Record-Theoretic Emergence of Spacetime Geometry and Gravitational Dynamics

Full Derivation Chain: Axioms → Invariant Speed → Lorentzian Structure → Information Geometry → Einstein Equations → TPB

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Summary for the General Reader

This paper asks a simple question: *what if the fabric of space and time isn't fundamental, but instead emerges from something deeper — the physics of irreversible information?*

Every measurement, every observation, every physical event leaves a permanent mark on the universe — a "record." A photon hits a detector. A particle decays. A signal arrives. These records, once created, cannot be undone. We start from five basic rules about how such records behave — they're irreversible, they're local, there's a limit to how fast they can be created, and the laws governing them look the same to all observers — and we show that the familiar structure of physics follows.

What we prove from these rules alone:

- There is a maximum speed at which any physical influence can travel — and it is the same for all observers. This is the speed of light, derived here not as a postulate but as a consequence of finite information-processing capacity.
- The way clocks slow down for moving observers (time dilation) and the way space and time mix together (Lorentz transformations) follow automatically. Einstein's special relativity emerges from information limits.
- The statistical distinguishability of records at different locations creates a natural notion of "distance" between physical states — a geometry that arises from probability rather than being assumed. We show that the bookkeeping of record creation — how many records are produced and in which direction they flow — is forced by the axioms to take the form of a four-component quantity (a "4-current"), and that this quantity provides exactly enough structure to build a four-dimensional spacetime geometry.

What we derive with additional physical assumptions:

- Einstein's equation of general relativity — the law governing gravity — can be obtained as a consistency condition: it is what you get when you demand that the information capacity of any region of space responds to energy flow in a thermodynamically sensible way. Gravity, in this picture, is not a fundamental force but the universe's bookkeeping — ensuring that information storage and energy flow remain consistent everywhere.

- In the weak-gravity regime, Newton's law of gravitation (the inverse-square law, falling apples) is recovered as a limiting case.

A new interpretive tool:

The paper introduces a quantity called Ticks-Per-Bit (TPB) — the computational cost, in fundamental time-steps, of creating one irreversible record. TPB increases for fast-moving objects and for objects deep in a gravitational field, providing an information-theoretic interpretation of why clocks run slow near massive bodies and at high speeds: it takes the universe more "work" to commit each bit of information. For speed, the mechanism is the causal-cone trade-off: the faster a system moves through space, the less substrate capacity remains for local record commitment. For gravity, the mechanism is commitment-density congestion: mass-energy sources record commitment, elevating the local information density and increasing the substrate cost per bit.

What remains open:

The paper is honest about its boundaries. The full derivation of gravity requires assumptions beyond the five basic rules — specifically, a thermodynamic relationship between information and horizon area that is motivated but not yet proven from first principles. A second route to gravity, based on how record-creation density varies from place to place, is worked through in full with the simplest plausible microscopic model (Poisson counting with spatial drift). The key step — that mass-energy sources record commitment — is not freely chosen but is tightly constrained by the record-accounting balance law and Lorentz symmetry: the source of record production must be a scalar, and the natural scalar that reduces to mass density is the local rest-frame energy. This route recovers Newton's law of gravitation as a variational consequence of the information action, with Newton's constant G emerging from the combination of substrate parameters. Beyond the weak-field limit, we show that under explicit infrared regularity assumptions (second-derivative dominance and covariance), Einstein's equation is the unique leading-order metric structure compatible with the constrained action — with possible corrections only at extremely small (Planck-scale) distances. The theory also predicts that gravitational waves carry exactly two polarisations and travel at the speed of light, matching what LIGO and Virgo have observed. The paper catalogues experimental signatures — deviations from standard physics at extremely small scales — that could test specific implementations of this framework, though these predictions depend on details of the underlying substrate that are not yet determined.

In short: the paper demonstrates that the architecture of spacetime — the stage on which all of physics plays out — can be understood as a consequence of how irreversible information is created, stored, and distinguished across the universe.

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Abstract

We present a record-theoretic route to Lorentzian causal structure and GR-compatible gravitational dynamics starting from five operational axioms governing irreversible record commitment. The construction proceeds in stages: (i) locality and finite distinguishability imply a finite maximal speed for record-relevant influence, yielding an invariant causal cone and a frame-invariant constant c; (ii) inertial symmetry plus cone preservation forces Lorentz transformations and the Minkowski interval; (iii) probability distributions over record outcomes induce a Fisher information metric on macrostate (model) space; (iv) when macrostates are promoted to local fields, Fisher geometry supplies a natural distinguishability scale which can be coupled to the already-derived causal (conformal) structure to construct a spacetime metric. Gravitational field equations are obtained via coarse-grained closures. Route A derives Einstein's equation in Jacobson form as a local equation of state for record entropy across causal horizons under explicitly stated thermodynamic assumptions. Route B provides a variational derivation of Newtonian gravity from an information action on the commitment-density field: the Poisson–drift micro-model, combined with canonical volume matching and a matter–record source term forced by the balance law and Lorentz symmetry, yields $\nabla^2\Phi = 4\pi G\rho_m$ with $G = \gamma_0 c^4/(8\pi\rho_{c0})$ expressed in terms of substrate parameters; a fully covariant constrained action $S_{tot} = S_{info} + S_{vol} + S_{src}$ generalises this to arbitrary curvature. A universality lemma shows the Newtonian result holds for any Poisson-like counting process with linear rate parameter, not just the specific micro-model. Under an explicit infrared truncation to second-derivative order, variation of the covariant action with respect to the metric, followed by elimination of the record fields via constraints, yields the Einstein equation as the unique leading-order structure (Lovelock uniqueness) — with possible higher-derivative corrections suppressed by the Fisher curvature scale. The linearised theory predicts exactly two transverse-traceless gravitational-wave polarisations propagating at speed c, consistent with LIGO/Virgo observations. In the regime where the record-thermodynamic assumptions hold, the macrodynamics reduce exactly to general relativity. The Ticks-Per-Bit (TPB) ratio inherits relativistic and gravitational scaling via TPB ∝ dt/dτ. We conclude with open problems — including extension of the matter sourcing beyond the Newtonian regime and into the full

quantum-gravitational domain — and outline candidate experimental signatures that arise in common discrete-substrate implementations.

I. Introduction and Scope

I.1 Axioms versus Closures (Roadmap)

From Axioms 1–5 alone we derive: bounded domain of dependence, a finite invariant record-influence speed c, Lorentzian causal (cone) structure, Lorentz transformations, the record-commitment 4-current $C^\mu = (\rho_c, J_c)$ as the minimal macrostate basis, Fisher metric on macrostate space, and TPB scaling via $dt/d\tau$.

From the canonical information-geometric closure (record-calibration principle): the conformal factor $\Omega(x)$ linking Fisher distinguishability to spacetime volume, yielding a concrete emergent metric $g_{\mu\nu} = \Omega^2 \tilde{\eta}_{\mu\nu}$ (Section VI.3).

To obtain gravitational field equations we adopt explicitly stated macroscopic closure assumptions:

- **(A)** A local horizon thermodynamic closure (Section VII) yielding Einstein's equation as an equation of state for record entropy, under four assumptions (A1–A4) stated in full.
- **(B)** An induced-gravity programme (Section VIII) in which curvature is sourced by spatial variation in commitment density. The matter–record source term is derived from the balance law and Lorentz symmetry; the Poisson–drift micro-model then yields Newtonian gravity variationally (Section VIII.7).

I.2 Relation to General Relativity

This framework does not compete with or seek to replace general relativity. In the macroscopic regime where the record-thermodynamic closure assumptions hold, the dynamics reduce exactly to GR. The framework's contribution is to identify operational, information-theoretic conditions under which GR emerges, to unify the kinematic (Lorentz) and dynamic (Einstein) structures under a single set of axioms, and to point toward regimes — principally near the Planck scale — where the discrete substrate structure might produce deviations from smooth GR.

II. Foundational Axioms

The framework rests on five axioms. Each has a precise operational meaning; together they are sufficient to derive relativistic causal structure and information geometry. Gravitational dynamics requires additional closure assumptions stated in Section VII.

Axiom 1 (Record Commitment). Physical events correspond to the creation of stable, irreversible records — classical distinctions that, once committed, cannot be erased by subsequent unitary evolution of the substrate. A record is operationally defined as an outcome that can be independently verified by spacelike-separated observers.

Axiom 2 (Reversible Substrate). Between record-commitment events, the microscopic dynamics of the substrate are reversible (unitary). Irreversibility enters only at the moment of record commitment. This separates the entropic arrow (tied to records) from the underlying dynamical law (which is time-symmetric).

Axiom 3 (Finite Distinguishability). Any finite spatial region can generate or export at most finitely many distinguishable records per unit substrate time. Formally: for any bounded region Ω and tick interval $[n, n+1]$, the number of orthogonal record states accessible to Ω is bounded by a constant $D(\Omega) < \infty$.

Axiom 4 (Local Coupling). The state update at any lattice site depends only on sites within a bounded neighbourhood of graph-distance radius r_0. No instantaneous action at a distance is permitted at the substrate level.

Axiom 5 (Inertial Symmetry). The laws governing record commitment are invariant under spatial translations, rotations, and boosts between inertial laboratories. Operationally, this axiom buys three things: (a) coordinate transformations between inertial frames must be **linear** (from spatial homogeneity and temporal uniformity); (b) the transformation must be **isotropic** (no preferred spatial direction); (c) successive boosts must compose — the set of transformations forms a **group**. These three properties, together with the invariant speed from Axioms 3–4, uniquely determine the Lorentz group (Section IV).

III. Finite Influence Speed and the Causal Cone

III.1 Time Conventions

To avoid ambiguity, we distinguish three time concepts:

- **Substrate ticks** $n = 0, 1, 2, \ldots$: the discrete update steps of the underlying lattice. These are not directly observable.
- **Tick duration** Δt_tick: the physical duration assigned to one substrate tick in the continuum limit.
- **Emergent coordinate time** t: the macroscopic time coordinate in an inertial laboratory, related to substrate ticks by the coarse-graining $t \approx n\, \Delta t\_tick$ in the rest frame of the lab.
- **Proper time** τ: the invariant time along a worldline, measured by local clocks (equivalently, by local record-commitment rate).

III.2 Discrete Domain of Dependence

Model the substrate as a graph $\mathcal{G}$ with sites {x} updated in discrete ticks. By Axiom 4, the update rule at site x and tick n depends only on sites within graph-distance r_0 of x.

Theorem 1 (Discrete Bounded Domain of Dependence). Let R(x, n) denote any record observable at site x and tick n. Then R(x, n) depends only on initial data at sites y satisfying $d\_\mathcal{G}(x, y) \leq n\, r_0$, where $d\_\mathcal{G}$ is the graph distance on $\mathcal{G}$.

Proof. By induction on n. At n = 0 the claim is trivial. Suppose R(x, n) depends only on data within graph-distance $n\, r_0$. At tick n+1, the update at x consults only sites within distance r_0 of x, each of which depends on data within distance $n\, r_0$. The union lies within distance $(n+1)\, r_0$ of x. ■

Corollary. Distinguishable causal influence cannot propagate faster than r_0 adjacency layers per tick.

III.3 Continuum Limit

Assign a physical lattice spacing *a* and tick duration Δt_tick. Define the maximal signal speed:

$c = (a\, r_0) / \Delta t\_tick$.

Take the continuum limit a → 0, Δt_tick → 0 with c held fixed.

Theorem 2 (Continuum Bounded Domain of Dependence). In the continuum limit, any record observable R(t, x) depends only on initial data within the region $|x - x_0| \leq c\, t$. The boundary $|x - x_0| = c\, t$ defines the causal cone.

Proof sketch. The discrete bound $d\_\mathcal{G}(x, y) \leq n\, r_0$ becomes, after rescaling, $|x - x_0| \leq (n\, \Delta t\_tick)(a\, r_0 / \Delta t\_tick) = c\, t$. The bound is preserved under refinement because c is held constant. ■

III.4 Invariance of c

We must establish that c is not merely a bound but a *frame-invariant* maximal speed for empirically meaningful signalling. The argument has three components.

Sharp bound, not bandwidth limit. By Axiom 4, any influence capable of altering future committed records must propagate through a bounded neighbourhood per tick. By Axiom 3, finite regions cannot export arbitrarily many distinguishable record-effects per tick. We therefore define c as the **maximal speed of record-relevant influence** — the fastest rate at which any effect that can alter future committed records propagates through the substrate. If the substrate graph is homogeneous (Axiom 5), this bound is generically saturated: there exist configurations in which record-relevant information propagates at exactly r_0 sites per tick, yielding signals at exactly c in the continuum limit.

Frame invariance. By Axiom 5, the operational record laws are identical in all inertial laboratories. Since c is determined by substrate capacity and lattice geometry alone — properties of the substrate, not of any observer — the maximal record-influence speed is the same in all inertial frames. Thus c is a frame-invariant limiting speed.

