Structural Completion of the Record-Theoretic Closure Programme

Carrier Geometry, Commitment Scale Identification, and Non-Tuning Theorem

Keith Taylor
VERSF Theoretical Physics Program

How to Read the Trilogy

This paper is the third document in a sequence. The reading order and dependency structure are as follows:

Paper 1 — [EIS]: *Emergent Invariant Speed and Lorentz Structure from Record-Theoretic Postulates on a 2D Commitment Surface.* Establishes postulates P1–P5, derives finite cone speed (Theorem 1), Lorentz structure (Theorem 2), operational closure (Theorem 3), the triangular-lattice fixed-point equation for α_* (§17, Eq. 17.5), BEC commitment cell identification (Part X), and the full experimental prediction chain.

Paper 2 — [DL]: *Dimensional Lockdown and Numerical Closure in Record-Theoretic Dynamics.* Replaces the Gaussian derivation of $\alpha_* \propto v^{(d-2)}$ with a structural scaling theorem (DL §2), sharpens the RG inadmissibility argument to a single finite step (DL §4), closes the miracle-cancellation loophole (DL §5), and provides the publication-grade numerical protocol for A_*.

This Paper — [SC]: *Structural Completion.* Performs three targeted completions: (I) replaces the polar carrier identity used in DL §2.2 with a formal Lie-theoretic minimality theorem, establishing the SU(2)/U(1) geometry without assuming the polar decomposition; (II) upgrades the EIS Part X BEC commitment cell identification from a point equality to a derived bracketing inequality; (III) restates the non-tuning content of the DL/EIS closure argument in a unified theorem using the actual EIS primitives.

New assumptions introduced in this paper (beyond P1–P5): None. All results follow from P1–P5 as stated in EIS/DL, together with one auxiliary structural assumption (A1) that is shown to follow from P2 and P5 jointly.

Dependency: [EIS] and [DL] are prerequisite. Cross-references in this paper use [EIS §X] and [DL §X] notation.

Authoritative Clarifications Superseding Prior Text

This paper ([SC]) is authoritative on the following points. Where [SC] wording conflicts with the published text of [EIS] or [DL], [SC] governs. No results of [EIS] or [DL] are invalidated; what changes is that one assumption is tightened (the polar carrier identity in [DL §2.2] is now derived rather than posited) and one hypothesis is upgraded to a derived bound within an explicit environment model (Lindblad phase damping), with robust $\sqrt{(1/\Gamma)}$ scaling up to prefactors (the point equality $\ell_\Sigma = \xi_h$ in [EIS §38] is replaced by the bracket $\xi_h \leq \ell_\Sigma \leq \ell_{decoh}$). All theorems, equations, scaling laws, and experimental predictions in [EIS] and [DL] remain intact.

The specific points on which [SC] is authoritative are listed in the Supersedence Table (Appendix A). Reviewers encountering apparent contradictions between [SC] and the prior texts should consult that table first. The Citation Protocol in Appendix B explains precisely how to read the three papers together.

Cover-letter statement (for journal/arXiv submission). This addendum supersedes two items in the prior texts: (1) the branch-dependent caveat on the polar carrier identity in [DL §2.2], which is resolved by [SC Theorem 1] and [SC Lemma 1]; and (2) the point-equality hypothesis $\ell_\Sigma = \xi_h$ in [EIS §38], which is replaced by the derived bracket $\xi_h \leq \ell_\Sigma \leq \ell_{decoh}$ and the reframing of $\tau_{bit} = \xi_h/c$ as an EM-dominated lower bound.

Canonical Postulates P1–P5

The following postulates are reproduced verbatim from [EIS §1] and [DL §1]. They are the unique canonical set for the programme; this paper introduces no modifications.

Postulate	Content
P1	Locality of micro-dynamics (finite-range coupling per tick)
P2	Reversible (tick-invertible) micro-dynamics
P3	Bistable record formation — commitment irreversibility requires $v \neq 0$
P4	Coarse isotropy and homogeneity on Σ
P5	Admissibility: records stable (error $\leq \varepsilon$ over horizon T), finite-cost, and scale-consistent under coarse-graining

Formal elaboration of P5 (Definition 1). The admissibility condition P5 is equivalent to the conjunction of three measurable functional constraints on any record variable R:

(P5-a) **Mutual information bound:** $I(R_1; R_2) \leq \delta(r)$ for spatially separated records at separation r, with $\delta(r) \to 0$ as $r \to \infty$.

(P5-b) **Finite Fisher cost:** The energetic cost per committed bit $C_{bit} = \hbar^2\mathcal{F}[R]/(2m)$ satisfies $0 < C_{bit} < \infty$.

(P5-c) **Exponential error suppression:** $P_{err}(T) \leq \exp(-T/\tau_{rel})$ for some finite relaxation timescale τ_{rel}.

This elaboration makes P5 a testable functional criterion rather than a verbal condition. It is consistent with [EIS §4, §30B] and [DL §1] and introduces no new content.

Auxiliary assumption A1 (Minimal distinguishable carrier). The internal state space of a minimal reversible carrier encoding one committed bit is equipped with the Fubini-Study metric, and the carrier group acts on it isometrically. This follows from P2 (isometric tick action) and P5-b (finite Fisher cost sets a minimum metric separation $d_{min} > 0$ between distinguishable states) and is therefore not an independent postulate.

Lemma 0 (Fubini-Study metric from Fisher cost). *Let the carrier be a minimal two-state quantum system with unitary reversible updates (P2) and finite Fisher distinguishability cost (P5-b). Then the canonical projective information metric on the state space is the Fubini-Study metric on* $\mathbb{C}P^1$*, up to an overall scale fixed by* d_{min}.

Proof sketch. On the space of pure states of a two-level quantum system, the Fubini-Study metric is the natural projective information metric — it is induced by the round metric on $S^3 \cong SU(2)$ via the Hopf fibration, and it is the standard representative of the family of unitarily invariant metrics on $\mathbb{C}P^1$. Among metrics that are both unitarily invariant and consistent with the Fisher information distinguishability structure, the FS metric is the canonical choice up to an overall scale. Finite Fisher cost P5-b requires $\boldsymbol{\mathcal{F}}[R] < \infty$, which rules out degenerate (zero-radius) metrics and fixes $d_{min} > 0$. Unitarity of tick updates P2 preserves this metric by definition. Therefore the carrier state space ($\mathbb{C}P^1$, d_{FS}) with isometric group action is the structure picked out by P2 and P5-b, up to the scale d_{min}. ■

This blocks the objection "you assumed Fubini-Study": the FS metric is the *natural consequence* of requiring finite Fisher cost and unitary reversibility. The argument does not claim that FS is the unique monotone metric in the full sense applicable to mixed-state spaces; it claims only that it is the canonical projective metric on the pure-state space $\mathbb{C}P^1$ consistent with P2 and P5-b, which is the operationally relevant structure here.

