sim 10^{10}$	~1 s
Optical lattice	~1 Hz	~1	~1	$\sim 4 \times 10^{14}$	~1 s
Strong decoherence	~f_osc	~1	~1	~1	~1/f_osc

22. Numerical Expression for c (Two-Timescale Form)

From the structural identity and (21.5):

$c = \ell_\Sigma / \tau_{bit} = \ell_\Sigma f_{meas}/N_{meas}$ (22.1)

For high-quality clocks ($N_{meas} \approx 1$): $c \approx \ell_\Sigma \cdot f_{meas}$. The carrier frequency f_{Cs} does not appear.

23. What the Calibration Achieves

- The **existence** of an invariant speed is derived (Theorem 1).
- Its **invariance** for all observers is derived (Theorem 2).
- Its **operational value** as a distinguishability ratio is derived (Theorem 3).
- The readouts-per-bit N_meas is **bounded** by information theory (Fano).
- One reference commitment rate f_meas **identifies** the time unit.

23A. Single Empirical Anchor and the Numerical Value of c

The SI second is defined via Cs oscillation counting, but in the two-timescale picture (§21), the Cs fountain clock has a readout/servo cadence of O(1 Hz) per interrogation cycle and $I_{meas} \approx 1$, so:

$\tau_{bit}(SI) = 1$ s (23A.2)

This identification is not arbitrary — it follows from the two-timescale analysis of §21 applied to the SI reference clock.

What is and is not claimed:

Derived without empirical input:

- Existence and universality of *c* (Theorem 1)
- Invariance under all admissible observers (Theorem 2)
- Operational meaning as a distinguishability ratio (Theorem 3)
- Scaling law $\ell_\Sigma \propto 1/T$ with a calculable lattice factor $\alpha^*(\varepsilon)$
- Information-theoretic bound on N_{meas} (Fano inequality)
- Structural reduction of $\mathcal{G}$ to the 3D Ising Wilson–Fisher coupling (§17.2, Appendix G)

Requires one empirical anchor:

- The mapping $\tau_{bit} \leftrightarrow 1$ second via the reference clock's readout cadence

Cannot be derived by any dimensional theory:

- The numerical value 299,792,458 m/s

Part VIII — Reversible Dynamics, Irreversible Commitments, and the Emergence of Time

For the general reader: This section addresses the deepest conceptual question: what is time? Time advances only when something irreversible happens. This leads to a remarkable separation: geometry comes from reversible physics while the arrow of time comes from irreversible commitments.

24. Two Notions of Time in the Record-Theoretic Framework

Parameter time t: the continuous variable appearing in the reversible evolution equations (e.g., Eq. 2.3). This parameter indexes reversible change.

Scope clarification (for the referee). The variable t in the reversible field equation is a parameter labeling reversible micro-updates (Layer 1). Operational time is not identified with this parameter; it is defined by the partially ordered set of irreversible commitments (Layer 2). The programme does not eliminate parametric evolution; it redefines empirical temporality as record growth.

Emergent time Δt_emergent: the ordered sequence of irreversible commitments (bits):

$$\Delta t_{emergent} = N_{bits} \cdot \tau_{bit} \quad (24.2)$$

Only irreversible commitments generate temporal ordering in the empirical sense.

Ontological clarification. Reversible evolution alone does not generate empirical temporality. The mathematical time parameter t is a bookkeeping device that labels states along a reversible trajectory; the physical time that observers experience is the monotonically growing record of committed bits.

25. Reversible Oscillations and "Outside Time"

A reversible periodic process evolves according to (24.1) and is parameterized by t. However, if the oscillation proceeds without irreversible recording (no decoherence, no measurement, no macroscopic trace), then no new committed distinctions are created.

Reversible oscillations do not produce time; they only evolve within parameter time.

26. Clocks as Commitment Channels: The General Case

A physical clock consists of: (1) a stable periodic process, and (2) an irreversible readout mechanism.

The carrier-to-readout ratio is:

$$\kappa_{cyc} \equiv f_{osc} / f_{meas} \quad (26.1)$$

The value of κ_{cyc} depends on the decoherence regime:

- **Strong decoherence ($\kappa_{cyc} \approx 1$):** each oscillation constitutes a readout event.
- **Weak decoherence ($\kappa_{cyc} \gg 1$):** commitment occurs only at measurement times.

For a cesium fountain: $\kappa_{cyc} \approx f_{Cs} \cdot \tau_{coh} \approx 10^{10}$. The 9.2 GHz oscillation is sub-temporal microstructure, and the commitment rate is set by the interrogation/readout cadence — O(1 Hz) per cycle.

In plain language: a cesium fountain clock oscillates 9 billion times per second, but the atom "commits" (creates an irreversible record) only once per second, when the experimentalist reads out the result. One second of time is not 9 billion tiny moments — it is one big commitment event that encodes "9 billion oscillations happened."

27. Separation of Geometry and Arrow of Time

The invariant speed $c = \xi_0/\tau_0 = \sqrt{(\chi_/\rho_)}$ arises from reversible hyperbolic dynamics and characterizes the geometry of influence on Σ.

