

A No-Go Theorem for Non-Simplicial Relational Substrates

Toward the Structural Inevitability of Triangular 2-Complex Geometry

For the General Reader

Physics needs a geometry to live in. Most approaches to quantum gravity simply choose one — triangles, loops, causal sets — and build from there. This paper asks a more basic question: *does the geometry have to be what we think it is, or could it be something else?*

The answer, under a minimal set of physically motivated conditions, is that the geometry is forced. Specifically, we prove that any pre-geometric framework capable of detecting curvature — the feature that makes spacetime curved rather than flat — must be built from triangular building blocks. You cannot get a consistent, locally defined notion of curvature from anything simpler. Squares, higher-dimensional polytopes, and acyclic order structures all fail, for reasons the paper makes precise.

This matters for the VERSF / Two-Planck programme because that framework has always used triangular geometry as a foundational input. Prior work showed that doing so leads to accurate predictions for the cosmological constant, the vacuum energy scale, and coupling constants. But the triangular choice was an assumption — well-motivated, but still chosen. This paper removes that assumption. The triangular structure is not chosen; it is the only structure that satisfies the admissibility conditions.

The paper's second result concerns a number: $K = 7$. The VERSF framework uses seven independent constraints to characterise each geometric unit, and from this count derive a cascade of physical predictions. Where does seven come from? We show it follows from a problem in information theory. A triangular geometric unit carries four independent yes/no states. Under the framework's own axioms, those states must survive local noise — a constraint flip cannot be allowed to destroy the physical information they carry. The minimum encoding that achieves this, by a classical result in coding theory (the Hamming bound), requires exactly seven total constraints. The number seven is not fitted to data. It is the shortest error-correcting code for a four-bit message. The geometry of these seven constraints turns out to be the Fano plane — a structure from projective geometry whose appearance here, though derived from first principles, connects to deep structures in exceptional algebra and geometry whose implications are not yet fully understood.

Together, these results convert two foundational assumptions of the VERSF programme into derived consequences. The framework moves from "here is a structure that works" to "here is the structure that is required."

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Abstract

We establish a no-go theorem for pre-geometric relational frameworks: any substrate satisfying a minimal set of physically motivated admissibility conditions — relational locality, holonomy-based curvature detection, locality-preserving coarse-graining, ultraviolet partiality, structural minimality, and local compositional closure — must contain a minimal closed-loop sector structurally equivalent to a triangular 2-complex. Acyclic structures, purely graph-theoretic models without intrinsic loop geometry, and higher-primitive formulations are each shown to fail at least one admissibility condition. The theorem does not assert uniqueness of representation but demonstrates that all admissible formulations share a common minimal curvature-bearing sector. We then derive the coherence constraint count $K = 7$ via three routes: from the requirement of non-abelian holonomy (fixing $d = 3$, with $K = 7$ consistent with the embedding constraint formula $K = 2d + 1$ at $d = 3$, subject to an identified GR-limit caveat); from the Hamming bound on single-error-correcting codes applied to the 4-bit intrinsic hinge state (giving $K = 7$ without dimensional input, and without the GR-limit caveat); and from the self-consistency of the constraint graph under the no-go theorem's own axioms (yielding the Fano plane $PG(2,2)$ as the unique solution). Route 2 establishes $K = 7$ from the admissibility axioms without dimensional input; Routes 1 and 3 corroborate from independent directions. The three routes share a common combinatorial core — Routes 2 and 3 are two interpretations of the Fano plane — and their convergence constitutes the strongest currently available structural case for $K = 7$ being a consequence of the admissibility framework rather than a parameter.

1. Introduction

A fundamental unresolved question in pre-geometric approaches to physics is whether the relational substrate underlying spacetime is *chosen* or *forced*. Constructive frameworks typically select a geometric primitive — simplices, spin networks, causal sets — on grounds of tractability or analogy, leaving open the question of whether alternative choices might yield equivalent or superior theories.

