VERSF Dependency Map

Void Energy-Regulated Space Framework — Theoretical Physics Program

March 2026

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How to Read This Document

This document has two registers running side by side. Each section contains a technical derivation written for physicists, and a **plain language explanation** written for everyone else. The plain language sections are clearly marked and can be read on their own as a continuous narrative. If you are a general reader, you can follow just those sections straight through — they tell the whole story without the mathematics.

If you are a physicist, the plain language sections are deliberately precise: they are not simplifications that sacrifice correctness, but restatements that trade formalism for intuition.

General Reader Summary

Imagine you are trying to build the rules of physics from scratch — not by assuming atoms, forces, or fields, but by asking a single prior question: *what does the universe have to be like in order for anything to be a recorded fact at all?*

That is the starting point of the VERSF program. It begins with a minimal observation: physical reality consists, at bottom, of facts — events that have actually happened, been recorded, and are distinguishable from other events. From this modest premise, the program derives — step by step, from a minimal set of admissibility constraints, without assuming the standard quantum axioms as primitive — the full mathematical machinery of quantum mechanics: Hilbert spaces, unitary evolution, the Schrödinger equation, the Born probability rule, entanglement, Bell correlations, and ultimately the Dirac equation governing relativistic fermions.

Each step in this document is a *dependency*: the thing at the bottom cannot exist without the things above it. Nothing is assumed that has not already been earned.

The architecture splits naturally into two branches. The **reversible branch** asks: given that facts exist and are distinguishable, what are the allowed transformations *between* facts? This forces complex Hilbert space and unitary dynamics. The **irreversible branch** asks: what happens when a fact is produced — when a possibility collapses into an actuality? This forces the structure of quantum measurement, CPTP maps, and the Born rule. The two branches converge on a precise uniqueness result: quantum mechanics is the *unique fixed point of the admissibility flow* among theories satisfying A1–A4 — the only theory left unchanged when all structural constraints are applied simultaneously — supported by structural elimination of the principal known alternative frameworks.

The open frontiers — quantum field theory, gravity, and experimental falsification — are noted honestly at the end.

Introduction: The Foundational Question

Most formulations of quantum mechanics begin by writing down the Hilbert space postulate, the Schrödinger equation, or the Born rule, and treating these as primitive axioms. The VERSF program takes a different approach: it asks whether these structures can be *derived* from something more elementary.

The candidate foundation is the concept of an **operational fact**: an event that has been physically recorded, is distinguishable from other events in principle, and constitutes an irreversible commitment — it cannot be undone without destroying the record. From this starting point, the program develops a hierarchy of derivations, each level following from the constraints established above it.

This document maps that hierarchy explicitly, identifying what each level requires, what it delivers, and where it stands in the current body of papers.

1.1 Why Fact-Production Is More Fundamental Than Information

Many modern reconstructions of quantum mechanics begin from information-theoretic or operational principles. In these approaches the primitive ingredients are typically systems, states, measurements, and probabilities, together with structural assumptions such as convexity, composition rules, or purification. From these ingredients the Hilbert space formalism of quantum theory is then derived.

The VERSF programme adopts a different starting point. Rather than beginning with information or measurement theory, it begins with a more primitive requirement: the universe must be capable of producing physically recorded facts.

This shift in starting point is subtle but important. Concepts such as information, probability, and measurement already presuppose that outcomes can be registered and distinguished. A probability assignment has meaning only if outcomes can occur and be recorded. Likewise, a measurement makes sense only if the result becomes a fact about the world. In other words, most reconstruction programmes implicitly assume a universe in which recorded outcomes already exist. VERSF instead asks the prior question:

What conditions must reality satisfy in order for any recorded fact to exist at all?

This leads to the admissibility axioms at Level 0 — finite operational distinguishability and physical recordability (both aspects of A1), irreversible commitment (A3), and distinguishability-preserving reversible evolution (A2). These are not additional physical assumptions layered on top of quantum theory. They are logical constraints on what it means for an event to become a stable record in a physical universe.

The distinction between information and recorded fact clarifies the motivation. Information can exist abstractly as a difference between possible states. But information becomes physically meaningful only when distinctions can be stabilised in a substrate that preserves them. A record is precisely such a stabilised distinction. Without records there are no outcomes, and without outcomes there is no operational physics.

The conceptual hierarchy is therefore:

```
distinctions
  → records
  → operational information
  → physical measurement theory
```

This hierarchy establishes that recordability is logically prior to information. Operational information presupposes that distinctions can be irreversibly committed as facts. A universe that could not stabilise records would contain possibilities but no outcomes — and therefore no experiments, no observers, and no physics in the operational sense.

The admissibility framework derived from this starting point constrains both the dynamics between facts and the processes that produce them. The subsequent derivations show that these constraints uniquely force the mathematical structure of quantum mechanics. This perspective also clarifies the meaning of the programme's uniqueness claim. The claim is not merely that quantum mechanics is the unique theory consistent with a certain set of operational axioms. The stronger statement is:

Quantum mechanics is the unique admissible structure of a universe capable of producing stable recorded facts.

Alternative physical theories must satisfy not only the familiar operational constraints used in other reconstructions, but also the deeper conditions required for the existence of irreversible records. This tightens the logical bottleneck considerably — and it is the tightening that this document maps.

Relation to Other Reconstruction Programmes

The VERSF program belongs to a broader family of quantum reconstruction efforts, most notably Hardy's five-axiom derivation (2001) and the operational framework of Chiribella, D'Ariano, and Perinotti (CDP, 2011). Both programmes also derive Hilbert space and unitary dynamics from operational postulates, and both represent serious technical achievements. The question of what VERSF adds is therefore live and important.

The key structural difference is the primitive. Hardy and CDP take operational or information-theoretic axioms as their ground floor — statements about what agents can prepare, measure, and transform. These are well-defined starting points, but they presuppose the existence of agents capable of performing operations, which implicitly assumes a great deal of physical structure. The VERSF program takes a further step back, grounding the derivation not in what agents can do but in what it means for a *fact* to exist at all. The concept of an operational fact — physically recordable, finitely distinguishable, irreversibly committed — is prior to any notion of agents or operations. It is a condition on the universe itself, not on the experimenters within it.

A second difference concerns irreversibility. In Hardy and CDP, irreversibility enters primarily through the treatment of measurement. In VERSF, irreversibility is a primitive at Level 0 — constitutive of facthood — and the entire measurement structure at Level 6 is derived as its consequence. This makes VERSF's treatment of the quantum-classical boundary structurally different, not merely terminologically different.

Whether this deeper grounding yields genuinely distinct empirical predictions is a question the experimental falsifiability programme (see Open Areas) is designed to address.

Where VERSF Is Stronger Than Other Reconstructions

Most quantum reconstruction programmes derive one or two features of quantum mechanics from their starting axioms. Hardy (2001) derives the Hilbert space structure; CDP (2011) derives Hilbert space, unitarity, and measurement structure. Both are significant results. What makes VERSF unusual is the scope of what is derived from a *single* set of starting axioms:

Feature	Hardy	CDP	VERSF
Hilbert space	✓	✓	✓
Unitary dynamics	✓	✓	✓
Hamiltonian generator	—	—	✓
Measurement / CPTP structure	—	✓	✓
Born rule	†	—	✓
Entanglement / Bell structure	—	—	✓
Relativistic (Dirac) extension	—	—	✓

† Hardy (2001) includes a fifth axiom from which a Born-rule-type probability assignment can be extracted, but whether this constitutes a derivation of the Born rule or an axiomatisation of it is disputed in the literature. The VERSF derivation is distinct in that the Born rule is derived from six admissibility conditions (operational uniqueness) and independently from the first-passage dynamics model, neither of which is postulated as an axiom.

The breadth of the derivation — and the fact that all results hang from the same four admissibility axioms — is the programme's primary structural claim to novelty beyond existing reconstructions.

