Finite Distinguishability and Dimensional Reduction:

Structural Consequences for Macroscopic Dynamics

For the General Reader

At its core, this paper asks a question about information: **what are the geometric consequences of the fact that physical records must be finite?**

Any real record — whether stored in a memory chip, a fossil, or a rock stratum — requires finite energy, finite time, and finite physical resources to write and to read. This is not a technological limitation; it is a fundamental constraint. And it turns out to have structural consequences for the geometry of space itself.

The two starting ideas are:

1. The laws of physics at the microscopic level are reversible. If you filmed the motion of individual particles and played the film backwards, the reversed motion would also be physically valid. This is not controversial — it is a cornerstone of both classical mechanics and quantum mechanics.

2. No physical process can make infinitely fine distinctions. Any real record requires finite resources to write and read. This places a floor on how small a "unit of distinction" can be. You cannot record a difference that costs more to distinguish than the universe can supply.

From just these two ideas, together with careful reasoning about the geometry of information storage, we derive something surprising: **in regions of space that are thin enough, three-dimensionally independent stable record structure becomes geometrically impossible.** The system is effectively forced to behave as if it only has two spatial dimensions available to it. We call this *dimensional reduction*, and we prove it as a theorem — not as an assumption or a model choice.

This theorem stands on its own as a result in the foundations of physics. It says something precise about what kinds of distinctions can be stably maintained in thin physical regions — independent of any particular theory of gravity, fields, or matter.

We then show that when this theorem is applied to galactic disk systems — using an additional assumption about how record structure connects to observable dynamics — it produces a concrete and measurable consequence: **a universal threshold in the surface density of matter at which the transition occurs, and from that threshold a characteristic gravitational**

acceleration scale emerges automatically. That acceleration scale is consistent with the observed scale at which galaxy rotation curves stop falling and go flat — one of the longest-standing puzzles in astrophysics, conventionally addressed by invoking an undetected dark matter component.

Within the VERSF programme, the phenomena conventionally attributed to dark matter can instead emerge from the geometry of record-keeping — from the structural impossibility of maintaining three-dimensionally independent information storage in sufficiently thin regions. No new particles are required. In the regime where dimensional reduction activates, the effective macroscopic field description can change because the admissible record algebra changes — which can look like modified dynamics at the observational level. This is an effective-theory modification, not a claim that GR is false microscopically. The structural theorem is a proof; the galactic application is a testable hypothesis once the bridge principle is adopted. The key prediction is whether $\Sigma_b(r_v)$ clusters (or varies in a structured way) across galaxies.

For readers with a physics background who wish to follow the technical argument, the main text begins at §1. For those who want to understand how this result connects to the broader VERSF theoretical programme, see the following section.

This Paper and the VERSF Programme

The Void Energy-Regulated Space Framework (VERSF) is a theoretical physics programme developing the hypothesis that time, space, and physical phenomena emerge from entropy dynamics on a fundamentally structureless substrate, rather than being built into the fabric of reality from the start. Within VERSF, concepts like the flow of time, the geometry of space, and even the behaviour of gravity are not taken as primitive inputs — they are derived outputs of deeper informational and thermodynamic processes.

The VERSF programme encompasses several interconnected papers: a scalar-tensor action framework with post-Newtonian predictions, an emergent time formalism, applications to quantum measurement, and cosmological consequences including early galaxy formation. These papers, taken together, are ambitious and wide-ranging. They make specific physical predictions, but they also rest on interpretive foundations — concepts like the void substrate, commitment intensity, and the irreversible flux of entropy — that are distinctive to VERSF and require the reader to engage with a substantial theoretical framework before evaluating the claims.

The present paper serves a different and complementary function within the programme. Its purpose is to establish a result that does *not* require the reader to accept any VERSF-specific interpretive machinery. The Dimensional Reduction Theorem proved here rests on two assumptions — reversible microdynamics and finite distinguishability — that are accepted across essentially all of theoretical physics, independent of any particular framework. The surface-density threshold and acceleration scale that follow are derived from partition geometry alone.

This matters for the programme in four specific ways:

It provides an independently verifiable foundation. The prediction that Σ_b at galaxy transition radii clusters near ~137 M⊙ pc^{-2} can be tested directly against existing rotation-curve datasets such as SPARC, with no VERSF-specific parameters. A positive result would constitute evidence for the geometric activation mechanism on its own terms, before any commitment to VERSF's broader interpretive framework. A negative result would be cleanly informative without destabilising the rest of the programme.

It separates what is proven from what is interpreted. One of the recurrent challenges in developing a new theoretical framework is that specific derivations become entangled with interpretive choices, making it hard for external readers to identify exactly where the mathematical content ends and the physical narrative begins. This paper draws that line explicitly. The theorem is the theorem; the VERSF interpretation (commitment intensity as the physical realisation of distinguishability; the void substrate as the source of v_0) is an application of the theorem, not part of it. That separation strengthens both the theorem and the broader programme.

It provides a reusable structural foundation for further VERSF papers. The Dimensional Reduction Theorem and its corollaries are proved here once, in a self-contained form, from assumptions that any VERSF paper can cite without re-derivation. This prevents the same core argument from appearing in multiple places in slightly different forms — a source of inconsistency that weakens theoretical programmes over time.

It demonstrates that VERSF's most striking empirical prediction — the emergence of a_0 — has a proof that is framework-independent. Within VERSF, the acceleration scale a_0 arises because commitment intensity (the density of irreversible records) determines when the Σ-lockdown transition occurs. This paper shows that the same conclusion follows from much weaker assumptions. The VERSF interpretation therefore becomes one physical realisation of a more general structural result, rather than an ad hoc construction designed to fit a particular observation. That is a stronger scientific position.

In summary: this paper is the logical and rhetorical foundation of the VERSF programme's engagement with galaxy phenomenology. It should be submitted first, so that subsequent VERSF papers can build on an established result rather than carrying an internal derivation.

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Abstract

We develop a structural consequence of finite distinguishability under reversible microdynamics. Assuming (i) measure-preserving microscopic evolution and (ii) the existence of a minimum admissible coarse-graining cell measure $v_0 > 0$, we derive a minimum spatial distinguishability scale ℓ_0 and prove that regions whose physical thickness falls below ℓ_eff cannot support independent three-dimensional stable record partitions. This enforces effective dimensional reduction in sufficiently sparse regimes. We further show that this structural reduction implies a universal surface-density threshold $\Sigma*$ and an associated characteristic acceleration scale $a_0 = 2\pi G\Sigma*$. All results are independent of specific gravitational field models and follow solely from admissible partition geometry under the stated axioms. The connection to gravitational phenomenology requires an additional operational closure assumption (§4.4), stated explicitly and separately from the geometric results. The scope of the results and their conditions of applicability are stated precisely in §7.

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1. Substrate Axioms

We work within a minimal axiomatic framework designed to be as broadly applicable as possible. No specific dynamical law, field theory, or gravitational model is assumed in the main body of the paper.

1.1 Axiom 1 — Reversible Microdynamics

Let **$(\Omega, \mathcal{S}, \mu)$** be a measurable microstate space equipped with a σ-finite measure μ. Let **$\Phi_t : \Omega \to \Omega$** denote the microscopic time evolution. We assume measure preservation:

$\mu(\Phi_t(A)) = \mu(A)$ for all measurable $A \subset \Omega$ and all $t \in \mathbb{R}$.

This is a Liouville-type property, expressing reversibility at the microscopic level. It does not preclude effective irreversibility at the macroscopic level, which arises through coarse-graining.

1.2 Axiom 2 — Finite Distinguishability

There exists a minimum admissible coarse-graining cell measure **$v_0 > 0$** such that no physically realisable record partition resolves sets of measure smaller than v_0.

Formally, for any admissible partition $\pi : \Omega \to \{1, \ldots, M\}$, the preimage cells $C_i = \pi^{-1}(i)$ satisfy:

$\mu(C_i) \geq v_0$ for all i.

This postulate encodes finite distinguishability under bounded physical resources — finite energy, finite time, and finite action available to the record-forming process. It does not fix v_0 to any particular value; the results below hold for any $v_0 > 0$.

1.3 Axiom 3 — Stable Records

A **record** is a macroscopic label $r : \Omega \to \{0, 1\}$ that satisfies the following stability condition: there exists a readout duration $\tau > 0$ and a failure threshold $\varepsilon \in (0, 1/2)$ such that

$\mathrm{Prob}[r(\Phi_t(\omega)) \neq r(\omega)] < \varepsilon$ for all $t \in [0, \tau]$ and μ-almost all ω.

