Noether's Theorem Without Fundamental Time: Energy Conservation and Emergent Temporality in the VERSF Framework

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General Reader Abstract

One of the most beautiful results in all of physics is a theorem proved by the mathematician Emmy Noether in 1918. Noether showed that whenever the laws of nature have a symmetry — whenever they look the same from two different vantage points — there is always a conserved quantity hiding behind that symmetry. The symmetry that concerns us here is the simplest imaginable one: the laws of physics are the same today as they were yesterday and as they will be tomorrow. Physics does not care what time it is. Noether's theorem says this single fact guarantees that energy is conserved. Energy can never be created or destroyed, only transformed, precisely because the laws governing it are timeless.

But here is the puzzle. Noether's theorem seems to need time to exist in the first place. You cannot say "the laws look the same at different times" unless time is already there as a stage on which events play out. Yet a growing number of physicists suspect that time is not actually fundamental — that it does not exist as a background stage at all, but instead *emerges* from something deeper. In this view, what we experience as the flow of time is really the accumulation of irreversible events: facts about the world that, once true, can never be made untrue. The past exists because it has left permanent marks. Time's arrow points forward because those marks only ever accumulate — they never dissolve.

This paper addresses the tension directly. If time is something that emerges from irreversible events, how can Noether's theorem — which seems to require time as a given — still guarantee that energy is conserved?

The answer is that two very different things have been travelling under the name "time," and keeping them separate dissolves the puzzle entirely. The first is a *counting parameter* — a step index that labels each microscopic event in the reversible, information-preserving dynamics at the foundation of the framework. This step index is perfectly symmetric: running it forward or backward makes no difference to the underlying rules. Noether's theorem applies to this symmetric dynamics, and energy conservation follows. The second is the *arrow of time* — the directed, irreversible accumulation of permanent facts that we actually experience as the passage of time. This arrow emerges from the topology of the information substrate: specifically, from the existence of closed loops in the network of possible state transitions, which act as traps that lock in a fact permanently.

The paper proves that these two temporal structures — the symmetric counting parameter and the emergent arrow — are mutually consistent. An observer built entirely out of accumulated facts will always find the two orderings in agreement. Energy conservation and the arrow of time therefore arise from complementary aspects of the same underlying architecture, neither threatening the other.

Technical Abstract

Noether's theorem establishes that conservation of energy follows from invariance of the action under continuous time translation. This result is standardly interpreted as presupposing time as a fundamental background dimension. The Void Energy–Regulated Space Framework (VERSF) instead proposes that time is not fundamental but emerges from the ordered accumulation of irreversible informational commitments — *facts* — on a zero-entropy substrate. This raises a structural question: if time is emergent, how can time-translation symmetry ground energy conservation?

We resolve this apparent tension by distinguishing two levels of description that operate simultaneously. At the microscopic level, reversible *tick* dynamics is formulated as a symplectic discrete-time map on a finite sampling Ω of a smooth symplectic manifold (M, ω), restricted to those maps arising as time-ε maps of autonomous Hamiltonian flows. By backward error analysis and a discrete analogue of Noether's theorem, the dynamics formally preserves a modified Hamiltonian $\tilde{H}$ order by order in the tick duration ε. At the coarse-grained level, irreversible *bit* formation introduces a directional ordering corresponding to the monotonic accumulation of fixed informational distinctions.

Coarse-grained irreversibility requires the existence of a *monochromatic permutation cycle* — a cycle of the tick map lying entirely within one cell of the bit partition. We prove that this condition is necessary (Lemma 4.2) and constructively sufficient (Lemma 4.4) for locally accessible trapping. The first Betti number β_1 enters as a precise topological descriptor only at the level of the induced subgraph on the trapped cell; global cycle structure of the full permutation graph is insufficient to guarantee trapping.

The two temporal structures — the reversible step index supporting Noether symmetry, and the emergent ordering of fact accumulation — are non-conflicting by construction. Together with the monochromatic cycle threshold, this yields a complete structural picture: irreversible record formation is possible if and only if the permutation cycle structure of the tick map aligns with the coarse-graining partition, and when it does, the emergent temporal ordering is fully consistent with the underlying Hamiltonian dynamics. A discussion section draws structural correspondences between the VERSF tick–bit–fact hierarchy and the quantum mechanical framework of unitary evolution, projection operators, and decohered pointer states, presented as interpretive analogy rather than derivation.

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1. Introduction

Among the deepest organizing principles of modern physics is Noether's theorem [1]: every continuous symmetry of the action corresponds to a conserved quantity. Time-translation symmetry — invariance under $t \mapsto t + \Delta t$ — produces conservation of energy.

Plain language: Noether's theorem says that if you ran an experiment today and then ran the exact same experiment tomorrow, and got the same result, that simple fact — that physics is the same on both days — mathematically *forces* energy to be conserved. It is one of the most powerful connections in all of science: a symmetry of time produces an eternal conservation law.

This principle appears, at first sight, to require time as a primitive background parameter. The action must be defined as an integral *over* time; the symmetry is a shift *of* time; the conservation law is a statement *in* time. Yet a growing body of work in quantum gravity, causal set theory, and relational approaches to physics argues that time may not be fundamental at all, but an emergent, observer-dependent structure arising from deeper relational or informational degrees of freedom [2, 3, 4, 5].

The Void Energy–Regulated Space Framework (VERSF) adopts this position rigorously. In VERSF, the fundamental substrate carries no intrinsic temporal structure. Physical time emerges from the accumulation of *irreversible informational commitments*, called facts, that cannot be locally undone by the underlying reversible dynamics. The arrow of time is not a primitive feature of the laws but a consequence of the combinatorial topology of the information substrate.

This raises an immediate and genuine tension:

If time emerges from fact formation, how does Noether's theorem — which apparently requires time as a pre-existing parameter — still guarantee energy conservation?