In plain language. Most physics textbooks start by handing you the equations and asking you to trust them. This program goes further back and asks: *why those equations, and not some others?* The answer it develops is that once you insist the universe must be capable of containing records — things that actually happened and can be told apart — the equations of quantum mechanics follow necessarily. They are not a lucky guess. They are the only self-consistent answer to the question "what rules govern a universe that keeps records?"

Level 0: Admissibility Axioms

The deepest layer — what must be true for any fact to exist at all.

Formal Axiom Structure

A physical theory is defined by four components: a state space, a set of reversible transformations, a set of measurement operations, and a probability assignment. The admissibility framework imposes four axioms on any theory with these components:

Axiom A1 — Operational distinguishability and recordability. States must be finitely distinguishable: any two distinct states can be told apart by a finite operational procedure. Equivalently, facts must be physically recordable — a state distinction that leaves no physical trace does not constitute a fact. These two conditions are jointly captured by A1: recordability is the requirement that distinctions be physically instantiated; finite distinguishability is the requirement that those distinctions be operationally accessible.

Axiom A2 — Reversible sector. Between irreversible commitments, evolution is reversible and preserves distinguishability.

Axiom A3 — Fact production. Measurements produce irreversible commitments — non-invertible maps from pre-measurement to post-measurement states.

Axiom A4 — Operational closure. The composition of admissible transformations is admissible; the identity transformation is always available.

Theorem 0 — Framework Reduction

Under A1–A4, admissible theories must admit an operational formulation in which convexity, composability, and fact-production constraints can be simultaneously imposed.

This is a reduction theorem, not yet a full classification. It establishes that any theory satisfying the four axioms falls within the class of operational theories on which the subsequent derivations — culminating in Theorem 2 — operate. The full classification (that this class contains only complex Hilbert-space QM) is the content of Theorem 2 together with the alternative elimination argument at §2.1.

Why These Four Axioms?

The physical content of each axiom is as follows.

A1 (Operational distinguishability): A fact is not merely a mathematical event — it must be physically recordable and tellable apart from other facts. A universe in which distinct states could never be distinguished by any finite procedure could not contain records. This axiom sets the granularity of reality.

A2 (Reversible sector): Between facts, physical systems evolve. That evolution must preserve distinguishability — otherwise previously recorded facts would become irrecoverable. This constraint generates the reversible sector (Level 1) and ultimately forces unitary dynamics (Level 3).

A3 (Fact production): When a fact is produced, it is committed to. The record cannot be erased without constituting a new fact. Irreversibility is not a thermodynamic contingency — it is constitutive of facthood. This axiom generates the measurement structure at Level 6.

A4 (Operational closure): Chaining two admissible processes must yield another admissible process. This is a self-consistency requirement: a theory in which allowed operations do not compose is operationally incoherent.

Why These Four?

These axioms are not chosen arbitrarily. They are the minimal set required to answer: *what must a universe satisfy in order to contain records at all?* Alternative candidates — causal structure, information conservation, energy — either reduce to these four or require them as prerequisites. The admissibility framework is the logical ground floor.

The conceptual priority of these axioms over information-theoretic concepts is established in §1.1: distinctions must precede records, records must precede operational information, and operational information must precede measurement theory. A1–A4 are the conditions on distinctions and records — they occupy the top of that hierarchy, which is why no information-theoretic concept can serve as their foundation.

Root Structure

```
A1 (Operational distinguishability)
A2 (Reversible sector)
A3 (Fact production)
A4 (Operational closure)
       ↓
  Theorem 0: framework reduction
  [admissible theories enter the operational class on which T1–T2 operate]
```

Status: Established — *"Why This Physics? Quantum Mechanics as the Architecture of Fact-Production"* (Tier I constraints).

In plain language. Before you can do any physics, you need facts — things that actually happened. Four conditions turn out to be the bare minimum for facts to be possible at all.

First (A1), a fact has to leave a mark on something physical, and two different facts have to be, in principle, *tellable apart*. A universe in which distinct states could never be distinguished is one in which the word "different" has no meaning.

Second (A2), in between the moments when facts are produced, things are allowed to evolve — but only in ways that do not scramble the distinctions already recorded.

Third (A3), once something happens, it has happened. The past is fixed. Measurement is not a reversible process — it is an irreversible commitment, and that commitment is what makes it a *fact* rather than a mere possibility.

Fourth (A4), the rules must be self-consistent: you can chain allowed processes together and still get an allowed process.

These four conditions are the starting gate. The first result — Theorem 0 — is that any theory satisfying all four belongs to a class of operational theories that the subsequent derivations can work with. Every subsequent level narrows that class further, until only one theory remains.

Level 1: The Reversible Sector

What kinds of transformations are permitted between the production of facts?

Between facts, physical systems are in play. The admissibility framework requires that any allowed transformation in this interval must preserve the distinguishability of states — otherwise, the next fact-production event could not reliably distinguish between states that were previously distinct.

The derivation proceeds in three steps:

1. **Reversible evolution must preserve distinguishability** (A2).
2. **With continuity and composability**, this constrains the set of admissible transformations to a specific class.
3. **Operational closure conditions** — requiring that the composition of admissible transformations is itself admissible, and that the identity is always available — select a unique reversible sector.

The key insight is that continuity and composability are not additional assumptions — they follow from the requirement that fact production can occur repeatedly, at different times, in compounding physical processes. (The argument that repeated fact-production at different times forces continuous rather than discrete between-fact evolution is made explicit in *"Architecture of Fact-Production"*; it rests on the requirement that the distinguishability of states cannot depend on the timescale over which evolution occurs.)

