The Fold Interface Law

Deriving the Minimal Commitment Structure at the Void–Universe Boundary

Keith Taylor
VERSF Theoretical Physics Program

For the General Reader

Physics rests on facts—on stable records of what happened that can be compared, verified, and reproduced. But where do facts come from? What must the universe look like, at the most basic level, for stable facts to exist at all?

This paper addresses that question by asking what structure is forced on the boundary between a purely reversible substrate—what the VERSF framework calls the *Void*—and the committed physical world where facts are fixed. We call this boundary the *fold*.

Starting from three conditions that any fact-supporting universe must satisfy (things cannot be infinitely precise; some events must be irreversible; any bounded region can store only a finite amount of information), we show that the fold is not a free assumption but a derived necessity. The derivation also determines the fold's internal structure: it must be a two-dimensional surface carrying a four-dimensional state space described by complex numbers. Remarkably, even the use of complex numbers—rather than ordinary real numbers or the more exotic quaternions—turns out to be forced. Real arithmetic lacks the phase structure needed for consistent quantum correlations; quaternionic arithmetic introduces excess internal degrees of freedom that the minimal interface has no room for. Complex geometry is exactly what is needed, nothing more and nothing less.

A further consequence falls out of the topological argument: the minimal number of spatial dimensions needed to accommodate a network of interacting fold surfaces is three—recovering the dimensionality of the space we observe from the requirements of stable commitment alone.

The paper is a bridge paper within the VERSF programme. Its job is to close the logical gap between the programme's foundational axioms and the *One-Fold* architecture used in its downstream results on particles, gauge symmetry, and quantum correlations. Those results previously required the fold to be assumed. After this paper, the fold is derived.

What This Paper Proves

The paper establishes two main results within the VERSF axiomatic framework, each building on the last, with a further structural consequence.

The first result is that a commitment boundary must exist. If reversible distinctions never become irreversible, no stable physical facts exist — but the axioms presuppose facts. The boundary is therefore not a modelling choice; it is a logical necessity given A1–A3.

The second result is that this boundary must be a two-dimensional surface carrying a four-dimensional complex state space $\mathscr{H}_{\text{fold}} \cong \mathbb{C}^4$, and that the algebraic state structure is induced by, and serves as the minimal reversible representation of, the geometric fold data. A 2D commitment boundary intrinsically separates two domains (inducing commitment polarity $\sigma \in \{0,1\}$) and admits a local orientation-reversal symmetry from the manifold boundary structure (inducing orientation $\omega \in \{-1,+1\}$). Because polarity operates across the boundary and orientation operates along it, they are geometrically independent — the 2D fold already determines four distinct interface states. Section 7 derives the minimal reversible representation of that four-state structure: the state space over any admissible field is F^4; the minimally admissible field is $\mathbb{C}$, since real spaces lack the phase degree of freedom required for invariant projection geometry and quaternionic spaces carry non-commutative phase directions that constitute excess structure the minimal interface was never required to support. The field-selection argument is a minimality result internal to the VERSF axioms, not an impossibility theorem over all conceivable structures.

A further consequence of the topological argument is that the minimal natural embedding dimension for a network of interacting fold surfaces is three, recovering three spatial dimensions from the commitment structure alone.

The cumulative result is that the One-Fold architecture—the four-dimensional complex interface used throughout the VERSF programme to derive particles, gauge symmetry, and quantum correlations—was previously assumed as the minimal reasonable interface. This paper shows it is the unique minimal interface within the VERSF axiomatic framework, proceeding from axioms A1–A3 without additional commitments beyond the linear reversible state-space framework adopted in [T1]. That is the logical gap it closes.

Table of Contents

Abstract

The Void Energy–Regulated Space Framework (VERSF) reconstructs physics from operational constraints on distinguishability and irreversible record formation. Physical facts arise when reversible distinctions become irreversibly committed at the boundary separating the reversible Void from the committed universe.

Prior work in the programme introduced the *fold* as the minimal commitment interface, showing that it carries a four-dimensional internal Hilbert structure, supports closure defects interpretable as particles, and yields gauge symmetry and quantum correlations under additional structural assumptions. However, the existence and structure of the fold were stipulated as the minimal admissible interface rather than derived from the programme's foundational axioms.

This paper closes that gap. Starting from three unavoidable conditions for physics—**finite distinguishability**, **irreversible commitment**, and **finite localisation capacity**—we derive the minimal form of the commitment boundary from the programme's operational axioms. The result is a binary commitment interface possessing a reversible orientation degree of freedom and a minimal reversible state space isomorphic to $\mathbb{C}^4$. Crucially, the *complex* algebraic structure of this space is itself derived: real spaces lack sufficient phase structure for invariant projection geometry, while quaternionic spaces are not minimally admissible due to excess internal degrees of freedom. Complex structure is the unique minimally admissible choice within this framework. This structure matches precisely the One-Fold architecture used throughout the VERSF programme.

The derivation establishes the fold as the unique minimal boundary within the VERSF axiomatic framework compatible with the operational requirements for physical fact formation, providing a deeper foundation for the programme's results on spacetime emergence, quantum correlations, and particle closure structures. A further consequence is derived: the minimal natural embedding dimension for networks of interacting folds is three, recovering three spatial dimensions from topological constraints on commitment boundaries alone.

