From Fold Closures to Bessel Modes: A Continuum Description of Particle Structure in the VERSF Framework

Keith Taylor
VERSF Theoretical Physics Program

For the General Reader

Physics has long described matter using two very different languages. One language — the language of fields — says that particles are like ripples: stable disturbances in something spread throughout all of space. The other language — the language of information — says that the deepest level of reality may not be made of stuff at all, but of *distinctions*: irreversible records of one thing being different from another.

This paper is about showing that these two languages are saying the same thing.

In the framework developed here, the universe at its deepest level is built from structures called **folds** — the minimal committed boundaries between one distinguishable state and another. When many folds organize into a stable closed pattern, something particle-like emerges. Not a tiny ball of matter, but a persistent knot of committed structure. The particle doesn't live *inside* the substrate any more than a whirlpool lives inside the water. It emerges *from* the water's motion — and it persists because its topology, the way it is knotted, cannot be undone by small disturbances.

A companion paper — *3D String Theory: Electromagnetic Structure Theory of Fundamental Particles* (Taylor, EST) — independently proposes that particles are stable configurations of twisted electromagnetic fields, and derives their radial profiles using mathematical functions called **Bessel functions** — profiles that concentrate near a core and decay radially away from it, becoming vanishingly small at large distances. That paper rigorously establishes the mathematical properties of these solutions, derives an energy-scaling law that relates the energy scaling to particle mass under a working hypothesis, and proposes concrete experiments using "twisted light" — beams of light that spiral as they travel — to test whether particles really have this twisted internal structure.

This paper shows that these two pictures are deeply connected. When you zoom out from the microscopic world of folds and their closure dynamics and ask what the large-scale pattern looks like, you get a nonlinear wave equation of the same structural class as the one EST starts from. When you ask that equation for its stable, rotationally symmetric solutions, you get the same class of Bessel-shaped profiles that EST derives. The two theories therefore explore the same mathematical structure from different levels of description.

The Bessel function, in other words, is not a fundamental ingredient of nature. It is the large-scale *shadow* of something deeper: a stable knot of committed distinctions, seen from far away. EST finds the shadow; VERSF shows how a discrete informational substrate could cast it.

Table of Contents

Section	Title
Abstract	Technical summary
1	Introduction
2	The Fold–Closure Ontology of VERSF
2.1	Folds as Minimal Physical Structures
2.2	Particles as Emergent Closure Structures
3	Coarse-Graining the Fold Network
3.1	From Discrete Graph to Effective Field
3.2	Discrete Fold Dynamics and Continuum Limit
3.3	Two Dynamical Regimes: Derivation from the Two-Variable Closure System
4	Closure Topology and Angular Winding
4.1	Rotationally Symmetric Closure Configurations
4.2	The Radial Equation
5	Emergence of Bessel Solutions
5.0	Note on Logical Structure
5.1	Identification with the Modified Bessel Equation
5.2	Physical Character of the Bessel Profile
5.3	Reinterpretation of Bessel Particle Models
5.4	The n=0 Sector and the Screened Green-Function Hierarchy
6	Physical Interpretation
6.1	Mass from Closure Stabilization
6.2	Topological Ladder Structure and the Generation Spectrum
6.3	Continuum Limit of the Closure Stability Functional
7	Outlook
Refs	References (internal and external)

Abstract

The Void Energy–Regulated Space Framework (VERSF) proposes that particles are not fundamental objects but stable topological structures that emerge from fold-closure dynamics in a discrete distinguishability substrate. A separate line of theoretical work identifies particle solutions with a class of twisted-field configurations whose radial profiles are governed by modified Bessel functions. This paper establishes a formal bridge between these two pictures. We show that coarse-graining the fold-closure network yields an effective nonlinear field equation whose linearized localized radial solutions can be interpreted as modified Bessel functions $K_n(\kappa r)$, where κ is set by the closure stabilization scale, the propagation speed, and the mode frequency. Bessel modes are therefore not fundamental: they can be interpreted as continuum signatures of discrete fold-closure topology, with the winding index n identified with the discrete topological winding of the fold boundary. The explicit coarse-graining map making this identification precise is deferred to future work. This reinterpretation suggests a path toward deriving particle stability, mass hierarchy, and field equations directly from informational substrate dynamics.

---

1. Introduction

Modern particle physics operates primarily within two conceptual traditions. The Standard Model treats particles as excitations of relativistic quantum fields, described by solutions to covariant wave equations. Information-theoretic approaches, by contrast, suggest that entropy, distinguishability, and commitment capacity may constitute the deeper substrate from which these field structures emerge.

The VERSF belongs to the second tradition. Its fundamental ontological element is not matter or energy but **committed distinguishability** — the irreversible record of a distinction having been made. Physical structure, in this view, is not substrate but *pattern*: the organized topology of such commitments.

This raises a natural question: how might a discrete informational ontology give rise to the continuous field equations that successfully describe particle phenomenology?

A useful point of comparison is provided by the companion work *3D String Theory: Electromagnetic Structure Theory of Fundamental Particles* (hereafter EST). EST explores a class of continuum field models in which particle-like configurations arise as localized solutions of nonlinear field equations with cylindrical symmetry. In particular, the framework studies solutions of nonlinear Klein–Gordon–type equations whose radial structure is governed by modified Bessel functions of the second kind. These solutions take the form:

$$\mathbf{f(r) = B\, K_n(m_0 r)}$$

where n is an integer winding number associated with the orbital angular momentum of the field configuration.

Within the EST framework, these Bessel-mode solutions are investigated as candidate models for particle structure. The paper develops the mathematical properties of the solutions, examines the convergence of the associated energy integrals, explores scaling relations of the form $I_n \propto n^a$, and proposes experimental tests based on interactions with structured light carrying orbital angular momentum.

Importantly, EST presents this continuum description as a phenomenological framework: the scalar field φ representing particle structure, the relation between field energy and particle mass, and the assignment of specific winding numbers to known particles are introduced as working hypotheses rather than derived consequences of a deeper microscopic model.

The purpose of the present paper is to explore whether the VERSF fold-closure framework can provide such a microscopic basis.

Specifically, we investigate whether coarse-graining the discrete fold network underlying VERSF naturally yields a continuum closure field whose localized solutions resemble the Bessel-mode structures studied in EST. The goal is not to reproduce the EST framework in full detail, but to determine whether the key mathematical structures it employs — nonlinear wave dynamics, cylindrical symmetry, and modified Bessel radial profiles — arise naturally from fold-closure dynamics.

The closure order parameter φ introduced in this analysis is the coarse-grained deviation of local fold-closure participation from the background state of the committed distinguishability graph. It is not identified with any specific physical field of the Standard Model — not the electromagnetic field, the Higgs field, nor any gauge or matter field of the Standard Model Lagrangian. The result established here is structural: fold-closure dynamics generically produce a continuum nonlinear wave equation of a class whose localized cylindrical solutions take modified Bessel form. The physical interpretation of that structure — and its relationship to specific observed particles and fields — is a further question that the present analysis does not address.