Scope. This argument bounds the propagation of *record-relevant* influence — effects that can alter distinguishable committed records. It does not rule out "superluminal" correlations that carry no distinguishable record content (analogous to EPR correlations in quantum mechanics, which propagate no usable signal). Such correlations are undetectable by definition, since detection requires record commitment.

Inhomogeneous case. The saturation argument — that c is achieved, not merely an upper bound — relies on substrate homogeneity (Axiom 5). In the inhomogeneous regime arising from gravitational dynamics (Sections VII–VIII), c remains locally invariant (every local inertial frame measures the same maximal speed), but the effective light cone tilts from point to point — this is precisely the content of a curved spacetime metric $g_{\mu\nu}(x)$. The local invariance of c is preserved; what varies is the causal structure into which it is embedded.

IV. Emergence of Lorentzian Structure

IV.1 From Invariant Speed to Causal Cone

The existence of a finite, frame-invariant maximal speed c partitions spacetime events into causal (reachable within the cone) and acausal (unreachable) regions. The cone boundary is defined by:

$$c^2\, dt^2 - d\ell^2 = 0\ ,$$

where $d\ell^2 = dx^2 + dy^2 + dz^2$ is the spatial line element.

IV.2 Lorentz Transformations

Theorem 3 (Uniqueness of Cone-Preserving Linear Maps). The most general linear coordinate transformation between inertial frames that preserves the cone $c^2dt^2 - d\ell^2 = 0$, respects spatial isotropy and homogeneity, and composes as a group, is a Lorentz transformation.

Derivation. Consider a boost along the x-direction. Linearity follows from spatial homogeneity and temporal uniformity (Axiom 5a):

$$t' = A\, t + B\, x\ ,\quad x' = C\, t + D\, x\ .$$

Impose invariance of c: if $x = ct$ then $x' = ct'$, and if $x = -ct$ then $x' = -ct'$. These two conditions give:

$$C + D = c(A + B)\ ,\quad -C + D = c(A - B)\ .$$

Adding: $D = cA$. Subtracting: $C = cB$. The origin of the primed frame moves at $x = vt$, so $x' = 0$ when $x = vt$, giving $C = -vA$. Therefore $B = -v\,A/c^2$, $D = A$, and consistency with the inverse transformation (group property, Axiom 5c) fixes:

$$A = \gamma = 1 / \sqrt{(1 - v^2/c^2)} \ .$$

The resulting transformation is:

$$t' = \gamma(t - vx/c^2) \ , \ x' = \gamma(x - vt) \ , \ y' = y \ , \ z' = z \ .$$

IV.3 Invariant Interval and Proper Time

Define the invariant interval:

$$ds^2 = c^2\, dt^2 - dx^2 - dy^2 - dz^2 \ .$$

This is preserved under Lorentz transformations by construction. The proper time along a timelike worldline is:

$$d\tau/dt = \sqrt{(1 - v^2/c^2)} \ ,$$

which will be central to TPB scaling (Section X).

IV.4 Lorentz Group Structure

Define the Minkowski metric $\eta = \mathrm{diag}(+1, -1, -1, -1)$. A transformation Λ preserves the interval if and only if:

$$\Lambda^T\, \eta\, \Lambda = \eta \ .$$

The set of all such Λ forms the group $O(1,3)$. The proper orthochronous subgroup $SO^+(1,3)$ — preserving orientation and time direction — is the physical symmetry group of inertial record physics.

V. Information Geometry: Record Probabilities Force a Metric

V.1 Setup

Let $p(r \mid \theta)$ denote the probability of record outcome r given macrostate parameters $\theta = (\theta^1, \theta^2, \ldots, \theta^n)$. Here θ parametrises the statistical ensemble of records accessible at a given spacetime region — for example, local temperature, density, field amplitudes, or commitment rate.

V.2 Kullback–Leibler Divergence and the Fisher Metric

Lemma IG1 (KL Expansion). The Kullback–Leibler divergence between neighbouring models expands as:

$$D_{KL}(\theta \parallel \theta + d\theta) = \tfrac{1}{2}\, g_{ab}(\theta)\, d\theta^a\, d\theta^b + \mathcal{O}(|d\theta|^3) .$$

The leading quadratic form defines a unique metric on parameter space.

Lemma IG2 (Fisher Metric). The metric coefficients are:

$$g_{ab}(\theta) = \mathbb{E}[\, \partial_a \ln p(r|\theta) \cdot \partial_b \ln p(r|\theta)\,] ,$$

where $\partial_a \equiv \partial/\partial\theta^a$ and the expectation is over $r \sim p(\cdot|\theta)$. This is the **Fisher information metric**.

Proof. Expand $\ln p(r|\theta+d\theta)$ to second order in $d\theta$. Insert into $D_{KL} = \Sigma_r\, p(r|\theta) \ln[p(r|\theta)/p(r|\theta+d\theta)]$. The first-order term vanishes by normalisation ($\Sigma_r\, \partial_a p = 0$). The second-order term yields g_{ab} as stated. Uniqueness (up to scale) follows from the Čencov theorem: the Fisher metric is the only Riemannian metric on statistical models that is invariant under sufficient statistics (Markov embeddings). ■

V.3 Key Properties

Lemma IG3 (Reparametrisation Invariance). Under $\theta \to \tilde{\theta}(\theta)$, the Fisher metric transforms as a rank-(0,2) tensor:

$$\tilde{g}_{cd}(\tilde{\theta}) = g_{ab}(\theta)\, (\partial\theta^a/\partial\tilde{\theta}^c)(\partial\theta^b/\partial\tilde{\theta}^d) .$$

The line element $ds^2 = g_{ab}\, d\theta^a\, d\theta^b$ is coordinate-independent.

Lemma IG4 (Additivity / Composability). For independent record systems ($p = p_1 \cdot p_2$), the total Fisher metric is the direct sum: $g = g^{(1)} \oplus g^{(2)}$. Independent subsystems contribute additively to distinguishability.

Corollary (Riemannian Structure). The Fisher metric is symmetric ($g_{ab} = g_{ba}$) and positive semi-definite, with strict positivity when the model is identifiable. It therefore defines a Riemannian geometry on the space of record-probability models.

VI. Deriving the Macrostate Basis and Fixing the Conformal Factor

This section closes two gaps that have persisted through earlier versions: the physical identity of the macrostate fields, and the functional form of the conformal factor. Both are now derived —

the macrostate basis from the axioms, the conformal factor from a canonical information-geometric closure.

VI.1 The Macrostate Basis Is Forced to Be a Record 4-Current

Lemma 1 (Existence of a local commitment density). *From Axiom 1 (irreversible record commitment) and Axiom 3 (finite distinguishability per finite region per tick), any sufficiently small spacetime cell admits a finite expected number of committed distinctions. Hence there exists a coarse-grained scalar commitment density:*

$$\rho_c(x) \equiv \lim_{\Delta V, \Delta\tau \to 0} \mathbb{E}[\Delta N_b] / (\Delta V \cdot \Delta\tau) ,$$

where ΔN_b is the number of committed record-bits in the cell, ΔV its proper 3-volume, and $\Delta\tau$ its proper time thickness. This definition is operational: it is measurable by local counting of committed record events. ■

Lemma 2 (Locality forces a flux term). *Axiom 4 (local coupling) implies that changes in the number of commitments in a region Ω can only occur by (i) transport of record-relevant influence across $\partial\Omega$ and (ii) local production of commitments within Ω. Therefore there exists a spatial commitment flux $J_c(x)$ and a (possibly nonzero) source $s_c(x)$ such that for any region Ω:*

$$(d/d\tau) \int_\Omega \rho_c \, dV = -\int_{\partial\Omega} J_c \cdot dA + \int_\Omega s_c \, dV .$$

In differential form (in a local inertial chart):

$$\partial_\tau \rho_c + \nabla \cdot J_c = s_c .$$

Thus, once ρ_c exists, J_c is not optional: it is the boundary term demanded by locality. ■

Proposition 1 (Inertial symmetry forces a 4-current macrostate). *From Sections III–IV, inertial symmetry plus an invariant causal cone yields Lorentz transformations as the coordinate changes between inertial laboratories. The local commitment balance law must be form-invariant under these transformations (Axiom 5).*

Why a 4-vector and not some other tensor object: ρ_c is defined per proper 3-volume per proper time — it is a scalar density as seen by local comoving observers. In curved spacetime, the balance law must be expressible in covariant divergence form $\nabla_\mu C^\mu = s_c$; in local inertial coordinates this reduces to $\partial_\mu C^\mu = s_c$. This identifies C^μ as a contravariant vector field (not a tensor density or higher-rank object). The unique Lorentz-covariant packaging of "scalar density + spatial flux" satisfying this divergence law is therefore a 4-current:

$$C^\mu(x) \equiv (\rho_c(x), J_c^1(x), J_c^2(x), J_c^3(x)) ,$$

in terms of which the balance law becomes:

$\partial_\mu C^\mu = s_c$,

with s_c a scalar. Therefore the minimal non-degenerate macrostate basis compatible with inertial symmetry is the record 4-current C^μ. In particular, even if a given rest frame has $J_c = 0$, boosts generically produce $J_c \neq 0$, so the four components are required globally. ■

Conclusion (macrostate basis). We set $\theta^a \equiv C^\mu$, $a = 0, 1, 2, 3$, as the minimal macrostate field content forced by record commitment + locality + inertial symmetry.

VI.2 Fisher Metric on the 4-Current Space

With the derived macrostate basis $\theta^a = C^\mu$, the Fisher metric on the 4-dimensional macrostate space is:

$$g_{ab}(C) = \mathbb{E}[\, \partial_a \ln p(r|C) \cdot \partial_b \ln p(r|C) \,] ,$$

where $p(r|C)$ is the probability of record outcome r given local commitment 4-current C^μ. This is a 4×4 positive-definite matrix at each spacetime point, with all the properties established in Section V (reparametrisation invariance, additivity, Čencov uniqueness).

The pullback to spacetime is:

$$\hat{g}_{\mu\nu}(x) = g_{ab}(C(x))\, \partial_\mu C^a(x)\, \partial_\nu C^b(x) .$$

Since the macrostate space now has dimension $n = 4$, $\hat{g}_{\mu\nu}$ generically has full rank 4 — the degeneracy problem is resolved.

Important clarification on signature. The pullback $\hat{g}_{\mu\nu}$ is positive semi-definite (inheriting the positive-definiteness of the Fisher metric g_{ab}). It is therefore a *Riemannian* object — a measure of statistical distinguishability between nearby macrostates, not the physical spacetime metric. The physical spacetime metric $g_{\mu\nu}$ is Lorentzian, with signature inherited from the causal cone (Sections III–IV). The relationship between them is that $\hat{g}_{\mu\nu}$ supplies the *volume element* (conformal factor) of $g_{\mu\nu}$, not its signature or conformal class. In short: $\hat{g}_{\mu\nu}$ measures how different nearby record ensembles are; $g_{\mu\nu}$ measures spacetime intervals. They are distinct geometric objects linked by the conformal selection principle below.

VI.3 Fixing the Conformal Factor by Record-Volume Matching

At this stage the axioms fix the causal (conformal) structure via the causal cone, but not the overall local scale. We now show how to select the missing volume element from record statistics in an invariant way.

Lorentzian structure does not arise from Fisher geometry; it arises earlier, from the causal cone derived in Sections III–IV. The causal cone determines a conformal class $[\tilde{\eta}_{\mu\nu}]$ of Lorentzian metrics — all metrics related by $g_{\mu\nu} \mapsto \Omega^2 g_{\mu\nu}$ share the same causal cones. What

Fisher geometry supplies is a distinguishability scale: the missing volume element needed to pick a representative within the conformal class.

A Lorentzian metric is equivalent to a causal cone structure plus a volume element. Since the cone is fixed, specifying the volume element completes the metric.

Lemma 3 (Canonical invariant volume density from record probabilities). *The Fisher information metric $g_{ab}(\theta)$ on macrostate space admits a canonical reparametrisation-invariant volume element (Jeffreys measure):*

$$dV_J(\theta) = \sqrt{\det g_{ab}(\theta)}\, d^n\theta \,.$$

Define the associated invariant record-volume density:

$$J(x) \equiv \sqrt{\det g_{ab}(C(x))} \,.$$

For the derived macrostate basis $\theta^a = C^\mu$ (hence $n = 4$), this becomes $J(x) = \sqrt{\det g_{ab}(C(x))}$. Under reparametrisation $\theta \mapsto \tilde{\theta}(\theta)$, the product $J\, d^4\theta$ is invariant. ■

Proposition 2 (Conformal selection by volume-form matching). *Since the causal cone is fixed by Sections III–IV, we select the remaining freedom by choosing the representative $g_{\mu\nu}$ in the conformal class such that:*

$$\sqrt{|g(x)|} = \kappa_0 \cdot J(x) \,,$$

where κ_0 is a universal constant setting units (fixed by matching the Newtonian limit or horizon-entropy normalisation).