---

1. Introduction

Modern physics describes nature through spacetime geometry, quantum fields, and probabilistic measurement rules—a framework of extraordinary empirical success. Yet these structures presuppose the prior existence of stable physical facts: persistent, reproducible records that can be observed, compared, and communicated.

The VERSF programme begins from this observation and asks a more fundamental question: *what are the minimal structural conditions under which a universe can support physical facts at all?* This places it in dialogue with broader programmes of reconstructing quantum theory from operational or information-theoretic axioms [H01, CDP11, DB11, MM11], and with Wheeler's conception of physics as emerging from information [W90].

Three such conditions have been identified in earlier work:

1. **Finite distinguishability** — physical states must be finitely resolvable
2. **Irreversible commitment** — distinctions become facts only when irreversibly recorded
3. **Finite localisation capacity** — bounded regions support only finitely many independent records

Together these conditions imply that reversible distinctions cannot persist indefinitely. At some boundary, reversible information must become irreversibly committed. The purpose of this paper is to derive the minimal structure of that boundary from the programme's axioms and to show that it is unique.

---

2. The Epistemic Boundary and Interface Methodology

VERSF does not attempt to describe the internal structure of the Void. The Void is defined operationally as the domain in which persistent distinctions cannot exist. Because measurement, locality, and dynamical law all require stable distinctions, these concepts do not apply inside the Void in the ordinary sense. Any attempt to assign them internal Void dynamics would presuppose exactly the structure whose origin we seek to explain.

The programme therefore adopts a principled **interface methodology**: rather than modelling the Void's interior, we ask only what structure must exist at the boundary where reversible distinctions become irreversible physical records.

Definition (Fold). The *fold* is the minimal commitment interface—the boundary at which reversible distinctions in the Void substrate become irreversible physical facts in the committed universe.

The task of this paper is to derive the fold's structure from axioms A1–A3, rather than to assume it.

---

3. Foundational Axioms

Axiom A1: Finite Distinguishability

A physical universe cannot sustain infinitely precise distinctions. Any stable record must occupy a finite region of distinguishability space. Operationally, any physically accessible degree of freedom must admit only finitely resolvable distinctions within a bounded record context.

Motivation. Infinite precision would require infinite information storage per record, violating any finite bound on localisation capacity. Moreover, no physical measurement procedure can resolve infinitely fine distinctions.

Axiom A2: Irreversible Commitment

A distinction becomes a physical fact only when it is irreversibly exported beyond the local causal control of any agent. Reversible distinctions, however well-defined, do not constitute physical records.

Motivation. A distinction that could always be undone carries no stable informational content. Physical laws require facts that persist across independent observations, which necessitates irreversibility.

Axiom A3: Finite Localisation Capacity

A bounded spacetime region can support only a finite number of independent physical records.

Motivation. This is the physical content of entropy bounds (e.g., Bekenstein–Hawking [Bek73, B81, Haw75]), which constrain the information content of any bounded region. It also follows from A1 applied locally.

Together A1–A3 imply that record formation must occur at a finite-capacity commitment interface. An unbounded interface would violate A3; an interface admitting infinite resolution would violate A1.

---

4. The Necessity of a Commitment Boundary

Proposition 4.1. *If reversible distinctions exist and physical facts must exist, then there must be a commitment boundary at which reversible distinctions become irreversible records.*

Proof. Suppose reversible distinctions could remain indefinitely reversible. Then by A2 they never become physical facts. But physical laws—including A1 and A3—presuppose the existence of stable facts. Contradiction. Therefore any reversible distinction must eventually encounter a boundary at which it is irreversibly committed. \square

This boundary must satisfy three structural requirements:

R1. It must support a binary distinction between committed alternatives (the minimal distinguishable fact).

R2. It must allow reversible transformations *before* commitment (so that the substrate remains reversible up to the boundary).

R3. It must admit a stable orientation separating committed states from reversible substrate states (so that the direction of commitment is well-defined).

These requirements constrain the interface's structure and, as we show below, uniquely determine its minimal form within the VERSF axiomatic framework.

---

5. Binary Commitment

The smallest possible physical fact distinguishes exactly one alternative from another—it carries one bit of distinguishability. Let the committed alternatives be labelled $\sigma \in \backslash 0, 1$.

Observation 5.1 *(Minimality of binary commitment). The minimal commitment event is binary: it distinguishes exactly two alternatives and carries one bit.*

Any commitment event over more than two alternatives can be decomposed into binary commitments; by A3 the interface has finite capacity, so the minimal irreducible unit is binary. A one-alternative interface carries no information. The fold must therefore support a **binary commitment channel**.

---

6. Reversible Orientation

Commitment requires distinguishing the substrate side (reversible) from the record side (committed). However, by R2, reversible fluctuations must still be possible before commitment occurs. Therefore the interface itself must carry a degree of freedom that:

- distinguishes "pointing toward commitment" from "pointing away from commitment," and
- can be transformed reversibly before the commitment event.

Observation 6.1 *(Minimal orientation symmetry). The minimal reversible symmetry group for a binary orientation degree of freedom is* $\mathbb{Z}_2$.

An orientation is a binary attribute; the minimal group acting reversibly on a two-element set is $\mathbb{Z}_2$. Any larger group introduces excess degrees of freedom not required by R1–R3.

The minimal commitment interface therefore contains two independent binary structures:

Structure	Role
Commitment polarity	Distinguishes the two committed alternatives ($\sigma \in \{0,1\}$)
Reversible orientation	Allows reversible pre-commitment transformations

7. The Minimal Reversible State Space

The four-state structure derived in this section will be seen in Section 8.4 to follow from the 2D fold geometry itself; it is derived independently here to make the algebraic argument self-contained. Readers may prefer to read Section 8 first and return here for the representation-theoretic details.