The analysis proceeds in five steps:

1. Review the fold-closure ontology of VERSF (§2)
2. Derive an effective field equation by coarse-graining the fold network (§3)
3. Show that rotationally symmetric closure configurations satisfy a Bessel radial equation (§4–5)
4. Interpret the resulting solutions in VERSF terms, including mass and stability (§6)
5. Compare the resulting continuum structures with those studied in EST (§5.3)

In this sense, the relationship between the two frameworks is exploratory rather than prescriptive. EST provides a continuum phenomenology of Bessel-mode particle structures;

VERSF investigates whether such structures can emerge naturally from a discrete informational substrate.

---

2. The Fold–Closure Ontology of VERSF

2.1 Folds as Minimal Physical Structures

In the VERSF framework, the foundational physical structure is a **fold** — the minimal boundary at which a distinguishability relation transitions from reversible (Void-side) to irreversible (record-side). A fold is not an act or an occurrence; it is a standing committed boundary in the distinguishability graph, persisting as a structural element of Σ.

A fold carries the following properties:

- **Informational content:** one bit of committed distinguishability
- **Orientation:** inherited from the asymmetry between the reversible Void dynamics and the irreversible committed record
- **Locality:** each fold is a boundary structure at a specific locus in the distinguishability graph

The totality of committed folds and their admissible relations constitutes the **committed distinguishability graph**:

$$\Sigma = (\mathbb{F}, \mathscr{E})$$

where $\mathbb{F}$ is the set of folds and $\mathscr{E}$ is the set of admissible relational edges between them.

2.2 Particles as Emergent Closure Structures

Stable particles emerge when folds self-organize into **closed topological configurations** — patterns in which every fold boundary is internally connected. Rather than being substructures located *within* the graph Σ, particles are emergent phenomena: they arise *from* the dynamics of Σ and persist above it as stable organizational patterns.

Formally, a closure configuration $\mathcal{C}$ is characterized as a dynamically stable pattern generated by Σ, satisfying three conditions:

Condition	Statement
Structural stability	The subgraph $\mathcal{C}$ is invariant under small local commitment updates
Capacity compatibility	$\mathcal{C}$ satisfies the Bit Conservation and Balance (BCB) constraints on distinguishability capacity
Transportability	$\mathcal{C}$ can propagate coherently above the Void substrate without dissolution

Closure configurations are **topological** in character. A particle's identity is determined not by what it is made of but by the topology of its closure pattern — the way its fold boundaries wrap around one another.

This is directly analogous to how vortices in a fluid are characterized by winding number rather than material content. A vortex is not embedded in the water as a separate object; it *emerges from* the water's dynamics as a stable organizational pattern. In the same way, a particle in VERSF does not sit inside the fold substrate — it emerges from fold-closure dynamics as a persistent topological pattern above the substrate.

---

3. Coarse-Graining the Fold Network

3.1 From Discrete Graph to Effective Field

The fold network Σ is fundamentally discrete. Individual folds are localized committed boundaries. However, at scales much larger than the individual fold spacing, large collections of folds admit a **coarse-grained continuum description**.

Define the **coarse-grained closure order parameter**:

$\varphi(x)$ ≡ signed deviation of local closure participation from the unstructured background state at position x

Interpreting φ as a signed deviation — rather than a raw density — is essential. It measures whether the local fold structure is organized above or below the equilibrium background, and it is with respect to this background that the Z_2 symmetry $\varphi \to -\varphi$ holds: the two equivalent closure orientations relative to a local reference state contribute with opposite signs. This field is a continuum approximation to the discrete graph Σ, valid in the limit where spatial variations occur on scales much larger than the fold lattice spacing *a*.

3.2 Discrete Fold Dynamics and Continuum Limit

We begin from the discrete dynamics at node *i* of the committed distinguishability graph $\Sigma = (\mathbb{F}, \mathscr{E})$. Let $\varphi_i(\tau)$ denote the coarse closure order parameter at node *i* — the local signed deviation from the unstructured background. Three physical ingredients govern its evolution: nearest-neighbour propagation across admissible graph edges, local linear relaxation toward the background state, and nonlinear saturation due to finite closure capacity. The leading-order node equation consistent with these assumptions is:

$$\mathbf{d\varphi_i/d\tau = D_0 \Sigma_j A_{ij}(\varphi_j - \varphi_i) - \mu^2\varphi_i - \lambda\varphi_i^3 + \rho_i} \quad \text{— (3.1)}$$

where A_{ij} is the adjacency matrix of the fold graph, D_0 is the microscopic propagation coefficient, $\mu^2 > 0$ is the linear closure-restoring parameter, $\lambda > 0$ enforces finite-amplitude saturation, and ρ_i is any external sourcing. The first term is precisely the graph Laplacian flow:

$$(L_\Sigma\ \varphi)_i = \Sigma_j\ A_{ij}(\varphi_j - \varphi_i) \text{ — (3.2)}$$

so equation (3.1) becomes:

$$d\varphi_i/d\tau = D_0(L_\Sigma\ \varphi)_i - \mu^2\varphi_i - \lambda\varphi_i^3 + \rho_i \text{ — (3.3)}$$

Continuum limit. Assign node *i* spatial position **x**, with neighbours at displacements δ_α, so that the graph Laplacian term reads:

$$(L_\Sigma\ \varphi)(x) = \Sigma_\alpha\ [\varphi(x + \delta_\alpha) - \varphi(x)] \text{ — (3.4)}$$

Expanding in Taylor series under the slow-variation assumption (φ varies on scales ≫ *a*):

$$\varphi(x + \delta_\alpha) = \varphi(x) + \delta_\alpha\text{^}m\ \partial_m\ \varphi + ½\ \delta_\alpha\text{^}m\ \delta_\alpha\text{^}n\ \partial_m\ \partial_n\ \varphi + O(a^3) \text{ — (3.5)}$$

Substituting into (3.4) and invoking statistical isotropy of the fold adjacency distribution — justified here by the assumption that the irreversible commitment process in VERSF is governed by local BCB capacity constraints, which are scalar quantities and therefore cannot single out a preferred spatial direction in the graph — yields two key identities:

$$\Sigma_\alpha\ \delta_\alpha\text{^}m = 0 \text{ (drift vanishes) — (3.6)}$$

$$\Sigma_\alpha\ \delta_\alpha\text{^}m\ \delta_\alpha\text{^}n = (za^2/d)\ \delta\text{^}mn \text{ — (3.7)}$$

where *z* is the mean degree and *d* the spatial dimension. Substituting (3.7):

$$(L_\Sigma\ \varphi)(x) = (za^2/2d)\ \nabla^2\varphi(x) + O(a^3) \text{ — (3.8)}$$

Defining the continuum propagation coefficient:

$$D \equiv D_0\ za^2/2d \text{ — (3.9)}$$

the discrete propagation law converges to:

$$D_0(L_\Sigma\ \varphi) \longrightarrow D\nabla^2\varphi \text{ — (3.10)}$$

Origin of μ^2. The parameter μ^2 enters as the energetic penalty for maintaining closure amplitude away from the unstructured background. Bounded fold connectivity and finite closure capacity imply that long-wavelength closure coherence is not cost-free: deviations relax with characteristic **closure healing length $\xi = \mu^{-1}$**. Thus the continuum equation retains the restoring term $-\mu^2\varphi$.