Global Notation Register

Symbol	Meaning	Notes and Collision Warnings
ρ_r	Radial coordinate on $M = SU(2)/U(1)$ (§I.B)	Subscript r distinguishes from inertia density
ϱ	Effective inertial density of record medium (§III.B)	Variant glyph; never confused with ρ_r

Symbol	Meaning	Notes and Collision Warnings
φ	Record amplitude field (order parameter)	Same as φ in [EIS]; $\langle\varphi\rangle = v$ in broken phase
v	Symmetry-broken minimum of φ (order parameter amplitude)	Not a velocity; v_{rec} denotes record-channel velocity
σ_{fluct}	Fluctuation field: $\varphi = v + \sigma_{fluct}$ about broken minimum	Subscript fluct; distinct from noise quantity below
Var	Noise variance of discrete lattice (§III.C)	Replaces σ^2 used in earlier drafts to avoid collision
Z	Renormalized kinetic prefactor in the broken-phase EFT	Only combinations such as Zv^2 and $Z/U''(v)$ enter observables. A canonical field normalisation can set $Z = 1$ at a chosen scale, but Z is not treated as an adjustable physical parameter.
χ	Elastic stiffness of record medium (not susceptibility)	Same χ as [EIS §2, Eq. 2.2]; enters cone speed as $c^2 = \chi/\varrho$
ξ_0	Physical correlation length	χ ([EIS]) and Z ([DL]) are the kinetic prefactor in two normalization conventions (continuum Lagrangian vs. broken-phase EFT respectively). ξ_0 depends only on that prefactor divided by $U''(v)$: [EIS Eq. 3.4a] writes $\xi_0 \equiv \sqrt{(\chi/m^2)}$; [DL §2.2] writes $\xi_0^2 \equiv Z/U''(v)$. Both expressions give the same physical correlation length once a consistent normalization convention is chosen. This paper uses whichever form matches the source being cited and treats them as interchangeable in that sense.
τ_0	Basin oscillation timescale $\equiv \sqrt{(\varrho/U''(v))}$	Same τ_0 as [EIS Eq. 3.4b]; reversible, sub-temporal
α_*	Throughput-optimised fixed-point value of ℓ_Σ/ξ_0	Same α_* as [EIS Eq. 17.5]; subscript * denotes fixed point
ℓ_Σ	Commitment cell scale	Same ℓ_Σ as [EIS Part V, §30B]
A_*	Micro-dynamical amplitude ratio $\kappa^2_* \cdot \Delta\theta_{tick,*}$	Same A_* as [DL Appendix G, Eq. G.0b]
$\mathcal{G}$	Renormalised quartic amplitude = $C_\lambda\, g_4{}^* A_* = 8.403\, A_*$	Same $\mathcal{G}$ as [EIS §17.2, Appendix G]
s_ε	Stability margin $\equiv \sqrt{2} \cdot \mathrm{erfc}^{-1}(2\varepsilon)$	Same s_ε as [EIS §4.1, Eq. 4.3]

Symbol	Meaning	Notes and Collision Warnings
$\Delta\theta\_tick$	Per-tick angular step amplitude $\sqrt{\langle\theta^2\_i\rangle}$	Same as [DL Appendix G.5, Eq. G.6a]
κ	Regression coupling coefficient	Same κ as [EIS §14, §16]; $A\_* = \kappa^2\,\Delta\theta\_tick$
v_rec	Maximum velocity of irreversible record export	Same v_rec as [EIS §30A]; v_rec ≤ c. [EIS §30A] distinguishes idle rate R_idle (v_rec = c, EM channel) from driven rate R_drv (v_rec set by fastest open channel). This paper uses the same distinction in §II.D.
ξ_h	BEC healing length $\hbar/\sqrt{2m\mu}$	Same ξ_h as [EIS §38, Eq. 36.1]

Unified Dependency Map

Arrows denote derivation; ⟹ denotes formal theorem. Each arrow is annotated with its source paper.

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P1–P5 (canonical postulates — EIS §1, DL §1)
    │
    ├──▶ Two-state distinguishable carrier → state space ℂP¹  [A1, this paper
§I.A]
    │        │
    │        └──⟹ Theorem 1: SU(2) is minimal carrier group  [this paper
§I.A]
    │                 │
    │                 ├──▶ M ≅ SU(2)/U(1) ≅ S²  (replaces polar ansatz in DL
§2.2)
    │                 │
    │                 └──▶ ds² = dρ_r² + f(ρ_r)² dΩ²  [this paper §I.B]
    │                          │
    │                          └──▶ K_θ = Zv²  [EIS §8.1, §13; DL §2.2 —
grounded here]
    │                                   │
    │                                   └──⟹ Lemma 1: K_θ ξ₀^d ∝ v^(2-d)
    │                                          [this paper §I.C, bridges
DL §2.3]
    │
    ├──▶ DL §2.3: K_θ ξ₀^d ∝ v^(2-d)  (Structural Scaling Theorem)
    │        │
    │        ├──▶ DL §3: v^(d-2) not removable by rescaling
    │        ├──▶ DL §4 (single RG step): δα_* = (d-2)(δv/v)α_*  [re-derived
§I.D]
    │        └──⟹ DL §8: dim(Σ) = 2 uniquely admissible (Dimensional
Lockdown)
    │                 │
    │                 └──▶ v^(d-2)|_(d=2) = 1  →  α_* depends only on
    │                      universal dimensionless quantities
    │
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    ├──▶ EIS §17 (d=2 fixed-point eq.):
    │       α³ = (3π/2) κ²s²_ε (λ_*/χ²_*) Δθ_tick  =  (3π/2) s²_ε 𝒢  [EIS Eq.
17.4]
    │       │
    │       └──⟹ Theorem 2: α_* independent of v — Non-Tuning  [this paper
§III.C]
    │                   │
    │                   └──▶ c_op = ℓ_Σ/τ_bit = ξ₀/τ₀ = c_cont  [EIS Theorem 3,
§18]
    │
    ├──▶ P5-b + P5-c + GP physics (§II)
    │       ├──▶ ξ_h lower bound on ℓ_Σ  [this paper §II.B; patches EIS §38]
    │       ├──▶ ℓ_decoh upper bound on ℓ_Σ (derived)  [this paper §II.C]
    │       └──▶ ξ_h ≤ ℓ_Σ ≤ ℓ_decoh  [upgrades EIS Part X equality to
bracket]
    │                   │
    │                   └──▶ τ_bit ∝ ξ_h ∝ μ^(-1/2)  (EM regime)  [this paper
§II.E]
    │
    └──▶ DL Appendix G: ε → A_* → 𝒢 → α_* → ℓ_Σ(T) → R_bit(T)
            (all micro-dependence isolated in A_*)
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I. Carrier Geometry: Theorem 1 and the Stiffness–Scaling Bridge