7.1 The Two Independent Binary Structures

The preceding sections established that the commitment interface must carry two logically distinct binary structures:

- **Commitment polarity** $\sigma \in \{0, 1\}$ — the minimal informational content of a physical fact, distinguishing which of two alternatives has been committed.
- **Reversible orientation** $\omega \in \{-1, +1\}$ — the interface's directional degree of freedom, distinguishing the reversible substrate side from the committed-record side.

These two variables are not interchangeable. Commitment polarity *selects* the physical record; reversible orientation *locates* the interface relative to the substrate. Conflating them would assign orientation-flips the status of physical commitment events, which contradicts A2—reversible operations cannot produce facts.

Lemma 7.1. *Commitment polarity and reversible orientation are independent variables.*

Proof. Suppose they were the same variable. Then flipping the interface orientation (a reversible operation) would simultaneously change the committed physical record. But A2 requires that only irreversible operations produce physical facts. A reversible orientation flip therefore cannot alter the committed alternative. The two variables must therefore be independent. \square

The joint state space is the Cartesian product:

$$(\sigma, \omega) \in \{0,1\} \times \{-1,+1\}$$

yielding **four distinct interface states**.

7.2 The Minimal Reversible State Space Has Four Basis States

To support reversible dynamics among four distinguishable states, the state space must admit continuous reversible transformations. This motivates a linear state-space framework as adopted in the programme [T1]. We note that the choice of such a framework is a substantive assumption, encoding linearity and a notion of reversible evolution; it is not derived here but inherited from the programme's foundational papers. The task of the present section is more limited: to show

that, given any admissible linear reversible framework over an algebraic field F, the minimal state space has exactly four basis states.

Proposition 7.2 *(Four-state minimal structure). Within the adopted linear reversible framework, the minimal state space supporting two independent binary structures has four basis states — that is, it is of the form F^4 for an admissible field F.*

Proof. Each binary degree of freedom requires a two-dimensional state space F^2 to represent its two distinguishable values and support reversible (invertible linear) dynamics between them. Independence requires that the two degrees of freedom be encoded in distinct, non-overlapping subspaces. This requires the tensor product structure:

$F^2\_\sigma \otimes F^2\_\omega \cong F^4$

We verify no smaller space suffices:

- **F^2:** Supports only a single binary degree of freedom. Cannot simultaneously encode independent σ and ω without conflating them, violating Lemma 7.1.
- **F^3:** Any embedding of two independent binary variables into three basis states forces at least one pair to share a basis element, again violating independence.

Therefore F^4 is the minimal state space, regardless of the field. Section 7.3 determines which field F is minimally admissible. \square

The four basis states correspond to the four joint states $(\sigma, \omega) \in \{0,1\} \times \{-1,+1\}$ derived in Section 7.1. Once Section 7.3 establishes $F = \mathbb{C}$, the full result $\mathscr{H}\_{fold} \cong \mathbb{C}^4$ follows.

7.3 Field Selection: Why the Fold Interface Requires Complex Structure

Proposition 7.2 established that the minimal reversible state space has four basis states, of the form F^4 for some admissible field F. The present section determines which field is minimally admissible. This is a *minimality selection result* internal to the VERSF axioms and interface requirements—not an impossibility theorem over all conceivable algebraic frameworks. Real structure is excluded because it is insufficient for the required projection geometry; quaternionic structure is excluded because it is non-minimal, not because it is impossible in principle; complex structure is the unique choice satisfying both sufficiency and minimality.

Three candidates are mathematically natural: real ($\mathbb{R}$), complex ($\mathbb{C}$), and quaternionic ($\mathbb{H}$) vector spaces.

7.3.1 Requirements on the Reversible Interface

Any reversible state space for the fold must satisfy four conditions:

Requirement	Content
(i) Binary distinguishability	Represent two independent binary structures: commitment polarity and reversible orientation
(ii) Continuous reversibility	Support continuous transformations between pre-commitment states, not merely discrete flips
(iii) Compositional stability	Combined fold interfaces preserve local distinguishability, reversible composition, and stable projection
(iv) Invariant projection statistics	Outcome statistics depend only on invariant overlaps between local contexts—not on coordinate choices

Requirement (iv) is especially significant: it is the interface-level precursor of both Lorentz invariance and the Tsirelson-bound structure. Any admissible field must support a projection law whose statistics are coordinate-independent.

7.3.2 Real Vector Spaces Are Insufficient

Suppose $\mathscr{H}\_\text{fold}(\mathbb{R}) \cong \mathbb{R}^4$. The possibility and limitations of real Hilbert space quantum mechanics have been studied formally [S60], and recent experiments confirm that real-number quantum theory is empirically distinguishable from—and inferior to—complex quantum theory [RTV22].

A real space represents four basis states and admits real inner products, but it cannot support a nontrivial phase degree of freedom independent of amplitude. Signed amplitudes alone provide an overlap structure that is too weak for the required contextual geometry:

- Interference structure is impoverished—no relative phase exists as an intrinsic reversible degree of freedom.
- Projection overlaps cannot encode rotationally stable contextual relations across independent fold interfaces.
- Composition laws lack the invariant geometry needed for the quantum-correlation sector.