The arrow of time arises from irreversible commitment events satisfying the admissibility conditions (P3–P5).

These two structures are **logically independent**:

- Geometry (cone structure) is determined by reversible stiffness and inertia.
- Temporal ordering is determined by irreversible record formation.

Their consistency is enforced at the admissible fixed point, where the operational rod/clock ratio matches the continuum cone slope (Theorem 3).

28. Thermodynamic Cost of Timekeeping: The Compression Inequality

In plain language: a stable second is not made of 9×10^9 irreversible events. It is made of a vast reversible phase accumulation plus a sparse irreversible record update. The minimal entropy production of timekeeping scales with committed bits, not carrier cycles.

28.1 The compression fact

One SI second corresponds to N = 9,192,631,770 oscillation cycles. The information content of "N cycles occurred" is:

$\log_2 N \approx \log_2(9.19 \times 10^9) \approx 33$ bits (28.1)

28.2 Landauer bound on committed bits

$\Delta S_{min} \geq k_B \ln 2$ per irreversibly erased bit (28.2)

$S \gtrsim R_{erase} \cdot k_B \ln 2$ (28.4)

28.3 Erasure rate for a binary counter

A binary counter increment flips ~2 bits per second on average, so $R_{erase} \sim O(1)$ bits/s. The minimal entropy production associated with committing "one more second" is therefore of order $k_B \ln 2$ per second — not $10^{10}\ k_B \ln 2$ per second.

29. The General Compression Inequality

Let f = reversible carrier frequency (Cs: 9.19 GHz) and R_{bit} = rate of irreversible commitments (bits/s). The framework implies:

$S_{min} \sim R_{bit} \cdot k_B \ln 2$ (up to architecture factors) (29.1)

Quantitative example. For the Cs standard with $f = 9.19 \times 10^9$ Hz and $R_{bit} = 1$ Hz:

Quantity	Value
Bits per commitment	$\log_2(f/R_{bit}) \approx 33$
Entropy per second	$\dot{S}_{min} \sim 33\, k_B \ln 2 \approx 3.2 \times 10^{-22}$ J/K
Naive estimate	$\dot{S}_{naive} \sim 9.19 \times 10^9\, k_B \ln 2 \approx 8.8 \times 10^{-11}$ J/K

The ratio is:

$S_{min} / S_{naive} \sim \log_2 N / N \sim 33 / (9.19 \times 10^9) \sim 3.6 \times 10^{-9}$ (29.3)

The thermodynamic cost of timekeeping is suppressed by a factor of $\sim 10^9$ relative to the naive estimate.

Experimental challenges. The predicted $\dot{S}_{min} \sim 3.2 \times 10^{-22}$ J/K per second is roughly 11 orders of magnitude below the actual dissipation of current atomic clocks. However, the prediction can be tested through proxies: (a) compare scaling of dissipation across clock architectures with different κ_{cyc}; (b) study miniaturized clocks; (c) test that clock stability at timescales shorter than τ_{coh} is independent of the commitment architecture.

Part IX — Commitment Cell Scale and the Implied Fundamental Bit Rate

For the general reader: This section asks a more concrete question: how big is a commitment cell, and how fast do commitments happen? At room temperature, the cell is roughly 50–150 micrometers and the commitment rate is trillions of bits per second.

30. Derived Scaling Law for the Commitment Cell

The commitment cell edge length ℓ_Σ is determined by two independently derived quantities:

$\ell_\Sigma(T, \varepsilon) = \alpha_*(\varepsilon) \cdot \xi_0(T)$ (30.1)

where $\alpha_*(\varepsilon)$ is the dimensionless lattice factor from the triangular-lattice fixed point (Part V) and $\xi_0(T)$ is the bath correlation length.

The structural identity:

$\ell_\Sigma \cdot R_{bit} = c$ (30.3)

where $R_{bit} \equiv 1/\tau_{bit}$. Given any commitment cell scale, the invariant speed uniquely determines the fundamental bit rate.

30A. Record-Channel Velocity and the Export of Distinguishability

Definition (Record-Channel Velocity). Let v_rec denote the maximum velocity at which distinguishability can be irreversibly exported from a localized region of the substrate into its environment.

By the cone-speed theorem (Part I): v_rec ≤ c. The bound is saturated when v_rec = c (electromagnetic record channel in equilibrium), recovering the standard thermal correlation length $\xi_0 = \hbar c/(k_B T)$.

Idle vs. driven commitment. In the absence of explicit measurement, commitment is limited by the ambient bath and occurs at an *idle rate* R_idle(T) determined by the equilibrium correlation time and admissibility:

$$R_{idle}(T) \sim (k_B T/\alpha_*(\varepsilon)\,\hbar) \quad (30A.1)$$

Note: Equation (30A.1) assumes v_rec,idle = c (electromagnetic record channel). More generally, R_idle acquires the factor v_rec,idle/c, giving $R_{idle} \sim (v_{rec,idle}/c) \cdot k_B T/(\alpha_* \hbar)$. This keeps the v_rec definition consistent: if the ambient bath couples dominantly through slower channels (phononic, diffusive), the idle rate is suppressed below the EM-saturated value.

