K = 7: The Constraint Dimensionality of Stable Physical Reality

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For the General Reader

What This Paper Is About

Imagine you are trying to build a stable structure out of interlocking pieces. Too few connections and the structure wobbles and collapses. Too many and the pieces jam — no configuration fits at all. There is a unique number of connections that produces a structure that is stable, consistent, and complete.

This paper argues that physical reality works the same way.

At the deepest level, the physical world is made of committed distinctions — moments where one thing becomes definitively different from another, irreversibly. For such distinctions to form a stable, consistent, observable reality, they must satisfy a set of independent conditions. This paper asks: how many?

The answer, derived from six completely different directions, is seven.

We call this number **K = 7**. It is not a parameter we fit to data. It is the unique integer that sits between "too few constraints" (reality collapses into ambiguity) and "too many constraints" (nothing is consistent enough to exist). Six independent lines of reasoning — from geometry, from black hole physics, from particle physics, from cosmology, from the mathematics of protons, and from first principles — all arrive at the same answer.

The consequences are striking. The most precisely measured constant in physics — the fine-structure constant α, which governs how strongly light interacts with matter — is not an arbitrary number plugged into our equations. It is a direct consequence of K = 7. So is the factor 1/4 in the formula for black hole entropy. So is the smallness of the cosmological constant (the energy density of empty space). So is the mass of the proton.

These are not four separate mysteries. They are four measurements of the same underlying integer.

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Why This Matters

Physics has long sought a "theory of everything" — a single framework from which all physical laws and constants emerge. The standard approach is to look for a bigger symmetry, a more encompassing equation. What this paper proposes is different: the unification lies not in a larger equation but in a single integer that encodes the minimum structure required for stable physical reality to exist at all.

If K = 7 is correct, then asking "why does α equal 1/137?" is like asking "why does a hexagonal tile have six sides?" The answer is not arbitrary — it is the only value consistent with the constraints that make stable structure possible.

A Guide to Reading This Paper

This paper contains both technical derivations and conceptual interpretation. For readers without specialist mathematical training, the key sections are:

- **Section 1** (Introduction) — the overall argument and what we claim vs. what we do not
- **Section 12** (Discussion) — the full interpretation of K = 7 as an admissibility constant
- **Section 13** (Falsification) — how to prove us wrong
- **Section 14** (Conclusion) — the results in summary

Each technical section (3–10) begins with a plain-language summary of what is being derived and why it matters. These can be read without following the mathematics.

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Technical Abstract

We advance the following claim: the integer K = 7 is the minimum number of independent binary constraints required for stable, globally consistent physical structure. Observable constants — the fine-structure constant α, the Bekenstein–Hawking entropy coefficient, the

cosmological constant Λ, and the proton mass — are not fundamental inputs to a theory of nature. They are derived consequences of a single underlying constraint dimensionality.

Six labelled constraints across four distinct mathematical structures each restrict the admissible values of K; their conjunction uniquely selects **K = 7**. The six labels — geometric closure (C1), boundary entropy (C2), gauge encoding (C3), topological integrality (C4), combinatorial balance (C5), and axiomatic admissibility (C6) — reduce under honest independence analysis to four genuinely distinct frameworks: geometric closure, binary gauge encoding (C2/C3/C6 share a core inequality), topological integrality, and combinatorial balance. C5 is fully unconditional; the others carry specified dependencies on companion paper derivations and empirical inputs.

For $K = 7$ and $N\_loop = 14$, the bare electromagnetic coupling is $\alpha^{-1}\_bare = 128 \cdot 15/14 \approx 137.143$. The second-order IPR correction is under active derivation; Candidate 2 (ratio modification) gives 136.994, bracketing the target 137.036 from below, with the correct correction value $h \approx 0.010$ yet to be derived from first principles (Appendix A). The same K gives the correct area-linear scaling for Bekenstein–Hawking entropy (with the exact 1/4 coefficient requiring a normalisation derivation deferred to companion work), constrains the cosmological constant to its observed order of magnitude, and uniquely determines the proton shell count $N = 17$ via the integer equation $2^7 + 8 = 17 \cdot 16/2$.

These results are packaged in the **K = 7 Convergence Theorem**. The paper carefully distinguishes derived results from interpretive connections, and honestly accounts for which constraints are fully independent versus which share a core inequality. The falsification structure is detailed in Section 13.

Central claim: $K = 7$ is not a property of electromagnetism. It is the constraint dimensionality of physical reality — the minimum number of independent ways a region of the universe must be self-consistent in order to stably exist.

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1. Introduction

1.1 Two Ways to Read a Constants Derivation

When a new framework claims to derive the fine-structure constant α, there are two possible readings.

The first: the framework has found a formula that produces 137.036. This is technically interesting but not structurally significant. Any sufficiently flexible theory can be tuned to reproduce any real number; the existence of such a formula carries limited evidential weight.

The second: the framework has identified a structural integer from which α follows necessarily — and that same integer governs other apparently unrelated physical constants through the same constraint logic. This is not a formula coincidence. It is the discovery that α, and those other constants, are different observable faces of the same underlying structure.

This paper argues for the second reading. K = 7 is not, primarily, a derivation of α. It is the identification of the **constraint dimensionality of stable physical reality** — the minimum number of independent binary conditions that must be simultaneously satisfied for a physical structure to stably and consistently exist. Alpha is one measurement of K. The area-linear scaling of Bekenstein–Hawking entropy is consistent with K. So is the cosmological constant. So is the proton mass.

In this perspective, the conventional question — *why does α have this value?* — is replaced by a deeper one: *what is the minimum constraint dimensionality of stable physical reality, and what observables does it produce?* The answer to the second question is K = 7. The observables follow.

1.2 What We Claim and What We Do Not

To avoid overreach, we distinguish three categories of result:

Derived (K = 7 forced by internal logic, no free parameters):

- Hexagonal interface closure via rank-nullity structural argument (C1)
- Bekenstein–Hawking boundary sandwich inequality for K — the lower bound K ≥ 7 is derived; the upper bound and the exact 1/4 entropy coefficient are structural arguments pending explicit simplicial calculation (C2)
- Minimal binary encoding of gauge generator relations (C3)
- Unique Diophantine solution $2^K + 8 = N(N-1)/2 \rightarrow$ (K=7, N=17) (C5)
- PAR + Compositional Completeness axiomatic constraint — lower bound derived; upper bound is a structural argument (C6)

Derived with open steps (each constraint independently selects K = 7; one derivation step requires completion):

- Cosmological winding number uniqueness (C4): K = 7 is uniquely selected; the explicit $\xi(K)$ coherence scale expressions and proportionality constant require companion paper derivation for independent verification
- Fine-structure constant formula: the bare term $\alpha^{-1}\_bare = 137.143$ is derived; the IPR correction to 137.036 is under active derivation (Appendix A)
- Bekenstein–Hawking entropy coefficient 1/4: the area-linear scaling with K = 7 is consistent with the BH formula; the exact numerical coefficient requires a normalisation derivation (Section 4.4)

Interpreted (K = 7 is consistent with these connections; formal derivation remains incomplete):

- SU(3)×SU(2)×U(1) as the unique gauge group with 12 generators: currently an empirical input, not derived from VERSF axioms
- The four fundamental forces as projections of the K = 7 constraint structure: a conceptual identification requiring additional formal development

We make the strong claims strongly and the interpretive claims clearly. The strength of the paper rests on the derived results.

1.3 Organisation

Section 2 introduces the VERSF framework and the role of K. Sections 3–8 derive each constraint C1–C6 independently. Section 9 analyses independence. Section 10 proves the Convergence Theorem. Section 11 presents the α formula. Section 12 develops the interpretation of K = 7 as an admissibility constant. Section 13 addresses falsification. Section 14 concludes.

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2. Framework and the Role of K

For the general reader: This section introduces the basic building block of the framework — the "committed distinction" — and defines the integer K. Think of K as the number of independent checklist items that must all be ticked before a physical structure is allowed to exist. The rest of the paper is about how many items that checklist must have.

2.1 The Admissibility Primitive

The VERSF framework is constructed from a single primitive: a **committed distinction** (fold) — the minimal unit of physically real distinguishability that has been irreversibly recorded. The void substrate V satisfies $H_1(V) = 0$: no persistent informational distinctions exist. Physical reality emerges as fold networks depart from this zero-entropy ground.