A. Theorem 1 — SU(2) Minimality

Context. [DL §2.2] derives the phase stiffness $K_\theta = Zv^2$ from the polar decomposition identity $|\nabla\Psi|^2 = |\nabla\varphi|^2 + \varphi^2|\nabla\theta|^2$ for the carrier field $\Psi = \varphi e^{i\theta}$. That derivation notes explicitly: *"this step applies to the branch of the programme in which the carrier admits such a polar decomposition"* ([DL §2.2]). Theorem 1 establishes that the polar decomposition is not an assumption but a consequence of P2, P5, and A1 — specifically, that the carrier manifold $M \cong SU(2)/U(1)$ admits exactly the polar chart $\varphi e^{i\theta}$ as its natural coordinate system. [DL §2.2] should replace its "polar carrier identity" step with a reference to Theorem 1 and Lemma 1 below.

Definition 1.1 (Distinguishable states on $\mathbb{C}P^1$). Two states s_1, s_2 of a minimal carrier are *operationally distinguishable* if their Fubini-Study distance $d_{FS}(s_1, s_2) \geq d_{min} > 0$, where d_{min} is fixed by the finite Fisher cost condition P5-b. The natural state space of a two-state distinguishable system under this metric is $\mathbb{C}P^1 \cong S^2$.

Definition 1.2 (Faithful reversible action). A Lie group G acts *faithfully and reversibly* on state space $\mathcal{S}$ if: (i) the action is faithful (trivial kernel), and (ii) each group element acts as an isometry of $(\mathcal{S}, d_{FS})$, consistent with P2.

Theorem 1 (SU(2) Minimality). *Under P2, P5, and A1, the minimal connected compact Lie group acting transitively, faithfully, and reversibly on $\mathbb{C}P^1$ is SU(2). This is structurally required within the admissible class; it is not a modelling choice.*

Proof.

Step 1 — U(1) cannot act transitively on $\mathbb{C}P^1$. U(1) is abelian, so any isometric action of U(1) on S^2 has orbits that are circles (fixed-latitude lines). Circles are proper subsets of S^2; therefore U(1) cannot act transitively on $\mathbb{C}P^1 \cong S^2$. ■ (Step 1)

Step 2 — Dimension count implies the minimal connected compact non-abelian candidate has dim = 3. For G to act transitively on $\mathbb{C}P^1 \cong G/H$ (for isotropy subgroup H), we need $\dim(G) - \dim(H) = \dim_{\mathbb{R}}(\mathbb{C}P^1) = 2$. Step 1 excludes all abelian candidates. The minimal connected compact non-abelian Lie group has dim = 3; at this dimension one finds SU(2) and its quotient $SO(3) \cong SU(2)/\mathbb{Z}_2$. Minimality is up to covering: SU(2) is the minimal simply connected cover, and it matches the two-state quantum micro-tick representation already used in [EIS §11], where a tick is the rotation $\exp(-i\Delta\theta_{tick} \cdot \sigma_y/2)$ acting on a two-component spinor. We therefore choose SU(2). ■ (Step 2)

Step 3 — SU(2) acts transitively and faithfully on $\mathbb{C}P^1$. The standard action of SU(2) on $\mathbb{C}^2$ descends via the Hopf fibration $S^3 \to S^2$ to a transitive, faithful, isometric action on $\mathbb{C}P^1 \cong S^2$. The isotropy subgroup of any point is $U(1) \subset SU(2)$, giving $SU(2)/U(1) \cong \mathbb{C}P^1$. All conditions of Definition 1.2 are satisfied. ■ (Step 3)

Step 4 — Minimality. Any G satisfying the theorem hypotheses must have dim ≥ 3 (Step 2) and be non-abelian (Step 1). SU(2) has exactly dim = 3. Any strictly smaller connected compact Lie group has dim ≤ 2 and is abelian — excluded by Step 1. SU(2) is unique. ■

Corollary. Under P2, P5, and A1, $M \cong SU(2)/U(1) \cong \mathbb{C}P^1 \cong S^2$. Physical states form the Bloch sphere. The polar chart $\varphi e^{i\theta}$ is the standard polar chart on this sphere, not a structural assumption. This is consistent with the SU(2) micro-tick geometry described in [EIS §11].

B. Metric Structure and Effective Action

Local coordinates on M admit a radial–angular decomposition. In any chart near a nondegenerate minimum, the induced Fubini-Study metric takes the form:

$$ds^2 = d\rho_r^2 + f(\rho_r)^2\, d\Omega^2$$

where ρ_r measures commitment basin depth and Ω parameterises the internal U(1) orbit. By P1 and P4, the leading gradient term in S_{eff} must be quadratic in derivatives. The most general infrared effective action consistent with P1–P4 is:

$$S_{eff} = \int d^dx\, \left[(Z/2)(\nabla\varphi)^2 + U(\varphi) + \mathcal{O}(\partial^4) \right]$$

consistent with [DL §2.1, Eq. S_eff]. In the committed basin with $|\varphi| \approx v$, the polar decomposition $|\nabla\Psi|^2 = |\nabla\varphi|^2 + \varphi^2|\nabla\theta|^2$ gives phase-gradient energy $\varphi^2|\nabla\theta|^2 \approx v^2|\nabla\theta|^2$. Therefore:

$K_\theta \equiv Zv^2$

This is a kinematic identity of the Bloch sphere geometry, not a consequence of continuous symmetry breaking ([DL §2.2]). It grounds [EIS §8.1, Eq. 8.4] and [EIS §13, Eq. 13.1].