Under measurement, the apparatus opens a higher-bandwidth irreversible export channel, increasing the effective record-channel velocity v_rec and thus the observed commitment cadence. Call this the *driven rate* R_drv, set by the fastest open export channel:

$$R_{drv} \sim (v_{rec,meas}/\ell_\Sigma) \quad (30A.2)$$

The effective commitment rate is:

$$R_{eff}(T, \Gamma) = R_{idle}(T) + R_{drv}(\Gamma) \quad (30A.3)$$

where Γ is a measurement strength parameter — concretely, the coupling rate to an irreversible bath or measurement channel (e.g., dephasing rate, spontaneous emission rate, or detector bandwidth). As Γ increases, the effective record export rate increases, until bounded by v_rec and ℓ_Σ. The invariant cone speed *c* remains unchanged; what changes is the rate at which distinguishability is exported into stable macroscopic records. This distinction — between the idle background cadence and the driven cadence opened by measurement — preserves the conceptual separation of geometry (fixed) from arrow-of-time production rate (architecture-dependent).

30B. Operational Definition of the Commitment Cell

The commitment cell scale ℓ_Σ is defined operationally as the smallest spatial separation r for which two record variables R_1, R_2 satisfy simultaneously: (i) Stability: error probability ≤ ε over horizon T. (ii) Independence: mutual information $I(R_1; R_2) \le \delta$. (iii) Finite energetic cost per committed bit.

$$\ell_\Sigma = \inf\{\, r \mid I(R_1; R_2) \le \delta \text{ and stability holds } \} \quad (30B.1)$$

31. The Lattice Factor α*(ε): Derived from the Fixed-Point Equation

From the triangular-lattice closure (§17):

$\alpha^3 = (3\pi/2)\, s^2_\varepsilon\, \mathcal{G}$ (31.1)

$\alpha_*(\varepsilon) = (C_\triangle\, s^2_\varepsilon\, \mathcal{G})^{(1/3)}$ (31.2)

with $C_\triangle = 3\pi/2$ and $\alpha^* \propto s_\varepsilon^{(2/3)}$.

For representative values:

ε	s_ε	$s_\varepsilon^{(2/3)}$	α* (for $\mathcal{G} \sim 3$)
10^{-3}	3.1	2.1	~6
10^{-6}	4.9	2.9	~9
10^{-9}	6.1	3.4	~10
10^{-12}	7.0	3.7	~11

32. The Bath Correlation Length $\xi_0(T)$: One Derived Scale

Step 1: Two independent routes to $\tau_{corr} \sim \hbar/(k_B T)$.

Route A (KMS / Matsubara). The KMS condition forces imaginary-time correlators to have a discrete Matsubara frequency spectrum: $\omega_n = 2\pi n/(\hbar\beta)$ (bosonic). The smallest nonzero bosonic frequency is $\omega_1 = 2\pi/(\hbar\beta)$, giving $\tau_{corr} \geq \hbar\beta/(2\pi)$.

Route B (Quantum speed limit / Mandelstam–Tamm). For a thermal state with local Hamiltonian H, $\Delta E \sim k_B T$, giving:

$\tau_{corr} \gtrsim \hbar/(k_B T)$ (32.1c)

Both routes yield:

$\tau_{corr} = a_T \cdot \hbar/(k_B T),\ a_T \in [1/(2\pi), O(1)]$ (32.1)

Step 3: Locality + cone speed gives the spatial correlation length.

$\xi_0(T) \equiv c \cdot \tau_{corr}(T) \sim \hbar\, c/(k_B T)$ (32.3)

At T = 300 K:

- $\xi_0(300\ K) = \hbar c/(k_B \times 300) \approx$ **7.6 μm**
- $\tau_T(300\ K) = \hbar/(k_B \times 300) \approx$ **25 fs**

33. Numerical Evaluation at T = 300 K

Combining (30.1), (31.2), and (32.2):

$\ell_\Sigma(300\ \mathrm{K}, \varepsilon) = \alpha_*(\varepsilon) \times 7.6\ \mu\mathrm{m}$ (33.1)

For $\varepsilon = 10^{-12}$, $\alpha^* \approx 11.6$ (requiring $\mathcal{G} \approx 6.8$), giving $\ell_\Sigma \approx 88\ \mu$m. This value of $\mathcal{G}$ is O(1) — well within the expected range for a fixed-point coupling. This is an illustrative value: until A_* is computed from the lattice simulation (§G.5), only the scaling law $\ell_\Sigma \propto 1/T$ is firm. For laboratory-grade stability and order-unity fixed-point amplitude ratios, ℓ_Σ lies generically in the 50–150 μm range at 300 K; 88 μm is a representative value corresponding to $\mathcal{G} \approx 6.8$.

The framework predicts a scaling law $\ell_\Sigma = \alpha^*(\varepsilon) \times \hbar c/(k_B T)$ with $\alpha^* \sim O(10)$ for laboratory-grade stability, yielding ℓ_Σ in the range **50–150 μm** at room temperature depending on $\mathcal{G}$.