A real interface can represent *alternatives* but cannot represent *relative phase* as an intrinsic reversible quantity. This is fatal for Requirements (ii)–(iv).

Real spaces are not minimally sufficient.

7.3.3 Quaternionic Spaces Are Not Minimally Admissible

Now suppose $\mathscr{H}\_\text{fold}(\mathbb{H})$ is quaternionic [A95].

Quaternionic Hilbert spaces are mathematically consistent but fail the minimality requirement in two ways:

- **(i) Over-parameterisation.** Quaternionic amplitudes carry multiple imaginary directions (\mathbfi, \mathbfj, \mathbfk), introducing internal phase freedom beyond what the two binary fold structures require. The minimal interface should carry no excess degrees of freedom beyond those demanded by R1–R3 and A1–A3.
- **(ii) Excess invariant labels.** The fold principle requires that observable statistics depend only on quadratic contextual overlaps. Quaternionic structure generically supports richer relational data—non-commutative phase relations—that cannot be removed by any local re-description. These constitute additional accessible labels absent from the minimal interface specification. The quaternionic option is therefore not ruled out as impossible, but as *non-minimal*: it introduces more structure than the interface conditions require.

Quaternionic spaces are not minimally admissible.

7.3.4 Complex Structure Is Minimally Sufficient

A complex Hilbert space $\mathscr{H}\_\text{fold} \cong \mathbb{C}^4$ sits exactly between the two extremes:

- It provides nontrivial reversible phase relations (unlike $\mathbb{R}$),
- without introducing excess non-commutative invariant structure (unlike $\mathbb{H}$),
- its natural quadratic inner product $\langle u, v \rangle$ is the minimal invariant overlap law,
- it is stable under local tensor-product composition,
- projection statistics depend only on this quadratic overlap.

The minimal admissible overlap law is therefore the complex inner product. Its associated geometry—the projective space $\mathbb{C}P^3$—is the fold's physical state manifold.

7.3.5 Inner Product from Projection Covariance

The fold principle requires that projection outcomes depend only on invariant relations between local interface contexts u, v. The admissible projection law must take the form:

$P(u \to v) = f(\Gamma(u, v))$, where Γ is invariant under local re-description

Since the interface statistics are required to be invariant under local re-description and compositionally stable under reversible transformations, the lowest-order nontrivial invariant is bilinear—quadratic in the interface amplitudes. Linear invariants vanish by the symmetry of orientation reversal. Higher-order invariants—quartic or above—would introduce amplitude-dependent statistics that go beyond what the binary commitment structure supplies: a quartic overlap law $|\langle u,v\rangle|^4/(\|u\|^2\|v\|^2)$ varies with intermediate-state normalisation in a way that is not determined by the one bit per commitment event the interface carries, constituting additional observable data the minimal interface specification never required. On minimality grounds, these are therefore excluded, and the minimal admissible invariant is quadratic:

$\Gamma(u,v) = \langle u, v \rangle$

This is the structure of a complex inner product. Real inner products exist but lack the phase degree of freedom required by (ii)–(iv); quaternionic inner products satisfy a richer algebra than the minimal interface demands. The **complex inner product is the minimal invariant projection geometry consistent with the fold requirements.**

7.3.6 Minimality Selection Result for the Fold Field

Proposition 7.3 *(Minimality-based field selection for the fold interface). Let the commitment interface satisfy: two independent binary structures, continuous reversible transformations, compositional stability, invariant projection statistics depending only on contextual overlaps, and no excess invariant labels beyond those required for commitment and orientation. Then the minimally admissible field is $F = \mathbb{C}$, giving:*

$\mathscr{H}$_fold $\cong \mathbb{C}^4$, projective states $\in \mathbb{C}P^3$

Real spaces are excluded as insufficient: they cannot support the phase structure required for invariant projection geometry. Quaternionic spaces are excluded as non-minimal: they are mathematically consistent but introduce internal structure—non-commutative phase directions—exceeding the minimal interface requirements. Complex structure is the unique choice satisfying both sufficiency and minimality.

This is a minimality selection result within the VERSF axiomatic framework. It is not an impossibility theorem over all conceivable algebraic structures.

7.3.7 Significance for the Programme

This result closes a foundational gap that ran through earlier VERSF work. Previously, complex geometry entered the One-Fold analysis as an inherited or companion assumption—imported from quantum mechanics rather than derived from the framework's own axioms. The logical chain is now complete and sequential:

A1, A2, A3 $\Longrightarrow$ commitment boundary $\Longrightarrow$ $\dim(\Sigma) = 2 \Longrightarrow (\sigma,\omega) \in \{0,1\} \times \{-1,+1\} \Longrightarrow F^4 \Longrightarrow F=\mathbb{C}$ $\Longrightarrow$ $\mathscr{H}$_fold $\cong \mathbb{C}^4$

The complex geometry used in the Tsirelson-bound derivation, the gauge-channel construction, and the entropy-curvature route to quantum structure is no longer imported from quantum mechanics as prior input. It is inherited from the minimal reversible requirements of the commitment interface itself.

---

8. Minimal Topology of the Fold Boundary

With the internal state space established, we now turn to the geometry of the commitment interface itself. The task is to determine the minimal dimension of an interface that can stably separate reversible substrate structure from committed physical record.

