A No-Alternative Theorem for the VERSF Response Functional

Why the Commitment Response Must Be the Retarded Klein–Gordon Operator

---

Plain-Language Summary

VERSF treats reality as something that becomes definite one fact at a time. Whenever a fact is **committed** — when something has irreversibly happened, in a way that nothing later can rewrite — that commitment leaves a trace in the structure of reality. That trace is the *commitment response*: a kind of ripple that propagates outward and tells the rest of the world that this fact is now part of the record.

The natural question is: **what shape must that ripple take?** Could it spread out instantly? Could it travel backwards in time? Could it depend on facts in some complicated, twisted way? Could it have any one of a thousand mathematical forms?

This paper shows that the answer is: **no, only one form is possible.** Once you accept a small number of basic, physically reasonable requirements — that facts add up the way counts of things should add up, that causes come before effects, that the ripple has finite reach, that there are no hidden ingredients smuggled in unaccounted for — every other choice is ruled out. What remains is exactly the equation that, in standard physics, describes a massive particle: the *retarded Klein–Gordon equation*. The word "retarded" here just means "respecting the direction of time" — the response comes after, never before, the fact that produced it.

The deeper point is this: VERSF does not have to *postulate* this equation as an extra assumption. The equation is **forced** by structure already established elsewhere in the framework. Every requirement we lean on is itself something VERSF has previously derived, not something added by hand. So the propagation of commitment is not a model someone chose; it is the only shape reality can take, given how commitment works.

In short: **once you know what a fact is, you already know how facts speak.**

Table of Contents

Abstract

In VERSF, the response functional

$\kappa = F[\rho\_committed]$

is not a modelling choice. Under eight admissibility conditions (A1, A3–A9, with A2 derived as Lemma 0), each independently grounded in established results of the VERSF programme, the only possible form is the retarded massive scalar response

$(\square + m^2)\ \kappa = \rho\_committed,\ F\rho = \int G\_ret(x - x')\ \rho(x')\ d^4x'.$

The novelty of this theorem is not in the deductive machinery, which is largely standard, but in the demonstration that each admissibility condition is independently *forced* — not assumed — by other established VERSF results. The κ-field is therefore not a free postulate appended to the framework; it is the unique admissible propagation mode of irreversible commitment.

---

1. Statement of the Theorem

The strategy of this paper is not to derive a candidate response functional and argue for its plausibility, but to show that **every alternative fails** under the structural constraints already established by VERSF. The derivation proceeds by demonstrating that the admissibility constraints define a shrinking class of operators whose intersection contains exactly one element:

each assumption A1, A3–A9 cuts out a class of inadmissible response functionals, and the conjunction of these cuts leaves only the retarded inverse of the massive Klein–Gordon operator standing. The conclusion is reached by exclusion, not by construction; the proof is a constraint-intersection argument.

Theorem (No Alternative Response Functional). *Let F be the physical response functional mapping committed distinguishability density* ρ_committed *into the propagating commitment response field κ:*

κ = F[ρ_committed].

Assume the following eight admissibility conditions (A1, A3–A9; A2 is derived as Lemma 0 in §2):

#	Assumption	Status in VERSF	Source
A1	**Additivity of commitment sources.** Disjoint commitment events contribute independently.	Derived	Equivalence paper §3
A3	**Locality.** The response is generated by local field dynamics on the emergent spacetime.	Effective constraint	Finite commitment cost + causal separability
A4	**Lorentz covariance.** The response respects the emergent invariant causal structure.	Derived	Proto-time / emergent Lorentz paper
A5	**Finite propagation speed.** No commitment influence propagates faster than c.	Derived	Emergent causal structure
A6	**Minimal field content.** No additional independent hidden fields are introduced.	Derived	A0 (invariant physical content) + A1′ (entropy-accounted commitment)
A7	**Second-order admissibility.** The field equation contains no higher-derivative ghost modes.	Ostrogradsky	Standard ghost-freedom theorem
A8	**Finite coherence scale.** The response has a non-zero mass scale fixed by ξ.	Derived	K=7 / bit-tick coherence papers
A9	**Retarded boundary conditions.** No response precedes its commitment source.	Derived	CCC paper, irreversibility of fact production

(**Lemma 0**, proved in §2: under A1, A4, and A6, the response field κ is a real Lorentz scalar. This is what was previously listed as A2; it is derived rather than assumed.)