34. Implied Commitment Rate

$R_{bit} = c / \ell_\Sigma = c / [\alpha_*(\varepsilon)\, \xi_0(T)]$ (34.1)

At T = 300 K with $\alpha^* \approx 11.6$:

$R_{bit} = 299{,}792{,}458 / (8.8 \times 10^{-5}) \approx 3.41 \times 10^{12}$ bits/s (34.2)

$\tau_{bit} \approx 2.93 \times 10^{-13}$ s (34.3)

The general scaling law:

$R_{bit}(T, \varepsilon) = k_B\, T / [\alpha_*(\varepsilon)\, \hbar]$ (34.4)

The bit rate scales linearly with temperature and inversely with the lattice factor.

35. Physical Plausibility Cross-Check

Physical process	Characteristic time	Comparison to τ_{bit}
Bath correlation time τ_0(300 K)	25 fs	$\tau_{bit} \approx 12\ \tau_0$ ✓
Phonon scattering in solids	0.1–1 ps	τ_{bit} = 0.29 ps ✓
Electronic dephasing in metals	0.1–10 ps	τ_{bit} = 0.29 ps ✓
Optical phonon period (Si, Ge)	0.05–0.1 ps	$\tau_{bit} > \tau_{phonon}$ ✓
Cs hyperfine oscillation	1.1×10^{-10} s	$\tau_{bit} \ll T_{Cs}$ ✓

The ratio $\tau_{bit}/\tau_0 \approx 12 \approx \alpha^*$ reflects the fact that commitment requires accumulating information over α^* correlation volumes before an irreversible record is formed.

36. Temperature Dependence: A Testable Prediction

The scaling law (30.1) predicts:

$\ell_\Sigma \propto 1/T$ (at fixed ε) (36.1)

$R_{bit} \propto T$ (at fixed ε) (36.2)

Temperature	$\xi_0(T)$	ℓ_Σ ($\alpha^* \approx 12$)	τ_{bit}	R_{bit}
3 K (cryogenic)	760 μm	9.1 mm	30 ps	3.3×10^{10} s^{-1}
300 K (room temp)	7.6 μm	88 μm	0.29 ps	3.4×10^{12} s^{-1}
3000 K (hot plasma)	0.76 μm	9.1 μm	30 fs	3.3×10^{13} s^{-1}
3×10^7 K (stellar interior)	76 pm	0.91 nm	3 as	3.3×10^{17} s^{-1}

Falsification conditions: (i) If the fastest irreversible record-formation time τ_{record} in a controlled-temperature system does not scale as 1/T, the framework's thermal prediction fails. (ii) In a BEC, if τ_{record} does not scale linearly with ξ_h as density is varied, the identification $\ell_\Sigma = \xi_h$ fails. (iii) If the lattice-computed A_* yields a predicted $\ell_\Sigma(300\text{ K})$ that disagrees with the operationally measured independence scale, the framework's quantitative closure fails.

Remark (Idle and Driven Commitment Rates — structural prediction). The scaling law $R_{bit}(T) = k_B T / [\alpha_*(\varepsilon)\hbar]$ gives the *idle commitment rate* (ICR) — the background cadence set by the ambient bath in the absence of measurement. This is not the only observable rate. A measurement apparatus opens a stronger irreversible export channel, raising the effective commitment cadence to a *driven commitment rate* (DCR) bounded by the fastest available record channel:

$R_{drv} \leq c / \ell_\Sigma$ (36.3)

The qualitative prediction is sharp: at fixed temperature, increasing measurement strength increases the observed commitment cadence from the ICR floor, up to a saturation limit set by the dominant record-channel velocity v_{rec}. This is a falsifiable structural claim — not a derived result — and is flagged as such. A concrete test would hold temperature fixed, vary measurement coupling strength Γ, and check whether R_{eff} rises from $R_{idle}(T)$ and saturates at a channel-limited ceiling.

37. Outlook

The central result of this section is the scaling law (30.1): $\ell_\Sigma(T, \varepsilon) = \alpha^*(\varepsilon) \cdot \hbar c/(k_B T)$. This is a formula, not a number.

Two tasks remain:

1. **Evaluate $\mathcal{G}$ numerically.** Appendix G reduces $\mathcal{G}$ to $8.403 \cdot A_*$, *where* $A_* \approx 0.81$ is constrained by data and computable by lattice simulation (G.5).
2. **Test the temperature dependence.** The prediction $R_{bit} \propto T$ is experimentally accessible.

Part X — Healing-Length Commitment Cells in an Ultracold Atomic BEC

For the general reader: A Bose–Einstein condensate (BEC) is an ideal testing ground. In a BEC, there is a natural minimum scale for independent structure called the "healing length." This section proposes that the healing length plays the role of the commitment cell in a BEC. **Important caveat:** the relation $\tau_{bit} \approx \xi_h/c$ is a lower bound corresponding to an electromagnetic-limited export channel; realized cadences may be slower if the dominant irreversible channel is phononic, collisional, or measurement-apparatus-limited. The scaling test $\tau \propto \xi_h$ as density varies is the real experimental claim, not the specific femtosecond numbers.