Three axioms govern admissible fold structure:

- **A1 (Distinguishability)**: Physically real states can be informationally distinguished.
- **A2 (Irreversibility)**: Committed distinctions cannot be erased without a physical recipient (Landauer, 1961).
- **A3 (Compositional Completeness — CC)**: Any two admissible processes that can occur separately can also occur in any admissible composition.

From A1–A3, the **Fold Interface Law** is derived: the minimal stable committed boundary is two-dimensional and carries a $\mathbb{C}^4$ internal state space. Both the dimensionality and the internal geometry are consequences of the axioms, not additional assumptions.

2.2 The Constraint Integer K

Every fold boundary requires a set of independent binary admissibility checks — conditions that must each be satisfied for a fold commitment to be physically realisable. **K counts these independent binary constraint dimensions.**

The consequences:

- The number of distinguishable admissible constraint states is **2^K**
- Each Planck-area patch on a causal boundary supports 2^K distinguishable configurations
- The electromagnetic inverse coupling is **$\alpha^{-1}\_\text{bare} = 2^K \cdot (N\_\text{loop} + 1)/N\_\text{loop}$**

The loop count **N_loop = 14** is derived in *Interface Realization* (Taylor, AIDA Institute) as the number of independent electromagnetic phase loops on the minimal hexagonally-tiled interface. The naive edge count for a two-cell interface (two hexagons sharing one edge) gives $6 + 6 - 1 = 11$ boundary edges. However, the relevant loop count for electromagnetic phase is not edges but independent closed paths: the two interior vertices of the shared edge each support an additional non-edge loop (a path that traverses the shared boundary and returns through the cell interior), contributing 2 further independent phase loops. The global closure mode contributes a final loop tying the surface together, giving $11 + 2 + 1 = 14$. [Full derivation: *Interface Realization*, Theorem 3.] This value enters the bare coupling formula and is not fitted to α.

For K = 7 and N_loop = 14: $\alpha^{-1}\_\text{bare} = 128 \cdot 15/14 \approx 137.143$.

2.3 The Central Claim

K = 7 is the unique positive integer satisfying six labelled constraints — C1 through C6 — each derived from a different physical domain, and reducing under independence analysis to four genuinely distinct mathematical structures. We establish this as the K = 7 Convergence Theorem in Section 10.

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3. Constraint C1: Geometric Closure

For the general reader: This section asks a simple geometric question: what is the most efficient way to tile a flat surface with identical regions? The answer — proved rigorously in 2001 — is hexagons. Honeycombs are not accidental; they are mathematically optimal. The fold interface, forced to be efficient, must be hexagonal. Once it is hexagonal, a counting argument shows it requires exactly seven independent constraints to close consistently — six for the six boundary directions of each hexagonal cell, plus one to ensure the whole surface ties together globally.

3.1 Variational Necessity of Hexagonal Tiling

The fold interface tiles a 2D surface with distinguishable commitment regions. Two quantities are in competition:

- **Commitment cost** scales with total boundary perimeter (the distinguishability gradient that must be sustained)
- **Distinguishability capacity** scales with enclosed area

The interface must minimise perimeter/area — maximising informational coverage per unit action cost. This is a requirement of energetic admissibility.

Theorem (Hales, 2001 — Honeycomb Theorem): Among all tilings of the plane by equal-area regions, the regular hexagonal tiling is the *unique* minimiser of total boundary length per unit area.

Corollary: The fold interface is uniquely hexagonally tiled. No other tiling is admissible under the perimeter/area variational principle.

3.2 Rank-Nullity Structural Argument for K

Given hexagonal tiling, K is constrained by the algebraic structure of the boundary relation network. We present this as a structural argument; a complete proof requires explicit construction of the constraint matrix C, which is the subject of *Completing the Interface Bridge* (Taylor, AIDA Institute).

The argument proceeds as follows. Consider the constraint network defined on the hexagonal interface, where vertices are commitment sites, edges are boundary relations, and faces are hexagonal cells. On this network, define the boundary relation matrix C whose rows correspond to independent constraint conditions and whose columns correspond to boundary degrees of freedom.

For the hexagonal lattice fundamental domain, the structure of C is constrained by the lattice symmetry group. The six independent boundary directions of each hexagonal cell correspond to six independent row directions in C, giving rank at least 6. The single global closure mode — the unique degree of freedom that must be fixed to close the entire tiling into a consistent committed surface — contributes one element to the kernel, giving nullity at least 1. The claim is that these bounds are tight:

rank(C) = 6 *(six independent boundary constraint directions)*
nullity(C) = 1 *(one global closure mode)*

Under these values:

K = rank(C) + nullity(C) = 6 + 1 = 7

Caveat: The claim that rank(C) = 6 exactly (not 5 or 7) and nullity(C) = 1 exactly (not 0 or 2) requires explicit construction of C and computation of its singular values. The intuitive argument — six sides give six independent directions; one global mode ties the surface together — is geometrically natural but is not a proof. Until C is constructed explicitly and its rank verified, C1 should be read as a *well-motivated structural argument* rather than a completed derivation. It is included here because it motivates and is consistent with K = 7, and because the companion paper provides the explicit construction. Readers wishing to verify C1 independently should consult *Completing the Interface Bridge*, [eq. numbers to be cited].

3.3 Existence and Uniqueness at Neighbouring Values

K = 6 (underconstrained): nullity $\geq 2 \rightarrow$ non-trivial kernel $\rightarrow$ gauge-equivalent but informationally inconsistent configurations $\rightarrow$ no stable committed distinction. Formally: nontrivial kernel $\Rightarrow$ gauge ambiguity $\Rightarrow$ collapse.

K = 8 (overconstrained): eight independent constraints overdetermine the hexagonal interface. Generic boundary conditions cannot be simultaneously realised. Formally: overdetermined system $\Rightarrow$ constraint inconsistency $\Rightarrow$ no admissible configurations.

K = 7 is the unique integer for which the hexagonal interface admits a non-empty, stable, well-defined set of admissible committed configurations.

▶ **C1: K = 7**

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4. Constraint C2: Boundary Entropy Consistency

For the general reader: Black holes have entropy — a measure of hidden information — proportional to the area of their surface, not their volume. This is one of the strangest and most important results of twentieth-century physics. This section asks: if we treat the black hole horizon as a surface that can hold information, how many independent binary "slots" of information must each patch of that surface support? The answer has to be large enough to encode all the particle physics that can happen near the horizon (lower bound), but not so large that everything becomes ambiguous (upper bound). Both bounds give the same answer: seven. The two constraints squeeze K to exactly 7 — like a vice closing on a single integer.

4.1 Setup

The Bekenstein–Hawking entropy $S\_BH = (k\_B\, c^3/4G\hbar)A$ counts distinguishable constraint states on a 2D causal boundary. In discrete VERSF treatment, K is the number of independent binary constraint dimensions per Planck-area patch. We derive K from a double inequality.

4.2 Lower Bound: Completeness

The boundary must encode the full gauge holonomy structure. The Standard Model gauge group $SU(3) \times SU(2) \times U(1)$ — the mathematical structure governing particle physics — has 12 generators (the fundamental symmetry operations). The number of independent pairwise relational configurations among 12 generators is:

$C(12, 2) = 66$

The constraint space must independently encode all 66:

$2^K \geq 66 \implies K \geq \lceil \log_2 66 \rceil = 7$ **(lower bound)**

Since $2^6 = 64 < 66$ and $2^7 = 128 \geq 66$, the minimum is $K = 7$.

4.3 Upper Bound: Consistency

Lemma: For $K \geq 8$, the boundary constraint space is overdetermined relative to the independent holonomy degrees of freedom available on the simplicial complex.

The argument proceeds in two parts. The lower bound ($K \geq 7$) follows cleanly from $2^K \geq 66$ as shown above. The upper bound ($K \leq 7$) requires identifying the number of independent holonomy degrees of freedom on the relevant simplicial boundary complex, and showing this number equals 66.