Justification. Since the causal cone fixes the conformal structure but not the volume form, completing the metric requires choosing a scalar density from record statistics. Reparametrisation invariance eliminates coordinate-dependent candidates (e.g., $\mathrm{tr}(g_{ab})$ relative to a fixed basis), but does not single out $\det(g_{ab})$ uniquely: other symmetric polynomials of the eigenvalues of g_{ab} are also invariant. What selects the Jeffreys density $\sqrt{\det g_{ab}}$ over these alternatives is **composability**: for independent record subsystems with $g = g^{(1)} \oplus g^{(2)}$, the determinant factorises ($\det g = \det g^{(1)} \cdot \det g^{(2)}$), making $\log J$ additive. No other symmetric polynomial of the eigenvalues has this multiplicative property under direct sums. Combined with locality (J depends only on the Fisher metric at a point) and the requirement that J have the correct density weight to serve as a volume element, the Jeffreys measure is the unique candidate satisfying all three constraints: reparametrisation invariance, composability, and correct density weight. We therefore adopt the calibration that identifies the spacetime volume element with this canonical record-counting density (up to a unit-setting constant κ_0).

Existence and uniqueness. Given any conformal representative $\tilde{\eta}_{\mu\nu}$ compatible with the cone, there exists a unique local conformal factor $\Omega(x)$ (up to boundary conditions) such that $g_{\mu\nu} = \Omega^2 \tilde{\eta}_{\mu\nu}$ satisfies $\sqrt{|g|} = \kappa_0 J(x)$. In local inertial coordinates where $\tilde{\eta}_{\mu\nu} = \eta_{\mu\nu}$ (Minkowski) and hence $\sqrt{|\tilde{\eta}|} = 1$, this reduces to:

$\Omega(x) = (\kappa_0 J(x))^{1/4}$,

which provides a concrete, gauge-fixed expression in the natural local frame.

Interpretation. The causal cone fixes which directions are timelike / null / spacelike (signature and conformal class), while record distinguishability fixes the local spacetime "volume per coordinate cell" via $\sqrt{|g|}$. This provides a non-arbitrary, reparametrisation-invariant route from record statistics to the metric scale. Operationally: the amount of spacetime measure assigned to a region tracks how much distinguishable record structure it can support.

Consistency check (composability). For independent record subsystems, Fisher metrics add as direct sums $g = g^{(1)} \oplus g^{(2)}$, so $\det g = (\det g^{(1)})(\det g^{(2)})$ and $J = J_1 \cdot J_2$. Thus log J is additive — the correct behaviour for a local capacity density.

VI.4 What This Closes, and What Remains Open

Closed: The minimal macrostate basis $C^\mu = (\rho_c, J_c)$ is forced by record accounting + locality + Lorentz covariance (Lemma 1, Lemma 2, Proposition 1). The dimensionality ≥ 4 requirement is derived, not assumed.

Closed (up to one constant): The conformal factor Ω(x) is fixed by a canonical invariant record-volume density $J(x) = \sqrt{\det g_{ab}(C(x))}$ via $\sqrt{|g|} = \kappa_0 J$ (Lemma 3, Proposition 2). This replaces "some monotone f" with a specific invariant.

Still open: The micro-to-macro model p(r|C) that determines $g_{ab}(C)$, and the matching that fixes κ_0 (e.g., Newtonian limit, Bekenstein–Hawking η).

What this enables for Route B: Since Ω depends on $\det g_{ab}(C)$, and C depends on ρ_c, the gravitational potential Φ can be derived from an explicit choice of p(r|C). Section VIII.7 carries this out with the Poisson–drift micro-model, recovering Newton's law variationally with G expressed in terms of substrate parameters.

VII. Route A — Einstein Equations as Record-Thermodynamic Equations of State

This derivation follows the Jacobson (1995) strategy, recast in record-theoretic language. It shows that Einstein's field equations are the macroscopic consistency condition ensuring record-entropy responds to energy flux across causal boundaries in a manner compatible with local equilibrium and conservation laws.

Important framing. Route A is a conditional derivation: it requires four explicit thermodynamic closure assumptions (A1–A4 below) in addition to Axioms 1–5. These assumptions are physically motivated but not deducible from the axioms alone. The Einstein equation is therefore derived as the unique field equation *given* that the thermodynamic conditions hold — not as an axiomatic inevitability.

VII.1 Thermodynamic Closure Assumptions (Record Language)

A1 (Local Causal Boundary). Around any spacetime point p, an accelerated observer defines a local causal horizon — a boundary of maximal causal influence — with associated Rindler geometry.

A2 (Record Entropy–Area Law). The record capacity associated with a local causal horizon patch is proportional to its area:

$$S_{rec} = \eta A ,$$

where η is a universal constant (to be identified with $1/(4\ell_P^2)$ upon matching to Bekenstein–Hawking entropy). This assumption is motivated by the holographic bound: the maximum number of distinguishable records attributable to a region scales with its boundary area, not its volume.

A3 (Local Clausius Relation). For energy flux δQ crossing a horizon patch, record entropy obeys:

$$\delta Q = T \, \delta S_{rec} ,$$

where T is the Unruh temperature associated with the local acceleration (Unruh 1976).

A4 (Local Equilibrium). The horizon generator congruence is chosen so that expansion θ and shear $\sigma_{\mu\nu}$ vanish at p to leading order.

VII.2 Local Rindler Geometry and Energy Flux

Let k^μ be tangent to the null generators of the local horizon, with affine parameter λ ($\lambda = 0$ at p). The approximate boost Killing vector is:

$$\chi^\mu = -\kappa \lambda k^\mu ,$$

where κ is the surface gravity. The energy flux through the horizon patch is:

$$\delta Q = \int T_{\mu\nu} \chi^\mu d\Sigma^\nu .$$

For a null horizon element $d\Sigma^\nu = k^\nu d\lambda \, dA$:

$$\delta Q = -\kappa \int \lambda T_{\mu\nu} k^\mu k^\nu d\lambda \, dA .$$

VII.3 Entropy Change via Raychaudhuri

The area evolution of the null generator bundle is governed by the expansion θ through $dA/d\lambda = \theta A$. The Raychaudhuri equation for a null congruence reads:

$$d\theta/d\lambda = -\tfrac{1}{2} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} - R_{\mu\nu} k^\mu k^\nu .$$

Under local equilibrium (A4), $\theta = 0$ and $\sigma_{\mu\nu} = 0$ at $\lambda = 0$. To leading order:

$$\theta(\lambda) \approx -\lambda R_{\mu\nu} k^\mu k^\nu .$$

The area change is therefore:

$$\delta A = \int \theta \, d\lambda \, dA \approx -\int \lambda R_{\mu\nu} k^\mu k^\nu \, d\lambda \, dA .$$

Using $S_{rec} = \eta A$:

$$\delta S_{rec} = \eta \, \delta A \approx -\eta \int \lambda R_{\mu\nu} k^\mu k^\nu \, d\lambda \, dA .$$

VII.4 Clausius Relation Yields the Field Equation

The Unruh temperature for acceleration κ is $T = \kappa/(2\pi)$. Substituting δQ and δS_{rec} into $\delta Q = T \delta S_{rec}$:

$$-\kappa \int \lambda T_{\mu\nu} k^\mu k^\nu \, d\lambda \, dA = (\kappa/2\pi)(-\eta) \int \lambda R_{\mu\nu} k^\mu k^\nu \, d\lambda \, dA .$$

Cancel the common factors (κ, the integral measure, and the λ-weighting):

$$T_{\mu\nu} k^\mu k^\nu = (\eta/2\pi) R_{\mu\nu} k^\mu k^\nu .$$

Since this holds for **all** null vectors k^μ at p, we invoke the following standard result:

Lemma (Null-vector characterisation). *If a symmetric tensor $S_{\mu\nu}$ satisfies $S_{\mu\nu} k^\mu k^\nu = 0$ for all null vectors k^μ, then $S_{\mu\nu} = f g_{\mu\nu}$ for some scalar f.* (Proof: decompose k^μ into timelike and spacelike parts and use the freedom to vary the null direction independently; see Wald (1984), §4.2.)

Applying this to $S_{\mu\nu} = R_{\mu\nu} - (2\pi/\eta) T_{\mu\nu}$, the tensor equation must take the form:

$$R_{\mu\nu} + \Phi g_{\mu\nu} = (2\pi/\eta) T_{\mu\nu} ,$$

where Φ is an undetermined scalar function (terms proportional to $g_{\mu\nu}$ vanish under null contraction and are therefore invisible to the null-vector argument alone).

VII.5 Fixing Φ via Conservation and Bianchi

Apply the contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$, where $G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu}$, together with local energy-momentum conservation $\nabla^\mu T_{\mu\nu} = 0$. These jointly fix:

$\Phi = -\frac{1}{2} R + \Lambda$,

where Λ is an integration constant (the cosmological constant). The result is the **full Einstein field equation**:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa_G T_{\mu\nu} ,$$

with gravitational coupling:

$$\kappa_G = 2\pi/\eta .$$

Upon identifying $\eta = 1/(4 \ell_P^2) = c^3/(4G\hbar)$, this reproduces $\kappa_G = 8\pi G/c^4$, the standard Einstein coupling.

VII.6 Record-Theoretic Interpretation

Einstein's equation is not a fundamental dynamical law in this framework. It is a **macroscopic equation of state**: the condition that record capacity (proportional to horizon area) responds to energy flux across local causal boundaries in a way compatible with local equilibrium thermodynamics and the conservation laws inherited from the substrate's unitarity (Axiom 2). Gravity is the entropic response of the record substrate to energy redistribution.

VIII. Route B — Curvature from Commitment-Density Variation and Fisher Compatibility

Route B takes a complementary approach: rather than deriving the Einstein equation thermodynamically, it constructs the metric and curvature directly from spatial variation of a commitment-density field. Sections VIII.1–VIII.6 establish the framework; Section VIII.7 completes the Newtonian pipeline; Sections VIII.8–VIII.9 define the covariant action and prove universality; Section VIII.10 shows that the full Einstein equation emerges as the unique leading-order strong-field metric equation.

VIII.1 Commitment Density as a Macrostate Field

Let $\rho_c(x)$ denote the **commitment density** — the number of irreversible record commitments per unit proper time per unit proper volume. By Axiom 3, ρ_c is bounded from above; by Axiom 1, it is the fundamental source of irreversibility and hence of entropic time.

The local record distribution $p_x(r \mid \rho_c(x))$ depends on the local commitment density. The single-parameter Fisher information is:

$$g_{\rho\rho}(x) = \mathbb{E}[(\partial_{\rho_c} \ln p_x)^2] .$$

VIII.2 Emergent Potential from Fisher Information

In the weak-field regime, the time-time component of the emergent metric responds to local statistical distinguishability:

$g_{00}(x) = 1 + 2\Phi(x)/c^2$,

where $\Phi(x) = F(g_{\rho\rho}(x))$ for some functional F mapping Fisher information to gravitational potential. The physical content: regions of higher commitment density (greater record distinguishability) have deeper gravitational potential wells. More record activity corresponds to stronger gravitational binding.

VIII.3 Weak-Field Metric and Geodesic Motion

Adopt the standard weak-field metric (signature +,−,−,−):

$g_{00} = 1 + 2\Phi/c^2$, $g_{0i} = 0$, $g_{ij} = -(1 - 2\Psi/c^2)\, \delta_{ij}$,

with $|\Phi|/c^2, |\Psi|/c^2 \ll 1$. In the Newtonian regime with negligible anisotropic stress, $\Phi \approx \Psi$.

For slow motion ($v \ll c$), the spatial geodesic equation reduces to:

$d^2x^i/dt^2 \approx -\Gamma^i_{00}\, c^2$.

Compute Γ^i_{00} to leading order:

$\Gamma^i_{00} = \frac{1}{2} g^{ij}\, \partial_j\, g_{00} \approx -(1/c^2)\, \partial^i\Phi$.

Therefore:

$\mathbf{d^2x^i/dt^2 = -\partial^i\Phi}$,

which is Newton's law: gravitational acceleration equals the negative gradient of the potential.

VIII.4 Field Equation and Poisson Limit

The 00-component of Einstein's equation in the weak-field, slow-motion limit gives:

$\boldsymbol{\nabla^2\Phi = 4\pi G\, \rho_m}$,

where ρ_m is the mass-energy density ($T_{00} \approx \rho_m c^2$).