That is, record basins must remain non-mixing under the forward dynamics Φ_t over the readout interval, with failure probability bounded strictly below one-half. Stable differentiation therefore requires partitions into non-mixing basins of measure $\geq v_0$.

Remark 1.1. The condition $\varepsilon < 1/2$ is the natural threshold for a binary record to carry positive information. The specific value of ε enters only in the stability analysis of Appendix B.4 and does not affect the main dimensional reduction argument.

1.4 Axiom 4 — Local Record Dynamics

There exists a finite **dynamical correlation length** $\xi(\tau, \varepsilon) > 0$ such that any record basin whose minimum linear dimension falls below ξ cannot satisfy the stability condition of Axiom 3 over the readout interval τ with failure probability $< \varepsilon$.

More precisely: $\xi(\tau, \varepsilon)$ is the minimum spatial scale over which the dynamics Φ_t can maintain distinct, non-mixing basins for time τ with failure probability $< \varepsilon$. It is determined jointly by the readout parameters (τ, ε) and the fastest available influence propagation in the system over that interval.

Axiom 4 is satisfied by any dynamics with finite signal propagation speed over the readout interval τ. If the maximum speed at which Φ_t can transport influence across a basin boundary is c_*, then $\xi(\tau, \varepsilon) \geq c_* \cdot \tau \cdot f(\varepsilon)$ for some positive function f of the failure threshold. Causal influence cannot mix two basins separated by more than $c_*\tau$ within the readout interval, so basins smaller than this scale cannot be guaranteed to remain non-mixing over τ under generic local dynamics. Axiom 4 is therefore an admissibility condition: it defines the minimum scale at which stable basins exist for the record channel being modelled. Axiom 4 does not require short-range forces; it requires only that influence cannot propagate arbitrarily far in arbitrarily small time.

Remark 1.2 — Independence from Axiom 1. Axiom 4 is independent of Axiom 1. Measure preservation (Liouville) places no constraint on the spatial scale of mixing: a measure-preserving flow can mix arbitrarily fine spatial regions arbitrarily quickly. Axiom 4 is therefore a genuinely separate, minimal locality statement. It is not derivable from reversibility alone.

Remark 1.3 — Gravity and collisionless dynamics. Two natural objections arise.

Does long-range Newtonian gravity violate Axiom 4? No — but the key point is not relativistic retardation. The relevant ξ in galactic disk systems is set by baryonic record-stability processes — molecular cloud formation, gravitational collapse, radiative cooling — not by the propagation speed of gravitational influence. These processes define a characteristic mixing timescale τ and a minimum stable spatial scale $\xi(\tau, \varepsilon)$ that are both finite and macroscopic. Finite signal speed c provides only the ultimate upper bound on ξ; in practice, baryonic dynamics sets a much larger effective ξ.

Does collisionless stellar dynamics violate Axiom 4? Collisionless systems (Vlasov dynamics) are measure-preserving and have formally infinite mixing time in some modes. However, the relevant question for Axiom 4 is not whether mixing eventually occurs, but whether it occurs within τ at scales below ξ. For the baryonic record processes relevant to galactic disks — molecular cloud formation, gravitational collapse, radiative cooling — the dynamical timescale for small-scale mixing is long compared to τ, so ξ remains well-defined and finite. Axiom 4 is applied to the record channel, not to the collisionless dark matter fluid.

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2. Lemma I — Minimum Spatial Distinguishability Scale

The cell measure v_0 is defined on phase space Ω, which in a mechanical system is a space of positions *and* momenta. To derive spatial consequences, we introduce an explicit and physically motivated mapping from a minimum phase-space cell measure to a minimum real-space cell volume.

2.1 Finite Control-Bandwidth Postulate

Physical record formation requires resources to resolve and stabilise a spatial configuration: finite energy, finite time, and bounded momentum control. We encode this through a **maximum resolvable momentum-space volume per record cell**, denoted $\mathbf{P_0}$.

Operational definition. P_0 is a property of the *record channel* — the physical process by which the partition label is written and read — not a property of the system being partitioned. Specifically:

P_0 is the largest momentum-space volume over which the readout map $r(\cdot)$ remains stable with failure probability $< \varepsilon$ over the readout duration τ.

Equivalently, P_0 is the momentum-space coarse-graining that the channel can sustain without confusion; finer momentum discrimination would require greater action per readout than the channel budget permits. This definition ties P_0 to the channel's impulse resolution — not to any property of a particular galaxy or environment.

Given this, each admissible phase-space cell satisfies:

$v\_cell \leq V\_cell \cdot P_0$

where V_cell is the associated real-space volume of the cell.

Remark 2.1. P_0 is a *maximum* (upper bound on momentum-space volume per stable cell). Coarser momentum resolution means larger P_0; finer resolution requires smaller P_0. Because P_0 is defined by the *channel physics* — the atomic and molecular processes that underpin stable record formation — and not by the galaxy hosting the records, a universal upper bound on P_0 across baryonic disk systems is physically motivated by the universality of the underlying readout physics. This is central to the universality argument in §5.3.

2.2 Lemma I — Statement and Proof

Lemma I (Spatial Distinguishability Volume Bound). *Under Axiom 2 and the finite control-bandwidth postulate, every admissible real-space record basin satisfies:*

$$\mathbf{V\_cell \geq V_0 \equiv v_0 / P_0}$$

Consequently, a minimum spatial distinguishability length exists:

$$\boldsymbol{\ell_0 \equiv V_0^{(1/3)} = (v_0 / P_0)^{(1/3)}}$$

Proof. By Axiom 2, every admissible phase-space cell satisfies $v\_cell \geq v_0$. By the finite control-bandwidth postulate, $v\_cell \leq V\_cell \cdot P_0$. Combining:

$v_0 \leq v\_cell \leq V\_cell \cdot P_0$

Dividing through by $P_0 > 0$:

$V\_cell \geq v_0 / P_0 \equiv V_0$. □

Remark 2.2. All model dependence is localised in the pair (v_0, P_0). The ratio v_0/P_0 is the only quantity that enters the spatial bound. Lemma I is therefore a structural result: it holds for any physical system admitting the two postulates, regardless of the specific values of v_0 and P_0.

Remark 2.3. The minimum length ℓ_0 is not a Planck-scale quantity by assumption. Its value depends on the physical context and the available control bandwidth. In macroscopic systems with limited control resources, ℓ_0 may be astronomically large relative to atomic scales.

3. Lemma II — Partition Capacity of a Finite-Thickness Slab

We now derive the geometric constraints that finite distinguishability places on spatial regions of finite physical thickness.

3.1 Geometric Setup

Let **D** $\subset \mathbb{R}^3$ denote a slab region of the form:

$D = A \times [-h/2, h/2]$

where **A** $\subset \mathbb{R}^2$ is a connected two-dimensional domain in the horizontal plane and $\mathbf{h > 0}$ is the vertical thickness. We assume that any admissible stable record partition of D consists of disjoint real-space cells, each of volume at least V_0, as established in Lemma I.

3.2 Geometric Regularity: Corollary of Axiom 4

Volume $\geq V_0$ alone does not prevent a cell from being a thin pancake — a cell of arbitrarily small vertical extent but large lateral area can satisfy $V\_cell \geq V_0$ without occupying any meaningful vertical thickness. This loophole is closed by Axiom 4 directly, without any additional geometric postulate.

Lemma 3.1 (Minimum Vertical Extent from Locality and Stability). *Under Axioms 3 and 4, every admissible stable record basin has minimum linear dimension* $d\_min \geq \xi(\tau, \varepsilon)$*. In particular, the vertical extent of any admissible basin is bounded below by* ξ*, regardless of its lateral extent.*

Intuition. A basin thinner than ξ in any direction is crossed and mixed by the local dynamics within the readout interval τ. Once mixed, the record label becomes unreliable before it can be read, violating Axiom 3. Thin "pancake" basins therefore cannot be stable records: they are too narrow to maintain distinct states against the local flow. Only basins with spatial footprint $\geq \xi$ in every direction can sustain the non-mixing condition required by Axiom 3.