The resolution requires careful separation of two temporal concepts that are conflated in standard presentations:

1. **The symmetric step parameter.** The reversible microscopic dynamics possesses a group of step-index translations. Because the generating Hamiltonian is autonomous (no explicit step-index dependence), a discrete Noether theorem guarantees a conserved quantity. This parameter is symmetric: forward and backward evolution are equally valid.
2. **The emergent temporal ordering.** The accumulation of irreversible coarse-grained commitments produces a directed partial order on system histories. This ordering constitutes the *arrow of time* — it is irreversible, observer-relative, and topologically constrained.

These two structures are complementary. The reversible substrate satisfies the Noether conditions; the irreversible coarse-graining produces temporal direction. Section 5 shows they are order-compatible for macroscopic observers. Section 7 includes a discussion of structural correspondences with standard quantum mechanics.

The distinctive contribution. The distinction between a reversible evolution parameter and an emergent physical arrow is not new in itself; it appears in various relational, covariant, and causal-set approaches to quantum gravity [2, 3, 4, 5]. The novelty of the present framework is stronger and more specific: VERSF identifies a *concrete combinatorial criterion* for when irreversible record formation becomes possible — namely, the existence of monochromatic permutation cycles relative to a coarse-graining partition. This criterion is structurally derived, not postulated, and it identifies the onset of temporal direction with a sharp combinatorial threshold rather than with a probabilistic or entropic tendency.

Paper organization. Section 2 defines the state space and the tick–bit–fact hierarchy. Section 3 derives the Hamiltonian structure and the discrete Noether conservation law. Section 4 establishes the topological threshold theorem with a constructive proof. Section 5 clarifies the relationship between the two temporal orderings. Section 6 discusses structural correspondences with quantum measurement theory. Section 7 develops the broader physical implications: the macrostate-stabilization interpretation of the threshold theorem, a quantitative treatment of partition-crossing and fact-production rates, connections to causal set theory and relational time, and potential empirical directions. Section 8 concludes.

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2. The Informational State Space and Tick–Bit–Fact Hierarchy

Plain language: This section builds the vocabulary of the framework from the ground up. We start with a collection of possible states a system can be in. We define the simplest possible dynamics: a reversible rule that sends each state to exactly one successor. We then define the coarser concepts — a *bit* (a yes/no question about the state), a *fact* (a yes/no question whose answer has become permanently locked in), and a *fact counter* (the tally of permanently decided questions, which only ever goes up). That rising tally is what we will identify with the arrow of time.

2.1 The State Space: Finite Sampling of a Symplectic Manifold

Let (M, ω) be a smooth $2n$-dimensional symplectic manifold — the ambient phase space. We do not work on M directly but on a finite subset

$$\Omega \subset M, \; |\Omega| < \infty,$$

which we call the set of *informational microstates*. The symplectic structure ω of M is inherited by Ω operationally: the discrete tick map F (defined below) is required to extend to a symplectomorphism $\tilde{F}: M \to M$ such that $\tilde{F}|\_\Omega = F$.

Plain language: Think of M as the full continuous space of all possible physical states — like the space of all possible positions and velocities of every particle in a system. Ω is a finite selection of landmark points in that space, like a grid of sample locations. The condition that F extends to a symplectomorphism of M is the precise mathematical requirement that the discrete rule governing these landmark points is consistent with a well-behaved physics on the whole continuous space.

Scope note. The extendability condition — that F is the restriction of a symplectomorphism of M — is a structural assumption, not a derived result. In this paper we restrict attention to those finite bijections $F: \Omega \to \Omega$ that admit extension to a smooth symplectomorphism of an ambient symplectic manifold. This is a defining constraint on the class of admissible tick dynamics studied here, not a claim that every finite bijection possesses such an extension. Not every permutation of a finite set is compatible with a smooth symplectic ambient dynamics; the framework applies specifically to those that are. The canonical instance is $M = \mathbb{R}^{2n}$ with standard symplectic form $\omega = \Sigma_i \, dq_i \wedge dp_i$, and Ω a finite lattice in a Liouville-measure-preserving region — exactly the setting of symplectic integrators [6], which requires no new mathematical infrastructure.

The *permutation graph* of the tick map is defined in Section 2.2. It plays the central role in the topological analysis of Section 4.

2.2 Reversible Tick Dynamics and the Permutation Graph

Definition 2.1 (Tick Map). A *tick* is a bijection $F: \Omega \to \Omega$ that is the restriction of a symplectomorphism $\tilde{F}: M \to M$ to the finite sample Ω. Explicitly, $\tilde{F}^*\omega = \omega$ on M.

The bijective character of F ensures each microstate has a unique predecessor and a unique successor. No information is created or destroyed at the microscopic level.

Plain language: A tick is a single step of the underlying reversible dynamics. The word "bijection" is the key: every state maps to exactly one successor, and every state has exactly one predecessor. Nothing is created or destroyed; the dynamics is a perfect shuffle of states. The symplectic condition means this shuffle is the kind that can arise from a physical Hamiltonian — it is not an arbitrary rearrangement.

Definition 2.2 (Permutation Graph). The *permutation graph* of F is the directed graph

$$G\_F = (\Omega, E\_F), \quad E\_F = \{(x, F(x)) : x \in \Omega\}.$$

Each vertex has out-degree exactly 1 and in-degree exactly 1, because F is a bijection. By the classification of permutations of finite sets, G_F decomposes uniquely into a disjoint union of directed cycles:

$$G\_F = C_1 \sqcup C_2 \sqcup \cdots \sqcup C_r,$$

where each C_j is a directed cycle of length $\ell_j \geq 1$ and $\Sigma_j\, \ell_j = |\Omega|$.

Plain language: Draw an arrow from each state to its successor. Because F is a perfect bijection, you get a network where every dot has exactly one arrow going out and one arrow coming in. The only shape that can result is a collection of closed loops — circles of states chasing each other's tails. This is a basic fact of combinatorics: any bijection on a finite set decomposes completely into such loops. The permutation graph is just the picture of those loops.