The key requirement is not merely the existence of noncontractible loops. It is stronger: **the interface must support intrinsic region-boundary structure.** That is, the interface must allow closed curves that bound enclosed subregions on the interface itself, so that one region can function as the committed side and the complementary region as the reversible side. This is the geometric realisation of requirement R3: the interface must intrinsically distinguish "sides" of commitment. Throughout this section the interface is assumed to be locally compact, so that region boundaries and enclosed domains are well defined and pathological cases are excluded.

8.1 Stable Facts Require Intrinsic Separation

A physical fact must remain stable against local reversible fluctuations in the Void-adjacent substrate. This requires more than a local label. It requires a commitment boundary that can separate two domains: a region associated with committed record, and a region associated with reversible pre-commitment structure.

If that separation is only extrinsic—imposed by an ambient embedding rather than supported by the interface itself—then the distinction is not encoded in the interface structure and can be erased without violating the interface's own topology. Accordingly, the fold must support closed boundaries of enclosed subregions.

Structural Requirement 8.1. *Stable commitment requires an interface on which closed curves can bound enclosed subregions intrinsically.*

A commitment boundary must itself encode the separation between reversible and committed domains—it cannot rely on ambient context to supply this structure. This requires the interface to admit closed curves that function as boundaries of regions *within* the interface. Without such intrinsic region-boundary structure, there is no topologically stable notion of "this side committed, that side reversible," and requirement R3 is violated.

8.2 Why One-Dimensional Interfaces Are Insufficient

A one-dimensional structure can carry order and even support closed loops, but it cannot support the intrinsic domain separation required by Structural Requirement 8.1. There are two cases.

Case 1: contractible 1D interface. If the interface is an open curve or interval, it has no closed boundary structure at all. Any local distinction can be erased by reversible contraction or translation along the curve.

Case 2: closed 1D loop. A closed loop S^1 does possess nontrivial first homology, $H_1(S^1) = \mathbb{Z}$. So noncontractible loops alone are not the decisive criterion. The problem is different: a one-

dimensional loop does not support enclosed subregions on the interface itself. A loop is already all boundary and no interior—it cannot host a closed curve that divides the interface into a committed region and a reversible region intrinsically. Any "inside" or "outside" associated with a 1D loop is determined only after embedding it in a higher-dimensional ambient space; it is not a structural property of the interface itself.

But the fold must itself be the commitment boundary. Its separating power must be intrinsic, not borrowed from an external embedding.

Proposition 8.2. *A one-dimensional interface cannot serve as the minimal commitment boundary.*

Proof. A 1D interface cannot support intrinsic region-boundary structure. Open 1D manifolds have no closed enclosing boundaries; closed 1D manifolds are themselves boundaries but contain no enclosed subregions of their own. Therefore a 1D interface cannot intrinsically distinguish committed and reversible domains. This violates requirement R3. \square

8.3 Two-Dimensional Interfaces Are Minimally Sufficient

A two-dimensional interface is the first dimension in which closed curves can bound enclosed regions within the interface itself.

On a 2D surface Σ, a closed curve can divide the surface into distinct subregions. One region can then be interpreted as committed, the complementary region as reversible, with the curve functioning as the intrinsic commitment boundary between them. This gives exactly the structure missing in 1D: closed boundary curves, enclosed subregions, and intrinsic separation of domains. Moreover, those subregions can be topologically protected against local reversible deformations, provided the enclosing structure is preserved.

Proposition 8.3. *The minimal interface dimension supporting intrinsic commitment separation is two.*

Proof. By Proposition 8.2, a 1D interface cannot support intrinsic region-boundary structure. A 2D surface can: closed curves on a surface may bound enclosed subregions, thereby providing an intrinsic separation between two domains on the interface itself. Higher-dimensional interfaces also permit this, but are not minimal. Therefore the minimal interface dimension is $\dim(\Sigma) = 2$. \square

8.4 From Fold Geometry to Fold State Structure

Proposition 8.3 establishes that the fold boundary must be two-dimensional. The results of Section 7—the four-state structure, field selection, and Hilbert space $\mathbb{C}^4$—can now be seen not as a parallel derivation from the same axioms, but as flowing from the geometry itself.

A two-dimensional commitment interface with intrinsic region-boundary structure necessarily carries two independent geometric binary degrees of freedom.

First, commitment polarity from region separation. Any 2D commitment boundary intrinsically separates two domains on the interface: one domain associated with committed record and the complementary domain associated with reversible pre-commitment structure (Proposition 8.3). This separation is a geometric fact about the 2D surface, not an imposed label. It induces a binary commitment polarity:

$\sigma \in \{0, 1\}$

Second, orientation from local surface ordering. A two-dimensional boundary admits a local orientation-reversal symmetry. Any locally oriented 2D region induces an orientation on its boundary curve via the manifold boundary operation ∂: the boundary inherits a canonical ordering from the region's own orientation. Reversing that induced orientation defines a canonical $\mathbb{Z}_2$ operation on the boundary ordering—clockwise versus counterclockwise—which is intrinsic to the surface structure and not a chosen representation. This gives a binary orientation degree of freedom:

$\omega \in \{-1, +1\}$

These two structures are geometrically independent. Reversing boundary orientation (a local traversal operation) does not interchange committed and reversible domains; conversely, exchanging committed and reversible domains does not reverse local traversal orientation. Polarity concerns which domain is selected *across* the boundary; orientation concerns the local ordering structure *along* the boundary. They are distinct operations living at different geometric levels.