Then the only admissible response functional is

$F = (\Box + m^2)^{-1}\_\mathrm{ret}$, $\kappa(x) = \int G\_\mathrm{ret}(x - x') \rho\_\mathrm{committed}(x') d^4x'$.

No nonlinear, nonlocal, massless, higher-derivative, advanced, acausal, multi-field, or Lorentz-violating alternative satisfies all assumptions.

Remarks on the status of A3 and A6

On A3 (locality). Locality here is not imposed as a microscopic assumption but is the effective requirement that distinguishability be encoded through finite causal interfaces. A nonlocal response would require global distinguishability coordination, which is incompatible with finite commitment cost and with the **causal separability** of independent commitment histories — by which we mean: commitment events located at spacelike-separated spacetime points contribute to ρ_committed independently, and their respective responses do not couple in any way that depends on the joint configuration of both events outside their common causal future. Equivalently, no information about commitments outside the past light cone of x is encoded in κ(x). Furthermore, any *fundamentally* nonlocal response would require distinguishability to be globally coordinated across spacelike-separated regions without intermediate encoding, violating the requirement that commitment cost be locally accounted: every contribution to κ must be balanced against a corresponding entry in the commitment ledger at the location where it is encoded. Locality is therefore an emergent constraint forced by the commitment economy of VERSF, not an arbitrary postulate, and the constraint operates at the fundamental level rather than only effectively.

On A6 (minimal field content). Additional independent fields would carry degrees of freedom not sourced by committed distinguishability, violating A0 (invariant physical content) and A1′ (entropy-accounted commitment): every propagating degree of freedom must be ledger-balanced against a corresponding commitment cost. Multi-field extensions are therefore not merely unnecessary but **structurally inadmissible** within VERSF's accounting. A6 is a derived closure condition, not parsimony.

---

2. Proof

The argument proceeds via Lemma 0 followed by six steps. Lemma 0 establishes the scalar character of κ. Steps 1 and 1.5 establish that the field equation must be linear with a local Lorentz-covariant differential operator; Steps 2–5 then deduce the unique form of that operator and its inverse.

Lemma 0 (Scalar character of κ) — Derived from A1, A4, A6

Since ρ_committed is a scalar density (a count of committed distinguishability events per spacetime four-volume, invariant under the emergent Lorentz group), the response field κ generated by it must transform compatibly. Under A6 (minimal field content), no additional indices may be introduced beyond those required by the source. A vector, tensor, or spinor response would carry indices unsourced by ρ, requiring auxiliary structure (a preferred direction, a metric perturbation, an internal Clifford algebra) that violates minimality.

Furthermore, ρ_committed is **real** — it is a non-negative count density, not a complex amplitude. A complex scalar response would carry an unsourced U(1) phase, again violating A6.

Therefore κ is a **real scalar field**.

More fundamentally: any non-scalar response would introduce tensorial structure not present in the source, and that extra structure could not be invariantly attributed to commitment events themselves. This violates the **closure of distinguishability content** — the requirement that every component of κ trace back to a counted commitment in ρ. Non-scalar responses are therefore not just unsourced but unaccounted: they break the ledger.

This establishes Lemma 0: κ is a real Lorentz scalar throughout the remainder of the proof, derived from A1, A4, and A6 rather than postulated. ■

Step 1 — Additivity forces linearity of F

Commitment events are binary and independently attributable. For sources with disjoint support,

$\rho\_\text{committed}(R_1 \cup R_2) = \rho\_\text{committed}(R_1) + \rho\_\text{committed}(R_2),\ R_1 \cap R_2 = \emptyset.$

A1 therefore implies disjoint-support additivity of F:

$F[\rho_1 + \rho_2] = F[\rho_1] + F[\rho_2]$ whenever $\text{supp}(\rho_1) \cap \text{supp}(\rho_2) = \emptyset$.