VIII.5 Mapping to Commitment Density

To reproduce Newtonian gravity from commitment density, identify an effective mass density proportional to ρ_c:

$\rho_m(x) = \alpha\, \rho_c(x)$,

where α is a conversion constant (units: kg m^{-3} per commitment-density unit). The Poisson equation becomes:

$$\boldsymbol{\nabla^2 \Phi = 4\pi G\alpha\, \rho_c} \, .$$

Combined with the Fisher bridge $\Phi = F(g_{\rho\rho}(\rho_c))$, this gives the empirical constraint:

$$\nabla^2 F(g_{\rho\rho}(\rho_c)) = 4\pi G\alpha\, \rho_c \, .$$

In the linearised regime, $F \approx F_0 + F_1\, \delta g_{\rho\rho}$, and potential gradients are tied to gradients in Fisher information — spatial variation in record distinguishability acts as a gravitational source.

VIII.6 Status and Open Problems for Route B

Route B in Sections VIII.1–VIII.5 demonstrates that BCB/TPB variables *can* reproduce Newtonian gravity with appropriate identifications. To promote this to a **derivation** — showing that gravity *must* take this form — two additional ingredients are needed:

(i) A micro-to-macro model specifying $p_x(\cdot|\rho_c)$, from which $g_{\rho\rho}(\rho_c)$ follows as a derived quantity rather than a free function.

(ii) A closure principle selecting the functional form F — candidates include least-description-length, maximum record entropy subject to boundary constraints, or a variational principle minimising information loss under coarse-graining.

Once (i) and (ii) are supplied, Route B would independently determine gravitational dynamics from record statistics alone, without invoking the Jacobson thermodynamic argument. The Newtonian limit then fixes α in terms of the micro-model. Section VIII.7 carries out this programme explicitly for the simplest plausible micro-model.

VIII.7 Worked Example: From Minimal Micro-Model to Newton's Law (Derived, Not Imposed)

With the macrostate basis C^μ and conformal selection $\sqrt{|g|} = \kappa_0 J(C)$ fixed, Route B reduces to a concrete pipeline: specify a local record micro-model $p(r|C)$, compute the Fisher metric $g_{ab}(C)$ and Jeffreys density $J(C) = \sqrt{\det g_{ab}(C)}$, use volume matching to select the Lorentzian metric $g_{\mu\nu}$ within the cone-determined conformal class, and then derive the static weak-field limit. This section carries out the calculation for the simplest plausible micro-model and shows that Newtonian gravity is recovered.

Step 1: Minimal local record micro-model $p(r|C)$ and Fisher metric scaling.

A local record outcome in a small spacetime cell must encode (i) how many commitments occur and (ii) how commitment activity is transported (directional bias / drift). The minimal exponential-family realisation is a Poisson counting process for the number of commitments plus a drift family for the spatial displacement of committed events. Two equivalent implementations are commonly useful:

Directional-channel model (Poisson + softmax drift). Six independent Poisson channels $n_k \sim \mathrm{Poisson}(\mu_k)$ for $k \in \{\pm x, \pm y, \pm z\}$ with $\mu_k = \lambda\, q_k(u)$, $\lambda = \rho_c\, \Delta V\, \Delta\tau$, and drift weights $q_k(u) = \exp(\beta\, b_k \cdot u/c) / Z(u)$, where $b_k \in \{\pm e_x, \pm e_y, \pm e_z\}$.

Displacement model (Poisson count + Gaussian displacement). A Poisson count $N \sim \mathrm{Poisson}(\lambda)$ with $\lambda = \rho_c\, \Delta V\, \Delta\tau$, and conditional on N, an i.i.d. displacement distribution for committed events with mean drift velocity $u^i = J^i_c / \rho_c$ and finite variance σ^2 per spatial direction.

Both are maximum-entropy (least-informative) distributions consistent with the axiom-derived macro-observables (ρ_c, J^i_c), and both lie in the same universality class: near equilibrium ($u \approx 0$) they produce the same Fisher scaling structure:

- the "rate" direction satisfies $g_{\rho\rho}(\rho_c) \propto 1/\rho_c$,
- the drift/flux block scales as $g_{ij}(\rho_c) \propto \rho_c\, \delta_{ij}$.

For clarity, in the near-equilibrium rest frame ($J^i_c \approx 0$) we use the compact representative Fisher metric:

$$g_{ab}(C) \approx \mathrm{diag}(A/\rho_c , \rho_c/\sigma^2 , \rho_c/\sigma^2 , \rho_c/\sigma^2) ,$$

where $A > 0$ absorbs cell-scale and unit conventions and σ is the (micro) dispersion scale of record transport.

Step 2: Jeffreys density and volume selection.

The Jeffreys density is:

$$J(C) = \sqrt{\det g_{ab}(C)} = \sqrt{ (A/\rho_c)(\rho_c/\sigma^2)^3 } = (\sqrt{A} / \sigma^3)\, \rho_c .$$

Thus, in the weak-drift regime, $J \propto \rho_c$. The conformal selection principle (Section VI.3) chooses the Lorentzian metric representative within the cone-determined conformal class by:

$$\sqrt{|g(x)|} = \kappa_0\, J(C(x)) .$$

Choose the background commitment density ρ_{c0} so that $\sqrt{|g|} = 1$ in the reference state; equivalently $\kappa_0\, J(\rho_{c0}) = 1$. Then the volume constraint becomes:

$$\boldsymbol{\sqrt{|g(x)|} = \rho_c(x) / \rho_{c0}} .$$

Step 3: Weak-field bridge to the Newtonian potential.

In standard weak field (with $\Phi \approx \Psi$):

$g_{00} = 1 + 2\Phi/c^2$, $g_{ij} = -(1 - 2\Phi/c^2)\,\delta_{ij}$,

which implies to first order:

$\sqrt{|g|} \approx 1 - 2\Phi/c^2$.

Equate with $\sqrt{|g|} = \rho_c / \rho_{c0}$ and linearise $\rho_c = \rho_{c0} + \delta\rho_c$:

$1 - 2\Phi/c^2 \approx 1 + \delta\rho_c/\rho_{c0} \Longrightarrow \boldsymbol{\Phi(x) \approx -(c^2 / 2\rho_{c0})\,\delta\rho_c(x)}$.

So the Newtonian potential is directly proportional to departures in commitment density.

Step 4: Static limit of the covariant action yields an elliptic operator (not imposed).

The covariant Route B action is defined in Section VIII.8. In the static weak-field regime, with small drift and dominant scalar degree ρ_c, the sigma-model term reduces at leading derivative order to a gradient-energy functional for ρ_c. Using the Fisher scaling $g_{\rho\rho} \propto 1/\rho_c$ and the volume constraint $\sqrt{|g|} \propto \rho_c$, the static functional becomes (up to an overall constant):

$S_{\text{info, static}}[\rho_c] \propto \int d^3x\, |\nabla\rho_c|^2$,

whose Euler–Lagrange equation is the Laplace-type operator:

$\nabla^2\rho_c = (\text{source})$.

This is the key point: the elliptic operator arises from the information action in the static limit, not by fiat.

Step 5: The source is fixed by record balance + Lorentz symmetry (Newtonian limit).

The record-current balance law is $\nabla_\mu C^\mu = s_c$. Locality and inertial symmetry require s_c to be a local Lorentz scalar. In the Newtonian regime (slow motion, dust-dominated), the natural operationally motivated scalar that reduces to mass density is the local rest-frame energy density:

$\varepsilon(x) \equiv T_{\mu\nu} u^\mu u^\nu \approx T_{00} \approx \rho_m c^2$,

so we take the minimal constitutive form:

$\boldsymbol{s_c(x) = \gamma_0\, \varepsilon(x)}$,

with γ_0 a universal energy→commitment conversion constant.

Scope note. There is one other Lorentz scalar linear in $T_{\mu\nu}$ that also reduces to $\rho_m c^2$ for pressureless dust: the trace $T = g^{\mu\nu} T_{\mu\nu}$ (since $T \approx \rho_m c^2 - 3p \approx \rho_m c^2$ when $p \approx 0$). The two candidates diverge for relativistic matter (e.g., radiation: $\varepsilon = \rho_{rad} c^2$ but $T = 0$). In the Newtonian regime treated here, both give the same result; the choice between ε, T, or a linear combination is precisely what the strong-field extension (O2) needs to resolve. We adopt ε as the default because it has a direct operational interpretation (energy density measured by a comoving record-counting observer), but the trace T has a significant structural advantage: it requires no auxiliary 4-velocity field u^μ, making it well-defined for all matter types including field-theory stress-energy tensors (e.g., electromagnetic $T_{\mu\nu}$) where no unique timelike eigenvector exists.

Combining Steps 4–5 gives the static sourced field equation:

$$\nabla^2\rho_c = -\gamma_0\, \rho_m c^2 .$$

Step 6: Poisson gravity follows.

Apply ∇^2 to the bridge $\Phi \approx -(c^2/2\rho_{c0})\, \delta\rho_c$:

$$\nabla^2\Phi \approx -(c^2/2\rho_{c0})\, \nabla^2\rho_c = -(c^2/2\rho_{c0})(-\gamma_0\, \rho_m c^2) = (\gamma_0\, c^4 / 2\rho_{c0})\, \rho_m .$$

Matching $\nabla^2\Phi = 4\pi G\, \rho_m$ yields:

$$G = \gamma_0\, c^4 / (8\pi\, \rho_{c0}) .$$

Thus Newton's law is recovered as the weak-field static limit of Route B, with G expressed in record parameters.

VIII.8 Covariant Constrained Action (Relativistic Extension)

Section VIII.7 derived the Newtonian limit using weak-field and static assumptions. We now define the fully covariant Route B action from which the Newtonian result follows by reduction.

Fields.

- Lorentzian metric $g_{\mu\nu}$ (conformal class fixed by the record-influence cone; volume element selected by the constraint below).
- Macrostate fields $\theta^a(x) \equiv C^\mu(x)$, $a = 0, 1, 2, 3$.
- Lagrange multipliers $\chi(x)$ (volume selection) and $\lambda(x)$ (record sourcing).

Matter enters via $T_{\mu\nu}(x)$ and a timelike unit field $u^\mu(x)$ representing the local rest frame (standard in relativistic matter; for dust it is the matter 4-velocity). The use of u^μ does not introduce additional structure: for any timelike matter flow, u^μ is the timelike eigenvector of $T^\mu{}_\nu$. One could alternatively use the trace $T = g^{\mu\nu} T_{\mu\nu}$ (avoiding u^μ entirely), but T vanishes for radiation — the choice between them is a strong-field question (see Step 5 scope note above and O2).

Action.

$S\_tot[\theta, g, \chi, \lambda; T, u] = \frac{1}{2} \int d^4x \sqrt{|g|}\, g^{\mu\nu} g\_ab(\theta) \nabla\_\mu\theta^a \nabla\_\nu\theta^b$

- $\int d^4x\, \chi(x) (\sqrt{|g|} - \kappa_0 \sqrt{(\det g\_ab(\theta))})$
- $\int d^4x \sqrt{|g|}\, \lambda(x) (\nabla\_\mu C^\mu - \gamma_0 T\_\alpha\beta u^\alpha u^\beta)$.

All covariant derivatives ∇_μ are defined with respect to the Lorentzian metric $g\_\mu\nu$. This metric is itself selected by the cone (fixing the conformal class) and by the χ-constraint (fixing the volume element). There is no circularity: S_tot is a functional of (θ, g, χ, λ), and variation with respect to each field yields its determining equation simultaneously.

Constraints from variation.

(1) Vary χ → volume selection:

$\delta S\_tot/\delta\chi = 0 \Longrightarrow \sqrt{|g|} = \kappa_0 \sqrt{(\det g\_ab(\theta))}$.

(2) Vary λ → sourced balance law:

$\delta S\_tot/\delta\lambda = 0 \Longrightarrow \nabla\_\mu C^\mu = \gamma_0 T\_\alpha\beta u^\alpha u^\beta$.

(3) Vary θ^a → macrostate dynamics:

Variation yields a sigma-model (harmonic-map) equation with source terms induced by the constraints. In the weak-field static regime, this reduces to a Laplace-type operator on ρ_c with the sourced right-hand side, reproducing Section VIII.7.

(4) Vary g_μν → effective stress-energy.

Two closures are available:

- **Route A / EFT closure:** Add an Einstein–Hilbert term $S\_EH = (1/16\pi G) \int \sqrt{|g|} R\, d^4x$ (motivated by the Jacobson derivation in Section VII). Then varying g_μν gives a standard Einstein equation with the record fields as matter content.
- **Induced-gravity closure:** Treat g_μν as having no independent degrees of freedom. The causal cone fixes the conformal class; the χ-constraint fixes the volume element. Together, these fully determine g_μν (up to boundary conditions) without adding gravitational degrees of freedom beyond those already present in C^μ and T_μν.

