From Quantum Speed Limits to Cosmological Horizons: The Quartic Capacity Parameter of Bounded Spacetime Regions

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For the General Reader

Physics contains many deep limits — on how fast things can change, how much information a region of space can hold, how many computations a system can perform. These limits come from different corners of physics: quantum mechanics, thermodynamics, gravity, information theory. They were discovered independently, motivated by different problems, and expressed in different mathematical languages.

This paper asks a simple question: is there a single quantity that underlies all of them?

The answer appears to be yes. The combination $\boldsymbol{\rho L^4}$ — where ρ is energy density and L is the size of a bounded region of spacetime — emerges from at least four independent physical limits. But before cataloguing where it appears, this paper first gives it a direct microscopic meaning.

A bounded region of spacetime contains a finite action budget — the total energy inside it multiplied by the time available for causal coordination (L/c). Each irreversible physical event, by the laws of quantum mechanics, consumes at least one quantum of action $\hbar$. Dividing total action budget by minimum action cost gives:

$\chi(L) = \rho L^4 / \hbar c$ = (total action in causal region) / (minimum action per irreversible event)

This is the *commitment capacity* of the region: the maximum number of distinguishable irreversible events it can support in one light-crossing time. When $\chi < 1$, the region lacks the action budget to produce even a single stable classical fact. When $\chi \sim 1$, it is at the threshold. When $\chi \gg 1$, it is deep in the classical regime.

This microscopic meaning makes sense of the four physical limits that independently yield the same combination:

- the **Margolus–Levitin bound**: the maximum rate at which quantum states can evolve,
- the **Bekenstein entropy bound**: the maximum information a region can contain,

- the **causal diamond action**: the total quantum phase accumulated within a spacetime region,
- **Lloyd's computational bound**: the maximum number of logical operations a physical system can perform.

When made dimensionless by dividing by $\hbar c$, this combination defines a parameter:

$$\boldsymbol{\chi(L) = \rho L^4 / \hbar c}$$

which we call the *causal capacity* of the region.

In the companion paper *The Quartic Capacity of Causal Regions* we derived this parameter from quantum and thermodynamic first principles. In this paper we show it is far more universal than that derivation suggested.

Evaluated at a **black hole horizon**, χ equals the black hole's entropy — the amount of information stored in the most information-dense objects in the universe.

Evaluated at the **cosmological horizon**, $\chi \sim 10^{122}$ — the same number that appears in the total computational capacity of the observable universe, in cosmological horizon entropy, and in the Planck–Λ hierarchy of the cosmological constant problem.

The **threshold $\chi \sim 1$** marks the smallest region of space capable of producing a stable, irreversible physical fact. Applied to the energy density of the vacuum, this threshold gives a length scale $\xi \approx 80$ microns — a scale that coincides, intriguingly, with the characteristic size of the brain's basic computational units.

The paper traces this single invariant across 120 orders of magnitude of physics, from the Planck scale to the edge of the observable universe, and discusses what this structural unity might mean.

Abstract

The dimensionless quantity $\chi(L) = \rho L^4/\hbar c$ arises independently from the Margolus–Levitin quantum speed limit, the Bekenstein entropy bound, the action content of causal diamonds, and Lloyd's computational bound on logical operations. We show that this convergence reflects a direct microscopic meaning: $\chi(L)$ equals the total action available in a causal region of size L divided by the minimum action cost $\hbar$ of an irreversible update, giving the maximum number of distinguishable irreversible commitments the region can support in one light-crossing time. This microscopic interpretation — developed from minimal postulates about discrete degrees of freedom, action costs, causal coordination, and energy density — provides the foundation for all subsequent applications.

We demonstrate this universality across five domains. (i) For a Schwarzschild black hole, χ evaluated at the horizon scale yields $\chi(R) \sim R^2/L_p^2 \sim$ S_BH, reproducing the Bekenstein–

Hawking entropy. We show this is not a dimensional accident: it follows from the fact that black holes saturate the Bekenstein bound at every scale, and that χ is the causal-region entropy capacity. The result is situated within Bousso's covariant entropy bound and Jacobson's thermodynamic derivation of general relativity, where the quartic structure appears independently. (ii) At the cosmological horizon, $\chi(R_H) \sim R_H^2/L_p^2 \sim 10^{122}$, coinciding with the horizon entropy, Lloyd's estimate of the universe's total computational capacity, and the Planck–Λ hierarchy. (iii) The threshold $\chi \sim 1$, applied to the vacuum energy density ρ_Λ, defines the VERSF coherence scale $\xi = (\hbar c/\rho_\Lambda)^{(1/4)} \approx 80$ μm — the smallest causal region capable of supporting irreversible classical record formation. This scale coincides with the experimentally observed upper boundary of the mesoscopic regime, where quantum coherence gives way to classical behaviour in laboratory systems; the physical content of both descriptions — irreversible entropy production at a characteristic causal scale — are closely aligned. A speculative numerical coincidence with biological neural-processing scales is discussed separately in Section 12.

Together these results support the interpretation that $\chi(L) = \rho L^4/\hbar c$ is a universal invariant governing the information-processing capacity of bounded spacetime regions, spanning 120 orders of magnitude from the Planck scale to the cosmological horizon.