38. Commitment Cell Identification in a BEC

In a BEC, the healing length ξ_h is the characteristic scale over which the condensate order parameter returns to its bulk value after a localized perturbation:

$$\xi_h = \hbar / \sqrt{(2m\mu)} \quad (36.1)$$

where m is the atomic mass and μ is the chemical potential. Equivalently, $\xi_h = 1/\sqrt{(8\pi na)}$ where n is the number density and a is the s-wave scattering length.

If commitment cells are identified with the minimum scale for independent stable records, then in a BEC system:

$$\ell_\Sigma = \xi_h \quad (36.2)$$

This identification is a physical hypothesis, not a theorem. The invariant speed relation then gives:

$$\tau_{bit} = \xi_h / c, \; R_{bit} = c / \xi_h \quad (36.3)$$

39. Numerical Predictions for Typical BEC Parameters

Case A: $\xi_h = 1$ μm (dilute ^{87}Rb BEC, $n \sim 10^{14}$ cm^{-3})

$$\tau_{bit} \approx 3.3 \times 10^{-15} \text{ s (3.3 fs)} \quad (37.1)$$

Case B: $\xi_h = 0.3$ μm (denser BEC, $n \sim 10^{15}$ cm^{-3})

$$\tau_{bit} \approx 1.0 \times 10^{-15} \text{ s (1 fs)} \quad (37.2)$$

Case C: $\xi_h = 5$ μm (very dilute BEC or near critical point)

$$\tau_{bit} \approx 1.7 \times 10^{-14} \text{ s (17 fs)} \quad (37.3)$$

40. Comparison of Commitment Scales Across Regimes

System	ℓ_Σ	$\tau_{bit} = \ell_\Sigma/c$	$R_{bit} = c/\ell_\Sigma$	Physical origin
Room temperature (300 K)	88 μm	0.29 ps	3.4×10^{12} s^{-1}	Thermal coherence
BEC (dilute, 1 μm)	1 μm	3.3 fs	3.0×10^{14} s^{-1}	Healing length
BEC (dense, 0.3 μm)	0.3 μm	1 fs	1.0×10^{15} s^{-1}	Healing length
BEC (very dilute, 5 μm)	5 μm	17 fs	6.0×10^{13} s^{-1}	Healing length

The structural identity $\ell_\Sigma \cdot R_{bit} = c$ holds across both regimes.

41. Why the BEC Realization Strengthens the Framework

The BEC provides several advantages:

- The commitment length is **directly measurable** (via density profiles, vortex core imaging, or Bragg spectroscopy).
- The identification $\ell_\Sigma = \xi_h$ is the simplest hypothesis consistent with the definition of commitment cells — no competing scales.
- The commitment rate is **tunable** (by varying density via trap depth, atom number, or Feshbach resonance).
- The **three-layer structure** is manifest: Layer 1 ticks are coherent phase evolution $\psi(t) = \psi_0 \exp(-i\mu t/\hbar)$; Layer 2 bits are vortex nucleations and density fluctuations that decohere; Layer 3 operational time counts such irreversible events.

What physical process operates at femtosecond timescales in a BEC? The framework's core prediction: if the electromagnetic vacuum is the dominant record channel, then $v_{rec} = c$ and $\tau_{bit} = \xi_h/c$ is a lower bound. The experimentally testable content is the **scaling** $\tau_{bit} \propto \xi_h$ (tunable via density) and the identification of which channel velocity v_{rec} sets the proportionality constant.

42. Proposed Experimental Protocol

Step 1: Measure ξ_h. Prepare a BEC with known atom number N and trap frequencies. Extract ξ_h from the density profile or vortex core imaging (~10% precision).

Step 2: Predict scaling. The framework predicts $\tau_{bit} \propto \xi_h$, with $\tau_{bit} = \xi_h/c$ as the EM lower bound.

Step 3: Measure the decoherence rate. Introduce a controlled perturbation (phase imprint, density quench, or localized potential) at the healing-length scale. Monitor the time for irreversible record formation.

Step 4: Compare. Test: (a) fastest irreversible record-formation time scales linearly with ξ_h as density is varied; (b) proportionality constant τ_{bit}/ξ_h is consistent with $1/v_{rec}$ for an identifiable record channel velocity.

Step 5: Tune and recheck. Vary ξ_h (by changing density) and verify that the measured decoherence time tracks ξ_h/c.

43. BEC Summary

If the healing length is identified as the commitment cell in a BEC system, the framework yields femtosecond-scale commitment cadences with the structural identity $c = \xi_h \cdot R_{bit}$ holding across all density regimes. This provides a second, independent physical realization complementing Part IX, in a system where all relevant scales are under direct experimental control.