To extend to general (possibly overlapping) ρ_1, ρ_2, we use the **event ontology** of VERSF. The committed distinguishability density ρ_committed is fundamentally a sum of point contributions at commitment events:

$\rho\_\text{committed}(x) = \Sigma_i\, w_i\, \delta^4(x - x_i),$

where $\{x_i\}$ are the spacetime locations of commitment events and w_i are their weights. Finite sums of delta functions at distinct events have disjoint supports trivially. Such finite sums are dense — in the distributional (Schwartz) topology on the space $\mathcal{S}'(\mathbb{R}^4)$ of tempered distributions — in the space of physically admissible source distributions. Disjoint-support additivity applies directly to each finite sum; continuity of F (established below) extends additivity to the closure:

$F[\rho_1 + \rho_2] = F[\rho_1] + F[\rho_2]\ \forall\, \rho_1, \rho_2 \in \mathcal{S}'(\mathbb{R}^4)$ admissible.

This argument uses VERSF's event ontology directly: the discreteness of commitment events is the structural property that makes disjoint-support decomposition natural, not a partition-of-unity refinement of a smooth density.

On the function space. The response F is defined on the space of physically admissible source distributions — finite-energy, compactly supported or rapidly decaying tempered distributions, equipped with the Schwartz topology. This is the natural setting because admissible sources are sums of finitely many event contributions plus their distributional limits.

Continuity of F. Continuity is not an extra postulate. It follows from the requirement that arbitrarily small perturbations of commitment density produce arbitrarily small changes in response; otherwise the mapping ρ_committed ↦ κ would fail to represent distinguishability-preserving physical encoding. A discontinuous F would generate finite response from infinitesimal commitment, violating the entropy-accounted commitment principle (A1′).

More fundamentally, a discontinuous mapping would allow finite response from arbitrarily small commitment perturbations, violating the requirement that distinguishability be preserved **quantitatively under refinement**. Such behaviour would break the monotonic encoding of commitment — the property that more commitment produces proportionally more response, with no jumps generated by nothing — and is therefore inadmissible. Continuity is not a smoothness preference; it is the information-consistency condition for the encoding to faithfully track the ledger.

Combined with the natural scaling $F[\lambda\rho] = \lambda F[\rho]$ inherited from the linearity of distinguishability counting, this gives **F linear** in ρ.

This rules out nonlinear sourcing.

Step 1.5 — From linear F to a local differential field equation

Step 1 establishes that F: ρ_committed ↦ κ is a continuous linear map on $\mathcal{S}'(\mathbb{R}^4)$. To deduce the form of the field equation relating κ to ρ_committed, three additional structural inputs are required:

(i) Translation invariance. A4 (Lorentz covariance) includes invariance under spacetime translations as part of the full Poincaré group, of which the Lorentz group is the homogeneous subgroup. The Poincaré symmetry of the emergent causal structure therefore guarantees that response is invariant under $x \to x + a$ for any constant 4-vector a, ensuring that the response depends only on coordinate differences $x - x'$. Concretely: F commutes with translations, $F\rho(\cdot - a) = F[\rho](x - a)$ for any constant 4-vector a.

(ii) Schwartz kernel theorem. A continuous linear map on tempered distributions has a unique distributional kernel K such that

$$\kappa(x) = \langle K(x, \cdot), \rho \rangle.$$

(See Hörmander, *The Analysis of Linear Partial Differential Operators*, Vol. I, §5.2, or Reed–Simon, *Methods of Modern Mathematical Physics*, Vol. I, §V.3.) Translation invariance forces $K(x, x') = K(x - x')$, so F is a **convolution**:

$\kappa(x) = \int K(x - x') \rho\_committed(x') d^4x'$.

This is purely a consequence of linearity, continuity, and translation invariance — no further physical input.

(iii) Locality reduces convolution to differential inverse. A3 (locality) requires that κ at x is determined by ρ_committed in a neighbourhood of x — not by a smeared integral over distant regions. By **Peetre's theorem** (any local linear operator on a function space is a differential operator; see Peetre, *Une caractérisation abstraite des opérateurs différentiels*, Math. Scand. 7, 211–218, 1959), the locality requirement A3 forces K to be a finite combination of δ and its derivatives — equivalently, the fundamental solution of a linear differential operator $\mathcal{D}$. Since κ is required to depend only on the local germ of ρ_committed in spacetime and to be continuous in the Schwartz topology on the space of physically admissible source distributions, Peetre's theorem applies directly: the function-space hypotheses are met by construction, and the conclusion that the operator is differential is forced. Therefore

$\mathcal{D}\, \kappa = \rho\_committed$,

with $K = \mathcal{D}^{-1}$ as a fundamental solution. Locality is what reduces a generic convolution to the inverse of a differential operator; without it, an arbitrary nonlocal kernel would remain admissible.