Proof. By Axiom 4, any basin with minimum linear dimension $< \xi$ fails the stability condition of Axiom 3 over the readout interval $[0, \tau]$. Therefore no such basin is admissible. □

Corollary 3.2 (Effective Minimum Vertical Thickness). *Combining Lemma I (*$V\_cell \geq V_0$*, giving volumetric lower bound* $\ell_0 = V_0$^(1/3)*) and Lemma 3.1 (*$d\_min \geq \xi$*, giving locality lower bound* ξ*), the effective minimum vertical extent of any admissible stable record basin is:*

$\mathbf{d\_vert \geq \ell\_eff \equiv \max(\ell_0, \xi)}$

We keep $\ell_0 = (v_0/P_0)^{(1/3)}$ and $\ell_{eff} = \max(\ell_0, \xi)$ as distinct symbols throughout. The Σ-lockdown activation condition $h(r_s) = \ell_{eff}$ uses ℓ_{eff}, not ℓ_0 alone. Since both ℓ_0 and ξ are determined by channel physics and substrate properties rather than by galaxy-specific parameters, ℓ_{eff} is universal under the same conditions that make ℓ_0 and ξ individually universal (see §5.3).

Remark 3.1. Bounded aspect ratio is not assumed; it is a *consequence* of Axiom 4. The aspect ratio of any admissible cell is bounded above by $V_{cell} / (\xi^3) \leq V_{cell}/\xi^3$, which is finite. The thin-pancake loophole is therefore closed by the local dynamics axiom, not by geometric fiat.

3.3 Vertical Stacking Bound

Each admissible stable record basin has vertical extent at least ℓ_{eff} (Corollary 3.2). Therefore the maximum number of vertically independent admissible layers that can be stacked within D satisfies:

$$N_z \leq \lfloor h / \ell_{eff} \rfloor$$

where $\lfloor\cdot\rfloor$ denotes the integer floor function.

3.4 Lemma II — Statement

Lemma II (Vertical Stacking Bound). *If $h < \ell_{eff}$, then $\lfloor h / \ell_{eff} \rfloor = 0$, so $N_z = 0$. In particular, no two vertically independent admissible stable record basins can coexist within D.*

Consequently, admissible stable partitions of D cannot encode independent three-dimensional structure along the vertical direction.

Proof. By Corollary 3.2, each admissible stable basin has vertical extent at least ℓ_{eff}. If $h < \ell_{eff}$, the slab D cannot contain two disjoint basins with this minimum vertical extent without their combined extent $2\ell_{eff}$ exceeding h. Therefore no two admissible basins can be simultaneously disjoint and vertically separated within D. □

Remark 3.2. The bound $N_z \leq \lfloor h/\ell_{eff} \rfloor$ combines both the volumetric constraint (ℓ_0) and the locality constraint (ξ) through ℓ_{eff}. It depends on the microdynamics Φ_t only through $\xi(\tau, \varepsilon)$; in regimes where $\xi < \ell_0$, the bound is dominated by the volumetric term alone.

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4. The Dimensional Reduction Theorem

4.1 Definitions

Let $D = A \times [-h/2, h/2] \subset \mathbb{R}^3$ be a slab as above. An **admissible stable record partition** of D is a partition into disjoint subsets each satisfying:

- Real-space volume $\geq V_0$ (from Lemma I);
- Dynamical stability over the readout duration τ (from Axiom 3, with failure probability $< \varepsilon$).

The **induced partition on A** is defined by projecting any partition of D onto the horizontal base region A: a basin $B \subset D$ maps to its horizontal projection $B\_A \subset A$.

4.2 Theorem — Dimensional Reduction Under Finite Distinguishability

Theorem 4.1. *Assume Axioms 1–4, Lemma I, and Corollary 3.2. If the slab thickness satisfies $h < \ell\_eff$, then:*

(i) Any admissible stable record partition of D contains at most one independent layer in the vertical direction.

(ii) The induced partition on A uniquely determines the partition of D up to controlled partition error.

(iii) The macroscopic distinguishable record structure of D is fully representable by a partition of the two-dimensional base region A alone.

Proof.

Part (i). By Lemma II, $N\_z \leq \lfloor h/\ell\_eff \rfloor = 0$ when $h < \ell\_eff$. No two disjoint admissible cells can be stacked in the vertical direction within D.

Part (ii). Let π be any admissible partition of D. Since no two basins B_i, B_j with $i \neq j$ are vertically separated within D, the projection map $B \mapsto B\_A$ is injective on the basins of π. The induced partition π_A on A therefore uniquely identifies each basin of π.

Part (iii). By parts (i) and (ii), any macroscopic observable that depends only on the record structure of D — that is, on which record basin contains the system microstate — is fully determined by the projected partition π_A on A. The effective macroscopic record dynamics is therefore representable as dynamics on A. □

4.3 Interpretation

Theorem 4.1 does not assume any specific gravitational or field dynamics. It is a purely structural consequence of admissible partition geometry under finite distinguishability and local dynamics. Dimensional reduction emerges as a necessary constraint: it is not imposed by hand, not a symmetry assumption, and not a perturbative approximation. It follows from the inability to fit two vertically independent admissible record basins into a region thinner than ℓ_eff.

Remark 4.1 (Controlled error). Part (ii) asserts representability "up to controlled partition error." The theorem applies exactly in the idealised slab limit; real systems have varying $h(r)$. Define the relative thickness variation on an annulus of width Δr:

$\varepsilon_h \equiv \sup_{r \in \text{annulus}} |h(r) - \bar{h}| / \ell_{eff}$

where $\bar{h}$ is the mean scale height on the annulus. All 2D reduction conclusions hold up to fractional corrections of $O(\varepsilon_h)$. When $h(r)$ varies slowly ($\varepsilon_h \ll 1$), corrections are modest and the activation condition $h(r_h) = \ell_{eff}$ shifts radially by at most $\Delta r \lesssim \ell_{eff} / |\partial_r h|$, which is small wherever the scale height gradient is order-unity or smaller. Empirical estimation of ε_h from scale-height measurements at transition radii is part of the planned analysis described in Appendix F.

4.4 Bridge Principle: From Record Geometry to Effective Dynamics

The Dimensional Reduction Theorem establishes a constraint on admissible stable record partitions in regions where $h < \ell_{eff}$. It does not, by itself, alter microscopic field equations. To connect the structural result to macroscopic gravitational phenomenology, an additional assumption is required. We state it as a formal assumption, separate from and independent of Axioms 1–4.

Assumption BP (Operational Closure).

(i) The effective action governing macroscopic dynamics is defined on the σ-algebra generated by admissible stable records (Axiom 3).

(ii) Integrating out degrees of freedom that are not representable by stable record partitions over the readout interval τ yields an effective theory defined on the base manifold A of the stable record algebra.

Physical rationale. This is the standard logic of Wilsonian effective field theory: macroscopic laws govern the degrees of freedom that remain stable under coarse-graining. If a putative degree of freedom cannot form stable, non-mixing macrostates over τ, it is not a macroscopic state variable and does not appear in the effective action. Assumption BP is therefore not an epistemic claim about observers; it is a statement about which macrostates exist as stable dynamical sectors — the same logic by which phonons, not atomic coordinates, appear in solid-state effective actions. The claim is not that observers cannot see a 3D field; it is that the macroscopic state space available for stable, coarse-grained dynamics loses independent vertical degrees of freedom when $h < \ell_{eff}$, so the effective action cannot retain them.

Assumption BP does not remove microscopic degrees of freedom; it removes independent macroscopic state variables. When no stable, non-mixing partitions exist along a coordinate direction, that direction cannot support an independent macroscopic field component in the effective action. This is analogous to dimensional reduction in finite-temperature field theory, where compactification of the Euclidean time direction eliminates independent low-energy Matsubara modes: the compact direction still exists microscopically, but it no longer contributes independent macroscopic dynamical degrees of freedom at energies below the compactification scale.

Lemma BP.1 (Dimensional reduction of the effective theory). *If the only stable macrostates in a slab D with $h < \ell_{eff}$ are representable by partitions on A (Theorem 4.1), then under Assumption BP, any effective local functional of stable macrostates can be written as an integral over A with fields defined on A.*

Proof sketch. By Theorem 4.1, every admissible stable record basin in D projects injectively onto A. Under Assumption BP(ii), the effective action is a functional of these projected fields. Locality on A is assumed as an additional condition on the effective theory — integrating out vertical modes can in principle generate non-local terms on A, but these are suppressed by EFT power-counting at scales large compared to ℓ_{eff}. Under this assumption, locality of the original dynamics restricted to record-representable variables then gives a local functional on A. □

Lemma BP.1 licenses the replacement of 3D field equations by effective 2D field equations in the Σ-regime, as follows.

Assumption BP does not assert that integrating out the vertical direction eliminates its influence; it asserts that any residual influence must be encoded in local operators on A. Provided that no new long-range nonlocal kernel is generated at scales $\gg \ell_{eff}$ — which is the EFT power-counting condition stated in the proof sketch above — the dimensionality of the independent macroscopic field algebra is reduced. Vertical physics continues to affect the effective theory on A through its contribution to the local operator coefficients (e.g., renormalising λ in the 2D Poisson equation of §6.5), but it does not add independent field degrees of freedom beyond those supported by A.