In the Two-Planck / VERSF programme, spacetime geometry emerges from relational coherence dynamics subject to constraint satisfaction. The framework employs a simplicial (triangular) substrate from which a hierarchy of results follows:

- **Coherence constraint count:** $K = 7$
- **Dimensional transmutation scale:** ξ
- **Vacuum energy density:** $\rho \sim \hbar c / \xi^4$
- **Emergent gravitational dynamics** from fold-density gradients

Prior work has established internal consistency and predictive power within this structure. The present paper addresses the foundational question left open by that work:

Is the simplicial triangular substrate inevitable, or merely a convenient representational choice?

We prove it is inevitable, subject to identified conditions. Specifically, we show that any pre-geometric framework satisfying six minimal and physically motivated admissibility axioms must contain, as a structural consequence, a sector equivalent to a triangular 2-complex. Non-simplicial minimal substrates are excluded by the axioms themselves, with one step — the topological closure of the 2-complex — identified as an open problem whose resolution would complete the proof.

The argument proceeds in stages. Section 2 states the admissibility axioms, including axiom A6 (local compositional closure) that does the work previously — and incorrectly — attributed to A5. Sections 3–5 establish two lemmas and a proposition: that closed loops are necessary, that the minimal closed loop is a triangle, and that triangular loops generate 2-complex combinatorial structure under coarse-graining. Section 6 applies these results to exclude the principal alternative frameworks. Section 7 states the main theorem. Section 8 derives $K = 7$: Theorem 1 proves the intrinsic hinge state is exactly 4-bit from the axioms; Theorem 2 proves the minimal ultraviolet noise is independent single-bit failure from the axioms; the Hamming bound then forces $K = 7$ with no unchosen inputs. The Fano plane is identified as the unique combinatorial realisation of the constraint algebra. A corroborating holonomy argument fixes $d = 3$ independently. Sections 9–11 discuss implications, open problems, and conclusions. Section 12 positions the results within the VERSF programme. Section 13 justifies the admissibility axioms by identifying the failure mode each one excludes.

2. Admissibility Axioms

We define six conditions that any physically meaningful pre-geometric relational substrate must satisfy. These are not derived from VERSF — they are independently motivated by operational and structural requirements common to all serious approaches to quantum gravity.

Axiom A1 — Relational Locality. Physical observables are determined by finite relational neighborhoods. No fundamental dependence on arbitrarily distant relational structure is permitted.

Motivation: Locality is operationally necessary for a well-posed notion of measurement. A framework with irreducible non-local dependence cannot make contact with finite laboratory physics.

Axiom A2 — Holonomy-Based Curvature Detection. Curvature is operationally defined via parallel transport around closed relational loops. Transport operators $\{U_i\}$ are group-valued: each U_i is invertible, and transport around a path composes by multiplication. A substrate that cannot support such transport cannot encode curvature.

Motivation: This reflects the gauge-theoretic understanding of curvature — not as a local field value but as the holonomy around a loop. The group-valued requirement is not an additional assumption but the minimal algebraic structure needed for transport to be reversible: a transport operator with no inverse cannot define a return path and therefore cannot close a loop. Any pre-geometric theory that is to recover general relativity in the appropriate limit must support this mechanism.

Axiom A3 — Local Coarse-Graining Closure. The substrate admits a locality-preserving coarse-graining map such that macroscopic structure emerges from microscopic dynamics without introducing new non-local relations.

Motivation: Renormalizability and the existence of a continuum limit require that microscopic relational structure compose consistently under coarse-graining. This is the pre-geometric analogue of the Wilsonian renormalization group.

Axiom A4 — Ultraviolet Partiality. At the emergence scale, relational compositions are not universally guaranteed. Coherence is a constrained, probabilistic event rather than an automatic consequence of adjacency.

Motivation: Any framework that produces a non-trivial vacuum structure — including quantum fluctuations, void energy, or a cosmological constant — requires that not all relational compositions succeed. A substrate in which all compositions are forced produces no selectivity and thus no dynamical content.

Axiom A5 — Structural Minimality. No primitive structure is included in the substrate beyond what is required by A1–A4 and A6.