Remark on the Betti number. For the permutation graph G_F, we have $|E\_F| = |V| = |\Omega|$ and $c(G\_F) = r$ (the number of cycles). Therefore:

$$\beta_1(G\_F) = |E\_F| - |V| + c(G\_F) = |\Omega| - |\Omega| + r = r \geq 1.$$

The first Betti number β_1 counts independent loops in the graph. Since G_F is always a union of loops, $\beta_1 \geq 1$ automatically. The operative question for fact formation — addressed in Section 4 — is not whether $\beta_1 \geq 1$ globally (it always is), but whether a specific loop lies entirely within one cell of a given bit partition.

2.3 Coarse-Graining and Bit Formation

Definition 2.3 (Bit). A *bit* is a surjective map $b: \Omega \to \{0, 1\}$ partitioning Ω into two cells $\Omega_0 = b^{-1}(0)$ and $\Omega_1 = b^{-1}(1)$.

Plain language: A bit is any yes/no question you can ask about the current state of the system. "Is the particle on the left side of the box?" is a bit. "Is the spin pointing up?" is a bit. Each bit divides the full set of states into two groups: those where the answer is yes, and those where the answer is no.

A bit b is *dynamically fixed* on a trajectory $\{F^k(x_0)\}_{k \geq 0}$ if there exists k_0 such that for all $k \geq k_0$:

$$b(F^k(x_0)) = b(F^{k_0}(x_0)).$$

The fixing event at step k_0 constitutes an *irreversible commitment* for that trajectory.

Plain language: A bit is fixed when the system settles on one answer — yes or no — and stays there for all future time. After that point, every time you ask the question, you get the same answer. Something has been decided.

Definition 2.4 (Fact). A *fact* is a bit b that is dynamically fixed and whose complementary cell is *inaccessible* from the current trajectory under F without nonlocal reconfiguration of the transition structure. The precise formalization of inaccessibility via local trapping is given in Section 4.

Plain language: A fact is a decision that cannot be reversed by any local intervention. It is not just that the system happens to keep giving the same answer — it is that the alternative answer has become structurally unreachable. The other possibility has been locked out. Note that this is a *model* of irreversible record formation. The framework demonstrates how such locking can arise from the topology of the underlying dynamics; it does not claim to derive the full thermodynamic arrow of time from first principles in a single step.

2.4 The Fact Counter and Emergent Ordering

Let $\mathscr{B} = \{b_1, \ldots, b_m\}$ be a finite collection of bits on Ω. The *fact count* at step k along trajectory $\{F^k(x_0)\}$ is:

$$\mathscr{F}(k) = |\{ i : b_i \text{ is dynamically fixed by step } k \}|.$$

Since bits, once fixed under the trapping condition of Section 4, cannot unfix, $\mathscr{F}(k)$ is monotonically non-decreasing. This monotone quantity generates the emergent temporal ordering developed in Section 5.

Plain language: Keep a running tally of how many yes/no questions have become permanently decided. This tally can only go up — decisions don't get un-made. That ever-rising count is the embryo of what we experience as the forward direction of time. Notice that it is defined relative to a collection of questions and an observer who can ask them; it is not a property of the microstate alone.

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3. Hamiltonian Structure and the Discrete Noether Theorem

Plain language: This section shows that the reversible microscopic dynamics has a conserved energy. The key tool is a version of Noether's theorem adapted for discrete steps rather than continuous time. The technical device — called the *modified Hamiltonian* — is a corrected energy function that the discrete dynamics preserves formally at every order. This is the VERSF counterpart of the standard statement that unitary quantum evolution preserves total energy.

3.1 Generating Hamiltonian and Symplectic Discretization

In this paper we consider symplectomorphisms $\tilde{F}$ arising as the time-ε maps of autonomous Hamiltonian flows — equivalently, those produced by a standard symplectic discretization of such a flow. For this class of maps, $\tilde{F}$ is governed by a smooth autonomous Hamiltonian $H: M \to \mathbb{R}$, and the trajectory of any point in Ω evolves via the discrete Hamilton equations. In canonical coordinates $(q_i, p_i)_{i=1}^n$ on M, the discrete equations are:

$$q_i(k+1) = q_i(k) + \varepsilon \cdot (\partial H/\partial p_i)|_{(q(k), p(k))} + O(\varepsilon^2)$$

$$p_i(k+1) = p_i(k) - \varepsilon \cdot (\partial H/\partial q_i)|_{(q(k), p(k))} + O(\varepsilon^2)$$

where $\varepsilon > 0$ is the tick duration. This is a symplectic (Störmer–Verlet type) discretization [6].

3.2 The Modified Hamiltonian and Formal Preservation

The key tool is *backward error analysis* [6, Ch. IX]:

Proposition 3.1 (Modified Hamiltonian). For an autonomous symplectic map $\tilde{F}$ arising as the time-ε map of a Hamiltonian flow generated by H, backward error analysis produces a formal power series

$$\tilde{H} = H + \varepsilon H_1 + \varepsilon^2 H_2 + \cdots$$

where each H_j is a smooth function determined by H and its derivatives, such that $\tilde{F}$ is formally the time-ε flow of the Hamiltonian vector field generated by $\tilde{H}$ [6, Theorem IX.3.1]. Consequently, $\tilde{H}$ is formally preserved by $\tilde{F}$:

$$\tilde{H}(\tilde{F}(x)) = \tilde{H}(x) \text{ for all } x \in M, \text{ in the sense of the formal series.}$$

Plain language: The original energy H is not perfectly conserved step-by-step — discretization introduces tiny errors at each tick. But there is a corrected energy function $\tilde{H}$ (a systematic adjustment of H, order by order in the tick duration ε) that the dynamics preserves in the strongest available sense for an asymptotic series: every correction term is accounted for. Choose any precision level you like; $\tilde{H}$ truncated at that level is conserved up to an error smaller than that level. For physical purposes, this provides the appropriate discrete analogue of energy conservation.