Proposition 4.1 (2D Poisson and logarithmic Green's function). Proposition 4.1 is conditional: it shows that *if* the reduced effective theory is a local, isotropic, scalar potential theory with a second-order field operator, *then* the Green's function is logarithmic. Explicitly:

If the effective gravitational response in the reduced regime is mediated by a local scalar potential φ on A, and the effective theory is local, isotropic, and second-order (i.e., the field operator involves at most second derivatives of φ), then the effective field operator on A is the 2D Laplacian ∇^2_A, and the Green's function is:

$$G_A(r, r') = (1/2\pi) \ln|r - r'| + C$$

where C is an additive constant (fixed by boundary conditions; on bounded domains, up to harmonic functions determined by those conditions).

Proof. By the classification of isotropic second-order local operators on $\mathbb{R}^2$: the only operator satisfying locality, isotropy, linearity, and second-order in derivatives is (a multiple of) ∇^2_A. Its fundamental solution satisfies $\nabla^2_A G = \delta^2(r - r')$, giving $G = (1/2\pi) \ln|r - r'|$ up to a harmonic function, fixed by the boundary condition of vanishing field at spatial infinity on unbounded domains. □

Remark. We restrict to a scalar potential description because the thin-disk limit of GR reduces to a single scalar potential governing circular motion in the weak-field regime; tensor degrees of freedom are suppressed at the level of accuracy relevant for galactic rotation curves. The argument extends straightforwardly to any theory whose weak-field circular-motion limit is governed by a single scalar potential; this includes GR in the Newtonian regime. The second-order assumption rules out higher-derivative operators such as ∇^4 (which appear in AQUAL and related modified gravity theories). Such operators would alter the Green's function at scales comparable to their associated length scale. We assume higher-derivative corrections are suppressed at scales large compared to ℓ_{eff}; this is consistent with standard EFT power-counting in the 2D effective theory on A, and is an additional hypothesis whose violation would modify the functional form of the acceleration profile without invalidating the dimensional reduction theorem.

All gravitational phenomenology claims in §6 and Appendix C rely explicitly on Assumption BP, Lemma BP.1, and Proposition 4.1. Without these, Theorem 4.1 establishes a constraint on record geometry but does not alter the effective field equations. With them, dimensional reduction of the admissible record algebra translates directly into dimensional reduction of the macroscopic field theory, and the logarithmic Green's function follows from symmetry alone.

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5. Corollary I — Universal Surface-Density Threshold

5.1 Geometric Activation Condition and Transition Radii

Consider a disk-like physical system in which the vertical scale height h varies with radial position r. Represent a radial annulus at position r as a slab:

$D(r) = A(r) \times [-h(r)/2, h(r)/2]$

By Theorem 4.1, dimensional reduction activates where h(r) first falls to ℓ_{eff}. We define two distinct transition radii that must not be conflated:

Geometric transition radius r_h: the radius at which the vertical scale height equals the effective minimum spatial distinguishability scale:

$\mathbf{h(r_h) = \ell_{eff}}$

This is the *theoretically defined* activation radius. For $r > r_h$, effective dynamics on D(r) reduces to the 2D base region A(r); for $r < r_h$, full 3D record structure is supportable. The radius r_h is not directly observable because ℓ_0 is not independently measured; it must be inferred indirectly.

Observational estimator r_v: the radius inferred from the rotation curve as the point where $d \ln v / d \ln r$ first falls below a threshold δ (e.g., $\delta = 0.05$), signalling the onset of flat rotation. This is the *empirically accessible* quantity, but it is a lagged indicator — the rotation curve responds to the threshold crossing over a dynamical timescale, so $r_v \gtrsim r_h$ with an offset of order R_d for an exponential disk.

The extraction protocol in Appendix C treats r_v as an estimator for r_h with a corresponding systematic uncertainty. Future work should model the lag explicitly to sharpen the connection between the geometric prediction and the observational proxy.

5.2 Definition of the Surface-Density Threshold

Let $\Sigma_b(r)$ denote the baryonic surface density at radius r, directly observable from luminous matter profiles. Define the **threshold surface density** in terms of the geometric transition radius r_h (§5.1):

$$\boldsymbol{\Sigma_* \equiv \Sigma_b(r_h)}$$

This is the theoretical definition. In practice r_h is not directly observable; the observable proxy is $\Sigma_b(r_v)$, where r_v is the rotation-curve estimator defined in §5.1. Clustering of $\Sigma_b(r_v)$ across a galaxy sample is therefore the empirical test of whether Σ_* is universal. The gap between r_h and r_v introduces a systematic offset that should be modelled in any precision comparison.

5.3 Universality Scenarios (Hypotheses) and Test Plan

The theory does not *derive* universality of ℓ_{eff}. The theorem establishes that a geometric threshold exists wherever $h(r) = \ell_{eff}$; it does not show that ℓ_{eff} is the same across all disk systems. Whether Σ_* is universal is therefore an empirical question, not a theorem. The framework offers two falsifiable scenarios — either outcome is scientifically informative; failure of both falsifies the activation mechanism in disk systems.

The universality of Σ_* depends on whether $\ell_{eff} = \max(\ell_0, \xi)$ is universal across disk systems. Since ℓ_{eff} is set by two independent channel properties — the volumetric bound $\ell_0 = (v_0/P_0)^{(1/3)}$ and the locality bound $\xi(\tau, \varepsilon)$ — universality of ℓ_{eff} requires both to be approximately universal. In the regime where $\xi > \ell_0$, it is ξ that sets the threshold, and universality of ξ (through universality of the baryonic record channel dynamics) is the relevant condition. In disk galaxies, the appropriate τ is the characteristic baryonic record-stability timescale — the timescale over which a stable macroscopic record (e.g., a molecular cloud or gravitationally collapsed structure) forms and remains readable — rather than the dynamical orbital period, which governs collisionless matter but not the record channel. If τ scales with the local baryonic free-fall or cooling timescale — which itself scales weakly with surface density at the transition radius, and therefore varies little across disk systems at the point where $\Sigma_b \approx \Sigma_*$ — then $\xi(\tau, \varepsilon)$ and hence ℓ_{eff} remain approximately constant across galaxy types, supporting the strong universality case. Whether this scaling holds quantitatively is an open question, but it provides a concrete physical mechanism by which τ need not track galaxy mass strongly.

Hypothesis H1 (Strong universality). ℓ_eff is a fixed value across all baryonic disk systems, making Σ∗ a single universal constant.

Empirical signature: Σ_b(r_v) clusters tightly across diverse galaxy types (dwarfs, spirals, massive disks) with scatter attributable only to measurement noise, mass-to-light ratio uncertainty, and the r_h–r_v lag. No systematic trend with galaxy mass, metallicity, star formation rate, or morphology.

Hypothesis H2 (Weak universality). ℓ_eff varies between galaxy types because either ℓ_0 or ξ varies between environments.

For example, if the dominant baryonic record-forming processes differ between gas-rich dwarfs and red early-type disks, shifting either P_0 or the effective mixing timescale, then ℓ_eff varies systematically and so does Σ∗. The theory still predicts a well-defined Σ∗ per class of disk system, but with scatter following predictable trends.

Empirical signature: Σ_b(r_v) shows systematic trends with galaxy properties (e.g., linear scaling with metallicity, surface brightness, or gas fraction) consistent with identifiable variations in ℓ_eff.

Pre-registered falsification conditions:

Observation	Conclusion
Tight clustering of Σ_b(r_v) near 137 M⊙ pc^{-2}, no mass trend	Strong universality supported
Systematic trend in Σ_b(r_v) with galaxy mass or type	Weak universality; ℓ_eff varies; identify the driver
No clustering; Σ_b(r_v) broadly distributed	Σ-lockdown mechanism falsified in disk systems
Σ_b(r_v) clusters but at a value ≠ 137 M⊙ pc^{-2}	Mechanism supported; revise a_0 calibration

This pre-registration distinguishes between outcomes that would refine the theory and outcomes that would falsify it, preventing the analysis from being unfalsifiable by post-hoc adjustment.

5.4 Falsifiability

If empirical disk systems do not exhibit clustering of Σ_b at their transition radii, then either (i) ℓ_eff is not approximately universal in the relevant class of systems — because either v_0/P_0 or ξ varies significantly — (ii) the slab approximation fails systematically, or (iii) the effective dynamics is not record-dominated in the relevant regime. All three conditions are physically discriminable.

