

The Spacetime Coherence Length: Vacuum-Energy Crossover, Information Throughput, and Scale Hierarchy in the VERSF Framework

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For the General Reader

Physics has identified a handful of fundamental length scales — distances at which the rules governing nature change dramatically. The smallest is the Planck length ($\sim 10^{-35}$ m), where gravity and quantum mechanics are expected to merge. A little larger are the scales of particle physics ($\sim 10^{-18}$ m), where the fundamental constituents of matter interact. At the other extreme lies the cosmic horizon ($\sim 10^{26}$ m), the boundary of the observable universe. Between particle physics and the cosmos, classical physics — Einstein's spacetime — is assumed to hold without interruption across thirty-plus orders of magnitude, with no structural threshold in between.

This paper examines a new candidate scale that emerges from the Void Energy–Regulated Space Framework (VERSF): a *spacetime coherence length* of roughly 10^{-4} m, about the width of a human hair. This scale is not arbitrary — it is fixed by the measured energy density of the vacuum of space, and follows from two independent physical arguments that reach the same answer: one based on the energy a region of space must contain to sustain quantum fluctuations, and one based on how quickly a region can "make a decision" — stabilise an irreversible physical fact — before information escapes its causal horizon. Within VERSF, this scale represents the minimal spacetime region capable of stabilising an irreversible physical record within its own causal horizon.

What makes this scale particularly striking is where it sits. Place the six fundamental scales — Planck length, particle physics, atomic scale, this new coherence length, planetary distances, and cosmic horizon — on a logarithmic ruler (where each tick mark represents a factor of ten), and they are spaced almost evenly, like rungs on a ladder. The Earth–Sun distance turns out to be almost exactly the geometric average of the coherence scale and the cosmic horizon — the sharpest of the regularities found. The particle physics scale lies near the geometric average of the Planck length and the coherence scale, a weaker but suggestive echo of the same pattern. The coherence scale sits near the centre of the entire sixty-two-decade span of known physics.

Whether these regularities are deep physical structure or coincidence is the central question this paper addresses. We present the evidence carefully and honestly, assess its precision, identify its weaknesses, and describe what experimental tests could distinguish the two possibilities.

Abstract

The Void Energy–Regulated Space Framework (VERSF) predicts the existence of a spacetime coherence scale $\xi \sim 10^{-4}$ m. We propose that ξ emerges as the unique length at which a region contains enough vacuum energy to complete one irreversible update within its own causal crossing time. We identify this scale via two independent physical derivations, reinforced by two complementary causal-cell reformulations that recover the same quartic threshold. The primary derivations are: first, a coherence domain of linear size L supports a minimum-wavelength mode with zero-point energy $\sim \hbar c/L$, implying a substrate energy density $\rho_{\text{sub}}(L) \sim \hbar c/L^4$ — structurally consistent with Casimir vacuum-mode scaling — giving a crossover at $\rho_{\text{sub}}(\xi) = \rho_{\Lambda}$; and second, by the Margolus-Levitin throughput bound, a cell of size L can complete one irreversible commitment within its causal crossing time only if $\rho L^4 \gtrsim \hbar c$ — independently reproducing the quartic structure. These are reinforced by: the action budget of a causal spacetime cell, $S_{\text{avail}} \sim \rho L^4/c \gtrsim \hbar$, which recovers the same threshold and admits a natural causal-diamond interpretation; and the Bekenstein entropy bound applied to the cell, $S_{\text{max}} \sim \rho L^4/(\hbar c) \gtrsim k_B \ln 2$, which gives the quartic from entropy capacity alone. A Minimal Commitment Cell Lemma shows that all four conditions reduce to the same dimensionless threshold $\chi(L) = \rho L^4/\hbar c \gtrsim 1$, making ξ the unique scale where energetic self-consistency, causal completion, minimum action budget, and record-formation entropy capacity are simultaneously satisfied — equivalently, where a causal cell first becomes capable of supporting order-unity irreversible record production within one crossing time. When ξ is placed within the full hierarchy of fundamental length scales, two geometric-mean relationships emerge of notably different strength: the Earth–Sun distance arises approximately as the geometric mean of ξ and the cosmological horizon (factor ~ 2 ; the strongest numerical coincidence but the weakest independent evidence, due to anthropic selection effects), while the electroweak scale arises approximately as the geometric mean of the Planck length and ξ (factor ~ 30 ; a weaker numerical echo not subject to the same anthropic concern). The resulting hierarchy echoes Dirac's large-number coincidences and suggests a possible approximate discrete scale symmetry in the architecture of physical law.

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1. Introduction

Modern physics contains several fundamental length scales that appear disconnected from one another. The Planck length,

$$L_P = \sqrt{(\hbar G/c^3)} \approx 1.616 \times 10^{-35} \text{ m},$$

marks the threshold below which quantum gravitational effects become non-negligible. The electroweak scale,

$$L_{EW} \sim \hbar c / (m_{EW} c^2) \approx 10^{-18} \text{ m},$$

governs particle physics interactions. The cosmological horizon,

$$R_H \sim 8.8 \times 10^{26} \text{ m},$$

bounds the observable universe. Between these extremes lie atomic, molecular, and astrophysical scales — each the domain of distinct physical laws.

No currently accepted framework derives these separations from common first principles. The hierarchy problem — why the electroweak scale is so much larger than the Planck scale — remains among the deepest open questions in fundamental physics.

Within the VERSF framework, an intermediate scale emerges naturally from the requirement that the spacetime substrate be consistent with observed vacuum energy. This substrate coherence length ξ is not a free parameter but is fixed by \hbar , c , and the measured cosmological vacuum energy density. Its deepest interpretation is as the unique length at which a region contains enough vacuum energy to complete one irreversible update within its own causal crossing time. We call such a region a *minimal commitment cell*: the smallest spacetime region within which one irreversible record can be completed before causal separation prevents the region from acting as a single commitment-supporting unit. When placed within the full hierarchy of physical scales, ξ reveals geometric relationships that suggest a possible logarithmic organisation of nature's fundamental length scales.

2. Derivation of the Substrate Coherence Scale

Two independent physical derivations within VERSF converge on the same quartic formula for ξ , reinforced by two complementary causal-cell reformulations. The first primary derivation proceeds from the mode structure of coherence domains and vacuum energy balance (Sections 2.1–2.3). The second primary derivation proceeds from the information-theoretic requirements of irreversible record formation and the Margolus-Levitin throughput bound (Section 2.4). These are reinforced by an action-budget reformulation (Section 2.5) and a Bekenstein entropy-capacity reformulation (Section 2.6), both of which recover the same quartic threshold from different starting points. A complementary stability argument shows why a finite preferred scale should

generically exist even without pinning its numerical value (Section 2.7). Section 2.9 then presents a Minimal Commitment Cell Lemma that unifies all four perspectives.

2.1 Energy Density of a Coherence Domain

Consider a coherence-supporting substrate domain of characteristic linear size L . Within VERSF, such a domain supports reversible substrate disturbance modes — the same modes whose overlap integral

$$C(t) = \int \Phi_L \Phi_R^* d^3x$$

governs interference visibility in the measurement framework. The longest-wavelength mode the domain can support has wavelength of order L . At leading order, substrate disturbance modes propagate at the emergent invariant speed c , so the minimum mode energy scales as $E \sim \hbar\omega \sim \hbar c/L$:

$$E(L) \sim \hbar c/L.$$

Distributed over the coherence volume $V(L) \sim L^3$, this yields an effective substrate energy density

$$\rho_{\text{sub}}(L) \sim E(L)/V(L) \sim \hbar c/L^4.$$

Three physical commitments underlie this scaling: (i) the domain supports a minimum irreducible mode of wavelength $\sim L$; (ii) that mode carries zero-point energy $\sim \hbar c/L$; (iii) the energy is distributed over the coherence volume L^3 . The L^{-4} law follows from these properties and is not assumed from dimensional analysis. Note that this scaling is structurally consistent with the vacuum-mode energy density in Casimir systems, where confined vacuum modes between parallel plates also yield $\rho \sim \hbar c/L^4$. The analogy lends physical plausibility to the domain mode picture — the L^{-4} scaling is independently established in a laboratory context — but the physical origins differ: Casimir systems involve boundary-modified zero-point spectra with geometry- and material-dependent coefficients, whereas the coherence domain here has a single minimum-wavelength mode set by its own size. The scaling is shared; the derivation is not.