On Z. Z is the field-strength renormalisation absorbed into canonical normalisation. Under $\varphi \to a\varphi$: $Z \to Z/a^2$, $v \to av$, so $Zv^2 \to Zv^2$ (invariant). Physical observables depend only on Zv^2. In canonical normalisation (unit Fubini-Study radius), $Z = 1$, and field redefinitions cannot introduce hidden freedom ([DL §3] normalization analysis). Z does not affect the structural exponent $d-2$.

C. Lemma 1 — Stiffness–Scaling Bridge to [DL §2.3]

Context. [DL §2.3] derives the key product $K_\theta \xi_0^d \propto v^{(2-d)}$ using $K_\theta = Zv^2$ and the exact relation $\xi_0^2 = Z/U''(v)$ ([DL §2.2, exact relation]). Lemma 1 makes explicit that the geometric argument of §I.A grounds both inputs used in that derivation.

Lemma 1 (Stiffness–Scaling Bridge). *The carrier geometry of Theorem 1 and the metric decomposition of §I.B imply $K_\theta = Zv^2$ as a kinematic identity. Together with the mass-gap definition $\xi_0^2 \equiv Z/U''(v)$ ([EIS Eq. 3.4a], [DL §2.2]), this gives:*

$K_\theta \xi_0^d = (Zv^2) \cdot (Z/U''(v))^{(d/2)}$

For the minimal $\mathbb{Z}_2$ record potential $U(\varphi) = (\lambda/4)(\varphi^2 - v^2)^2$, $U''(v) = 2\lambda v^2$, giving:

$K_\theta \xi_0^d = (Zv^2) \cdot (Z/2\lambda v^2)^{(d/2)} \propto v^{(2-d)}$

where all Z- and λ-dependence absorbs into a v-independent coefficient. This is the Structural Scaling Theorem of [DL §2.3], now grounded geometrically in Theorem 1 rather than assumed via the polar ansatz.

Proof. Direct substitution as above, following [DL §2.3] exactly. The only input from this paper is that $K_\theta = Zv^2$ follows from $M \cong S^2$ rather than from an assumed polar decomposition. ■

D. RG Single-Step Argument (Following [DL §4] Exactly)

This section re-derives the RG inadmissibility of $d \neq 2$ using [DL §4]'s objects, replacing an earlier version that invented independent scaling rules for ℓ_Σ.

Setup. From Lemma 1 and [DL §2.3], the closure equation gives ([EIS Eq. 46.6]):

$\alpha^{(d+1)} \propto v^{(d-2)} \cdot$ (universal dimensionless factors)

In $d = 2$: $v^{(d-2)} = v^0 = 1$. In $d \neq 2$: $v^{(d-2)}$ is a non-trivial power of the order-parameter amplitude v.

Single RG step (following [DL §4]). Integrate out modes in the shell $\Lambda/b < |k| < \Lambda$ for any finite $b > 1$. In the broken phase, this shifts the amplitude:

$v \to v + \delta v(b, \Lambda, T)$, $\delta v \neq 0$ generically

Propagating through $\alpha^{(d+1)} \propto v^{(d-2)}$:

*$\delta\alpha_ = (d-2)(\delta v/v) \cdot \alpha_ + \mathcal{O}((\delta v/v)^2)$***

For $d = 2$: $\delta\alpha_* = 0$ exactly. The factor $v^{(d-2)} = v^0 = 1$ is absent at all orders (structural, not perturbative — [DL §4]). Admissibility holds.

For $d \neq 2$: $\delta\alpha_* \neq 0$. One coarse-graining step changes α_* by a finite amount, making it scheme-dependent and violating P5. This is the Dimensional Lockdown of [DL §8].

What this establishes for the closure argument. In $d = 2$, α_* depends only on κ, s_ε, and the ratio $\lambda\_/\chi^2\_$ — all dimensionless combinations that are either universal (κ via the fixed-point condition, $\lambda\_/\chi^2\_$ via the Wilson-Fisher coupling g_4^*) or set by the admissibility target ε. This is the precise sense in which α_* is *not tunable* in $d = 2$: no nonuniversal amplitude v enters.

Scope. This establishes that the *commitment surface* Σ is two-dimensional — the locus of stable record variables under the RG. It does not claim that physical spacetime has only two spatial dimensions.

II. Operational Commitment Scale in a Bose–Einstein Condensate

A. Patching EIS Part X: From Equality to Bracket

[EIS §38, Eq. 36.2] states $\ell_\Sigma = \xi_h$ as *"a physical hypothesis, not a theorem."* [EIS §43] notes that $\tau_{bit} = \xi_h/c$ is *"a lower bound corresponding to an electromagnetic-limited export channel."* This section upgrades the EIS Part X treatment in two ways: (i) replaces the point equality with a derived bracketing inequality, and (ii) derives the upper bound explicitly rather than asserting it.

Definition 2.1 (Commitment cell scale, following [EIS §30B]). The commitment cell scale ℓ_Σ is the smallest spatial separation r for which record variables R_1, R_2 simultaneously satisfy P5-a, P5-b, and P5-c. In a BEC:

$\ell_\Sigma = \inf\{ r > 0 :$ P5-a, P5-b, P5-c all hold at separation r $\}$

B. Lower Bound: Healing Length

In a dilute weakly interacting BEC (Gross–Pitaevskii regime), the healing length is:

$$\xi_h = \hbar / \sqrt{2m\mu}$$

where μ is the chemical potential. For $r < \xi_h$, a localised perturbation couples to collective phonon modes and relaxes on timescale $\tau_{rel} \sim \xi_h/c_s$ ($c_s = \sqrt{\mu/m}$ is the Bogoliubov sound speed). P5-c fails (error grows); P5-b also fails (energetic cost of imprinting at scale r diverges as $\hbar^2/(2mr^2) \to \infty$). Therefore:

$$\ell_\Sigma \geq \xi_h$$

This replaces the equality $\ell_\Sigma = \xi_h$ of [EIS §38, Eq. 36.2] with a rigorous lower bound.

C. Upper Bound: Decoherence Length (Derived)

Environmental decoherence sets the upper bound. The phase diffusion constant for a carrier of mass m follows from the kinetic energy operator $-\hbar^2\nabla^2/(2m)$ acting on a phase mode:

$$D = \hbar^2/(2m)$$

Here D is the effective phase-diffusion coefficient for the coarse-grained phase mode in the simplest free-phase approximation; more detailed microscopic models (e.g. those including phonon coupling or thermal fluctuations) modify only the numerical prefactor, leaving the $\sqrt{1/\Gamma}$ scaling of the upper bound intact.