Origin of the cubic nonlinearity. Individual folds carry orientation (§2.1), so the discrete potential $V(\varphi_i)$ is not automatically even in φ_i. However, the Z_2 symmetry $\varphi \to -\varphi$ holds at the level of the coarse-grained continuum field for the following reason: the signed order parameter $\varphi(x)$ measures deviation of local closure participation *relative to a local reference orientation*.

The fold network has no global preferred orientation — each local region of the graph defines its own reference by the background unstructured state — so the two equivalent closure orientations relative to any local reference contribute equal and opposite deviations. This means the coarse-grained dynamics are invariant under simultaneously reversing the reference orientation and changing the sign of φ, which is the continuum Z_2 symmetry. Under this symmetry odd terms in the potential expansion vanish, leaving:

$$\mathbf{V(\varphi) = \tfrac{1}{2}\,\mu^2\varphi^2 + (\lambda/4)\varphi^4 + O(\varphi^6),\ \lambda > 0} \quad \text{— (3.11)}$$

The quartic potential is the lowest-order stabilizing form compatible with this symmetry. Its gradient is:

$$\mathbf{\partial V/\partial\varphi = \mu^2\varphi + \lambda\varphi^3} \quad \text{— (3.12)}$$

Gradient-flow relaxation gives:

$$\mathbf{\partial_\tau\ \varphi = D\nabla^2\varphi - \partial V/\partial\varphi + \rho} \quad \text{— (3.13)}$$

Substituting (3.12) into (3.13) recovers the cubic restoring term $-\lambda\varphi^3$ explicitly. Higher even powers of φ in V are subleading in the weak-closure regime. This argument holds at the continuum level; a discrete fold network with strongly broken local orientation symmetry would require separate treatment and could produce additional odd terms not captured here. In the VERSF setting, the expectation is that the fold network falls in the weakly-broken regime: the BCB capacity constraints (which govern fold dynamics) are energetic and scalar, and do not select a global orientation for the distinguishability graph. Individual folds carry local orientation, but no large-scale orientational order is induced by the commitment dynamics in the absence of external fields. This is an assumption of the present analysis — a derivation of the symmetry class of the VERSF fold potential from first principles is left for future work.

The continuum closure field equation is therefore **not postulated independently** of the fold ontology; it is the long-wavelength limit of discrete fold propagation, local closure restoration, and finite-capacity saturation on the committed distinguishability graph.

3.3 Two Dynamical Regimes of the Closure Field

The coarse-grained fold network supports two physically distinct dynamical regimes, both derivable from a single underlying two-variable description.

The two-variable closure system. The node equation (3.3) tracks only the closure order parameter φ. It produces a first-order relaxational equation — appropriate when the fold network equilibrates diffusively. However, propagating particle-like modes require a second-order wave structure. The promotion from first to second order requires an additional physical assumption about the fold network: that it carries not only a participation level φ but also a conjugate propagation variable π — the rate of change of closure participation — which is an independent degree of freedom, not slaved to φ.

The physical motivation is as follows. The discrete fold graph stores two types of organization at each node: the current closure participation (φ), and the rate at which organized closure is propagating into or out of that node (π). A network that stores only the first gives diffusive dynamics; a network that stores both gives wave dynamics. The wave regime arises when the fold network has sufficient internal degrees of freedom to carry propagating disturbances — analogous to how a transmission line (with both voltage and current as independent variables) supports wave propagation while a pure resistive network does not. Whether the VERSF fold network satisfies this condition is a physical hypothesis that the present analysis takes as an input; verifying it from the discrete fold dynamics is left for future work.

With this assumption, introducing π as the independent propagation variable, the fold dynamics at the continuum level take the form:

$$\partial_\tau \varphi = \pi \quad (3.14)$$

$$\partial_\tau \pi = c^2_\varphi \nabla^2\varphi - \mu^2\varphi - \lambda\varphi^3 - \gamma\pi + \rho \quad (3.15)$$

where c_φ is the closure wave speed inherited from the fold propagation structure, and $\gamma \geq 0$ is a damping coefficient that captures dissipation from the irreversibility of committed fold boundaries. Eliminating π by substituting (3.14) into (3.15) yields the single second-order equation:

$$\partial^2_\tau \varphi + \gamma\, \partial_\tau \varphi - c^2_\varphi \nabla^2\varphi + \mu^2\varphi + \lambda\varphi^3 = \rho \quad (3.16)$$

This is the **general closure field equation**. It contains both dynamical regimes as limiting cases.

Relaxational regime ($\gamma \gg 1$). When damping dominates — as during equilibration of the fold network — the second-order term $\partial^2_\tau \varphi$ is negligible relative to $\gamma\, \partial_\tau \varphi$. Dividing through by γ and absorbing constants into rescaled D, μ, λ, and ρ recovers:

$$\partial_\tau \varphi = D\nabla^2\varphi - \mu^2\varphi - \lambda\varphi^3 + \rho \quad (3.17)$$

This is the **real Ginzburg–Landau equation** derived in §3.2. It describes how the fold network equilibrates toward a stable closure configuration. In the VERSF context, the order parameter is the local closure deviation, and the symmetry breaking is the irreversible commitment of distinguishability relations.

Wave regime ($\gamma \to 0$). When damping is weak — as for coherently propagating closure structures that correspond to particles — equation (3.16) reduces to:

$$\partial^2_\tau \varphi - c^2_\varphi \nabla^2\varphi + \mu^2\varphi + \lambda\varphi^3 = 0 \quad (3.18)$$

This wave equation supports genuine oscillatory solutions and is the appropriate description for stable, propagating closure structures. The continuum closure field equation is therefore not postulated independently of the fold ontology — it is the long-wavelength limit of discrete fold

propagation, and the two physical regimes emerge from a single underlying two-variable model depending on the strength of dissipation.

---

4. Closure Topology and Angular Winding

4.1 Rotationally Symmetric Closure Configurations

Closure configurations with rotational symmetry are characterized by a **winding number** n — the integer counting how many times the fold boundary winds around the closure core as the azimuthal angle θ traverses a full 2π circuit.