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1. Introduction

Several fundamental limits constrain what can occur within a bounded region of spacetime. Despite originating in different areas of physics, these limits share a common mathematical skeleton. We distinguish two primary derivations that yield the quartic structure from independent physical starting points, and two further reformulations that recast the same quantity in geometric and computational language.

Primary derivation 1 — Margolus–Levitin quantum speed limit [1]. A quantum system of energy E can undergo at most $E/\pi\hbar$ distinguishable transitions per second. Over a light-crossing time $\tau = L/c$, and for a region of energy $E \sim \rho L^3$, this gives a transition count proportional to $\rho L^4/\hbar c$. This derivation draws on quantum mechanics and the time-energy uncertainty relation; it makes no reference to thermodynamics or gravity.

Primary derivation 2 — Bekenstein entropy bound [2]. The entropy of any physical system satisfies $S \leq 2\pi ER/\hbar c$, where R is the system's spatial extent and E its energy. Substituting $E \sim \rho L^3$ yields $S \lesssim \rho L^4/\hbar c$, up to numerical prefactors. This derivation draws on thermodynamics and the generalised second law; it is logically independent of the quantum speed-limit route above.

Structural reformulation — causal diamond action. The spacetime volume integral of the Lagrangian density within the causal future-past intersection of a region of size L scales as $\mathcal{A} \sim \rho L^4$. The dimensionless ratio $\mathcal{A}/\hbar \sim \rho L^4/\hbar c$ is the Lorentz-invariant measure of quantum phase accumulated within the diamond. This is best understood as a geometric restatement of the same quantity rather than an independent derivation: it expresses the quartic in the language of causal structure, but draws on the same underlying physics as the quantum speed-limit route.

Computational reinterpretation — Lloyd's bound [9]. Lloyd showed that the maximum number of logical operations a physical system of energy E can perform in time t is $N\_ops \leq 2Et/\pi\hbar$. Applying this to a causal region with $E \sim \rho L^3$ and $t \sim L/c$ again yields $\rho L^4/\hbar c$. This is closely related to the Margolus–Levitin logic — both follow from the time-energy uncertainty relation — and is best understood as a computational reinterpretation of the first derivation rather than an independent result. It is nonetheless valuable because it connects the quartic directly to information-processing capacity.

In each case the relevant quantity is:

$\boldsymbol{\chi(L) = \rho L^4 / \hbar c}$

Before extending this to gravitational and cosmological contexts, Section 2 provides a direct microscopic interpretation of χ from minimal postulates about action cost, causal coordination, and bounded energy — showing that it equals the total action budget of a causal region divided by the minimum action cost of an irreversible update. This grounds subsequent applications in a clear physical definition rather than a pattern of dimensional recurrence.

The companion paper argued that χ(L) measures the *causal capacity* of a bounded spacetime region — the number of distinguishable irreversible records it can support. In this paper we extend the argument to gravitational horizons and show that $\chi(R) \sim S\_BH$ for Schwarzschild black holes.

However, we do so carefully. A naive substitution of $\rho \sim M/R^3$ into the formula is not sufficient to establish a deep connection, for three reasons:

1. In general relativity, energy density inside a horizon is not a well-defined frame-independent quantity.
2. The result $\rho L^4 \sim R^2/L_p^2$ follows from dimensional analysis alone and does not distinguish the quartic form from other expressions with the same dimensions.
3. Black holes saturate the Bekenstein bound by definition, so any expression proportional to the bound will agree with S_BH at $L \sim R$.

We address all three concerns in turn before presenting the core result.

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2. Microscopic Foundations of the Quartic Parameter

The Introduction presented χ(L) as a quantity that emerges from several distinct physical limits. Before applying it to black holes and cosmological horizons, we give it a direct microscopic interpretation. This serves two purposes: it explains *why* the combination $\rho L^4/\hbar c$ is the natural capacity measure rather than some other expression, and it provides a clear physical definition that the subsequent structural arguments can stand on.

2.1 Microscopic Postulates

We work with four minimal assumptions about bounded spacetime regions.

P1 — Finite degrees of freedom. A bounded region of linear size L contains finitely many local degrees of freedom over any finite resolution scale. We make no commitment to the nature or spacing of these degrees of freedom beyond their finiteness.

P2 — Action cost of irreversibility. We take as a microscopic postulate, motivated by quantum speed limits and the requirement of distinguishable state change, that an irreversible update carries a minimum action cost of order $\hbar$ per event:

$$s^* \sim \hbar$$

This is the key microscopic postulate. A transition that is genuinely irreversible — one that cannot be undone by the local dynamics and that produces a distinguishable record — must orthogonalise the system state relative to its prior configuration. By the Margolus–Levitin bound, such a transition requires energy ε and time Δt satisfying $\varepsilon \cdot \Delta t \geq \hbar/2$, so the minimum action cost per distinguishable irreversible event is of order $\hbar$. We note that this argument is motivating rather than definitive: the time-energy uncertainty relation is not an operator relation in the same sense as position-momentum, and the connection between irreversibility and orthogonalisation depends on the context. The postulate is adopted as the simplest assumption consistent with known quantum limits.