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6. Corollary II — Emergent Acceleration Scale

6.1 Thin-Disk Gravitational Scaling

For a thin disk of surface density Σ, the gravitational acceleration near the disk plane satisfies the scaling relation:

$a \sim 2\pi G\Sigma$

This follows from integrating Poisson's equation over the disk volume and is exact for an infinite uniform sheet; for finite exponential disks it holds up to order-unity geometric factors that depend on the ratio r/R_d.

6.2 Definition of the Acceleration Threshold

Evaluating the gravitational scaling at the universal surface-density threshold yields the **characteristic acceleration scale**:

$\mathbf{a_0 \equiv 2\pi G\Sigma_*}$

If Σ_* is universal (§5.3), then a_0 is universal by the same argument. This scale is not inserted as a phenomenological parameter: it emerges from the geometric activation of dimensional reduction under finite distinguishability.

6.3 The Structural Chain

The complete logical derivation can be stated as a chain:

Finite distinguishability (v_0) ⟶ maximum momentum bandwidth (P_0)
⟶ volumetric minimum scale $\ell_0 = (v_0/P_0)^{(1/3)}$
⟶ locality lower bound $\xi(\tau, \varepsilon)$ from Axiom 4
⟶ effective threshold $\ell_{eff} = \max(\ell_0, \xi)$
⟶ dimensional reduction when $h(r) < \ell_{eff}$
⟶ universal transition surface density $\Sigma_* = \Sigma_b(r_h)$
⟶ universal acceleration scale $a_0 = 2\pi G\Sigma_*$

Each arrow from Axioms 1–4 through the dimensional reduction step is a logical consequence of the preceding structural assumptions under those axioms and the finite control-bandwidth postulate alone. The subsequent identification of the acceleration scale as a macroscopic observable additionally requires Assumption BP and the symmetry hypotheses of Proposition 4.1 (§4.4).

6.4 Calibration Target and Empirical Test

The theory predicts that if Σ_* is universal, then $a_0 = 2\pi G\Sigma_*$ is also universal and will appear as a characteristic scale in galaxy rotation phenomenology. This is a *structural prediction*: the theory says such a scale must exist, and predicts that it is set by the Σ-lockdown threshold.

The observed acceleration scale in galaxy phenomenology — inferred from the flattening of rotation curves across many galaxy types — is empirically:

$a_0^{(obs)} \approx 1.2 \times 10^{-10}$ m s^{-2}

We use this observed value as a **calibration target**: inverting $a_0 = 2\pi G\Sigma_*$ gives the value of Σ_* the theory must produce in order to be consistent with observations:

$\Sigma_*^{(cal)} = a_0^{(obs)} / (2\pi G) \approx 0.286$ kg m$^{-2} \approx 137$ M⊙ pc^{-2}

This is not a free-parameter fit. It is a consistency check: the theory has no adjustable parameters in this expression. We do not claim to predict Σ_* from microphysics here; the numerical value 137 M⊙ pc^{-2} is a calibration target derived by inverting the observed a_0. The real empirical test is whether $\Sigma_b(r_v)$, measured directly from baryonic surface-density profiles at observed transition radii, clusters near this calibration value across a diverse galaxy sample. If it does, the theory is supported. If $\Sigma_b(r_v)$ clusters at a significantly different value — say 300 or 50 M⊙ pc^{-2} — then either the calibration of a_0 is wrong or the dimensional reduction mechanism is not the source of the observed flattening. See §5.4 and the pre-registered falsification conditions in §5.3 for the full decision tree.

6.5 Corollary — Baryonic Tully–Fisher Scaling (Outline)

In the dimensionally reduced (Σ) regime beyond r_s, the effective dynamics on A(r) is governed by a two-dimensional Poisson equation:

$\nabla^2_\Sigma \varphi = -\lambda \Sigma_b(r)$

where λ is an EFT coupling (with dimensions of acceleration per surface density) set by matching the 2D effective potential to the 3D Newtonian field at r_s. The resulting scalar-mediated acceleration scales as $a_\varphi(r) \sim \kappa\lambda M_b / (2\pi r)$, where κ is a dimensionless geometric coupling determined by the same matching, giving asymptotically flat circular velocity for $r \gg r_s$.

To recover the baryonic Tully–Fisher relation $v^4_{flat} \propto G a_0 M_b$, one requires $v^2_{flat} \approx r_s \cdot a(r_s)$, with $a(r_s) \approx a_0$ at the threshold. For an exponential disk $\Sigma_b(r) = \Sigma_0 \exp(-r/R_d)$, the transition radius satisfies:

$r_s = R_d \ln(\Sigma_0 / \Sigma_*)$ (when $\Sigma_0 > \Sigma_*$)

Expressing $\Sigma_0 = M_b/(2\pi R_d^2)$ and substituting the exponential disk Newtonian acceleration (expressible in terms of Bessel functions I_0, I_1, K_0, K_1 evaluated at $r_s/2R_d$) yields, in the limit where $r_s \gg R_d$:

$v^4\_flat \approx G a_0 M\_b + \mathcal{O}(R\_d/r\_s)$

The full analytic derivation, including the Bessel function matching at r_s and the sub-leading corrections, is deferred to future work. The present result establishes that the structural chain of §6.3 is *consistent* with Tully–Fisher scaling and identifies the one additional input required to close it: the disk Newtonian field at r_s.

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7. Scope, Limits, and Non-Claims

This section gives a precise account of what the preceding results do and do not establish. Clarity on scope is essential for the paper to serve its function as a structural foundation for further physical applications.

7.1 What Is Proven

Under Axioms 1–4 and the finite control-bandwidth postulate, the following results are established:

- **Lemma I:** Every admissible real-space record basin has volume $\geq V_0 = v_0/P_0$, defining a volumetric minimum length $\ell_0 = V_0$^(1/3).
- **Lemma 3.1 / Corollary 3.2:** Axiom 4 (local record dynamics) implies every admissible stable basin has minimum vertical extent $\geq \xi$; the effective threshold is $\ell\_eff = \max(\ell_0, \xi)$. Bounded aspect ratio is a corollary of locality and stability, not an independent postulate.
- **Lemma II:** A slab of thickness $h < \ell\_eff$ cannot contain two vertically independent admissible stable record basins.
- **Theorem 4.1:** Effective macroscopic record dynamics in a slab with $h < \ell\_eff$ reduces to two-dimensional partition structure on the base region A.
- **Corollary I:** A universal surface-density threshold $\Sigma*$ exists wherever the activation condition $h(r\_h) = \ell\_eff$ is met, under universality of ℓ_eff.
- **Corollary II:** A universal acceleration scale $a_0 = 2\pi G\Sigma*$ follows from thin-disk gravitational scaling evaluated at $\Sigma*$ (as a geometrically defined scale; its observational significance as the rotation-curve flattening scale requires Assumption BP).

7.2 What Is Not Proven

- No specific gravitational field equations, scalar–tensor realizations, or screening mechanisms are assumed or derived in the main body.
- No claim is made that finite distinguishability uniquely determines the dynamics of gravity or cosmology.
- The explicit values of v_0 and P_0 from microscopic physics are not derived here and remain an open problem.

- The application of Theorem 4.1 to specific astrophysical systems requires empirical validation; universality of Σ_* is a prediction, not a theorem.
- The full analytic derivation of Tully–Fisher scaling from the threshold matching (§6.5) is not completed here.

7.3 Conditions of Applicability

Theorem 4.1 and its corollaries apply only where:

1. The system admits stable record partitions in the sense of Axiom 3 with $\varepsilon < 1/2$;
2. The region of interest can be approximated locally as a slab of approximately uniform thickness h;
3. The relevant physical dynamics respects the admissible partition constraints imposed by Axioms 1–2.

In highly turbulent regimes, strongly non-planar geometries, or systems where record-stability conditions fail, no dimensional reduction conclusion is asserted.

7.4 Structural Position

The results of this paper are geometric and structural. They establish a consequence of admissible partition geometry under finite distinguishability. Any physical interpretation — gravitational, cosmological, or information-theoretic — is an *application* of the theorem, not part of the theorem itself. Appendix A provides one such application (a scalar–tensor embedding) as an existence proof; it is illustrative, not unique.

7.5 Relation to MOND-like Phenomenology and Modified Gravity Frameworks

The empirical phenomena addressed in this paper — flat rotation curves, the baryonic Tully–Fisher relation (BTFR), and the existence of a characteristic acceleration scale — have been extensively studied in the literature.

Modified Newtonian Dynamics (MOND) introduces a phenomenological acceleration scale a_0 and postulates an interpolation between Newtonian and deep-MOND regimes. Relativistic extensions such as AQUAL and TeVeS provide field-theoretic embeddings of this idea. More recent approaches (e.g., emergent gravity proposals) attempt to derive MOND-like scaling from entropic or holographic considerations.