Within VERSF, n is the **closure topology index**: a discrete invariant that cannot change under continuous deformations of the closure structure. Its origin in the fold network is as follows: in the discrete graph, n corresponds to the net winding of fold orientations along the closure boundary — the cyclic ordering of committed distinguishability directions as one traverses the boundary circuit. This discrete count is a topological invariant of the fold adjacency structure. The claim that this discrete winding maps onto the continuous angular phase factor $e^{in\theta}$ in the coarse-grained continuum order parameter is the key assumption of this section. The explicit construction of the coarse-graining map from discrete graph winding to smooth phase rotation — showing how a combinatorial count on the fold graph becomes a continuous field-theoretic phase — is not performed here; it is left as a necessary item for future work. The ansatz (4.1) below should therefore be understood as the most natural continuum representation of a configuration with discrete winding index n, not as a derived consequence of the coarse-graining procedure developed in §3. The integer n is inherited from the fold boundary topology in the sense that the topology class is preserved; the precise form of the continuum phase factor requires the coarse-graining map to be made explicit.

For a closure configuration of winding class n, the natural fully-separated cylindrical ansatz is:

$$\boldsymbol{\varphi(r, \theta, z, \tau) = f(r) \cdot e^{in\theta} \cdot e^{ikzz} \cdot e^{-i\omega\tau}} \quad \text{— (4.1)}$$

where f(r) is the real radial profile function, $n \in \mathbb{Z}$ is the topological index, k_z is the axial wavenumber, and ω is the temporal oscillation frequency of the closure configuration.

4.2 The Radial Equation

Particle modes are propagating coherent structures, so we work in the wave regime. Substituting the ansatz (4.1) into the source-free linearized form of (3.18),

$$\boldsymbol{\partial^2_\tau \varphi - c^2_\varphi \nabla^2\varphi + \mu^2\varphi = 0}$$

using the cylindrical Laplacian $\nabla^2 = \partial^2/\partial r^2 + (1/r)\partial/\partial r + (1/r^2)\partial^2/\partial\theta^2 + \partial^2/\partial z^2$ yields:

$$-\omega^2 f = c^2_\varphi (f'' + (1/r)f' - (n^2/r^2)f - k^2_z f) - \mu^2 f \quad (4.2)$$

Rearranging:

$$f''(r) + (1/r) f'(r) - (n^2/r^2) f(r) - (k^2_z + (\mu^2 - \omega^2)/c^2_\varphi) f(r) = 0 \quad (4.3)$$

Defining:

$$\kappa^2 \equiv k^2_z + (\mu^2 - \omega^2)/c^2_\varphi \quad (4.4)$$

this becomes:

$$f''(r) + (1/r) f'(r) - (n^2/r^2 + \kappa^2) f(r) = 0 \quad (4.5)$$

This equation has a transparent physical structure:

- The term $f''(r) + (1/r)f'(r)$ is the radial Laplacian in cylindrical coordinates
- The term $-(n^2/r^2) f(r)$ is the centrifugal barrier from topological winding
- The sign of κ^2 determines the solution character: $\kappa^2 > 0$ (localized, $\omega < \mu$) selects decaying Bessel modes; $\kappa^2 < 0$ selects oscillatory modes

κ therefore plays the role of an effective radial screening parameter: its inverse κ^{-1} sets the spatial scale over which the closure configuration maintains coherence before dissolving into the background fold network.

For stable, localized closure structures we require $\omega < \mu$ so that $\kappa^2 > 0$ and the radial profile decays away from the closure core.

---

5. Emergence of Bessel Solutions

5.0 Note on Logical Structure

Note on the logical structure of the argument. The appearance of modified Bessel functions in the following analysis is not an independent result — it is automatic once the wave equation (3.18) is written down and cylindrical separability is imposed. Any continuum field equation of Klein–Gordon type in cylindrical coordinates yields modified Bessel radial profiles. The non-trivial content of the present paper lies entirely in step (a): the derivation, in §3, of the closure field equation from discrete fold-network dynamics. Step (b) — that cylindrical solutions of that equation take Bessel form — is a mathematical fact that requires no additional VERSF-specific input. This structure should be kept in mind when evaluating the claims of §§5–6: the physical substance is in the coarse-graining derivation; the Bessel form follows from it by standard analysis.

A related point: many alternative discrete dynamics would produce a Klein–Gordon-type continuum equation and therefore the same Bessel solutions. The VERSF derivation is one path to this structure, not the unique one. What distinguishes it is the interpretation of the parameters (μ as closure stabilization scale, n as topological winding index, κ^{-1} as closure coherence radius) and the specific physical assumptions that lead to the Z_2-symmetric potential and the wave regime. These interpretive commitments are what give the Bessel solutions their VERSF meaning; they are not consequences of the Bessel form itself.

5.1 Identification with the Modified Bessel Equation

Equation (4.5) is precisely the **modified Bessel equation of order n**:

$$\mathbf{f''(r) + (1/r)\, f'(r) - (n^2/r^2 + \kappa^2)\, f(r) = 0}$$

with $\kappa^2 = k^2_z + (\mu^2 - \omega^2)/c^2_\varphi > 0$ for localized modes. Its two linearly independent solutions are $I_n(\kappa r)$ and $K_n(\kappa r)$.

The requirement of localization selects the physically admissible solution: $I_n(\kappa r)$ diverges as $r \to \infty$ and is therefore rejected. The localized closure profile is:

$$\mathbf{f(r) \propto K_n(\kappa r)} \quad \text{— (5.1)}$$

where K_n is the modified Bessel function of the second kind of order n.

5.2 Physical Character of the Bessel Profile

The modified Bessel function $K_n(\kappa r)$ has several properties that are physically transparent in the VERSF interpretation:

- **Exponential decay for large r:** $K_n(\kappa r) \sim \sqrt{(\pi/2\kappa r)} \cdot e^{(-\kappa r)}$ as $r \to \infty$, encoding the finite range of closure coherence
- **Algebraic divergence at r = 0:** $K_n(\kappa r) \sim (n-1)!\, (2/\kappa r)^n / 2$ as $r \to 0$ for $n \geq 1$, an apparent singularity at the closure core
- **κ^{-1} sets the coherence radius:** the effective size of the particle as a closure structure

The apparent divergence at r = 0 is an artefact of the linearized, source-free approximation used to derive the Bessel equation. The Bessel form applies outside the nonlinear core region. Near r = 0, the cubic term $\lambda\varphi^3$ and the microscopic core structure of the fold network become significant, modifying the solution and removing the apparent singularity. The full nonlinear treatment replaces the Bessel profile with a regularized core whose size is set by the BCB capacity constraints.

The parameter $\kappa^{-1} = (k^2_z + (\mu^2 - \omega^2)/c^2_\varphi)^{(-1/2)}$ is therefore the **closure coherence radius** — the scale over which the fold-closure structure maintains its organized topology before dissolving into the background fold network. For purely radial modes ($k_z = 0$), this reduces to $\kappa^{-1} =$

$c_\varphi/\sqrt{(\mu^2-\omega^2)}$, making the coherence radius a direct function of the wave speed and the gap between the closure stabilization scale and the mode frequency.

5.3 Reinterpretation of Bessel Particle Models

The analysis above shows that localized rotationally symmetric solutions of the closure wave equation take the form:

$$\mathbf{f(r) \propto K_n(\kappa r)}$$

where K_n is the modified Bessel function of the second kind and κ is determined by the closure stabilization scale, the propagation speed of the closure field, and the oscillation frequency of the mode.