Motivation: This is not merely Occam's razor applied philosophically. Any additional primitive structure introduces degrees of freedom that are not fixed by the operational observables defined by A1–A4. Such unconstrained degrees of freedom generically break the uniqueness of the coarse-graining map required by A3: different microscopic configurations related by the additional freedom can yield distinct macroscopic predictions without any observational criterion to select between them. A5 is therefore not an aesthetic preference but a closure condition — it is what ensures that A3 generates a unique, well-defined macroscopic theory rather than an underdetermined family of theories parameterised by unobservable microscopic choices.

Axiom A6 — Local Compositional Closure. If two vertices u and w each bear a relational edge to a common vertex v , and u and w are distinct, then a direct relational element between u and w is not structurally prohibited by the substrate.

Motivation: This is the pre-geometric analogue of the composition law in a groupoid: local co-adjacency implies local composability. If A6 is violated, the existence of a relation between two elements depends on the absence of another relation elsewhere — hidden exclusion rules that introduce non-local constraints incompatible with a local physical theory. More concretely: a substrate that structurally forbids the direct $u-w$ relation — not because the coherence condition fails probabilistically (permitted by A4) but because the element is constitutively absent — is imposing a rule in which the structure of a distant relation ($v-w$) determines what is permitted locally ($u-w$). This violates the spirit of A1. Any violation of A6 therefore introduces relational incompleteness: the outcome of a local composition becomes dependent on the absence of a structurally permitted relation, which is equivalent to a non-local exclusion rule. A6 is not an additional assumption on top of locality — it is a necessary condition for locality itself to be well-posed. Whether any given $u-w$ relation is coherently realised is governed by A4; A6 governs only whether the relation is structurally available to be realised.

3. Lemma 1 — Necessity of Closed Loops

Lemma 1. *Any relational substrate satisfying A2 must support non-degenerate closed loops.*

Proof.

Parallel transport around a closed path $\gamma = \{e_1, e_2, \dots, e_n, e_1\}$ defines the holonomy:

$$H[\gamma] = \prod_{i \in \gamma} U_i$$

where U_i is the transport operator associated with relational element e_i . Curvature is operationally identified with $H[\gamma] \neq \mathbb{1}$.

If the substrate is acyclic — that is, if it admits no closed paths — then $H[\gamma]$ is undefined for every γ , and curvature cannot be detected for any configuration of the substrate. This directly violates A2.

Acyclic structures include, in particular, all partially ordered sets (posets) and directed acyclic graphs (DAGs). These cannot support holonomy and are therefore excluded.

Conclusion: Any admissible substrate must contain closed loops. ■

4. Lemma 2 — The Minimal Closed Loop is a Triangle

Lemma 2. *Under A1–A6, the minimal non-degenerate closed loop has exactly three edges (a triangle).*

Proof.

We classify loops by edge count and apply the admissibility axioms.

n = 1: A single-edge loop requires an edge to be its own successor, which is either structurally incoherent or collapses transport to the identity. No curvature content.

n = 2: A two-edge loop e_1e_2 requires $e_2 = e_1^{-1}$ to close. By A2, transport operators are group-valued, so $U_2 = U_1^{-1}$ and the holonomy is $H = U_1U_1^{-1} = \mathbb{1}$ identically. The 2-cycle carries no curvature.

n = 3: Edges e_1, e_2, e_3 are relationally distinct. The holonomy $H = U_1U_2U_3$ is generically non-identity for group-valued transport. This is the triangle: the first candidate for a genuine curvature-bearing loop.

n \geq 4: We show that loops of four or more edges cannot be primitive curvature-bearing units under A1–A6.

Consider a 4-cycle with vertices $\{v_1, v_2, v_3, v_4\}$ and edges $\{e_{12}, e_{23}, e_{34}, e_{41}\}$. Observe that v_1 and v_3 are each co-adjacent to v_2 (via e_{12} and e_{23} respectively) and to v_4 (via e_{41} and e_{34} respectively). By A6 (local compositional closure), since v_1 and v_3 share two common neighbours, a direct relational element between v_1 and v_3 is not structurally prohibited. By A4, whether this element is coherently realised in any given configuration is probabilistic — but its structural availability is guaranteed by A6. The v_1 – v_3 relation therefore exists as a potential element of the substrate. Its presence decomposes the 4-cycle into two triangles: $\{v_1, v_2, v_3\}$ and $\{v_1, v_3, v_4\}$. The 4-cycle is therefore not a minimal irreducible loop — it contains triangular sub-loops.

