The Quartic Capacity of Causal Regions: Why Energy Density, Action, Entropy, and Information Throughput Converge on ρL^4

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How this paper relates to the main VERSF coherence scale paper. The companion paper "The Spacetime Coherence Length" derives and interprets a specific physical scale — $\xi \approx 8.2 \times 10^{-5}$ metres — from within the VERSF framework, using two independent physical arguments and placing it within the hierarchy of fundamental length scales. A reader of that paper might reasonably ask: is the quartic formula $\xi = (\hbar c/\rho)$^(1/4) a special feature of VERSF, or does it reflect something more general about physics? This paper answers that question directly. The quartic structure is not invented by VERSF. It emerges from quantum mechanics, thermodynamics, and the causal geometry of spacetime independently of any particular framework. VERSF's contribution is a single physical identification: that the energy density relevant to spacetime substrate dynamics is the cosmological vacuum energy density. Once that identification is made, the scale ξ is fixed by universal physics, not by theoretical choice. The two papers therefore work in sequence: this paper establishes the universality of the quartic capacity measure and the physical meaning of the threshold $\chi = 1$; the coherence scale paper applies that threshold to the cosmological vacuum and develops its consequences for spacetime structure, measurement, and experiment. This paper can be read first, as foundational context, or after, as a derivation-independent confirmation that the coherence scale rests on ground broader than any single theoretical framework.

For the General Reader

Physics usually talks about energy, space, and time as separate ingredients of reality. But when we ask a deeper question — *how much can actually happen inside a region of spacetime?* — these ingredients combine in a surprising way.

Again and again, when physicists study the limits of what can occur within a bounded region, the same mathematical combination appears:

ρL^4

Here ρ is energy density — the amount of energy packed into each cubic metre — and L is the linear size of the region. Multiply the energy density by the fourth power of the size, and you get a quantity with a specific physical meaning: it measures the total capacity of a bounded region of spacetime to support physical processes.

Why the fourth power? Three of the four come from volume (L × L × L), which converts energy density into total energy. The fourth power of L comes from time: a region of size L has a natural clock built in — light takes a time L/c to cross it, where c is the speed of light. Multiplying energy by this crossing time gives action, the fundamental currency of quantum mechanics. So ρL^4 is, in essence, the amount of quantum action available within a causally bounded region.

What makes this surprising is that four very different physical theories — quantum mechanics, thermodynamics, information theory, and the geometry of spacetime — each arrive at this same combination when they ask what a bounded region can do. The quantum limit on how fast physical changes can occur gives ρL^4. The maximum entropy a region can hold gives ρL^4. The minimum action required for a quantum event gives ρL^4. The capacity to produce irreversible physical records gives ρL^4.

All four perspectives converge on a single threshold condition:

$$\rho L^4 \sim \hbar c$$

where $\hbar$ is Planck's constant. Below this threshold, a region is too small or too sparse in energy to support even one irreversible physical event before light can escape and take information with it. Above this threshold, a region is capable of "making something happen" — stabilising a permanent physical record within its own causal boundary.

This threshold has a direct application. In the Void Energy–Regulated Space Framework (VERSF), the measured energy density of the cosmological vacuum — the background energy of empty space — can be substituted for ρ. The threshold condition then predicts a characteristic length scale of about 10^{-4} metres, roughly the width of a human hair, as the smallest region of spacetime capable of stabilising a classical fact. This companion paper shows that this scale is not a coincidence or an arbitrary VERSF parameter: it is the natural length at which the universal causal capacity measure ρL^4 reaches the quantum threshold $\hbar c$, as demanded simultaneously by quantum mechanics, thermodynamics, and the causal geometry of spacetime.