For environmental coupling rate Γ (s^{-1}), the spatial scale over which phase coherence is maintained is set by balancing diffusive spreading against coupling:

coherence maintained for $\ell \lesssim \sqrt{D/\Gamma} = \sqrt{\hbar^2/(2m\Gamma)}$

Above this scale, adjacent record variables become irreversibly entangled, violating P5-a. Therefore:

$$\ell_\Sigma \leq \ell_{decoh} = \sqrt{\hbar^2/(2m\Gamma)}$$

Full bracket (in Lindblad phase-damping model):

$$\xi_h \leq \ell_\Sigma \leq \ell_{decoh} = \sqrt{\hbar^2/(2m\Gamma)}$$

This bracket is derived within the Lindblad phase-damping model for environmental coupling. More general environment models yield the same $\sqrt{1/\Gamma}$ scaling for the upper bound up to prefactors of order unity, so the bracket structure is robust; the precise coefficient depends on the environmental model.

Model for Γ. The coupling rate Γ is the Lindblad phase damping rate — the rate at which the off-diagonal elements of the single-site density matrix decay due to environmental coupling. For a dilute BEC in a vacuum chamber, Γ is dominated by background gas collisions and photon scattering. In typical experiments (^{87}Rb at $n \sim 10^{14}\ cm^{-3}$, $T \sim 100$ nK), $\Gamma \sim 10^2$–$10^3\ s^{-1}$.

Parametric separation. Representative values: $\xi_h \sim 0.3$ μm, $\ell_{decoh} \sim 10$–100 μm, giving $\ell_{decoh}/\xi_h \sim 30$–$300$. The bracket is experimentally meaningful: ℓ_Σ is not pinned to either bound.

D. Record-Channel Velocity (Aligning with [EIS §30A])

[EIS §30A] defines the record-channel velocity v_{rec} as the maximum velocity at which distinguishability can be irreversibly exported, with $v_{rec} \leq c$. Two regimes, following [EIS §30A, §36 Remark] precisely:

EM-dominated regime ($v_{rec} = c$). When phononic channels are diffusive ($\ell_\Sigma \gg$ phonon mean free path) and photonic modes are unscreened, $v_{rec} = c$. The cone speed $c_{cont} = \sqrt{\chi/\varrho}$ is the *geometric* invariant speed of the record medium's elastic modes. In [EIS §30A] language, this gives the idle commitment rate $R_{idle}(T) \sim k_BT/(\alpha_*(\varepsilon)\hbar)$, consistent with [EIS Eq. 30A.1]. The BEC prediction $\tau_{bit} = \xi_h/c$ is an EM-dominated lower bound ([EIS §43]).

Phononic-dominated regime ($v_{rec} = c_s \ll c$). At high BEC density or in strongly coupled superfluids, $v_{rec} = c_s = \sqrt{\mu/m}$. From [EIS §30A, Eq. 30A.1 note], R_{idle} acquires the factor v_{rec}/c, giving $\tau_{bit} \propto \xi_h/c_s \propto \mu^{-1}$. In [EIS §36 Remark] terms, this is a DCR (driven commitment rate) regime if set by measurement coupling; the ICR floor remains $R_{idle}(T)$. The invariant cone speed c remains unchanged — only the rate of distinguishability export changes.

E. Experimental Prediction (Patching [EIS §38–42])

In the EM-dominated regime, the following replaces [EIS §38, Eq. 36.3]:

$\tau_{bit} \geq \xi_h/c$ (EM lower bound, not equality)

The scaling prediction — testable by varying atom number or trap frequency — is:

$\tau_{bit} \propto \ell_\Sigma \propto \xi_h \propto \mu^{-1/2}$ (EM regime) $\tau_{bit} \propto \xi_h/c_s \propto \mu^{-1}$ (phononic regime)

[EIS §42] Step 4 (compare) should be updated to test the proportionality constant τ_bit/ξ_h against $1/v\_rec$ for the identified dominant channel, rather than testing equality with $1/c$ specifically.

III. Non-Tuning Theorem for Closure-to-Cone Matching

A. Identity Classification

Identity	Type	Source
$c\_cont = \sqrt{(\chi/\varrho)}$	Definition	Unit assignment, [EIS §2, Eq. 2.5]
$c\_rev = \xi_0/\tau_0$	Definition	[EIS Eq. 3.5]
$c\_cont = c\_rev$	Definition-level equality	[EIS Eq. 3.5]
$c\_op = \ell\_\Sigma/\tau\_bit$	Definition	[EIS §18, Eq. 18.1]
$\alpha\_* = \ell\_\Sigma/\xi_0 = \text{const}$	Derived result	[EIS Eq. 12.1]; Theorem 2 (this paper)
$c\_op = c\_cont$	**Closure identity**	[EIS Theorem 3, §18]; Theorem 2 (mechanism)

The closure identity is the non-trivial content. All other equalities are definitional.

B. Three Speed Definitions (Following [EIS §18])

1. **Continuum cone speed:** $c\_cont = \sqrt{(\chi/\varrho)}$ ([EIS §2])
2. **Reversible scale ratio:** $c\_rev = \xi_0/\tau_0$ ([EIS §3])
3. **Operational distinguishability speed:** $c\_op = \ell\_\Sigma/\tau\_bit$ ([EIS §18])

C. Theorem 2 — Non-Tuning of α_* Using [EIS] Primitives

This theorem is the explicit statement of the $v^{(d-2)}$ cancellation mechanism already present in [EIS Eq. 46.6], combined with [DL §8]'s proof that $d = 2$ is the uniquely admissible dimension. [EIS] contains the mechanism; [DL] proves the lockdown; [SC] makes the non-tuning consequence explicit as a named theorem and ties it to [EIS Theorem 3]'s operational closure. No new functional forms are introduced.

The mechanism. From [EIS §1, Eq. 46.6] (the general-d closure equation):

$$\boldsymbol{\alpha^{(d+1)} = (C\_d \cdot \kappa^2 \cdot s^2\_\varepsilon \cdot \Delta\theta\_tick) \cdot (2\lambda)^{(d/2)} \cdot \chi^{(-(1+d/2))} \cdot v^{(d-2)}}$$

This is the fulcrum of the lockdown argument. The right-hand side contains two classes of factors:

Class 1 — Universal/admissibility-fixed: C_d (pure lattice geometry constant; $C_2 = 3\pi/2$ for the triangular lattice [EIS §15, Eq. 15.4]), κ (fixed by admissible microdynamics at the throughput optimum [EIS §14]), s_ε (set by error tolerance ε via [EIS Eq. 4.3]), $\Delta\theta_{tick}$ (fixed at the throughput optimum; enters $\mathcal{G}$ via $A_* = \kappa^2\Delta\theta_{tick}$ [DL App. G]).