The minimal fold therefore carries the geometric state set

$(\sigma, \omega) \in \{0,1\} \times \{-1,+1\}$

with four distinct states—not by assumption, but because the 2D fold geometry forces it.

The algebraic state space of Section 7 is therefore not an independently postulated object. It is the minimal reversible representation of the geometric data carried by the fold boundary. Section 7.2 determines the minimal reversible realisation of this four-state geometric structure (F^4); Section 7.3 selects the minimally admissible field ($F = \mathbb{C}$). The result $\mathscr{H}_{\text{fold}} \cong \mathbb{C}^4$ follows.

This closes the geometry–algebra gap: the 2D fold geometry induces the binary data; the Section 7 argument determines their minimal reversible representation. The two derivations are not parallel conclusions from a common set of axioms—the geometric derivation is upstream of the algebraic one.

8.5 Topological Stability and Closure

The preceding result identifies the minimal interface dimension. We now connect it to topological stability.

Once the interface is two-dimensional, closed curves can act as commitment boundaries of enclosed regions. These boundaries are stable precisely because local reversible deformations cannot remove the distinction unless they destroy the enclosing structure itself.

In this sense, the relevant topology is not merely the existence of nontrivial homology, but the existence of stable enclosed regions bounded intrinsically on the interface. Homological language remains useful as a supporting description—stable commitment structures correspond to nontrivial boundary relations on the interface—but the decisive property is intrinsic region separation, not abstract H_1 by itself.

8.6 Three Spatial Dimensions as the Minimal Natural Ambient Setting

Once multiple folds exist, they must coexist and interact without generically collapsing into one another. A single fold is a two-dimensional interface; a network of interacting folds consists of multiple closed 2D structures. The following observation identifies the minimal natural ambient setting for such a network.

Observation 8.4 *(Minimal ambient dimension for fold networks). Three dimensions is the minimal natural ambient setting for a nontrivial network of interacting fold surfaces.*

Supporting argument. A fold is a 2D surface. A nontrivial network of extended 2D commitment surfaces requires a minimal ambient dimension allowing distinct extended surfaces to coexist without generic degeneracy—distinct surfaces must be able to maintain independent embedding and interact without collapsing into a single coincident structure. The minimal such ambient dimension is three. This is not a full embedding theorem but a minimal structural observation: an ambient dimension of three is the smallest in which a nontrivial network of distinct 2D surfaces can be arranged without generic intersection.

This observation identifies three dimensions as a minimal natural ambient setting, consistent with the three-dimensional spatial structure observed physically. A fuller derivation of spatial dimensionality—including dynamic and stability considerations—is developed in [T4]. The present argument establishes consistency with that result from the fold network structure alone.

8.7 Combined Interface Structure

Combining Sections 7 and 8, and in particular the bridge established in Section 8.4, the minimal commitment interface is:

$(\Sigma, \mathscr{H}\_\text{fold}) = (\text{2D surface}, \mathbb{C}^4)$

Geometrically, the fold is a two-dimensional interface; algebraically, it carries a four-dimensional complex reversible state space. These are not independent conclusions: the 2D geometry induces the binary state data whose minimal reversible representation is $\mathbb{C}^4$.

Theorem 8.5 *(Minimal fold structure). Within the VERSF axiomatic framework, the unique minimal commitment interface is a two-dimensional boundary carrying a minimally admissible*

reversible state space $\mathscr{H}_{fold} \cong \mathbb{C}^4$. The 2D geometry induces the binary state data whose minimal reversible representation is precisely the $\mathbb{C}^4$ structure. This is the One-Fold architecture used throughout the downstream VERSF programme.

---

9. Consequences for the VERSF Programme

The structure derived here provides an upstream grounding for the following downstream results already developed elsewhere in the programme. Each of those results previously required the fold structure to be assumed; they now inherit it as a derived consequence.

9.1 Closure defects and particles. When fold topology closes on itself, defects arise at closure points. These defects carry quantised internal charges and are naturally interpreted as fundamental particles. The present derivation grounds this identification in the unique structure of the minimal commitment boundary.

9.2 Quantum correlations and the Tsirelson bound. Projection constraints arising from the $\mathbb{C}^4$ Hilbert structure are consistent with the Tsirelson bound on quantum correlations derived in earlier VERSF work [C80, T5]. The derivation of $\mathscr{H}_{fold} \cong \mathbb{C}^4$ from axioms A1–A3 therefore provides a structural basis for the specific bound on Bell-inequality violations [B64, CHSH69] observed in quantum mechanics.

9.3 Gauge symmetry. The geometric decomposition of the fold's internal channels produces gauge degrees of freedom. Because the fold structure is now derived rather than assumed, gauge symmetry emerges as a necessary consequence of the minimal commitment interface rather than an additional postulate.

9.4 Fermion generations. Admissible closure configurations of the fold—topological constraints on how the boundary can close—reproduce the pattern of fermion generations. The present derivation shows these constraints are not arbitrary but are consequences of the fold's unique minimal structure.