Abstract

The quantity ρL^4, where ρ is energy density and L a characteristic spatial scale, appears repeatedly across seemingly independent physical limits governing bounded spacetime regions. We identify two independent derivations showing that this quartic structure is the natural measure of the physical capacity of a causal region. The first proceeds from the Margolus–Levitin bound on distinguishable quantum transitions: a cell of energy $E \sim \rho L^3$ and causal crossing time $t \sim L/c$ can complete at most $N \sim \rho L^4/(\hbar c)$ distinguishable updates within one causal interval, with $N \sim 1$ marking the threshold for irreversible record formation. The second proceeds from the Bekenstein entropy bound applied to the cell: the maximum thermodynamic entropy $S_{max} \sim \rho L^4/(\hbar c)$ must meet or exceed the one-bit record-formation requirement $\Delta S \geq k_B \ln 2$. These two derivations are independent because the first invokes the energy-time uncertainty relation and orthogonality, while the second invokes only the Bekenstein bound and

a thermodynamic record condition. Both are grounded in the causal-diamond action framework, which shows that $\rho L^4/c$ is the total action contained within a causal spacetime cell of four-volume $V_4 \sim L^4/c$ — making ρL^4 the natural action measure of any bounded causal region. All three perspectives reduce to the same dimensionless parameter

$$\chi(L) = \rho L^4 / (\hbar c)$$

which measures the physical capacity of a causal region to support distinguishable irreversible events. The threshold $\chi \sim 1$ marks the point at which a causal region first contains sufficient energy, action, and entropy capacity to produce order-unity irreversible record formation within one causal crossing time. This quartic structure provides a structural explanation for the appearance of the coherence scale $\xi \sim (\hbar c/\rho)^{(1/4)}$ in the VERSF framework, and suggests that ρL^4 represents a fundamental invariant governing the information-processing capacity of bounded spacetime regions.

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1. Introduction

Many fundamental limits in physics constrain what can occur within a bounded region of spacetime. These include the Margolus–Levitin bound on quantum evolution rates, the Bekenstein bound on entropy capacity, holographic entropy bounds, causal-diamond thermodynamics, and quantum energy-time uncertainty limits. Each arises from apparently different physical principles — quantum mechanics, thermodynamics, gravitational physics — yet when expressed in terms of energy density ρ and characteristic size L, they repeatedly reduce to the same mathematical structure: ρL^4.

This paper asks why.

The answer is not that these limits secretly derive from a common source. It is rather that ρL^4 is the *natural measure of the physical capacity of a bounded causal region* — the quantity that converts energy density and causal geometry into a count of how much can happen. Just as entropy measures the number of accessible microscopic states, and action measures the cost of a dynamical trajectory, ρL^4 measures the capacity of a causal region to support irreversible physical events.

We make this precise through two independent derivations and one geometric framework. The geometric framework (Section 2) establishes why ρL^4 is the action measure of a causal diamond — not by coincidence but by the structure of relativistic causality. The first independent derivation (Section 3) approaches the threshold from quantum mechanics: the Margolus–Levitin theorem constrains how many distinguishable transitions a region of energy E can complete within its causal crossing time, yielding $N \sim \rho L^4/(\hbar c)$. The second independent derivation (Section 4) approaches from thermodynamics: the Bekenstein entropy bound constrains the maximum entropy available for record formation, yielding $S_{max} \sim \rho L^4/(\hbar c)$. Because the first derivation invokes the energy-time uncertainty relation and quantum orthogonality while the second invokes only a thermodynamic entropy bound and a one-bit record condition, the two are genuinely independent at the level of physical assumptions.

Both derivations, and the geometric framework, converge on the dimensionless parameter $\chi(L) = \rho L^4/(\hbar c)$ (Section 5). The threshold $\chi \sim 1$ marks the point at which a causal region first becomes capable of producing one irreversible record within its own causal boundary. The natural length scale at which this threshold is crossed — for a given energy density ρ — is $\xi \sim (\hbar c/\rho)^{(1/4)}$.