P3 — Causal coordination time. A region of linear size L can function as a unified causal processor only over a coordination time:

$$T(L) \sim L/c$$

Events separated by more than this time cannot be causally coordinated within the region during a single interval.

P4 — Homogeneous energy density. The total energy inside the region is $E \sim \rho L^3$, where ρ is a characteristic mean energy density. This enters only as an order-of-magnitude tool.

2.2 Derivation of the Commitment Capacity

The total action available inside a region of size L over one coordination interval is:

$$S\_avail(L) \sim E \cdot T(L) \sim \rho L^3 \cdot (L/c) = \rho L^4/c$$

By P2, each irreversible commitment consumes at least $s^* \sim \hbar$ of this budget. The maximum number of distinguishable irreversible commitments — the *commitment capacity* of the region — is therefore:

$$\mathbf{N\_max(L) \sim S\_avail(L) / \hbar \sim \rho L^4 / \hbar c}$$

We define:

$$\boldsymbol{\chi(L) \equiv N\_max(L) = \rho L^4 / \hbar c}$$

This is more than dimensional analysis: within the present microscopic model, χ is a ratio of two physically defined quantities with direct operational meaning:

$\chi(L)$ = total action available in a causal region / minimum action per irreversible update

A bounded spacetime region is an *action-limited commitment processor*. Its causal capacity is the total action budget of the region expressed in units of the quantum of action.

2.3 The Distributed Clock Derivation

An equivalent formulation makes the locality of the argument more explicit. Suppose each microscopic degree of freedom *i* has a local energy ε_i and an update rate bounded by the Margolus–Levitin constraint:

$$\nu_i \leq \varepsilon_i / \hbar$$

Over a coordination interval $T = L/c$, the maximum number of updates at node *i* is:

$$n_i \leq \varepsilon_i \cdot L / \hbar c$$

Summing over all degrees of freedom within the region:

$$N\_max \leq \Sigma_i n_i \leq (L/\hbar c) \cdot \Sigma_i \varepsilon_i = (L/\hbar c) \cdot E(L) \sim \rho L^4/\hbar c$$

This derivation makes no reference to a global action integral. Each local degree of freedom ticks at a rate set by its own energy; the total number of ticks that can be causally coordinated across the region in one light-crossing is $\chi(L)$. The distributed and integral formulations agree because both ultimately count the same thing: quanta of action, divided by the minimum cost of an irreversible event.

2.4 The Minimal Commitment Cell

The threshold $\chi(L) \sim 1$ now has a precise microscopic meaning:

A region of size L is a *minimal commitment cell* if and only if $\chi(L) \gtrsim 1$.

The three regimes are:

Regime	Condition	Interpretation
Sub-threshold	$\chi(L) < 1$	Insufficient action budget for one irreversible update per causal interval — region is quantum-coherent
Threshold	$\chi(L) \sim 1$	Minimally capable of one irreversible commitment per causal interval

Regime	Condition	Interpretation
Super-threshold	$\chi(L) \gg 1$	Many commitment opportunities — region behaves as a composite of smaller commitment-capable units

This is considerably more specific than saying "the quartic equals one." It defines a phase boundary in the space of causal regions between those that can and cannot generate stable classical facts within their own causal boundary.

2.5 The VERSF Division of Labour

The microscopic argument above is generic — it applies to any physical system satisfying P1–P4. The specifically VERSF step is the identification of the relevant background energy density with the cosmological vacuum energy ρ_Λ. This gives the present-epoch minimal commitment scale:

$$\xi = (\hbar c / \rho_\Lambda)^{1/4} \approx 80\ \mu\text{m}$$

The VERSF framework does not derive the quartic form — that follows from P1–P4 alone. Its specific contribution is the identification of ρ_Λ as the energy density of the substrate against which commitment thresholds are measured in the current cosmological epoch. This is the honest division of labour: the quartic is generic; the scale ξ is VERSF-specific.

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3. On the Well-Definedness of Black Hole Energy Density

A rigorous treatment of energy density inside a Schwarzschild horizon requires care. The standard stress-energy tensor $T_{\mu\nu} = 0$ everywhere in the exterior vacuum, and the concept of a uniform interior density has no covariant meaning.

We adopt two complementary approaches that avoid this problem.

3.1 The ADM mass approach. In asymptotically flat spacetimes, the ADM mass M_{ADM} is a well-defined global conserved quantity [3]. One may define an *effective* mean energy density by distributing this mass over the coordinate volume of the horizon:

$$\rho_{eff} = M_{ADM} c^2 / V_{coord} = (3c^6) / (32\pi G^3 M^2)$$

This is explicitly a heuristic quantity — a global average, not a local density — and it enters the calculation only as an order-of-magnitude tool. We do not claim it represents a physical field inside the horizon.

3.2 The Brown–York quasilocal energy approach. For a surface at radius $r = R$, the Brown–York quasilocal energy [4] gives a surface energy density that encodes the gravitational binding of the region. At the horizon, this reduces to an expression of order c^4R/G per unit area, consistent with the ADM estimate when integrated.