2.2 The Coherence Length as a Vacuum-Energy Crossover

The VERSF coherence length is defined as the domain size at which the intrinsic substrate fluctuation energy density equals the observed cosmological vacuum energy density:

$$\rho_{\text{sub}}(\xi) = \rho_{\Lambda}.$$

This condition has a direct physical meaning. For $L < \xi$, the substrate energy density exceeds the cosmological background ($\rho_{\text{sub}} \gg \rho_{\Lambda}$): the domain is self-sustaining, with intrinsic fluctuation energy dominating the local vacuum. For $L > \xi$, the domain energy density falls below the background ($\rho_{\text{sub}} \ll \rho_{\Lambda}$): the cosmological vacuum dominates and substrate

coherence cannot be self-maintained. The scale ξ is therefore the crossover boundary between self-sustaining quantum-coherent domains and classically passive spacetime.

Substituting the L^{-4} scaling into the crossover condition:

$$\hbar c / \xi^4 \sim \rho_{\Lambda} \quad \Rightarrow \quad \xi \sim (\hbar c / \rho_{\Lambda})^{1/4}.$$

It is useful to define the dimensionless ratio

$$\chi(L) = \rho L^4 / (\hbar c),$$

which measures the degree to which a domain of size L can support its own vacuum fluctuations against the cosmological background. Then:

$$\begin{aligned} \chi(L) < 1 &\rightarrow \text{domain energy insufficient; vacuum-dominated,} \\ \chi(L) > 1 &\rightarrow \text{domain self-sustaining; coherence supported,} \\ \chi(L) = 1 &\rightarrow \text{crossover; defines the coherence scale } \xi. \end{aligned}$$

This dimensionless form makes the physics transparent: ξ is the unique scale where $\chi = 1$.

2.3 Numerical Result

From observational cosmology,

$$\rho_{\Lambda} = \Lambda c^2 / (8\pi G) \approx 6.9 \times 10^{-10} \text{ J m}^{-3}.$$

Substituting:

$$\begin{aligned} \xi &= ((1.055 \times 10^{-34}) (3.00 \times 10^8) / (6.9 \times 10^{-10}))^{1/4} \\ &\approx (4.59 \times 10^{-17})^{1/4} \\ &\approx 8.2 \times 10^{-5} \text{ m.} \end{aligned}$$

The scale is fixed entirely by \hbar , c , and ρ_{Λ} — no free VERSF parameters enter.

2.4 Second Route: Commitment Throughput and the Margolus-Levitin Bound

A coherence cell of linear size L has available substrate energy

$$E^*(L) \sim \rho^* L^3,$$

where ρ^* is the relevant substrate energy density. The threshold condition for commitment can be stated directly in terms of the energy-time uncertainty bound. A distinguishable quantum transition — the minimum physical act underlying an irreversible record — requires

$$E \cdot t \gtrsim \hbar.$$

Strictly, the Margolus-Levitin theorem gives $t \geq \pi\hbar/2E$; at scaling level this reduces to $E \cdot t \sim \hbar$, with order-unity factors suppressed consistently throughout.

Setting $E = \rho^* L^3$ (the cell's energy content) and $t = L/c$ (the causal crossing time):

$$\rho^* L^3 \cdot (L/c) \gtrsim \hbar \quad \Rightarrow \quad \rho^* L^4 \gtrsim \hbar c,$$

which is exactly $\chi(L) \gtrsim 1$. This form makes the derivation manifestly model-independent at the scaling level: it rests on (i) the energy content of the cell and (ii) the time available before causal fragmentation, connected by the Heisenberg bound. No specific substrate model is required.

The Margolus-Levitin theorem sharpens this. A record requires a *distinguishable* quantum transition: the post-record state must be orthogonal to the pre-record state. ML states that the maximum rate at which a quantum system of average energy E^* can execute such distinguishable transitions is

$$\Gamma_{\max}(L) \leq 2E^*(L) / (\pi\hbar) \sim \rho^* L^3 / \hbar.$$

The ML theorem is stated for a quantum system transitioning from a pure state to an orthogonal state. A substrate cell of size L has many degrees of freedom, not a single two-level quantum. The application here is therefore by analogy: we treat the effective single irreversible record being laid down as the relevant quantum transition, with the cell's total energy $E^* = \rho L^3$ setting the bound. The energy-time uncertainty argument above ($E \cdot t \sim \hbar$) captures the same physics without this subtlety; ML is cited as a sharper version of the same bound that connects directly to orthogonality and record distinguishability.

Dropping order-unity factors (including $2/\pi$) — consistent with working at scaling level throughout — and requiring at least one such transition within the causal crossing time L/c :

$$\Gamma_{\max}(L) \cdot (L/c) \gtrsim 1.$$

This is equivalent to $E \cdot t \gtrsim \hbar$ at scaling level, and recovers $\chi(L) \gtrsim 1$ identically. The logical chain is: quantum information (a distinguishable transition is required) \rightarrow causality (it must complete within L/c) \rightarrow record formation (the transition constitutes an irreversible *commitment*, the VERSF term for an event in which the environment accrues $\Delta S_{\text{env}} \geq k_B \ln 2$ and the quantum state is irreversibly distinguished). The ML bound constrains the minimum time for the distinguishable-transition component of that process; the thermodynamic irreversibility — the environmental entropy increase — is a further condition imposed by the VERSF measurement framework and is not itself derived from ML. The entropy condition does not change the quartic scaling of the threshold: it amounts to a fixed energy cost per record (of order $k_B T \ln 2$, which at the substrate level is subsumed in the order-unity prefactor) and does not introduce any additional L -dependence. The coherence scale ξ is therefore set by the quartic alone. A commitment-supporting cell must be large enough to complete at least one such update before causal separation fragments it into dynamically disconnected subregions.

Substituting:

$$\rho^* L^4 / (\hbar c) \gtrsim 1 \quad \Rightarrow \quad L \gtrsim (\hbar c / \rho^*)^{1/4}.$$

In terms of the dimensionless ratio introduced above, this is simply $\chi(L) \gtrsim 1$ — the same threshold condition reached by route 1.

The minimal commitment-capable cell — the smallest spatial region that can complete one irreversible record within its own causal horizon — therefore has linear size

$$L_{\text{coh}} \sim (\hbar c / \rho^*)^{1/4}.$$

Identifying ρ^* with ρ_{Λ} recovers

$$\xi \sim (\hbar c / \rho_{\Lambda})^{1/4} \sim 10^{-4} \text{ m}.$$

This derivation is independent of the zero-point mode argument: the quartic structure $\rho L^4 \sim \hbar c$ emerges here from the throughput condition alone, which is why convergence on the same formula is non-trivial.

A clean interpretation of the threshold $\chi(\xi) = 1$ follows directly. Rearranging $\rho \sim \hbar c / L^4$ and using $E = \rho L^3$, $t = L/c$:

$$\text{update density} \sim (1/L^3) \cdot (1/(L/c)) = c/L^4.$$

At $\chi = 1$, $\rho = \hbar c / L^4$, so $\rho / \hbar = c / L^4$ — exactly the update density. In other words: at the coherence threshold, the vacuum energy density corresponds to the threshold at which a causal cell becomes capable of sustaining order-unity distinguishable quantum updates within a single crossing time. The cosmological constant is not an arbitrary energy density; within VERSF it is the energy density at which spacetime reaches this minimum throughput for irreversible record formation. This ties ρ_{Λ} directly to the information throughput of spacetime rather than to any other physical scale.

The relaxation timescale appearing in the VERSF measurement framework,

$$\tau_s = \xi / c \approx 2.8 \times 10^{-13} \text{ s},$$

is therefore not an inserted parameter but the causal crossing time of the minimal commitment cell.

2.5 Third Route: Action-Budget Derivation

A third independent route to the quartic scaling follows from the minimum action required to complete one irreversible distinguishable update within a bounded causal cell. The argument uses only the causal structure of the cell and the quantum of action \hbar — no substrate model is required.

Consider a region of characteristic linear size L . Its causal crossing time is $t_c \sim L/c$, so the corresponding minimal spacetime cell has four-volume

$$V_4 \sim L^3 \cdot (L/c) = L^4/c.$$

If the local energy density is ρ , the total action available in that cell is

$$S_{\text{avail}} \sim \rho \cdot V_4 \sim \rho L^4/c.$$

A necessary condition for the cell to support one irreversible distinguishable update is that this available action meet or exceed the quantum of action:

$$S_{\text{avail}} \gtrsim \hbar \quad \Rightarrow \quad \rho L^4/c \gtrsim \hbar \quad \Rightarrow \quad \rho L^4 \gtrsim \hbar c.$$

This is again $\chi(L) \gtrsim 1$. The threshold coherence scale is therefore

$$L_{\text{coh}} \sim (\hbar c/\rho)^{1/4},$$

reproducing the same quartic scaling obtained from the zero-point mode energy density (route 1) and the Margolus-Levitin throughput argument (route 2). The common structure across all three routes is now explicit: ξ is the size at which a causal spacetime cell first contains enough action to complete one irreversible update.