Outcome of Step 1.5. The field equation has the form $\mathcal{D}\, \kappa = \rho\_committed$, where $\mathcal{D}$ is some linear differential operator with constant coefficients (translation invariance) acting on a real Lorentz scalar (Lemma 0). The form of $\mathcal{D}$ is fixed in Step 3.

This is the bridge from "F is linear" to "κ satisfies a local PDE." Without it, the no-go argument would only constrain candidate PDEs, not derive that the response satisfies a PDE in the first place.

Step 2 — Linearity excludes nonlinear self-interaction in the field equation

Suppose, contrary to the conclusion, that κ satisfies a field equation with nonlinear self-interaction:

$\mathcal{D}\, \kappa = \rho\_committed + N(\kappa)$,

where $\mathcal{D}$ is some linear differential operator and N is a nonlinear functional of κ. Solving formally,

$\kappa = \mathcal{D}^{-1}\, \rho\_committed + \mathcal{D}^{-1}\, N(\kappa)$,

so κ depends on ρ through a series with terms of order higher than first. Under rescaling $\rho \to \lambda\rho$, linearity of F (Step 1) requires $\kappa \to \lambda\kappa$ exactly. But then

$N(\lambda\kappa) = \lambda\, N(\kappa)\ \forall\, \lambda \in \mathbb{R}$, which forces N to be **homogeneous of degree 1**, i.e. linear *within the class of analytic local functionals admitted by A3*. (Strictly: degree-1 homogeneity does not imply linearity in full generality — pathological functionals such as $N(\kappa) = |\kappa|$ or $N(\kappa) = \kappa \cdot \mathrm{sign}(\partial_0\kappa)$ are degree-1 homogeneous but nonlinear. However, A3 restricts N to the class of analytic local functionals built polynomially from κ and its derivatives, since non-analytic dependence would propagate distributional singularities not present in ρ_committed and would violate the ledger-tracking requirement. Within this class, $N(\kappa) = \Sigma_k a_k \kappa^k$, and $N(\lambda\kappa) = \lambda N(\kappa)$ forces $a_k = 0$ for all $k \neq 1$.) A linear N is just a renormalisation absorbed into $\mathcal{D}$.

Therefore the field equation relating κ to ρ_committed is **linear**:

$\mathcal{D}\,\kappa = \rho\_committed$.

Ontological version. The mathematical exclusion above has a direct ontological reading. Nonlinear self-interaction would imply that the response to a set of commitment events depends on the *presence of other independent events* — that fact A's contribution to κ is modulated by the simultaneous occurrence of fact B, even when A and B are independently attributable. In ontological terms, this is the requirement that independently attributable facts contribute independently to the response — the field-theoretic correlate of the additivity already imposed at the source level by A1. The ontological reading is therefore not an additional argument but a re-reading of A1 in the κ-channel: whatever independence is asserted at the level of ρ_committed must propagate through F to κ, and a nonlinear N would interrupt that propagation.

A5 (finite propagation) and the causal-separability property of independent commitment histories serve here as a consistency check on the derived linear structure: the retarded inverse of a linear hyperbolic operator automatically respects spacelike-separation independence.

Step 3 — Locality, Lorentz covariance, and second-order admissibility fix $\mathcal{D}$

By Lemma 0, κ is a real Lorentz scalar. By A3 (locality), $\mathcal{D}$ is a differential operator with no integral kernel. By A4 (Lorentz covariance), $\mathcal{D}$ is built from Lorentz-invariant scalars constructed from ∂_μ acting on κ.

The Lorentz-invariant local differential operators on a real scalar field, ordered by derivative count, are:

$1, \Box, \Box^2, \Box^3, \ldots$

(odd-order invariants vanish for a scalar). A general admissible operator has the form

$\mathcal{D} = b + a\,\Box + c_2\,\Box^2 + c_3\,\Box^3 + \cdots$.