The present framework differs in conceptual starting point and logical structure:

No interpolation function is introduced. The appearance of an acceleration scale follows from a geometric activation condition $h(r_h) = \ell_{eff}$, not from modifying the force law by hand.

The threshold is surface-density based. The fundamental structural quantity is the baryonic surface density at the geometric transition radius, $\Sigma_* = \Sigma_b(r_h)$, rather than acceleration per se. The acceleration scale $a_0 = 2\pi G\Sigma_*$ emerges secondarily.

Dimensional reduction mechanism. The mechanism proposed here is geometric and information-theoretic: when three-dimensionally independent stable record structure becomes impossible, the effective macroscopic field description reduces to two dimensions. MOND postulates modified dynamics; this framework postulates modified admissible coarse-graining.

Explicit structural theorem. The Dimensional Reduction Theorem is independent of any specific gravitational Lagrangian. Field-theoretic realizations (Appendix A) are existence proofs, not foundational inputs.

The present work does not claim to supersede MOND or its relativistic completions. Rather, it proposes a structural activation mechanism that could underlie MOND-like phenomenology, and makes a distinct empirical prediction: clustering (or structured variation) of $\Sigma_b(r_v)$ at transition radii.

Empirical discriminators. The following predictions differ from MOND and provide clean observational tests. The core conceptual distinction is this: MOND posits an acceleration interpolation; here the trigger is geometric ($h(r)$ crossing ℓ_{eff}), so two galaxies with the same acceleration field but different thickness profiles are not predicted to behave identically.

D1 — Thickness dependence. The activation condition $h(r_h) = \ell_{eff}$ depends explicitly on the disk vertical scale height. MOND does not. At fixed Σ_b profile, larger h shifts the activation radius outward because the condition $h(r_h) = \ell_{eff}$ is reached at larger r — this geometric dependence does not appear in acceleration-interpolation frameworks. This predicts a correlation between disk thickness and the radius at which rotation curves flatten that has no MOND counterpart. Scale height data for individual SPARC galaxies is limited; D1 will be tested on a thickness-measured subset, using published photometric scale heights where available and edge-on disk axis ratios as a proxy for the remainder.

D2 — Kinematic lag between r_h and r_v. The geometric transition radius r_h and the kinematically inferred transition radius r_v are distinct (§5.1), with the offset of order R_d set by the record-stability timescale τ. MOND's interpolation function does not naturally generate a lag between geometry-defined and kinematic-defined transition points. A systematic offset between the photometric transition and the kinematic flattening, scaling with R_d, would support the Σ-lockdown mechanism over a pure acceleration-interpolation picture. For a Milky Way-scale disk ($R_d \sim 3$ kpc), the expected offset $r_v - r_h \sim 0.5$–$1\ R_d$ corresponds to 1.5–3 kpc — resolvable at the radial resolution of existing SPARC rotation curves for nearby galaxies (~ 0.3–1 kpc per point at distances < 10 Mpc). The lag should scale with R_d across the sample, providing a regression test independent of the overall clustering signal.

D3 — Environment sensitivity via weak universality. If $\xi(\tau, \varepsilon)$ is set by baryonic process timescales, then star formation rate, gas fraction, and metallicity can shift ℓ_{eff} in predictable ways (Hypothesis H2 of §5.3). MOND's a_0 is conventionally taken as universal across environments. Structured variation of $\Sigma_b(r_v)$ correlating with gas fraction or metallicity — distinct from random scatter — would constitute evidence for weak universality in the Σ-lockdown framework and would be difficult to accommodate within standard MOND.

Whether this geometric activation picture offers explanatory or predictive advantages over existing modified gravity frameworks is an empirical question.

Appendix A — Example Physical Embedding (Scalar–Tensor Realization)

This appendix provides one explicit field-theoretic realization consistent with the structural results of the main text. The Dimensional Reduction Theorem does not depend on this realization; it is supplied as an existence proof that conventional gravitational dynamics can embed the structural threshold.

A.1 Example Action

$$S = \int d^4x \sqrt{-g} \left[(F(\varphi)/16\pi G)R - (1/2)(\partial\varphi)^2 - V(\varphi) \right] + S_m[\Psi; A^2(\varphi)g_{\mu\nu}]$$

Here φ is a scalar field, $A(\varphi)$ controls matter coupling, and $V(\varphi)$ may include environment-dependent screening structure. In high-density regimes, an effective mass $m_{eff}(\rho)$ suppresses scalar deviations from GR; in low-density regimes, scalar effects may become active.

A.2 Weak-Field Limit and PPN Parameters

In the weak-field limit, expanding around a background φ_0 with $\ln A(\varphi) = \alpha_0(\varphi - \varphi_0) + (1/2)\beta_0(\varphi - \varphi_0)^2 + \mathcal{O}[(\varphi - \varphi_0)^3]$, the leading Parameterized Post-Newtonian parameters are:

$$\gamma - 1 = -2\alpha_0^2 / (1 + \alpha_0^2) \approx -2\alpha_0^2 \quad (\text{for } \alpha_0^2 \ll 1)$$

$$\beta - 1 = (1/2)\, \alpha_0^2\, \beta_0 / (1 + \alpha_0^2)^2 \approx (1/2)\, \alpha_0^2\, \beta_0 \quad (\text{for } \alpha_0^2 \ll 1)$$

The Cassini constraint $|\gamma - 1| < 2.3 \times 10^{-5}$ then requires $\alpha_0 \lesssim 3.4 \times 10^{-3}$, consistent with chameleon screening suppressing α_0 to an effective value $\alpha_0^{(eff)} = \alpha_0\Delta$ where the thin-shell parameter $\Delta \ll 1$ in high-density environments. Specific PPN parameter values depend on the choice of coupling $A(\varphi)$ and are beyond the scope of the present paper.

A.3 Connection to the Dimensional Reduction Threshold

The transition radius r_h defined by $\Sigma_b(r_h) = \Sigma_*$ maps, in this realization, to the radius where the scalar effective mass m_{eff} drops sufficiently that scalar-mediated forces activate. The dimensional reduction is therefore physically encoded as a screening transition: inside r_h, GR is recovered; outside, the two-dimensional Σ-regime scalar dynamics govern circular velocities.

Appendix B — Detailed Proofs

This appendix provides formal versions of the proof sketches in the main body. No new assumptions are introduced beyond Axioms 1–4 and the finite control-bandwidth postulate.

B.1 Proof of Lemma I

Claim: $V_{cell} \geq V_0 \equiv v_0/P_0$.

By Axiom 2, every admissible phase-space cell satisfies $\mu(C_i) \geq v_0$, so $v_{cell} \geq v_0$. By the finite control-bandwidth postulate, $v_{cell} \leq V_{cell} \cdot P_0$. Therefore:

$$v_0 \leq v_{cell} \leq V_{cell} \cdot P_0$$

Dividing by $P_0 > 0$:

$$V_{cell} \geq v_0 / P_0 = V_0$$

Setting $\ell_0 = V_0^{(1/3)}$ completes the proof. □

B.2 Proof of Lemma II

Claim: If $h < \ell_{eff}$, no two vertically independent admissible stable basins exist in D.

By Corollary 3.2, each admissible stable basin has vertical extent at least $\ell_{eff} = \max(\ell_0, \xi)$, where $\ell_0 = V_0^{(1/3)}$ is the volumetric lower bound and ξ is the locality lower bound from Axiom 4. If $h < \ell_{eff}$, the slab D cannot contain two disjoint basins each of vertical extent $\geq \ell_{eff}$, since their combined vertical extent $2\ell_{eff} > h$ exceeds the slab thickness. Therefore no two admissible basins can be simultaneously disjoint and vertically separated within D. □

B.3 Proof of Theorem 4.1

Part (i) follows from Lemma II: $N_z \leq \lfloor h/\ell_{eff} \rfloor = 0$ for $h < \ell_{eff}$.

Part (ii): Let π be any admissible partition of D into basins $\{B_i\}$. Since no two basins are vertically separated within D (part i), the horizontal projection map $B_i \mapsto (B_i)_A$ is injective on the basins. The induced partition π_A on A therefore recovers π uniquely, up to the controlled error introduced by approximating the true boundary geometry as perfectly slab-like.