Remark on the status of preservation. The series $\tilde{H}$ is generally asymptotic, not convergent. The correct formulation is: the discrete dynamics formally preserves $\tilde{H}$ order by order in ε. For any truncation order N, the truncated modified Hamiltonian $\tilde{H}^{(N)} = H + \varepsilon H_1 + \cdots + \varepsilon^N H_n$ is conserved up to an error of order O(ε^(N+1)). In this precise sense, energy conservation survives discretization: formally exact at the level of the asymptotic series, and accurate to arbitrarily high finite order at any truncated level. We will use "$\tilde{H}$ is preserved" as shorthand for this formal statement throughout.

3.3 The Discrete Noether Theorem

Theorem 3.2 (Discrete Noether [7, Theorem 2.3]). Let $\tilde{F}: M \to M$ be a symplectic map generated by an autonomous discrete Lagrangian $L: M \times M \to \mathbb{R}$ (no explicit dependence on the step index k). Then the discrete energy function

$$E(x_k, x_{k+1}) = (\partial L/\partial x_k) \cdot (x_{k+1} - x_k) - L(x_k, x_{k+1})$$

satisfies $E(x_{k+1}, x_{k+2}) = E(x_k, x_{k+1})$ along all trajectories — i.e., E is conserved. The function E coincides with H to leading order in ε and with $\tilde{H}$ order by order in the formal series.

Proof. Autonomy of L means $L(x_k, x_{k+1})$ is invariant under the shift $k \mapsto k + 1$, which is the discrete time-translation symmetry. The discrete Noether identity (see [7, §2]) applied to this symmetry yields conservation of the discrete energy E. By backward error analysis, $E = \tilde{H}^{(N)} + O(\varepsilon^{N+1})$ for any N, which in the formal series sense is the statement that E and $\tilde{H}$ agree order by order. (The discrete result is consistent with the continuous Noether theorem: any autonomous discrete map $\tilde{F}$ can be embedded in a suspension flow whose time-translation symmetry recovers the same conservation law [8, §1.1].) □

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4. The Topological Threshold Theorem for Fact Formation

Plain language: This is the mathematical heart of the paper. We ask: under what conditions can a yes/no question about the system become permanently decided? The answer turns out to be purely about the *shape* of the network of state transitions — specifically, whether any of the closed loops in that network happen to lie entirely on one side of the yes/no divide. If they do, a trajectory that enters such a loop is trapped there permanently: the question is decided forever. If no loop lies entirely on one side, the trajectory will eventually visit both sides, and no decision ever sticks.

The core claim is:

A bit b on Ω admits a locally constructible trap for the tick map F if and only if the permutation graph G_F contains a cycle lying entirely within one cell of the bit partition.

We call such a cycle *monochromatic*. We carefully distinguish what this condition implies about necessity versus sufficiency.

4.1 Definitions

Definition 4.1 (Local Trap). Let $b: \Omega \to \{0, 1\}$ be a bit and G_F the permutation graph of F. A *local trap* for b in cell Ω_1 is a subset $T \subseteq \Omega_1$ satisfying:

1. **Forward-invariance**: $F(T) \subseteq T$.
2. **Closure**: No edge in E_F leads from T to Ω_0.
3. **Local constructibility**: Membership in T is determined by the subgraph of G_F induced on a bounded neighborhood of T — not by global properties of Ω.

Condition 3 distinguishes *local* trapping (accessible to a physically situated observer) from global trapping (which would require surveying the entire state space).

Plain language: A trap is a region of state space that the system can enter but never leave — at least not without some globally coordinated intervention from outside. Forward-invariance means every state inside the trap maps to another state inside the trap. Closure means there is no escape hatch. Local constructibility means the trap can be identified by examining only the immediate neighbourhood of the trap — an observer does not need to know the entire state of the universe to recognize that they are trapped.

4.2 The Necessary Condition

Lemma 4.2 (Necessity). If $T \subseteq \Omega_1$ is a local trap for bit b, then the subgraph of G_F induced on T contains at least one directed cycle, i.e., has $\beta_1 \geq 1$ on the induced subgraph.

Proof. Since T is finite and F|_T is a bijection from T to itself (by forward-invariance and closure), F|_T is a permutation of T. By the classification of permutations of finite sets, the permutation graph of F|_T decomposes into disjoint directed cycles. Therefore the induced subgraph on T consists entirely of cycles: $\beta_1 \geq 1$. □

Corollary 4.3. If every permutation cycle of F crosses the bit boundary between Ω_0 and Ω_1 — i.e., no monochromatic cycle exists — then no local trap for b exists.

This is the sharp necessary condition: the absence of a monochromatic cycle rules out local trapping for that bit entirely.

4.3 The Sufficient Condition

Lemma 4.4 (Sufficiency, Constructive). If the permutation graph G_F contains a directed cycle C lying entirely within Ω_1, then T = C is a local trap for b.

Proof. Let $C = (v_1 \to v_2 \to \cdots \to v_m \to v_1)$ with all $v_i \in \Omega_1$. We verify Definition 4.1:

1. **Forward-invariance**: $F(v_i) = v_{i+1 \bmod m} \in C \subseteq \Omega_1$. So $F(C) = C \subseteq C$. ✓
2. **Closure**: Since C is a permutation cycle of F, no edge of G_F exits C — each v_i has its unique successor v_{i+1} within C, and all $v_i \in \Omega_1$. No edge leads to Ω_0. ✓
3. **Local constructibility**: Identifying C requires only the edges of G_F incident to the vertices $\{v_1, \ldots, v_m\}$, a finite bounded neighborhood. ✓

Any trajectory initialized at $v_j \in C$ satisfies $F^k(v_j) \in C \subseteq \Omega_1$ for all $k \geq 0$, permanently fixing bit b at value 1. The alternative cell Ω_0 is inaccessible from C, constituting a fact. □

4.4 The Role of β_1: Necessity vs. Sufficiency

Important clarification. The condition $\beta_1(G\_F) \geq 1$ is automatically satisfied by any bijection on a finite set. It is therefore *not* by itself the operative condition for trapping. The operative condition is:

A cycle of F is monochromatic with respect to the bit partition b.