The action-budget argument also clarifies why a purely holographic area-capacity argument gives different scaling. A static holographic count of degrees of freedom scales as $\sim \xi^2/L_P^2$ — an area law — and does not on its own produce the quartic law. The quartic arises specifically when the relevant primitive is not static storage capacity but the action budget $\rho L^4/c$ of a dynamical causal cell. Record formation is a process requiring action, not merely a state requiring area.

This action-budget route admits a natural causal-diamond interpretation. A causal region of linear size L has four-volume $V_4 \sim L^4/c$, so the total action contained in the cell is $S_{\text{cell}} \sim \rho L^4/c$. Requiring $S_{\text{cell}} \gtrsim \hbar$ gives the same quartic threshold $\rho L^4 \gtrsim \hbar c$. The significance of this perspective is not that it introduces a new numerical result, but that it shows ρL^4 is the natural action measure of a bounded causal spacetime region — making $\chi(L) = \rho L^4/(\hbar c)$ the ratio of the cell's action to the quantum of action, rather than an ad hoc dimensional grouping.

2.6 Fourth Route: Bekenstein Entropy Bound

A fourth independent route uses the Bekenstein entropy bound and the record-formation condition, without invoking the energy-time uncertainty principle or the ML theorem. The starting principles are therefore genuinely distinct from routes 2 and 3.

The Bekenstein bound states that for any physical system of energy E confined to a region of size R , the maximum thermodynamic entropy is

$$S_{\text{max}} \leq 2\pi ER/(\hbar c).$$

For a substrate cell of size L with energy $E \sim \rho L^3$ and confinement radius $R \sim L$:

$$S_{\max}(L) \leq 2\pi \cdot \rho L^3 \cdot L / (\hbar c) = 2\pi\rho L^4 / (\hbar c).$$

Dropping the order-unity factor 2π — consistent with working at scaling level throughout — this gives

$$S_{\max}(L) \sim \rho L^4 / (\hbar c).$$

A record — an irreversible commitment in the VERSF sense — requires the cell to undergo a distinguishable transition accompanied by an entropy increase of at least one bit:

$$\Delta S \geq k_B \ln 2.$$

For this to be physically possible, the maximum available entropy must meet or exceed the minimum required by record formation:

$$S_{\max}(L) \gtrsim k_B \ln 2 \quad \Rightarrow \quad \rho L^4 / (\hbar c) \gtrsim k_B \ln 2.$$

At scaling level (dropping $k_B \ln 2$ as an order-unity factor):

$$\rho L^4 \gtrsim \hbar c,$$

recovering $\chi(L) \gtrsim 1$ for the fourth time. The Bekenstein route is independent of routes 2 and 3 because it uses neither energy-time uncertainty nor the ML bound — it uses only the Bekenstein entropy capacity of the cell and the single-bit record-formation requirement. The quartic structure $\rho L^4 \sim \hbar c$ appears because the Bekenstein bound is itself linear in both energy ($\propto \rho L^3$) and size ($\propto L$).

A note on the holographic area bound. One might instead try to use $S_{\text{holographic}} \sim L^2/L_P^2$ — the area-law entropy capacity — as the starting point. This would not produce the quartic. Setting $L^2/L_P^2 \gtrsim 1$ gives only $L \gtrsim L_P$, which is trivially satisfied at all macroscopic scales and carries no information about ξ . The quartic emerges from the Bekenstein bound specifically because it contains the energy density ρ ; the holographic area bound does not. This is why the action-budget argument (route 3) explicitly noted that "the quartic arises when the relevant primitive is not static storage capacity but the action budget of a dynamical causal cell" — the Bekenstein route makes the same point from the entropy side.

2.7 Stability Selection (Supporting Context)

The two primary derivations and two complementary reformulations above carry the physics of this paper. This section provides supporting context only: it shows that once small- L and large- L penalties coexist in any substrate dynamics, a finite preferred scale is generically expected. This argument does not determine ξ independently — it cannot, since A , B , m , n are not yet derived from the substrate action. Its sole purpose is to confirm that the existence of a preferred scale is structurally expected rather than coincidental.

Consider the effective free energy of a substrate coherence domain of size L :

$$F(L) = A/L^m + BL^n, \quad A, B, m, n > 0.$$

The first term penalises small domains: a cell below some threshold size lacks sufficient degrees of freedom and entropy-handling capacity to record even one irreversible bit. The second penalises large domains: maintaining phase coherence across an overly large region incurs growing coordination costs — internal mode dispersion, inhomogeneous fluctuations, and phase fragmentation all increase with L . The competition between these penalties selects a unique minimising scale,

$$\xi = (mA/nB)^{1/(m+n)},$$

which is finite and positive for any $m, n > 0$. The existence of a preferred domain size is therefore a structural consequence of opposing penalties, not a coincidence requiring fine-tuning.

The detailed exponents m and n — and the coefficients A and B — remain to be derived from the VERSF substrate action. This argument is therefore schematic. It supports the conclusions of routes 1 and 2 by showing that the dynamical structure they motivate generically selects a finite scale; it does not stand alongside them as an independent derivation.

2.8 Uniqueness and the Composite-Cell Argument

The commitment-throughput bound establishes that ξ is a lower limit. What prevents taking $L \gg \xi$ as the fundamental scale?

For $L \gg \xi$, $\chi(L) \gg 1$ is comfortably satisfied, but such a domain is not fundamental — it is a composite of many minimal commitment cells, each of size ξ . The large domain can be decomposed into $\sim (L/\xi)^3$ independent ξ -cells, each capable of stabilising its own record. A larger domain may still exhibit coherent macroscopic behaviour as an effective structure, but its irreducible commitment-supporting substructure is built from ξ -cells. Therefore ξ is the fundamental scale of irreversible fact production even when larger coherence patterns exist.

For $L \ll \xi$, $\chi(L) \ll 1$. Such domains cannot complete one irreversible commitment within their causal horizon — they are sub-threshold and cannot function as record-stabilising units.

The uniqueness statement is therefore:

- $L < \xi$: $\chi < 1$; dynamically insufficient for irreversible record formation,
- $L > \xi$: $\chi > 1$; composite aggregates of ξ -cells, not fundamental,
- $L = \xi$: $\chi = 1$; the minimal stable commitment-supporting coherence domain.

Within VERSF, ξ represents the minimal spacetime region capable of stabilising an irreversible physical record within its own causal horizon.

2.9 Minimal Commitment Cell Lemma

The two primary derivations and two complementary reformulations above can be unified into a single statement.

Lemma. A region of the VERSF substrate that functions as an elementary unit of irreversible record formation must satisfy the following threshold condition, which can be derived from several independent physical considerations:

Energetic self-consistency: $\rho_{\text{sub}}(L) \gtrsim \rho_{\Lambda}$,

Causal completion: $\Gamma_{\text{max}}(L) \cdot L/c \gtrsim 1$.

Substituting $\rho_{\text{sub}}(L) \sim \hbar c/L^4$ and $\Gamma_{\text{max}}(L) \sim \rho L^3/\hbar$, both conditions reduce identically to

$$\chi(L) = \rho L^4 / (\hbar c) \gtrsim 1.$$

The action-budget route (Section 2.5) arrives at the same threshold from a third starting point: the causal four-volume action $S_{\text{avail}} \sim \rho L^4/c \gtrsim \hbar$ gives $\rho L^4 \gtrsim \hbar c$ directly. The Bekenstein route (Section 2.6) arrives from a fourth direction: $S_{\text{max}}(L) = \rho L^4/(\hbar c) \gtrsim k_B \ln 2$ (one record) gives $\rho L^4 \gtrsim \hbar c$ using neither energy-time uncertainty nor ML. The unique scale at which this threshold is exactly met is

$$\xi = (\hbar c / \rho_{\Lambda})^{1/4}.$$

Corollary. The convergence of both primary derivations and both complementary reformulations on the same quartic condition is not a numerical coincidence. The two primary derivations — energetic self-consistency (route 1) and quantum throughput against a causal deadline (route 2) — are independent at the level of physical commitments: one proceeds from vacuum mode energy, the other from distinguishable-transition dynamics. The two complementary reformulations — available action per causal cell (route 3) and entropy capacity for record formation (route 4) — recover the same threshold from the perspectives of the cell's action budget and its entropy capacity respectively, neither introducing new physics but both clarifying what $\chi(L) \gtrsim 1$ means geometrically and informationally. All four reduce to $\chi(L) = \rho L^4/(\hbar c) \gtrsim 1$. The dimensionless ratio $\chi(L)$ is the order parameter for commitment-capable coherence.

The physical starting points of routes 2, 3, and 4 are distinct at the level of what each characterises. Route 2 specifies what type of process is required: a distinguishable quantum transition, with the ML bound connecting orthogonality to energy and time. Route 3 specifies the minimum action budget of the spacetime cell in which any such process must occur, without invoking the nature of the process itself. Route 4 specifies the minimum entropy capacity required for the outcome of that process to constitute a record, using the Bekenstein bound and the single-bit record condition without invoking dynamics at all. The three characterise the process, the cell, and the record respectively — and all three recover $\chi(L) \gtrsim 1$.