---

10. Discussion

10.1 What has been derived

Starting from three operational axioms—finite distinguishability (A1), irreversible commitment (A2), and finite localisation capacity (A3)—we have derived:

- the necessity of a commitment boundary (Proposition 4.1),
- the binary character of the minimal commitment event (Observation 5.1),
- the $\mathbb{Z}_2$ reversible orientation symmetry (Observation 6.1),

- the independence of commitment polarity and reversible orientation (Lemma 7.1),
- the four-state minimal reversible structure F^4 over a generic admissible field (Proposition 7.2),
- the minimality-based field selection $F = \mathbb{C}$, giving $\mathscr{H}\_\text{fold} \cong \mathbb{C}^4$ (Proposition 7.3),
- the intrinsic separation requirement for stable commitment (Structural Requirement 8.1),
- the two-dimensional geometric fold boundary (Propositions 8.2–8.3),
- three dimensions as the minimal natural ambient setting for fold networks (Observation 8.4),
- the combined minimal fold structure (Theorem 8.5).

Each step is forced within the VERSF axiomatic framework: no weaker structure satisfies A1–A3 and R1–R3, and no additional structure is required beyond what the axioms demand. The logical chain can be summarised:

A1–A3 $\Longrightarrow$ commitment boundary $\Longrightarrow$ 2D fold surface $\Longrightarrow$ $(\sigma,\omega) \in \{0,1\} \times \{-1,+1\}$ $\Longrightarrow$ F^4 $\Longrightarrow$ $F=\mathbb{C}$ $\Longrightarrow$ $\mathscr{H}\_\text{fold} \cong \mathbb{C}^4$

The key structural advance over prior formulations is the third arrow: the 2D fold geometry itself induces the binary state data, so the algebraic derivation of Section 7 is not a parallel thread but a downstream consequence of the geometric result.

10.2 Logical status

The derivation closes the logical gap between VERSF's foundational axioms and the One-Fold architecture used throughout the programme. Previously, the fold was introduced as "the minimal admissible interface"—a reasonable heuristic, but one that needed justification. That justification is now provided.

It is important to be precise about what the uniqueness claim does and does not assert. The paper does not show that every conceivable ontology reduces to the fold, nor that alternative frameworks cannot produce different commitment structures. It shows that *within the VERSF axioms*, any minimal commitment interface must take the fold form. The uniqueness is relative to A1–A3 and the operational requirements R1–R3. It is that constrained uniqueness which grounds the programme's downstream results.

10.3 Scope and limitations

The derivation does not describe the internal dynamics of the Void. It does not explain *why* Axioms A1–A3 hold—these are treated as operational necessities for any universe capable of supporting physical laws, not as conclusions derived from something more primitive. Whether a deeper derivation of A1–A3 is possible remains an open question.

This paper does not derive Hilbert space itself. It derives the minimal dimensional and field structure given the linear reversible state-space framework adopted in the programme's foundational papers [T1]. The Hilbert-space formulation—including linearity, compatible inner product, and unitary evolution—is inherited structure, not a conclusion of the present derivation.

The paper derives the minimal geometric and algebraic structures of the fold from a shared axiomatic base and, through Section 8.4, shows that the 2D fold geometry induces the binary state data whose minimal reversible representation is $\mathbb{C}^4$. This constitutes a structural bridge: the geometric derivation is upstream of the algebraic one, not merely parallel to it. What remains open is a deeper *unification theorem*—a single formal result establishing that 2D topology and $\mathbb{C}^4$ structure are not just sequentially related but jointly necessitated by a single geometric principle. Establishing that connection more directly remains a target for future work in the programme (Open Problem 7).

---

11. Open Problems

The present results suggest several natural next steps for the programme:

1. **Commitment barrier.** Derive the commitment barrier Φ_c—the energetic threshold at which reversible distinctions become irreversible facts—directly from the fold geometry, rather than introducing it as a separate parameter.
2. **Inner product from symmetry.** Prove that the complex inner-product structure on $\mathscr{H}_{\text{fold}}$ follows from fold projection symmetry alone, without invoking quantum mechanics as a prior assumption.
3. **Coarse-graining to Lorentzian geometry.** Show that fold-admissible configurations generically coarse-grain to Lorentzian spacetime with the correct signature, providing a derivation of macroscopic spacetime geometry from the commitment boundary structure.
4. **Chirality.** Derive the chirality of fundamental fermions from the asymmetry of the fold boundary geometry, explaining the left-right asymmetry of the weak interaction without additional input.
5. **Uniqueness in higher dimensions.** Determine whether $\mathscr{H}_{\text{fold}} \cong \mathbb{C}^4$ remains uniquely minimal in the presence of additional structural constraints, or whether higher-dimensional interfaces become admissible under extended axiom sets.
6. **Strengthening the quaternionic exclusion.** Proposition 7.3 excludes quaternionic structure on minimality grounds—it is a minimality argument, not a dynamical exclusion. A stronger result would show that quaternionic fold interfaces are *dynamically unstable*—that they generically decay to complex interfaces under the commitment process—making the exclusion thermodynamic rather than merely structural.
7. **Geometry–algebra unification theorem.** Section 8.4 establishes that the 2D fold geometry induces the binary state data whose minimal reversible representation is $\mathbb{C}^4$, making the geometric derivation upstream of the algebraic one. What remains is a deeper *unification theorem*: a single formal result showing that 2D topology and $\mathbb{C}^4$ structure are jointly necessitated by a single geometric principle, rather than merely sequentially related. This is the natural next step in tightening the foundational structure of the fold derivation.

---

12. Conclusion

We have derived the minimal structure of the commitment boundary separating the reversible Void substrate from the committed physical universe. Starting from the unavoidable operational conditions for physics—**finite distinguishability (A1)**, **irreversible commitment (A2)**, and **finite localisation capacity (A3)**—the derivation proceeds from the foundational axioms within the linear reversible state-space framework of [T1].