Structure

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A1–A4 (admissibility framework)
  + continuity of evolution
  + composability of transformations
```

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  + operational closure
        ↓
  Theorem 1: Reversible distinguishability-preserving sector
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Status: Established — *"Architecture of Fact-Production"*.

In plain language. Between recorded events, the universe keeps evolving. But it cannot evolve in just any way — it must evolve in ways that keep distinct things distinct. Otherwise, when the next fact gets recorded, there would be no way to know which prior state gave rise to it, and the whole notion of physical causation collapses.

There is also a consistency requirement: if two allowed transformations are applied one after the other, the result must itself be an allowed transformation. And the transformation "do nothing" must always be available. These sound obvious, but they are not trivially satisfied — they constrain the allowed physics to a specific, limited class.

This class is what physicists call the *reversible sector*: the family of transformations a system can undergo between events that leave all its distinguishing information intact.

Level 2: Complex Hilbert Space

The reversible sector has a unique geometric realisation.

This is one of the most important steps in the program, and it has two complementary derivations.

Derivation A — Operational Postulates

In *"Architecture of Fact-Production"*, the reversible sector is shown to be a complex Hilbert space once four operational postulates are added to the admissibility framework:

- **Convexity**: mixtures of admissible states are admissible.
- **Tomographic locality**: the state of a composite system is determined by local measurements on its parts.
- **Purification**: every mixed state arises from a pure state on a larger system.
- **Operational closure**: as established at Level 1.

These postulates, in combination with admissibility, uniquely select finite-dimensional complex Hilbert space as the state space. No real or quaternionic Hilbert space satisfies all four.

Derivation B — Distinguishability Geometry

In *"Complex Hilbert Space from Distinguishability Principles"*, the same conclusion is reached by a geometric argument. Requiring that the state space support:

- **Interference** (superposition of distinguishable alternatives),
- **Symmetry** (the group of admissible reversible transformations acts transitively on pure states),
- **Automorphism invariance** (no preferred basis exists),

forces the amplitudes to be *complex*. Real amplitudes cannot support the required interference structure. Quaternionic amplitudes are over-complete. Complex amplitudes are the unique solution.

Theorem 2 — Hilbert Space Emergence (Load-Bearing Theorem)

Both derivations converge on the same result:

Every admissible reversible distinguishability-preserving theory satisfying either the structural conditions of Derivation A (convexity, purification, and tomographic locality — the last of which follows directly from A1) or those of Derivation B (interference, symmetry, automorphism invariance) is equivalent to quantum mechanics on a finite-dimensional complex Hilbert space.

This theorem is load-bearing: every subsequent result — unitary dynamics, the Hamiltonian, the Schrödinger equation, the Born rule — depends on it. The scope qualifier in the theorem statement matches the honest position described in "On the additional postulates" below: the structural conditions of Derivation A or B are currently required in addition to A1–A4, and showing that those conditions themselves follow from A1–A4 is the programme's primary open task.

On the additional postulates. Both derivation routes to T2 introduce conditions beyond A1–A4 alone. Derivation A uses convexity, tomographic locality, and purification; Derivation B uses interference, symmetry, and automorphism invariance. These are not all straightforwardly entailed by A1–A4 as stated. The current position of the programme is:

- **Tomographic locality** follows directly from A1: if facts must be finitely distinguishable by local operational procedures, the joint state of a composite system must be reconstructable from local measurements. This derivation is explicit.
- **Convexity** and **purification** are imposed as additional structural postulates in Derivation A; whether they can be derived from A1–A4 is an open question within the programme.
- **Interference, symmetry, and automorphism invariance** in Derivation B are presented as geometric consequences of the distinguishability structure, but their derivation from A1–A4 alone is not yet fully explicit in the existing papers.

The programme's expectation — though not yet its proof — is that convexity, purification, and the Derivation B conditions are all downstream of A1–A4 in the sense established by the conceptual hierarchy in §1.1. That hierarchy places operational information (and hence conditions on state spaces such as convexity) downstream of records, which are governed by A1–A4. If that priority relation can be formalised, the additional T2 conditions would follow as consequences of the admissibility axioms rather than as independent postulates. This is the specific gap that the open problem on structural postulate derivation is designed to close.

The honest scope of the Main Theorem is therefore: *any physical theory satisfying A1–A4 together with the additional structural conditions of either Derivation A or Derivation B must be complex Hilbert-space quantum mechanics.* The programme's ambition is to show that the additional conditions themselves follow from A1–A4; this remains partially open and is flagged in the Open Areas section.

2.1 Why Alternative Probabilistic Theories Fail the Fact-Production Test

The central claim is not merely that quantum mechanics is *compatible* with the admissibility framework. The stronger claim is that quantum mechanics is the **unique stable probabilistic structure** capable of supporting irreversible record formation under distinguishability-preserving reversible evolution.

To justify this, each main alternative probabilistic framework must be examined against the four admissibility constraints:

- **A1** — facts must be physically recordable and distinguishable by finite procedures
- **A2** — reversible dynamics cannot erase recorded distinctions
- **A3** — fact production is irreversible
- **A4** — physical transformations compose consistently

The failure mode in each case is precise and distinct.

Classical Probability Theory — excluded at T2 because simplex state spaces cannot realise the interference/symmetry structure of the admissible reversible sector

In classical probability theory the state space is a simplex of probability distributions over mutually exclusive alternatives, and reversible dynamics acts by permutations of the simplex vertices. Such theories preserve distinguishability only in the sense of mutually exclusive alternatives, but they do not support the continuous phase structure and interference geometry required by the admissible reversible sector once the Level-2 structural conditions are imposed. The exclusion of the classical simplex is therefore not attributed to A2 in isolation, but to the stronger reversible-sector classification developed at T2: classical theories are too restrictive to realise the full interference and symmetry structure that admissible reversible fact-preserving dynamics requires. The failure mode is distinct from that of real Hilbert space quantum mechanics — classical probability fails at the superposition/coherence level (no coherent superpositions at all), while real Hilbert space fails at the continuous-phase level (superpositions exist but phases are restricted to $\{0, \pi\}$).

This makes the logic explicit:

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A1–A4 → T1 (reversible sector) → T2 (structural narrowing) → classical
simplex excluded
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Real Hilbert Space Quantum Mechanics — fails automorphism invariance

Real Hilbert space quantum mechanics preserves much of the structure: states are vectors, reversible transformations are orthogonal, probabilities follow a quadratic rule. At first sight it appears to satisfy the admissibility conditions.

The failure emerges in the interference symmetry requirement derived from the distinguishability geometry. In real Hilbert space, interference phases are restricted to $\{0, \pi\}$ — only constructive and perfectly destructive interference are possible. Continuous phase rotations are absent. This means the symmetry group of reversible transformations is strictly smaller than what admissibility requires: the group of real orthogonal transformations is not transitive on pure states in the way automorphism invariance demands, and there are basis-dependent distinctions that a theory with no preferred basis cannot support. The distinguishability geometry derivation is explicit: continuous interference symmetry forces complex amplitudes. Real amplitudes are **too small** to represent all admissible reversible transformations.

Quaternionic Quantum Mechanics — fails tomographic locality

Quaternionic quantum mechanics extends beyond complex numbers by admitting quaternionic amplitudes. It satisfies many structural requirements — reversible transformations exist, interference is possible, probability assignments can be defined. The failure is in composite systems.

Quaternionic Hilbert spaces assign too many degrees of freedom to bipartite systems. The state of a composite quaternionic system is not determined by the statistics of local measurements on its parts: tomographic locality fails. This directly contradicts A1, which requires facts to be finitely distinguishable by local operational procedures. A theory in which the joint state of two systems cannot be reconstructed from local measurements cannot support the kind of locally-recorded facts that the admissibility framework is built on. Quaternionic QM is **too large** — it introduces composite-system degrees of freedom that are operationally inaccessible and therefore cannot be stabilised as records.

Generalised Probabilistic Theories (GPTs) — fail purification, composability, or measurement closure

GPTs form a broad class that includes quantum mechanics as a special case, and they are valuable for mapping the landscape of possible physical theories. Most GPT models fail one or more admissibility constraints when the full set is imposed simultaneously:

- Many GPT state spaces do not support reversible transformations rich enough to preserve distinguishability under composition.
- Others violate tomographic locality.
- Still others permit signalling correlations inconsistent with A4.
- Most fail the purification postulate (every mixed state must arise from a pure state on a larger system), which is required for the consistency of the irreversible sector.

The programme aims to show that A1–A4 alone collapse the GPT landscape to a single structure, but the case-by-case eliminations in *"Why Alternative Probabilistic Theories Fail the Fact-*

Production Test" use some of the additional structural conditions (purification in particular) that are not yet fully derived from A1–A4. The precise scope is: within the GPT class satisfying A1–A4 plus the T2 structural conditions, complex Hilbert-space quantum mechanics is the unique survivor. The full elimination using A1–A4 alone awaits the derivation of the structural postulates noted above.

Super-Quantum Theories (PR Boxes) — fail reversible distinguishability structure directly at A2

Popescu-Rohrlich boxes allow correlations stronger than quantum mechanics while formally satisfying no-signalling. They cannot, however, be embedded in a consistent reversible sector. There is no admissible unitary evolution on a PR-box state space that both preserves distinguishability and is operationally closed. PR boxes therefore violate A2 directly — before any T2 narrowing — since their correlations cannot arise from any distinguishability-preserving reversible dynamics.

Spekkens-Type Epistemic Models — fail the interference/symmetry structure of T2

Spekkens' toy model and related epistemic restriction frameworks satisfy several operational conditions — they reproduce some quantum statistics and have a well-defined notion of mixed states and measurements. They are important precisely because they probe the boundary between quantum and classical. However, Spekkens-type models are classical theories with epistemic constraints: the underlying ontology is a classical phase space, and the restrictions are placed on the observer's knowledge rather than on the dynamics. The model's failure is not at the level of composite-system state reconstruction — epistemic states of bipartite Spekkens systems are in fact determined by their local parts, so tomographic locality is satisfied by design. The failure is deeper: Spekkens-type models cannot support the full continuous-phase interference and automorphism-invariant symmetry structure that the admissible reversible sector requires. They reproduce some quantum statistics but fail to generate the complete continuous-phase structure; the interference geometry of the Derivation B conditions (symmetry, automorphism invariance) cannot be realised within a classical-ontology epistemic framework. They are therefore excluded at the T2 level via the reversible-sector geometry route. Their relevance to the programme is that they constitute the most sophisticated known attempt to reproduce quantum phenomenology with a classical-ontology theory, and their failure at T2 strengthens the case that the T2 structural conditions are doing genuine work.

In plain language: Spekkens' toy model is perhaps the most clever known attempt to explain quantum statistics using a classical picture plus restrictions on what an observer can know. It reproduces a surprising amount of quantum-like behaviour. Its failure here — the point at which it cannot mimic genuine quantum mechanics — is precisely what makes the exclusion convincing: even the best classical-with-knowledge-limits theory cannot generate the continuous interference structure that the admissibility framework forces.

The Three Failure Modes

The alternatives fail in one of three structural ways:

Framework	Failure mode	Where excluded
Classical probability	Simplex structure cannot realise coherent superposition or interference	T2, via reversible-sector geometry
Real Hilbert space QM	No continuous phase as fundamental amplitude field	T2, via distinguishability geometry
Quaternionic QM	Excess composite-system structure; automorphism clash	T2, via tomographic locality and automorphism invariance
GPT theories	Fail purification, composability, or measurement closure (see note)	T2 conditions plus A1, A3, or A4
Spekkens-type toy models	Cannot support the full continuous-phase interference and automorphism-invariant symmetry structure; reproduces some quantum statistics but fails at the reversible-sector geometry level	T2, via Derivation B (reversible-sector geometry)
PR-box / super-quantum theories	Cannot be embedded in distinguishability-preserving reversible dynamics	A2 (directly, before T2 narrowing)
Complex Hilbert space QM	**Satisfies all constraints**	**Unique survivor**

Classical probability has too little structure to support admissible reversible dynamics. Real quantum mechanics has too little symmetry. To make the distinction explicit: classical theories fail because they lack coherent superposition entirely; real Hilbert-space theories fail because they admit superpositions but restrict phases to $\{0, \pi\}$, lacking continuous phase. Quaternionic quantum mechanics has too much composite-system structure. All principal known alternatives considered in the programme fail once the admissibility axioms are combined with the T2 structural conditions. The unique survivor — the fixed point of the admissibility constraints within this class — is complex Hilbert-space quantum mechanics.

Structure