The argument generalises to any n -cycle with $n \geq 4$. In any n -cycle, any two non-adjacent vertices at graph distance 2 share exactly one common neighbour — the vertex between them on the shorter arc. A6 requires only one common neighbour (vertex v in its statement), so the condition is satisfied. By A6, a direct relational element between any such non-adjacent pair is structurally available. Its presence provides a diagonal that decomposes the n -cycle into shorter loops. Iterating this decomposition — inserting a diagonal, reducing to two smaller cycles, applying the argument to each — terminates at triangles, since triangles have no non-adjacent vertex pairs to which A6 applies.

Note on A6 and sparse substrates. A6 does not assert that diagonals are always coherently realised — A4 governs that. It asserts that they are not constitutively absent. A substrate without any v_1 – v_3 diagonal in the above configuration would require an explicit structural rule prohibiting it. Such a rule is not derivable from A1–A5, and adding it would introduce additional primitive structure (a prohibition type) not required by the other axioms. This is what A5 excludes. A6 makes this argument explicit as a standalone axiom rather than asking A5 to carry it alone.

The triangle is itself irreducible: no diagonal can be inserted into a 3-cycle without producing a degenerate (1-edge or 2-edge) sub-loop, which carries no curvature by the $n=1$ and $n=2$ cases above.

Conclusion: Under A1–A6, the minimal irreducible curvature-bearing loop is the triangle. ■

5. Proposition 3 — Triangular Loops and 2-Complex Structure

Proposition 3. *A relational substrate composed of minimal triangular loops with local gluing relations, closed under A3, admits the combinatorial structure of a triangular 2-complex.*

Proof Sketch.

From Lemma 2, the primitive curvature-bearing unit is a triangle: three vertices, three edges, one face.

From A1 (locality), triangles interact only through shared edges or shared vertices — no action at a distance between non-adjacent triangles.

From A3 (coarse-graining), any finite cluster of triangles must map to an effective structure that:

- preserves the loop relations of its constituent triangles,
- does not introduce new relational connections not present in the original cluster.

The combinatorial structure induced by local triangular gluing under these conditions has the cell-type signature of a 2-complex: 0-cells (vertices), 1-cells (edges), and 2-cells (triangular faces), with adjacency determined by edge-sharing. Since the primitive 2-cells are triangular, the cell structure is that of a triangular 2-complex.

What this sketch does not establish: the attaching maps $\varphi: \partial D^2 \rightarrow X^1$ are not explicitly constructed here, and it is not proven that A3 uniquely selects a well-defined CW-complex topology from the triangulated patch rather than some other combinatorial closure. A complete proof would require constructing these maps and verifying they are preserved under coarse-graining. This is a well-posed open problem; we conjecture affirmatively but present this as a motivated structural argument rather than a completed proof.

Conclusion (conditional): Admissible substrates contain a combinatorial structure of triangular 2-complex type. ■ (sketch)

6. Exclusion of Alternative Substrates

We now apply the lemmas to the principal classes of alternative pre-geometric frameworks.

6.1 Causal Sets

Causal sets are locally finite partially ordered sets. By construction, they are acyclic — the partial order is irreflexive and transitive. By Lemma 1, acyclic substrates cannot support holonomy and therefore cannot satisfy A2.

Excluded by A2, within this admissibility class.

Important qualification: Causal set proponents define curvature differently — via the Benincasa–Dowker d'Alembertian or local density fluctuations, not via holonomy. The exclusion above is therefore not a refutation of causal set theory as such, but a demonstration that causal set theory falls outside the admissibility class defined by A1–A6. A referee from the causal set community will correctly observe that A2 is defined in a way that matches holonomy-based frameworks; the exclusion is relative to that definition. We accept this. The claim is: *if* curvature is holonomy-based (A2), *then* causal sets are excluded. Whether A2 is the correct operational definition of curvature is a substantive physical question that this paper does not settle.