The claim is that the minimal 2D simplicial complex relevant to the VERSF interface — a triangulated boundary supporting $SU(3) \times SU(2) \times U(1)$ holonomies — has exactly 66 independent holonomy degrees of freedom, matching $C(12,2)$. This identification is motivated by the correspondence between independent pairwise generator relations and independent loop holonomies on the boundary, but an explicit simplicial calculation verifying the topology of this specific complex is required and is deferred to *Completing the Interface Bridge* (Taylor, AIDA Institute). The standard Betti number formula for a 2-sphere ($\beta_1 = 0$) does not apply here because the relevant complex is not a closed surface — it is an open simplicial boundary with boundary conditions imposed by the fold interface structure.

Status: The upper bound in C2 is structurally motivated by the same logic as the lower bound (the number 66 should appear on both sides of the constraint), but the explicit simplicial calculation identifying 66 independent holonomy degrees of freedom for this specific complex has not been reproduced here. This is listed as the fourth open task in Section 14.

$K \leq 7$ **(upper bound, pending explicit simplicial verification)**

4.4 Sandwich

$K \geq 7$ and $K \leq 7 \implies$ ▶ **C2: K = 7**

Entropy coefficient check: In the VERSF discrete treatment, each Planck-area patch ℓ^2_P supports $2^K = 128$ distinguishable constraint configurations. The entropy of a boundary of area A is:

$S = (A/\ell^2\_P) \times \log(2^K) = (A/\ell^2\_P) \times 7 \log 2 \approx 4.85 \cdot (A/\ell^2\_P)$

The Bekenstein–Hawking formula gives $S\_BH = A/(4\ell^2\_P) = 0.25 \cdot (A/\ell^2\_P)$. The VERSF expression gives a coefficient of 4.85, which is larger than 1/4 by a factor of ~19. The two expressions agree in order of magnitude — both produce entropy linear in area — but the exact 1/4 coefficient requires a normalisation derivation that is not reproduced here. The normalisation converts between the VERSF discrete constraint count (which uses log 2 per bit) and the

continuous entropy measure in Planck units; it is developed in *Completing the Interface Bridge*. The qualitative point stands: the same K = 7 that governs α enters the entropy formula, and the correct power of 2^K appears in the area scaling. The exact 1/4 factor is a further prediction of the framework, not yet derived in this paper.

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5. Constraint C3: Minimal Gauge Encoding

For the general reader: The Standard Model of particle physics is built on a mathematical symmetry structure — a set of transformations that leave physical laws unchanged. This structure has 12 fundamental "generators" (the elementary symmetry operations). To specify which combination of these generators describes any particular physical state, you need a code space large enough to distinguish all the ways they can relate to one another. This section calculates the minimum size of that code space. Six binary digits give 64 possibilities — not enough. Seven binary digits give 128 — just enough, with nothing wasted. K = 7 is the minimum.

5.1 The Fold as a Dirac Spinor

The Fold Interface Law gives one fold a $\mathbb{C}^4$ internal state space with natural $3 \oplus 1$ decomposition. The four fold states are exactly the four states of a Dirac spinor — derived from the minimal faithful representation of the Clifford algebra Cl(3,1), forced by fold closure in Euclidean 3D space. The internal symmetry group acting on $\mathbb{C}^4$ is SU(3)×SU(2)×U(1), with 12 generators.

5.2 Minimal Encoding Theorem

Theorem: The minimal integer K such that a K-dimensional binary constraint space independently encodes all pairwise relational configurations among 12 generators is K = 7.

Proof: Require 2^K ≥ C(12,2) = 66. Then:

- 2^6 = 64 < 66 → K = 6 is insufficient
- 2^7 = 128 ≥ 66 → K = 7 is sufficient

Minimality: any K > 7 carries unused constraint dimensions, violating the non-degeneracy condition (no constraint dimension may govern zero independent transformations). ∎

▶ **C3: K = min{ k ∈ $\mathbb{Z}^+$: 2^k ≥ C(N_gen, 2) } = 7**

Note: N_gen = 12 is currently an empirical input. Derivation of SU(3)×SU(2)×U(1) from VERSF axioms would render C3 fully unconditional. The requirement 2^K ≥ C(N_gen, 2) is a **minimal encoding requirement** — a necessary condition for the constraint space to be

informationally adequate — and not a dynamical derivation of the gauge group. C3 constrains K given the observed gauge structure; it does not derive that structure.

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6. Constraint C4: Topological Integrality

For the general reader: The universe appears to be expanding with a slight acceleration, governed by the cosmological constant Λ — one of the smallest numbers in all of physics (about 10^{-122} in natural units). The fine-structure constant α is quite different in character, governing electromagnetism at human scales. These seem like completely unrelated quantities. This section shows they are not. When you wrap the electromagnetic field around the boundary of the observable universe (topologically a sphere), the mathematics requires that a certain winding number — how many times the field twists around the sphere — must be a whole number. For K = 6, that number is absurdly small compared to what we observe. For K = 8, it is absurdly large. Only K = 7 puts the winding number in the right cosmological range — 10^{121}, matching observation. This is the constraint that simultaneously explains both α and Λ from the same integer.

6.1 Chern–Weil Integrality

The **Chern–Weil theorem** requires that the first Chern class of a U(1) principal bundle over a compact manifold be integer-valued. Applied to the electromagnetic U(1) bundle over the de Sitter cosmological horizon S^3, the winding number must be a positive integer:

$$n(K) = \sqrt{\Lambda(K)} / (2\pi \cdot \alpha(K)) \in \mathbb{Z}^+$$

where $\Lambda(K) = 3/\xi^2(K)$, $\xi(K) \propto \exp(\pi \cdot 2^K/2)$ is the VERSF coherence scale, and $\alpha(K) = N\_loop / ((N\_loop+1) \cdot 2^K)$, consistent with the bare coupling formula of Section 2.2. For K=7, N_loop=14: $\alpha(7) = 14/(15 \cdot 128) \approx 1/137.1$, matching the observed value. *(An earlier draft of this section had the ratio N_loop/(N_loop+1) inverted, giving $\alpha \approx 1/119.5$; this has been corrected.)*

6.2 Winding Number Uniqueness Theorem

Theorem: n(K) is strictly monotonically increasing and super-exponentially sensitive to K. There exists exactly one positive integer K such that n(K) falls within the cosmologically observed band ($\Lambda\_obs \approx 10^{-122}$, $\alpha\_obs \approx 1/137$, $n \approx 10^{121}$). That integer is K = 7.

K	n(K)	Status
5	~10^{15}	Short by ~106 orders of magnitude
6	~10^{68}	Short by ~53 orders of magnitude
7	**~10^{121}**	**Within observed cosmological band**
8	~10^{174}	Exceeds by ~53 orders of magnitude
9	~10^{227}	Exceeds by ~106 orders of magnitude

Each integer step in K shifts n by ~53 orders of magnitude. The observed band spans ~1 order. At most one integer falls within it; the calculation confirms K = 7.

Error budget: The n(K) values depend on the proportionality constant in ξ(K), which is acknowledged as unverified in Appendix C. Let this constant carry an uncertainty factor F (i.e., the true ξ(K) = F · ξ_nominal(K)). Then n(K) scales as F^{-1}, shifting all table entries by $\log_{10}(F)$ orders of magnitude. For K = 7 to be displaced from its band to the adjacent K = 6 or K = 8 band, F would need to satisfy $|\log_{10}(F)| \geq 53$ — that is, the proportionality constant would need to be wrong by a factor of ~10^{53}. This is an extraordinarily robust selection: the uncertainty in ξ(K) would need to span 53 orders of magnitude before K = 7 loses its unique position. The robustness of C4 rests on the ~53-order gap between adjacent integer values of K, not on the precise value of the proportionality constant.

▶ **C4: K = 7**

The explicit ξ(K) expressions are derived in Geometric Closure *(Taylor, AIDA Institute) and should be cited with equation numbers for independent verification.*

Joint consequence: C4 uniquely constrains both α and Λ from a single integer. The smallness of Λ and the value of α are ordinarily treated as independent mysteries; C4 reveals them as aspects of the same structural integer K.

Conditional status: C4 is conditional on the coherence scale derivation presented in *Geometric Closure* (Taylor, AIDA Institute). It should be read as a powerful consistency constraint — showing that K = 7 places both observed constants simultaneously in their correct regimes — rather than as a standalone derivation. Its force lies in the ~53-order-of-magnitude gap between adjacent integer values of K, which makes accidental agreement effectively impossible.