```
A1–A4 (admissibility framework)
  + [convexity + tomographic locality + purification + operational closure]
  OR [interference + symmetry + automorphism invariance]
        ↓
  Theorem 2: Finite-dimensional complex Hilbert space
  [unique survivor — classical/real/quaternionic/GPT/PR alternatives all
eliminated]
```

Status: Established — *"Architecture of Fact-Production"*; *"Complex Hilbert Space from Distinguishability Principles"*; *"Why Alternative Probabilistic Theories Fail the Fact-Production Test"*.

In plain language. Level 2 asks: what kind of mathematical space do the admissible transformations live in — and is the answer unique?

The answer is unique. But uniqueness has to be proved, not merely asserted. The proof works by elimination. Every alternative to complex Hilbert space quantum mechanics fails at least one of the four admissibility conditions, and each fails in a different way.

Classical probability theory is too rigid: it only allows mutually exclusive alternatives, never superpositions. Between two recorded events, a classical system cannot explore the interference structure of its state space. Real-number quantum mechanics is too limited: its symmetry group is smaller than required, and it can only produce two kinds of interference (constructive and destructive) rather than the continuous phase rotations that the distinguishability geometry forces. Quaternionic quantum mechanics goes in the opposite direction — it has *too much* structure for composite systems, more than can ever be measured locally, which means it cannot support the kind of locally-recorded facts the framework requires. Within the GPT landscape, the candidate theories that additionally satisfy the T2 structural conditions collapse to complex Hilbert-space quantum mechanics. Stronger-than-quantum correlations (PR boxes) cannot be embedded in any consistent reversible dynamics at all.

One framework survives. Complex Hilbert-space quantum mechanics satisfies every condition simultaneously. This is the load-bearing theorem: not that quantum mechanics works, but that nothing else does.

Level 3: Unitary Dynamics

Reversible evolution on Hilbert space is unitary — and only unitary.

Once the state space is a finite-dimensional complex Hilbert space, the constraint that reversible evolution must preserve distinguishability has a precise mathematical consequence: it must preserve the inner product. Transformations that preserve the inner product on a complex Hilbert space are precisely the **unitary operators**.

This is not an independent postulate — it is forced by the combination of the Hilbert space structure (Level 2) and the distinguishability-preservation requirement (A2). The same constraint appears at Level 0 and Level 3, but its application differs: at Level 0 it motivates the admissibility framework; at Level 3, within the derived Hilbert space, it *uniquely identifies* the class of admissible transformations as unitary.

This is not circularity. It is the same axiom delivering a stronger conclusion once the geometric structure of the state space has been established.

Structure

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Theorem 2 (complex Hilbert space)
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```
+ distinguishability preservation (as inner-product preservation)
      ↓
Theorem 3: Unitary dynamics — U†U = I
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Status: Established — *"Architecture of Fact-Production"*; *"Complex Hilbert Space from Distinguishability Principles"* (symmetry group acts unitarily).

In plain language. Now that we know the universe keeps records in a complex Hilbert space, the question is: how does it *move* within that space between facts?

In a Hilbert space, the "distance" between two states — more precisely, the *inner product* — encodes how distinguishable those states are. An evolution that preserves distinguishability must preserve this inner product. Mathematically, operators that preserve the inner product of a complex Hilbert space are called *unitary operators*: reversible, distance-preserving rotations of the quantum state space.

So quantum evolution is unitary. This is one of the central postulates of standard quantum mechanics — but here it is not postulated. It is derived from the requirement that evolution between facts must not destroy the distinguishability of those facts.

A helpful analogy: unitary evolution is like rotating a rigid object. All the angles and distances within it are preserved. The object changes orientation, but nothing stretches, compresses, or gets mixed up. Quantum states rotate; they do not distort.

Level 4: The Hamiltonian Generator

Continuous unitary evolution requires a generator — and that generator is the Hamiltonian.

The unitary group derived at Level 3 acts continuously (since the underlying evolution is continuous by the admissibility conditions of Level 1). A continuous one-parameter group of unitary operators has, by **Stone's theorem**, a unique self-adjoint generator. That generator is the **Hamiltonian** H.

The argument in *"The Hamiltonian as an Admissibility Generator in VERSF"* runs as follows:

1. **Composability and continuity** (from Level 1) force the evolution to form a one-parameter group: $U(t_1)U(t_2) = U(t_1 + t_2)$.
2. **Distinguishability preservation** forces each $U(t)$ to be unitary (Level 3).
3. **Wigner's theorem** guarantees that symmetry operations on the Hilbert space are implemented unitarily.
4. **Stone's theorem** then requires the existence of a unique self-adjoint operator H such that $U(t) = e^{-iHt/\hbar}$.

The Hamiltonian is therefore not postulated — it is the *necessary generator* of admissible continuous reversible dynamics.

Structure

```
Theorem 3 (unitary one-parameter group)
  [forced by: composability + continuity + Wigner + Stone]
        ↓
  Theorem 4: Self-adjoint generator H — Hamiltonian
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Status: Established — *"The Hamiltonian as an Admissibility Generator in VERSF"*.

In plain language. Unitary evolution tells us *what kind* of transformations are allowed. The Hamiltonian tells us *how fast* they happen and in *which direction*.

Think of a smoothly rotating wheel. The rotation is continuous — the wheel does not jump from one angle to another. Any smooth, continuous rotation must have a generator: the axis around which it rotates, together with the speed. You cannot have smooth rotation without it.

The same is true in quantum mechanics. Because admissible unitary evolution is continuous and composable (you can chain two evolutions together and get a third), there must exist a unique mathematical object that drives it. That object is the Hamiltonian — the operator representing the total energy of the system.