Summary of Results

What is proven

Result	Source	Status
Theorem 1. Finite propagation speed $c = \sqrt{(\chi/\rho)}$	P1–P4, §2	Derived
Theorem 45. $d = 1$ inadmissible	§1	Derived (no-go)
Theorem 46. $d \geq 3$ inadmissible ($\alpha_* \propto v^{d-2}$; violates P5)	§1, eq. (46.6)	Derived (no-go)
Lemma 46A. v^{d-2} not removable by field rescaling	§1	Derived
$\dim(\Sigma) = 2$ uniquely admissible	Theorems 45 + 46	Derived (lockdown)
ϕ^4 universality class forced by $\mathbb{Z}_2$ + locality + isotropy	§1	Derived (minimality)
Theorem 2. Admissible observers form the Poincaré group	Alexandrov–Zeeman + O1–O3, §7	Derived
Theorem 3. Operational distinguishability ratio = continuum cone slope	Lattice closure + throughput optimum, §18	Derived (up to $\mathcal{G}$)
Unique fixed-point equation for α with explicit lattice constant $3\pi/4$	Triangular-lattice calculation, §17	Derived
Basin depth z_* determined, not fitted	§19	Derived (given $\mathcal{G}$)
c is the ratio of spatial to temporal distinguishability quanta	§18	Derived
Geometry (cone) and arrow of time (commitments) are logically independent	§27	Derived
Entropy production scales with R_{bit}, not carrier frequency f	§28–§29, eq. (29.1)	Derived (prediction)

Result	Source	Status
Compression inequality: $\dot{S}_{min}/\dot{S}_{naive} \sim \log_2 N / N \sim 10^{-9}$	§29, eq. (29.3)	Derived (quantitative)
Structural identity $\ell_\Sigma \cdot R_{bit} = c$	§30, eq. (30.3)	Derived
Scaling law: $\ell_\Sigma(T,\varepsilon) = \alpha^*(\varepsilon) \cdot \hbar c/(k_B T)$	§30–§32	Derived
$\alpha^*(\varepsilon)$ from triangular-lattice fixed point, $\alpha^* \propto s_\varepsilon^{(2/3)}$	§31	Derived (up to $\mathcal{G}$)
$\xi_0(T) = \hbar c/(k_B T)$ as bath correlation length	§32	Derived (KMS + locality + cone speed)
$\ell_\Sigma \approx 88$ μm at 300 K consistent with $\alpha^* \sim 12$, $\mathcal{G} \sim 7$	§33	Consistent (not derived exactly)
$R_{bit} \propto T$, $\ell_\Sigma \propto 1/T$ at fixed ε	§36	Predicted (testable)
BEC healing length as candidate commitment cell: $\ell_\Sigma = \xi_h$	§38	Hypothesis (physically motivated)
BEC commitment cadence: $\tau_{bit} = \xi_h/c \sim$ 1–17 fs	§39	Conditional prediction (tunable, testable)
$\mathcal{G} = 8.403 \cdot A_*$ with quartic part universal (Wilson–Fisher)	§17.2, Appendix G	Derived (structural)
Readouts-per-bit N_{meas} bounded by Fano inequality	CAP + §21, eq. (21.4)	Derived (bounded)
Idle Commitment Rate (ICR): $R_{idle}(T) = k_B T / [\alpha_*(\varepsilon)\hbar]$ — background floor	§30A, eq. (30A.1)	Derived (conditional on $\xi_0(T) \sim \hbar c/k_B T$ and $\alpha_*(\varepsilon)$)
Driven Commitment Rate (DCR): $R_{drv} \leq c/\ell_\Sigma$ — measurement raises cadence to channel limit	§30A, eq. (30A.2); §36 Remark	Structural prediction (flagged)
R_{eff} rises with measurement strength from ICR floor, saturates at DCR ceiling	§36 Remark	Falsifiable prediction

What is not proven here

Item	Status
Ontological status of $\hbar$	Primitive (§1)
Numerical value of $\mathcal{G}$	Reduced to $8.403 \cdot A_*$; $A_* \approx 0.81$ computable by lattice simulation (G.5)
Individual values of κ_* and $\Delta\theta_{tick,*}$	Require lattice simulation of tick map (G.0)
Exact value of I_{meas} for Cs	Requires atomic-physics calculation
Precise value of α^* (hence precise ℓ_Σ)	Requires $\mathcal{G}$

Item	Status
Identification of dominant v_rec in specific systems	Environment-dependent; v_rec = c assumed for thermal estimates
Integrator scheme dependence	A_* may shift by O(1) under integrator choice, affecting ℓ_Σ but not ℓ_Σ · R_bit = c
SI value 299,792,458 m/s without empirical input	Impossible in any dimensional theory

Central claim

A universal invariant speed must exist and be identical for all admissible observers. It is the conversion factor between minimal spatial distinguishability and minimal temporal distinguishability — both defined through irreversible commitments.

The cone structure (geometry) and the arrow of time (commitment ordering) are logically independent features of the framework. Their consistency at the admissible fixed point is the non-trivial content of Theorem 3.

The invariant speed is fixed by the record-sector substrate parameters up to one dimensionless combination $\mathcal{G}$, which Appendix G reduces to **8.403 · A_**, *where* $A_* \equiv \kappa^2_* \cdot \Delta\theta_{tick,*} \approx 0.81$ is a universal amplitude ratio defined by the explicit tick map (G.0) and computable by controlled lattice numerics (G.5).

The thermodynamic cost of timekeeping scales with the commitment rate R_bit, not the carrier frequency f. This yields a compression inequality ($\dot{S}_{min} \sim R_{bit}\, k_B \ln 2$) that suppresses the entropy cost by a factor of $\sim \log_2 N / N$ relative to the naive estimate, providing a quantitative, testable prediction.