Route B Closure (Closed Model Statement). Fix a micro-model p(r|C) for local record outcomes. This uniquely determines the Fisher metric g_ab(C) and Jeffreys density $J(C) = \sqrt{(\det g\_ab(C))}$. The physical Lorentzian metric g_μν is selected within the cone-determined conformal class by the volume constraint $\sqrt{|g|} = \kappa_0 J(C)$. The record current satisfies the sourced balance law *$\nabla\mu C^\mu = \gamma_0 T\_\mu\nu u^\mu u^\nu$. Dynamics follow from the covariant constrained sigma-model action S_tot = S_info + S_vol + S_src. In the weak-field, slow-motion limit this yields $\nabla^2\Phi = 4\pi G\rho\_m$ with $G = \gamma_0 c^4/(8\pi\rho${c0}).*

Constant calibration. The remaining free constants (κ_0, γ_0, and micro-model parameters such as the drift sensitivity β) are fixed by one-time matching:

- **Newtonian limit** determines G (and hence the ratio γ_0/ρ_{c0}).
- **Redshift/time-dilation normalisation** fixes the background ρ_{c0}.
- **Horizon entropy** (optional) links to Route A's $\eta = 1/(4\ell^2_P)$, providing a consistency check between the two routes.

After calibration, Route B makes definite predictions for how geometry responds to changes in record statistics.

Strong-field note. The choice of source scalar $\varepsilon = T_{\mu\nu} u^\mu u^\nu$ is forced in the Newtonian/dust limit but is not unique for relativistic matter (e.g. radiation). Theorem VIII.3 shows that the Einstein equation emerges regardless of this choice at leading derivative order; the source scalar ambiguity affects only the detailed matter coupling. There is a structural argument favouring the trace $T = g^{\mu\nu} T_{\mu\nu}$ over ε for the covariant extension: (i) T requires no auxiliary 4-velocity field u^μ, making S_{src} well-defined for all matter content including field-theory stress-energy tensors where $T^\mu{}_\nu$ has no unique timelike eigenvector; (ii) T connects directly to the trace of the Einstein equation, $R = -\kappa_G T$ (in 4D with $\Lambda = 0$), suggesting that if Route B reproduces GR, the natural source scalar is the one already appearing in GR's trace. The cost is that T vanishes for pure radiation ($T = 0$ for a traceless stress-energy), implying that radiation would not directly source record commitment — an interesting physical prediction in its own right, if borne out.

VIII.9 Universality of the Newtonian Limit (Precise Conditions)

The worked example above used a particular minimal micro-model. Here we state the precise sense in which the Newtonian result is universal.

Lemma VIII.1 (Universality class for Fisher scaling near equilibrium). Consider a local record-counting model in a small cell with parameter ρ_c such that:

- **(U1) Rate parametrisation:** the expected number of commitments in the cell satisfies $E[N] = \rho_c \, \Delta V \, \Delta\tau$.
- **(U2) Finite-variance counting:** Var(N) exists and scales linearly with E[N] for small cells (Poisson-like independent-increment regime).
- **(U3) Regularity:** $p(N|\rho_c)$ is differentiable in ρ_c and identifiable.
- **(U4) Transport block:** any additional drift/flux parameters enter through a finite-variance displacement family whose Fisher block is proportional to ρ_c near equilibrium (as in exponential-family drift models / CLT regime).

Then, near equilibrium drift ($J^i_c \approx 0$), the Fisher information metric on the macrostate (ρ_c, drift) has the scaling form:

$g\_{\rho\rho}(\rho\_c) \sim A/\rho\_c$, $g\_{drift} \sim \rho\_c \times$ (constant matrix) ,

and therefore:

$\sqrt{(\det g\_{ab})} \propto \rho\_c$ to leading order .

Sketch of justification. Under conditions (U1)–(U3), the score $\partial\_{\rho\_c} \ln p$ scales like $(N - E[N])/\rho\_c$ in Poisson-like regimes, giving $g\_{\rho\rho} \sim \mathrm{Var}(N)/\rho^2\_c \sim E[N]/\rho^2\_c \sim 1/\rho\_c$. Drift parameters contribute through sums of i.i.d. increments whose Fisher information scales with event count, hence with ρ_c.

Corollary VIII.2 (Universal Newtonian limit under canonical volume matching). If (i) the micro-model lies in the universality class of Lemma VIII.1 and (ii) the canonical volume selection $\sqrt{|g|} = \kappa_0 \sqrt{(\det g\_{ab}(C))}$ is adopted, then in the weak-field near-equilibrium regime:

$\sqrt{|g|} \propto \rho\_c$, $\Phi \propto -\delta\rho\_c$,

and the static limit of the covariant action yields a Laplace-type operator $\nabla^2\rho\_c$ sourced by ρ_m via $s\_c = \gamma_0 \rho\_m c^2$. Hence:

$\nabla^2\Phi = 4\pi G \rho\_m$ with $G = \gamma_0 c^4 / (8\pi \rho\_{c0})$

independently of the detailed form of the micro-model within the universality class.

What is micro-model dependent. The constant A (which enters the information cost and the multiplier field, but not G itself), the drift sensitivity β, and the detailed form of g_ab away from equilibrium all depend on the specific choice of p(r|C). These affect the strong-field predictions and the detailed spatial profile of ρ_c, but not the Newtonian gravitational coupling.

Processes outside the universality class. Condition (U2) requires Var(N) to scale linearly with E[N] (Poisson dispersion). Counting processes with over-dispersion — e.g., negative binomial models where $\mathrm{Var}(N) \propto E[N]^\alpha$ with $\alpha > 1$ — violate (U2) and produce different Fisher scaling ($g\_{\rho\rho} \sim \rho\_c^{1-\alpha}$ rather than $1/\rho\_c$). Such models would modify the volume-matching relation and hence the Newtonian coupling. This is either a constraint on the physics (ruling out over-dispersed record commitment as inconsistent with observed Newtonian gravity) or, more conservatively, a prediction: if the substrate's counting statistics were over-dispersed, the gravitational coupling would deviate from $G = \gamma_0 c^4/(8\pi\rho\_{c0})$ in a calculable way. The Poisson dispersion condition (U2) is thus not merely a technical convenience but a physically meaningful requirement tied to the observed form of gravity.

VIII.10 Strong-Field Metric Equation (Structural Resolution of O2)

Section VIII.8 defined the covariant constrained action S_tot. Here we state and prove (structurally) the conditional theorem that the Einstein equation is the unique leading-order metric equation arising from this action.

Theorem VIII.3 (Route B Strong-Field Completion: Einstein Form as Unique Second-Order Limit).

Assume:

(H1) Cone-fixed conformal class. Axioms 1–5 determine a Lorentzian causal cone structure, hence a conformal class $[g]$ of Lorentzian metrics.

(H2) Canonical volume selection (record calibration). The representative metric $g_{\mu\nu} \in [g]$ is selected by the local volume constraint $\sqrt{|g|} = \kappa_0 J(C)$, where $J(C) = \sqrt{\det g_{ab}(C)}$ is the Jeffreys density of the Fisher metric on macrostate space. (This is the minimal reparametrisation-invariant, composable choice — Proposition 2.)

(H3) Covariant record sourcing (balance-law scalar). The record current satisfies the sourced balance law $\nabla_\mu C^\mu = s_c$, with $s_c = \gamma_0 \varepsilon$, $\varepsilon = T_{\mu\nu} u^\mu u^\nu$, where u^μ is the local matter rest frame. In the dust/Newtonian regime $\varepsilon \approx \rho_m c^2$.

(H4) Locality + finite-derivative truncation (IR completeness). At macroscopic scales $L \gg \ell_*$ (for some micro/record scale ℓ_*), *the effective dynamics obtained from the constrained sigma-model action $S_{tot} = S_{info} + S_{vol} + S_{src}$ admits a leading truncation to second derivatives of $g_{\mu\nu}$, with higher-derivative corrections suppressed by powers of ℓ_*/L.*

(H5) No additional propagating gravitational degrees of freedom. The constraints (H2)–(H3) render the record fields C^μ auxiliary at the IR level: after imposing the constraints and solving the C^μ Euler–Lagrange equations, the remaining independent geometric content is entirely in $g_{\mu\nu}$ (within the cone-fixed conformal class, with volume fixed by $J(C)$).

Then, in the strong-field IR regime $L \gg \ell_*$, the metric field equation obtained by varying S_{tot} and eliminating C^μ must take the form:

$$G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = \kappa_{eff}\, T_{\mu\nu} + \ell_*^2\, H_{\mu\nu}[g, T] ,$$

where $G_{\mu\nu}$ is the Einstein tensor; Λ_{eff} is an effective cosmological term (induced by the volume constraint sector); κ_{eff} matches the Newtonian limit, hence $\kappa_{eff} = 8\pi G/c^4$ once G is fixed by the weak-field derivation (Section VIII.7); and $H_{\mu\nu}$ is a covariant, divergence-controlled correction built from higher-curvature invariants, suppressed by $(\ell_*/L)^2$.

In particular, if the higher-derivative corrections are negligible in the regime of interest, the strong-field equation reduces exactly to:

$$\mathbf{G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = \kappa_{eff}\, T_{\mu\nu}} .$$

Proof sketch.

(1) Diffeomorphism covariance and locality constrain the allowed metric equation. Because S_{tot} is built from covariant scalars (volume density, covariant derivatives) and is local at

macroscopic scales (H4), the resulting metric equation is a local, covariant symmetric 2-tensor equation.

(2) Second-derivative leading order is uniquely Einstein in 4D. Under the second-derivative truncation (H4), the only divergence-free symmetric rank-2 tensors built from the metric and up to second derivatives are $G_{\mu\nu}$ and $g_{\mu\nu}$ (Lovelock's uniqueness theorem in 4D). Therefore the leading IR field equation must be of Einstein + Λ form with some effective coupling.

(3) Matter coupling normalisation is fixed by the Newtonian limit. The weak-field Route B derivation (Section VIII.7) yields $\nabla^2\Phi = 4\pi G\rho_m$ with $G = \gamma_0 c^4/(8\pi\rho_{c0})$. Matching this fixes $\kappa_{eff} = 8\pi G/c^4$ in the IR equation.

(4) All remaining model dependence appears as suppressed higher-derivative corrections. Nonlinearities from the Fisher sector $g_{ab}(C)$, elimination of C^μ, and any non-Poisson microstructure contribute higher-gradient terms in the effective metric dynamics. Under (H4), these enter as $H_{\mu\nu}$ suppressed by ℓ^2_*/L^2, completing the proof of the claimed form.

Explicit variation (supporting detail). The three components of $\delta S_{tot}/\delta g^{\mu\nu}$ are:

(A) From S_{info}: $T^{(info)}_{\mu\nu} = g_{ab}(C)(\nabla_\mu C^a \nabla_\nu C^b - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha C^a \nabla_\beta C^b)$. Standard sigma-model stress-energy, second-order in derivatives of C.

(B) From S_{vol}: $\delta S_{vol}/\delta g^{\mu\nu} = -\frac{1}{2} \chi \sqrt{|g|}\, g_{\mu\nu}$. Contributes a term $\propto g_{\mu\nu}$ (cosmological-constant generator).

(C) From S_{src}: $T^{(src)}_{\mu\nu} = \lambda(\nabla_\mu C_\nu + \nabla_\nu C_\mu - g_{\mu\nu} \nabla_\alpha C^\alpha) - \gamma_0 \lambda(T_{\alpha\beta} u^\alpha u^\beta)$ $g_{\mu\nu}$. Arises from variation of $\sqrt{|g|}$ and the connection $\Gamma^\mu_{\alpha\mu}$ in $\nabla_\mu C^\mu$. Remains second-order or lower.

The raw metric equation $T^{(info)}_{\mu\nu} + T^{(src)}_{\mu\nu} + \frac{1}{2}\chi\, g_{\mu\nu} = 0$ holds with all fields present. The constraints eliminate χ, λ; volume matching renders $\rho_c \propto \sqrt{|g|}$ (H5). The following lemma bridges the substitution:

Lemma VIII.4 (Auxiliary-field elimination implies curvature basis). After imposing the volume constraint $\sqrt{|g|} = \kappa_0 J(C)$ and solving the C^μ Euler–Lagrange equation, the constrained solution $C^\mu = C^\mu(g, T)$ depends on $g_{\mu\nu}$ and $T_{\mu\nu}$ only. Any covariant symmetric rank-2 tensor built at leading (second) derivative order from $\nabla_\mu C^a(g, T)$ $\nabla\nu$ *$C^b(g, T)$ therefore reduces to the curvature basis $\{R_{\mu\nu}, R\, g_{\mu\nu}, g_{\mu\nu}\}$ plus terms involving $T_{\mu\nu}$ and its derivatives — plus higher-derivative corrections suppressed by* $(\ell^*/L)^2$. This is because: (i) $C^\mu(g, T)$ inherits its derivative structure from g (through the determinant and connection); (ii) at second derivative order in g, the only available symmetric rank-2 tensors are $R_{\mu\nu}$ and $R\, g_{\mu\nu}$ (by the classification of local curvature invariants); (iii) matter enters algebraically through the sourcing constraint.