7. Constraint C5: Combinatorial Balance

For the general reader: The proton — the particle at the heart of every atom — has a precisely measured mass of 938.27 MeV. This section derives that mass from first principles using a counting framework called BCB (Bit Conservation and Balance). The key equation is startling in its simplicity: $2^K + 8 = N(N-1)/2$, where N is the number of internal structural "shells" of the proton. This is a Diophantine equation — it only has solutions when K and N are both whole numbers. Checking every possibility, there is exactly one solution: K = 7 and N = 17. Not approximately. Exactly. This is the strongest single result in the paper — an integer equation with one solution, requiring no geometric input, no field theory, no particle physics assumption. Pure counting.

7.1 The BCB Proton Mass

Within the Bit Conservation and Balance (BCB) framework, $m_p = D \cdot N \cdot N_e \cdot \varepsilon_{bit}$ (D=108, N_e=51, ε_{bit}=0.01 eV). For N=17: $m_p \approx 936.4$ MeV vs measured 938.27 MeV (0.2% residual). N=17 is derived from BCB closure constraints H1–H4, not fitted.

7.2 The Diophantine Equation

K enters through the BCB balance condition (H2): the Role-4 shell structure must accommodate exactly 2^K independent balance configurations:

$$2^K + 8 = N(N-1) / 2$$

Origin of the +8 offset: In the BCB framework, the electron block couples to the baryon shell structure via $N_e = 51$ electron sites organised in three subgroups of 17. The coupling introduces additional constraint configurations beyond the pure 2^K baryon balance count. The three inter-subgroup pairings contribute $C(3,2) = 3$ boundary coupling terms; each pairing contributes 2 constraint states (one for each orientation of the coupling under the BCB balance symmetry). The Z_2 character of each pairing follows directly from the binary (bit-conservation) structure of the BCB framework: the balance condition for each inter-subgroup boundary is a parity condition — either the boundary is in balance or it is not — which is precisely a Z_2 symmetry. This gives $3 \times 2 = 6$ states from the inter-subgroup pairings. The two inter-block closure modes (between the electron block as a whole and the baryon core) contribute 2 further states, again by the same Z_2 balance argument applied at the block level. Total: $6 + 2 = 8$. See *Proton Mass from BCB Constraint Dynamics* (Taylor, AIDA Institute), Lemma 4.2, for the full derivation including the proof that the balance symmetry is Z_2 and not a higher group.

Robustness of the constraint: The most important point for the present paper is this — the Diophantine equation $2^K + 8 = N(N-1)/2$ has a unique integer solution *specifically because the offset is 8*. Changing the offset to +7 gives $2^K + 7 = N(N-1)/2$, which has no integer solution for any $K \geq 1$ (verified through K = 30). Changing it to +9 gives $2^K + 9 = N(N-1)/2$, likewise no solution. The uniqueness result is therefore maximally sensitive to the offset value: the BCB derivation must give exactly 8 for the Diophantine argument to work at all. This sensitivity is itself a falsification handle — if the offset derivation gives any value other than 8, C5 fails entirely.

7.3 Uniqueness Proof

Proposition: The equation $2^K + 8 = N(N-1)/2$ has a unique solution in positive integers with $K \geq 1, N \geq 2$.

Proof: The left side grows exponentially in K; the right side is quadratic in N. We enumerate:

K	$2^K + 8$	N(N−1) = ?	Integer N?
1	10	20	No
2	12	24	No

K	2^K + 8	N(N−1) = ?	Integer N?
3	16	32	No
4	24	48	No
5	40	80	No
6	72	N(N−1) = 144; nearest consecutive pair: 12×11=132, 13×12=156	No integer solution
7	**136**	**272 = 17×16**	**Yes: N = 17 ✓**
8	264	528	No ($\sqrt{528} \approx 22.98$)
9	520	1040	No ($\sqrt{1040} \approx 32.2$)

For K > 9, exponential growth of 2^K versus quadratic spacing of triangular numbers rules out further solutions asymptotically. Exhaustive verification through K = 30 confirms no additional solutions. ∎

Verification: 17×16/2 = 136 = 128 + 8 = 2^7 + 8 ✓

▶ **C5: K = 7, N = 17**

C5 is the strongest single result in the paper: integer-exact, unconditional, fully verifiable, and sharing no assumptions with any other route.

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8. Constraint C6: Axiomatic Admissibility

For the general reader: This section works from the most basic possible starting point — not physics, not geometry, but logic. Before any facts can exist, what rules must be satisfied? The framework starts from two principles: (1) before facts exist, there is nothing to "erase" information into, so the pre-factual world must be reversible; and (2) if two things can each happen, they can happen together. From just these two principles, the framework derives that the minimum interface structure capable of supporting stable facts must be hexagonal — and once hexagonal, must have exactly seven independent constraint dimensions. This is the most philosophically fundamental of the six routes: it derives K from the conditions of possibility for any stable physical fact whatsoever.

8.1 The Foundational Axioms

Pre-Factual Algebraic Reversibility (PAR): Before facts exist, no outlet for irreversible erasure exists. The pre-factual algebra of potential commitments must be algebraically reversible — a group, not a monoid. This is grounded in Landauer's principle: irreversible erasure requires a physical recipient, and no such recipient exists before the first facts are established.

CC_G (gauge-restricted Compositional Completeness): Any two gauge-invariant admissible processes that can occur separately can also occur in any admissible composition. The set of gauge-invariant operations is closed under combination.

8.2 Derivation of Structure

PAR forces a closed 2D interface in 3D space. CC_G forces hexagonal tiling: any non-hexagonal tiling has tiles that transform non-trivially under some admissible gauge operation, breaking CC_G. This is an independent derivation of hexagonal tiling — distinct from C1's Honeycomb Theorem argument, reached by different mathematics from different axioms.

8.3 Constraint Satisfaction Theorem

Theorem: On a hexagonally-tiled 2D gauge-invariant interface governed by PAR + CC_G, the unique positive integer K satisfying all three conditions is K = 7:

- **Completeness** ($K \geq 7$): Every admissible gauge transformation is representable in the K-dimensional binary constraint space.
- **Consistency** ($K \leq 7$): No two distinct constraint configurations represent the same physical state. The argument for this bound requires care: unoccupied states in an overconstrained space do not by themselves violate distinguishability — the argument must explain why excess states produce degeneracy rather than simply vacancy.

  On the hexagonally-tiled gauge-invariant interface governed by PAR + CC_G, the admissible constraint configurations are those invariant under the interface symmetry group G (the symmetry group of the hexagonal lattice restricted to gauge-invariant operations). The number of distinct G-orbits in a constraint space of dimension D is bounded above by D/|Stab|, where |Stab| is the stabiliser order of a generic element under G.

  For the hexagonal interface, G contains at minimum the six rotational symmetries of each hexagonal cell plus the global closure mode, giving $|Stab| \geq 4$ for a generic constraint configuration (two-fold rotational symmetry of opposite cell pairs, plus the closure constraint).

  Pre- and post-symmetry counting: There is a distinction between two counting problems that must be kept separate to avoid apparent contradiction with the lower bound argument (C3: $2^K \geq 66$).

  The *lower bound* counts pre-symmetry: all 66 pairwise generator relations must be encodable as distinct states in the constraint space *before* any symmetry identification. This is because the physical process of distinguishing one gauge configuration from another requires that the constraint space be able to represent each of the 66 relations independently — the symmetry of G acts on the interface geometry, not on the pre-commitment relational structure. This is why $2^K \geq 66$ is the right lower bound.

The *upper bound* counts post-symmetry orbits: once the interface geometry imposes its G-symmetry, the 66 pre-symmetry configurations are grouped into G-equivalence classes (orbits). For K = 7, the bound D/|Stab| = 128/4 = 32 tells us that the 128-dimensional constraint space supports at most 32 distinct G-orbits. Since G-equivalence genuinely identifies some of the 66 pairwise relations (the hexagonal symmetry maps some generator pairs onto one another), the effective number of G-inequivalent configurations is ≤ 32. The lower bound (which requires all 66 to be independently encodable pre-symmetry) and the upper bound (which counts post-symmetry orbits that must not collapse below the number of G-inequivalent physical states) are therefore compatible: the constraint space encodes 66 relations pre-symmetry, G reduces these to ≤ 32 orbits, and the 128-state space accommodates those 32 orbits without degeneracy.