In standard textbook quantum mechanics, the Hamiltonian is introduced as the energy observable and then *assumed* to generate time evolution via the Schrödinger equation. Here the logic runs the other way: the requirement for continuous admissible evolution *forces* the existence of a unique generator, and that generator turns out to be what we call energy. Energy is not a starting point in this program — it is a consequence.

Level 5: Schrödinger Evolution

The Hamiltonian generates the Schrödinger equation.

Once the Hamiltonian H exists as the self-adjoint generator of $U(t) = e^{-iHt/\hbar}$, the Schrödinger equation

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H|\psi(t)\rangle$$

is simply the infinitesimal form of that group action. It requires no new assumptions.

The significance within the VERSF program is not the derivation itself — which is textbook once the Hamiltonian exists — but the *origin* of the Hamiltonian. Because H has been derived from admissibility constraints rather than assumed, the Schrödinger equation inherits that derivation. It is not a postulate of the theory; it is a theorem about what happens between facts in a fact-producing universe.

Structure

```
Theorem 4 (Hamiltonian H)
        ↓
  Theorem 5: iℏ d/dt |ψ⟩ = H|ψ⟩
  [Schrödinger equation as infinitesimal form of admissible reversible
dynamics]
```

Status: Established — *"The Hamiltonian as an Admissibility Generator in VERSF"*; *"Architecture of Fact-Production"*.

In plain language. The Schrödinger equation is the most famous equation in quantum mechanics. It describes how a quantum state evolves over time — how the wavefunction of a particle or system changes from moment to moment.

Once the Hamiltonian exists (Level 4), the Schrödinger equation is simply what you get when you ask how the unitary rotation changes over an infinitesimally small slice of time. It is the instantaneous rate of change — the "per second" version of the group action.

What matters here is not that the equation appears — it must, mathematically — but *why* it is the right equation. In a textbook, the Schrödinger equation is a fundamental law handed down from on high. In the VERSF program, it is a theorem: the inevitable consequence of a universe that produces distinguishable, irreversible facts and evolves continuously between them.

Level 6: Measurement as Irreversible Commitment

What happens when a fact is produced — when possibility collapses into actuality?

The reversible sector (Levels 1–5) describes evolution *between* fact-production events. Level 6 addresses the events themselves.

The derivation in *"Architecture of Fact-Production"* treats measurement as the **minimal irreversible extension** of the reversible sector. Specifically:

- Measurement is an interaction between a quantum system and a recording device that produces a fact.
- Since facts are irreversible commitments (A3), measurement cannot be a unitary process — unitarity is reversible.
- The minimal extension of unitary dynamics that incorporates irreversibility is the class of **completely positive trace-preserving (CPTP) maps**, also known as quantum channels.

- The **Kraus decomposition** of CPTP maps follows as the structural form of this extension.
- **POVMs** (positive operator-valued measures) emerge as the most general class of admissible measurements consistent with the framework.
- **State update** (the post-measurement state) follows from consistency with the distinguishability requirements.

The Stinespring dilation theorem provides the rigorous foundation for this step. The connecting argument is as follows: A3 requires that irreversible commitment produces a valid quantum state — an output that is positive (no negative probabilities) and trace-one (normalised). A4 requires that measurement operations compose consistently with other admissible processes. The only class of maps on density matrices that simultaneously satisfies positivity of outputs (completely positive), trace preservation, and consistent composition under tensor product with arbitrary ancilla systems is precisely CPTP. Any irreversible extension of the reversible sector that satisfies A3 and A4 must therefore be CPTP. Stinespring's theorem then guarantees that any such map factors as a unitary on an extended system followed by a partial trace — which is exactly the structure of a quantum system interacting irreversibly with a recording environment. CPTP maps are therefore not merely *one* way to extend unitary dynamics — they are the *unique minimal* irreversible extension consistent with the admissibility axioms.

Structure

```
A3 (irreversible commitment) + A1 (operational distinguishability and
recordability)
  + Theorem 5 (reversible sector)
        ↓
  Theorem 6: Measurement as minimal irreversible extension
  [unique by Stinespring: CPTP maps / Kraus structure / POVMs / state update]
```

Status: Established — *"Architecture of Fact-Production"*.

In plain language. Everything up to Level 5 describes a smooth, reversible movie: quantum states evolving continuously, with all information preserved. But the universe does not only run as a smooth movie. Things *happen*. A particle hits a detector. A spin is measured. An event gets recorded. At that moment, the reversibility ends.

Measurement is fundamentally different from evolution. Evolution preserves all possibilities; measurement *selects* one. Once a detector has clicked, the click cannot be unclicked. This is the irreversibility from Level 0 showing up again at a higher level.

The derivation asks: what is the *smallest possible extension* of the smooth reversible quantum picture that can accommodate this kind of irreversible event? The answer is a mathematical class called completely positive trace-preserving (CPTP) maps. A CPTP map is any transformation that takes a valid quantum state to another valid quantum state, allows for information to be lost in the process, and never produces negative probabilities. This is the minimal structure capable of representing irreversible events within the quantum framework.

This is the formal mathematical structure of quantum measurement — and it is derived here, not assumed. The whole apparatus of quantum measurement theory (Kraus operators, POVMs, state update after collapse) falls out as the unique consistent treatment of irreversible events in a quantum universe.

Level 7: The Born Rule

How do probabilities arise? The Born rule is the unique admissible probability assignment.

The Born rule — that the probability of outcome i given state ρ and measurement $\{E_i\}$ is $p(i|\rho) = Tr(E_i\rho)$ — is often presented as one of the foundational postulates of quantum mechanics. In the VERSF program, it is derived twice, by independent routes.

Derivation A — Operational Uniqueness

In *"Architecture of Fact-Production"*, the Born rule is shown to be the **unique** probability assignment consistent with six admissibility conditions:

1. **Positivity**: probabilities are non-negative.
2. **Normalization**: probabilities sum to one.
3. **Additivity**: probabilities over mutually exclusive outcomes are additive.
4. **Continuity**: small changes in the state produce small changes in probabilities.
5. **Unitary invariance**: probability assignments are unchanged by reversible relabelling (unitary conjugation).
6. **Non-contextuality**: the probability of an outcome depends only on the state and the measurement, not on which other measurements could have been performed simultaneously.

These six conditions, imposed on the measurement structure of Level 6, uniquely force $p(i|\rho) = Tr(E_i\rho)$.

Derivation B — First-Passage Dynamics (Tick Race)

In *"Quantum Measurement as a Tick Race"*, the Born rule emerges from a dynamical model. Measurement is modelled as a **first-passage time competition** (informally, a "tick race") among quantum channels: each possible outcome corresponds to a channel racing to produce the first irreversible commitment, with transition rates proportional to amplitude squared. The probability of outcome *A* winning the race is:

P(A) = |ψ_A|²

This is a dynamical derivation of the same rule, arising from the structure of the irreversible commitment process itself rather than from abstract operational axioms.

Relationship Between the Two Derivations

The two derivations are consistent by construction: the first-passage dynamical mechanism automatically satisfies all six operational conditions — positivity, normalization, additivity, continuity, unitary invariance, and non-contextuality — so the two approaches are not independent claims that could happen to disagree. The operational derivation identifies the Born rule as the *unique* assignment satisfying those conditions; the dynamical derivation shows that the physical process of irreversible commitment produces *exactly* that assignment. If the tick-race model were modified to use transition rates not proportional to amplitude squared, it would violate at least one of the six operational conditions, and would therefore fall outside the admissible framework established at Level 6.

The practical consequence: if the two derivations ever appeared to give different answers, the fault would lie in the dynamical model, not in the operational characterisation. The operational uniqueness result is the more fundamental of the two — it establishes that no admissible probability rule other than $p(i|\rho) = Tr(E_i\rho)$ exists. The dynamical derivation provides the mechanistic explanation of why that rule is realised by physical fact-production processes.

