Admissibility Closure and the Uniqueness of Physical Entropy: A Derivation of η = 1 in the VERSF Framework

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For the General Reader

Physics has long assumed that entropy—the quantity measuring disorder, irreversibility, and the arrow of time—can be defined in multiple equivalent ways, with the choice of which microstates to count left as a free input to the theory. This paper shows that, within a specific theoretical framework called VERSF, only one entropy definition is compatible with the framework's operational structure. But the argument goes further: part of it applies to *any* physical theory of records, not just VERSF.

The paper delivers two results.

The first is **framework-independent**. Starting from four minimal assumptions about what it means for something to be a physical record—that it can be re-identified without being recreated, that physical distinctions must be measurable, that reversible processes preserve record content, and that records must be locally readable—the paper proves that any entropy governing irreversible processes must respect exactly the operational structure of the theory. It cannot invent distinctions no experiment can detect, and it cannot erase distinctions that experiments genuinely reveal. This holds for any theory satisfying these four assumptions.

The second result is **specific to VERSF**. VERSF proposes that physical reality emerges from a pre-temporal structure called the *void substrate*, through irreversible events called *commitment events*. The framework defines a partition of states called the *closure partition*, grounded in the algebraic dynamics of the void. The paper proves that this substrate-defined partition coincides exactly with the operationally-defined one from the first result. Combined, the two results force a unique entropy: the logarithmic closure entropy.

Every alternative fails. An entropy that distinguishes states no measurement can separate invents physics. An entropy that merges states experiments can tell apart suppresses physics. An entropy that breaks the symmetry structure of the framework produces contradictory probability predictions. No alternative survives.

The consequence is $\eta = 1$: the entropy conversion factor is not a free parameter but a structural necessity. And the broader lesson—reaching beyond VERSF—is that microstate counting is never truly free. It is always constrained by what a theory's operational structure can and cannot distinguish.

Technical Abstract

A central open question in the Void Energy-Regulated Space Framework (VERSF) has been the formal identification between closure entropy—defined over distinguishability classes on the pre-temporal void substrate—and thermodynamic entropy governing irreversible physical processes. Prior work demonstrated that if this identification holds, the entropy conversion factor η equals unity and the primitive commitment barrier takes a normalized, structurally interpretable value. However, the identification itself has remained a conditional assumption rather than a derived result.

This paper establishes two results at different levels of generality.

Framework-independent. From four primitive assumptions about what physical records are—recordhood, observability, reversible covariance, and local retrievability—we derive that any entropy governing irreversible record formation must coincide exactly with the operational equivalence partition of the theory: it can neither separate states that no measurement can distinguish nor merge states that are genuinely distinguishable. This conclusion holds in any theory of irreversible record formation that satisfies these assumptions, independently of VERSF.

Within VERSF. Extending the admissibility methodology previously applied to quantum probability structure, amplitude field geometry, and Hamiltonian existence, we prove that the operational equivalence partition coincides exactly with the closure partition of the void substrate—a result that depends on the VERSF closure structure. Combined with the framework-independent result, this uniquely identifies the physical entropy as the logarithmic closure entropy. All alternative entropy assignments violate at least one of six admissibility conditions. A central Equivalence-Class Dependence Lemma closes the equal-cardinality loophole, ensuring that no partition of the same size as the closure partition but different equivalence structure can survive.

Together these results establish a **No-Alternative Partition Theorem**: within VERSF, the closure partition and the thermodynamic partition must coincide. The entropy conversion factor is thereby fixed uniquely to $\eta = 1$ by structural necessity rather than normalization convention, and the primitive commitment barrier $\Phi\_c = 1$ is established as a forced property of the framework rather than a free parameter. The broader implication—applicable beyond VERSF—is that microstate counting is not a free input to a physical theory: it is constrained by the operational distinguishability structure of whatever framework is under consideration.

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1. Introduction

The VERSF programme proposes that physical reality emerges through irreversible commitment events acting on a pre-temporal substrate of distinguishability relations—the void. Within this framework, entropy appears in two operationally distinct roles:

1. **Closure entropy** $\tilde{S} = \ln N$, defined by counting distinguishability classes over the void substrate
2. **Thermodynamic entropy** $S = k_B \ln \Omega$, governing the irreversibility of physical commitment events

These two quantities are related by a dimensionless conversion factor:

$$\eta = S / (k_B \cdot \tilde{S})$$

Previous results in the VERSF programme have shown that if the microstates counted by $\tilde{S}$ coincide with those counted by S—the **Class Identification Assumption (CIA)**—then $\eta = 1$ follows immediately. But the CIA has remained the one formally unproven step in an otherwise increasingly derivation-driven framework.

The strategy of this paper is to replace the CIA with a proof. Specifically, we show that any entropy functional that could, in principle, serve as the physical entropy of irreversible records is subject to a set of necessary admissibility conditions, and that these conditions uniquely select the closure entropy. No alternative partition survives.

This result completes the entropy sector of the VERSF admissibility programme, which has now established unique admissibility results for:

Physical Structure	Result
Probability rule	Born rule uniquely forced
Amplitude field	Necessarily $\mathbb{C}$ (not $\mathbb{R}$ or $\mathbb{H}$)
Hilbert space geometry	Uniquely derived
Hamiltonian existence	Structurally necessary
Spinor algebra	Forced by closure symmetry
Entropy partition	**Uniquely the closure partition (this paper)**

1.1 Organisation

Section 2 recaps the admissibility methodology. Section 3 establishes the primitive assumptions (R1–R4) underlying physical records, derives definitions of operational and record-equivalence, and proves Propositions 1–2 and Theorem 4.3, which establish the Operational Record Condition (E5′) and Distinguishability Consistency Condition (E6′) as theorems rather than postulates, and identify the closure partition with the operational equivalence partition without circularity. Section 4 states the six formal admissibility conditions. Section 5 enumerates candidate entropy classes. Section 6 eliminates each alternative systematically. Section 7 states and proves the No-Alternative Partition Theorem. Sections 8–10 derive $\eta = 1$, fix the commitment barrier, and discuss implications. Appendix C addresses five anticipated objections in their strongest form, including the circularity concern, the scope of the framework-independent result, and the relationship to Shannon–Jaynes uniqueness theorems.

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2. The Admissibility Methodology

The central methodological principle of the programme is:

Physical structure is determined by admissibility, not by assumption.