The Betti number β_1 enters as follows:

- **Necessity**: A trap requires a monochromatic cycle, which requires $\beta_1 \geq 1$ on the induced subgraph on the trapped cell (Lemma 4.2).
- **Sufficiency**: A single monochromatic cycle suffices for a local trap (Lemma 4.4).
- **Generic dynamics**: For a randomly chosen bit partition on a system with multiple permutation cycles, whether a monochromatic cycle exists depends on the geometry of the partition relative to the cycle structure. Trapping requires alignment between the loops of F and the boundary of b — $\beta_1 \geq 1$ globally is not sufficient on its own.

Plain language: β_1 counts the loops in the dynamics. All bijections have loops, so $\beta_1 \geq 1$ is always true and by itself tells us nothing about fact formation. What matters is whether any loop happens to stay entirely on one side of your yes/no question — the monochromatic cycle condition. The β_1 description becomes precise and useful only when applied to the induced subgraph on the trapped cell.

Theorem 4.5 (Topological Threshold). A bit b on (Ω, F) admits a locally constructible trap if and only if there exists a permutation cycle of F lying entirely within one cell of the partition induced by b.

Interpretation: coarse-grained macrostate stabilization. Theorem 4.5 can be read as a combinatorial instance of a phenomenon familiar from statistical mechanics: *macrostate stabilization*. The bit partition b is precisely a coarse-graining of phase space into macroregions — the same mathematical object used to define macroscopic observables such as temperature, magnetization, or pressure. The monochromatic cycle condition says the microscopic dynamics has entered a loop that never escapes the corresponding macroregion. This is the minimal formal structure underlying familiar examples of macroscopic stability: a ferromagnet whose spins circulate among aligned microstates without ever reaching the disordered region; a chemical system that cycles among product configurations without revisiting the reactant basin; a decohered quantum system that cycles among pointer-state microstates without re-entering superposition. In each case, the underlying microdynamics is reversible and periodic, but the coarse partition confines the trajectory to a single macroregion, producing a stable observable record. The monochromatic cycle condition can therefore be interpreted as the minimal combinatorial expression of a mechanism analogous to familiar macrostate stabilization in thermodynamics and decoherence theory.

Corollary 4.6. If F has a single permutation cycle of length $|\Omega|$ — a cyclic permutation of all states — then every trajectory visits every state, crossing every bit boundary, and no monochromatic cycle can exist. Such a dynamics admits no local trap for any bit. Global cycle structure ($\beta_1 = 1$ for a single cycle) does not guarantee trapping; multiple shorter cycles whose loops respect the bit boundary are required.

Plain language: Corollary 4.6 is a warning sign: if the dynamics is a single giant loop through all states, no question ever gets permanently decided. The system visits every state and therefore answers "yes" and "no" alternately to every question, forever. To form facts — to have an arrow of time — you need the dynamics to be broken into multiple smaller loops, some of which happen to stay on one side of the relevant question.

Ergodicity breaking, novelty, and the scope of the framework. Corollary 4.6 shows that irreversibility in this framework is incompatible with globally ergodic single-cycle dynamics on the sampled state space. The mechanism of fact formation therefore requires a form of *effective ergodicity breaking* relative to the coarse-graining of interest: the dynamics must decompose into multiple cycles, some of which respect the bit boundary. This should not be interpreted as a derivation of generic irreversibility from arbitrary reversible dynamics. Rather, it is a structural criterion identifying when irreversible record formation is possible at all. Whether a given physical system satisfies this criterion is a separate empirical question. Thermodynamics offers a partial answer: systems with basins of attraction naturally satisfy the monochromatic cycle condition for macroscopic observables, because microstates inside a basin cycle endlessly while remaining within the same macroregion. The VERSF framework does not derive this basin structure; it identifies the monochromatic cycle condition as the combinatorial signature of its existence.

A related point concerns novelty. Because Ω is finite and F is a bijection, every microscopic trajectory is ultimately periodic. A skeptical reader may ask: if everything cycles, where does genuine novelty come from? The answer is that emergent novelty in VERSF is not identified with the appearance of previously unseen microstates but with the *irreversible stabilization of coarse-grained distinctions relative to locally accessible partitions*. A fact is not the creation of a new microstate; it is the permanent commitment of a trajectory to one cell of a bit partition, from which no locally available operation can dislodge it. The periodic character of the microdynamics is not in tension with this — the cycle that traps a trajectory is itself periodic, and that is precisely what makes the trap permanent. The framework's claim is that monochromatic cycle structure is the minimal combinatorial ingredient for such trapping, not that it is guaranteed to appear in all physically relevant dynamics.

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5. Two Temporal Concepts and Their Order Compatibility

5.1 The Symmetric Step Parameter

The tick dynamics indexed by k ∈ ℤ_{≥0} is governed by the autonomous Hamiltonian H (or $\tilde{H}$). Autonomy means the map F is identical at every step: $F_k = F$ for all k. This is the discrete analogue of time-translation invariance. By Theorem 3.2, it implies formal preservation of $\tilde{H}$.

The step index k is a *symmetric* parameter: reversing the sign of the step index gives F^{-1}, which is equally well-defined (since F is a bijection). The reversible dynamics defines no preferred direction.

5.2 The Emergent Temporal Ordering

The fact counter $\mathscr{F}$(k) is monotonically non-decreasing. Define the *emergent time* of a fact-formation event as the step index k* at which a new bit becomes fixed. The ordering of fact-formation events defines a directed partial order ≺ on the history of the system:

$e_1 \prec e_2 \Leftrightarrow k_{e_1} < k_{e_2}$.

This ordering is irreversible by construction: no fact-formation event can be un-formed once it occurs (by the trap closure condition). It constitutes the *arrow of time* at the coarse-grained level.

Remark (Order Compatibility). The emergent ordering $\prec$ is defined as a sub-ordering of the step-index order: $e_1 \prec e_2$ means $k_{e_1} < k_{e_2}$ by definition. The two orderings therefore cannot conflict. This is a clarification of the framework's internal consistency, not an independent theorem. The emergent ordering is *partial* (bits may be fixed simultaneously at the same step), while the step-index ordering is total; macroscopic coarse-graining recovers an approximate total order by grouping simultaneous events.