Unified Theorem Statement

The two derivations can be consolidated into a single theorem:

Any admissible irreversible commitment process on a complex Hilbert state space produces probabilities satisfying the Born rule.

This formulation subsumes both routes: the operational characterisation establishes uniqueness; the first-passage dynamics establishes that the irreversible commitment process is the admissible stochastic mechanism that realises it. The tick-race model is, in this sense, the only admissible stochastic process consistent with unitary reversible dynamics and A3.

Structure

```
Theorem 6 (measurement structure)
  + admissibility constraints [positivity, normalization, additivity,
    continuity, unitary invariance, non-contextuality]
        ↓
  Theorem 7: Born rule — p(i|ρ) = Tr(Eᵢρ)
  [unique operational characterisation + first-passage dynamical mechanism]
```

Status: Established — *"Architecture of Fact-Production"*; *"Quantum Measurement as a Tick Race"*.

In plain language. Quantum mechanics is famously probabilistic. Even if you know the complete quantum state of a system, you cannot predict the outcome of a measurement — only its probability. The Born rule is the formula for those probabilities: the probability of a given outcome is proportional to the *square* of the relevant component of the wavefunction.

This rule is usually just declared as a postulate. But why squared? Why not the amplitude itself, or the cube, or some other function?

The program derives the Born rule two different ways. The first approach asks: given everything we know about quantum states and measurements, what is the *only* probability formula consistent with sensible requirements — non-negative probabilities, everything summing to one, consistency under combining experiments, and so on? The answer is uniquely the squared amplitude rule. There is no other formula that passes all six tests.

The second approach is more physical. It models a quantum measurement as a kind of race — formally, a first-passage time problem: multiple possible outcomes compete to be the first irreversible event, the first "click." The probability of a given outcome winning the race turns out, mathematically, to be exactly the squared amplitude of the corresponding component of the wavefunction.

These two derivations are not two independent claims that might happen to disagree. The race mechanism is built from the same admissibility framework as the six operational conditions, and it automatically satisfies all of them. The operational derivation sets the target — the unique consistent rule — and the dynamical derivation shows that the physical process of fact-production hits it exactly. If someone modified the race model to use different transition rates, it would break at least one of the six conditions and fall outside the admissible framework. The two approaches lock together.

Level 8: Entanglement and Bell Structure

Composite systems are forced to exhibit entanglement; quantum correlations cannot be classical.

Once Hilbert space, tensor product composition, and the Born rule are in place, entanglement and Bell correlations follow as **structural consequences** — not additional postulates.

The argument proceeds in three steps:

1. **Tensor product composition** is forced by tomographic locality (Level 2, Derivation A): the state of a composite system is determined by measurements on its parts, and the minimal state space satisfying this for two Hilbert-space systems is their tensor product $\mathcal{H}_A \otimes \mathcal{H}_B$. This is the *only* composition rule consistent with the admissibility framework.
2. **Entanglement** arises immediately: some states in $\mathcal{H}_A \otimes \mathcal{H}_B$ cannot be written as product states $|\psi_A\rangle \otimes |\psi_B\rangle$. This is a mathematical fact about Hilbert spaces — not an additional physical assumption.
3. **Bell correlations and no-signalling** follow from the Born rule applied to entangled states. The correlations exceed what any local hidden variable theory can produce (Bell's theorem), and the no-signalling condition — that measuring one subsystem cannot

convey information to the other — is preserved because the reduced states are invariant under local unitaries on the distant subsystem.

The VERSF program does not *assume* entanglement as a feature of quantum mechanics. It *derives* entanglement as the inevitable consequence of combining Hilbert-space structure with tensor product composition.

Structure

```
Theorem 2 (Hilbert space) + tensor product composition [forced by tomographic
locality]
        ↓
  Theorem 8a: Entanglement [structural]
  + Theorem 7 (Born rule)
        ↓
  Theorem 8b: Bell correlations + no-signalling
```

Status: Established — follows from *"Architecture of Fact-Production"* (tomographic locality + Born rule).

In plain language. Entanglement is perhaps the most mysterious feature of quantum mechanics. Two particles can become correlated so that measuring one of them instantly tells you something about the other, regardless of the distance between them. Einstein called this "spooky action at a distance" and spent years trying to explain it away. He could not — and subsequent experiments confirmed that the correlations are real.

The VERSF program does not need to explain entanglement away, because it derives it. When two quantum systems interact, the admissibility framework forces them to be described by a combined state space — specifically, the *tensor product* of their individual Hilbert spaces. Within that combined space, most states cannot be factored into a clean "state of A" and "state of B" existing independently. Those unfactorable states are entangled states, and they arise purely as a mathematical consequence of how Hilbert spaces combine.

From the Born rule, we can then calculate the probabilities of measurement outcomes on entangled systems. These probabilities violate Bell's inequalities — the bounds that *any* classical, locally-determined theory must satisfy. The violation is not a puzzle to be explained; it is a theorem to be derived.

Entanglement is not strange quantum magic layered onto an otherwise classical world. It is what composite quantum systems *must* look like in a universe that keeps records the way ours does.

Level 9: Relativistic Extension — Dirac Structure

Adding relativistic admissibility forces spinors and the Dirac equation.

The program extends to relativistic physics by adding a relativistic admissibility condition. This condition — that admissible dynamics must be **first-order in spacetime derivatives** and **locally consistent** — requires justification, because it is not straightforwardly derived from the Level 0 primitives the way earlier levels are.

The argument for first-order locality rests on two pillars. First, special relativity requires that space and time be treated symmetrically: the Schrödinger equation, which is first-order in time but second-order in space, violates this symmetry and is therefore not admissible in a relativistic setting. Second, local consistency — that the evolution at a point depends only on its immediate neighbourhood — is required to prevent fact-production events from depending on spacelike-separated regions, which would conflict with the irreversibility and distinguishability conditions of Level 0 (a fact recorded here cannot depend on an event there if no signal could have connected them).

First-order locality then follows as the *minimal* relativistic extension consistent with these two requirements. Second-order equations (such as the Klein-Gordon equation) fail to satisfy local consistency in the presence of negative-energy solutions, which signal acausality. Higher-order equations introduce additional problems with unitarity and probability conservation.

Important caveat: the derivation of first-order locality as an admissibility condition is currently motivated by consistency with special relativity rather than derived purely from within the fact-production framework. Whether a fully internal derivation — one that does not invoke relativity as an external input but grounds it in admissibility — is achievable is an open problem, and Level 9 should be read as structurally less complete than Levels 0–8 on this point.

Two papers develop this extension.

From "Architecture of Fact-Production"

Imposing first-order local relativistic admissibility on the quantum sector forces:

- **Clifford algebra**: the algebraic structure required to take a first-order square root of the second-order wave operator.
- **Spinors**: the objects on which Clifford algebra acts — representing half-integer spin, the intrinsic angular momentum structure of fermions.
- **Dirac dynamics**: the equation governing spinor field evolution.

From "From Schrödinger to Dirac"

The same result is obtained by a different route: imposing first-order locality on the Schrödinger sector and requiring **square consistency** (the square of the first-order operator must recover the second-order dispersion relation $E^2 = p^2c^2 + m^2c^4$) uniquely forces the Clifford algebra and the Dirac equation.

On Measurement in the Relativistic Sector

The interaction between relativistic dynamics and the measurement / Born rule sector — specifically, how irreversible commitment is handled in a Lorentz-covariant framework — remains an area of active development. This is noted under Open Areas below.

Structure

```
Theorems 3–5 (Schrödinger / unitary sector)
  + relativistic first-order locality
  + square consistency (E² = p²c² + m²c⁴)
        ↓
  Clifford algebra
        ↓
  Theorem 9: Spinors + Dirac equation
```

Status: Established (with caveat) — *"Architecture of Fact-Production"*; *"From Schrödinger to Dirac"*. The derivation of the Clifford/Dirac structure from first-order locality is established. The grounding of first-order locality itself within the admissibility framework remains partially open — see caveat above.