Proposition (era coherence ratio). If $L_i = (\hbar c / \rho_i)^{1/4}$ and $L_j = (\hbar c / \rho_j)^{1/4}$ are the coherence scales at any two epochs i and j , then

$$L_i / L_j = (\rho_j / \rho_i)^{1/4}.$$

Scale spacing in the hierarchy is therefore equivalent to energy-density spacing between epochs. A sequence of epochs whose energy densities form a geometric progression — ρ_1, ρ_2, ρ_3 with $\rho_1/\rho_2 = \rho_2/\rho_3$ — produces coherence scales L_1, L_2, L_3 with equal logarithmic spacing. The approximate logarithmic regularity of the physical scale hierarchy is, in this light, a statement about the ratios of successive epoch energy densities rather than a free-standing geometric pattern.

Status of the Coherence-Scale Claim

It is worth distinguishing three levels of claim made in this paper:

- **Derived at scaling level:** $\xi \sim (\hbar c/\rho_\Lambda)^{1/4}$, established by two independent physical derivations (vacuum-energy crossover and Margolus-Levitin commitment throughput) reinforced by two complementary causal-cell reformulations (action budget and Bekenstein entropy capacity), all converging on $\chi(\xi) = 1$.
- **Physically interpreted:** ξ as the size of a minimal commitment cell — the smallest spacetime region capable of completing one irreversible record within its own causal horizon.
- **Suggestive but unproven:** the geometric-mean hierarchy relations, which are retrospective numerical regularities rather than forward predictions of the framework.

This distinction matters for reading the paper: Sections 2–3 carry the load-bearing physics; Sections 4–11 develop suggestive large-scale pattern and its caveats.

Relation	Status	Comment
$\chi(\xi) = 1$	Defined threshold	Exact once ξ is defined
$\xi \sim (\hbar c/\rho_\Lambda)^{1/4}$	Derived at scaling level	Two primary derivations + two complementary reformulations
$L_i = (\hbar c/\rho_i)^{1/4}$	Algebraic scaling law	Epoch generalisation; consistency checks at L_P, L_{EW}
$\rho_P/\rho_\Lambda = (\xi/L_P)^4$	Algebraic identity	The CC problem exponent equals the fourth power of the length hierarchy
$L_{AU} \approx \sqrt{(\xi \cdot R_H)}$	Numerical regularity	Suggestive; horizon-sensitive (see §10.4)
$L_{EW} \approx \sqrt{(L_P \cdot \xi)}$	Weak numerical echo	Reinterpreted via era scaling
Order-unity prefactor	Not yet derived	Requires full substrate mode spectrum
$f(L/\xi)$ functional form	Not yet derived	Requires microscopic substrate action

3. Physical Significance of the Coherence Scale

The deepest interpretation of ξ is as the size of a minimal commitment cell: the smallest spacetime region within which one irreversible record can be completed before causal separation prevents the region from acting as a single commitment-supporting unit. This single statement ties together quantum information, causality, vacuum energy, and measurement theory. Its implications extend well beyond the numerical value of the scale itself.

Modern physics currently contains no established fundamental length scale between the electroweak regime and macroscopic classical behaviour. The Planck length $L_P \sim 10^{-35}$ m marks the domain where quantum gravity becomes significant; the electroweak scale $L_{EW} \sim 10^{-18}$ m characterises particle interactions. Classical spacetime geometry is ordinarily treated as valid at all larger scales without any intermediate structural threshold. The emergence of ξ introduces a new regime — one with both cosmological and laboratory-accessible implications.

3.1 A Bridge Between Cosmology and Laboratory Physics

The coherence length is fixed entirely by the cosmological vacuum energy density:

$$\xi = (\hbar c / \rho_{\Lambda})^{1/4}.$$

This implies that a cosmological parameter — the vacuum energy density governing the large-scale expansion of the universe — determines a length scale that falls squarely within the mesoscopic domain of laboratory physics. Such a connection is unusual: cosmological constants ordinarily influence phenomena only at astronomical scales. If ξ corresponds to a genuine physical property of the spacetime substrate, it would constitute a direct bridge between cosmology and microscopic dynamics, playing a role analogous to the Planck length but linking vacuum energy to mesoscopic processes rather than linking gravity to quantum mechanics.

3.2 The Quantum–Classical Transition

Within the VERSF measurement framework, the same scale appears as the substrate relaxation timescale $\tau_s = \xi/c \sim 10^{-13}$ s — the characteristic time over which substrate disturbances relax and irreversible records become stabilised. The recurrence of ξ in both the cosmological derivation and the measurement-theoretic context suggests it marks the substrate scale at which reversible quantum distinctions become capable of producing stable classical records. Within VERSF, ξ is interpreted not as a strict upper bound on quantum coherence length, but as the characteristic substrate scale above which stable record formation becomes generically supported and below which irreversible commitment becomes dynamically underpowered:

- below ξ , $\chi(L) < 1$: irreversible commitment is dynamically underpowered within a single causal cell,
- at and above ξ , $\chi(L) \geq 1$: stable record formation becomes generically supportable within a minimal commitment-capable domain.

The coherence scale thus provides a natural substrate boundary between quantum possibilities and classical facts — grounding the VERSF measurement framework in a cosmologically determined constant rather than an ad hoc parameter.

3.3 Spacetime as a Coherent Medium

A complementary interpretation is that ξ represents a correlation length of the spacetime substrate itself. In condensed-matter physics, coherence lengths arise when collective behaviour organises microscopic degrees of freedom into macroscopic phases — superconductors, superfluids, and Bose–Einstein condensates all exhibit characteristic coherence scales set by the underlying microphysics. If spacetime possesses an analogous coherence structure, ξ would correspond to the spatial region over which the substrate behaves as a unified dynamical system. Below this scale, substrate fluctuations remain coherent; above it, the medium can be treated as effectively classical spacetime. This picture is broadly consistent with emergent-spacetime approaches in which geometry arises from collective microscopic dynamics.

3.4 Experimental Outlook and a Falsifiable Prediction

If ξ represents a genuine physical property of spacetime, it may influence observable phenomena in the mesoscopic domain. Casimir experiments currently probe separations in the range 10^{-7} – 10^{-5} m — approaching ξ from below. This makes the Casimir regime the most immediately accessible experimental window.

In standard quantum field theory, the Casimir energy density between parallel plates scales as

$$E(L) \propto L^{-3}.$$

VERSF identifies $L \sim \xi$ as the natural regime in which departures from this standard vacuum-mode scaling would first become plausible, as $\chi(L)$ approaches 1 from below and the substrate transitions from self-sustaining to vacuum-dominated. A minimal phenomenological representation is

$$E(L) = E_{\text{QED}}(L) \cdot f(L/\xi),$$

where $f(L/\xi) \rightarrow 1$ for $L \ll \xi$ and departs from unity as $L \rightarrow \xi$.

On the sign of the expected deviation. While the precise functional form of f requires the microscopic substrate action (not yet available), a qualitative argument suggests *suppression* rather than enhancement of the Casimir force near ξ . For $L < \xi$, the cell is self-sustaining ($\chi > 1$): it supports its own coherent vacuum modes independently of any external boundary condition. As L approaches ξ from below, the substrate transitions toward the vacuum-dominated regime, where fewer independent substrate modes can be maintained against the cosmological vacuum. If the total vacuum fluctuation density is partially replaced by the cosmological background rather than supplemented by it, the available mode density for Casimir summation would decrease, producing a reduction in Casimir force relative to the QED prediction. A heuristic expectation is suppression, but this should be regarded as a heuristic expectation pending

derivation of the substrate action — not a prediction. The sign of $f - 1$ near $L = \xi$ is the first quantity that a full substrate-action calculation should determine.

Order-of-magnitude estimate. Without the substrate action, the scale of the correction cannot be derived. However, the natural dimensionless parameter is $\chi(L) = \rho \Lambda L^4 / (\hbar c)$, which reaches 1 at ξ and scales as $(L/\xi)^4$. At $L = \xi/2 \approx 41 \mu\text{m}$, $\chi \approx 0.06$ — suggesting that the fractional deviation $|f - 1|$ may be of order $\chi \sim$ a few percent at scales 40–80 μm , provided the departure is gradual. This is speculative but sets a scale for instrumental sensitivity required.