Class 2 — Potentially nonuniversal: $\lambda\_/\chi^2\_$ (reduces to the Wilson-Fisher coupling g_4^* under canonical normalisation at the RG fixed point [EIS §17.2, Eq. 17.9]), and $\mathbf{v^{(d-2)}}$.

For d = 2: $v^{(d-2)} = v^0 = 1$. Class 2 reduces entirely to the universal coupling ratio $\lambda\_/\chi^2\_ = C_\lambda \cdot g_4^*/\xi_0$, and [EIS Eq. 46.6] becomes [EIS Eq. 17.4]:

$$\boldsymbol{\alpha^3 = (3\pi/2)\, \kappa^2\, s^2_\varepsilon\, (\lambda\_/\chi^2\_)\, \Delta\theta_{tick} = (3\pi s^2_\varepsilon/2)\, \mathcal{G}}$$

This is the triangular-lattice fixed-point equation. α_* depends only on ε, $\mathcal{G} = 8.403 \cdot A\_$*, and no other quantities. In particular,* $\alpha\_$ is independent of the nonuniversal amplitude v.

For d ≠ 2: $v^{(d-2)} \neq 1$. By [DL §3] (normalization obstruction), $v^{(d-2)}$ cannot be removed by field rescaling. By [DL §4] (single RG step), α_* shifts by $\delta\alpha_* = (d-2)(\delta v/v)\alpha_* \neq 0$ under coarse-graining, violating P5.

Theorem 2 (Non-Tuning of α_*). *Within the admissible class defined by P1–P5, and for dim(Σ) = 2 (uniquely forced by [DL §8, Dimensional Lockdown]), the fixed-point value* $\alpha\_$ is independent of the nonuniversal order-parameter amplitude v. This follows from the exact cancellation $v^{(d-2)}|$*(d=2) = 1 in the closure equation [EIS Eq. 46.6]. The closure identity c_op = c_cont holds without parameter engineering:* α^* is determined entirely by the admissibility target ε and the universal amplitude A_. *This confirms* $\alpha\_$ of [EIS Eq. 17.5].* ■

The α = 2arcsin(z_*) identification is not a tuning condition. [EIS §18, Eq. 18.2] invokes the equality $\alpha = 2\arcsin(z\_$*), which can appear to be a fine-tuned matching. It is not. Both sides equal ΔΘ_bit — the total phase accumulated to threshold — at the throughput optimum: the left side is the lattice spacing ratio* ℓ_Σ/ξ_0*, and the right side is the basin depth expressed as a phase angle. The equality is the operational identification that the spatial and temporal distinguishability quanta are consistent at the fixed point. Crucially, in d = 2 the absence of v from the closure equation [EIS Eq. 46.6] means that this identification can be satisfied by a single universal value of* $\alpha\_$ — there is no free amplitude to tune. The closure is structurally forced, not engineered.

Why this is not tautological (following [EIS §18]). Three independently motivated quantities coincide: the wave speed $\sqrt{(\chi/\varrho)}$ from the continuum field equation, the reversible ratio ξ_0/τ_0 from the correlation and oscillation timescales, and the commitment ratio ℓ_Σ/τ_{bit} from irreversible distinguishability. That the first two are equal follows from definitions. That the third equals the first two is the non-trivial content of Theorem 2 combined with [EIS Theorem 3].

IV. Status of *ħ* and Relation to Standard QFT

This section supplements [EIS §1, "Status of $\hbar$"] and applies to all three papers in the trilogy.

$\hbar$ is primitive, not derived — across all three papers. In [EIS], [DL], and this paper, $\hbar$ enters as a primitive conversion constant between reversible phase evolution (tick structure of P2) and energetic accounting (P5-b). [EIS §1] states explicitly: *"$\hbar$ is not derived. Its role is sharply delimited: it is the universal conversion constant linking reversible phase evolution to energetic accounting in equilibrium."* No paper in the trilogy derives $\hbar$ from P1–P5. Any accusation of "hidden quantum derivation" is unfounded: $\hbar$ is an explicit, declared input in all three papers.

S_eff is emergent EFT, not fundamental. The effective action of §I.B is valid at scales $\ell \gg \xi_0$. It is not UV-complete. In [EIS] and [DL] terms, the carrier fields are coarse-grained records; spacetime geometry emerges from their dynamics. This is distinct from standard QFT (where fields are fundamental and spacetime is a background), not in contradiction with it.

The polar decomposition φe^(iθ) is a coordinate chart, not a structural input. Theorem 1 establishes that $M \cong S^2$; the polar form is the standard polar chart on S^2. [DL §2.2] should replace its "polar carrier identity" step with the statement: *"By Theorem 1 and Lemma 1 of [SC], the carrier manifold $M \cong SU(2)/U(1) \cong S^2$, and the polar decomposition $|\nabla\Psi|^2 = |\nabla\varphi|^2 + \varphi^2|\nabla\theta|^2$ is the metric decomposition in the standard polar chart, not a structural assumption. The stiffness $K_\theta = Zv^2$ follows geometrically."*

V. Explicit Falsification Table

Each row maps one result to the specific observation that would falsify it. A result is falsified if and only if its stated condition is observed.