An effective mean density associated with the horizon scale may be defined by distributing the ADM mass over the horizon volume. This does not represent a local stress-energy tensor — the Schwarzschild interior is vacuum in the $T_{\mu\nu}$ sense — but serves as an order-of-magnitude proxy for the gravitational energy contained within the causal region. In both the ADM and Brown–York approaches this gives, to leading order:

$$\boldsymbol{\rho_{eff} \sim c^4 / (GR^2)}$$

We use this solely as a heuristic order-of-magnitude tool in what follows.

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4. Uniqueness of the Quartic Form

Before evaluating $\chi(R)$, we address the concern that any expression with correct dimensions will reproduce S_{BH} at $L \sim R$.

Consider the general class of dimensionless combinations constructible from ρ, L, $\hbar$, c, and G:

$$\Xi(\alpha, \beta) = \rho^\alpha L^\beta / (\hbar^\gamma c^\delta G^\varepsilon)$$

For Ξ to be dimensionless, the exponents must satisfy four constraint equations (one per independent dimension: M, L, T). This leaves a one-parameter family of solutions. Three physically motivated criteria select the quartic form from this family:

1. **Extensivity:** χ scales as volume ($\propto L^3$) when ρ is held fixed, reflecting that capacity grows with region size.
2. **Independence from G:** the two primary derivations in the companion paper — from quantum speed limits and thermodynamic entropy — make no reference to gravity. For χ to arise from those arguments, it must be G-free.
3. **Planck-unit normalization:** $\chi = 1$ at the Planck density $\rho_P = c^5/\hbar G^2$, marking the natural quantum gravity threshold.

These criteria strongly constrain the possible dimensionless combinations that can appear in causal-capacity measures. Among the admissible combinations, the quartic form ρL^4 is the only physically natural G-free extensive candidate that simultaneously (i) scales extensively with region size, (ii) does not explicitly depend on the gravitational coupling G, and (iii) reproduces the natural Planck-density normalization $\chi = 1$ at $\rho = \rho_P$. This is not a proof of uniqueness in the strict mathematical sense — it is a statement about which combinations are physically natural given the derivations in the companion paper. The quartic form is the only member of the admissible family that satisfies all three criteria simultaneously, which is why it arises from the quantum and thermodynamic derivations and why it reappears in the gravitational context.

5. Evaluating the Quartic Capacity at the Horizon

Using the heuristic density estimate from Section 2:

$\rho \sim c^4 / (GR^2)$

and setting $L = R$:

$\chi(R) = \rho R^4 / \hbar c \sim [c^4 / (GR^2)] \cdot R^4 / \hbar c = c^3 R^2 / (G\hbar)$

Using the Planck length $L_p^2 = \hbar G/c^3$:

$\chi(R) \sim R^2 / L_p^2$

The Bekenstein–Hawking entropy is:

$S\_BH = A / (4L_p^2) = 4\pi R^2 / (4L_p^2) = \pi R^2 / L_p^2$

Therefore:

$\chi(R) \sim (1/\pi) \cdot S\_BH$

The numerical prefactor $1/\pi$ is within the order-of-magnitude accuracy of the heuristic density estimate. The structural identification $\chi(R) \sim S\_BH$ holds up to a pure numerical coefficient of order unity.

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6. Situating the Result: The Bousso Bound and Jacobson's Derivation

Rather than treating the $\chi \sim S\_BH$ correspondence as a new result in isolation, we situate it within two established frameworks that independently support the same quartic structure.

6.1 Bousso's Covariant Entropy Bound. Bousso [5] showed that the entropy passing through any null hypersurface L satisfies:

$S(L) \leq A(\partial L) / (4L_p^2)$

where $A(\partial L)$ is the area of the bounding surface. This constrains the maximal entropy of null-bounded regions to scale with boundary area, not volume. Independently, the Bekenstein bound constrains the entropy-energy relation for matter systems: $S \leq 2\pi ER/\hbar c$. When these two bounds are applied jointly to a causal diamond of size L — using $E \sim \rho L^3$ and $A \sim L^2$ — they together imply that the natural entropy variable of such a region is:

$$S_{diamond} \sim \rho L^4 / \hbar c \sim \chi(L)$$

The quartic form is not produced by the Bousso bound alone; it emerges from the combination of the area bound with the Bekenstein energy-entropy relation. This confirms that $\chi(L)$ is the correct entropy variable for causal diamonds, and that the quartic structure is consistent with both holographic constraints simultaneously.

6.2 Jacobson's Thermodynamic Derivation. Jacobson [6] derived the Einstein field equations by applying the Clausius relation $\delta Q = T\, dS$ to local Rindler horizons, where:

- the heat flux δQ is the boost energy crossing the horizon: $\delta Q \sim T_{\mu\nu} k^\mu k^\nu\, d\lambda\, dA$ (with k^μ a null generator)
- the temperature is the Unruh temperature $T = \hbar a/2\pi c k_B$
- the entropy is proportional to horizon area: $dS = (dA)/(4L_p^2)$

In this framework, the entropy associated with a local causal wedge of size L and energy density ρ takes the same quartic form at order of magnitude:

$$S_{local} \sim \rho L^4 / \hbar c \sim \chi(L)$$

The quartic form therefore appears as the natural entropy variable in the derivation of gravitational dynamics itself, not merely as an approximate measure. This provides the deepest justification for the $\chi \sim S_{BH}$ identification: it is the entropy variable from which general relativity emerges thermodynamically.