Experimental accessibility. Current precision Casimir measurements (torsion pendulum and AFM-based) typically operate at separations of 0.1–10 μm , well below $\xi = 82 \mu\text{m}$. Reaching ξ is technically non-trivial: at separations ~ 50 –80 μm , Casimir forces between typical geometries fall to the \sim fN range, requiring either larger plate areas or substantially improved force sensitivity beyond current state of the art. The ξ scale sits at the far-infrared / THz boundary ($f_\xi = c/\xi \approx 3.66$ THz), and THz spectroscopy of vacuum fluctuations may offer a complementary approach. The prediction is not immediately falsifiable with existing apparatus but provides a concrete instrumental target.

Additional experimental contexts include:

- **Decoherence measurements** in mesoscopic quantum systems, where the VERSF entropy threshold $S_{\text{crit}} = k_B \ln 2$ predicts a minimum decoherence time $\tau_s \approx 2.8 \times 10^{-13}$ s — a potentially falsifiable lower bound distinct from standard quantum mechanics.
- **Precision interferometry** probing environmental entropy accumulation near the record-formation threshold.

What the VERSF framework provides, which generic quantum mechanics does not, is a definite physical scale at which substrate coherence transitions from self-sustaining to vacuum-dominated — giving experimental programmes a concrete target rather than an unbounded parameter space. The prediction would be strengthened by: (1) deriving the substrate action and recovering $f(L/\xi)$ from first principles, and (2) identifying an experimental geometry capable of reaching the 50–100 μm separation range with sufficient force sensitivity.

4. The Physical Scale Hierarchy

We collect the fundamental physical length scales spanning known regimes.

Level	Scale	Approximate Value	Physical significance
Planck geometry	L_P	1.6×10^{-35} m	Quantum gravity threshold
Electroweak / particle	L_{EW}	$\sim 10^{-18}$ m	Particle physics scale
Atomic structure	L_{atom}	$\sim 10^{-10}$ m	Bohr radius
VERSF substrate	ξ	$\sim 10^{-4}$ m	Spacetime coherence length

Level	Scale	Approximate Value	Physical significance
Planetary orbits	L_AU	$\sim 1.5 \times 10^{11}$ m	Earth–Sun separation
Cosmic horizon	R_H	$\sim 8.8 \times 10^{26}$ m	Observable universe radius

In \log_{10} units these scales occupy approximately $-35, -18, -10, -4, +11, +27$. The gaps between consecutive levels span roughly 17, 8, 6, 15, and 16 orders of magnitude. The grouping is not perfectly uniform, but five of the six scales cluster into approximately equal logarithmic steps when the atomic scale is recognised as intermediate between the electroweak and coherence levels.

5. Geometric Mean Relationships

Once ξ is in hand, a natural question is whether it occupies any geometrically significant position within the known scale hierarchy. Two relationships emerge. They are not equal in strength, and that inequality runs in two directions simultaneously: the planetary relationship is the *strongest numerical coincidence* — holding to within a factor of 2 over a thirty-one-decade span — but simultaneously the *weakest evidential datum*, because anthropic selection effects (Section 10.3) substantially contaminate it as independent evidence. The electroweak relationship is a weaker numerical echo (factor ~ 30) but is not subject to the same anthropic concern. These two dimensions of strength should not be conflated. We present the planetary result first, with its caveats made explicit.

5.1 Planetary Orbital Distances

Consider the geometric mean of ξ and the cosmic horizon:

$$\begin{aligned} L_{\text{mid},1} &= \sqrt{(\xi \cdot R_H)} \\ &= \sqrt{(8.2 \times 10^{-5}) (8.8 \times 10^{26})} \\ &\approx 2.7 \times 10^{11} \text{ m.} \end{aligned}$$

The Earth–Sun distance $L_{\text{AU}} \approx 1.5 \times 10^{11}$ m lies within a factor of 2 of this value. In logarithmic terms:

$$\log_{10}(L_{\text{AU}}) = 11.2, \quad \log_{10}\sqrt{(\xi \cdot R_H)} = 11.5.$$

The AU sits $\sim 1\%$ below the exact logarithmic midpoint of $[\xi, R_H]$ — a span of thirty-one decades.

$\begin{aligned} L_{\text{AU}} &\approx \sqrt{(\xi \cdot R_H)} \\ &(\text{factor } \sim 2; \sim 1\% \text{ logarithmic offset}) \end{aligned}$
--

The relation can be inverted to read $\xi \approx L_{\text{AU}}^2/R_{\text{H}}$. Inserting numbers: $(1.5 \times 10^{11})^2 / (8.8 \times 10^{26}) \approx 2.6 \times 10^{-5}$ m, within a factor of 3.2 of ξ . This is algebraically identical to the geometric-mean form — no new information is added — but the rearrangement is structurally reminiscent of see-saw relations in particle physics ($mv \sim mD^2/M_R$), where a small scale is generated as the square of an intermediate scale divided by a large one. The analogy is suggestive and is noted here without further claim.

5.2 The Electroweak Scale

A second, weaker relationship appears when examining the geometric mean of the Planck length and ξ :

$$\begin{aligned} L_{\text{mid},2} &= \sqrt{L_{\text{P}} \cdot \xi} \\ &= \sqrt{(1.6 \times 10^{-35}) (8.2 \times 10^{-5})} \\ &\approx 3.6 \times 10^{-20} \text{ m.} \end{aligned}$$

The electroweak scale $L_{\text{EW}} \sim 10^{-18}$ m lies within a factor of ~ 30 of this value. In logarithmic terms:

$$\log_{10}(L_{\text{EW}}) = -18, \quad \log_{10}\sqrt{L_{\text{P}} \cdot \xi} = -19.5.$$

The electroweak scale sits $\sim 5\%$ above the exact logarithmic midpoint of $[L_{\text{P}}, \xi]$. This is suggestive but considerably looser than the AU relationship — a factor of 30 offset versus a factor of 2.

$L_{\text{EW}} \approx \sqrt{L_{\text{P}} \cdot \xi}$ <p>(factor ~ 30; $\sim 5\%$ logarithmic offset)</p>
--

The two relationships, with their respective precisions, are visualised below:

Log ₁₀ (L/m) :					
-35	-18	-10	-4	+11	+27
L _P	L _{EW}	L _{atom}	ξ	L _{AU}	R _H
	↑			↑	
	$\sqrt{L_{\text{P}} \cdot \xi} \approx -19.5$			$\sqrt{(\xi \cdot R_{\text{H}})} \approx +11.5$	
	[offset: 1.5 dec / 31 dec]			[offset: 0.3 dec / 31 dec]	
	[factor ~ 30 , weaker]			[factor ~ 2 , tight]	

6. Dirac Large-Number Coincidences and the Cosmological Constant Hierarchy

Dirac famously noted that several dimensionless ratios in physics cluster around powers of 10^{40} . For instance, the ratio of the electromagnetic to gravitational force between a proton and electron is approximately 10^{40} , as is the ratio of the cosmic age to the atomic unit of time.

The VERSF hierarchy produces a set of dimensionless ratios worth examining in this context.

Ratio	Approximate value
ξ / L_P	$\sim 5 \times 10^{30}$
R_H / ξ	$\sim 10^{31}$
R_H / L_P	$\sim 5 \times 10^{61}$
ρ_P / ρ_Λ	$\sim 7 \times 10^{122}$

The ratios $\xi/L_P \sim 10^{31}$ and $R_H/\xi \sim 10^{30}$ are approximately equal, suggesting that ξ lies near the logarithmic midpoint of the full Planck-to-horizon span. This approximate symmetry is consistent with ξ sitting close to the centre of the sixty-two-decade span, with the AU relationship providing the sharper observational test.

6.1 The Planck– Λ See-Saw

The most important large-number observation concerns the ratio of energy densities. By the definitions of $\rho_P = \hbar c/L_P^4$ and $\xi = (\hbar c/\rho_\Lambda)^{1/4}$:

$$\rho_P / \rho_\Lambda = (\hbar c/L_P^4) / (\hbar c/\xi^4) = (\xi/L_P)^4.$$

This is an algebraic identity — it follows immediately from the two definitions and contains no free parameters. Its content is that the notorious 10^{123} hierarchy between the Planck energy density and the observed vacuum energy density is not an independent number: it is exactly the fourth power of the $\sim 10^{31}$ length-scale hierarchy between ξ and L_P .

Numerically:

$$\rho_P / \rho_\Lambda \approx 6.7 \times 10^{122} \quad \text{and} \quad (\xi/L_P)^4 \approx 6.6 \times 10^{122} \quad [\text{factor } 1.01].$$

The cosmological constant problem — why is ρ_Λ so much smaller than ρ_P ? — is therefore equivalent, within VERSF, to the question: why is ξ so much larger than L_P ? These are the same question expressed in different variables. The quartic relationship

$$\rho_\Lambda = \rho_P (L_P/\xi)^4$$

is not a solution to the cosmological constant problem; it is a restatement of it in terms of the coherence scale. The value of this restatement is that it replaces the energy-density hierarchy (difficult to motivate) with a length-scale hierarchy (potentially derivable from substrate dynamics). Whether VERSF substrate dynamics can explain why $\xi \gg L_P$ is a well-posed question for future work.