Applying Lemma VIII.4, the substitution $C^\mu(g, T)$ converts $\nabla C \cdot \nabla C \rightarrow$ curvature terms, yielding the Einstein structure by step (2).

Verification of (H4) for the canonical micro-model. A potential concern is that the elimination $C^\mu \to C^\mu(g, T)$ could generate higher derivatives of $g_{\mu\nu}$ that are parametrically large rather than suppressed. For the Poisson-drift micro-model of Section VIII.7, this concern can be checked explicitly. The volume constraint gives $\rho_c \propto \sqrt{|g|}$, which depends on the metric determinant (zeroth derivative of g in the sense of the connection). The sourcing constraint $\nabla_\mu C^\mu = \gamma_0 \varepsilon$ involves Christoffel symbols $\Gamma^\mu_{\alpha\mu} = \partial_\alpha \ln\sqrt{|g|}$, which are first derivatives of g. The sigma-model stress tensor $T^{(info)}_{\mu\nu}$ involves products $\nabla_\mu C^a \nabla_\nu C^b$, *which — after substitution of $C^\mu(g)$ — are at most quadratic in first derivatives of g. This is precisely the derivative order of the Einstein–Hilbert action (the Ricci scalar R is also built from products of first derivatives of g plus second derivatives that enter linearly). No higher derivatives are generated because: (a) ρ_c depends algebraically on det(g), not on derivatives of g; (b) J^i_c is determined by the sourcing constraint, which involves at most first derivatives of g through the connection; (c) the Fisher metric $g_{ab}(C)$ depends on C algebraically (not on its derivatives). Therefore, in the canonical Poisson-drift near-equilibrium regime, the elimination $C^\mu \to C^\mu(g, T)$ introduces no derivatives beyond those already present in the Einstein–Hilbert structure at leading order. Higher-derivative terms arise only from (i) corrections to the near-equilibrium Fisher metric and (ii) genuinely nonlocal elimination effects at the micro scale, and are suppressed by* $(\ell^*/L)^2$.

Proposition VIII.5 (Coefficient-level strong-field completion). In the canonical Poisson-drift universality class, the explicit coefficients of the leading-order metric equation are determined as follows.

(i) Auxiliary-field elimination. The near-equilibrium Fisher metric takes the diagonal form $g_{ab}(C) \approx \mathrm{diag}(A/\rho_c, \rho_c/\sigma^2, \rho_c/\sigma^2, \rho_c/\sigma^2)$. Here the macro-parameters are taken as $\theta^a = (\rho_c, u^i)$ with $u^i \equiv J^i_c/\rho_c$ (drift velocity); in this coordinate choice the Fisher matrix is diagonal and the Jeffreys density remains proportional to ρ_c. Explicitly: $\det g_{ab} = (A/\rho_c)(\rho_c/\sigma^2)^3 = A \rho^2_c/\sigma^6$, and hence $J(C) = \sqrt{\det g_{ab}} = (\sqrt{A}/\sigma^3)\, \rho_c \equiv \alpha_J \rho_c$ — linear in ρ_c. The volume constraint then yields:

$$\sqrt{|g|} = \kappa_0\, \alpha_J\, \rho_c = \rho_c / \rho_{c0} \implies \boldsymbol{\rho_c = \rho_{c0} \sqrt{|g|}} ,$$

where $\rho_{c0} \equiv 1/(\kappa_0\, \alpha_J)$.

The commitment density is algebraically fixed by the metric determinant. The record current decomposes covariantly as $C^\mu = \rho_c v^\mu$ with $v^\mu v_\mu = 1$ (a density times a unit timelike flow direction, as in relativistic fluid theory). In the matter rest frame, $v^\mu \approx u^\mu$ at leading order. The balance equation becomes $\rho_{c0} \nabla_\mu(\sqrt{|g|}\, v^\mu) = \gamma_0 \varepsilon$ — a first-order constraint on v^μ given g and ε, introducing no higher derivatives of g.

(ii) Coupling constant. The Newtonian limit (Section VIII.7) gives $G = \gamma_0 c^4/(8\pi\rho_{c0})$, hence:

$$\boldsymbol{\kappa_{eff} = 8\pi G/c^4 = \gamma_0/\rho_{c0}} .$$

This is a clean record-theoretic identification: the gravitational coupling is the ratio of the sourcing strength γ_0 to the background commitment density ρ_{c0}. As in any emergent or effective theory of gravity, κ_eff is fixed by matching the weak-field limit to the Poisson equation; the record-theoretic novelty is that the matched constant is expressed directly in terms of substrate parameters (γ_0, $\rho\_\{c0\}$).

(iii) Cosmological term. The metric variation of the volume constraint contributes a term proportional to $g\_\mu\nu$ with coefficient χ. Decomposing χ into a constant mode plus fluctuations, the constant mode defines:

$$\boldsymbol{\Lambda\_eff \equiv -\chi_0/2}\ .$$

Fluctuations $\delta\chi(x)$ correspond to an effective spacetime-dependent vacuum term and are part of the cosmological-constant program (O5).

(iv) Leading-order metric equation. Combining:

$$\boldsymbol{G\_\mu\nu + \Lambda\_eff\ g\_\mu\nu = (\gamma_0/\rho\_\{c0\})\ T\_\mu\nu + O(\ell^2\_*)}\ ,$$

or equivalently, $G\_\mu\nu + \Lambda\_eff\ g\_\mu\nu = (8\pi G/c^4)\ T\_\mu\nu + O(\ell^2\_*)$. This is Einstein's equation with the coupling expressed entirely in terms of record-substrate parameters.

(v) What remains at next order. The leading correction to the effective action takes the form $S\_eff = \int \sqrt{|g|}\ (R/2\kappa\_eff - \Lambda\_eff + \ell^2\_*\ (a\ R^2 + b\ R\_\mu\nu\ R\^\mu\nu + \cdots))$, where the coefficients a, b, … depend on the higher-order structure of the Fisher metric away from equilibrium and on the relaxation dynamics of $v\^\mu$. These are calculable in principle for any specified micro-model.

Remarks.

(i) Conditional structure — verified for the canonical model. This is a conditional strong-field closure theorem: structural commitments (H1–H5) plus clearly stated regularity conditions yield the Einstein form. For the Poisson-drift near-equilibrium regime, (H4) is explicitly verified at leading order (see above), so the theorem applies without additional assumptions in that case. For more exotic micro-models or far-from-equilibrium configurations, (H4) remains a regularity assumption — but any violation would itself be a prediction of the framework, generating observable higher-derivative corrections.

(ii) What remains computational. With the leading coefficients now identified (Proposition VIII.5), the remaining strong-field work is: compute or bound the next-order correction coefficients (a, b, …) in $H\_\mu\nu$, determine the correct scalar sourcing for relativistic matter (ε vs T vs a combination), and classify whether the leading $H\_\mu\nu$ is of Gauss–Bonnet type or includes other invariants.

(iii) Λ_eff. The cosmological term arises naturally from the volume constraint sector (S_vol). Whether its value is determined by the framework or remains an integration constant is an open question (see O5).

VIII.11 Gravitational Wave Sector and Radiative Degrees of Freedom

Theorem VIII.3 establishes the Einstein form at leading derivative order. Here we examine the radiative sector to determine the propagating degrees of freedom and connect to precision gravitational-wave observations.

Linearisation. In the vacuum sector with Λ_{eff} and ℓ^2_* corrections neglected, write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$. In harmonic gauge ($\partial^\mu \bar{h}_{\mu\nu} = 0$, with $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h$), the linearised Einstein equation gives:

$$\Box \bar{h}_{\mu\nu} = 0 \,.$$

Gravitational perturbations propagate at speed c — the invariant record-influence speed derived from Axioms 3–5. This is not an external constraint the framework happens to satisfy: $c_{GW} = c$ is built into the axioms at the deepest level, since the causal cone sets the maximum propagation speed for all record-relevant influence, including metric perturbations. This distinguishes the framework from scalar-tensor alternatives (Horndeski, beyond-Horndeski) where $c_{GW} \neq c$ generically arises from non-minimal coupling and had to be tuned or discarded after the GW170817/GRB 170817A speed-of-gravity measurement.

Two polarisations. Of the ten components of $h_{\mu\nu}$, four are removed by harmonic gauge, four by residual gauge freedom, and one by the trace condition $\bar{h} = 0$. The remaining two independent components are the transverse-traceless (TT) modes h_+ and $h_\times$: exactly the two tensor polarisations predicted by GR. No additional scalar or vector gravitational modes propagate at leading order — this is a direct consequence of (H5), which renders the record fields C^μ auxiliary in the infrared.

Higher-derivative corrections. The $\ell^2_* H_{\mu\nu}$ correction generically introduces curvature-squared terms (R^2, $R_{\mu\nu} R^{\mu\nu}$), which can source additional propagating modes: an extra scalar (from R^2) and a massive spin-2 mode (from $R_{\mu\nu} R^{\mu\nu}$). Under (H4), these are suppressed by $(\ell\_/L)^2$*, where L is the gravitational wavelength. Since observed gravitational waves satisfy* $L \gg \ell\_$, these corrections are observationally negligible. If ℓ_* is of order the Planck length, deviations occur only near Planckian curvature — consistent with existing LIGO/Virgo/KAGRA bounds.

Observational constraints as structural consistency checks. Current multi-messenger observations constrain: (i) gravitational wave speed equals c to high precision (GW170817/GRB 170817A); (ii) only two independent tensor polarisations are detected within sensitivity. These are precisely the conditions encoded in (H4)–(H5). The framework does not merely accommodate these results — they follow from the same hypotheses that produce the Einstein equation.

Record-theoretic interpretation. In the BCB picture, gravitational waves are propagating perturbations in the commitment-density–metric system. Because the conformal factor is constrained by volume matching (Proposition 2) and C^μ is auxiliary in the infrared (H5), no independent scalar conformal mode propagates. Radiative gravitational degrees of freedom arise solely from tensorial deformation of the causal structure — not from independent record-density fluctuations. Physically: a passing gravitational wave modulates the local distinguishability geometry (the Fisher metric pullback) in a transverse-traceless pattern, stretching and compressing the information cost of record commitment in perpendicular directions — which is what interferometers detect as differential arm-length changes.

VIII.12 Black Hole Ringdown and Quasinormal Modes

Section VIII.11 tests the weak-field radiative sector. Black hole ringdown probes the strong-field vacuum: the quasinormal mode (QNM) spectrum of a remnant Kerr black hole is a precision test of the metric equation in high-curvature vacuum.

GR baseline. In GR, linear perturbations of Kerr satisfy a separable wave equation with quasinormal frequencies $\omega_{\ell mn}(M_f, a_f)$ determined entirely by remnant mass and spin. The spectrum contains two tensor polarisations and no additional propagating modes. Ringdown observations therefore test: (i) the existence of extra modes; (ii) deviations in the frequency spectrum; (iii) late-time tail behaviour.

Vacuum sector of Route B. In the vacuum exterior ($T_{\mu\nu} = 0$), the sourcing constraint reduces to $\nabla_\mu C^\mu = 0$, *and under (H1)–(H5) the metric equation becomes* $G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = \ell^2_* H_{\mu\nu}$. At leading IR order (neglecting $H_{\mu\nu}$), the stationary vacuum solution is Kerr (or Kerr–de Sitter), and the linearised perturbation equations coincide with GR. Therefore:

At leading order in the infrared truncation, Route B predicts the standard Kerr quasinormal mode spectrum with no additional radiative degrees of freedom.

Location of possible deviations. All corrections must arise from the $\ell^2_* H_{\mu\nu}$ sector. Two qualitatively distinct possibilities exist:

(A) Small frequency shifts (no new modes). If $H_{\mu\nu}$ contains only higher-curvature scalars (R^2, $R_{\mu\nu} R^{\mu\nu}$ combinations), the QNM frequencies receive corrections $\omega_{\ell mn} = \omega^{Kerr}_{\ell mn} [1 + \delta_{\ell mn}]$ with $\delta_{\ell mn} \sim O(\ell^2_*/r^2_+)$, where r_+ is the horizon radius. These are parametrically suppressed strong-curvature corrections.

(B) Extra propagating modes. If the correction sector introduces additional dynamical fields (e.g., a scalar from generic R^2 gravity), new damped oscillatory families appear in the ringdown signal. Under (H5), such extra modes are excluded at leading order. Any observed additional ringdown structure would signal breakdown of the IR truncation or the presence of new record-sector degrees of freedom beyond those retained — either of which would be a discovery.

Observational status. Current ringdown analyses constrain large deviations from Kerr spectra; no statistically robust evidence for non-Kerr QNM structure has been established. Consistency requires $\ell_* \ll r_+$ for astrophysical black holes, which is satisfied if ℓ_* is the fundamental record or substrate scale (expected to be of order ℓ_P).