This quartic structure is not coincidental. Section 6 identifies why this power and no other is selected by the combination of spatial volume, causal time, and quantum action. Section 7 applies the result to the VERSF framework, where the cosmological vacuum energy density ρ_Λ fixes ξ at approximately 10^{-4} m. The conclusion (Section 8) draws out the implication: ρL^4 is a fundamental invariant governing the information-processing capacity of bounded spacetime regions, placing the VERSF coherence scale within a broader organisational principle of physical law.

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2. The Causal-Diamond Action Framework

Before presenting the two independent derivations, we establish the geometric grounding that makes ρL^4 the natural object of study.

2.1 The Causal Diamond and Its Four-Volume

A causal diamond of linear scale L — the intersection of the future light cone of one event with the past light cone of another event separated by a time L/c — has a spatial extent of order L and a temporal duration of order L/c. Its four-volume is therefore

$$V_4 \sim L^3 \cdot (L/c) = L^4/c$$

This is the natural spacetime volume associated with a region that is causally self-contained at scale L: information cannot enter or leave the diamond faster than light, so any physical process occurring within it must complete within the causal crossing time.

2.2 The Action of a Causal Cell

If the region contains energy density ρ, the total action contained within the causal diamond is

$$\mathcal{S}\_\text{cell} \sim \rho \cdot V_4 \sim \rho L^4/c$$

This has dimensions of action. Comparing to the quantum of action $\hbar$ gives the dimensionless ratio

$$\mathcal{S}\_\text{cell} / \hbar \sim \rho L^4/(\hbar c) \equiv \chi(L)$$

The threshold $\chi \sim 1$ therefore identifies the minimal causal diamond whose action budget equals the quantum of action — the minimum causal region in which quantum-scale processes can occur.

2.3 Why This Framing Matters

The causal-diamond framework shows that ρL^4 is not an ad hoc dimensional combination. It is the action of a naturally defined spacetime object: the smallest causally self-contained region at scale L. This geometric grounding explains why ρL^4 appears so persistently — it is the quantity that connects energy density to causal structure, and causal structure is what all bounded-region limits have in common.

Note that ρL^4 differs structurally from the holographic entropy of the same region, which scales as L^2/L_P^2. The holographic count is a static area measure of storage capacity. The quartic ρL^4 is a dynamical action measure of process capacity. These are different physical questions, and the quartic arises specifically when the question is not "how much information can this region store?" but "how much can happen here within one causal crossing time?"

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3. First Independent Derivation: Quantum Throughput

The first derivation proceeds from quantum mechanics and asks: how many distinguishable physical updates can a causal region support within its own causal interval?

3.1 The Margolus–Levitin Bound

The Margolus–Levitin theorem states that a quantum system of average energy E can undergo distinguishable transitions — transitions to orthogonal states — at a maximum rate

$$\Gamma_{max} \leq 2E / (\pi\hbar)$$

At the scaling level used throughout this paper, order-unity numerical factors including $2/\pi$ are suppressed, giving $\Gamma_{max} \sim E/\hbar$. The physical content is that energy limits the speed of distinguishable quantum change: more energy means faster possible transitions, with $\hbar$ setting the conversion factor.

A note on the domain of application: the ML theorem is stated for a quantum system transitioning from a pure state to an orthogonal state, not for a macroscopic region with many degrees of freedom. The application here treats the effective single irreversible record being produced as the relevant quantum transition, with the cell's total energy $E \sim \rho L^3$ setting the bound. The energy-time uncertainty argument $E \cdot t \gtrsim \hbar$ — requiring a distinguishable transition to take at least a time $\hbar/E$ — captures the same physics without this subtlety and leads to the same threshold.

3.2 Update Capacity of a Causal Region

For a region of energy $E \sim \rho L^3$, the maximum number of distinguishable transitions within the causal crossing time $t \sim L/c$ is

$$N \sim \Gamma_{max} \cdot (L/c) \sim (E/\hbar) \cdot (L/c) \sim (\rho L^3/\hbar) \cdot (L/c) = \rho L^4/(\hbar c)$$

This is precisely $\chi(L)$. The threshold $N \sim 1$ yields

$$\rho L^4 \sim \hbar c$$

with the interpretation: a causal region of size L contains enough energy to support *one* distinguishable quantum update within one causal crossing time when $\chi(L) \sim 1$. Below this threshold, the energy is insufficient for even one distinguishable transition before causal fragmentation; above it, the region supports multiple updates per crossing time.