6.2 The Time-Scale Hierarchy

The commitment timescale $\tau_s = \xi/c$ and the Planck time $t_P = L_P/c$ satisfy

$$\tau_s / t_P = \xi / L_P \sim 10^{31}.$$

This is algebraically trivial — dividing both by c leaves the same ratio as the lengths. Its physical content is that the smallest irreversible-record clock (τ_s) exceeds the smallest causal interval (t_P) by the same factor that ξ exceeds L_P . In a relativistic framework this is expected: space and time hierarchies are locked together by c . We note it here because it reinforces the interpretation of ξ as a causal scale rather than an arbitrary length.

6.3 The Entropy Midpoint

Using the Bekenstein-Hawking formula, the holographic entropy of a sphere of radius ξ is

$$S(\xi) \sim (\xi / L_P)^2 \approx 2.6 \times 10^{61} \quad (\text{in units of } k_B).$$

The standard estimate of the observable-universe entropy budget (dominated by black holes and the cosmic microwave background) is $S_{\text{universe}} \sim 10^{122}$. Comparing:

$$S(\xi)^2 \approx 6.6 \times 10^{122} \quad \text{vs} \quad S_{\text{universe}} \sim 10^{122} \quad [\text{factor} \sim 6].$$

To within an order of magnitude, the square of the holographic entropy of a minimal commitment cell equals the total cosmic entropy budget. This relationship is equivalent to the length midpoint $\xi^2 \sim L_P \cdot R_H$ (see Section 5.1), since the holographic entropy of the cosmic horizon is $(R_H/L_P)^2 \sim 10^{123}$. The entropy framing is nonetheless worth stating because $S(\xi)$ and S_{universe} are independently measurable quantities: it says that the information capacity of a single commitment cell, squared, approximately matches the information content of the entire observable universe. Whether this reflects a genuine information-theoretic structure — connecting the local commitment-cell scale to the global cosmic information budget — is an open question that the VERSF framework's emphasis on irreversible record formation makes natural to ask.

Whether any of these large-number relationships are explained by VERSF dynamics or reflect a deeper symmetry remains an open question.

6.4 The Friedmann Identity for the Coherence Scale

Substituting the Friedmann equation into the definition of ξ yields an exact identity relating ξ , the Planck length, and the Hubble radius — connecting the VERSF coherence scale directly to the large-scale structure of the universe through known cosmological equations, without any free parameters.

The Friedmann equation at the present epoch gives the vacuum energy density as

$$\rho_{\Lambda} = 3H_0^2 c^2 / (8\pi G),$$

where H_0 is the Hubble constant. Substituting into $\xi = (\hbar c / \rho_{\Lambda})^{1/4}$ and using $L_P = \sqrt{\hbar G / c^3}$ and $R_H = c / H_0$:

$$\begin{aligned} \xi &= (\hbar c \cdot 8\pi G / 3H_0^2 c^2)^{1/4} \\ &= (8\pi G \hbar / 3H_0^2 c)^{1/4}. \end{aligned}$$

Now substituting $G\hbar = L_P^2 c^3$ and $H_0^2 = c^2 / R_H^2$:

$$\begin{aligned} \xi &= (8\pi L_P^2 c^3 R_H^2 / 3c^3)^{1/4} \\ &= (8\pi/3)^{1/4} \cdot (L_P^2 R_H^2)^{1/4} \\ &= (8\pi/3)^{1/4} \cdot \sqrt{L_P \cdot R_H}. \end{aligned}$$

This is an exact identity given the Friedmann equation and the standard definitions of L_P and $R_H = c / H_0$. The coefficient $(8\pi/3)^{1/4} \approx 1.70$ is not a numerical coincidence — it is fixed entirely by the Friedmann equation coefficients and the definitions of L_P and R_H . No free parameters enter.

The identity is exact in the limit $\Omega_{\Lambda} \rightarrow 1$ (a flat, Λ -dominated universe). In the actual universe, $\rho_{\Lambda} = \Omega_{\Lambda} \cdot 3H_0^2 c^2 / 8\pi G$ with $\Omega_{\Lambda} \approx 0.685$, so the identity acquires a correction factor of $\Omega_{\Lambda}^{1/4} \approx 0.91$. Numerically:

$$(8\pi/3)^{1/4} \cdot \sqrt{L_P \cdot R_H} \approx 8.0 \times 10^{-5} \text{ m},$$

matching $\xi = 8.2 \times 10^{-5} \text{ m}$ to within 2.5% — a residual attributable to the matter and radiation content of the universe rather than to any imprecision in the derivation.

The physical content of this identity is that the VERSF coherence scale is not an isolated prediction: it is algebraically embedded in the Friedmann cosmology. Given H_0 and G and \hbar , ξ is determined. This converts the "midpoint relation" $\xi \sim \sqrt{L_P \cdot R_H}$ from a numerical observation into a derived consequence of cosmological structure. The midpoint coefficient $(8\pi/3)^{1/4}$ is a cosmological number in the same sense that $3/8\pi$ is the Friedmann coefficient — both follow from the Einstein field equations applied to a homogeneous, isotropic universe.

This result also partially resolves the horizon ambiguity of Section 10.4. The algebraically natural horizon is the Hubble radius c / H_0 , not the particle horizon $R_H = 8.8 \times 10^{26} \text{ m}$. The earlier near-equality $\xi \approx \sqrt{L_P \cdot R_H}$ using the particle horizon gave a coefficient of ≈ 0.69 with no derivable interpretation. With $R_H = c / H_0$, the coefficient is $(8\pi/3)^{1/4}$ — derivable from first principles.

7. The Era Coherence Scale

The formula $\xi = (\hbar c / \rho_i)^{1/4}$ is not peculiar to today. At each cosmic epoch characterised by a dominant energy density ρ_i , the same expression defines an epoch coherence scale:

$$L_i(\text{epoch}) = (\hbar c / \rho_i)^{1/4}.$$

This is not a new postulate — it follows directly from applying the same $\chi(L) = 1$ threshold condition to the energy density prevailing at each epoch. The result is an algebraic consistency check at both endpoints of the known hierarchy:

At Planck energy density $\rho_P = \hbar c / L_P^4$ (by the definition of L_P):

$$L_i(\rho_P) = (\hbar c / (\hbar c / L_P^4))^{1/4} = L_P \quad [\text{algebraic consistency check}].$$

At electroweak energy density $\rho_{EW} \sim (m_W c^2)^4 / (\hbar c)^3$:

$$L_i(\rho_{EW}) = (\hbar c / \rho_{EW})^{1/4} = \hbar c / (m_W c^2) = L_{EW} \quad [\text{algebraic consistency check}].$$

Both results hold by construction once the epoch energy density is parameterised by the corresponding field scale. They show that the era coherence scale formula is consistent with known scales at both endpoints — the formula gives the right answer when fed the right input — but they are not independent derivations of L_P or L_{EW} from first principles.

The physical content of $L_i(\text{epoch}) = \hbar c / (k_{BT} \text{epoch})$ is that it is simply the de Broglie–thermal wavelength of the epoch. In this light, the formula $\chi(L) = \rho L^4 / (\hbar c) = 1$ is not a special VERSF condition but the universal statement that a coherence cell of size L is matched to its epoch when L equals the thermal de Broglie length of that epoch.

The era coherence scale sequence runs:

Epoch	ρ_i (J m ⁻³)	L_i (m)	Known scale
Planck	$\sim 10^{113}$	1.6×10^{-35}	L_P (algebraic)
Electroweak	$\sim 10^{45}$	2.5×10^{-18}	L_{EW} (algebraic)
QCD	$\sim 10^{34}$	1.3×10^{-15}	$\sim \text{fm}$
Today (Λ -dominated)	6.9×10^{-10}	8.2×10^{-5}	ξ

This reframes the geometric-mean observations of Section 5. The approximate relationship $\sqrt{L_P \cdot \xi} \sim L_{EW}$ is not an independent numerical coincidence — it reflects the fact that L_P , L_{EW} , and ξ are all instances of the same formula evaluated at successive epochs. If the epoch energy density runs as a power law in cosmological time, the resulting sequence of L_i values is automatically logarithmically spaced. The "geometric mean" structure then emerges from the logarithmic running of the era coherence scale rather than from a static pattern imposed on the hierarchy.

This interpretation carries a significant implication: the EW geometric-mean relationship is not a free-standing regularity requiring its own explanation. It may instead be a derived consequence of the era coherence scale law applied to three cosmological epochs. Whether this derivation can

be made precise within VERSF — by specifying the running of ρ_i and recovering the exact spacing — is a priority for future work.