Structural conclusion. Route B survives strong-field vacuum tests: under (H1)–(H5), black hole ringdown is indistinguishable from Kerr at leading order, with deviations parametrically controlled by $(\ell_*/r_+)^2$. The framework is not merely matching Newton's law — it matches gravitational spectroscopy.

IX. Tensor Derivation: Connection, Curvature, and the Sigma-Model Action

This section provides the explicit tensor-level steps connecting the information-geometric construction to standard differential geometry.

IX.1 Information Pullback Metric (Restated)

Given macrostate fields $\theta^a(x)$ with $a \in \{1, \ldots, n\}$ $(n \geq 4)$ — identified with the record-commitment 4-current C^μ in Section VI.1 — the **information pullback** to spacetime is:

$$\hat{g}_{\mu\nu}(x) = g_{ab}(\theta(x))\, \partial_\mu\theta^a(x)\, \partial_\nu\theta^b(x) .$$

As noted in Section VI.2, $\hat{g}_{\mu\nu}$ is positive semi-definite (Riemannian). It is not the physical spacetime metric but supplies the volume element that, combined with the causal cone's conformal class, determines the physical Lorentzian metric $g_{\mu\nu}$ via Proposition 2. For the remainder of this section, standard tensor operations (connection, curvature, sigma-model action) are presented in terms of the physical metric $g_{\mu\nu}$.

IX.2 Levi-Civita Connection

The unique torsion-free, metric-compatible connection is:

$$\Gamma^\rho_{\mu\nu} = \tfrac{1}{2} g^{\rho\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}) .$$

IX.3 Riemann Curvature Tensor

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma} .$$

Contraction yields the Ricci tensor $R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$ and Ricci scalar $R = g^{\mu\nu} R_{\mu\nu}$.

Curvature is non-zero whenever the macrostate gradients are non-uniform — physically, when the record statistics vary from point to point in a way that cannot be removed by a coordinate transformation.

IX.4 Sigma-Model Action and Macrostate Dynamics

The dynamics of the macrostate fields $\theta^a(x)$ on a given spacetime background are governed by the nonlinear sigma-model action:

$$S[\theta; g] = \tfrac{1}{2} \int d^4x \sqrt{|g|}\, g^{\mu\nu} g_{ab}(\theta)\, \partial_\mu\theta^a \partial_\nu\theta^b .$$

This is the unique two-derivative action for maps $\theta: (M, g) \rightarrow (\mathscr{M}, g_{ab})$ that is (a) diffeomorphism-invariant on M, (b) reparametrisation-invariant on $\mathscr{M}$, and (c) at most second-order in derivatives of θ. Within the record framework, these three properties follow from: (a) the emergent coordinate freedom of spacetime, (b) the Čencov invariance of the Fisher metric, and (c) the restriction to leading-order gradient dynamics in the macroscopic limit.

Variation with respect to θ^a yields the **harmonic map equation**:

$$\Box\theta^a + \Gamma^a{}_{bc}(\theta)\, g^{\mu\nu} \partial_\mu\theta^b \partial_\nu\theta^c = 0 ,$$

where $\Box = g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the spacetime d'Alembertian and $\Gamma^a{}_{bc}$ are the Christoffel symbols of the Fisher metric on target space.

IX.5 GR Closure Options

Option 1 (Effective Field Theory). Treat $g_{\mu\nu}$ as an independent dynamical field and add the Einstein–Hilbert action:

$$S_{total} = \int d^4x \sqrt{|g|}\, [\, (1/2\kappa_G)(R - 2\Lambda) + \tfrac{1}{2} g^{\mu\nu} g_{ab} \partial_\mu\theta^a \partial_\nu\theta^b \,] .$$

Variation with respect to $g^{\mu\nu}$ yields Einstein's equation with the sigma-model stress-energy tensor as source:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa_G T_{\mu\nu} .$$

Option 2 (Induced Gravity). Take the metric to be entirely determined by the macrostate fields via $g_{\mu\nu} = g_{ab} \partial_\mu\theta^a \partial_\nu\theta^b$, with no independent metric degrees of freedom. The Einstein–Hilbert term is then a derived functional of θ^a. This is the more ambitious approach, requiring the full closure of Route B (Section VIII.6).

X. TPB Integration

X.1 Definitions

The **Ticks-Per-Bit (TPB)** ratio measures the computational cost — in substrate ticks — of committing one irreversible distinction (one bit of record). Define:

- dN_b: number of bits committed in proper-time interval $d\tau$,
- dN_t: number of substrate ticks elapsed in coordinate-time interval dt.

By construction, $dN\_b \propto d\tau$ (record commitment is a proper-time process, as records are local and frame-independent by Axiom 1). The relation $dN\_t \propto dt$ is an **identification**, not a derivation: it defines coordinate time as the macroscopic proxy for substrate tick count in the laboratory frame (see Section III.1 for the mapping $t \approx n\ \Delta t\_tick$). This identification is what allows TPB to inherit the $dt/d\tau$ ratio; without it, TPB would require an independent operational definition of "tick count."

Therefore:

$$TPB = dN\_t / dN\_b \propto dt/d\tau = (d\tau/dt)^{-1} .$$

X.2 Velocity Dependence

From Section IV.3:

$$d\tau/dt = \sqrt{(1 - v^2/c^2)} .$$

Therefore:

$$\mathbf{TPB(v) = TPB_0 / \sqrt{(1 - v^2/c^2)} = TPB_0\ \gamma} .$$

Moving systems require more substrate ticks per committed record.

X.3 Gravitational Dependence

In the weak-field regime with both velocity and gravitational potential:

$$d\tau/dt = \sqrt{(1 + 2\Phi/c^2 - v^2/c^2)} .$$

Therefore:

$$\mathbf{TPB(\Phi, v) = TPB_0\ (1 + 2\Phi/c^2 - v^2/c^2)^{-1/2}} .$$

In a gravitational well ($\Phi < 0$), TPB increases: fewer records are committed per substrate tick, consistent with gravitational time dilation. Near a black hole horizon ($1 + 2\Phi/c^2 \to 0$), TPB diverges — record commitment freezes relative to the substrate, recovering the standard prediction of infinite redshift at the horizon.

X.4 General Relativistic TPB

For an arbitrary spacetime metric $g\_\mu\nu$ and a worldline with four-velocity $u\^\mu$:

$$d\tau^2 = g\_\mu\nu\ dx\^\mu\ dx\^\nu ,$$

and for a static observer ($u^i = 0$) in a stationary metric:

$TPB = TPB_0 / \sqrt{g_{00}}$.

X.5 Interpretation versus Prediction

An important distinction: the TPB scaling derived above **reinterprets** known relativistic and gravitational time-dilation effects in information-theoretic language. It does not, by itself, predict any deviation from standard GR. TPB becomes a source of new predictions only in regimes where the discrete substrate structure modifies the smooth proper-time formula — i.e., at scales approaching Δt_{tick} (see Section XI). In the continuum limit, TPB scaling is exactly equivalent to standard time dilation and carries no independent empirical content beyond the interpretive.

X.6 Speed in the Record-Theoretic Picture: Why Moving Costs More

The TPB formula $TPB(v) = TPB_0 \gamma$ states that a moving system requires more substrate ticks per committed record. What is the physical mechanism in the record-theoretic framework?

The causal-cone constraint. Axioms 1–5 establish that record-relevant influence propagates at most at speed c (Section III). A system moving at speed v has its worldline tilted relative to the substrate's rest frame: for each substrate tick (interval dt), the system traverses a spatial distance $v \cdot dt$. The causal cone geometry forces a trade-off between spatial traversal and temporal processing — the faster a system moves through space, the less substrate capacity is available for local record commitment per unit coordinate time.

Proper time as commitment time. Record commitment is a local, frame-independent process (Axiom 1): the number of bits committed along a worldline segment depends only on the proper time elapsed along that segment, not on the coordinate time. For a moving system, proper time runs slower: $d\tau = dt/\gamma$. The substrate, however, continues to tick at its own rate ($dN_t \propto dt$). The result is that γ substrate ticks elapse for every one proper-time tick of the moving system — each committed bit "costs" γ times as many substrate updates.

Approaching the speed of light. As $v \to c$, the Lorentz factor $\gamma \to \infty$ and TPB diverges. In the record-theoretic picture, this means the substrate cannot commit any records along a null worldline — all of its processing capacity is consumed by spatial propagation, with nothing left for local commitment. This is why massless particles do not have proper time: they propagate record-relevant influence at the causal limit but do not themselves commit records. The speed of light is the boundary between worldlines that can support record commitment (timelike, TPB finite) and those that cannot (null, $TPB \to \infty$).

The BCB balance law. In terms of the commitment 4-current $C^\mu = (\rho_c, J_c)$, a Lorentz boost mixes ρ_c and J_c: what appears as pure commitment density in one frame appears partly as commitment flux in another. The total commitment count along any worldline segment is Lorentz-invariant (it equals the number of committed bits, an integer), but its decomposition into "how many" (ρ_c) and "which direction" (J_c) is frame-dependent — exactly as energy and momentum are frame-dependent projections of the 4-momentum. The TPB increase for moving systems is the information-theoretic counterpart of relativistic energy increase: more substrate resources are required to maintain the same rate of local record commitment when the system is also traversing space.

X.7 Gravity in the Record-Theoretic Picture: Why Depth Costs More

The TPB formula $TPB(\Phi) = TPB_0 (1 + 2\Phi/c^2)^{-1/2}$ states that systems deeper in a gravitational well require more substrate ticks per committed record. Route B provides a concrete mechanism for this.

Commitment density and the gravitational potential. The weak-field bridge (Section VIII.7, Step 3) gives $\Phi(x) \approx -(c^2/2\rho_{c0}) \delta\rho_c(x)$. Since $\Phi < 0$ in a gravitational well, $\delta\rho_c > 0$: the commitment density near mass is elevated above its background value. More records are being committed per unit volume in the vicinity of massive objects. The gravitational potential is, in this framework, a direct measure of how the local information environment departs from the background.

Why elevated commitment density slows local clocks. The volume-matching principle (Proposition 2) ties the metric determinant to the commitment density: $\sqrt{|g|} \propto \rho_c$. Where ρ_c is higher, the spacetime volume element is larger — proper volumes and proper times are "stretched" relative to coordinate measures. For a static observer in a gravitational well, this means $g_{00} < 1$, so the proper time interval $d\tau = \sqrt{g_{00}}\, dt$ is shorter than the coordinate interval dt. The substrate ticks at its own rate ($dN_t \propto dt$), but the local record-commitment clock runs slower relative to it. The physical picture: the substrate near mass is already processing a higher density of commitments. Each new bit is committed into a richer, more densely populated information environment, and this congestion increases the substrate cost per record.

The sourcing mechanism. The balance law $\nabla_\mu C^\mu = \gamma_0 \varepsilon$ (Section VIII.7, Step 5) explains why commitment density is elevated near mass in the first place. Matter, through its rest-frame energy density $\varepsilon = T_{\mu\nu} u^\mu u^\nu$, acts as a source of record commitment. Mass-energy doesn't just exist in spacetime — it actively drives the creation of irreversible records. The resulting gradient in ρ_c, mediated by the information action (S_{info}), produces the Newtonian potential. Gravity, in the BCB picture, is the response of the commitment-density field to the presence of matter-sourced records.

Horizon behaviour. Near a black hole horizon ($g_{00} \to 0$), TPB diverges for static observers: the substrate cost per record becomes infinite. In the record-theoretic interpretation, this means a static observer at the horizon would require infinite substrate resources to commit a single bit — record commitment freezes from the perspective of a distant coordinate-time observer. A freely falling observer, by contrast, continues to commit records normally (their local TPB remains TPB_0), because proper time is smooth across the horizon. The TPB divergence is a coordinate effect, not a physical singularity — but it reflects a genuine feature of the information geometry: the mapping between substrate ticks and local record commitment becomes singular at the horizon boundary.

Gravity as information bookkeeping. Combining these elements, the record-theoretic picture of gravity is: mass-energy sources commitment density (balance law) → elevated ρ_c modifies the spacetime volume element (volume matching) → the modified metric produces curvature (Route B / Theorem VIII.3) → freely falling worldlines are those that maximise proper time between events, equivalently minimising the cumulative substrate cost $\int$ TPB dt along the worldline (since TPB $\propto$ $d t/d\tau$, minimising $\int$ TPB dt is equivalent to maximising $\int d\tau$). Newton's gravitational force is, at bottom, the substrate's response to spatial gradients in the information cost of record commitment.

XI. Candidate Experimental Signatures (Conditional on Substrate Realisation)

The axioms fix causal structure and constrain admissible macroscopic closures, but they do not uniquely determine the ultraviolet (substrate) completion. The signatures listed below are therefore conditional: they arise in broad classes of discrete or finite-capacity substrate realisations compatible with Axioms 1–5. Their purpose is to provide falsifiable targets once a specific substrate model (and hence numerical coefficients) is selected.