The procedure in each domain is:

1. Identify the physical role the structure must play
2. Formulate minimal conditions any candidate must satisfy to play that role
3. Enumerate the space of candidates
4. Show by elimination that exactly one candidate survives

The strength of this approach is that it converts free assumptions into forced consequences. The conclusion carries the weight not of an axiom but of a no-go theorem applied in reverse: because all alternatives fail, the surviving structure is not merely consistent—it is *necessary*.

This methodology has proven productive across multiple sectors of the framework. We apply it here to entropy.

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3. Physical Entropy as a Record Functional

3.1 The Role of Entropy in VERSF

In the entropic unfolding formalism, a commitment event selects outcome i with probability:

$P_i \propto a_i \cdot \exp(-\lambda\, \Delta S_i)$

where a_i is an admissibility amplitude and ΔS_i is the **entropy cost** of stabilising the record corresponding to outcome i. The exponential suppression of high-cost outcomes is not imposed—it emerges from the requirement that records be dynamically stable against environmental perturbation.

This makes entropy operationally concrete: it is not a measure of ignorance, but a measure of the **cost of irreversible record formation** on the void substrate.

3.2 Primitive Assumptions for Physical Records

Before stating any admissibility conditions, we ground the analysis in four primitive assumptions about what physical records are. These are not specific to VERSF; they are minimal requirements that any theory of irreversible record formation must satisfy.

Primitive Assumption R1 — Recordhood.
A physical record is a state-feature that can be re-identified at later times without requiring a new irreversible commitment merely to define its content. If something only "exists" because it is continuously recreated, it is not a record.

Primitive Assumption R2 — Observability.
A physical distinction is meaningful only if there exists an admissible measurement procedure that can, in principle, distinguish the alternatives by different outcome statistics. No measurement difference entails no physical distinction.

Primitive Assumption R3 — Reversible Covariance.
Reversible dynamics preserve physical content. They may transport or relabel a record, but they cannot create or destroy record content without irreversibility. Reversible evolution preserves what is already physically present; it does not generate new facts.

Primitive Assumption R4 — Local Retrievability.
A record counts as physical only if its content is retrievable by some finite admissible readout procedure acting on the system through physically available couplings. Hidden distinctions with no admissible readout are not physical records.

Definition (Admissible reversible evolutions). An admissible reversible evolution is a transformation of the substrate state that preserves the closure structure — mapping closure classes to closure classes — and corresponds to physically allowed reversible dynamics within the VERSF framework. Admissible reversible evolutions form a group; their action transports and relabels records without creating or destroying record content.

3.3 Derived Definitions

From R1–R4 we derive two equivalence relations that will be needed throughout.

Definition D1 — Operational equivalence.
Two states x, y are *operationally equivalent*, written $x \sim_{op} y$, if and only if every admissible measurement procedure yields the same outcome statistics on x and y.

Definition D2 — Record-equivalence.
Two states x, y are *record-equivalent*, written $x \sim_{rec} y$, if and only if no admissible future readout can distinguish them without first creating a new irreversible commitment.

Record-equivalence (D2) is slightly stronger than operational equivalence (D1): it is tailored specifically to the persistence of irreversible records rather than to instantaneous measurement discrimination. Both are equivalence relations. We will show that for the purpose of entropy partitioning, the two coincide.

3.4 Derivation of (E5′): The Operational Record Condition

We now derive the three properties that any partition supporting a physical record entropy must satisfy.

Lemma 1 — Reversible persistence of record content.
If r is a physical record and U is an admissible reversible evolution, then U(r) carries the same record content as r.

Proof. By R1, a record must be re-identifiable at later times without new irreversible commitment. By R3, reversible dynamics do not create or destroy physical content. Therefore the content associated with r must persist under U: if it did not, either reversible evolution would erase a record—creating a net irreversible change via a nominally reversible operation, contradicting R3—or it would generate new record content without irreversibility, also contradicting R3. Hence record content is invariant under admissible reversible dynamics, up to reversible re-expression. ■

Lemma 2 — Operational accessibility of records.
If a putative record cannot be accessed by any admissible finite readout procedure, then it does not define physical record content.

Proof. By R2, physically meaningful distinctions must be tied to observable statistical differences. By R4, a record must be retrievable through admissible coupling. If no such readout exists, the alleged distinction cannot enter any physical prediction and has no operational content. It is at best a mathematical label, not a physical record. ■

Lemma 3 — Perturbative stability of records.
A physical record must be stable under sufficiently small admissible perturbations.

Proof. By R1, a record must remain re-identifiable through time. If arbitrarily small admissible perturbations destroy the record content, then the feature cannot persist without continuous refresh. But a continuously refreshed structure is not a stored irreversible record; it is an actively maintained dynamical pattern. Therefore recordhood implies perturbative stability: a structure that collapses under infinitesimal perturbation cannot support persistent re-identification. ■

Proposition 1 — (E5′) follows from R1–R4.
Any physical record must be: (i) invariant in content under reversible evolution; (ii) retrievable by admissible finite readout; (iii) stable under small admissible perturbations.

Proof. Immediate from Lemmas 1, 2, and 3 respectively. ■

This is the Operational Record Condition (E5′). It is not postulated; it is a derived consequence of the primitive assumptions about what records are.

3.5 Derivation of (E6′): The Distinguishability Consistency Condition

We now derive the constraint on how entropy labels must relate to operational distinctions.

Lemma 4 — No entropy distinction without observable distinction.
If x ~_op y, then any physical entropy assignment must satisfy $\Delta S_x = \Delta S\_y$.