Part (iii): Any macroscopic observable depending only on which basin contains the current microstate ω is determined by $\pi_A(\omega)$. The effective macroscopic dynamics is therefore a dynamics on A alone. □

B.4 Stability Under Reversible Microdynamics

Under Axiom 1, Φ_t preserves μ. By Axiom 3, record basins satisfy $\text{Prob}[r(\Phi_t(\omega)) \neq r(\omega)] < \varepsilon < 1/2$ over $[0, \tau]$. If independent vertical layers existed under $h < \ell_{eff}$, they would require two disjoint admissible basins within D, which Lemma II prohibits. Therefore, dimensional reduction is dynamically stable under the measure-preserving microdynamics: no measure-preserving flow can restore vertical independence that partition geometry forbids. □

Appendix C — Application to Disk Systems

C.1 Activation and Transition Radius

For an exponential baryonic disk $\Sigma_b(r) = \Sigma_0 \exp(-r/R_d)$ with $\Sigma_0 = M_b/(2\pi R_d^2)$, the geometric activation condition $\Sigma_b(r_h) = \Sigma_*$ gives:

$$r_h = R_d \ln(\Sigma_0 / \Sigma_*)$$

This is defined only when $\Sigma_0 > \Sigma_*$; galaxies with central surface density below Σ_* lie entirely in the 3D regime and are not expected to show flat rotation curves driven by this mechanism. In the Bessel function matching of §C.2 we write r_s for the transition radius as a mathematical convenience; it should be understood as r_h.

C.2 Acceleration Scaling in the Σ-Regime

Using thin-disk scaling $a \sim 2\pi G\Sigma$:

$$a_0 = 2\pi G\Sigma_*$$

In the dimensionally reduced regime ($r > r_s$), 2D Green's function dynamics yields $a_\varphi(r) \propto 1/r$, producing asymptotically flat circular velocity. Matching at r_s and using $v^2_{flat} \approx r_s \cdot a_0$ reproduces the scaling $v^4_{flat} \propto G\, a_0\, M_b$ (to leading order for $r_s \gg R_d$); full derivation including Bessel function corrections is deferred to future work.

C.3 Empirical Extraction Protocol

1. For each galaxy in a rotation-curve sample, identify the observational transition radius r_v as the radius where $d \ln v / d \ln r$ first falls below a small threshold δ (e.g., $\delta = 0.05$).
2. Compute $\Sigma_b(r_v)$ from the combined stellar + gas surface-density profile, corrected for inclination and distance, with a mass-to-light ratio Υ_* treated as a per-galaxy fit parameter.
3. Test whether $\Sigma_b(r_v)$ clusters near the predicted value $\Sigma_* \approx 137\ M_\odot\ pc^{-2}$ across the sample, accounting for the systematic offset between r_v and r_h (of order R_d) when comparing to the theoretical threshold.

A statistically significant clustering near Σ∗ supports the geometric activation mechanism. A broad, mass-dependent distribution would falsify strong universality and motivate investigation of ℓ_{eff} dependence.

C.4 Self-Consistency of the Activation Radius Under Thickness Feedback

The activation condition $h(r_h) = \ell_{eff}$ determines the geometric transition radius. However, the vertical scale height h(r) is itself determined by vertical hydrostatic equilibrium in the gravitational potential. If dimensional reduction modifies the effective radial potential beyond r_h, it may also modify h(r), potentially shifting the activation radius.

This introduces a potential feedback loop:

1. Assume an initial baryonic disk model with vertical scale height $h_0(r)$.
2. Solve $h_0(r_h) = \ell_{eff}$ to obtain an initial transition radius $r_h^{(0)}$.
3. Compute the modified effective potential in the Σ-regime $r > r_h^{(0)}$.
4. Recompute the vertical equilibrium profile $h_1(r)$ under the updated potential.
5. Solve $h_1(r_h^{(1)}) = \ell_{eff}$ and iterate until convergence.

A consistent activation mechanism requires that this iteration converge to a stable fixed point r_h^*. Define the update map $F: r_h^{(n)} \mapsto r_h^{(n+1)}$ from steps 3–5 above. A sufficient condition for convergence is that F is a contraction near the fixed point:

$$|F'(r_h^*)| < 1$$

where $F'(r_h)$ is the derivative of the update map with respect to the input transition radius. Intuitively: if a small increase in r_h leads to a smaller increase in the updated r_h', the iteration is stable. If $F'(r_h) > 1$, the feedback is runaway — each updated potential pushes h(r) further from the original profile, driving r_h systematically outward or inward without settling.

Failure to converge, or systematic drift of r_h with galaxy parameters in contradiction to the predicted universality scenarios of §5.3, would falsify the Σ-lockdown hypothesis in disk systems. Computation of $F'(r_h)$ requires specifying a vertical equilibrium model (isothermal sheet, stellar hydrostatic balance, or gas pressure equilibrium) and is deferred to future work.

A full dynamical treatment of this feedback loop lies beyond the scope of the present structural paper. The purpose here is to identify the geometric activation condition and the observable surface-density test. Quantitative closure of the feedback problem is deferred to future work.

Appendix D — Parameter Anchoring and Scale Estimates

D.1 From Observed Acceleration Scale to Σ∗

Given $a_0 \approx 1.2 \times 10^{-10}$ m s^{-2} and $G \approx 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}:

$\Sigma_* = a_0 / (2\pi G) \approx 0.286$ kg m^{-2} ≈ 137 M⊙ pc^{-2}

D.2 Inferring ℓ_eff

If Σ_* corresponds to the activation condition h(r_h) = ℓ_eff, then ℓ_eff may be estimated from disk vertical scale-height measurements at r_h. Typical disk scale heights at r_h ~ 5–10 kpc are of order h ~ 0.3–1 kpc, suggesting:

ℓ_eff ~ 0.3–1 kpc

Since ℓ_eff = max(ℓ_0, ξ), this estimate constrains whichever of ℓ_0 or ξ is dominant in the galactic context. If ξ > ℓ_0 — as expected when the baryonic mixing timescale sets the effective threshold — then this range directly estimates ξ. In the opposite limit, using ℓ_eff ≈ $\ell_0 = (v_0/P_0)^{(1/3)}$, an empirical estimate of ℓ_eff provides a constraint on the fundamental ratio v_0/P_0:

$v_0 / P_0 \sim \ell\_\mathrm{eff}^3 \sim 10^{-2} - 1$ kpc^3

Microphysical plausibility of ξ ~ 0.3–1 kpc. If ξ is the dominant term in ℓ_eff, its value should be consistent with baryonic record-stability processes at galactic disk transition radii. The natural estimate is:

ξ ~ v_bary · τ_bary

where v_bary is a characteristic baryonic velocity scale (molecular cloud velocity dispersion, turbulent sound speed, or cooling front propagation speed) and τ_bary is the characteristic record-stability timescale (molecular cloud collapse or cooling time). At the outer disk conditions relevant to the transition radius:

- v_bary ~ 5–20 km s^{-1} (turbulent/thermal velocity in the cold neutral medium and giant molecular cloud envelopes)
- τ_bary ~ 10–100 Myr (giant molecular cloud free-fall or cooling timescale in outer disk conditions)

giving ξ ~ v_bary · τ_bary ~ 0.05–2 kpc, consistent with the observational estimate ℓ_eff ~ 0.3–1 kpc. The plausibility range is not fine-tuned: it follows from standard ISM conditions at the disk transition region and does not require any parameter to take an unusual value. This does not constitute a derivation of ℓ_eff from microphysics — that remains an open problem — but it shows that the inferred ℓ_eff is mechanistically reasonable.

D.3 From Astrophysical Observables to Foundational Parameters

The chain:

a_0 (observed) → Σ_* → ℓ_eff (from disk h at r_h) → max(ℓ_0, ξ) → v_0/P_0 or ξ provides a pathway from macroscopic astrophysical observables to the fundamental distinguishability parameters of the framework. Independent estimation of P_0 from microphysical considerations (minimum action cost of a stable baryonic record) would then constrain v_0 directly — connecting the astrophysical threshold to the quantum information-theoretic substrate.

Appendix E — Idle and Driven Record Formation Rates

A natural question arises when comparing the framework to laboratory physics: atomic and molecular systems routinely exhibit femtosecond-scale irreversibility — far faster than the galactic dynamical timescales relevant to dimensional reduction. Does this not imply that ℓ_0 should be sub-atomic rather than kiloparsec-scale?

The answer lies in distinguishing the *idle* commitment rate from the *driven* commitment rate of a physical system, and in recognising that P_0 is a property of the record channel, not the observed rate.

Idle commitment rate. Every physical system coupled to an environment produces irreversible records at a floor rate set by ambient decoherence — thermal fluctuations, vacuum coupling, background radiation. This rate is non-zero but typically very low in isolated macroscopic systems far from active readout apparatus. It sets a background record density $\mathcal{C}_{idle}$.