XI.1 Discrete Corrections to Time Dilation

Axiom 3 implies a finite tick rate and hence a minimum proper-time interval $\Delta\tau_{min} \sim \ell_P/c$. In discrete-substrate models, this modifies the smooth proper-time formula at leading order:

$$d\tau/dt = \sqrt{(1 - v^2/c^2)}\ [1 + \alpha_1 (\ell_P/L)^2 + \ldots]\ ,$$

where L is a characteristic scale of the measurement and α_1 is a dimensionless coefficient determined by the specific substrate geometry. Precision tests of time dilation (e.g., via optical lattice clocks at different heights, or relativistic ion traps) could in principle bound α_1.

XI.2 Modified Dispersion Relations

Finite distinguishability (Axiom 3) and discrete lattice structure (Axiom 4) generically produce modified dispersion relations at high energy in models where the substrate does not perfectly restore Lorentz invariance in the continuum limit:

$$E^2 = p^2c^2 + m^2c^4\ [1 + \beta (E/E_P)^n + \ldots]\ ,$$

where E_P is the Planck energy and $n = 1$ or 2 depending on whether the substrate preserves or breaks Lorentz invariance at the lattice scale. Gamma-ray burst timing data and ultra-high-energy cosmic ray observations already constrain $n = 1$ modifications and are beginning to probe $n = 2$.

XI.3 Anomalous Decoherence from Record Commitment

In substrate realisations where record commitment is an objective, irreversibility-inducing process, an additional decoherence channel proportional to commitment rate may appear:

$\Gamma_{decoherence} \propto \rho_c$,

independent of environmental coupling. Experimental searches for anomalous decoherence in well-isolated systems (e.g., matter-wave interferometry with increasingly massive particles) that cannot be attributed to known environmental interactions could test — or constrain — this class of models.

XI.4 Record Entropy–Area Scaling

Assumption A2 ($S_{rec} = \eta A$) predicts that the information capacity of any causal region scales with boundary area, not volume. While this is consistent with black hole thermodynamics, it could be tested in analogue gravity systems or in the scaling of entanglement entropy in quantum field theory on lattices with record-like irreversibility built in.

XII. Relation to Existing Programmes

The framework developed here intersects with, but is distinct from, several research traditions.

Jacobson (1995): Route A follows Jacobson's derivation closely. The new contribution is the identification of the entropy with record capacity and the embedding of the thermodynamic derivation within a broader axiomatic structure that independently derives the kinematic (Lorentz) sector.

Verlinde (2011): Entropic gravity proposals share the intuition that gravity arises from information-theoretic considerations. The present framework is more conservative in that it derives the causal structure (Lorentz invariance) independently before invoking thermodynamics, and more specific in that it identifies the microscopic degrees of freedom (records and commitments) rather than positing generic holographic screens.

Padmanabhan (2010): The interpretation of Einstein's equation as an equation of state is closely related to Padmanabhan's thermodynamic perspective on gravity. The record-theoretic framework adds a specific substrate ontology and derives the thermodynamic assumptions from operational axioms.

Wheeler's "It from Bit": The present work can be viewed as a concrete realisation of Wheeler's programme, with "bit" identified as an irreversible record distinction and "it" (spacetime geometry) derived from the statistical geometry of record distributions.

XIII. Status of Results

For reader and referee clarity, we classify each major result by its logical status within the framework.

Theorem-level (from Axioms 1–5):

- Theorem 1: Discrete bounded domain of dependence.
- Theorem 2: Continuum causal cone and invariant maximal record-influence speed c.
- Theorem 3: Lorentz transformations from cone preservation + inertial symmetry.
- Lorentz group O(1,3) as the kinematic symmetry group.
- TPB scaling TPB $\propto dt/d\tau$.

These results require no gravitational closure assumptions.

Standard results invoked: Fisher metric from KL expansion. Čencov uniqueness of the Fisher metric under sufficient-statistics invariance. Standard tensor calculus and variational principles.

Lemma-level (from Axioms 1–5): Existence of commitment density ρ_c (from Axioms 1 and 3). Existence of commitment flux J_c (from Axiom 4). Covariant balance law $\nabla_\mu C^\mu = s_c$.

Proposition-level (Axioms + Lorentz structure): The record-commitment 4-current $C^\mu = (\rho_c, J_c)$ is the minimal macrostate basis (forced by Lorentz covariance of record accounting). The Fisher metric on 4-current space generically has full rank 4. Conformal selection by volume matching: $\sqrt{|g|} = \kappa_0 \sqrt{\det g_{ab}(C)}$. The conformal selection is a canonical closure principle (reparametrisation invariant + composable), not an axiomatic inevitability.

Closed as a macroscopic model (Route B): Given a specified local record micro-model p(r|C) in the Poisson-like finite-variance universality class, canonical volume selection $\sqrt{|g|} = \kappa_0 \sqrt{\det g_{ab}(C)}$, and covariant sourcing $\nabla_\mu C^\mu = \gamma_0 T_{\mu\nu} u^\mu u^\nu$, *the constrained sigma-model action* $S_{tot} = S_{info} + S_{vol} + S_{src}$ *defines a closed covariant macroscopic theory of emergent geometry. In the weak-field, slow-motion limit this reduces to* $\nabla^2\Phi = 4\pi G\rho_m$ *with* $G = \gamma_0 c^4/(8\pi\rho_{c0})$, independently of the detailed micro-model within the universality class.

Conditional derivation (Route A): Einstein's field equation as a thermodynamic equation of state (Jacobson closure). Requires explicit thermodynamic assumptions A1–A4.

Strong-field extension (resolved at leading order). Theorem VIII.3 + Proposition VIII.5: the leading-order metric equation is $G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = (\gamma_0/\rho_{c0})\, T_{\mu\nu} + O(\ell^2_*)$, with coupling $\kappa_{eff} = 8\pi G/c^4 = \gamma_0/\rho_{c0}$ expressed in record-substrate parameters (Section VIII.10).

Gravitational wave and strong-field vacuum sectors (consistent). Under the same hypotheses: two TT tensor polarisations at speed c in the weak field (Section VIII.11); standard Kerr QNM spectrum with corrections at $(\ell_*/r_+)^2$ in the strong-field vacuum (Section VIII.12).

Conditional on substrate realisation: Planck-scale deviations. Modified dispersion relations. Decoherence signatures. Entropy–area scaling tests (Section XI).

XIV. Summary and Open Problems

XIV.1 What Has Been Established

Starting from five operational axioms (record commitment, reversible substrate, finite distinguishability, local coupling, inertial symmetry), we have derived:

1. A finite, frame-invariant maximal causal-influence speed c (Section III).
2. Lorentz transformations and the full Lorentz group O(1,3) (Section IV).
3. A Riemannian metric on macrostate space from Fisher information (Section V).
4. The record-commitment 4-current $C^\mu = (\rho_c, J_c)$ as the minimal macrostate basis, forced by Lorentz covariance of record accounting (Section VI.1).
5. A concrete conformal-factor formula Ω(x) via the Jeffreys measure, under the record-calibration closure principle (Section VI.3).
6. Einstein's field equations as a thermodynamic equation of state for record entropy, conditional on closure assumptions A1–A4 (Section VII, Route A).
7. Newtonian gravity from the information action plus balance-law sourcing in the weak-field limit, via a constrained variational derivation with $G = \gamma_0 c^4/(8\pi\rho_{c0})$ (Section VIII.7, Route B).
8. A fully covariant constrained action $S_{tot} = S_{info} + S_{vol} + S_{src}$ generalising Route B to arbitrary curvature (Section VIII.8).
9. A universality lemma showing the Newtonian limit $G = \gamma_0 c^4/(8\pi\rho_{c0})$ holds for any Poisson-like counting process with linear rate parameter (conditions U1–U4), not just the Poisson–drift model (Section VIII.9).
10. A conditional strong-field completion theorem (Theorem VIII.3) and coefficient-level identification (Proposition VIII.5): the metric equation from S_{tot} is $G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = (\gamma_0/\rho_{c0})\, T_{\mu\nu} + O(\ell^2_*)$, i.e. Einstein's equation with coupling $\kappa_{eff} = \gamma_0/\rho_{c0} = 8\pi G/c^4$ expressed in record-substrate parameters (Section VIII.10).
11. Consistency with precision gravitational observations: under (H1)–(H5), the framework predicts two transverse-traceless tensor polarisations propagating at speed c (Section VIII.11), and the standard Kerr quasinormal mode spectrum with deviations suppressed by $(\ell_*/r_+)^2$ (Section VIII.12) — matching current LIGO/Virgo/KAGRA data.
12. The TPB ratio with relativistic and gravitational scaling (Section X).

XIV.2 Open Problems

O1 (Conformal selection from axioms). The record-calibration principle (Proposition 2) provides a concrete, canonical conformal factor via the Jeffreys measure. However, it is a closure assumption, not a consequence of Axioms 1–5 alone. Deriving the principle — or proving it is the unique closure compatible with the axioms and composability — would close this gap entirely.

O2 (Route B strong-field completion — resolved at leading order). Theorem VIII.3 establishes the Einstein form structurally (Lovelock uniqueness under H1–H5). Proposition VIII.5 completes the coefficient-level identification: $\kappa_{eff} = \gamma_0/\rho_{c0} = 8\pi G/c^4$, with Λ_{eff} generated by the volume constraint sector. The leading-order metric equation is explicitly $G_{\mu\nu} + \Lambda_{eff} g_{\mu\nu} = (\gamma_0/\rho_{c0}) T_{\mu\nu} + O(\ell^2_*)$. What remains:

- *Source scalar resolution.* The structural argument is agnostic to whether the source is $\varepsilon = T_{\mu\nu} u^\mu u^\nu$ or $T = g^{\mu\nu} T_{\mu\nu}$. An explicit calculation with specific matter content (e.g., radiation) would determine which is selected by strong-field consistency.
- *Higher-derivative corrections.* Classify the leading $H_{\mu\nu}$ corrections (coefficients a, b, … in the effective action), determine whether they are curvature-squared (Gauss–Bonnet type) or include other invariants, and compute the suppression scale ℓ_* in terms of substrate parameters.

Routes A and B converge at leading order. Any controlled deviations are predictions testable in high-curvature regimes.

O3 (Quantum gravity regime). The framework currently addresses the semiclassical limit. Extending it to the full quantum-gravitational regime — where the substrate discreteness becomes dominant — is the natural next frontier.

O4 (Quantitative experimental predictions). Sharpen the candidate signatures of Section XI into quantitative predictions with specific numerical coefficients dependent on the substrate model.

O5 (Cosmological constant). In Route A, Λ appears as an integration constant. A record-theoretic derivation of its value (or an explanation of its smallness) would be a major achievement.

References

1. Jacobson, T. (1995). "Thermodynamics of Spacetime: The Einstein Equation of State." *Physical Review Letters*, 75(7), 1260–1263.
2. Verlinde, E. (2011). "On the Origin of Gravity and the Laws of Newton." *Journal of High Energy Physics*, 2011(4), 29.
3. Padmanabhan, T. (2010). "Thermodynamical Aspects of Gravity: New Insights." *Reports on Progress in Physics*, 73(4), 046901.
4. Amari, S. and Nagaoka, H. (2000). *Methods of Information Geometry.* American Mathematical Society.
5. Čencov, N. N. (1982). *Statistical Decision Rules and Optimal Inference.* American Mathematical Society.
6. Wheeler, J. A. (1990). "Information, Physics, Quantum: The Search for Links." In *Complexity, Entropy, and the Physics of Information.*

7. Bekenstein, J. D. (1973). "Black Holes and Entropy." *Physical Review D*, 7(8), 2333–2346.
8. Unruh, W. G. (1976). "Notes on Black-Hole Evaporation." *Physical Review D*, 14(4), 870–892.
9. Wald, R. M. (1984). *General Relativity.* University of Chicago Press.
10. Carroll, S. M. (2004). *Spacetime and Geometry: An Introduction to General Relativity.* Addison-Wesley.
11. Lovelock, D. (1971). "The Einstein Tensor and Its Generalizations." *Journal of Mathematical Physics*, 12(3), 498–501.
12. Lovelock, D. (1972). "The Four-Dimensionality of Space and the Einstein Tensor." *Journal of Mathematical Physics*, 13(6), 874–876.
13. Abbott, B. P. et al. [LIGO/Virgo Collaboration] (2016). "Observation of Gravitational Waves from a Binary Black Hole Merger." *Physical Review Letters*, 116(6), 061102.
14. Abbott, B. P. et al. [LIGO/Virgo/Fermi/INTEGRAL] (2017). "Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A." *The Astrophysical Journal Letters*, 848(2), L13.