Proof. Suppose instead $\Delta S_x \neq \Delta S\_y$. Since entropy enters the irreversible weighting law $P_i \propto a_i \exp(-\lambda\, \Delta S_i)$, the two states would in principle induce different probability weights under otherwise identical admissible conditions. That would produce a physically detectable distinction—different probabilities—not grounded in any admissible observable difference, directly violating R2. Therefore no physical entropy may separate operationally equivalent states. ■

Lemma 5 — No collapse of observable distinctions.
If x ≁_op y, a physical entropy partition cannot assign $\Delta S_x = \Delta S\_y$ *if the entropy is meant to track primitive irreversible record cost.*

Proof. If x and y are operationally distinguishable, they correspond to genuinely different measurable alternatives. If the entropy assignment forces $\Delta S_x = \Delta S\_y$ at the level of primitive record-cost classes, the entropy sector fails to register a real physical distinction. This does not automatically equalise total probabilities in every case, since amplitudes may differ. However, this observation does not rescue the assignment: the entropy partition is introduced precisely to track the irreversible cost structure of record formation independently of amplitude weighting. A partition that ignores real physical distinctions—allowing amplitude differences to compensate for its own failure to discriminate—is not functioning as a primitive record-cost partition. It is offloading to the amplitude sector a job the entropy sector was introduced to do. Such an assignment is therefore not admissible as the primitive entropy partition for irreversible record formation, regardless of whether total probabilities happen to differ. ■

Lemma 6 — Entropy partitions must coincide with the operational record partition.
A physical entropy partition for irreversible records must coincide with the partition induced by operational record-equivalence.

Proof. By Lemma 4, the entropy partition cannot be strictly finer than the operational equivalence partition—it may not separate states that are operationally indistinguishable. By Lemma 5, it cannot be strictly coarser—it may not merge states that are operationally distinguishable. Therefore it must coincide with the operational record partition. ■

Proposition 2 — (E6′) follows from R1–R4.
For any physical entropy governing irreversible record formation:

$$x \sim\_op\ y \Leftrightarrow \Delta S_x = \Delta S\_y$$

Equivalently, the entropy partition must match the operational record-equivalence classes.

Proof. Immediate from Lemma 6. ■

This is the Distinguishability Consistency Condition (E6′). Like E5′, it is not postulated; it follows from the primitive assumptions about observability and recordhood.

3.6 Exact Identification of Closure and Operational Equivalence

The derivations above establish that any physical entropy must respect the operational record-equivalence partition $\mathcal{P}$_op. We now prove, from the same primitive assumptions R1–R4, that $\mathcal{P}$_op coincides exactly with the closure partition $\mathcal{P}$_cl. This requires a precise substrate-side definition of closure-equivalence so that the theorem has genuine content rather than reducing to a definitional restatement.

Independent grounding of the closure relation. In VERSF closure theory [VERSF-CL-prior], the closure relation is defined at the level of the void substrate, prior to and independently of any measurement procedure or observer.

Definition (Closure-equivalence). Two substrate states x, y are *closure-equivalent*, written x ∼_cl y, if and only if they generate the same stable closure configuration under admissible reversible evolution and irreversible commitment structure.

This definition is substrate-side: it refers to which stable configuration a state produces under the dynamics of the void, not to what any measurement yields. Two states may be closure-equivalent even if no observer has yet measured them; and the definition does not invoke outcome statistics, readout procedures, or any operational notion. The *persistent distinguishability content* of a state is therefore its stable closure configuration — a structural property of the void substrate.

Theorem 4.3 below establishes that this substrate-defined equivalence relation coincides exactly with operational equivalence under R1–R4. The two directions are not trivially dual: one shows that structural indistinguishability entails operational indistinguishability; the other shows that any structural difference must be operationally accessible. Together they establish a non-trivial coincidence between the algebraic dynamics of the void and the observable record structure.

Proposition 4.1 — Closure equivalence implies operational equivalence.
If x ∼_cl y, then x ∼_op y.

Proof. By the substrate-side definition, x ∼_cl y means x and y produce the same stable closure configuration under all admissible reversible and irreversible dynamics. Suppose, for contradiction, that some admissible measurement procedure produces distinct outcome statistics on x and y. That statistical difference would constitute a physically retrievable distinction between x and y. By R2 (observability), a physically retrievable distinction is a real physical difference. By R4 (local retrievability), it must be accessible through admissible coupling. But any difference accessible through admissible physical coupling must be reflected in the stable closure configuration the states produce. By definition of admissible physical processes within VERSF, all physically real distinctions must arise from the substrate dynamics: a distinction not reflected in closure configurations would correspond to an observable effect without a substrate-level cause, violating the framework's closure of physical description. Therefore any retrievable statistical difference implies a difference in closure configuration, contradicting $x \sim_{cl} y$. Therefore $x \sim_{cl} y$ implies $x \sim_{op} y$. ■

Proposition 4.2 — Operational equivalence implies closure equivalence.
If $x \sim_{op} y$, then $x \sim_{cl} y$.

Proof. Suppose $x \sim_{op} y$ but $x \nsim_{cl} y$. Then x and y produce different stable closure configurations — there is a genuine substrate-level difference in their dynamics under admissible reversible evolution and commitment structure. By R1 (recordhood), any difference contributing to persistent record content must be stable under reversible evolution and re-identifiable at later times without new irreversible commitment. By R4 (local retrievability), any such stable difference must be accessible by some admissible finite readout — otherwise it would not qualify as physical record content at all. Therefore any closure-level distinction necessarily induces an operational distinction: the substrate-level difference between x and y must in principle be accessible by some admissible finite readout. That readout would produce distinct outcome statistics for x and y, contradicting $x \sim_{op} y$. Therefore $x \sim_{op} y$ implies $x \sim_{cl} y$. ■

Theorem 4.3 — Closure partition equals operational equivalence partition.
Under primitive assumptions R1–R4 and the substrate-side definition of closure-equivalence [VERSF-CL-prior]:

$$\mathcal{P}_{cl} = \mathcal{P}_{op}$$

Proof. Immediate from Propositions 4.1 and 4.2: $x \sim_{cl} y$ if and only if $x \sim_{op} y$, so the partitions are identical. ■

Remark. The non-trivial content of this theorem is the bridge in each direction: that substrate-level configurational sameness is exactly what admissible observation can and cannot distinguish, no more and no less. Adding one further proposition linking record-equivalence (D2) to operational equivalence (D1) — by the same R1+R4 argument applied to future readout — yields the full chain $\mathcal{P}_{cl} = \mathcal{P}_{rec} = \mathcal{P}_{op}$. For the entropy argument, Theorem 4.3 is sufficient: E5′ and E6′ constrain the physical entropy partition to be $\mathcal{P}_{op}$, and Theorem 4.3 identifies $\mathcal{P}_{op} = \mathcal{P}_{cl}$ with no circularity.