8. Logarithmic Structure and Physical Organisation

The approximate equal-spacing of these scales in log units is characteristic of systems governed by renormalisation group flows, where coupling constants and correlation lengths run logarithmically with energy scale. In quantum field theory, hierarchical separation of scales can arise from the slow running of couplings — as in the generation of the QCD confinement scale far below the Planck scale via dimensional transmutation.

Within VERSF, the logarithmic hierarchy may reflect a discrete scale symmetry of the substrate. If the substrate is organised into self-similar coherence domains at each hierarchical level, the geometric-mean relationship would follow from the self-similarity condition

$$L_{(n+1)} = \sqrt{L_n \cdot L_{(n+2)}},$$

which is simply the condition that consecutive scales are logarithmically equally spaced. This would be consistent with an approximate discrete scale symmetry or self-similar organisation of the substrate, connecting Planck geometry to cosmological structure through the pivot scale ξ .

9. Physical Interpretation of Each Level

Planck scale ($L_P \sim 10^{-35}$ m): The fundamental granularity of the spacetime substrate. Below this scale, the VERSF void-energy description breaks down and quantum gravity effects dominate.

Electroweak scale ($L_{EW} \sim 10^{-18}$ m): Quantum field interactions and particle mass generation. Within the era coherence scale picture (Section 7), the era-coherence formula reproduces the characteristic electroweak length once the electroweak epoch energy density is parameterised by the corresponding field scale. This shows algebraic consistency of the scaling law across epochs, though it is not an independent derivation of the electroweak scale from first principles. The apparent position of L_{EW} near the geometric mean of L_P and ξ may therefore reflect the logarithmic running of epoch coherence scales through successive cosmic eras rather than a free-standing geometric-mean relation.

Atomic scale ($L_{atom} \sim 10^{-10}$ m): Electromagnetic binding of electrons. This scale lies between the electroweak and coherence levels but is not directly predicted by the VERSF geometric-mean pattern — a gap the framework should eventually address.

Coherence scale ($\xi \sim 10^{-4}$ m): The central VERSF scale, where $\chi(L) = 1$ and vacuum energy density and causal throughput are commensurate. Governs measurement dynamics (entropy threshold paper) and marks the transition from quantum to classical spacetime behaviour.

Planetary scale ($L_{AU} \sim 10^{11}$ m): Gravitational structure formation. The geometric-mean relationship with R_H suggests that preferred scales for gravitational organisation may emerge where substrate coherence and cosmological boundary conditions intersect.

Cosmic horizon ($R_H \sim 10^{26}$ m): Set by the full expansion history of the universe (matter + radiation + Λ). Note that R_H (the particle horizon) is distinct from the de Sitter horizon $R_{dS} = \sqrt{3/\Lambda} \sim 1.45 \times 10^{26}$ m, which is the horizon scale determined directly by ρ_Λ alone. The geometric-mean relationship in Section 5.1 uses R_H ; the self-consistency of this choice is discussed in Section 10.4.

10. Critical Assessment

10.1 Precision

The planetary relationship ($L_{AU} \approx \sqrt{\xi \cdot R_H}$) holds to within a factor of 2 — a $\sim 1\%$ logarithmic accuracy over a sixty-two-decade span. The electroweak relationship holds within a factor of ~ 30 — about 5% logarithmic accuracy. Both are more precise than would be expected from random sampling of known scales, but this assessment depends on how one counts the "trials."

10.2 The Atomic Scale Gap

The atomic scale ($\sim 10^{-10}$ m) breaks the perfect logarithmic spacing between L_{EW} and ξ , and one sentence of acknowledgement is insufficient given that approximate logarithmic regularity is presented as a physical signature of the hierarchy.

The Bohr radius $a_0 = \hbar^2/(m_e e^2) = \hbar/(\alpha m_e c)$ is set by three constants — \hbar , m_e , and α — that do not obviously connect to the era-coherence-scale picture. Crucially, a_0 introduces matter parameters (electron mass m_e and fine-structure constant α) that play no role in the VERSF hierarchy. The hierarchy considered here concerns scales determined by spacetime energy densities — cosmological and field-theoretic — rather than bound-state structure constants. The atomic scale therefore belongs to a different class of scale, not a missing rung in the same ladder. In particular, a_0 does not take the form $(\hbar c/\rho_i)^{1/4}$ for any natural epoch energy density ρ_i . A rough check:

$$(\hbar c/\rho_{atom})^{1/4} = a_0 \quad \Rightarrow \quad \rho_{atom} \sim \hbar c/a_0^4 \sim 10^{72} \text{ J/m}^3.$$

No standard cosmic epoch corresponds to this energy density, which lies between the QCD scale ($\sim 10^{34}$ J/m³) and the electroweak scale ($\sim 10^{45}$ J/m³) by many orders of magnitude. Atomic binding does not fit into the era coherence scale sequence.

Two responses are possible. The first is to argue that atomic scales are determined by different physics — electromagnetic binding governed by α , m_e , and \hbar — and are therefore genuinely external to the VERSF hierarchy. This is plausible: the hierarchy is built from substrate coherence thresholds and cosmological energy densities, not from matter content. On this view, the atomic scale is an "extra" scale produced by a different physical mechanism (electromagnetic bound states), and its appearance in the table is a categorisation error, not a prediction failure.

The second response, more demanding but more satisfying, would be to derive where atomic-scale physics enters the VERSF framework through the entropy threshold condition or the quantum-to-classical transition. If the record-formation threshold at $\tau_s = \xi/c$ is connected to the thermal de Broglie wavelength at some temperature, a relationship to atomic scales might emerge. This is not yet developed and is identified as an open problem.

Until one of these responses is fully worked out, the atomic scale stands as the strongest argument against interpreting the hierarchy as reflecting a single underlying geometric principle.

10.3 Anthropic Considerations for the Planetary Scale

The Earth–Sun distance is not a freely determined cosmological scale, and the strength of this objection deserves explicit engagement. L_{AU} is constrained by at least three independent selection effects:

1. **Stellar physics:** The luminosity of a G2V star is set by nuclear physics and stellar structure equations, not by cosmological parameters. The habitable zone of a G-type star falls in a narrow range determined by this luminosity and the requirements for liquid water.
2. **Habitability:** The Earth–Sun distance is approximately the value required for a temperate planetary surface. Any observer capable of measuring L_{AU} is almost certainly within the habitable zone of their star.
3. **Sampling bias:** The geometric-mean relationship uses the specific planet-star pair that produced the observers. If one instead asks "is there any gravitationally bound orbital system within an order of magnitude of $\sqrt{(\xi \cdot R_H)}$?" the answer would almost certainly be yes regardless of the theory, given the density of stellar systems and the range of orbital separations they span.

These considerations mean that L_{AU} is not an independent cosmological datum in the sense required for the geometric-mean relationship to constitute non-trivial evidence. The relationship $L_{AU} \approx \sqrt{(\xi \cdot R_H)}$ should therefore be understood as a *suggestive numerical regularity subject to strong anthropic contamination*, not as a prediction or a coincidence requiring physical explanation.

The relationship would be considerably stronger evidence if it were shown to hold for:

- A broader class of stellar systems (other spectral types, different planetary architectures)
- A scale that does not carry the dual contamination of habitability and observer selection
- A theoretically derived scale not used in establishing ξ

Investigating whether the geometric-mean pattern extends to non-anthropically-selected scales is identified as a priority for future work. Until that is done, the AU relationship is treated here as the *weakest* of the numerical regularities, not the strongest.

10.4 Horizon Ambiguity: Partial Resolution via the Hubble Radius

The paper uses $R_H \approx 8.8 \times 10^{26}$ m — the comoving particle horizon — in the geometric-mean relationship $L_{AU} \approx \sqrt{(\xi \cdot R_H)}$. Section 6.4 shows, however, that a cleaner algebraic structure emerges with the Hubble radius $R_H = c/H_0 \approx 1.37 \times 10^{26}$ m. Substituting the Friedmann equation $\rho_\Lambda = 3H_0^2 c^2 / 8\pi G$ into the definition of ξ yields

$$\xi = (8\pi/3)^{1/4} \cdot \sqrt{(L_P \cdot c/H_0)},$$

exactly, given the definitions of L_P and $R_H = c/H_0$. The coefficient $(8\pi/3)^{1/4} \approx 1.70$ is derivable from the Friedmann equation, not arbitrary.

This partially resolves the earlier ambiguity. The algebraically natural cosmological horizon is the Hubble radius c/H_0 , not the particle horizon or the de Sitter horizon $R_{dS} = \sqrt{3/\Lambda}$. With this choice, the "midpoint relation" $\xi \sim \sqrt{(L_P \cdot R_H)}$ is exact in structure — the apparent near-equality with coefficient 0.69 using the particle horizon becomes a well-defined identity with coefficient $(8\pi/3)^{1/4}$ using the Hubble radius.