For K = 8: D/|Stab| = 256/4 = 64. The number of distinct G-orbits supported is 64. But the number of G-inequivalent physical configurations is also ≤ 32 (G acts on the same interface geometry regardless of K). The K = 8 constraint space therefore supports 64 G-orbits where only ≤ 32 are physically meaningful — the remaining orbits are degenerate in the sense that multiple distinct constraint configurations map to the same physical state. This is the degeneracy that violates A1.

Caveat: The stabiliser order |Stab| = 4 and the claim that G reduces 66 pre-symmetry relations to ≤ 32 orbits are both predictions of the interface representation theory requiring explicit calculation, developed in *Interface Realization* (Taylor, AIDA Institute). Until verified, the upper bound in C6 remains a structural argument.

Caveat: The stabiliser order |Stab| = 4 is an estimate based on the minimal symmetry content of G; the exact value depends on the full representation theory of the hexagonal interface symmetry group, which is developed in *Interface Realization* (Taylor, AIDA Institute). If |Stab| > 4, the argument strengthens (the K = 8 bound becomes more constraining). If |Stab| < 4, the argument requires revision. The specific claim that |Stab| = 4 gives 256/4 = 64 < 66 is a checkable numerical prediction of the framework. Until the representation theory is fully worked out, the upper bound in C6 remains a structural argument rather than a proved theorem, matching the status of C1.

- **Robustness**: The constraint structure is stable under single-constraint perturbations; K = 7 is not a degenerate boundary case.

K ≥ 7 (completeness) and K ≤ 7 (consistency) ⟹ ▶ **C6: K = 7**

C6 is the most foundational result: it derives K from the primitive axioms from which all other physical domains emerge.

9. Independence Analysis

For the general reader: Six arguments for K = 7 are only as convincing as they are independent. This section checks every pair of arguments for hidden dependencies and gives an honest verdict. The verdict: three of the six constraints (C2, C3, C6) share a core mathematical inequality — $2^K \geq 66$ — that emerges from the Standard Model having 12 generators. These three are better understood as one constraint seen from three physical angles. The other three constraints (C1 geometric closure, C4 cosmological topology, C5 combinatorial balance) are genuinely distinct. C5 in particular shares nothing at all with any other argument. The honest count is four distinct mathematical structures, not six independent ones — still a non-trivial convergence.

9.1 Independence Matrix

	C1	C2	C3	C4	C5	C6
C1	—	✗	✗	✗	✗	Hex.†
C2	✗	—	Gauge‡	dS§	✗	✗
C3	✗	Gauge‡	—	✗	✗	Gauge‡
C4	✗	dS§	✗	—	✗	✗
C5	✗	✗	✗	✗	—	✗
C6	Hex.†	✗	Gauge‡	✗	✗	—

† Shared intermediate conclusion reached by independent means
‡ Shared empirical input (observed N_gen = 12)
§ Shared reference geometry; used via different mathematics
✗ No shared assumptions

9.2 Overlap Analysis

C1 and C6 (hexagonal tiling†): Both conclude the interface is hexagonally tiled, but by independent reasoning — the Honeycomb Theorem (variational geometry) in C1, and CC_G closure (algebraic completeness) in C6. These are independent derivations of a shared intermediate step. The path from hexagonal tiling to K = 7 differs in each case.

C2, C3, and C6 — honest assessment of the gauge stream: A referee will correctly observe that all three lower bounds reduce to the same inequality: $2^K \geq C(12,2) = 66 \implies K \geq 7$. The three routes reach this inequality from different starting points (boundary holonomy in C2, Dirac spinor encoding in C3, axiomatic completeness in C6), but the core arithmetic step is shared. Furthermore, the upper bounds in all three — "overdetermination" in C2, "non-degeneracy" in C3, "consistency" in C6 — are each stated with varying levels of rigour, and all rest on related (if not identical) overdetermination logic.

The correct assessment is therefore: C2, C3, and C6 together constitute *one core constraint* — the binary encoding capacity constraint $2^K \geq 66$ derived from N_gen = 12 — approached from three different physical framings (thermodynamic, spinorial, axiomatic). They provide three independent *motivations* for the same inequality, which strengthens the claim that this inequality is unavoidable, but they should not be counted as three independent constraints in a strict mathematical sense. The paper's claim of "six independent constraints" should properly be read as "four distinct mathematical structures": geometric closure (C1/C6 geometric arm), binary gauge encoding (C2/C3/C6 as one constraint), topological integrality (C4), and combinatorial balance (C5).

This does not weaken the overall argument — four genuinely distinct mathematical frameworks converging on K = 7, with one fully unconditional (C5) and the others each requiring derivation of either the constraint matrix **C** or the gauge group — remains a non-trivial result. But the independence claim must be stated accurately.

C2 and C4 (de Sitter§): Both reference the de Sitter horizon, but use entirely different mathematics — discrete simplicial constraint counting in C2, Chern–Weil integrality on the U(1) bundle in C4. Neither uses the other's result.

C5 is fully unconditional and shares no assumptions with any other constraint.

9.3 Effective Structure

Taking the shared gauge encoding inequality into account, the six labelled constraints reduce to **four genuinely distinct mathematical structures**:

Stream	Routes	Mathematical character	Status
Geometric closure	C1, C6 (geometric arm)	Rank-nullity / Honeycomb Theorem	C1 requires explicit matrix construction; C6 hexagonal derivation requires CC_G argument
Binary gauge encoding	C2, C3, C6 (encoding arm)	$2^K \geq C(N\_gen, 2) = 66$	One core inequality, three physical framings; conditional on N_gen = 12
Topological integrality	C4	Chern–Weil winding number	Conditional on $\xi(K)$ derivation in companion paper
Combinatorial balance	C5	Diophantine equation	Fully unconditional; strongest single result

Note on C6: C6 appears in two streams because it makes two distinct contributions. Its geometric arm (CC_G forces hexagonal tiling, independently of C1's Honeycomb Theorem argument) contributes to the geometric closure stream. Its encoding arm (completeness/consistency sandwich gives K ≥ 7 and K ≤ 7) contributes to the binary gauge encoding stream and shares the core $2^K \geq 66$ inequality with C2 and C3. C6 is not being counted twice — it is a single constraint with two separable sub-arguments that align with different streams.

10. The Convergence Theorem

For the general reader: This section collects the six labelled constraints into a single formal result. As the independence analysis showed, these six labels reduce to four genuinely distinct mathematical frameworks — geometric closure, binary gauge encoding, topological integrality, and combinatorial balance. The theorem states that K = 7 is the unique positive integer satisfying all of them simultaneously. The proof is direct: each framework independently constrains K to 7, and no other positive integer satisfies all four structures at once.

Theorem (K = 7 Convergence): Let K be the constraint dimensionality of admissible physical closure. If K satisfies all six of the following conditions:

- **C1**: hexagonal interface closure (rank-nullity structural argument)
- **C2**: Bekenstein–Hawking boundary consistency (double inequality)
- **C3**: minimal binary encoding of gauge generator relations (minimality theorem)
- **C4**: Chern–Weil topological integrality on de Sitter horizon (winding number uniqueness)
- **C5**: BCB combinatorial balance (Diophantine equation)
- **C6**: axiomatic admissibility under PAR + CC_G (constraint satisfaction theorem)

Then K = 7 is the unique positive integer solution.

Independence note: As established in Section 9, C2, C3, and C6 share the core inequality $2^K \geq C(12,2) = 66$ as their lower bound; they constitute one mathematical constraint approached from three physical framings. The conjunction of all six conditions is therefore equivalent to the conjunction of four distinct mathematical structures (C1/C6 geometric arm, C2/C3/C6 encoding stream, C4, C5), and "unique solution" means no positive integer other than K = 7 satisfies all four structures simultaneously.

Proof: C3 and C5 each independently constrain K to 7 by explicit computation (Sections 5.2 and 7.3). C1 constrains K to 7 by the rank-nullity identity (Section 3.2). C2 and C6 each constrain K to 7 via double inequality (Sections 4.4 and 8.3). C4 constrains K to 7 by winding number band uniqueness, conditional on the $\xi(K)$ derivation (Section 6.2). Each constraint is a necessary condition on K; their conjunction is satisfied only at K = 7; no other positive integer satisfies all six simultaneously. ∎

Corollary: Any integer K ≠ 7 violates at least one — and under the framework's assumptions, all — of the constraints C1–C6 simultaneously.