3.7 Summary: What the Pre-formal Requirements Entail

The above derivations yield three operationally grounded requirements on any physical entropy:

- **Entropy must be defined over operationally distinguishable record classes.** (Propositions 1 and 2; identification with closure classes via Theorem 4.3)
- **Entropy must enter probability consistently.** (Lemma 4 and the Born-rule admissibility theorem [VERSF-BR])
- **Entropy must be additive over independent subsystems.** (Extensivity requirement)

These translate into the formal admissibility conditions of Section 4.

4. Admissibility Conditions for Physical Entropy

Let $\mathcal{S}[\mathcal{P}]$ denote an entropy functional defined over a partition $\mathcal{P}$ of the distinguishability space of a system.

We say $\mathcal{S}[\mathcal{P}]$ is **physically admissible** if and only if it satisfies all of the following:

(E1) Compositional Additivity

For any two independent systems A and B:

$\mathcal{S}[\mathcal{P}\_\{A\otimes B\}] = \mathcal{S}[\mathcal{P}\_A] + \mathcal{S}[\mathcal{P}\_B]$

Motivation: The thermodynamic entropy of a composite isolated system must equal the sum of entropies of its parts. Failure of additivity produces non-extensive thermodynamics inconsistent with the large-system limit and the observed extensivity of physical entropy.

(E2) Probability Structure Compatibility

The entropy functional must enter the probability assignment in a way that:

- **Preserves normalisation:** $\sum_i P_i = 1$ is maintained identically under changes of basis
- **Respects factorisation:** P_{ij}^$(A\otimes B) = P_i$^$A \cdot P_j$^B for product states
- **Introduces no basis-dependent correlations:** entropy cost cannot depend on a choice of representation

Motivation: The Born-rule admissibility theorem has established a unique probability structure for the framework. Any entropy that distorts or contradicts this structure is inadmissible.

(E3) Reversible Invariance

Between irreversible commitment events, under reversible (unitary) evolution $\psi(t)$:

$\mathcal{S}[\mathcal{P}\_\psi(t)]$ = constant

Motivation: Entropy must not be generated by reversible dynamics. This is the entropy analogue of TPB-consistency: only irreversible events—genuine commitment events—carry entropic cost. This condition is the partition-level expression of Lemma 1.

(E4) Minimal Distinguishability Quantisation

The entropy functional must respect the minimal distinguishability unit of the framework:

$\Delta\tilde{S}_{min} = \ln 2$

Motivation: Distinguishability on the void substrate is discrete. The minimal resolvable distinction is binary. No entropy assignment that implies sub-binary physical distinctions is realisable within the framework.

(E5′) Operational Record Condition

Derived from Primitive Assumptions R1–R4 via Proposition 1.

Any partition $\mathcal{P}$ whose classes are to support a physical entropy must correspond to structures that are: invariant in content under admissible reversible evolution; retrievable by finite admissible readout; and stable under small admissible perturbations.

Any $\mathcal{P}$ failing these properties does not define physical record classes and therefore cannot serve as the partition for a physical entropy functional.

(E6′) Distinguishability Consistency Condition

Derived from Primitive Assumptions R1–R4 via Proposition 2.

For any two states x, y in the distinguishability space:

$x \sim_{op} y \Leftrightarrow \Delta S_x = \Delta S_y$

The entropy partition must match the operational record-equivalence classes exactly — neither finer (which would posit distinctions no measurement can detect) nor coarser (which would suppress real physical distinctions from the entropy sector).

By Theorem 4.3 (§3.6), $\mathcal{P}_{op} = \mathcal{P}_{cl}$, so (E6′) constrains the entropy partition to be $\mathcal{P}_{cl}$.

4.1 A Foundational Lemma

Before enumerating candidates, we establish a lemma that will be essential to the uniqueness argument.

Lemma (Equivalence-Class Dependence). *Physical observables depend on equivalence relations, not merely on partition cardinality.*

Proof. Consider two partitions $\mathcal{P}$ and $\mathcal{P}'$ of the distinguishability space, with $|\mathcal{P}| = |\mathcal{P}'|$ but $\mathcal{P} \neq \mathcal{P}'$. Since the partitions differ, there exist states x, y such that $x \sim_{\mathcal{P}} y$ (same class under $\mathcal{P}$) but $x \nsim_{\mathcal{P}'} y$ (different classes under $\mathcal{P}'$). The entropy costs assigned by each partition are therefore:

$\Delta S_x^{\mathcal{P}} = \Delta S_y^{\mathcal{P}}$ but $\Delta S_x^{\mathcal{P}'} \neq \Delta S_y^{\mathcal{P}'}$

Since $P_i \propto a_i \exp(-\lambda\, \Delta S_i)$, the probability assignments under $\mathcal{P}$ and $\mathcal{P}'$ differ for outcomes x and y. These are physically distinct predictions. Therefore two partitions of equal cardinality but different equivalence structure produce different physics. Cardinality alone does not determine physical content. ■

This lemma has two uses. First, it closes a potential loophole in the uniqueness proof: one cannot argue that any partition with $|\mathcal{P}| = |\mathcal{P}_{cl}|$ is equivalent to the closure partition. Second, it connects to the upstream derivations of §3.4–3.5: what matters is not how many classes a partition has, but which states it groups together—and Propositions 1 and 2 have already fixed which groupings are physically admissible.

4.2 The Closure Partition as Operational Benchmark

The closure partition $\mathcal{P}_{cl}$ plays a dual role in the framework: it is the distinguishability partition inherited from the void substrate, and it is the candidate benchmark against which any entropy partition must be tested. The theorem below does not assume that thermodynamic entropy is defined on $\mathcal{P}_{cl}$; it shows that every alternative partition fails the operational admissibility conditions. The closure partition enters not as a privileged input but as the unique survivor of a process of elimination.

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5. Candidate Entropy Classes

Having stated the admissibility conditions, we enumerate the logically possible candidate entropy assignments:

Class I: Non-logarithmic functionals
Any entropy function f(N) with $f \neq c \ln N$ for some constant c.

Class II: Logarithmic entropy over a non-closure partition
Functions of the form $c \ln |\mathcal{P}|$ where $\mathcal{P} \neq \mathcal{P}_{cl}$. This class subdivides into:

- *Class IIa:* Finer-than-closure partitions ($|\mathcal{P}| > |\mathcal{P}_{cl}|$)
- *Class IIb:* Coarser-than-closure partitions ($|\mathcal{P}| < |\mathcal{P}_{cl}|$)

Class III: Closure entropy
$\tilde{S} = \ln N = \ln |\mathcal{P}_{cl}|$

The claim of this paper is that only Class III survives all six admissibility conditions.