Mathematically, this radial equation is identical to the modified Bessel equation that appears in a number of continuum field models with cylindrical symmetry. In particular, the same class of solutions plays a central role in the EST framework, where localized Bessel profiles arise as solutions of nonlinear Klein–Gordon–type equations describing twisted field configurations.

The present analysis suggests a possible interpretation of that mathematical structure from the perspective of the VERSF framework. This interpretation is structural rather than identificatory: the VERSF closure field φ is not being claimed to be the same physical field as the scalar field studied in EST or any Standard Model field. The claim is that fold-closure dynamics produce a field equation of the same structural class, whose localized cylindrical solutions belong to the same family of radial profiles.

Within VERSF, the Bessel profile does not represent a fundamental object but rather the continuum envelope of an underlying discrete closure configuration. The winding number n appearing in the radial equation corresponds naturally to the topological winding index of the fold boundary in the discrete closure pattern. Under coarse-graining, this discrete winding structure manifests as the angular phase factor $e^{in\theta}$ in the continuum order parameter.

From this viewpoint, Bessel-mode particle models may be interpreted as describing the large-scale field structure associated with stable fold-closure patterns.

In the EST framework, the parameter controlling the radial decay of the solution is written as m_0. In the VERSF derivation, the corresponding quantity is:

$$\boldsymbol{\kappa^2 = k^2_z + (\mu^2 - \omega^2)/c^2_\varphi} \quad \text{— (5.2)}$$

In the special case of purely radial modes ($k_z = 0$) and in the rest-frame limit ($\omega \to 0$), this expression reduces to:

$$\boldsymbol{\kappa = \mu/c_\varphi} \quad \text{— (5.3)}$$

Under this identification, the radial decay scale appearing in the EST solution corresponds to the closure stabilization scale of the fold network.

The relationship between the two frameworks can therefore be summarized as follows:

- **EST** begins with a continuum nonlinear field equation and investigates localized Bessel-mode solutions as candidate particle structures.
- **VERSF** begins with a discrete informational substrate and shows that coarse-graining of fold-closure dynamics yields a continuum field equation whose localized cylindrical solutions belong to the same class of radial solutions.

The two approaches therefore address the same mathematical structure from different starting points. EST analyzes the continuum phenomenology of Bessel-mode particle configurations, while VERSF explores whether such configurations can arise naturally as emergent structures of an underlying discrete closure network.

From the VERSF perspective, the modified Bessel function $K_n(\kappa r)$ can thus be viewed as the continuum signature of a topologically stabilized fold-closure configuration. The Bessel profile describes the spatial decay of the organized closure pattern away from its core, while the winding index n reflects the discrete topological structure of the underlying fold boundary.

In this interpretation, the Bessel function is not a fundamental constituent of particle structure but the large-scale field representation of a deeper informational topology.

5.4 The n=0 Sector and the Screened Green-Function Hierarchy

The n=0 closure sector deserves separate treatment because it connects the VERSF closure profile to a standard mathematical structure — the Green-function hierarchy of the modified Helmholtz operator — rather than to Yukawa physics by analogy.

In the n=0 case the angular momentum barrier vanishes, the closure order parameter carries no topological winding, and the radial equation (4.5) reduces to:

$$\mathbf{f''(r) + (1/r)\, f'(r) - \kappa^2 f(r) = 0} \quad \text{— (5.4)}$$

The localized solution is $K_0(\kappa r)$. This function is not merely analogous to a Green's function — it is one in a precise sense. The operator underlying the radial equation is the modified Helmholtz operator $(-\nabla^2 + \kappa^2)$. Its fundamental solution in d spatial dimensions satisfies:

$$\mathbf{(-\nabla^2_d + \kappa^2)\, G_d(r) = \delta^{(d)}(r)} \quad \text{— (5.5)}$$

and takes the closed form:

$$\mathbf{G_d(r) = (1/(2\pi)^{(d/2)}) \cdot (\kappa/r)^{(d/2-1)} \cdot K_{d/2-1}(\kappa r)} \quad \text{— (5.6)}$$

This single formula generates all the familiar screened kernels as special cases. For d=3, using $K_{1/2}(x) = \sqrt{(\pi/2x)}\, e^{-x}$, equation (5.6) recovers the standard Yukawa kernel:

$$\mathbf{G_3(r) = e^{-\kappa r} / (4\pi r)} \quad (5.7)$$

For d=2, equation (5.6) gives $d/2-1 = 0$, so:

$$\mathbf{G_2(r) = K_0(\kappa r) / (2\pi)} \quad (5.8)$$

The n=0 VERSF closure profile $K_0(\kappa r)/2\pi$ is therefore the d=2 member of the same Green-function family whose d=3 member is the Yukawa kernel. The Yukawa kernel and the K_0 kernel are dimensionally related members of the same screened Green-function family, both generated by equation (5.6).

Dimensional reduction. The exact identity (NIST DLMF §10.32):

$$\int_{-\infty}^{\infty} \mathbf{G_3(\sqrt{r^2+z^2})\, dz = G_2(r)} \quad (5.9)$$

is a direct consequence of this family structure. It states that integrating the three-dimensional point-source kernel along one spatial direction yields the two-dimensional point-source kernel in the transverse plane — which is exactly what cylindrical geometry of the fold-closure substrate should produce. The term "z-projection" used in earlier formulations of this result is better replaced by **axial integration** or **line-source Green function**: $K_0(\kappa r)/2\pi$ is the transverse Green function for a line source extending along the z-axis, where the 3D Yukawa kernel is the corresponding point-source Green function.

Asymptotic structure. At large r both kernels decay with the same exponential screening factor, but with different algebraic prefactors:

$$\mathbf{G_3(r) \sim e^{-\kappa r} / (4\pi r)} \text{ (3D, prefactor } 1/r)$$

$$\mathbf{G_2(r) \sim e^{-\kappa r}\, \sqrt{(1/(8\pi\kappa r))} / (2\pi)} \text{ (2D, prefactor } 1/\sqrt{r})$$

The screening length κ^{-1} is therefore dimension-independent — it is set entirely by κ and hence by the closure stabilization scale μ and the fold wave speed c_φ. The algebraic prefactor, by contrast, encodes the effective dimensionality of the source geometry. In VERSF terms: the exponential factor $e^{-\kappa r}$ encodes finite-range transmission of closure organization, while the power-law prefactor reflects whether the closure structure is being treated as a 3D point source or a cylindrically extended axial source.

Consequences for VERSF. This Green-function identification has two consequences.

First, the closure coherence radius κ^{-1} coincides formally with the Compton wavelength of a massive mediating scalar. In the rest-frame limit, $\kappa = \mu/c_\varphi$, so $\kappa^{-1} = c_\varphi/\mu$ — the natural length scale set by the BCB healing length and the fold wave speed. Provisionally identifying κ^{-1} with the Compton wavelength yields the mass-scale relation $m = \hbar\mu/(c \cdot c_\varphi)$. This identification

should be treated as a formal correspondence: it requires $c_\varphi = c$ and τ = physical time, neither of which has been established here.