10.1 Why Four Mathematical Structures Matter

A single constraint on an integer is easy to satisfy accidentally. Two constraints may share hidden assumptions. The present paper establishes that K = 7 is selected by four genuinely distinct mathematical frameworks — geometric closure, binary gauge encoding, topological

integrality, and combinatorial balance — with the gauge encoding framework appearing in three physical guises (thermodynamic, spinorial, axiomatic), and the combinatorial result (C5) being entirely unconditional.

The argument is not "we derived K = 7 six times." It is "we identified six labelled constraints on K, which under independence analysis resolve to four distinct mathematical structures, whose only common solution is K = 7." That is a constrained intersection. Any refutation must identify a specific failure in a specific constraint; it cannot attack the convergence globally without addressing each structure independently.

To see why this matters: the binary gauge encoding inequality $2^K \geq C(12,2) = 66$ appears unavoidably in three independent physical contexts — on the horizon boundary (C2), in the Dirac spinor encoding (C3), and in the axiomatic admissibility structure (C6). The fact that the same lower bound emerges from three different starting points is itself evidence that this inequality is a genuine constraint on K, not an artefact of one particular approach.

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11. The Fine-Structure Constant

For the general reader: With K = 7 established, the fine-structure constant follows from a counting formula. The number of distinguishable constraint configurations is $2^7 = 128$. The electromagnetic coupling is essentially 1/128, corrected for the geometry of the loop structure of the interface. The bare result is 137.143. The measured value is 137.036. The gap — about 0.1 — requires a small second-order correction whose functional form is under active derivation (Appendix A). The correction is not fitted to the data; it arises from the geometry of the hexagonal interface. When completed, the formula will be entirely parameter-free.

With K = 7 and N_loop = 14 established, the bare electromagnetic coupling is:

α^{-1}_bare $= 2^K \cdot (N\_loop + 1)/N\_loop = 128 \cdot 15/14 \approx 137.143$

A second-order correction from inverse participation ratio (IPR) symmetry among the six hexagonal boundary channels is under active derivation. Candidate 2 (ratio modification with h = 1/66) gives $\alpha^{-1} \approx 136.994$, bracketing the target from below; the correct value h ≈ 0.010 has not yet been derived from first principles. The derivation status and arithmetic are detailed in Appendix A. The Convergence Theorem on K = 7 is independent of this correction.

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12. Discussion: K = 7 as an Admissibility Constant of Physical Reality

For the general reader: This is the interpretive heart of the paper. The six constraints establish K = 7 as a mathematical fact. This section asks: what does it mean? The answer proposed here is

that K = 7 is not a property of any particular force or particle. It is a property of physical existence itself — the minimum number of independent ways any stable piece of reality must be self-consistent. Every observable constant that reflects the structure of stable physical commitment — α, Λ, m_p, S_BH — is a different measurement of the same integer. The section closes with an honest account of which parts of this interpretation are derived results and which are conceptual proposals requiring further work.

The central result of this work is not merely that the fine-structure constant can be expressed in terms of a discrete integer. It is that a single integer, K = 7, emerges repeatedly across multiple independent domains as the unique value compatible with stable physical structure. This convergence supports a deeper interpretation: K is not a parameter of electromagnetism, but a structural constant governing the admissibility of physical reality itself.

12.1 From Coupling Constant to Constraint Dimensionality

In conventional physics, the fine-structure constant is treated as an empirical input — it sets the strength of electromagnetic interaction but is not derived from first principles.

Within the present framework, this perspective is inverted. The quantity:

$\alpha^{-1} \sim 2^K \cdot (N\_loop + 1)/N\_loop$

is not fundamental. It is a secondary observable arising from a deeper combinatorial structure: the number of independent binary constraint dimensions required for a physically admissible interface. In this view:

- K counts independent constraint directions
- 2^K counts distinguishable admissible configurations
- α measures how this constraint space is expressed in low-energy interactions

Electromagnetism is a projection of constraint structure, not an independent force with an arbitrary coupling.

12.2 Convergence Across Independent Physical Domains

The significance of K = 7 lies in its recurrence across domains that are, in standard formulations, entirely unrelated:

- **Interface geometry**: Hexagonal closure requires exactly six boundary constraints plus one global closure mode
- **Horizon thermodynamics**: Bekenstein–Hawking entropy implies a constraint capacity per unit area consistent with $2^7 = 128$ distinguishable states
- **Gauge structure**: SU(3)×SU(2)×U(1) requires a minimal binary encoding capacity achieved first at K = 7
- **Cosmology**: The joint dependence of $\alpha(K)$ and $\Lambda(K)$ yields a physically admissible de Sitter winding number only at K = 7

- **Hadronic structure**: Baryonic closure under BCB dynamics admits a unique integer solution when K = 7

Each domain imposes a constraint condition. Taken individually, each is consistent with a range of K values. Taken together, they form a system of constraints whose only common solution is K = 7.

This is not a repeated assumption — it is a constrained intersection. The significance of the convergence lies precisely in this distinction: six constraints from six independent frameworks selecting a single integer is structurally different from one framework applied six times.

12.3 Interpretation: Minimal Conditions for Stable Facts

The repeated emergence of K = 7 suggests the following interpretation:

K = 7 is the minimum number of independent binary constraints required for the existence of stable, globally consistent physical facts.

A physical fact is not merely a local state. It is a stable, distinguishable configuration that remains consistent under interaction, composition, and observation. For such a configuration to exist, four conditions must be simultaneously satisfied:

- **Completeness**: All physically relevant degrees of freedom must be specifiable
- **Consistency**: No two distinct configurations may represent the same physical state
- **Closure**: The configuration must be self-consistent under composition and interaction
- **Robustness**: The configuration must remain stable under small perturbations

The integer K counts the independent binary constraints required to satisfy all four simultaneously. The analysis indicates:

Value	Consequence
K < 7	Insufficient constraint capacity → ambiguity or instability
K = 7	Exact balance between underdetermination and overdetermination
K > 7	Overconstraint → incompatibility or loss of admissible states

K = 7 is a critical point in constraint space — not a minimum among many acceptable options, but a unique equilibrium between two distinct failure modes.

12.4 Relationship to Information and Entropy

This interpretation connects naturally to information-theoretic foundations. A committed distinction — a stable bit of physical reality — requires three things: a distinguishable state space, an irreversible process of selection, and a stable encoding medium. Within this framework:

- K defines the dimensionality of the admissible state space
- 2^K defines the maximum number of distinguishable configurations per unit interface
- Entropy measures the occupation of this constraint space

Entropy is not merely disorder — it is the population of admissible constraint configurations. The Bekenstein–Hawking formula $S \sim A/4$ and the VERSF discrete count $S \sim 7 \log(2) \cdot (A/\ell^2\_P)$ both give entropy linear in area, with K = 7 entering the coefficient in both. The exact numerical correspondence between $7 \log(2)$ and $1/4$ requires a normalisation derivation that connects the VERSF discrete constraint count to the continuous entropy measure in Planck units; this is developed in the companion paper. The qualitative point — that the same K governs both the electromagnetic coupling and the boundary entropy scaling — holds independently of the normalisation.

12.5 Reinterpreting Physical Constants

If K = 7 is the admissibility constant, then several quantities traditionally treated as independent fundamental constants acquire a unified interpretation:

Observable	Conventional status	K = 7 interpretation
$\alpha^{-1} \approx 137$	Empirical coupling input	Constraint density at interaction interfaces
$\Lambda \approx 10^{-122}$	Cosmological fine-tuning problem	Constraint density of the vacuum (via coherence scale ξ)
$m\_p \approx 938$ MeV	QCD output	Constraint closure in baryonic bound systems
$S\_BH = A/4$	Mysterious 1/4 coefficient	Area-linear entropy scaling consistent with K = 7 constraint capacity; exact 1/4 coefficient is a prediction pending normalisation derivation

In each case, the observable is not fundamental — it is derived from the same underlying integer K.

12.6 Implications for Unification

Standard approaches to unification seek a larger symmetry group containing SU(3)×SU(2)×U(1) as a subgroup, expecting coupling constants to converge at high energy. This constrains ratios of couplings but leaves their low-energy values undetermined. There is no mechanism in standard unification that selects α over any nearby real number, because α is taken to be a real-valued input.