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6. Systematic Elimination

6.1 Elimination of Class I: Non-Logarithmic Functionals

Claim: Any entropy functional that is not logarithmic violates (E1).

Proof: The only continuous, monotone, normalised function satisfying

$f(mn) = f(m) + f(n)$ for all $m, n \in \mathbb{Z}^+$

is $f(n) = c \ln n$ for some constant $c > 0$. This is the Khinchin-type characterisation (Appendix A): additivity over independent systems (E1) requires that $|\mathcal{P}_{A\otimes B}| = |\mathcal{P}_A| \cdot |\mathcal{P}_B|$ maps to an additive entropy, and the only monotone measurable function achieving this is the logarithm. All power laws, polynomial entropies, and non-logarithmic forms are thereby excluded. ■

This eliminates Tsallis entropy ($q \neq 1$) and all other genuinely non-additive generalisations. The case of Rényi entropy requires more care and is addressed in Appendix B.

6.2 Elimination of Class IIa: Finer-Than-Closure Partitions

Claim: Any logarithmic entropy over a partition strictly finer than $\mathcal{P}_{cl}$ violates (E6′).

Proof: A partition $\mathcal{P}$ with $|\mathcal{P}| > |\mathcal{P}_{cl}|$ assigns distinct entropy costs to states x, y with $x \sim_{op} y$—states that cannot be operationally distinguished by any admissible measurement. The condition (E6′) requires $\Delta S_x = \Delta S_y$ for operationally indistinguishable states. Therefore any finer-than-closure partition violates (E6′) directly. Furthermore, since the additional classes have no physical correspondent in the void substrate, they also violate (E5′): no admissible local measurement operation can access them, so they cannot constitute record-distinguishing structure. ■

6.3 Elimination of Class IIb: Coarser-Than-Closure Partitions

Claim: Any logarithmic entropy over a partition strictly coarser than $\mathcal{P}_{cl}$ violates (E6′) and (E5′).

Proof: A coarser partition merges closure-distinguishable states $x \nsim_{op} y$ into a single entropy class, assigning $\Delta S_x = \Delta S_y$ despite their being operationally distinct. By (E6′), this is inadmissible: physically distinguishable states must receive distinct entropy costs. The consequence for probability is direct: when $a_x = a_y$, the merged entropy assignment collapses a real physical distinction directly in the probability weights. More generally, even when $a_x \neq a_y$, the entropy sector fails to respect the operational distinction that the partition is supposed to encode—the suppression factor $\exp(-\lambda\, \Delta S)$ becomes identical for operationally distinct outcomes, meaning the entropy makes no contribution to discriminating them. This also violates

(E5′): the operational accessibility requirement demands that entropy classes correspond to structures that local measurements can distinguish. A class that merges operationally distinct states fails this requirement because the entropy cost no longer tracks the actual distinguishability structure. ■

6.4 Probability Consistency Fails for All Non-Closure Partitions

Claim: Any entropy defined on $\mathcal{P} \neq \mathcal{P}\_cl$ violates (E2).

Proof: The Born-rule admissibility theorem [VERSF-BR] establishes that the probability structure of the framework is uniquely fixed by the symmetry group G_cl of the closure partition $\mathcal{P}\_cl$. Specifically, admissible transformations are those U ∈ G_cl that map closure classes to closure classes.

Now suppose entropy is defined on a partition $\mathcal{P} \neq \mathcal{P}\_cl$ with symmetry group $G\_\mathcal{P} \neq G\_cl$. There exists at least one transformation U ∈ G_cl that is not a symmetry of $\mathcal{P}$. For this U, and for some outcome i mapped to outcome U(i):

$$\Delta S_i \neq \Delta S\_\{U(i)\}$$

Since the probability assignment is:

$$P_i \propto |a_i|^2 \cdot \exp(-\lambda\, \Delta S_i)$$

this immediately yields:

$$P_i \neq P\_\{U(i)\}$$

even when $|a_i|^2 = |a\_\{U(i)\}|^2$ (which holds by the symmetry of the amplitude structure under G_cl). This means that a transformation U that is admissible—that maps the physical state space to itself—produces physically different probability predictions. The probability assignment is therefore not invariant under the symmetry group of the framework.

This violates (E2) in the strongest sense: it is not merely that normalisation fails or factorisation is broken, but that the theory predicts different outcomes under symmetry transformations that the framework identifies as physically equivalent. Such a theory is operationally inconsistent. ■

Remark: This argument shows that the entropy partition is not free to be chosen independently of the probability structure. Because the probability structure is already uniquely fixed, the entropy partition is constrained to carry the same symmetry group—and the unique partition with symmetry group G_cl is $\mathcal{P}\_cl$ itself.

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7. The No-Alternative Partition Theorem

We now collect the elimination results into a single theorem.

Theorem (Admissibility Closure of Entropy). *Let $\mathcal{S}[\mathcal{P}]$ be an entropy functional governing irreversible record formation in the VERSF framework. If $\mathcal{S}[\mathcal{P}]$ satisfies conditions (E1)–(E3), (E4), (E5′), and (E6′), then:*

$$\mathcal{S}[\mathcal{P}] = k_B \ln |\mathcal{P}_{cl}| = k_B \tilde{S}$$

That is, the physical entropy is uniquely the closure entropy, up to the conventional factor of k_B.

Proof.

Step 1 (Logarithmicity). By (E1) and the Khinchin characterisation, $\mathcal{S}[\mathcal{P}]$ must be of the form $c \ln |\mathcal{P}|$ for some $c > 0$. This eliminates Class I.

Step 2 (Partition identification). It remains to determine which partition $\mathcal{P}$ enters this logarithm.

- If $|\mathcal{P}| > |\mathcal{P}_{cl}|$: violates (E6′) and (E5′), as shown in §6.2.
- If $|\mathcal{P}| < |\mathcal{P}_{cl}|$: violates (E6′) and (E5′), as shown in §6.3.
- If $|\mathcal{P}| = |\mathcal{P}_{cl}|$ but $\mathcal{P} \neq \mathcal{P}_{cl}$: this case requires special treatment to close a potential loophole.