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7. Computational Capacity of Causal Regions

An additional perspective on the quartic capacity parameter emerges from the physics of computation, and provides a fourth independent derivation of the same structure.

7.1 Lloyd's Computational Bound

Lloyd [9] showed that the maximum number of elementary logical operations that a physical system of energy E can perform in time t is bounded by quantum mechanics:

$$N_{ops} \leq 2Et / \pi\hbar$$

At order of magnitude, ignoring numerical prefactors:

$$N_{ops} \sim Et / \hbar$$

This bound is related to — but distinct from — the Margolus–Levitin theorem used in Section 1. The Margolus–Levitin bound counts *distinguishable quantum state transitions*; Lloyd's bound counts *elementary logical operations*. They are not identical constraints, but they share the same dimensional structure and their ratio is a pure numerical factor. Both are derived from the time-energy uncertainty relation, and both yield the same scaling when applied to a causal region.

7.2 Applying Lloyd's Bound to a Causal Region

Consider a causal region of linear size L. The region contains energy $E \sim \rho L^3$, and its natural time scale is the light-crossing time $t \sim L/c$. Substituting into Lloyd's bound:

$$N\_ops \sim (\rho L^3/\hbar) \cdot (L/c) = \rho L^4/\hbar c$$

This is precisely the causal capacity parameter:

$$\boldsymbol{\chi(L) = \rho L^4 / \hbar c \sim N\_ops}$$

The quartic parameter therefore admits a direct computational interpretation: it equals the maximum number of logical operations that can occur within the causal region during one causal crossing time.

7.3 Four Convergent Capacity Constructions

We now have four physically distinct but mathematically convergent derivations of the same quartic structure. Each answers a different question about a causal region of size L and energy density ρ:

Question	Framework	Result
How fast can quantum states evolve?	Margolus–Levitin	$\chi \sim$ max distinguishable transitions
How much entropy can the region hold?	Bekenstein bound	$\chi \sim$ max entropy (in bits)
How much action does the region contain?	Causal-diamond geometry	$\chi \sim$ action / $\hbar$
How many operations can occur?	Lloyd bound	$\chi \sim$ max logical operations

These are not restatements of the same bound. The Margolus–Levitin and Lloyd bounds address dynamics; the Bekenstein bound addresses equilibrium information content; the causal-diamond action is a geometric quantity. The fact that all four yield the same combination $\rho L^4/\hbar c$ is a strong structural signal that this quantity captures something fundamental about the information-processing capacity of bounded spacetime regions.

7.4 The Threshold Condition Revisited

The threshold $\chi(L) \sim 1$ therefore admits four parallel interpretations simultaneously:

- the region contains one quantum of action ($\hbar$),
- it can support one distinguishable quantum transition,
- it has capacity for one bit of entropy increase,
- it can perform approximately one logical operation,

within one causal crossing time. These interpretations are physically distinct but mathematically equivalent, which is precisely what one expects of a fundamental capacity threshold rather than a limit specific to one domain.

7.5 Implication for the VERSF Coherence Scale

Applying $\chi(L) = 1$ to the cosmological vacuum energy density ρ_Λ yields the VERSF coherence scale $\xi = (\hbar c/\rho_\Lambda)^{(1/4)} \approx 10^{-4}$ m. The Lloyd bound now adds a fourth equivalent characterisation of this scale: it is the smallest causal region in which the vacuum energy density permits approximately one logical operation per light-crossing time. Below this scale, the causal capacity of the vacuum is sub-threshold in all four senses simultaneously.

7.6 Cosmological Computational Capacity

The quartic capacity parameter also appears naturally at the opposite extreme of scale: the information-processing capacity of the observable universe.

Lloyd showed that applying his bound to the observable universe — using its total energy and age — yields an estimated maximum number of operations:

$N_{ops} \sim 10^{120}$

Remarkably, the same scale emerges from $\chi(L)$ evaluated at the cosmological horizon. For a region of size $L \sim R_H$ (the Hubble radius), the causal capacity is:

$\chi(R_H) = \rho_\Lambda R_H^4 / \hbar c$

The cosmological constant satisfies the Friedmann relation $\rho_\Lambda \sim c^4/(G R_H^2)$ at the horizon scale — an order-of-magnitude estimate appropriate to the Hubble-scale horizon, not a local equation of state. At this scaling level:

$\chi(R_H) \sim c^3 R_H^2 / G\hbar = R_H^2 / L_p^2$

Numerically, with $R_H \sim 4.4 \times 10^{26}$ m and $L_p \sim 1.6 \times 10^{-35}$ m:

$\boldsymbol{\chi(R_H) \sim 10^{122}}$

This is identical in scale to three apparently independent quantities:

Quantity	Approximate value
Lloyd maximum universe operations	$\sim 10^{120}$

Quantity	Approximate value
Cosmological horizon entropy $S_H = A/4L_p^2$	$\sim 10^{122}$
Planck–Λ hierarchy (ρ_P/ρ_Λ)	$\sim 10^{122}$

The numerical coincidence across these three quantities is well known in cosmology but has lacked a unifying structural explanation. The quartic capacity parameter provides a natural one: all three expressions are evaluations of $\chi(R_H)$, the causal capacity of the observable universe. They agree because they all count the same thing — the total number of distinguishable causal events the universe can support within its horizon.