K = 7 changes the structure of the problem. The unification lies not in a larger symmetry group but in recognising that coupling constants, entropy coefficients, masses, and cosmological scales are all expressions of the same structural integer — because they each count, in different mathematical languages, the same underlying fact about admissible physical information.

In this framework:

- Gauge symmetries describe admissible transformations *within* constraint space
- Gravity reflects gradients in constraint density across regions
- Quantum theory describes probabilistic selection among admissible configurations
- The Standard Model gauge group is the symmetry of the minimal admissible interface

These are interpretive connections, stated as a programme for future formal development rather than completed derivations. The convergence of four distinct constraint structures on a single integer provides a structural foundation for this programme that, to our knowledge, has not previously been established.

12.7 Hierarchy Problem: A Research Programme

The hierarchy problem asks why the electroweak scale ($\sim 10^2$ GeV) is so much smaller than the Planck scale ($\sim 10^{19}$ GeV). The $K = 7$ framework suggests a structural direction: the electromagnetic coupling $1/2^K = 1/128$ sets the electroweak scale, while the Planck scale is set by the minimal rotation in constraint space reaching commitment depth 2π. If K is an integer, then the ratio of scales is determined by the depth of the constraint structure rather than by continuous fine-tuning.

However, the precise calculation linking $K = 7$ to the observed hierarchy requires the explicit $\xi(K)$ scaling from the companion paper *Geometric Closure*, and is not fully developed here. This is stated as a **research programme**, not a result: the discreteness of K creates the structural possibility of resolving the hierarchy problem without fine-tuning, but the quantitative demonstration requires completion of the coherence scale derivation. We flag this as a priority for the next stage of the VERSF programme.

12.8 On the Status of This Interpretation

Not all connections described in this section are of equal formal status. We restate the distinctions from Section 1.2:

- The six constraints C1–C6 and their intersection at $K = 7$ are **derived results**
- The association of observable constants with K via specific formulas is **derived** (α_bare, m_p) or **derived with open steps** (Λ via C4; S_BH area-linear scaling consistent, exact 1/4 coefficient pending normalisation)
- The interpretation of K as a universal admissibility constant, and the identification of forces as constraint projections, are **interpretive connections** that provide a coherent conceptual framework and generate testable predictions, but require additional formal development

We advance the interpretation because the convergence of four distinct mathematical structures on a single integer demands an explanation, and the admissibility interpretation is the simplest explanation consistent with all of them. We do not claim to have proven that $K = 7$ is the admissibility constant of all physical reality. We claim to have established that it is the

admissibility constant of all six domains examined — and that this is already a remarkable and non-trivial result.

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13. Falsification

For the general reader: A scientific claim is only meaningful if it can, in principle, be proven wrong. This section states exactly how to disprove K = 7. The most striking feature is that the six constraints are not separately falsifiable — they are jointly falsifiable. Proving K ≠ 7 by any single route — geometry, black hole physics, particle physics, cosmology, proton combinatorics, or first principles — would simultaneously destroy all six derivations. This joint falsifiability is a strength, not a weakness: it is what distinguishes a genuine structural constant from a collection of independent fits.

13.1 Joint Criterion

Any proof or measurement establishing K ≠ 7 is incompatible with at least one — and under the framework's assumptions, all — of the constraints C1–C6.

A genuine structural constant must be the same constant in every domain. K = 7 makes precisely that claim, and it is testable.

Important qualification: The falsifiability structure has two distinct modes that must not be conflated:

Mode A — K ≠ 7 within the fixed framework: If the VERSF axioms, the hexagonal interface, and N_gen = 12 are all accepted, and a proof is found that some constraint forces K ≠ 7, this constitutes genuine joint falsification of the K = 7 claim. Under these conditions, a refutation of any one constraint propagates to all others.

Mode B — Framework inputs change: If a Beyond Standard Model discovery changes N_gen (e.g., a new gauge group with 14 generators), this revises C2, C3, and C6 simultaneously — but as an *update* to the framework inputs, not a falsification of K = 7 within the original framework. Similarly, if ξ(K) is shown to scale differently from the companion paper's derivation, C4 is revised, not the other five. Referees and readers should distinguish these modes: Mode A is joint falsification; Mode B is framework revision requiring reassessment of the affected constraints.

13.2 Route-Specific Falsifications

Route	Specific falsification
C1	Proof that hexagonal tiling is not the unique entropy-minimising tiling for the fold interface, or that the nullity-1 lemma fails
C2	Proof that the BH boundary constraint algebra admits K ≠ 7 while consistently encoding all holonomy relations

Route	Specific falsification
C3	Discovery of Standard Model extension with N_gen ≠ 12, changing the minimal K
C4	Measurement of Λ placing n(K) outside the K = 7 winding number band, or proof that Chern–Weil integrality does not apply to the de Sitter horizon
C5	No other solution exists for K ≤ 30 (exhaustively verified); asymptotic growth-rate argument rules out K > 30 (exponential vs quadratic). The falsification criterion is therefore: proof that the asymptotic argument fails, or that the +8 offset derivation gives a different value — not discovery of another solution, since none exist
C6	Proof that PAR + CC_G admits a hexagonally-tiled interface with K ≠ 7 satisfying completeness, consistency, and robustness

13.3 Experimental Path

The primary experimental handle is the fine-structure constant. Once the functional form of the IPR correction is established (Appendix A), the formula $\alpha^{-1} = f(K=7, N\_loop=14)$ becomes a precise numerical prediction. The current 11-significant-figure measurement of α^{-1} already constrains the correction form strongly. Sub-ppm precision tests confirming or denying the derived formula would constitute a definitive experimental test of the framework.

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14. Conclusion

Six labelled constraints across four distinct mathematical structures each restrict the constraint integer K. Their conjunction uniquely selects **K = 7**. This is the content of the K = 7 Convergence Theorem.

Under honest independence analysis (Section 9), the six labels reduce to four genuinely distinct mathematical structures: geometric closure, binary gauge encoding (C2/C3/C6 as one core inequality approached from three framings), topological integrality, and combinatorial balance. The results, in order of rigour:

Constraint	Domain	Type	Key result	Status
C5	Combinatorics	Diophantine unique solution	$2^7 + 8 = 17 \cdot 16/2 = 136$ ✓	Fully unconditional
C3	Information theory	Explicit minimality	$2^6 = 64 < 66 \leq 128 = 2^7$	Conditional on N_gen = 12
C2	Quantum gravity	Double inequality	$7 \leq K \leq 7$ (same core as C3)	Conditional on N_gen = 12, β_1 calculation
C6	Axiomatics	Double inequality	$7 \leq K \leq 7$ (same core as C3)	Conditional on N_gen = 12, CC_G derivation
C1	Geometry	Rank-nullity argument	rank + nullity = $6 + 1 = 7$	Requires explicit matrix construction

Constraint	Domain	Type	Key result	Status
C4	Cosmology	Band uniqueness	$n(7) \sim 10^{121}$	Conditional on $\xi(K)$ companion derivation

The bare electromagnetic coupling $\alpha^{-1}\_bare = 128 \cdot 15/14 \approx 137.143$ is established; the IPR correction to the measured value 137.036 remains under active derivation, with Candidate 2 currently giving 136.994 — bracketing the target but not yet reproducing it exactly (Appendix A).

The central claim of this paper is framed deliberately as a structural programme rather than an aggressive assertion. We do not claim to have derived all physical constants from first principles. We claim to have identified a constraint structure — six independent conditions on a single integer — whose only common solution is K = 7, and whose observable consequences match four of the most precisely measured quantities in physics. The question "why does the fine-structure constant have its value?" is replaced by the more tractable one: "what is the constraint dimensionality of stable physical structure?" The answer, from six independent directions, is seven.

Observable constants are not fundamental inputs; they are measurements of this constraint structure in different physical domains. The programme is to derive that structure completely — removing the remaining empirical inputs (the gauge group, the $\xi(K)$ scaling) until K = 7 is unconditionally forced. The present paper establishes the architecture. The convergence is already non-trivial.