Second, the n=0 closure mode exhibits effective screened mediator behavior in the continuum limit. What has been established is a Green-function correspondence: the n=0 closure profile realizes the 2D member of the same screened operator family whose 3D member is the Yukawa kernel. This is not a derivation of a quantum field, propagator normalization, or interaction vertex structure — those require additional steps. What it establishes is that the screened-propagation form of the interaction arises from the cylindrical geometry of the fold-closure substrate rather than from a separate physical postulate.

Precise statement of the §5.4 result. The cylindrical fold-closure geometry, by itself, naturally leads to the static radial Green function of the closure propagation operator being $K_0(\kappa r)$ rather than the 3D Yukawa kernel $e^{-\kappa r}/r$. This is not a coincidence or an analogy: $K_0(\kappa r)/2\pi$ is the exact fundamental solution of $(-\nabla^2_2 D + \kappa^2)G = \delta^2(r)$, the modified Helmholtz equation appropriate to propagation in the transverse plane of a cylindrically symmetric source. The 3D Yukawa form is recovered precisely when the cylindrical extent of the source is integrated out along the z-axis, via identity (5.9). No additional physical input is needed: the distinction between the 2D and 3D forms follows from whether the effective source geometry is a transverse point (giving K_0) or a point in 3D space (giving $e^{-\kappa r}/r$), and the transition between the two is the dimensional reduction identity.

---

6. Physical Interpretation

The correspondence between the mathematical objects of the Bessel-mode description and the structural elements of VERSF is summarized below:

Mathematical Quantity	VERSF Interpretation
Winding index n	Closure topology index (topologically protected invariant)
$K_n(\kappa r)$	Radial closure coherence profile
$K_0(\kappa r) = 2\pi\, G_2(r)$	n=0 closure profile = d=2 member of the screened Green-function hierarchy (§5.4)
$\kappa^{-1} = (k^2_z + (\mu^2-\omega^2)/c^2_\varphi)^{(-1/2)}$	Closure coherence radius; effective screening length in the continuum limit
μ^{-1}	Closure healing length (set by BCB capacity constraints)
Field energy $E[\varphi] = \int d^3x\, [\frac{1}{2}(\partial_\tau\varphi)^2 + (c^2_\varphi/2)\lvert\nabla\varphi\rvert^2 + (\mu^2/2)\varphi^2 + (\lambda/4)\varphi^4]$	Closure stabilization energy
Soliton core ($r \to 0$ region)	Emergent closure locus — the organizing center of the fold pattern
Temporal frequency ω	Internal oscillation rate of closure configuration

The correspondence summarized below highlights how the parameters appearing in the Bessel-mode description acquire structural interpretations within VERSF.

6.1 Mass from Closure Stabilization

Within this framework, particle mass is related to the closure stabilization scale μ, which is set by the BCB capacity constraints. Different closure topologies (different values of n, or different internal fold organization) would correspond to different effective values of μ and hence different masses.

The connection to mass is made more precise through the full closure energy functional in the wave regime. For a localized, time-harmonic closure configuration, the energy is:

$$\mathbf{E[\varphi] = \int d^3x \left[\tfrac{1}{2}(\partial_\tau \varphi)^2 + (c^2_\varphi/2)|\nabla\varphi|^2 + (\mu^2/2)\varphi^2 + (\lambda/4)\varphi^4 \right]} \quad \text{— (6.1)}$$

Each term has a VERSF interpretation: the first is the kinetic cost of temporal oscillation, the second is the spatial gradient cost of maintaining closure coherence across neighboring folds, the third is the linear restoring cost set by BCB constraints, and the fourth is the nonlinear saturation cost.

Evaluated on the Bessel profile $f(r) \propto K_n(\kappa r)$ — valid in the linearized, source-free regime outside the nonlinear core — the dominant length scale is κ^{-1}. For purely radial modes ($k_z = 0$), dimensional analysis gives $\kappa = \sqrt{(\mu^2 - \omega^2)}/c_\varphi$, so the energy scales as an amplitude factor times a combination of c_φ, μ, and ω. In the rest-frame limit $\omega \to 0$, this combination reduces to a quantity proportional to μ. This is a dimensional scaling argument, not a mass derivation. The argument therefore establishes only that μ sets the characteristic mass scale; it does not determine the proportionality constant, which requires evaluating the energy integral on the regularized nonlinear core solution. That evaluation is left for future work. The VERSF-specific content is the interpretation of μ as the BCB-determined closure stabilization scale, and the prediction that different closure topologies (different n) correspond to different effective μ and hence different masses.

This interpretation has a natural point of contact with the energy analysis in EST, which numerically evaluates the integrals $I_n = \int_0^\infty s\, ds\, [K_{n+1}^2(s) + 2(n^2/s^2)K_n^2(s) + K_n^2(s)]$ and finds the empirical scaling $I_n \propto n^{1.479\pm0.016}$, consistent with the theoretical asymptotic prediction $I_n \propto n^{3/2}$ derived via uniform Bessel function expansions. EST introduces the proportionality between I_n and particle mass as a working hypothesis. If that hypothesis holds, then in the VERSF picture the energy scaling reflects the n-dependence of the closure stabilization cost: higher winding classes require more organized fold participation to maintain topological stability, and therefore carry greater energetic cost. The observed mass hierarchy of the lepton generations would, in this interpretation, reflect the increasing topological complexity of their corresponding closure configurations.

This provides a structural starting point for the **mass hierarchy problem**: why do different particles have the masses they do? In VERSF terms, the question becomes: what determines the BCB-compatible closure configurations and their associated stabilization scales?

6.2 Topological Ladder Structure and the Generation Spectrum

The modified Bessel functions $K_n(x)$ satisfy an exact three-term recursion relation:

$$\mathbf{K_{n+1}(x) = (2n/x)\, K_n(x) + K_{n-1}(x)} \text{ — (6.2)}$$

together with the differential recurrence relations:

$$\mathbf{-(d/dx + n/x)\, K_n(x) = K_{n+1}(x)} \text{ — (6.3a)}$$

$$\mathbf{(-d/dx + n/x)\, K_n(x) = K_{n-1}(x)} \text{ — (6.3b)}$$

Equation (6.3a) defines a **winding-raising operator** R_n = −(d/dx + n/x) that maps the radial profile of winding class n to that of winding class n+1. Equation (6.3b) defines the corresponding **winding-lowering operator** L_n = (−d/dx + n/x). Together they establish that the sequence $\{K_n(\kappa r) : n = 0, 1, 2, ...\}$ forms a **natural recurrence ladder of radial profiles**: starting from K_0 and K_1, every higher closure profile is generated by repeated application of R_n, with no free parameters.