Remark (Observer-relativity of $\mathscr{F}$). The fact counter $\mathscr{F}(k)$ depends on the chosen family of bits $\mathscr{B} = \{b_1, \ldots, b_m\}$. Different observers equipped with different bit families — different coarse-graining partitions — will in general induce different fact counters and different emergent orderings. The present paper does not attempt a full theory of inter-observer reconciliation or of preferred partitions. It establishes a minimal structural result: once a partition is fixed, irreversible record formation occurs if and only if the dynamics contains a monochromatic cycle relative to that partition. The observer-relativity of $\mathscr{F}(k)$ is a feature, not a flaw — it is the precise sense in which emergent time is relational.

5.3 The Two-Level Temporal Structure

Level	Mathematical Object	Temporal Character	Conservation
Tick dynamics	Symplectic bijection F: $\Omega \to \Omega$	Step-index symmetric (reversible)	$\tilde{H}$ formally preserved order by order
Bit formation	Coarse-graining map b: $\Omega \to \{0,1\}$	Conditionally irreversible (trap-dependent)	$\mathscr{F}(k)$ non-decreasing
Fact accumulation	Emergent partial order $\prec$	Strictly directed (arrow of time)	Order-compatible with step index by definition

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6. Discussion: Structural Correspondence with Quantum Measurement Theory

The core results of Sections 2–5 are formulated at the level of classical symplectic dynamics and combinatorial graph theory. This section discusses how the VERSF tick–bit–fact hierarchy relates to the standard quantum mechanical framework of unitary evolution, projection operators, and decoherence. The correspondences drawn here are structural and interpretive — they illuminate the physical meaning of the framework and motivate further development — but they are not derivations. Establishing a full quantitative connection between VERSF and quantum mechanics would require additional machinery (a quantization map, a Hilbert space representation, an environmental model) beyond the scope of this paper.

6.1 The Correspondence Table

VERSF Concept	Quantum Mechanical Counterpart
State space $\Omega \subset M$	Hilbert space $\mathscr{H}$; Ω indexes a basis or coherent state lattice
Tick map $F: \Omega \to \Omega$	Unitary evolution operator $U = e^{(-iH\varepsilon/\hbar)}$
Symplectic structure ω on M	Non-commutative symplectic structure on operator algebra
Bit $b: \Omega \to \{0,1\}$	Projection operator $\hat{P} = \hat{P}^2 = \hat{P}^\dagger$
Dynamically fixed bit	Stable pointer state of a measurement apparatus
Fact	Decohered eigenstate (definite, irreversible measurement outcome)
Modified Hamiltonian $\tilde{H}$	Effective Hamiltonian (Trotter–Suzuki approximation in QM)
Emergent ordering $\prec$	Quantum causal order of measurement events

6.2 Ticks and Unitary Evolution

The tick map F is the restriction of a symplectomorphism $\tilde{F}$ of the classical phase space (M, ω). In the quantum setting, symplectomorphisms of classical phase space correspond, via geometric quantization or Weyl quantization, to unitary operators U on the quantum Hilbert space $\mathscr{H}$— though this correspondence depends on quantization choices and is not canonical. The discreteness of the tick (ε-step evolution) is structurally analogous to the Trotter step of a discrete-time quantum simulation.

The formal preservation of $\tilde{H}$ by the discrete map F is structurally parallel to the exact conservation of energy by unitary quantum evolution: $\langle H \rangle_{\psi(t)}$ is constant whenever $\hat{H}$ has no explicit time dependence.

6.3 Bits and Projection Operators

A bit $b: \Omega \to \{0,1\}$ partitions Ω into two cells. In quantum mechanics, a binary observable is represented by a projection operator $\hat{P}$ with eigenvalues $\{0, 1\}$, partitioning the Hilbert space $\mathscr{H}$ into two orthogonal subspaces. The structural parallel is precise at the level of the partition: each induces a binary split of the state space. Establishing a full representation-theoretic equivalence requires specifying a quantization map, which goes beyond the scope of the present paper.

6.4 Facts and Decohered Pointer States

In decoherence theory [9, 10], a measurement outcome becomes definite through a dynamical process: system-apparatus-environment entanglement causes the off-diagonal elements of the reduced density matrix to vanish in the pointer basis. Once complete, recovering the alternative outcome requires access to the full environment. This process underlies the emergence of effective superselection sectors [11] that partition the accessible Hilbert space into stable, mutually inaccessible subspaces.

The structural parallel with VERSF facts is close. A fact is a bit commitment that is locally irrevocable because the trajectory is trapped in a monochromatic cycle; the complementary cell is inaccessible not because it is destroyed but because the local transition structure provides no path to it. The cycle plays the structural role of the environment — absorbing the alternative. The correspondence is:

Decoherence ↔ Monochromatic cycle trapping

Pointer state ↔ Dynamically fixed bit

Definite measurement outcome ↔ Fact

This parallel is suggestive and structurally clean, but it is not a derivation of decoherence from VERSF or vice versa.

6.5 The Arrow of Time Under Both Frameworks

The irreversibility of quantum measurement — that a decohered outcome cannot be "unmeasured" without global access to the environment — corresponds structurally to the irreversibility of fact formation in VERSF: the trap closure condition prevents re-entry into the complementary cell from within the trap. The quantum arrow of time (measurement irreversibility) and the VERSF arrow of time (fact accumulation) are structurally identified under this correspondence, though the identification remains at the level of analogy pending a fuller formal treatment.

6.6 Illustrative Example: Spin Systems

The following is an illustrative analogy, not a derivation. Establishing a rigorous quantitative connection between VERSF topological trapping and quantum decoherence rates would require a full environmental model.