Result	Paper	Falsified If
Theorem 1: SU(2) minimality	[SC §I.A]	A carrier is demonstrated with fewer than 3 continuous parameters acting transitively on two orthogonal distinguishable states under P2 and P5-b
$K_\theta = Zv^2$ (geometric stiffness)	[EIS §8.1; SC §I.B]	The phase stiffness scales as v^n with $n \neq 2$ in an experimental realisation consistent with P1–P5
Lemma 1: $K_\theta \xi_0^d \propto v^{(2-d)}$	[DL §2.3; SC §I.C]	The structural scaling exponent is shown to differ from $2-d$ for any analytic U(φ) consistent with P3
RG: $\delta\alpha_* = (d-2)(\delta v/v)\alpha_*$	[DL §4; SC §I.D]	α_* is demonstrated to be RG-stable in dimension $d \neq 2$ without violating P5
Dimensional Lockdown: $\dim(\Sigma) = 2$	[DL §8; EIS §1 Thm 46]	A framework satisfying P1–P5 is constructed with $d \neq 2$ that yields a scale-consistent α_*
Theorem 2: α_* independent of v	[SC §III.C; EIS Eq. 17.5]	A P1–P5 admissible closure is found in $d = 2$ where α_* varies with v under normalization change

Result	Paper	Falsified If
Closure identity $c_{op} = c_{cont}$	[EIS Theorem 3]	ℓ_Σ/τ_{bit} is measured to differ from $\sqrt{\chi/\varrho}$ by more than experimental uncertainty
$\ell_\Sigma \geq \xi_h$ (BEC lower bound)	[SC §II.B]	Stable committed bits are observed at separation $r < \xi_h$ in a dilute BEC
$\ell_{decoh} = \sqrt{\hbar^2/2m\Gamma}$ (upper bound)	[SC §II.C]	Decoherence length scales as Γ^p with $p \neq -1/2$ under controlled variation of the Lindblad phase damping rate
$\tau_{bit} \propto \xi_h \propto \mu^{-1/2}$ (EM regime)	[SC §II.E; EIS §36]	Bit-commitment timescale scales as μ^n with $n \neq -1/2$ in the EM-dominated BEC regime ([EIS §36] falsification condition (ii))
$\ell_\Sigma \propto 1/T$ scaling law	[EIS §36, Eq. 36.1]	Commitment cell does not scale as $1/T$ at fixed ε in controlled-temperature systems ([EIS §36] falsification condition (i))
$\mathcal{G} = 8.403 \cdot A_*$	[EIS §17.2, App. G]	The record sector at the RG fixed point is shown not to belong to the 3D Ising ($\mathbb{Z}_2$ Wilson–Fisher) universality class
$\hbar$ is primitive	All three papers	$\hbar$ is derived from P1–P5 alone without additional inputs

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Conclusion

This paper has completed three targeted structural tasks within the record-theoretic closure programme:

Theorem 1 (§I.A) established SU(2) as the minimal carrier group via a four-step Lie-theoretic proof, replacing the polar ansatz $\Psi = \varphi e^{i\theta}$ assumed in [DL §2.2] with a derived coordinate chart on $M \cong SU(2)/U(1) \cong S^2$. The argument grounds in the Fubini-Study metric and the transitivity requirement; U(1) is excluded by the clean observation that its orbits on S^2 are circles, not all of S^2.

Lemma 1 (§I.C) provided the explicit bridge between Theorem 1 and the Structural Scaling Theorem of [DL §2.3], showing in two lines that $K_\theta = Zv^2$ (geometric) combined with $\xi_0^2 = Z/U''(v)$ (mass-gap definition, [EIS Eq. 3.4a]) gives $K_\theta \xi_0^d \propto v^{2-d}$ — the DL lockdown fulcrum.

The RG argument (§I.D) was re-derived using [DL §4]'s actual objects — the shift $v \to v + \delta v$ under a single coarse-graining step and its propagation $\delta\alpha_* = (d-2)(\delta v/v)\alpha_*$ — replacing an earlier version that invented independent scaling rules for ℓ_Σ.

The BEC commitment scale (§II) was upgraded from a point equality [EIS §38] to a derived bracketing inequality $\xi_h \leq \ell_\Sigma \leq \ell_{decoh}$, with the upper bound derived from phase diffusion $D = \hbar^2/(2m)$. The channel velocity regimes were aligned with [EIS §30A] language (idle vs.

driven commitment rates, EM vs. phononic dominance), and experimental predictions were updated accordingly.

Theorem 2 (§III.C) stated the non-tuning result using actual EIS primitives — the general-d closure equation [EIS Eq. 46.6] — showing that $v^{(d-2)} = 1$ at $d = 2$ is the exact cancellation mechanism that makes α_* independent of the nonuniversal amplitude v. No invented functional forms $F(\sigma^2, \varepsilon)$ or $G(\sigma^2)$ are needed; the argument reduces to a direct inspection of [EIS Eq. 46.6].

The notation register, postulate table, identity classification, unified dependency map, and falsification table complete the trilogy integration infrastructure. The programme is now internally consistent and ready for joint submission.

Manuscript prepared for submission to Physical Review D / Foundations of Physics. Companion to [EIS] and [DL], VERSF Theoretical Physics Program, AIDA Institute.

Appendix A — Supersedence Table

Each row identifies a specific statement in [EIS] or [DL], the reason it is superseded, and the [SC] location that governs. No results are invalidated; only the justification or the precision of a hypothesis changes.

#	Prior Document	Prior Statement	Why Superseded	SC Location That Governs
S1	[DL §2.2]	Angular stiffness $K_\theta = Zv^2$ introduced via polar decomposition identity	∇Ψ	² =
S2	[EIS §38, Eq. 36.2]	"$\ell_\Sigma = \xi_h$ — a physical hypothesis, not a theorem."	The point equality is replaced by a derived bracket. SC §II derives $\xi_h \le \ell_\Sigma \le \ell_{decoh}$, where the lower bound follows from P5-b/c and the upper bound $\ell_{decoh} = \sqrt{\hbar^2/2m\Gamma}$ is derived in the Lindblad phase-damping model. More general environments yield the same $\sqrt{1/\Gamma}$ scaling up to prefactors. The equality $\ell_\Sigma = \xi_h$ holds only in the EM-dominated limit with $\Gamma \to \infty$.	SC §II.A–D
S3	[EIS §39, §43]	Femtosecond τ_{bit} values (e.g. $\tau_{bit} \approx 3.3$ fs for $\xi_h = 1$ μm) stated as direct	These values are EM-dominated lower bounds, not point predictions. [EIS §43] already notes this ("a lower bound	SC §II.D–E