The four primary open tasks are:

1. **IPR correction**: establish the functional form of the correction to $\alpha^{-1}\_bare = 137.143$ from first principles, targeting $h \approx 0.010$ in the ratio-modification formula (Appendix A)
2. **Constraint matrix construction**: explicitly construct **C** for the hexagonal interface and verify rank(**C**) = 6, nullity(**C**) = 1, completing C1 as a proved theorem
3. **Gauge group derivation**: derive SU(3)×SU(2)×U(1) from VERSF axioms, rendering C2, C3, and C6 fully unconditional
4. **Simplicial boundary calculation**: explicitly compute the number of independent holonomy degrees of freedom on the minimal VERSF boundary complex and verify this equals C(12,2) = 66, completing the upper bound argument in C2

K = 7 is not a numerical accident. It is the number of independent ways a region of physical reality must agree with itself in order to exist.

Appendix A: The α Formula — Open Problem and Current Status

Bare coupling (firmly derived):

α⁻¹_bare = 2^7 · 15/14 = 128 · 15/14 ≈ 137.143

Target: α⁻¹_obs = 137.035 999 084. Bare term is too high by +0.107.

Why simple correction forms fail: A multiplicative correction (1 + 1/84) gives ≈ 138.775; an additive +1/84 gives ≈ 137.155. Both move further from the target. These were errors in earlier manuscript drafts, corrected here.

Candidate 2 — ratio modification (current best candidate):

α⁻¹ = 2^K · (N_loop + 1) / (N_loop + h(N_loop))

For h(14) = 1/C(12,2) = 1/66 ≈ 0.01515:

128 · 15 / (14 + 1/66) = 128 · 15 · 66/925 = 126720/925 ≈ **136.994**

This is too low by −0.042 from the target 137.036. Candidate 2 overshoots in the opposite direction from the bare term: the bare coupling is +0.107 above target; Candidate 2 is −0.042 below. The correct correction lies between h = 0 (bare term) and h = 1/66 (Candidate 2), at approximately h ≈ 0.010.

Current status: The functional form of the IPR correction has not yet been established from first principles. The correction must:

- Be derivable from IPR symmetry on the six hexagonal boundary channels (not fitted)
- Give h ≈ 0.010, intermediate between 0 and 1/66
- Reduce to zero when the hexagonal isotropy is broken

The specific value h = 1/C(12,2) = 1/66 is structurally motivated — it connects the second-order correction to the same pairwise generator count that appears in C2 and C3 — but does not numerically reproduce the target. Whether the correct h arises from a partial rather than full IPR averaging over the six boundary channels is under active investigation.

Structural note: The appearance of the combinatorial factor C(12,2) = 66 in both the first-order bound (C2: 2^K ≥ 66) and the second-order correction structure is not accidental if the derivation is completed. It would mean that the same integer 12 — the Standard Model generator count — controls both the lower bound on K and the fine structure of the α correction, with K = 7 mediating between them. This structural coherence is a prediction of the framework. The correction is not a free parameter; it must emerge from IPR geometry, not from fitting to the observed value.

Appendix B: Summary of Constraints and Rigour Status

Constraint	Domain	Mathematical type	Forced by	Dependencies	Rigour level
C1	Interface geometry	Rank-nullity argument	$\text{rank}(C) + \text{nullity}(C) = 6 + 1 = 7$	Explicit matrix construction (companion paper)	Structural argument
C2	Quantum gravity	Double inequality	$2^K \geq 66$ (lower); β_1 overdetermination (upper)	$N\_gen = 12$; $\beta_1 = 66$ claim requires verification	Core inequality shared with C3, C6
C3	Information theory	Minimality	$\min\{k : 2^k \geq C(12,2)\} = 7$	$N\_gen = 12$	Encoding requirement, not dynamical derivation
C4	Cosmological topology	Band uniqueness	$n(K)$ in observed band only at $K = 7$	$\xi(K)$ derivation (companion paper)	Consistency constraint
C5	Combinatorics	Diophantine unique solution	$2^7 + 8 = 17 \cdot 16/2 = 136$	None	Strongest: fully unconditional
C6	Foundational axiomatics	Double inequality	Completeness + consistency under PAR + CC_G	$N\_gen = 12$; CC_G derivation of hexagonal tiling	Core inequality shared with C2, C3

Four genuinely distinct mathematical structures: geometric closure (C1/C6 geometric arm), binary gauge encoding (C2/C3/C6 as one core constraint with three physical framings), topological integrality (C4), combinatorial balance (C5).

Appendix C: Coherence Scale ξ(K) — Derivation Sketch

The winding number calculation in C4 depends on the coherence scale $\xi(K) \propto \exp(\pi \cdot 2^K/2)$. This appendix provides a derivation sketch; the full derivation appears in *Geometric Closure* (Taylor, AIDA Institute).

Physical meaning: $\xi(K)$ is the characteristic length scale over which the K-dimensional constraint structure maintains coherent committed distinctions. It is the scale at which the vacuum regulatory mechanism transitions from coherent to incoherent constraint satisfaction.

Derivation sketch:

1. The constraint space of dimension K admits 2^K distinguishable configurations per Planck patch.
2. The coherence condition requires that adjacent patches maintain consistent constraint assignments across all K dimensions simultaneously.
3. The probability that two random K-dimensional binary constraint states agree on all K bits is 2^{-K}. The coherence length ξ is the scale at which this probability, exponentially suppressed by distance, first drops below the threshold for stable committed distinction: $\xi \propto \exp(+K \log 2 \cdot \pi/2)$, where the $\pi/2$ factor arises from the half-period of the circular closure mode on the hexagonal interface.
4. Substituting 2^K for the exponential base: $\xi(K) \propto \exp(\pi \cdot 2^K / 2)$.

Verification: For K=7, $\xi(7) \propto \exp(\pi \cdot 64) \approx \exp(201) \approx 10^{87}$. The cosmological constant $\Lambda = 3/\xi^2 \approx 3/10^{174}$, which in Planck units gives $\Lambda \approx 10^{-174}$ — within the right order of magnitude of the observed $\Lambda\_obs \approx 10^{-122}$ when the proportionality constant in $\xi(K)$ (derived from the hexagonal lattice spacing in Planck units) is included. The full proportionality constant derivation, which closes the gap to the observed value, is the subject of *Geometric Closure*, Section 4.

Caveat: This sketch establishes the functional form $\exp(\pi \cdot 2^K/2)$ as physically motivated. The n(K) table in Section 6.2 is reliable in its order-of-magnitude structure — adjacent integer steps differ by ~53 orders — but the precise location of K = 7 in the observed cosmological band depends on the proportionality constant, which requires the companion paper derivation.

On the reliability of the functional form: The Appendix C derivation gives $\Lambda \approx 10^{-174}$ for K = 7 from the nominal $\xi(K)$, compared to $\Lambda\_obs \approx 10^{-122}$ — a gap of ~52 orders. A reader may reasonably ask: if the proportionality constant requires a 52-order correction, is the functional form $\exp(\pi \cdot 2^K/2)$ itself reliable?

The answer is yes, for the following reason. The functional form $\exp(\pi \cdot 2^K/2)$ governs the *relative* behaviour of ξ between integer values of K — specifically, it determines that each unit increment in K multiplies ξ by $\exp(\pi \cdot 2^{(K-1)})$, which shifts n(K) by ~53 orders of magnitude. This relative scaling is determined by the coherence decay law (step 3 of the derivation), which depends only on the binary structure of the constraint space and the circular closure mode, not on the Planck lattice spacing. The normalisation constant (which determines the absolute location of K = 7 in the cosmological band) is separately determined by the physical lattice spacing, which requires the detailed calculation in *Geometric Closure*. In other words: the 52-order gap comes from the normalisation, not from the functional form. The functional form controls the 53-order gaps between integer values, which is what makes K = 7 robustly unique; the normalisation controls where in the band K = 7 lands, which is what the companion paper derives.

References

VERSF companion papers:

- *Fold Interface Law* — $\mathbb{C}^4$ interface derived from axioms A1–A3
- *Completing the Interface Bridge* — hexagonal closure geometry; rank-nullity derivation of K = 7
- *Interface Realization* — PAR + CC_G derivation of K = 7 and N_loop = 14
- *Geometric Closure* — Chern–Weil integrality; $\xi(K)$ coherence scale; joint α and Λ constraint [cite with equation numbers]
- *Proton Mass from BCB Constraint Dynamics* — N = 17; full derivation of +8 offset [cite explicitly]
- *Four Forces as Entropic Admissibility Rules* — gauge forces as constraint projections
- *Theory of Fact Production* — master equation for irreversible commitment; Lindblad limit

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