3.3 Irreversible Record Formation

The ML bound constrains the quantum transition rate. An irreversible record — a commitment in the VERSF sense — requires not merely a distinguishable transition but one accompanied by environmental entropy increase $\Delta S_{env} \geq k_B \ln 2$, so the quantum state is permanently distinguished. The thermodynamic irreversibility condition is a further requirement addressed by the Bekenstein derivation below; it does not alter the quartic scaling, since the entropy cost per record is a fixed energy overhead (of order $k_B T \ln 2$) subsumed in the order-unity prefactor.

The threshold condition from this derivation is therefore: a causal region becomes capable of supporting one irreversible record within its own causal horizon when $\chi(L) \sim 1$.

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4. Second Independent Derivation: Entropy Capacity

The second derivation proceeds from thermodynamics and asks: does the causal region have enough entropy capacity to accommodate the thermodynamic cost of a single irreversible record? This derivation uses neither energy-time uncertainty nor the ML bound, making it genuinely independent of Section 3 at the level of physical assumptions.

4.1 The Bekenstein Entropy Bound

The Bekenstein bound states that for any physical system of energy E confined to a region of radius R, the maximum thermodynamic entropy is

$S_{max} \leq 2\pi ER / (\hbar c)$

For a causal cell of size L with energy $E \sim \rho L^3$ and confinement radius $R \sim L$:

$S_{max}(L) \sim 2\pi \cdot \rho L^3 \cdot L / (\hbar c) = 2\pi\rho L^4 / (\hbar c)$

Dropping the order-unity factor 2π, consistent with working at scaling level throughout:

$S_{max}(L) \sim \rho L^4 / (\hbar c)$

4.2 The One-Bit Record Condition

An irreversible record requires the region to undergo a distinguishable transition accompanied by an environmental entropy increase of at least one bit:

$\Delta S \geq k_B \ln 2$

For this to be thermodynamically possible, the maximum available entropy must meet or exceed the record-formation requirement:

$S_{max}(L) \gtrsim k_B \ln 2$

Substituting and dropping $k_B \ln 2$ as an order-unity numerical factor:

$\rho L^4/(\hbar c) \gtrsim 1 \Longrightarrow \rho L^4 \gtrsim \hbar c$

This is again $\chi(L) \gtrsim 1$.

4.3 Independence from the ML Derivation

The Bekenstein derivation is independent of Section 3 in the following sense. Section 3 asks: how quickly can a quantum system with energy E evolve to an orthogonal state? Its central input is the ML inequality connecting energy to transition rate. Section 4 asks: does this region have enough thermodynamic entropy capacity to accommodate a one-bit record? Its central input is the Bekenstein inequality connecting energy and size to maximum entropy. The two inequalities are distinct physical results from distinct theoretical frameworks. That both yield $\chi \sim 1$ at the same threshold is non-trivial: it says the quantum speed limit and the thermodynamic capacity limit coincide at the same causal scale.

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5. The Dimensionless Causal Capacity Parameter

The three perspectives above — causal-diamond action, quantum throughput, and entropy capacity — converge on a single dimensionless parameter.

Definition. The *causal capacity parameter* of a region of size L at energy density ρ is

$$\chi(L) = \rho L^4 / (\hbar c)$$

This parameter simultaneously measures:

- the ratio of the cell's action budget to the quantum of action: $\chi = S_{cell} / \hbar$
- the maximum number of distinguishable quantum updates per causal crossing time: $\chi \sim N$
- the ratio of the cell's entropy capacity to the one-bit record cost: $\chi \sim S_{max} / (k_B \ln 2)$

The threshold $\chi \sim 1$ is therefore the condition at which a causal region is simultaneously *action-sufficient*, *quantum-update capable*, and *entropy-adequate* for one irreversible record per causal crossing time.