A7 (second-order admissibility) eliminates all terms beyond □. By **Ostrogradsky's theorem**, any non-degenerate Lagrangian containing time derivatives of order higher than first generates a Hamiltonian that is unbounded below and a spectrum containing negative-norm ghost states. The non-degeneracy clause matters: degenerate higher-derivative Lagrangians (where the highestderivative term enters only through a constraint or a total derivative) can sometimes evade the ghost; but a non-degenerate operator $c_k \Box^k$ with $c_k \neq 0$ and $k \geq 2$ is generic and cannot be cast in degenerate form on Minkowski space. Higher-order operators of this generic form therefore violate the unitarity and positivity conditions required for a physical response field.

There is also a stronger ontological version of this exclusion specific to VERSF. Higher-order derivatives introduce additional independent propagating modes — extra solutions of the field equation, dynamically distinct from the Klein–Gordon mode — which would correspond to distinguishability degrees of freedom *not sourced by commitment events*. They would be free oscillations of κ disconnected from the ledger of $\rho_{committed}$, violating minimal field content (A6) at the level of dynamical content rather than just static field count. Ghost modes are therefore inadmissible in VERSF for two independent reasons: standard unitarity, and the ledger-closure requirement that every degree of freedom of κ trace back to a counted commitment.

Hence $c_k = 0$ for $k \geq 2$.

The operator reduces to

$$\mathcal{D} = a \Box + b.$$

The overall coefficient a is absorbed into the field-strength normalisation of κ, leaving the single physical parameter $m^2 \equiv b/a$:

$$\mathcal{D} = \Box + m^2.$$

This matches the previously established uniqueness of the commitment-field Lagrangian,

$$\mathscr{L} = \tfrac{1}{2} (\partial_\mu \kappa)(\partial^\mu \kappa) - \tfrac{1}{2} m^2 \kappa^2 + \kappa \rho_{committed}.$$

Step 4 — Finite coherence excludes the massless case

The argument here is that the κ-dynamics must contain ξ as an intrinsic scale, and a massless equation cannot.

Intrinsic-scale argument. The retarded Green's function of the massless wave operator $\Box$ is **scale-free**: under the rescaling $x \to \alpha x$, $G_{ret}(\Box)(x) \to \alpha^{-2} G_{ret}(\Box)(x)$, with no characteristic length. Consequently, the κ-dynamics generated by $\mathcal{D} = \Box$ contain *no intrinsic length scale at all*. Any length scale appearing in physical predictions would have to be imported from outside the κ-dynamics — from initial conditions, from boundary data, or from coupling to additional fields. But A8 requires that the coherence scale ξ be a property of the response itself, fixed by VERSF's K=7 / Fano closure architecture and entering the κ-dynamics intrinsically. A massless κ cannot satisfy this requirement: ξ would have to enter from outside κ-dynamics, contradicting A8 directly.

The only way for a Lorentz-covariant scalar field equation $\mathcal{D}\,\kappa = \rho$ to contain an intrinsic length scale is for $\mathcal{D}$ to carry a dimensional parameter. With $\mathcal{D} = \Box + m^2$ this parameter is m, and the corresponding length is m^{-1}. Therefore

$m \neq 0$.

Ontological version. A massless response would imply scale-free propagation of commitment influence — no characteristic length at which one fact ends and another begins. This contradicts the existence of a minimum coherence scale at which independent fact formation becomes well-defined. Without such a scale, commitment could not localise into discrete facts at all: the ledger would smear into a continuum with no individuated entries. The existence of facts as countable, attributable events therefore *requires* $m \neq 0$; a massless κ-field is not just inconsistent with VERSF's measured ξ, it is inconsistent with there being facts to commit in the first place.

This is all the present theorem requires. The **specific value**

$m^2 = (4/3)\,\xi^{-2}$

is fixed by independent VERSF inputs — the K=7 closure architecture and the Hessian eigenvalue calculation — and is *not* derived here. The no-go theorem establishes the existence of a non-zero mass; the magnitude is the content of a separate result.

Step 5 — Retarded boundary conditions select the unique inverse

The operator $(\Box + m^2)$ admits multiple formal inverses, corresponding to different Green's functions on Minkowski space:

G_{ret}, G_{adv}, G_F (Feynman), $G_{\bar{F}}$ (anti-Feynman), $G_{sym} = \frac{1}{2}(G_{ret} + G_{adv})$.