The centrifugal weight (2n/x) in (6.2) is large at small r (near the closure core) and small at large r (far from the core). Physically, this encodes the increasing angular momentum cost of adding each additional unit of winding: the inner region of the closure configuration must reorganize substantially to accommodate the increased centrifugal stress, while the outer exponential envelope is less affected. This is consistent with the core regularization requirement of §5.2: higher-n profiles have more severely singular linearized cores precisely because the centrifugal weight (2n/κr) diverges faster at the origin.

The recurrence structure therefore suggests a possible reframing of the VERSF interpretation of the particle generation spectrum. In quantum mechanics, raising and lowering operators generate a tower of states from a base state. Analogously, the Bessel recurrence generates a tower of closure profiles from the lowest-winding sector (n=0). This suggests that the question *why do particles organize into discrete winding classes?* may be reformulated rather than answered: the mathematics of the Bessel recurrence admits no continuous interpolation between integer n, so discreteness is automatic. The question that remains is *which rungs of the recurrence ladder are BCB-compatible stable configurations?*

That is, the observed particle generations may correspond not to the entire ladder but to a sparse subset of it — the winding classes n for which the BCB capacity constraints on the fold network permit a stable closure configuration. If the EST energy scaling $I_n \propto n^{(3/2)}$ correctly describes the closure stabilization cost, then it predicts the relative mass of every BCB-compatible rung, and the generation structure problem may be reformulated as an intrinsic question about the fold network: which topological sectors are accessible given the distinguishability budget of the substrate?

This reframes EST's quantum number fitting problem in structural terms. EST finds empirically that the three charged lepton generations correspond to n = 1, 37, 248 (Approach A) or n = 1, 15,

59 (Approach B), without deriving these values from first principles. The basis for taking these assignments seriously is that they reproduce observed lepton mass ratios under the EST energy scaling hypothesis — they are phenomenologically motivated, not theoretically derived. In the VERSF picture, if those assignments are correct, these winding numbers would be the rungs of the recurrence ladder that satisfy the BCB capacity constraints. However, this reframing is not itself an advance: identifying the generation problem as a constraint satisfaction problem on the fold network does not solve it. The reframing becomes substantive only when the BCB constraints are actually computed and compared against the EST winding assignments. Until then, the correspondence is a structural suggestion, not a result.

The raising operator (6.3a) also provides a natural candidate for a **topological winding transition** in the VERSF framework: a process in which a closure configuration of winding class n acquires one additional unit of fold-boundary winding, transitioning to class n+1. The energy cost of this candidate transition, $\Delta E_n = E_{n+1} - E_n$, is formally estimable from the EST energy integrals as $\Delta E_n \propto I_{n+1} - I_n$, and grows with n. Whether these transitions correspond to physical dynamical processes — and whether the EST I_n values are the relevant VERSF excitation energies — are open questions that the present analysis does not resolve.

6.3 Continuum Limit of the Closure Stability Functional

Earlier VERSF work introduced a closure stability functional of the form:

$$\mathcal{S}[\mathcal{C}] = \alpha\, n_{\mathcal{C}} + \beta\, \kappa_{\mathcal{C}} \quad (6.4)$$

where $n_{\mathcal{C}}$ is the fold count of the closure configuration and $\kappa_{\mathcal{C}}$ is its boundary curvature. This functional assigns an energetic cost to each closure configuration based on its discrete geometric properties.

The coarse-graining procedure of §3 establishes how this functional maps onto the continuum Bessel description:

Discrete quantity	Continuum limit
Fold count $n_{\mathcal{C}}$	Closure amplitude $\varphi(x)$
Boundary curvature $\kappa_{\mathcal{C}}$	Radial gradient energy $\lvert\nabla\varphi\rvert^2$
Stability functional $\mathcal{S}[\mathcal{C}]$	Bessel energy functional $\int(D\lvert\nabla\varphi\rvert^2 + \mu^2\varphi^2)\, d^3x$

In the continuum limit, the linearized localized analysis identifies $K_n(\kappa r)$ as the natural localized linearized profile associated with each topological sector n. The two-parameter family (n, κ) of the discrete stability functional maps onto the two-parameter family $(n, \kappa = \sqrt{k^2_z + (\mu^2-\omega^2)/c^2_\varphi})$ of Bessel solutions in the linearized regime.

This correspondence completes the continuum-to-discrete comparison developed in the present analysis: the closure stability functional of the discrete VERSF framework and the Bessel energy functional of the continuum field description are the same object viewed at different scales.

7. Outlook

This paper has explored the relationship between fold-closure topology in the discrete VERSF substrate and the Bessel-mode particle structures studied in the companion EST framework. The central result is:

Coarse-graining the fold-closure network yields a continuum closure field equation whose localized rotationally symmetric solutions take the form $K_n(\kappa r)$ — the same class of modified Bessel profiles that arises in the EST framework and in other continuum models of Bessel-mode particle structure. The winding index n is identified with the discrete topological winding of the fold boundary; the coarse-graining map making this identification explicit is deferred to future work. The radial decay scale κ is determined by the closure stabilization scale, wave speed, and mode frequency of the underlying fold dynamics.

The relationship between VERSF and EST is exploratory rather than prescriptive: EST establishes that Bessel-mode particle structures are mathematically coherent and proposes experimental tests for them; VERSF investigates whether the discrete fold-closure substrate naturally generates the field structures that EST postulates. The analysis suggests that several of the key mathematical ingredients of EST — including the nonlinear wave structure, localized Bessel radial profiles, and topological winding sectors — arise naturally from fold-closure dynamics. Whether this correspondence extends to the specific quantum number assignments, energy scaling constants, mass ratios, and generation structure proposed by EST remains an open question that motivates the research directions below.

Several research directions follow naturally from this result:

1. **Stability functionals and the recurrence base.** Derive the closure stability functional for each topological sector, beginning from the n=0 base sector established in §5.4 and ascending the recurrence ladder of §6.2. Determine whether the minimizers reproduce the energy values I_n computed numerically in EST ($I_1 = 2.094$, $I_2 = 5.247$, $I_3 = 10.848$).
2. **BCB-compatible ladder rungs.** Determine which winding classes n are accessible under the BCB capacity constraints of the fold network. The topological ladder generates a closure profile for every integer n; the question is which rungs correspond to BCB-compatible stable configurations. This computation would either derive or rule out the quantum number assignments proposed by EST (n = 1, 37, 248 or n = 1, 15, 59), reducing the generation problem from an empirical fitting exercise to a constraint satisfaction problem on the fold network.
3. **Screened closure propagation and interactions.** Use the Green-function hierarchy of §5.4 to develop the interaction picture for spatially separated closure configurations. The n=0 closure mode provides the screened propagation kernel $G_2(r) = K_0(\kappa r)/2\pi$ in the cylindrical geometry; determine whether the effective potential between two such configurations reproduces $G_3(r) = e^{-\kappa R}/(4\pi R)$ in the 3D limit, and compute the coupling strength from fold network parameters.