Proposition (causal capacity threshold). The unique length scale at which a causal region first satisfies all three conditions simultaneously is

$$\xi = (\hbar c / \rho)^{1/4}$$

For $L < \xi$, $\chi < 1$: the region is sub-threshold in all three measures. For $L > \xi$, $\chi > 1$: the region is super-threshold — it is a composite of multiple ξ-cells, each capable of independent record formation. At $L = \xi$, $\chi = 1$: the region is minimally capable, with precisely one unit of action, one possible update, and one bit of entropy capacity per causal crossing time.

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6. Why the Quartic and Not Some Other Power?

The appearance of L^4 rather than L^2, L^3, or L^5 is not arbitrary. It is forced by the combination of three structural features that any causal-region capacity measure must respect.

Spatial volume. The total energy within a region of linear size L scales as $E \sim \rho L^3$. Three powers of L come from the volume.

Causal time. A causally self-contained region of size L has a natural internal timescale $t \sim L/c$ — the light-crossing time. This is the maximum duration over which the region can act as a unified physical system before causal fragmentation separates its parts.

Action = energy × time. Quantum mechanics is governed by action, not energy alone. The action available within the causal region is $E \cdot t \sim \rho L^3 \cdot (L/c) = \rho L^4/c$. The fourth power of L is the product of the three spatial dimensions (giving volume and hence energy) with the one temporal dimension (the causal crossing time).

No other combination of ρ and L produces the causal action of the region. ρL^3 is the energy, but says nothing about what can happen dynamically. ρL^5 would require an additional factor of L with no causal justification. ρL^4 is exactly $E \cdot t$ for a causally bounded region — no more and no less.

The deeper point is this: any bounded physical process requires both spatial containment and causal duration. Spatial containment converts energy density to total energy (ρL^3). Causal containment imposes a finite dynamical interval (L/c). Their product gives the action available to the region, $\rho L^4/c$. The quartic therefore arises whenever the relevant limit concerns *process capacity* rather than static storage. This is why ρL^4 appears across quantum mechanics, thermodynamics, and causal geometry simultaneously — all three are asking the same question ("what can this region *do*?"), and the answer always involves the same causal action budget. The appearance is not a coincidence to be explained after the fact; it is the inevitable consequence of applying any dynamical limit to a causally bounded region.

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7. Application: The VERSF Coherence Scale

The dimensionless parameter $\chi(L) = \rho L^4/(\hbar c)$ applies to any energy density and any length scale. The VERSF coherence scale arises from a specific and physically motivated choice of ρ. Crucially, the VERSF framework does not introduce the quartic capacity measure — $\chi(L)$ is established on independent physical grounds in Sections 2–4. What VERSF contributes is a single physical identification: that the energy density governing spacetime substrate dynamics is the cosmological vacuum energy density ρ_Λ. Once that identification is made, the scale at which $\chi = 1$ is determined entirely by the measured value of ρ_Λ and the constants $\hbar$ and c. The logic chain is therefore: physics produces the invariant χ; the threshold $\chi = 1$ defines minimal causal process capacity; cosmology supplies the relevant energy density; the length scale follows.

Within the VERSF framework, spacetime is modelled as a substrate whose behaviour is regulated by the cosmological vacuum energy density ρ_Λ. The coherence scale ξ is defined as the length at which $\chi(L) = 1$ for $\rho = \rho_\Lambda$:

$$\xi = (\hbar c / \rho_\Lambda)^{(1/4)}$$

From the measured value $\rho_\Lambda \approx 6.9 \times 10^{-10}$ J m^{-3}:

$$\xi \approx 8.2 \times 10^{-5} \text{ m}$$

The physical interpretation of this scale is now completely transparent from the analysis above. The VERSF coherence length is not a free parameter, nor a coincidence. It is the length at which the cosmological vacuum energy density — the background energy of empty space — is exactly sufficient to support one irreversible physical record per causal crossing time, as measured simultaneously by:

- the action budget of the causal diamond (Section 2)
- the ML quantum throughput limit (Section 3)
- the Bekenstein entropy capacity (Section 4)

All three measures agree because all three are expressions of the same underlying causal capacity parameter χ.