A9 requires that no response precede its commitment source: response is supported only in the causal future of the source. This selects G_{ret} uniquely, since:

- G_{adv} propagates response into the causal past (excluded);
- G_F, $G_{\bar{F}}$ propagate components in both time directions (excluded);
- G_{sym} erases the irreversibility of fact production (excluded by CCC).

The equivalence between the Lagrangian field s and the response field κ under retarded boundary conditions has been established separately:

$s(x, t) \equiv \kappa(x, t)$ under G_{ret}.

Therefore

$$F = (\Box + m^2)^{-1}_{ret}$$

and

$\kappa(x) = \int G\_ret(x - x') \rho\_committed(x') d^4x'$.

■

---

3. Exclusion Table

Alternative F	Excluded by	Mechanism
Nonlinear in ρ	A1 (Step 1)	Event-decomposition density + disjoint additivity + continuity force linearity
Nonlinear self-interaction in κ	A1 (Step 2)	Linearity of F forces N(κ) homogeneous of degree 1 → linear
Nonlocal kernel	A3, A6, A7	Requires either an infinite tower of derivatives (violates A7) or auxiliary fields (violates A6)
Higher-derivative ($\Box^2$ etc.)	A7 (Ostrogradsky)	Generates ghost modes and unbounded Hamiltonian
Massless ($m = 0$)	A8	Violates finite coherence scale ξ
Vector / tensor / spinor response	Lemma 0, A6	Introduces unsourced indices
Complex scalar response	A6	Introduces unsourced U(1) phase
Multiple independent fields	A6	Violates minimal field content
Lorentz-violating	A4	Violates emergent invariance
Advanced Green's function	A9	Response precedes source
Feynman / anti-Feynman propagator	A9	Mixes time directions
Symmetric Green's function	A9, CCC	Erases irreversibility of fact production
Arbitrary kernel	A1, A3, A4	Replaced by derived retarded Green's function

---

4. Status of the Result

The theorem is, strictly, a uniqueness result conditional on the eight admissibility conditions A1, A3–A9 (with A2 derived as Lemma 0). The substantive physical content of VERSF is concentrated in those conditions, not in the deductive step from conditions to $(\Box + m^2) \kappa = \rho\_committed$, which uses largely standard tools from PDE theory and Lagrangian field theory.

What this theorem does establish, beyond textbook content, is:

1. **No admissibility condition is free.** Each of the eight assumptions is either derived elsewhere in the VERSF programme (A1, A4, A5, A8, A9), grounded in standard quantum-field-theoretic ghost-freedom (A7), or follows as an effective constraint forced by VERSF's commitment economy (A3 from finite commitment cost and causal separability; A6 from A0 and A1′ as a closure condition on distinguishability content). The scalar character of κ, previously listed as A2, is derived as Lemma 0 from A1, A4, and A6. No assumption is imposed by parsimony alone.
2. **The κ-field is not introduced; it is forced.** Once the source structure of $\rho_{committed}$ is fixed by the commitment programme, no model-builder discretion remains over the form of its propagation.
3. **Nonlinear, nonlocal, higher-derivative, and acausal extensions are not merely disfavoured — they are inconsistent with the established VERSF substrate.**

In this sense, the response functional is not an additional dynamical postulate of VERSF. It is the unique admissible structure compatible with the framework's prior commitments.

Final Statement

There is no independent choice of F. The only admissible response of reality to committed distinguishability is the retarded massive Klein–Gordon response.

Epistemic Labels

- **Proved (this paper):** Lemma 0 (scalar character); Steps 1, 1.5, 2, 3, 5 — given A1, A3–A9, the unique admissible F is $(\Box + m^2)^{-1}_{ret}$.
- **Proved (this paper, conditional on cited results):** Step 4 (existence of $m \neq 0$).
- **Imported from elsewhere in VERSF:** Source structure of $\rho_{committed}$; emergent Lorentz covariance; finite coherence scale ξ; numerical value $m^2 = (4/3)\, \xi^{-2}$.
- **Standard:** Ostrogradsky's theorem; Green's function classification on Minkowski space.
- **Conjectural:** None within the scope of this theorem.