4. **Twisted-light experimental signatures.** EST predicts resonant scattering enhancement when an orbital angular momentum beam of OAM ℓ is directed at a particle with winding index $n = \ell$. In the VERSF interpretation, this is the signature of selective coupling between an external field carrying topological charge and a fold-closure configuration of matching winding class. This is testable with current technology and is the most immediate experimental test of the VERSF-EST correspondence.
5. **Candidate winding transition spectrum.** Use the raising operator (6.3a) to characterize candidate transitions between adjacent winding classes and estimate the transition energy $\Delta E_n = E_{n+1} - E_n$ from the EST energy integrals. Determine whether these transitions correspond to physical dynamical processes in the fold network, and whether the resulting spectrum is consistent with known particle phenomenology.
6. **Core regularization and the centrifugal weight.** The centrifugal weight $(2n/\kappa r)$ in the ladder recursion (6.2) grows with n, implying that higher-n closure cores have more severely singular linearized profiles and require larger nonlinear corrections. Connect the BCB capacity constraints to a winding-class-dependent regularization scale $\sigma(n)$, providing a principled derivation of core size as a function of generation number.
7. **Lorentz invariance from fold dynamics.** The closure wave equation (3.18) is written in a fixed reference frame with a fold propagation speed c_φ. The identification $\kappa = \mu/c_\varphi$ with the EST parameter m_0 requires $c_\varphi = c$. More fundamentally, the modified Bessel solutions have their familiar particle-physics interpretation only in a Lorentz-covariant context — without Lorentz invariance, the rest-frame limit $\omega \to 0$ and the mass-shell condition are not frame-independent. Whether and how Lorentz invariance emerges from the fold-closure network — either as an exact symmetry of the continuum limit or as an approximate symmetry at low energies — is not addressed in the present analysis and constitutes one of its central open problems. No claim is made here that the closure field equation is Lorentz covariant; this is flagged as a necessary item for any future connection to relativistic particle physics.

On falsifiability. A referee-level concern with the correspondence established here is that it may be unfalsifiable at the level of this paper. If fold dynamics can produce any Klein–Gordon-type equation, and such equations always have Bessel solutions, then the VERSF-to-Bessel correspondence is a consequence of structural choices made in the coarse-graining — not a distinguishing prediction of VERSF specifically. The correspondence becomes falsifiable at the following levels: (i) the specific form of the closure field equation — in particular, the Z_2 symmetry, the wave regime, and the cylindrical symmetry assumption — makes predictions about which Bessel mode families and which symmetry classes of solutions arise; a fold network without these properties would not yield this structure; (ii) the identification of μ with the BCB healing length makes a prediction about the relationship between the closure stabilization scale and particle mass that could be tested if BCB constraints are computed independently; (iii) the winding number program (direction 2 above) would either derive the EST quantum number assignments from fold network constraints or produce different values, with the latter constituting a falsification of the VERSF interpretation of EST's assignments. At the present stage the correspondence is best understood as a structural compatibility result rather than a predictive one.

The program suggested here positions VERSF not merely as an alternative ontology but as a potential derivation of particle field theory from informational first principles. Two structural results in this paper are particularly suggestive. The Green-function analysis of §5.4 shows that the n=0 closure mode is not merely reminiscent of Yukawa behavior — it realizes the d=2 member of the same screened Green-function family whose d=3 member is the standard Yukawa kernel, with the dimensional reduction identity (5.9) explaining the connection as a direct consequence of the cylindrical geometry of the fold-closure substrate. The recurrence structure of §6.2 shows that the entire tower of Bessel-mode radial profiles is generated from the lowest-winding sector by a single differential operator, suggesting that the generation structure problem may be reformulated as a question about BCB-compatible rungs of a mathematically determined recurrence ladder. The EST framework demonstrates that Bessel-mode particle physics is mathematically coherent and experimentally testable; VERSF investigates where that structure comes from. The present paper suggests that the answer may lie in the discrete topology of the fold-closure substrate.

Manuscript prepared for the VERSF Theoretical Physics Program, AIDA Institute. Comments and critical correspondence welcome.

References

[EST] Taylor, K. *3D String Theory: Electromagnetic Structure Theory of Fundamental Particles — A Comprehensive Mathematical Framework.* VERSF Theoretical Physics Program, AIDA Institute. The companion continuum framework studied throughout the present paper. EST investigates localized solutions of nonlinear Klein–Gordon–type equations with cylindrical symmetry, develops the mathematical properties of modified Bessel function solutions including energy convergence proofs and asymptotic scaling analysis, and proposes experimental tests based on orbital angular momentum interactions with structured light. The energy scaling $I_n \propto n^{1.479}$ and the quantum number assignments to particle generations (n = 1, 37, 248 or n = 1, 15, 59) are presented in EST as working hypotheses rather than derived results.

[VERSF-BCB] Taylor, K. *Bit Conservation and Balance in the VERSF Framework.* VERSF Theoretical Physics Program, AIDA Institute. Establishes the BCB capacity constraints on the committed distinguishability graph that set the closure stabilization scale μ and the closure healing length $\xi = \mu^{-1}$ used throughout the present paper.

[VERSF-SF] Taylor, K. *The Closure Stability Functional in VERSF.* VERSF Theoretical Physics Program, AIDA Institute. Introduces the discrete stability functional $\mathcal{S}[\mathcal{C}] = \alpha n_{\mathcal{C}} + \beta\kappa_{\mathcal{C}}$ whose continuum limit is identified with the Bessel energy functional in §6.3 of the present paper.

External mathematical references

[Watson] Watson, G. N. *A Treatise on the Theory of Bessel Functions*, 2nd ed. Cambridge University Press, 1944. Standard reference for the recurrence relations, differential recurrences, asymptotic expansions, and integral representations of modified Bessel functions used throughout §§5–6 of the present paper.

[DLMF] Olver, F. W. J. et al. (eds.) *NIST Digital Library of Mathematical Functions*. Release 1.1.12, 2023. https://dlmf.nist.gov. Chapter 10 provides the Green-function identity $K_0(\kappa r)/2\pi = \int G_3 D\, dz$ cited in §5.4 (§10.32) and the asymptotic uniform expansions used in the EST energy scaling analysis.

[Rajaraman] Rajaraman, R. *Solitons and Instantons: An Introduction to Solitons and Instantons in Quantum Field Theory.* North-Holland, 1982. Standard reference for topological soliton solutions of nonlinear field equations, including vortex-type solutions with winding number structure analogous to the closure configurations described in §§2–4.

[Manton-Sutcliffe] Manton, N. and Sutcliffe, P. *Topological Solitons.* Cambridge University Press, 2004. Comprehensive treatment of localized field configurations with topological charge, relevant to the stability arguments of §6.2 and the closure stability functional of §6.3.

[Rubinstein] Rubinstein, J. "Sine-Gordon equation." *Journal of Mathematical Physics* 11, 258 (1970). Early reference for the Ginzburg–Landau and nonlinear Klein–Gordon equations as models of topological defect dynamics, situating the relaxational and wave regimes of §3.3 in the broader field-theory literature.