Closing the equal-cardinality loophole. Suppose $|\mathcal{P}| = |\mathcal{P}_{cl}|$ but $\mathcal{P} \neq \mathcal{P}_{cl}$. Since the partitions differ while having equal cardinality, there exist states x, y such that $x \sim_{\mathcal{P}} y$ (same class under $\mathcal{P}$) but $x \nsim_{\mathcal{P}_{cl}} y$ (different classes under $\mathcal{P}_{cl}$), and correspondingly states u, v with $u \sim_{\mathcal{P}_{cl}} v$ but $u \nsim_{\mathcal{P}} v$.

By the Equivalence-Class Dependence Lemma (§4.1), physical predictions depend on which states are grouped together, not merely on how many groups exist. The pair (x, y) is operationally distinguishable: by Theorem 4.3 (§3.6), $\mathcal{P}_{cl} = \mathcal{P}_{op}$, so $x \nsim_{\mathcal{P}_{cl}} y$ implies $x \nsim_{op} y$ directly, as a proved result rather than a consequence of the present theorem. Yet (x, y) are assigned equal entropy cost under $\mathcal{P}$—violating (E6′). The pair (u, v) is operationally indistinguishable: by the same Theorem 4.3, $u \sim_{\mathcal{P}_{cl}} v$ implies $u \sim_{op} v$. Yet they are assigned distinct entropy cost under $\mathcal{P}$—also violating (E6′).

For the probability violation: since $\mathcal{P} \neq \mathcal{P}_{cl}$ with equal cardinality, there exists at least one admissible symmetry $U \in G_{cl}$ that preserves closure classes but does not preserve $\mathcal{P}$-classes. Hence there exists an outcome i for which U(i) lies in the same closure class as i but a different $\mathcal{P}$-class, or vice versa. Therefore $\Delta S_i^{\mathcal{P}} \neq \Delta S_{U(i)}^{\mathcal{P}}$, while the amplitude structure remains symmetry-related ($|a_i|^2 = |a_{U(i)}|^2$ under G_{cl}), yielding $P_i \neq P_{U(i)}$ for configurations related by an admissible closure symmetry, which should therefore receive identical physical predictions—violating (E2) and (E3).

Therefore no partition of equal cardinality to $\mathcal{P}_{cl}$ but different equivalence structure is admissible.

Step 3 (Uniqueness). The only partition of the correct cardinality that does not violate any condition is $\mathcal{P} = \mathcal{P}_{cl}$ itself.

Step 4 (Unit matching). Steps 1–3 establish that the partition entering the physical entropy is uniquely $\mathcal{P}_{cl}$, and that the entropy must be of the form $c \ln |\mathcal{P}_{cl}|$ for some constant $c > 0$. The partition identification is structurally forced. The constant c is fixed to k_B by the requirement that $\mathcal{S}$ matches thermodynamic entropy in the classical limit, where $S = k_B \ln \Omega$ with $\Omega = |\mathcal{P}_{cl}|$. This is a conventional unit-matching between the information-theoretic natural-log scale and the thermodynamic Kelvin scale, fixed by the historical definition of the Boltzmann constant. It does not affect the partition identification, which is the structural content of the theorem.

Therefore $\mathcal{S}[\mathcal{P}] = k_B \tilde{S}$, and the theorem is proved. ■

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8. Consequence: η = 1

With the No-Alternative Partition Theorem established, the derivation of $\eta = 1$ is immediate.

Recall:

$\eta = S / (k_B \cdot \tilde{S})$

The theorem establishes that S and $k_B \tilde{S}$ count the same microstates—the closure classes of $\mathcal{P}_{cl}$. Therefore:

$\boldsymbol{\eta = 1}$

Two things are being claimed here and it is worth distinguishing them. The **partition identification**—that thermodynamic entropy and closure entropy count exactly the same microstates—is structurally forced by the No-Alternative Partition Theorem. The **k_B factor** is a conventional unit-matching between the information-theoretic scale and the thermodynamic Kelvin scale, fixed by the historical definition of the Boltzmann constant.

The statement "$\eta = 1$" is therefore a claim at both levels: structurally, the partitions coincide; conventionally, the unit scales are aligned. The non-trivial content—the part that is not merely a normalisation convention—is the partition identification. The k_B factor follows once that identification is made and the unit system is fixed.

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9. The Commitment Barrier is Structurally Fixed

From prior VERSF results, the primitive commitment barrier is:

$\Phi_c = \eta \cdot r$

where r is the dimensionless resolution parameter of the discrete spacetime structure, previously derived to satisfy $r = 1$ by separate admissibility arguments [VERSF-R].

With $\eta = 1$ now established:

$\boldsymbol{\Phi_c = 1}$

The commitment barrier is not a free parameter to be fit to data. It is a structural invariant of the framework: the cost of the minimal irreversible commitment event is exactly one natural unit of entropy, corresponding to one bit of physical distinguishability.

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10. Discussion

10.1 The Significance of the Proof Structure

The argument of this paper has a specific logical character that is worth making explicit. We have not shown that the closure entropy is a *good* entropy, or a *natural* entropy, or the *simplest* entropy. We have shown that every alternative is *contradictory*—that is, every alternative entropy assignment, taken seriously, leads to physical predictions that are internally inconsistent with other uniquely-derived structures of the framework.

This makes $\eta = 1$ a theorem in the strong sense: it cannot be otherwise without abandoning the framework entirely.

10.2 Relation to the Boltzmann Programme

The classical Boltzmann programme sought to derive thermodynamic entropy from the counting of microstates. The VERSF result sharpens this: the microstates that must be counted are not arbitrary—they are uniquely fixed by the admissibility conditions that govern physical distinguishability. The Boltzmann formula $S = k_B \ln \Omega$ is correct, but Ω is not a free input. It is the cardinality of the closure partition.

10.3 Implications for Information-Theoretic Interpretations

The identification $\eta = 1$ places VERSF squarely in alignment with the Landauer principle: one bit of logical irreversibility costs exactly $k_B \ln 2$ of thermodynamic entropy. In the VERSF framework, this is not a separate postulate but a corollary of the No-Alternative Partition Theorem, since the minimal closure unit is $\Delta\tilde{S}_{min} = \ln 2$ (condition E4).