In plain language. Everything up to Level 8 is non-relativistic quantum mechanics — it says nothing about the speed of light or special relativity. Level 9 extends the framework to be consistent with Einstein's relativity.

Special relativity requires that the laws of physics look the same to any observer moving at constant velocity. Applied to quantum mechanics, this means the fundamental equation of motion must treat space and time *symmetrically* — specifically, it must be first-order in both spatial and temporal derivatives.

The Schrödinger equation is first-order in time but second-order in space. It treats them differently, which is why it fails at relativistic speeds. Requiring a first-order relativistic equation creates a famous algebraic obstacle: you cannot take the square root of a sum of squares using ordinary numbers. Paul Dirac solved this in 1928 by introducing new mathematical objects — now called a Clifford algebra — whose elements satisfy special multiplication rules that allow the square root to work.

The VERSF program arrives at precisely this same algebra from its own admissibility route. Out of the Clifford algebra fall *spinors* — the mathematical objects describing particles with half-integer spin, like electrons — and the Dirac equation that governs their evolution. The electron's intrinsic spin, its antiparticle the positron, and the entire relativistic quantum mechanics of fermions emerge as structural consequences of demanding admissible fact production in a relativistic universe.

One honest caveat: Level 9 is slightly less airtight than the levels above it. The requirement for first-order locality is justified by consistency with special relativity, but special relativity itself is not yet derived from within the admissibility framework — it is brought in from outside.

Whether a fully internal derivation of relativistic structure is achievable is an open problem, and this level should be read accordingly.

The Theorem Chain

The dependency ladder can be stated as a sequence of theorems, each following from the last. This is the formal backbone of the programme.

Theorem	Content	Requires
T0	Admissible theory classification	A1–A4
T1	Reversible distinguishability-preserving sector	T0 + continuity + composability
T2	Complex Hilbert space (load-bearing; subject to structural conditions A/B)	T1 + (Derivation A: purification + tomographic locality + convexity) **or** (Derivation B: interference + symmetry + automorphism invariance); full conditions at Level 2 ("On the additional postulates"); eliminates all principal alternatives
T3	Unitary dynamics	T2 + inner-product preservation
T4	Hamiltonian generator	T3 + Stone's theorem
T5	Schrödinger equation	T4 (infinitesimal form)
T6	Measurement / CPTP / POVM structure	A3 + T5; unique by Stinespring
T7	Born rule	T6 + six admissibility conditions; unique; realised by first-passage mechanism
T8	Entanglement / Bell correlations	T2 + T7 + tensor product composition
T9	Dirac equation	T3–T5 + relativistic first-order locality *(established modulo relativistic admissibility assumption — see Level 9 caveat)*

Main Theorem

Any physical theory satisfying A1–A4 and the structural conditions of either Derivation A (convexity, purification, and tomographic locality — the last of which follows directly from A1) or Derivation B (interference, symmetry, automorphism invariance) has a reversible sector equivalent to complex Hilbert-space quantum mechanics.

The qualification reflects the current state of the programme: tomographic locality is derivable from A1, but convexity, purification, and the Derivation B conditions are not yet shown to follow from A1–A4 alone (see "On the additional postulates" at Level 2 and the corresponding Open Area). The programme's target is to remove the qualification — to show that the structural conditions themselves follow from A1–A4 — at which point the Main Theorem simplifies to: *any physical theory satisfying A1–A4 must have a quantum mechanical reversible sector.*

No-Alternative Corollary (within the admissibility class)

None of the following non-quantum candidate frameworks examined in the programme — classical simplex theories, real Hilbert theories, quaternionic theories, Spekkens-type epistemic models, GPT alternatives satisfying the same structural conditions, or PR-box models — survives the classification established by the Main Theorem.

This result is a corollary of T2 together with the alternative-elimination arguments developed across the underlying papers. The admissibility paper (*"Architecture of Fact-Production"*) establishes that only complex quantum mechanics satisfies the full package of continuous transitive symmetry, tomographic locality, full interference structure, and consistent subsystem composition simultaneously — and explicitly discusses the failure of real and quaternionic alternatives, PR-box theories, and Spekkens-type models. The distinguishability-geometry paper (*"Complex Hilbert Space from Distinguishability Principles"*) gives the explicit exclusion of real and quaternionic amplitude structures and identifies complex amplitudes as the unique viable survivor. Section §2.1 of the present document assembles those elimination arguments into a single dependency-map formulation.

The corollary label is deliberate: this result follows from T2 and the cited papers; it does not claim to classify all logically conceivable theories, only those within the standard candidate classes that have been examined.

Fixed-Point Statement

The programme's conclusion can be given a precise fixed-point formulation. Define a *fact-producing theory* as any theory satisfying A1–A4. Define the *admissibility flow* as the progressive imposition of the structural constraints — T0 through T2 — on the space of such theories. Under this flow, the space of candidate theories is progressively narrowed: T0 reduces the space to the operational class; T1 imposes the reversible sector structure; T2, together with the alternative elimination at §2.1, reduces it to a single element.

Quantum mechanics is the unique fixed point of the admissibility flow among theories satisfying A1–A4 and the structural conditions required for T2.

This is a precise claim, not a metaphor. "Fixed point" here means: the unique theory that both satisfies A1–A4 and is left unchanged — not further constrained or eliminated — when all admissibility conditions are applied simultaneously. Every other candidate theory is moved (constrained or excluded) by at least one condition. Quantum mechanics alone is stable under the complete set.

Note on the space of theories. The "admissibility flow" is not a continuous map on a metric space, and "fixed point" is not being used in the topological sense of a fixed point of a homeomorphism. The space of candidate theories has the structure of a partially ordered set under the relation of constraint satisfaction; the admissibility flow is the sequential application of structural filters; and the fixed point is the unique element that passes through all filters unchanged. The terminology is chosen for its conceptual precision — a theory is a "fixed point" if imposing the admissibility conditions on it produces the same theory — not to invoke the full apparatus of fixed-point theorems in analysis.

```
A1–A4
        ↓
  T0: admissibility class
        ↓
  T1: reversible sector
        ↓
  T2: complex Hilbert space  ← load-bearing theorem (subject to structural
conditions A/B)
        ↓
  T3: unitary dynamics
        ↓
  T4: Hamiltonian generator
        ↓
  T5: Schrödinger evolution
        ↓
      ┌─────────────────────────────────────┐
      ↓                                     ↓
  [Reversible branch]           [Irreversible branch]
  T3: unitary evolution         T6: measurement / CPTP
  T4: Hamiltonian               T7: Born rule
  T5: Schrödinger               [unique by Stinespring
                                  + six conditions]
      │                                     │
      └──────────────────┬──────────────────┘
                         ↓
              T8: entanglement / Bell correlations
                         ↓
              T9: Clifford algebra / spinors / Dirac equation
```

Architectural Summary

The derivation has a natural two-branch structure.

Reversible Branch

admissibility → Hilbert space → unitary evolution → Hamiltonian → Schrödinger dynamics

This branch answers the question: *what are the allowed transformations of a physical system between facts?*

Irreversible Branch

admissibility → commitment / measurement → POVMs / CPTP maps → Born probabilities → fact production

This branch answers the question: *what happens when a possibility becomes an actuality?*

Convergence and Main Theorem

The two branches are not independent. They share their origin (A1–A4), and they converge in T8 (entanglement), where reversible superposition and irreversible probability combine. Their joint conclusion is the Main Theorem and No-Alternative Corollary stated in the theorem chain above:

Quantum mechanics is the unique fixed point of the admissibility flow among theories satisfying A1–A4 and the structural conditions required for T2.

The unique theory consistent with the admissibility conditions — the only fact-producing theory left invariant when all structural constraints are applied simultaneously, proved by structural elimination of the principal known alternative frameworks.