This also illuminates the cosmological constant problem from a new angle. The ratio $\rho_P/\rho_\Lambda \sim 10^{122}$ is conventionally described as a fine-tuning puzzle. Within the capacity framework it has a different character: it is the ratio $\chi(R_H)/\chi(L_p)$, i.e. the total causal capacity of the universe expressed in Planck units. The problem is not why ρ_Λ is so small, but why the universe operates at a causal capacity scale of 10^{122}.

The quartic parameter therefore spans the full range of physical scales:

Scale	χ(L)	Physical interpretation
Planck length L_p	~ 1	Quantum gravity threshold
VERSF coherence scale ξ	~ 1	First classical record formation
Black hole of radius R	$\sim R^2/L_p^2$	Horizon entropy
Observable universe R_H	$\sim 10^{122}$	Total universe causal capacity

The same invariant appearing across 120 orders of magnitude in a structurally consistent way is precisely the kind of evidence that distinguishes a fundamental capacity measure from a dimensional coincidence.

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8. The Saturation Argument: Why Black Holes Are Special

We noted in Section 1 that black holes saturate the Bekenstein bound, which might appear to make the $\chi \sim S_{BH}$ identification circular. Here we clarify the logical structure.

The Bekenstein bound states $S \leq 2\pi ER/\hbar c$ for any system of energy E and size R. For a Schwarzschild black hole:

$$E = Mc^2 = c^4R / (2G)$$

Substituting:

$$S_{max} = 2\pi \cdot (c^4R/2G) \cdot R / \hbar c = \pi c^3R^2 / G\hbar = \pi R^2 / L_p^2$$

This matches S_BH exactly. Black holes are not merely consistent with the Bekenstein bound — they are the unique objects that *saturate* it at every scale. This is the statement that black holes are maximally entropic for their size and energy.

The connection to $\chi(R)$ is then not circular but complementary: $\chi(R)$ is the quantity that, by the companion paper's derivations, equals the maximum entropy of a causal region. Black holes achieve this maximum. Therefore $\chi(R)$ = S_BH not because we constructed χ to reproduce black hole entropy, but because:

1. χ is the causal-region entropy capacity (established independently in the companion paper),
2. black holes saturate the entropy capacity at every scale (established by the Bekenstein–Hawking result),
3. therefore $\chi(R)$ and S_BH must agree at $L \sim R$.

This is not circular; it is a consistency theorem: the maximum entropy of a causal region equals the entropy of the black hole that fills it.

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9. Summary of Convergence

The recurrence of $\chi(L) = \rho L^4/\hbar c$ across physical frameworks is the central claim of this paper. The frameworks are not all of equal logical strength, and it is important to present them in their proper hierarchy.

Two primary derivations — logically independent starting points that both yield the quartic structure:

Framework	Physical meaning of χ	Reference
Quantum speed limits (ML)	Distinguishable transition count per light-crossing	Margolus–Levitin [1]
Thermodynamic entropy bound	Entropy capacity of a bounded region	Bekenstein [2]

Two structural reformulations — the same quantity recast in geometric and holographic language:

Framework	Physical meaning of χ	Reference
Causal diamond action	Action content of a causal diamond / $\hbar$	Companion paper
Holographic null bounds	Natural entropy variable for null surfaces	Bousso [5] + Jacobson [6]

One computational reinterpretation — closely related to the ML bound, distinct in framing:

Framework	Physical meaning of χ	Reference
Lloyd computational bound	Maximum logical operations per causal crossing	Lloyd [9]

Strong structural extensions — the quartic evaluated at gravitational and cosmological scales:

Framework	Physical meaning of χ	Reference
Gravitational horizons	Bekenstein–Hawking entropy ~ R^2/L_p^2	This paper
Cosmological horizon	Total universe causal capacity ~ 10^{122}	This paper

The G-independence and extensivity arguments of Section 3 show that the quartic form is strongly selected among physically natural combinations — not merely sufficient, but the only candidate satisfying the criteria that emerge from the two primary derivations. The convergence across all tiers reflects a common underlying constraint: bounded spacetime regions possess an information-processing capacity determined by the combination of energy density and causal crossing time.

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10. Implications for the VERSF Framework

Within the VERSF framework, the threshold condition:

$\rho L^4 \sim \hbar c \Leftrightarrow \chi(L) \sim 1$

marks the boundary below which a causal region cannot support a single irreversible quantum transition. As shown in Section 6.5, applying this to the cosmological vacuum energy density ρ_Λ gives the VERSF coherence scale $\xi \approx 10^{-4}$ m — the smallest region in which the causal capacity reaches the quantum threshold for irreversible record formation, in all four senses developed in Section 6.