10.4 Remaining Open Questions

The present result closes the entropy sector of the admissibility programme. Open directions include:

- **Dynamical entropy generation:** A full account of how entropy is generated during commitment events, beyond the static counting argument
- **Entropy in curved backgrounds:** Whether the closure partition is modified near strong curvature (relevant to black hole thermodynamics)
- **Multi-party entanglement:** How the No-Alternative theorem extends to systems with irreducible multi-party closure relations

10.5 Scope, Limitations, and Anticipated Objections

The argument of this paper operates at two levels — framework-independent and VERSF-specific — and the strength of the result at each level is distinct. The framework-independent result (that physical entropy must match the operational equivalence partition) is robust: it follows from R1–R4 alone and applies to any theory of irreversible record formation. The VERSF-specific result (that this partition is the closure partition) depends on the substrate structure of the void and the identification established in Theorem 4.3. Readers should not read the VERSF-specific conclusion as a claim about entropy in all physical theories; they should read it as showing what follows once a theory commits to the closure structure.

Five objections are sufficiently predictable to warrant pre-emption. They are addressed in detail in Appendix C. In brief: the argument is not circular (closure is defined substrate-side, independently of operational notions; Theorem 4.3 proves coincidence rather than assuming it); the completeness of closure is a consistency condition of the framework rather than an additional axiom; the scope claim is precisely calibrated to distinguish what is universal from what is VERSF-specific; R1–R4 are genuinely minimal in the sense that dropping any one of them allows pathological entropy assignments; and the relationship to Shannon–Jaynes uniqueness theorems is complementary rather than redundant, since those results fix the functional form of entropy given a partition, while the present result fixes the partition itself.

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11. Conclusion

We have removed the final conditional step in the VERSF entropy framework. The Class Identification Assumption—that the closure partition and thermodynamic partition coincide—is not an assumption. It is the unique solution to a system of physical admissibility constraints.

The No-Alternative Partition Theorem shows that:

1. Physical entropy must be logarithmic (by compositional additivity)

2. The partition entering that logarithm must be exactly the closure partition (by distinguishability consistency, probability compatibility, and the operational conditions required of physical records)

From this it follows that:

$\eta = 1, \Phi_c = 1$

The entropy-sector identification problem is now reduced to a no-alternative admissibility theorem within the VERSF framework. Together with prior results on probability structure, amplitude geometry, Hilbert space emergence, and spinor algebra, this means that each major structural element of the framework is determined by admissibility rather than assumption—a result that substantially strengthens the theoretical foundations of the programme.

Appendix A: The Khinchin Characterisation

For completeness, we recall the result used in Step 1 of the proof of Theorem 7.

Theorem (Khinchin-type characterisation). *The only monotone and measurable function $H : \mathbb{Z}^+ \to \mathbb{R}$ satisfying:*

1. *$H(mn) = H(m) + H(n)$ (multiplicativity → additivity)*
2. *$H(1) = 0$*

is $H(n) = c \ln n$ for some $c > 0$.

This result underlies the uniqueness of entropy in both classical statistical mechanics and information theory, and is the foundation for Step 1 of the No-Alternative Partition Theorem.

Appendix B: Consistency with Rényi and Tsallis Entropies

One might ask whether generalised entropies—Rényi entropy H_α or Tsallis entropy S_q—could be considered as candidates. The two cases require separate treatment.

- **Tsallis entropy** (S_q, $q \neq 1$) is explicitly non-additive for independent systems, satisfying instead a q-additivity relation that is incompatible with thermodynamic extensivity. It therefore violates (E1) directly and is excluded by the same argument as Class I.
- **Rényi entropy** (H_α) requires more careful handling. For product distributions, Rényi entropy is additive at fixed α, retaining logarithmic form. The admissibility issue is therefore not additivity alone. Rather, for $\alpha \neq 1$, Rényi entropy is not the entropy functional that appears in ordinary thermodynamic or Shannon–Landauer record-cost

settings: the α-weighted moment structure enters the probability suppression in a way that is incompatible with the uniquely fixed probability-weighting structure of the framework (E2). Specifically, the record-cost interpretation $P_i \propto \exp(-\lambda\, \Delta S_i)$ requires an entropy that enters linearly in the exponent with a consistent weighting across outcomes. Rényi entropy for $\alpha \neq 1$ depends on the full α-moment structure of the outcome distribution rather than a linear record-cost assignment over individual outcome classes. That makes it unsuitable as the primitive irreversible cost entering $P_i \propto \exp(-\lambda\, \Delta S_i)$, and introduces a self-referential dependence of the entropy on the very probabilities it is supposed to determine. This violates (E2) and (E3) on grounds distinct from non-additivity.

Both families are therefore excluded, though by different mechanisms. Their potential physical relevance in other contexts (non-extensive statistical mechanics, quantum information) lies outside the regime where the admissibility conditions of the present framework apply.

References

The following prior VERSF results are cited in this paper. Each entry is marked with the role it plays in the present argument. For submission, these descriptions should be replaced with full bibliographic entries.

[VERSF-BR] *Born-rule admissibility theorem.*
Establishes that the probability structure of the VERSF framework is uniquely fixed by the symmetry group G_cl of the closure partition. Used in §6.4 and §3.7. *(Replace with full citation.)*

[VERSF-CL] *Closure partition as operational record-equivalence partition.*
Proved internally in this paper as Propositions 4.1, 4.2 and Theorem 4.3 (§3.6), from primitive assumptions R1–R4 and the structural definition of the closure relation [VERSF-CL-prior]. No external citation required for the identification result itself.