Driven commitment rate. When a measurement apparatus actively reads out a system — opening a high-bandwidth irreversible export channel — the rate of record formation can increase dramatically, toward a ceiling set by the fastest available channel (e.g., the atom-photon coupling rate in an optical cavity). This gives a driven record density $\mathcal{C}_{driven} \gg \mathcal{C}_{idle}$.

Key point. What determines ℓ_0 and therefore the Σ-lockdown threshold is not the *rate* of record formation but the *momentum bandwidth* P_0 available to stable records — a property of the channel architecture, not the channel throughput. A femtosecond laser pulse creates records very rapidly but does not change P_0; it changes how often records are created, not the minimum spatial scale at which they can be stably localised.

Consequence for galactic systems. In the low-commitment-density outer regions of a galactic disk, the dominant record-forming processes are baryonic — gravitational dynamics of gas and stars, radiative cooling, molecular cloud formation. The minimum phase-space cell volume v_0 for a stable baryonic macroscopic record is vastly larger than for a laboratory quantum record: it corresponds to the phase-space region required to stably localise a molecular cloud, not an atom. This large v_0 dominates the ratio v_0/P_0, making $V_0 = v_0/P_0$ large even if P_0 is also large, and therefore making $\ell_0 = V_0^{(1/3)}$ correspondingly large — consistent with the kiloparsec-scale ℓ_{eff} inferred from galactic disk heights at transition radii.

To be explicit about the scaling: a femtosecond laser record has a tiny v_0 (quantum phase-space cell) and a large P_0 (high momentum bandwidth), giving a tiny ℓ_0. A galactic baryonic record has an enormous v_0 (macroscopic phase-space cell) and a moderate P_0 (gravitational/molecular bandwidth), giving a large ℓ_0. The ratio v_0/P_0 — not P_0 alone — controls ℓ_0, and it is v_0 that differs by many orders of magnitude between the two contexts.

Two cautions. (1) Driving a system with a measurement apparatus changes the recording *rate*, not the invariant propagation speed or the causal cone structure. Nothing in the driven-commitment picture implies that information or causal influence propagates faster. (2) One cannot "create more time" by measuring more rapidly. What changes under driven readout is the frequency of irreversible updates in a subsystem, not the external time parameter or the structure of the causal order.

A full treatment of how P_0 depends on the physical record channel — including the contrast between laboratory and astrophysical contexts — is deferred to future work on the microphysical anchoring of the distinguishability parameters.

Appendix F — Statistical Analysis Plan: SPARC Test of Σ_b(r_v) Clustering

This appendix pre-registers the planned statistical analysis for testing the Σ-lockdown mechanism against the SPARC rotation curve dataset (Lelli et al. 2016). The plan is stated before analysis to prevent post-hoc parameter adjustment.

F.1 Dataset

The SPARC sample (Spitzer Photometry and Accurate Rotation Curves) provides 175 late-type galaxies with measured rotation curves and 3.6 μm surface photometry. We restrict to galaxies with inclination $30° < i < 80°$ (to control projection systematics) and at least 5 measured rotation curve points beyond $r = 2R_d$, leaving an expected subsample whose size will be determined by applying the selection cuts to the current SPARC catalogue. If fewer than 50 galaxies satisfy the selection criteria, the clustering test will be flagged as underpowered and the null model comparisons reported as exploratory rather than confirmatory.

F.2 Identification of r_v

The observational transition radius r_v is defined as the smallest radius at which:

$$d \ln v_c / d \ln r < \delta = 0.05$$

This threshold corresponds to rotation curves declining slower than $v_c \propto r^{0.05}$ — essentially flat to within 5% per decade in radius, consistent with the observational definition of a "flat" rotation curve used in the MOND and RAR literature. The choice $\delta = 0.05$ is the primary analysis value; robustness will be tested against $\delta = 0.03$ and $\delta = 0.08$ as secondary checks.

The criterion is evaluated on a smoothed rotation curve (Gaussian kernel width $\sigma = 0.3R\_d$). Sensitivity to this choice will be tested with $\sigma = 0.2R\_d$ (higher resolution, more noise sensitivity) and $\sigma = 0.5R\_d$ (smoother, less sensitive to small-scale features) as robustness checks. Results that change substantially between kernel widths will be flagged as unreliable for that galaxy. If no radius satisfying the criterion exists within the observed radial range, the galaxy is classified as "not yet transitioned" and excluded from the clustering test (but retained for upper bound analysis).

F.3 Surface Density Measurement

The baryonic surface density Σ_b(r_v) is computed as:

$$\Sigma\_b(r\_v) = Y_* \cdot \Sigma\_\bigstar(r\_v) + \Sigma\_gas(r\_v)$$

where Σ_★ is the stellar surface density from 3.6 μm photometry, Σ_gas from H I + H_2 maps where available, and Y_* is the stellar mass-to-light ratio. We use two treatments:

Fixed Y∗: $Y_* = 0.5$ M⊙/L⊙ (standard 3.6 μm value from McGaugh & Schombert 2014). *Fitted Y∗:* Y_* fitted per galaxy to minimise residuals in the inner Newtonian regime ($r < r\_v$), treated as a nuisance parameter.

F.4 Clustering Statistic and Null Model Comparison

The primary statistic is the log-variance of Σ_b(r_v) across the sample:

$$S \equiv \mathrm{Var}[\log_{10} \Sigma\_b(r\_v)]$$

Rather than comparing S against fixed thresholds, the analysis will evaluate S against three null models designed to test whether any observed clustering is mechanistically connected to the transition radius rather than an artifact of smoothing or sample selection:

Null 1 (shuffled r_v): Assign each galaxy a randomly drawn r_v from the sample distribution, breaking the coupling between Σ_b and the geometric transition.

Null 2 (shuffled Σ_b profiles): Preserve each galaxy's r_v but assign it a randomly drawn surface-density profile from another sample galaxy.

Null 3 (ΛCDM baseline): Synthetic rotation curves generated from NFW + disk models fitted to each galaxy's baryonic profile, without a Σ-lockdown mechanism.

The primary conclusion will be stated as: "We observe clustering tighter than null model N by a factor X (or: we cannot reject null model N at significance p)." The decision tree of §F.6 uses these effect-size comparisons as its empirical basis.

F.4b Uncertainty Propagation

Measurement uncertainties in $v(r)$, distance, inclination, and Υ_* will be propagated via bootstrap resampling (500 iterations per galaxy). Each bootstrap draw perturbs the rotation curve within its measurement errors, resamples the distance within its uncertainty, and resamples inclination within ±5°. The resulting distribution of S and of the mean $\log_{10} \Sigma_b(r_v)$ across bootstrap iterations yields 68% and 95% confidence intervals reported alongside the point estimates.

F.4c Robustness Grid (Pre-registered)

The full analysis pipeline will be repeated across the following parameter grid:

Parameter	Values
Slope threshold δ	0.03, 0.05, 0.07
Smoothing kernel σ	0.2R_d, 0.3R_d, 0.4R_d
r_v alternative	"first r where v within 5% of outer median"
Υ_* treatment	fixed (0.5), fitted per galaxy

Conclusions that change qualitatively across this grid will be flagged as parameter-sensitive and excluded from the primary result claims. Only conclusions that survive the full grid will be reported as robust.

F.5 Systematic Offsets

The geometric transition radius r_h and the kinematic estimator r_v are expected to differ by approximately R_d (§5.1). We correct for this by computing Σ_b at both r_v and r_v − R_d and reporting both. The primary test uses r_v; the corrected estimate is secondary.

F.6 Pre-registered Decision Tree

Conclusions are based on comparing S against the null models of §F.4 and on the mean of $\log_{10} \Sigma_b(r_v)$. The effect-size thresholds below are illustrative; the primary criterion is rejection of null models at $p < 0.05$ in two-sided tests.

Outcome	Conclusion
S significantly below all three nulls; mean near 137 $M_\odot\ pc^{-2}$	H1 (strong universality) supported
S below nulls 1–2 but with trend on galaxy type	H2 (weak universality); identify ℓ_eff driver
S below nulls 1–2 with trend on gas fraction or metallicity	H2 with environment-sensitive ξ
S indistinguishable from all three nulls	Σ-lockdown mechanism not supported in this sample
Mean Σ_b(r_v) clusters but significantly ≠ 137 $M_\odot\ pc^{-2}$	Mechanism supported; recalibrate a_0

This decision tree is pre-registered and will not be modified post-analysis. All code used for the SPARC analysis will be publicly released at the time of publication.