The results of this paper reinforce this interpretation at both extremes of scale. At the black hole horizon, $\chi(R) \sim S_{BH} \sim R^2/L_p^2$ shows that the quartic parameter correctly identifies the maximum entropy of a gravitational object. At the cosmological horizon, $\chi(R_H) \sim 10^{122}$ shows that the same parameter recovers the total causal capacity of the observable universe. The VERSF coherence scale sits between these extremes as the threshold at which the vacuum first becomes capable of classical fact formation.

Together, these results support the view that $\chi(L)$ is a fundamental capacity measure for spacetime regions — one whose physical content spans from the Planck scale to the cosmological horizon across 120 orders of magnitude. The connection between the coherence scale and the gravitational and cosmological limits established here is currently structural rather than derived from first principles within VERSF; a rigorous derivation is deferred to future work.

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11. Mesoscopic Quantum Systems and the VERSF Coherence Scale

A more experimentally grounded observation follows from the same coherence scale $\xi \approx 80\ \mu m$.

The boundary between quantum and classical behaviour has been pushed to ever larger scales in laboratory experiments. The current experimental frontier sits at a well-defined threshold that coincides closely with ξ.

11.1 The Experimental Frontier of Quantum Coherence

Quantum interference and coherent superposition have been demonstrated for progressively larger objects:

System	Characteristic coherence size
Electrons	$10^{-12} - 10^{-10}$ m
Atoms	$\sim 10^{-10}$ m
Large molecules (C_{60}, macromolecules)	$10^{-9} - 10^{-7}$ m
Optomechanical oscillators	$10^{-8} - 10^{-5}$ m

The largest systems for which clean quantum coherence has been demonstrated in interferometers today are roughly **10–50 microns**. Maintaining phase coherence for significantly larger structures becomes increasingly difficult regardless of experimental isolation technique.

That frontier sits immediately below ξ.

11.2 Environmental Decoherence Correlation Lengths

Decoherence theory predicts that classical behaviour emerges when a system's interaction with its environment irreversibly entangles quantum phase information into inaccessible environmental degrees of freedom. The characteristic length scales at which this occurs in realistic laboratory environments are:

Environment	Typical decoherence scale
Gas collisions	$\sim \mu m$
Thermal radiation	$\sim$ tens of μm
Phonon environments	$\sim$ tens of μm

The experimentally observed decoherence correlation lengths span approximately **$10^{-6} - 10^{-4}$ m** — a range that brackets ξ from below.

11.3 The Mesoscopic Regime

Physics has a name for the domain in which these phenomena occur. The *mesoscopic scale* occupies the intermediate regime between microscopic quantum systems and macroscopic classical systems:

Regime	Characteristic scale
Quantum microscopic	nm – μm
Mesoscopic (quantum–classical transition)	μm – 100 μm
Macroscopic classical	> 100 μm

The VERSF coherence scale $\xi \approx 80$ μm sits at the **upper boundary of the mesoscopic regime** — precisely where systems transition from predominantly quantum to predominantly classical behaviour.

11.4 Physical Alignment with Decoherence Theory

The coincidence with the mesoscopic boundary is not only numerical. The physical content of the two descriptions is closely aligned.

Decoherence occurs when a system's interaction with its environment generates irreversible entropy production — when quantum phase information is permanently lost to environmental degrees of freedom. This is precisely what the quartic capacity threshold describes: $\chi(L) \sim 1$ marks the scale at which a causal region first becomes capable of supporting *irreversible* physical events within its own causal boundary.

The ingredients of both descriptions are the same: causal crossing time, quantum action, entropy production, and distinguishable state transitions. The VERSF framework and decoherence theory appear to characterise closely related aspects of the same transition: the onset of irreversible entropy production at a characteristic causal scale.

If the VERSF interpretation is correct, the quantum–classical transition observed experimentally at 10–100 μm would correspond to the scale at which causal regions acquire sufficient capacity to generate stable classical records. Below ξ, the causal capacity $\chi(L) < 1$ and classical facts are difficult to stabilise; above ξ, $\chi(L) > 1$ and irreversible records accumulate reliably. The resulting picture is:

Scale	Expected behaviour
$L \ll \xi$	Predominantly reversible quantum evolution
$L \sim \xi$	Onset of irreversible classical record formation
$L \gg \xi$	Classical world dominates

This stratification matches the experimentally observed structure of the quantum–classical transition.

11.5 Scope of the Claim

We are careful to state what this observation does and does not establish. Decoherence in laboratory systems arises from specific environmental interactions — gas molecules, photons, phonons — whose detailed dynamics are not modelled within the VERSF framework. A derivation showing that those interactions produce decoherence preferentially at the scale ξ would require substantially more work.

What the present observation establishes is structural: the theoretical threshold defined by $\chi(L) \sim 1$ and the experimentally observed mesoscopic boundary coincide numerically, and the physical content of both descriptions — irreversible entropy production at a characteristic causal scale — is the same.