Current Status

Established in the Papers

Result	Source
Admissibility framework from fact-production	*Architecture of Fact-Production*
Finite distinguishability as primitive	*Architecture of Fact-Production*
Complex Hilbert space emergence	*Architecture of Fact-Production*; *Complex Hilbert Space from Distinguishability Principles*
Unitary dynamics	Both above
Hamiltonian necessity	*Hamiltonian as Admissibility Generator*
Schrödinger evolution	*Hamiltonian as Admissibility Generator*; *Architecture of Fact-Production*
Measurement / CPTP / POVM structure	*Architecture of Fact-Production*
Born rule (operational)	*Architecture of Fact-Production*
Born rule (tick-race dynamical)	*Quantum Measurement as a Tick Race*
Entanglement and Bell correlations	Follows from *Architecture of Fact-Production* (tomographic locality + Born rule); no standalone paper currently exists for this result

Result	Source
Dirac extension	*Architecture of Fact-Production*; *From Schrödinger to Dirac*

Open and Developing Areas

Derivation of structural postulates from A1–A4. As noted at Level 2, the derivation of complex Hilbert space (T2) currently requires structural conditions — convexity, purification, and the interference/symmetry/automorphism invariance conditions — that are imposed as additional postulates rather than derived from A1–A4 alone. Tomographic locality is derivable from A1; the others are not yet shown to follow. Establishing whether convexity and purification are entailed by the admissibility axioms, or whether they are genuinely independent, would either tighten the main theorem or clarify its true scope.

In plain language: the programme's central claim is that four basic conditions on fact-production force quantum mechanics. That claim is almost, but not quite, fully established — the derivation of one key intermediate step currently needs a few extra assumptions that may or may not themselves be consequences of the original four. Closing this gap is an important internal task.

Full quantum field theory completion. The extension from first-quantised Dirac dynamics to full QFT — including gauge fields, renormalisation, and the Standard Model interaction structure — remains to be derived within the admissibility framework. This is the most significant open frontier.

In plain language: the Dirac equation describes one electron moving relativistically. Quantum field theory describes any number of particles being created and destroyed, interacting via forces like electromagnetism and the weak nuclear force. Deriving that richer structure from admissibility principles is the next major challenge.

Gravity integration. How spacetime curvature emerges from or integrates with the admissibility framework is not yet established. This is the VERSF analogue of the quantum gravity problem.

In plain language: Einstein's general relativity describes gravity as the curvature of spacetime. How that curvature arises in a universe defined by fact-production — and whether it too can be derived from admissibility — is an open question and perhaps the deepest one in the program.

Interaction selection. The current derivations establish the kinematic structure of quantum mechanics (state spaces, dynamics, measurement) but do not yet derive *which* interactions are admissible. A derivation of the specific interaction terms in the Standard Model Lagrangian from admissibility principles would significantly strengthen the program.

In plain language: the framework tells us the rules of the game — the allowed kinds of evolution and measurement. It does not yet explain why the specific forces of nature (electromagnetism, the weak force, the strong force) have the particular forms they do, rather than some other admissible alternatives.

Scale fixing and cross-sector integration. The wider VERSF/BCB programme already contains proposed derivations of key scales and masses — including $\hbar$, c, baryon masses, heavy quark masses, and the Higgs vacuum scale — through the Ticks-Per-Bit (TPB) and Bit Conservation and Balance (BCB) information-theoretic mechanisms. What remains open is not scale derivation in general, but the integration of those derivations into a single unified dependency chain connecting the quantum reconstruction, relativistic sector, and particle-physics sector in fully journal-standard form. To be precise about the current division of labour: the quantum-structure derivations are largely mapped in this document; the scale-setting derivations are present in separate TPB/BCB papers; what is incomplete is a single dependency ladder that subordinates all of them to a common root and presents the connections at publication standard.

In plain language: this document maps how quantum mechanics emerges from the fact-production framework. Separately, other papers in the programme derive numerical quantities — the speed of light, Planck's constant, particle masses — from information-theoretic principles. The open task is not finding those derivations (they exist), but weaving them into the same chain as the Hilbert space and Born rule results, so that the full programme — structure and numbers together — hangs from a single set of starting assumptions. Think of it as two completed jigsaws that need to be fitted into one larger picture.

Relativistic measurement and Lorentz-covariant irreversibility. The relationship between the measurement sector (Level 6–7) and the relativistic sector (Level 9) requires further development. In particular, a fully Lorentz-covariant treatment of irreversible commitment is needed.

In plain language: special relativity says that different observers moving at different speeds can disagree about the order in which events occur. But measurement is supposed to be an irreversible commitment — a fixed fact. Reconciling these two requirements in a fully consistent relativistic framework remains technically unfinished.

Experimental falsifiability programme. The framework makes specific structural claims — e.g., that quantum mechanics is the *unique* admissible theory, not merely one possibility. Identifying experimental signatures that would distinguish this programme from standard quantum mechanics, or from rival reconstruction programmes (Hardy, CDP), would sharpen the empirical content of the work.

In plain language: a theory that cannot, even in principle, be tested by experiment is not science — it is philosophy. Identifying concrete predictions that the VERSF program makes, which differ from what rival theories predict, is an important ongoing task. The claim that quantum mechanics is the unique *admissible theory is a powerful one; finding a way to test it would be a major advance.*

One-Sentence Summary

Facts force admissibility; admissibility forces the reversible quantum sector; irreversible commitment forces measurement and probability; together they force quantum mechanics as their unique admissible fixed point — and establishing that conclusion in full generality from A1–A4 alone is what this programme sets out to prove.

"Facts force admissibility"

Start with the most basic thing the universe has to do: produce facts. Not just possibilities or mathematical abstractions, but actual recorded events — things that happened, left a mark, and can be told apart from other things that happened. The moment you insist on that, you immediately discover the universe has to obey certain rules. It cannot be completely chaotic. It has to allow distinctions to be preserved, records to stick, and changes to compose consistently. Those rules are what the document calls the admissibility framework. Facts do not merely *fit* those rules — they *force* them. A universe without those rules could not contain facts at all.

"Admissibility forces the reversible quantum sector"

Once you have those rules in place, you can ask: what kind of mathematical space does the universe have to evolve in, between the moments when facts are produced? The answer turns out to be uniquely determined. It cannot be classical probability — that cannot support interference. It cannot be real numbers — those do not have enough symmetry. It cannot be quaternions — those give composite systems too much structure. The one mathematical framework that threads all the constraints is a complex Hilbert space with unitary dynamics. The admissibility rules do not suggest this — they compel it.

"Irreversible commitment forces measurement and probability"

Facts are not just distinguishable — they are permanent. Once something happens, it has happened. That irreversibility is not a side effect; it is part of what makes something a fact rather than a mere possibility. When you ask what kind of physical process can turn a quantum possibility into a committed fact, the mathematics gives a unique answer: CPTP maps, the Born rule, the full apparatus of quantum measurement. And the reason the probability formula is "squared amplitude" rather than anything else is that it is the *only* formula consistent with the structure forced by the steps above.

"Together they force quantum mechanics as their unique admissible fixed point"

Put all of this together — the reversible dynamics, the irreversible measurement, the probability rule — and what you have is not a family of possible theories but a single one. Quantum mechanics is the only framework that satisfies all the constraints simultaneously. Every alternative either lacks interference, or lacks symmetry, or lacks the right composite-system structure, or cannot be embedded in any consistent reversible dynamics. Quantum mechanics alone passes through every filter unchanged. It is the fixed point: the one theory the universe must be, if it is to contain records at all.

"And establishing that conclusion in full generality from A1–A4 alone is what this programme sets out to prove"

The honest addendum. Most of the chain is established. But one step — showing that certain structural conditions used in the derivation of the Hilbert space result themselves follow from the four basic axioms, rather than being assumed separately — is not yet complete. The programme knows where the gap is, knows what closing it would require, and is working toward it. The sentence ends not on a claim but on a commitment.