[VERSF-CL-prior] *Structural definition of the closure relation.*
Defines the closure relation in terms of the algebraic symmetry structure of the void substrate — the orbit equivalence under the symmetry group of the pre-physical distinguishability algebra — independently of any measurement procedure. Used as the foundational input to Propositions 4.1 and 4.2. *(Replace with full citation to the VERSF paper in which the void substrate symmetry structure and closure relation are defined.)*

[VERSF-R] *Resolution parameter r = 1 derivation.*
Establishes by separate admissibility arguments that the dimensionless resolution parameter r = 1. Used in §9 to fix the commitment barrier Φ_c = 1. *(Replace with full citation.)*

[VERSF-SP] *Spinor algebra from closure symmetry.*
Establishes that spinor structure is algebraically forced by closure symmetry. Referenced in §1 admissibility programme table. *(Replace with full citation.)*

[VERSF-HS] *Hilbert space emergence.*
Establishes that Hilbert space geometry is uniquely derived within the framework. Referenced in §1 admissibility programme table. *(Replace with full citation.)*

[VERSF-AF] *Amplitude field necessarily ℂ.*
Establishes that the amplitude field must be complex rather than real or quaternionic. Referenced in §1 admissibility programme table. *(Replace with full citation.)*

[Khinchin-1957] A. I. Khinchin, *Mathematical Foundations of Information Theory*, Dover, 1957.
Provides the characterisation theorem used in §6.1 and Appendix A: the only monotone and measurable function $H : \mathbb{Z}^+ \to \mathbb{R}$ satisfying $H(mn) = H(m) + H(n)$ and $H(1) = 0$ is $H(n) = c \ln n$ for some $c > 0$.

Appendix C: Anticipated Objections

This appendix addresses five objections that a careful referee is likely to raise. They are stated in their strongest form and answered directly.

Objection 1 — Circularity: the closure relation is defined using operational notions, so Theorem 4.3 is trivial.

Response. The closure relation is defined substrate-side, in terms of which states generate the same stable closure configuration under admissible reversible evolution and irreversible commitment structure (Definition, §3.6). This definition refers to the dynamics of the void substrate — a structural, pre-physical notion — and makes no reference to observers, measurements, or outcome statistics. Two states may be closure-equivalent even if no observer has ever interacted with them.

The operational equivalence relation (D1, §3.3) is defined entirely differently: $x \sim_{op} y$ iff every admissible measurement procedure yields the same outcome statistics. This definition is observer-facing and makes no reference to substrate dynamics.

Theorem 4.3 proves that these two independently-defined relations coincide. The proof is non-trivial in both directions: Proposition 4.1 shows that substrate-level indistinguishability entails observational indistinguishability (using the completeness of the closure description as a consistency condition); Proposition 4.2 shows that any substrate-level distinction is operationally accessible (using R1 and R4). Neither direction is definitional. The two sides of the identification are genuinely separate, and the theorem bridges them.

Objection 2 — Completeness of closure: why must the substrate dynamics capture all physically accessible structure? This seems like an extra axiom.

Response. The completeness claim in Proposition 4.1 — that a distinction not registered in the closure configuration cannot be physically accessible — is not an additional axiom. It is a consistency condition of the VERSF framework.

VERSF is a framework in which physical reality emerges from the void substrate through its dynamics. By the framework's own commitments, there is no physical structure over and above what the substrate dynamics produce. A distinction not reflected in closure configurations would be a physically observable effect without a substrate-level cause — which is inconsistent with the framework's foundational claim that the substrate is the complete source of physical structure. A referee who rejects this consistency condition is not accepting VERSF as a framework; the objection is not to the proof but to the framework's premises.

This is no different from the situation in any foundational physical theory: one cannot object to a result in general relativity by denying that spacetime curvature is the source of gravitational effects. The completeness of the description is internal to the framework, not an additional assumption layered on top of it.

Objection 3 — Scope: you claim the R1–R4 result is framework-independent, but R1–R4 may themselves be VERSF-specific assumptions in disguise.

Response. Each of R1–R4 is stated in terms that are neutral with respect to any specific physical framework:

- R1 (Recordhood) asserts only that a record must be re-identifiable without continuous recreation. This is the ordinary notion of a stored record in any physical or information-theoretic context.
- R2 (Observability) asserts that meaningless physical distinctions — those producing no observable difference — should not enter physical descriptions. This is a standard operationalist commitment shared by quantum mechanics, thermodynamics, and information theory.
- R3 (Reversible Covariance) asserts that reversible processes do not create new physical facts. This is entailed by the definition of reversibility in any Hamiltonian or unitary framework.
- R4 (Local Retrievability) asserts that records without any admissible readout procedure are not physical records. This is a minimal anti-magic condition: physically real content must in principle be accessible.

None of these invoke the void substrate, closure structure, or any other VERSF-specific concept. The framework-independent result — that physical entropy must match the operational equivalence partition — holds in any theory satisfying R1–R4. The VERSF-specific contribution is Theorem 4.3: identifying *which* partition is the operational equivalence partition within

VERSF. That step genuinely depends on VERSF structure and is clearly labelled as such throughout the paper.

Objection 4 — Minimality of R1–R4: could we drop one of the assumptions and still get the result?

Response. Each assumption is load-bearing. Dropping any one allows pathological entropy assignments that the remaining three cannot exclude:

- *Without R1*: entropy could be assigned to dynamical features requiring continuous refresh. Lemma 3 (perturbative stability) and the record persistence argument fail.
- *Without R2*: entropy could distinguish states no experiment can separate. Lemma 4 — no entropy distinction without observable distinction — fails directly.
- *Without R3*: entropy could be generated by reversible dynamics, making record cost path-dependent rather than content-dependent. Lemma 1 and condition (E3) fail.
- *Without R4*: entropy could be assigned to hidden variables with no admissible readout. Lemma 2 and the key R1+R4 bridge in Proposition 4.2 fail.

R1–R4 are therefore not merely sufficient — they are individually necessary for the derivation. The set is minimal in the sense that no proper subset suffices.

Objection 5 — Relation to existing uniqueness results: Shannon (1948) and Jaynes already prove entropy is uniquely the logarithmic form. What does this paper add?

Response. The Shannon–Khinchin uniqueness theorems and the Jaynes maximum-entropy programme answer a different question from the one addressed here.

Shannon's theorem shows that, *given a probability distribution over a fixed set of outcomes*, the only measure of uncertainty satisfying certain regularity conditions is the logarithmic entropy. Jaynes' programme shows that, *given constraints*, the maximum-entropy distribution is the unique unbiased choice. Neither result addresses which outcomes should count as distinct microstates. Both take the partition of microstates as a given input.

This paper's contribution is to prove that this input is not free: the partition is uniquely forced by the operational distinguishability structure of the theory. In Shannon's language, the paper proves that the sample space is not arbitrary — it is the closure partition, and no other.

The two results are therefore complementary. Shannon fixes the functional form of entropy given the partition; the No-Alternative Partition Theorem fixes the partition itself. Together they fully determine the physical entropy functional. Neither subsumes the other.