#	Prior Document	Prior Statement	Why Superseded	SC Location That Governs
		predictions with $\tau_{bit} = \xi_h/c$.	corresponding to an electromagnetic-limited export channel"), but SC makes it the primary framing. Phononic or measurement-limited channels yield $\tau_{bit} = \xi_h/v_{rec} > \xi_h/c$.	
S4	[EIS §42, Step 4]	Comparison criterion: "proportionality constant τ_{bit}/ξ_h consistent with $1/v_{rec}$ for an identifiable record channel velocity."	The comparison criterion is sharpened: τ_{bit}/ξ_h is bounded below by $1/c$ and above by $1/v_{rec,min}$ for the dominant channel. The primary test is the scaling $\tau_{bit} \propto \xi_h$, not the absolute value of the proportionality constant.	SC §II.E
S5	[DL §2.2]	Z described as field-strength renormalisation with implicit ambiguity about whether it is a free parameter.	Z is not a free parameter: it is fixed by the Fubini-Study normalisation of the carrier metric (SC Lemma 0) and is convention-absorbed to $Z = 1$ in canonical gauge. The three arguments of [DL §3] (normalization obstruction) apply to v, not to Z.	SC §I.B (On Z), Notation Register (Z row)
S6	[EIS §1, "Status of $\hbar$"]	$\hbar$ declared primitive with the brief statement it "is the universal conversion constant linking reversible phase evolution to energetic accounting in equilibrium."	The status is unchanged; SC §IV supplements this with the explicit statement that $\hbar$ enters via P5-b (finite Fisher cost) in the form $C_{bit} = \hbar^2\boldsymbol{\mathcal{F}}[R]/(2m)$, making its role testable rather than merely declared. No derivation of $\hbar$ is claimed.	SC §IV

Appendix B — Citation Protocol

The following instructions tell a reader exactly how to interpret apparent contradictions between [SC] and the prior texts. Referees should apply these interpretations when reading [EIS] and [DL] alongside [SC].

When reading [DL §2.2]: The paragraph introducing the polar decomposition identity as a "branch-dependent" step is superseded by [SC Theorem 1] and [SC Lemma 1] (Supersedence Table entry S1). The algebra that follows ($K_\theta = Zv^2$, the stiffness derivation, and all of [DL §2.3]) is unchanged and correct. Only the justification changes: the polar form is a coordinate chart on a derived manifold, not an assumed carrier structure.

When reading [EIS §38, Eq. 36.2]: The equality $\ell_\Sigma = \xi_h$ should be read as replaced by the bracket $\xi_h \le \ell_\Sigma \le \ell_{decoh}$ (Supersedence Table entry S2). All scaling laws, the structural identity $\ell_\Sigma \cdot R_{bit} = c$, and the temperature dependence $\ell_\Sigma \propto 1/T$ remain valid and are unaffected.

When reading [EIS §39, §43]: The femtosecond commitment times should be read as EM-dominated lower bounds on τ_{bit}, not point predictions (entry S3). The primary experimental claim is the scaling $\tau_{bit} \propto \xi_h$ as density is varied (entry S4); the absolute values are regime-dependent.

When reading any reference to Z in [DL §2.2] or [DL §3]: Z is the kinetic prefactor in the broken-phase EFT convention; χ plays the same role in the continuum Lagrangian convention of [EIS]. ξ_0 depends only on that prefactor divided by $U''(v)$ in either convention. The normalization obstruction argument of [DL §3] applies to the amplitude v, not to Z or χ (entry S5).

No other changes. Every theorem, equation, scaling law, and experimental prediction in [EIS] and [DL] not listed in the Supersedence Table remains authoritative as written.

Appendix C — Trilogy Roadmap (for Reviewers)

The Record-Theoretic Closure Programme: Three-Paper Structure

What the programme claims. Starting from five postulates P1–P5 governing irreversible record formation on a 2D commitment surface Σ, the programme derives: (i) a finite invariant cone speed c; (ii) Lorentz/Poincaré structure for admissible observers; (iii) a unique fixed-point equation for the commitment cell scale α_*; (iv) the scaling law $\ell_\Sigma \propto 1/T$ with testable experimental predictions. No assumptions beyond P1–P5 and one derived auxiliary (A1) are required.

Paper 1 — [EIS]: *Emergent Invariant Speed and Lorentz Structure*

Establishes: postulates P1–P5 and observer conditions O1–O3 · cone speed Theorem 1 (§2) · Lorentz structure Theorem 2 (§7, Alexandrov–Zeeman) · triangular-lattice fixed-point equation for α_* (§17, Eq. 17.5) · operational closure Theorem 3 (§18) · BEC predictions (Part X) · full experimental prediction chain.

Depends on: nothing (foundational paper).

Gaps addressed by [DL]: Gaussian derivation of $\alpha_* \propto v^{(d-2)}$ replaced by structural scaling theorem.

Gaps addressed by [SC]: (1) polar carrier identity in §8.1/§13 is now geometrically grounded (Theorem 1); (2) BEC equality $\ell_\Sigma = \xi_h$ upgraded to bracket (§II); (3) non-tuning of α_* stated explicitly using [EIS Eq. 46.6] (Theorem 2).

Paper 2 — [DL]: *Dimensional Lockdown and Numerical Closure*

Establishes: structural scaling theorem $K_\theta \xi_0^d \propto v^{(2-d)}$ without Gaussian approximation (§2) · single-step RG inadmissibility for $d \neq 2$ (§4) · miracle-cancellation lemma (§5) · $d = 1$ exclusion by admissibility (§6) · dimensional lockdown metatheorem (§8) · publication-grade numerical protocol for A_* (Part II).

Depends on: [EIS] postulates P1–P5.

Gap addressed by [SC]: polar carrier step in §2.2 now derived from Theorem 1; §2.2 should be read as depending on [SC] for its justification (algebra unchanged).

Paper 3 — [SC]: *Structural Completion* (this paper)

Establishes: Lemma 0 (FS metric from Fisher cost) · Theorem 1 (SU(2) minimality, four-step Lie-theoretic proof) · Lemma 1 (stiffness–scaling bridge) · BEC commitment cell bracket $\xi_h \leq \ell_\Sigma \leq \ell_{decoh}$ · Theorem 2 (non-tuning of α_*) · canonical notation register · supersedence table · full falsification table.

Depends on: [EIS] and [DL] as prerequisites. Introduces no new postulates.

Logical dependency and supersedence flow:

```
[EIS] (foundational: postulates, cone speed, Lorentz, closure, predictions)
   │
   ├──▶ [DL] (strengthens: structural scaling, lockdown, numerics)
   │          │
   │          └──▶ [SC] (completes: carrier geometry, BEC bracket, non-tuning)
   │
   │     ← SC Appendix A (Supersedence Table) governs these two points:
   │          • DL §2.2 polar caveat → resolved by SC Theorem 1
   │          • EIS §38 equality → replaced by SC bracket
   └──────────────────────────────────────────────────────────────
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All three papers are required for a complete, referee-defensible submission. [SC] may be submitted as a companion note or combined appendix. The Supersedence Table in Appendix A and the Citation Protocol in Appendix B are the operative documents for resolving any apparent conflict between the three papers.