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12. A Speculative Biological Coincidence

The following observation is explicitly speculative. It is presented as a coincidence of potential interest, not as evidence for the VERSF framework or any causal claim.

The same coherence scale $\xi \approx 80$ μm appears in an unrelated biological context. Several characteristic length scales of information processing in mammalian cortex fall within the same range:

Biological structure	Typical scale
Cortical column diameter	30–80 μm
Average neuron spacing in cortex	20–50 μm
Axon bundle and dendritic cluster diameter	50–100 μm
Capillary spacing in cortical tissue	50–100 μm

Cortical columns are widely regarded as the basic computational units of the neocortex. The proximity of these scales to ξ admits two interpretations.

The first is coincidence. Biological length scales are determined by diffusion rates, metabolic requirements, axon conduction velocities, and structural mechanics — constraints that are independent of fundamental physics and could plausibly produce a characteristic scale near 80 μm without any connection to vacuum energy.

The second is physical constraint. If stable classical record formation requires a causal capacity above the threshold $\chi \sim 1$, then biological systems that process information reliably must operate at or above ξ — not because biology has access to VERSF physics, but because physics constrains the minimum size of any region capable of producing a robust classical fact. Under this interpretation, evolutionary pressure toward reliable information processing would naturally select for organisational scales at or above ξ.

No causal mechanism is proposed here, and the numerical agreement spans only one order of magnitude. What makes the observation worth noting is that ξ is derived from three fundamental constants of entirely different origin — $\hbar$ (quantum mechanics), c (relativity), and ρ_Λ (cosmology) — and the resulting scale coincides with the spatial granularity of the only known structures in the universe that generate irreversible records in the most complex sense. Whether this is physics or numerology remains an open question.

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13. Conclusion

This paper has examined the quartic capacity parameter $\chi(L) = \rho L^4/\hbar c$ across a wide range of physical contexts. The claims made vary substantially in strength, and we summarise them in explicit order of evidential weight.

Most foundational — microscopic interpretation of χ. From four minimal postulates (finite degrees of freedom, minimum action cost $\hbar$ per irreversible update, causal coordination time L/c, and homogeneous energy density ρ), the commitment capacity of a bounded region follows: $\chi(L) = \rho L^4/\hbar c$ = (total action budget) / (minimum action per irreversible event). This is more than dimensional analysis: within the microscopic model, χ is a ratio of two physically defined quantities. It defines a phase boundary between regions that can and cannot generate stable classical facts within their own causal boundary.

Most rigorous — convergence of primary derivations. The Margolus–Levitin quantum speed limit and the Bekenstein entropy bound are logically independent approaches — one from quantum dynamics, one from thermodynamics — that both recover χ as the natural capacity measure. Their convergence on the same combination is the strongest external validation of the microscopic interpretation. The causal diamond action and Lloyd computational bound provide two further structural reformulations consistent with the same quantity.

Strong structural extension — gravitational and cosmological horizons. When χ is evaluated at the Schwarzschild horizon using a well-motivated heuristic density (ADM mass over horizon volume), it reproduces the Bekenstein–Hawking entropy up to numerical prefactors. This is supported by the saturation of the Bekenstein bound at every scale, by Bousso's covariant entropy bound, and by Jacobson's thermodynamic derivation of general relativity. Evaluated at the cosmological horizon, $\chi(R_H) \sim 10^{122}$ coincides with horizon entropy, Lloyd's universe computation estimate, and the Planck–Λ hierarchy. These are structural correspondences, not derivations, but they are embedded within established holographic frameworks.

Suggestive empirical coincidence — mesoscopic quantum boundary. The threshold $\chi \sim 1$ defines a coherence scale $\xi \approx 80$ μm that coincides numerically with the upper boundary of the mesoscopic regime, where quantum coherence gives way to classical behaviour in laboratory experiments. The physical content of the $\chi \sim 1$ threshold and standard decoherence theory — irreversible entropy production at a characteristic causal scale — are aligned, making this more than a numerical accident. A full derivation from first principles within the VERSF framework is not yet available.

Speculative coincidence — biological scales. The same scale $\xi \approx 80$ μm is numerically close to the characteristic size of cortical columns and related neural processing structures. This is noted as a coincidence of potential interest. No causal mechanism is proposed, and the observation should not be interpreted as evidence for the VERSF framework.

The present work does not claim that black hole entropy, cosmological horizon entropy, Lloyd capacity, and the VERSF coherence scale have been derived from a single microscopic theory. The narrower and more defensible claim is structural: when bounded spacetime regions are analysed in terms of quantum evolution limits, thermodynamic entropy bounds, causal action, and computational capacity, the same quartic parameter $\chi(L) = \rho L^4/\hbar c$ repeatedly appears as the natural dimensionless measure of region-level capacity. The significance of this recurrence is that it suggests a common organisational principle across otherwise distinct domains of physics — one that spans 120 orders of magnitude from the Planck scale to the cosmological horizon.

Whether this reflects a deep structural invariant or a set of related dimensional coincidences is a question for future work — ideally through a microscopic derivation of the quartic form from first principles within a theory that unifies quantum mechanics and gravity.

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