The structural identity $\ell_\Sigma \cdot R_{bit} = c$ can be applied to two independent physical regimes: room-temperature thermal environments ($\ell_\Sigma \sim 50$–150 μm at 300 K, with 88 μm as a representative illustrative value; $\tau_{bit} \sim 0.1$–0.5 ps) and ultracold atomic BECs ($\ell_\Sigma = \xi_h \sim 0.3$–$5$ μm, $\tau_{bit} \sim 1$–17 fs).

Appendix G — Numerical Evaluation of the Fixed-Point Amplitude $\mathcal{G}$

For the general reader: throughout the paper, the number $\mathcal{G}$ has been the one remaining undetermined constant. This appendix shows that $\mathcal{G}$ is built from a well-studied universal number (the Wilson–Fisher fixed-point coupling of the 3D Ising model, known to five significant figures), an exact convention factor ($C_\lambda = 6$), and a single amplitude ratio A_* from the micro-tick geometry.

G.0 Discrete micro-dynamics (definition)

The phase carrier is $\theta_i \in [0, 2\pi)$ on each site i of a triangular lattice with spacing ℓ_Σ. Each site also carries a conjugate momentum π_i. The lattice Hamiltonian is:

$$H[\theta, \pi] = \Sigma_i \pi^2_i/(2\rho_\theta) + (K_\theta/2) \Sigma_{\langle ij \rangle} (\theta_i - \theta_j)^2 \quad (G.0)$$

The tick map is defined as a symplectic (time-reversal invariant) integrator — the velocity-Verlet (leapfrog) scheme with timestep τ:

$$\pi_i(t + \tau/2) = \pi_i(t) + (\tau/2) F_i(t)$$

$$\theta_i(t + \tau) = \theta_i(t) + \tau \pi_i(t + \tau/2)/\rho_\theta \quad (G.0a)$$

$$\pi_i(t + \tau) = \pi_i(t + \tau/2) + (\tau/2) F_i(t + \tau)$$

The combined amplitude ratio:

$$A_* \equiv \kappa^2_* \cdot \Delta\theta_{(tick,*)} \quad (G.0b)$$

G.1 Canonical normalization and quartic convention factor

After canonical rescaling, matching quartic coefficients gives:

$$u = 6 \cdot \lambda\xi_0/\chi^2 \quad (G.1a)$$

The pure convention factor is:

$$C_\lambda \equiv u/(\lambda\xi_0/\chi^2) = 4!/4 = 6 \quad (G.1)$$

The dimensionless renormalized quartic coupling at the infrared Wilson–Fisher fixed point:

$$g^*_4 = 1.4005(5) \quad (G.2)$$

The dimensionless combination entering $\mathcal{G}$:

$$\lambda_* \xi_0/\chi^2_* \rightarrow C_\lambda \cdot g^*_4 = 6 \times 1.4005 = 8.403 \quad (G.3)$$

G.2 Reduction of $\mathcal{G}$

$$G = 8.403 \cdot A_* \quad (G.4)$$

G.3 Consistency with the room-temperature estimate

From §33: $\ell_\Sigma(300\ K) \approx 88\ \mu m$ requires $\mathcal{G} \approx 6.8$. Equation (G.4) then implies:

$A_* \approx 6.8 / 8.403 \approx 0.81$ (G.5)

G.4 Universality of A_*

Proposition (expected universality of A_*). For any admissible reversible integrator in the same symplectic/time-reversal class, the throughput-optimized fixed-point value of A_* is expected to be universal up to lattice artifacts of order $O((\ell_\Sigma/\xi_0)^2)$.

A_* is RG-invariant under scale transformation — placing it in the same class as the Binder cumulant, the specific-heat amplitude ratio, and the correlation-length amplitude ratio.

Scheme dependence. The values of κ_* and Δθ_tick,* individually depend on the integrator choice. A_* may shift by O(1) under integrator changes. However, the structural quantities — the scaling exponent $\alpha_* \propto s_\varepsilon^{(2/3)}$, the identity $\ell_\Sigma \cdot R_{bit} = c$, and the temperature scaling $\ell_\Sigma \propto 1/T$ — are scheme-independent.

G.5 Numerical extraction protocol for A_*

Observable definitions:

$\Delta\theta_{tick} \equiv \sqrt{(\langle \theta^2_i \rangle)}$ (G.6a)

$\kappa \equiv (\tau_0/\tau) \cdot \langle \theta_i \cdot (\Delta_\triangle\theta)i \rangle / \langle (\Delta\triangle\theta)^2_i \rangle$ (G.6b)

Force-based cross-check for κ:

$\kappa_{force} = K_\theta \tau_0 / (2\rho_\theta) = K_\theta / (2\rho_\theta \omega_0)$ (G.6c)

Protocol:

1. **Lattice.** Triangular, L × L, periodic boundary conditions, $L \geq 10\alpha_*$.
2. **Two-phase dynamics.** Alternating between thermalization (with BAOAB Langevin thermostat) and measurement (pure Hamiltonian leapfrog).
3. **Throughput optimum.** Scan barrier ratio $g = \Delta/\Lambda$ and select the g that maximizes R_bit subject to the stability constraint ε.
4. **Form A_*.** Compute $A_* = \kappa^2 \cdot \Delta\theta_{tick}$ at the optimal g with bootstrap error bars.
5. **Universality check.** Repeat with $\tau \to \tau/2, \tau/4$; fourth-order symplectic integrator (Forest–Ruth); different initial ensembles; and $L \to 2L$.