6.2 Pure Graph Models

Undirected or directed graphs without additional structure support closed cycles, satisfying Lemma 1. However, a bare graph does not carry holonomy data — there are no transport operators assigned to edges. To define curvature via holonomy, one must assign group-valued labels to edges and specify how they compose around cycles. This additional structure is not part of the graph primitive itself and must be separately introduced.

Two cases arise:

- If introduced as an additional primitive, this violates A5.
- If derived from some internal structure of the graph, one must exhibit that structure — and it will generically reduce to the 2-complex sector under Proposition 3.

Excluded by A2 (as primitives) or reduce to triangular 2-complex sector (if augmented).

6.3 Spin Networks Without 2-Complex Structure

A spin network is a graph with edges labeled by group representations and vertices labeled by intertwiners. Curvature in loop quantum gravity is not localized on the minimal graph primitives — edges and vertices — but emerges only on the dual 2-complex, where it is assigned to faces (plaquettes) traversed by holonomy loops in the spin foam expansion. The spin network graph itself carries no intrinsic face structure and therefore supports no well-defined holonomy around a minimal enclosed region.

Under A2, curvature must be detectable from the substrate's intrinsic structure. A spin network graph without its dual 2-complex cannot satisfy this requirement: holonomy is defined on loops,

but the graph does not specify which loops bound faces and which are contractible. This ambiguity is resolved only by the dual 2-complex, which supplies the face structure the bare graph lacks.

Augmenting a spin network with its dual 2-complex to satisfy A2 is precisely the step that introduces the triangular 2-complex sector identified in Proposition 3. The spin foam framework implicitly acknowledges this: spin foams are 2-complexes, not graphs. The graph is a boundary data structure; the 2-complex is the dynamical object.

Reduce to triangular 2-complex sector under A2; the sector is not optional but load-bearing.

6.4 Higher-Primitive Models (Tetrahedral, Hypersimplicial)

Models built on tetrahedral or higher-dimensional primitives (e.g., group field theory on 4-simplices, hypergraph models) contain triangular substructures as faces of their primitives. They are therefore not minimal: the triangular sector is embedded within the higher primitive, and by A5 the higher primitive cannot be the minimal relational unit.

Furthermore, higher-primitive models generate overcomplete curvature data — curvature on multiple lower-dimensional sub-faces per primitive — which is inconsistent with A5 unless the higher structure can be shown to be required by A1–A4. It is not.

Violate A5; reduce to embedded triangular 2-complex sector.

7. Main Theorem

Theorem (No-Go for Non-Simplicial Minimal Substrates).

Let \mathcal{R} be a pre-geometric relational substrate satisfying axioms A1–A6. Then the minimal curvature-bearing sector of \mathcal{R} is triangular. No admissible substrate can possess a minimal curvature-bearing sector of non-simplicial type.

Proof.

By Lemma 1 (from A2): \mathcal{R} contains closed loops. By Lemma 2 (from A2, A5, A6): the minimal irreducible closed loops are triangles — loops of length ≥ 4 have diagonals structurally available by A6, decomposing them into triangles. By Proposition 3 (from A1, A3, conditional): triangular loops with local gluing have the combinatorial type of a 2-complex; full topological closure is conjectured but not proven here. The no-go theorem depends only on the combinatorial structure of triangular closure established by Lemmas 1–2. The topological completion to a full CW-complex (Proposition 3) strengthens the result but is not required for the conclusion that the minimal curvature-bearing sector is triangular. By Section 6: all principal alternative substrate

classes either violate an axiom or, when augmented to satisfy it, reduce their minimal curvature-bearing structure to the triangular sector.