The result is a **binary commitment interface** carrying a $\mathbb{Z}_2$ reversible orientation degree of freedom and a minimal internal Hilbert space $\mathscr{H}$_fold $\cong \mathbb{C}^4$, realised geometrically as a two-dimensional fold. The *complex* algebraic structure is not assumed but derived: it is the unique minimally admissible field within the VERSF framework—real spaces lack the phase structure required for invariant projection geometry; quaternionic spaces are mathematically consistent but introduce excess internal degrees of freedom the minimal interface has no room for. Moreover, the topological requirements for stable commitment loops force the fold boundary to be two-dimensional, and the minimal embedding space for networks of interacting folds is three-dimensional—recovering three spatial dimensions as a structural consequence rather than an assumption.

This structure coincides with the One-Fold architecture used throughout the VERSF programme to derive particle closure structures, gauge symmetry, quantum correlations, and fermion generations. The present derivation establishes that the fold is not merely a convenient minimal choice, but the **unique minimal boundary within the VERSF axiomatic framework** consistent with the operational requirements of physical fact formation. The geometric structure (2D boundary) and the algebraic structure ($\mathbb{C}^4$ state space) are not independent conclusions: the 2D fold geometry induces the binary state data whose minimal reversible representation is $\mathbb{C}^4$. A deeper unification theorem—a single formal result showing why these two aspects are jointly necessitated—remains an open problem and the natural next target for the programme.

The fold is, in this precise sense, what a universe must look like at the point where the reversible becomes real.

References

VERSF Programme Papers

[T1] Taylor, K. *The Void Energy–Regulated Space Framework: Foundational Axioms and the Emergence of Physical Structure.* VERSF Technical Report, AIDA Institute.

[T2] Taylor, K. *The One-Fold Architecture: Closure Defects, Gauge Symmetry, and Particle Generation.* VERSF Technical Report, AIDA Institute.

[T3] Taylor, K. *Entropy-Momentum Foundations of Quantum Mechanics.* VERSF Technical Report, AIDA Institute.

[T4] Taylor, K. *Discrete Spacetime Structure from Information-Theoretic Constraints.* VERSF Technical Report, AIDA Institute.

[T5] Taylor, K. *The Tsirelson Bound from Fold Projection Geometry.* VERSF Technical Report, AIDA Institute.

Information-Theoretic Reconstructions of Quantum Theory

[CDP11] Chiribella, G., D'Ariano, G. M., and Perinotti, P. (2011). Informational derivation of quantum theory. *Physical Review A*, 84, 012311.

[DB11] Dakić, B. and Brukner, Č. (2011). Quantum theory and beyond: is entanglement special? In *Deep Beauty: Understanding the Quantum World through Mathematical Innovation*, ed. H. Halvorson. Cambridge University Press.

[H01] Hardy, L. (2001). Quantum theory from five reasonable axioms. arXiv:quant-ph/0101012.

[MM11] Masanes, L. and Müller, M. P. (2011). A derivation of quantum theory from physical requirements. *New Journal of Physics*, 13, 063001.

Bell Inequalities and Quantum Correlations

[B64] Bell, J. S. (1964). On the Einstein Podolsky Rosen paradox. *Physics*, 1(3), 195–200.

[C80] Cirel'son, B. S. (1980). Quantum generalizations of Bell's inequality. *Letters in Mathematical Physics*, 4(2), 93–100.

[CHSH69] Clauser, J. F., Horne, M. A., Shimony, A., and Holt, R. A. (1969). Proposed experiment to test local hidden-variable theories. *Physical Review Letters*, 23(15), 880–884.

Real, Complex, and Quaternionic Quantum Mechanics

[A95] Adler, S. L. (1995). *Quaternionic Quantum Mechanics and Quantum Fields*. Oxford University Press.

[RTV22] Renou, M.-O., Trillo, D., Weilenmann, M., Le, T. P., Tavakoli, A., Gisin, N., Acín, A., and Navascués, M. (2022). Quantum theory based on real numbers can be experimentally falsified. *Nature*, 600, 625–629.

[S60] Stueckelberg, E. C. G. (1960). Quantum theory in real Hilbert space. *Helvetica Physica Acta*, 33, 727–752.

Entropy Bounds and Localisation Capacity

[Bek73] Bekenstein, J. D. (1973). Black holes and entropy. *Physical Review D*, 7(8), 2333–2346.

[Haw75] Hawking, S. W. (1975). Particle creation by black holes. *Communications in Mathematical Physics*, 43(3), 199–220.

[B81] Bekenstein, J. D. (1981). Universal upper bound on the entropy-to-energy ratio for bounded systems. *Physical Review D*, 23(2), 287–298.

It from Bit and Operational Foundations

[W90] Wheeler, J. A. (1990). Information, physics, quantum: the search for links. In *Complexity, Entropy, and the Physics of Information*, ed. W. H. Zurek. Addison-Wesley.

[Z03] Zurek, W. H. (2003). Decoherence, einselection, and the quantum origins of the classical. *Reviews of Modern Physics*, 75(3), 715–775.

Topology and Algebraic Structure

[H02] Hatcher, A. (2002). *Algebraic Topology*. Cambridge University Press.

Spatial Dimensionality

[E17] Ehrenfest, P. (1917). In what way does it become manifest in the fundamental laws of physics that space has three dimensions? *Proceedings of the Amsterdam Academy*, 20, 200–209.