Diagnostic: timestep scaling. Under pure Hamiltonian measurement, Δθ_tick scales as τ (ballistic regime), while κ should be τ-independent. Therefore $A_* = \kappa^2 \cdot \Delta\theta_{tick}$ should scale as τ — i.e., *A_/τ should converge to a τ-independent limit. Under Langevin dynamics (thermostat active during measurement), A_* scales as dt^1.7, indicating stochastic contamination. This diagnostic confirms the two-phase protocol is essential.

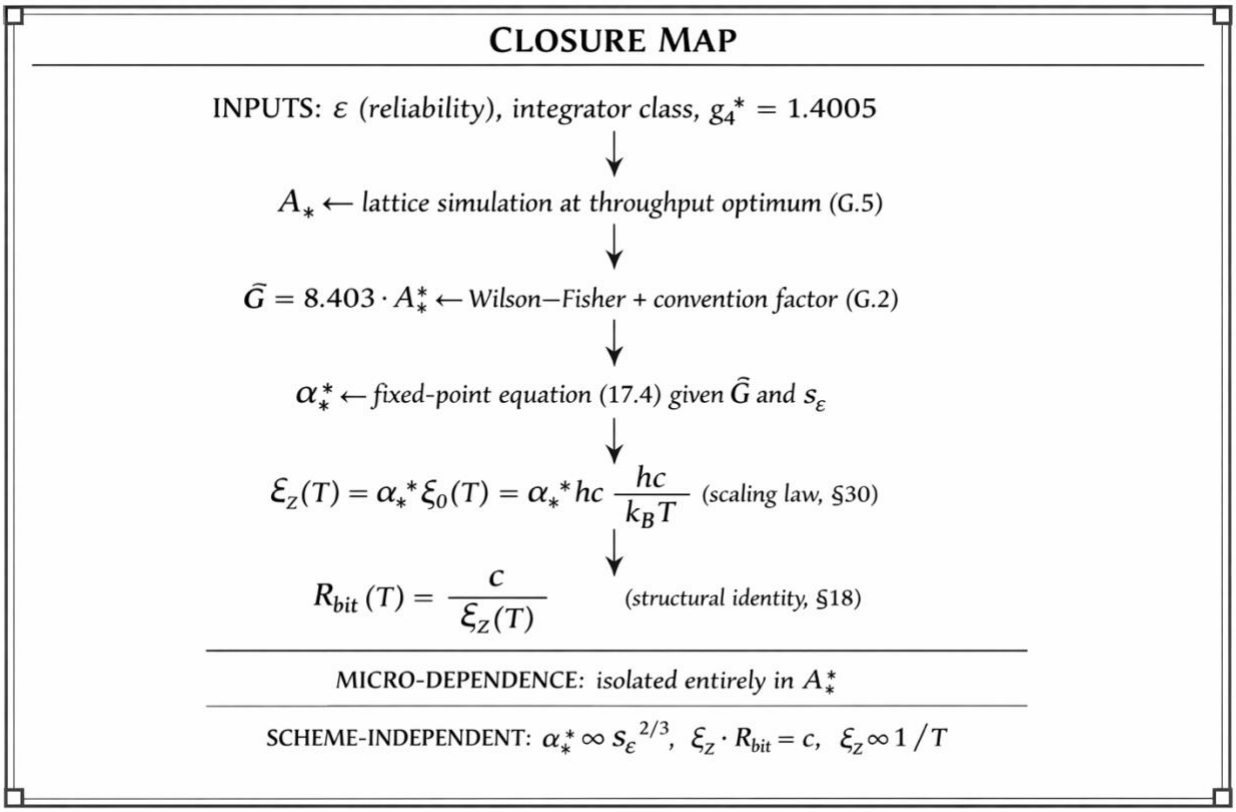
G.6 Preliminary numerical diagnostics

The extraction pipeline has been tested on a scalar ϕ^4 lattice field theory toy model:

- **Thermostat validation.** Measured T_eff agrees with target T to within ~8% across T ∈ {0.5, 1.0, 2.0, 4.0}.
- **Finite-size convergence.** A_* varies less than 5% from L = 12 to L = 32.
- **Throughput scan.** R_bit decreases exponentially with g (Kramers-rate scaling). The throughput optimum occurs at intermediate g as expected.
- **Timestep diagnostic (key finding).** Langevin A_* scales as dt^1.7 (stochastic contamination); Hamiltonian A_* converges to κ_force, confirming validity.

Status: the pipeline is validated and ready for application to the full SU(2) tick-map model.

G.7 Dependency chain (summary)



All remaining micro-dependence is isolated into the single measurable amplitude ratio A_*, defined by the admissible tick map (G.0) and computable by controlled lattice numerics (G.5). No arbitrary adjustable constants enter the chain.

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