The remaining ambiguity is why the Hubble radius, rather than R_{dS} or the particle horizon, appears in the AU geometric-mean relation. The Hubble radius c/H_0 mixes Λ , matter, and radiation through H_0 , so it too encodes more than ρ_Λ alone. What the algebraic identity shows is that the combination of inputs entering ξ via the Friedmann equation naturally produces $\sqrt{(L_P \cdot R_H)}$ with a clean coefficient — suggesting that the Hubble radius is the cosmologically appropriate partner scale for ξ .

The numerical AU relationship $L_{AU} \approx \sqrt{(\xi \cdot R_H)}$ holds to a factor of ~ 2 with $R_H = 8.8 \times 10^{26}$ m and to a factor of ~ 3 with $R_H = c/H_0$. Both are approximate; neither constitutes a derivation. Anthropic selection concerns (Section 10.3) apply in either case.

10.5 Status of the Derivation of ξ and the Path Forward

Section 2 presents two independent derivations of ξ — vacuum-energy crossover and commitment throughput — reinforced by two complementary causal-cell reformulations (action budget and Bekenstein entropy capacity), unified by the Minimal Commitment Cell Lemma, together with a stability-selection argument. The Lemma shows that all four conditions reduce to the same dimensionless threshold $\chi(L) \gtrsim 1$. Section 7 then identifies ξ as the current-epoch instance of the universal era coherence scale $L_i = (\hbar c / \rho_i)^{1/4}$, which holds exactly at the Planck and electroweak endpoints.

Four gaps remain before this constitutes a complete derivation. First, the order-unity numerical prefactor is not yet derived from the full substrate mode spectrum — though Section 6.4 shows that the Friedmann equation fixes it as $(8\pi/3)^{1/4}$ when $R_H = c/H_0$, reducing this gap to deriving the specific mode-spectrum coefficient from the substrate action. Second, the stability functional $F(L)$ in Section 2.6 is schematic — the exponents m and n need to be derived from VERSF substrate dynamics rather than chosen to illustrate the mechanism. Third, the geometric-mean relationships of Sections 5.1–5.2 remain retrospective identifications rather than forward predictions; the era scaling picture of Section 7 offers a physical mechanism for the EW relationship but requires that the epoch-density running be derived, not assumed. Fourth, the remaining horizon question — why the Hubble radius c/H_0 is the appropriate partner for ξ in the AU relation, given that H_0 encodes matter and radiation as well as Λ — is not yet answered.

For the hierarchical relationships to become genuine physical predictions they require: (1) deriving the epoch-density running from VERSF substrate dynamics and recovering the spacing from first principles; (2) a falsifiable prediction for a scale not used in establishing the hierarchy; or (3) resolving which cosmological horizon scale (R_H or R_{dS}) enters the AU geometric-mean relation and why. The Casimir deviation prediction in Section 3.4 represents one concrete path toward falsifiability independent of the hierarchy questions.

11. Conclusion

The VERSF framework yields a substrate coherence length $\xi \approx 8.2 \times 10^{-5}$ m, fixed by \hbar , c , and ρ_Λ . Its deepest interpretation is as the size of a minimal commitment cell: the unique length at which a region contains enough vacuum energy to complete one irreversible update within its own causal crossing time. This is captured by the dimensionless condition $\chi(\xi) = \rho_\Lambda \xi^4/(\hbar c) = 1$, established by two independent derivations and reinforced by two complementary reformulations, unified by a Minimal Commitment Cell Lemma. In this sense, $\chi(L)$ is the order parameter for commitment-capable coherence.

A broader perspective emerges from the era coherence scale and from the large-number structure of the hierarchy. The formula $L_i = (\hbar c/\rho_i)^{1/4}$, applied to each cosmic epoch, recovers L_P and L_{EW} as algebraic identities at the Planck and electroweak epochs respectively, and ξ today. The approximate geometric-mean relationship between L_P , L_{EW} , and ξ may therefore reflect the logarithmic running of epoch coherence scales through successive cosmic eras rather than an independent static regularity. Separately, the algebraic identity $\rho_P/\rho_\Lambda = (\xi/L_P)^4$ converts the cosmological constant hierarchy — 10^{123} in energy density — into the fourth power of the length-scale hierarchy between ξ and L_P . This does not solve the CC problem, but reframes it as the question of why $\xi \gg L_P$, which VERSF substrate dynamics may eventually address. A further result (Section 6.4 — the Friedmann identity for the coherence scale) shows that substituting the Friedmann equation into the definition of ξ gives $\xi = (8\pi/3)^{1/4} \cdot \sqrt{(L_P \cdot c/H_0)}$ exactly — so the midpoint relation between ξ and the Planck and Hubble scales is not a numerical coincidence but a derived consequence of Friedmann cosmology, with a coefficient fixed entirely by the Einstein field equations applied to a homogeneous isotropic universe.

When ξ is placed within the full hierarchy of fundamental physical scales, it sits at the approximate logarithmic centre of the Planck-to-horizon span and reveals two geometric-mean relationships of notably different precision:

$$L_{AU} \approx \sqrt{(\xi \cdot R_H)} \quad (\sim 1\% \text{ logarithmic accuracy; factor } \sim 2) \quad [\text{strongest numerical coincidence; weakest evidential datum}]$$

$$L_{EW} \approx \sqrt{(L_P \cdot \xi)} \quad (\sim 5\% \text{ logarithmic accuracy; factor } \sim 30) \quad [\text{reinterpreted as era scaling}]$$

The AU relationship is presented as a suggestive numerical regularity subject to the horizon-ambiguity caveat of Section 10.4. The EW relationship, while numerically weaker, acquires potential physical grounding through the era coherence scale picture. Establishing either as a genuine physical prediction requires deriving the relevant running from VERSF substrate dynamics. The Casimir deviation prediction outlined in Section 3.4 — a predicted departure from $E(L) \propto L^{-3}$ scaling as L approaches ξ from below — provides a concrete experimental target for future investigation independent of the hierarchy questions.

Appendix: Numerical Summary

Relationship	Predicted	Observed	Factor	Log accuracy	Epistemic status
$\sqrt{(\xi \cdot R_H)}$ vs. L_{AU}	$2.7 \times 10^{11} \text{ m}$	$1.5 \times 10^{11} \text{ m}$	$\sim 1.8\times$	$\sim 99\%$	Anthropically contaminated regularity
$\sqrt{(L_P \cdot \xi)}$ vs. L_{EW}	$3.6 \times 10^{-20} \text{ m}$	$\sim 10^{-18} \text{ m}$	$\sim 28\times$	$\sim 95\%$	Suggestive numerical regularity
ξ/L_P vs. R_H/ξ	10^{31}	10^{30}	$\sim 10\times$	$\sim 98\%$	Suggestive numerical regularity
$\chi(\xi) = \rho_\Lambda / \xi^4 / (\hbar c)$	1 (by definition)	—	exact	—	Derived threshold
ρ_P/ρ_Λ vs. $(\xi/L_P)^4$	6.6×10^{122}	6.7×10^{122}	$1.01\times$	algebraic	Algebraic identity
$S(\xi)^2$ vs. $S_{universe}$	6.6×10^{122}	$\sim 10^{122}$	$\sim 6\times$	order-of-magnitude	Suggestive numerical regularity
$L_i(\rho_P)$ vs. L_P	$1.6 \times 10^{-35} \text{ m}$	$1.6 \times 10^{-35} \text{ m}$	$1.00\times$	algebraic	Consistency check
$L_i(\rho_{EW})$ vs. L_{EW}	$2.47 \times 10^{-18} \text{ m}$	$2.47 \times 10^{-18} \text{ m}$	$1.00\times$	algebraic	Consistency check

Epistemic status key — Derived threshold: follows from the definition of ξ ; Algebraic identity: exact by algebra from the relevant definitions; Consistency check: the formula gives the right answer when fed the correct epoch parameterisation, not an independent prediction; Suggestive

numerical regularity: numerically close but not derived; Anthropically contaminated regularity: numerically close but subject to observer-selection effects (see §10.3).

Log accuracy = fraction of the full 62-decade span within which the prediction falls. Rows ordered by precision, strongest first. Era coherence scale rows (last two) are algebraic consistency checks — the formula gives the right answer when fed the correct epoch parameterisation, not independent predictions. Horizon note: the AU relationship uses $R_H = 8.8 \times 10^{26} \text{ m}$ (particle horizon). With $R_{dS} = \sqrt{3/\Lambda} = 1.45 \times 10^{26} \text{ m}$, $\xi/\sqrt{(L_P \cdot R_{dS})} \approx 1.69$ and the near-midpoint identity does not hold — see Section 10.4.