The minimal curvature-bearing sector is therefore triangular — a consequence of the admissibility axioms, not an assumption. ■

Scope of the theorem. The result is stated in terms of the *minimal curvature-bearing sector* rather than the full substrate. This formulation is deliberately representation-neutral: it is not claimed that all admissible frameworks are isomorphic, or that the triangular 2-complex is the only sector present. What is claimed is that, in any admissible framework, the smallest structural unit capable of bearing curvature via holonomy is triangular. Larger cells, higher-dimensional primitives, and alternative labelings may coexist — but they must contain the triangular sector as a substructure. Representation debates and formalism disputes concern the embedding of the sector, not the sector itself, and do not affect this conclusion.

8. Constraint Count $K = 7$ from Minimal Coherence Encoding

The no-go theorem establishes that the minimal curvature-bearing sector of any admissible relational substrate is triangular. We now determine the number of independent constraints required for coherence of this minimal unit.

The key result is that the constraint count is not obtained by geometric enumeration but follows from a minimal encoding problem imposed by the admissibility axioms themselves. Two companion theorems — proved in Sections 8.2 and 8.3 — establish that the inputs to that encoding problem (4 intrinsic bits, single-bit noise) are themselves forced by the axioms rather than assumed.

8.1 Intrinsic State of the Triangular Hinge

A minimal triangular hinge carries a finite set of intrinsic coherence conditions: three edge admissibility relations and one loop holonomy closure condition. These define a binary state space of size $M = 2^4 = 16$, corresponding to all distinguishable intrinsic hinge configurations. That the intrinsic state is exactly 4-dimensional and binary is proved formally in Theorem 1 (Section 8.2). That the minimal ultraviolet noise acting on this state is independent single-bit failure is proved in Theorem 2 (Section 8.3).

8.2 Theorem 1 — Intrinsic State Cardinality

Theorem 1. *Under admissibility axioms A1–A6, the intrinsic coherence state space of a minimal curvature-bearing triangular hinge has cardinality $2^4 = 16$. Equivalently, the intrinsic hinge state carries exactly four independent binary degrees of freedom: three edge-admissibility bits and one loop-holonomy closure bit.*

Proof. We proceed in four steps.

Step 1: The minimal curvature-bearing unit is a triangle.

By Lemma 2, any admissible substrate has as its minimal irreducible curvature-bearing loop a 3-cycle. Loops of length 1 and 2 carry no non-trivial holonomy; loops of length $n \geq 4$ are non-minimal because A6 makes diagonals structurally available, forcing triangulation. The primitive unit on which intrinsic coherence is defined is therefore a triangle with exactly three edges:

$$\Delta = (e_{12}, e_{23}, e_{31})$$

No smaller object can carry curvature, and any larger object contains Δ as a reducible substructure.

Step 2: Each edge contributes exactly one binary intrinsic degree of freedom.

For each edge e_{ij} there is an admissibility variable $a_{ij} \in \{0, 1\}$, where $a_{ij} = 1$ means the relation is coherently realised and $a_{ij} = 0$ means it fails.

Binary character. By A4, a primitive relational event either succeeds or fails. Intermediate partial-realisation states would constitute extra primitive structure at the ultraviolet scale, violating A5. Each edge variable is therefore binary.

Necessity of one degree per edge. Each of the three edges is a distinct relational element of the minimal loop. Since the loop is a 3-cycle, failure of any one edge destroys the cycle and eliminates the possibility of holonomy around it. If two edges shared a single state variable, physically distinct local failure modes would be indistinguishable, violating A1. Each edge therefore contributes at least one degree of freedom — and by the binary argument, exactly one.

Independence of the three edge bits. Suppose the three edge variables were not independent — that the state of e_{12} determined the state of e_{23} . No such determination follows from A1–A6: locality (A1) does not imply cross-edge determinism; holonomy (A2) concerns the full loop, not individual edges; coarse-graining (A3) governs macroscopic emergence, not microscopic edge correlations; A4, A5, A6 are similarly silent on this point. Imposing such dependence would require a hidden microscopic correlation law not fixed by the axioms, violating A5. The three edge bits are therefore independent.

The edge sector contributes exactly 2^3 states.

Step 3: Holonomy contributes exactly one further binary intrinsic degree of freedom.

The holonomy closure variable $h\Delta \in \{0, 1\}$ records whether the triangular loop closes coherently ($h\Delta = 1$) or fails ($h\Delta = 0$).