Consider N spin-½ systems with classical state space $\Omega = \{0,1\}^N$. The permutation graph of a generic spin-chain dynamics is a subgraph of the N-dimensional hypercube Q_N, which has $\beta_1(Q\_N) = 2^N - N - 1$ independent cycles for $N \geq 2$. A bit b corresponds to measuring the j-th spin: $b(\sigma) = \sigma_j$. In the quantum analogue, decoherence with an environment localizes the system in eigenstates of the j-th spin operator, producing stable pointer states. The VERSF counterpart is a permutation of Ω with cycles confined to $\{\sigma : \sigma_j = 0\}$ and $\{\sigma : \sigma_j = 1\}$ — the monochromatic cycle condition of Theorem 4.5. The analogy suggests that decoherence rates could in principle be related to the density of monochromatic cycles; developing this connection quantitatively is a topic for future work.

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7. Broader Implications and Potential Empirical Directions

7.1 Energy Conservation and Emergent Time

The discrete dynamics formally preserves $\tilde{H}$ order by order in ε. Energy conservation is therefore not threatened by the emergence of time — it is grounded in the symplectic structure of the reversible substrate, and the monochromatic cycle trapping that generates temporal direction operates at the coarse-grained level, orthogonal to the Hamiltonian structure below it.

7.2 The Combinatorial Origin of Record Formation

Theorem 4.5 identifies a sharp combinatorial condition for irreversible record formation: the existence of a monochromatic permutation cycle relative to a bit partition. This is not a statistical statement about entropy increase but a structural statement about the alignment between the dynamics F and the observational coarse-graining b. The framework provides a model of how irreversible record formation arises from reversible substrate dynamics; the full connection to thermodynamic time asymmetry remains a topic for further development.

This yields a conceptual separation between two layers of description: topology determines whether irreversible fixation is even possible (the monochromatic cycle condition), while entropy and statistics determine how often and how robustly it happens. Section 7.3 makes this separation quantitative via partition-crossing and fact-production rates.

The complete causal architecture of emergent time within this framework is therefore: symplectic dynamics (Hamiltonian substrate) → permutation cycle topology → coarse-graining partition (macroscopic observable) → monochromatic cycle trapping (macrostate stabilization) → irreversible bit commitment (fact) → emergent temporal ordering.

Within the present record-formation model, absence of monochromatic cycles precludes irreversible fact formation independently of thermodynamic considerations taken in isolation. This differs qualitatively from standard thermodynamic treatments, where temporal asymmetry is probabilistic rather than structurally contingent.

7.3 Cycle Confinement, Partition-Crossing Suppression, and Coarse-Grained Record Entropy

The previous section separated the topological condition (possibility of irreversibility) from the statistical one (probability and rate). This section makes that separation quantitative by introducing two observable quantities that bridge the combinatorial structure of Section 4 to physically measurable behaviour.

Partition-crossing rate. Given a bit $b: \Omega \to \{0,1\}$ and a trajectory $\{F^k(x_0)\}$, define the *boundary-crossing indicator* at step k:

$$J\_b(k) = 1 \text{ if } b(F^\wedge(k+1)(x_0)) \neq b(F^k(x_0)), \text{ else } 0.$$

This records whether the trajectory crossed the bit boundary at step k. The *mean crossing frequency* over N steps is:

$$\Gamma_b(N) = (1/N) \Sigma_{k=0}^{N-1} J_b(k).$$

If $\Gamma_b(N)$ remains positive, the bit keeps alternating — no fact forms. If $\Gamma_b(N) \to 0$ and all future crossings vanish, the trajectory has entered a monochromatic cycle: the bit is fixed, and a fact has formed. Fact formation therefore corresponds precisely to *partition-crossing suppression*:

$$\Gamma_b(N) \to 0 \text{ (with eventual permanence) } \Leftrightarrow \text{ bit b is fixed as a fact.}$$

This gives a rate-observable signature of Theorem 4.5: a bit transitions from free to fixed exactly as its crossing frequency drops to zero and stays there. The parenthetical "with eventual permanence" is the operative qualifier: on a finite state space with a bijective F, every trajectory is ultimately periodic, so if the trajectory is not confined to a monochromatic cycle it will eventually revisit states in both cells and the crossing frequency will not vanish permanently. By the classification of permutations of finite sets (Section 2.2), $\Gamma_b(N) \to 0$ permanently if and only if the trajectory has entered a monochromatic cycle — connecting the rate condition directly back to Theorem 4.5. The crossing rate is therefore not an independent empirical criterion but an observable expression of the same combinatorial condition.

Fact-production rate. For a bit family $\mathscr{B} = \{b_1, \ldots, b_m\}$, the *fact-production rate* at step k is:

$$R_{\mathscr{F}}(k) = \mathscr{F}(k+1) - \mathscr{F}(k),$$

the number of bits newly fixed at that step. The cumulative rate over N steps is:

$$\bar{R}_{\mathscr{F}}(N) = (1/N)[\mathscr{F}(N) - \mathscr{F}(0)].$$

When $R_{\mathscr{F}}(k) > 0$, a new irreversible distinction has been committed. In the Ticks-Per-Bit (TPB) language of the broader VERSF programme, the emergent flow of time is identified with the rate of fact production: time flows at the rate distinctions become permanent. The present paper explains the structural condition under which $R_{\mathscr{F}}(k)$ can become positive at all — namely, the monochromatic cycle condition of Theorem 4.5.