The result has a clean interpretive consequence. The cosmological constant ρ_Λ is the energy density at which the minimum-capacity causal commitment cell is approximately 10^{-4} m in size — a mesoscopic length scale squarely within the laboratory domain. This means that the measured value of the cosmological constant encodes a direct relationship between cosmological vacuum structure and the scale of irreversible record formation in spacetime, a connection that would be invisible without the causal capacity framing developed here.

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8. Conclusion

The quartic combination ρL^4 appears persistently across quantum mechanics, thermodynamics, and causal spacetime geometry because it is the fundamental measure of the physical capacity of a bounded causal region. It is the action budget of a causal diamond, the maximum quantum update count per crossing time, and the entropy capacity available for record formation — all simultaneously, and all expressed by the same dimensionless parameter $\chi(L) = \rho L^4/(\hbar c)$.

The two derivations presented here are independent at the level of physical assumptions: the Margolus–Levitin throughput argument invokes quantum orthogonality and the energy-time uncertainty principle; the Bekenstein entropy argument invokes only a thermodynamic capacity bound and a one-bit record condition. That both yield $\chi \sim 1$ as the threshold for irreversible record formation is non-trivial, and their convergence provides structural confirmation that the quartic is not an artefact of any single theoretical framework.

The threshold $\chi \sim 1$ defines a universal characteristic length for any energy density ρ: the minimal causal region in which one irreversible record can be produced within one causal crossing time. Applied to the cosmological vacuum energy density ρ_Λ, this length is $\xi \approx 8.2 \times 10^{-5}$ m — the VERSF spacetime coherence scale. The present paper shows that this scale is not a prediction of VERSF alone. It is the length at which the universal causal capacity measure of spacetime reaches its quantum threshold.

The broader implication is that ρL^4 represents a fundamental organisational invariant of physical law: the quantity that determines where causal structure, quantum mechanics, and thermodynamics simultaneously become capable of producing classical facts. Physical scales that appear "coincidentally" positioned may, on closer examination, correspond to the condition $\chi \sim 1$ evaluated at the relevant epoch energy density — suggesting that the quartic capacity measure is a unifying thread in the architecture of physical law across energy scales.

Appendix: Comparison of the Two Independent Derivations

Feature	Section 3: ML Throughput	Section 4: Bekenstein Entropy
Physical input	Margolus–Levitin bound on transition rate	Bekenstein bound on entropy capacity
Central inequality	$\Gamma_{max} \leq E/\hbar$	$S_{max} \leq 2\pi ER/(\hbar c)$
What it counts	Distinguishable quantum transitions per unit time	Entropy bits available for record formation
Record condition	At least one orthogonal transition per crossing time	$S_{max} \geq k_B \ln 2$
Result	$N \sim \rho L^4/(\hbar c) \gtrsim 1$	$S_{max} \sim \rho L^4/(\hbar c) \gtrsim k_B \ln 2$
Threshold	$\chi \sim 1$	$\chi \sim 1$
Independent of other derivation?	Yes — uses energy-time uncertainty; no entropy bound invoked	Yes — uses Bekenstein bound; no ML theorem invoked

The causal-diamond action framework (Section 2) provides the geometric grounding shared by both derivations: $\rho L^4/c$ is the action contained within a causal diamond of size L, making $\chi(L) = \rho L^4/(\hbar c)$ the ratio of the cell's action budget to the quantum of action. This is not a third independent derivation of the threshold but the structural explanation for why both derivations produce the same quartic combination.