Coarse-grained entropy and record structure. The partition-crossing picture connects naturally to a coarse-grained entropy, though doing so requires shifting from individual trajectories to an ensemble. For an ensemble of trajectories with initial distribution over Ω, define the bit occupation probabilities at step k:

$$p_0(k) = \Pr[b(x_k) = 0],\ p_1(k) = \Pr[b(x_k) = 1],$$

and the associated binary Shannon entropy:

$$S_b(k) = -p_0(k) \ln p_0(k) - p_1(k) \ln p_1(k).$$

While the fine-grained (Liouville) entropy of the symplectic dynamics is conserved — F preserves phase-space volume — the coarse-grained entropy $S_b(k)$ can decrease. The connection to the single-trajectory picture is this: for an ensemble initially spread across both cells, progressive trapping of individual trajectories into monochromatic cycles decreases the fraction of trajectories that still cross the boundary. As more trajectories commit to one cell, $p_1(k) \to 1$ for the trapped subensemble, driving $S_b \to 0$ for those branches. Each individual branch commits irreversibly — the trap closure condition of Definition 4.1 guarantees no locally available operation can return it to the complementary cell — so the ensemble-level entropy reduction is the aggregate of individually irrevocable commitments, not a statistical fluctuation that could reverse. This is not microscopic information destruction: F remains bijective and fine-grained entropy is conserved. It is *progressive stabilization of record structure* — the observer's coarse-grained description becomes more committed, and that commitment is irreversible relative to locally accessible operations.

This points toward a VERSF-native interpretation of entropy production:

Coarse-grained entropy change, in the sense of effective entropy reduction at the level of a committed branch, is associated not with destruction of microscopic information, but with the progressive confinement of trajectories to partition-defined cycle basins, increasing the observer's irreversible record structure.

The complete logical chain from substrate dynamics to emergent time is:

(i) $F: \Omega \to \Omega$ bijective and symplectic
(ii) $b: \Omega \to \{0,1\}$ coarse partition
(iii) $\exists$ monochromatic cycle $C \subseteq b^{-1}(1) \implies$ b becomes fixed on C
(iv) b fixed $\implies \Gamma_b \to 0$ permanently
(v) at onset of permanent crossing suppression: $R_{\mathscr{F}}(k)$ increases by 1
(vi) each such increment increases $\mathscr{F}(k)$
(vii) accumulation of increments in $\mathscr{F}(k) \implies$ emergent temporal ordering

Topology determines the existence of irreversible record channels; entropy-like coarse-graining determines their occupation statistics and effective production rates. The two are complementary layers of description, not competing explanations.

7.4 Connection to Causal Set Theory

In causal set theory [2], spacetime is replaced by a locally finite partial order generated by a sequential growth process. The emergent ordering $\prec$ in VERSF is structurally analogous — a partial order generated by irreversible local events. The key difference is that in VERSF the generating mechanism (monochromatic cycle trapping) is derived from first principles rather than postulated.

7.5 Connection to Relational and Thermal Time

Rovelli's relational approach [3] holds that time emerges from correlations between physical subsystems. The Connes–Rovelli thermal time hypothesis [4] identifies time with the modular flow of the KMS state of a system at equilibrium. In VERSF, the monotone sequence of fact-formation events plays an analogous role — generating a directed emergent structure without a primitive time coordinate. The observer-relativity of $\mathscr{F}(k)$ noted in Section 5 aligns with Rovelli's relational perspective: different observers with different partitions generate different emergent orderings, all equally valid.

7.6 Potential Empirical Directions

The following are not sharp discriminating predictions but structural consequences of the framework that point toward testable conditions, pending model completion and quantitative development.

1. **Minimal irreversibility condition**: A physical system whose effective dynamics admits no monochromatic cycles for the relevant macroscopic bit partition should exhibit no irreversible behavior at that observational scale. Observation of irreversibility constrains the permutation cycle structure of the microscopic dynamics.
2. **Decoherence and cycle structure**: The spin system analogy of Section 6.6 suggests that decoherence rates may be related to the density of monochromatic cycles relative to the relevant observable. This would provide a geometric criterion for which observables decohere rapidly versus slowly — a quantitative connection requiring further development.

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8. Conclusion

We have presented a mathematically grounded formulation of the VERSF framework that resolves the apparent conflict between Noether symmetry and emergent time. The core structural advances are:

Clarification 1 (State Space). The finite microstate set Ω is embedded in a smooth symplectic manifold (M, ω). Tick maps are restricted to those arising as time-ε maps of autonomous Hamiltonian flows — a stated scope restriction, not a claim about all bijections on finite sets.

Clarification 2 (Permutation Graph). The transition graph is the *permutation graph* G_F of the bijection F, whose components are literal permutation cycles. Graph topology and permutation algebra are kept carefully distinct.

Clarification 3 (Threshold Theorem). The operative condition for fact formation is the existence of a *monochromatic cycle* — a permutation cycle of F lying entirely within one cell of the bit partition. The necessary condition (Lemma 4.2) and constructive sufficient condition (Lemma 4.4) are proved separately. The β_1 description applies precisely at the level of the induced subgraph on the trapped cell, not globally.

Clarification 4 (Honest Scope). The framework establishes a minimal structural criterion for irreversible record formation, not a derivation of thermodynamic time asymmetry. The consistency of the two temporal orderings is a definitional observation, clearly labeled as such. The QM correspondence is structural and interpretive, explicitly not a derivation.

Together, the principal results establish a *minimal mechanism for irreversible record formation compatible with reversible substrate dynamics*:

- **Noether symmetry governs the reversible substrate** — the discrete dynamics formally preserves $\tilde{H}$ order by order in ε, grounded in the discrete Noether theorem applied to step-index autonomy;
- **Monochromatic cycle trapping produces irreversible facts** — the existence of a permutation cycle aligned with a bit partition is the sharp combinatorial condition for local irreversibility;
- **Emergent time is partition-relative and order-compatible** — the emergent ordering is a sub-ordering of the step index by definition; different observers with different partitions generate different but internally consistent emergent orderings.

The framework does not claim to have derived the full thermodynamic arrow of time. It demonstrates that a reversible, symplectic-consistent substrate dynamics can give rise to locally irrevocable informational commitments precisely when its cycle structure aligns with a macroscopic partition. That structural result is the paper's central contribution.

Future work includes: extending the analysis to continuous state spaces and quantum field theory; exploring connections to Lorentzian geometry via the emergent partial order; quantifying the decoherence–cycle density connection introduced in Section 6; and exploring early-universe cosmology [12], where the onset of monochromatic cycles in the information substrate would correspond to the emergence of temporal direction and may connect to proposals for cosmological time in the causal set programme.

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