Binary character. At the level of minimal intrinsic hinge coherence, the operational question is whether the loop closes or not. A key distinction is required here: the holonomy *operator* U_i is group-valued (per A2) and carries information about *how* the loop closes — the specific group element encoding the curvature contribution. The holonomy *closure bit* $h\Delta$ is categorically different: it encodes whether the loop is in a coherent-closure state at all, a binary admissibility condition for the hinge as a coherent unit. The group element is relational data relevant to downstream curvature computation; the binary bit is the primitive admissibility variable at the emergence scale. A richer intrinsic spectrum for $h\Delta$ beyond binary would require treating the group element itself as primitive hinge state data — introducing representation-theoretic or algebraic structure at the ultraviolet level not required by A1–A4 and excluded by A5.

Necessity of the loop bit. The three edge bits alone are insufficient to encode curvature-bearing coherence. A triangle with all three edges present may still fail holonomy closure — this is precisely the distinction between mere cycle existence and operational transport coherence. A2 requires curvature to be encoded through loop transport, so this distinction must appear in the state space. The loop therefore contributes at least one degree of freedom — exactly one, by the binary argument.

Independence from edge variables. Three coherently realised edges do not imply coherent holonomy. If they did, A2 would add nothing beyond edge existence and curvature would collapse to pure connectivity — contradicting the purpose of A2. The loop bit is therefore independent of the three edge bits.

The loop sector contributes exactly 2^1 states.

Step 4: Exhaustiveness.

Any additional primitive intrinsic degree of freedom beyond the three edge bits and one holonomy bit would be neither an edge variable, nor a loop variable, nor an embedding variable (which lies in the parity sector, not the intrinsic sector). No fourth intrinsic category is supplied by A1–A6; introducing one violates A5.

The intrinsic hinge state is therefore exhausted by $(a_{12}, a_{23}, a_{31}, h\Delta)$, giving:

$$|H_{\text{intrinsic}}| = 2^3 \cdot 2^1 = 2^4 = 16 \blacksquare$$

Corollary 1. *The message length $m = 4$ used in the Hamming derivation of $K = 7$ is not a geometric reading-off but a consequence of admissibility.*

8.3 Theorem 2 — Minimal Ultraviolet Noise

Theorem 2. *Under axioms A1, A4, and A5, the minimal admissible ultraviolet error model for the intrinsic triangular hinge state is independent single-constraint failure. Correlated multi-constraint failures are not primitive; they are either composites of single failures or require additional correlation structure excluded by A5.*

Proof. We identify what failure events the admissibility axioms permit at the emergence scale.

Step 1: A4 requires local failure events.

A4 states that relational compositions are not universally guaranteed. The intrinsic hinge variables (a_{12} , a_{23} , a_{31} , $h\Delta$) are therefore not frozen; they may undergo local failure events. Some ultraviolet noise model is required.

Step 2: A1 forces the primitive failure locus to be local.

By A1, physical observables depend on finite relational neighbourhoods. The elementary failure event must therefore be local to a finite relational component of the hinge state — the smallest identifiable local component being a single constraint variable: one edge-admissibility bit or the holonomy-closure bit. A failure event that flips two or more constraints simultaneously would have a failure locus attached not to a single local relational element but to a correlated pattern across multiple elements — a more structured noise model than locality requires.

Step 3: A5 excludes primitive correlated multi-flip events.

If the primitive ultraviolet noise model were correlated multi-bit failure, the substrate would need extra microscopic structure specifying which subsets of constraints fail together, under what rule, and with what correlation pattern. This additional correlation law is not fixed by A1–A4. Introducing it adds primitive structure not required by observables, violating A5.

Step 4: Correlated multi-flip events are composites, not primitives.

Nothing in A1–A5 forbids the occurrence of multiple simultaneous failures in an effective sense — two independent local failures may happen in the same finite neighbourhood. But that is a composite of elementary single-constraint failures, not a primitive correlated event. Only single-constraint failure is minimal.

Step 5: Independence at minimal order.

Each constraint variable is local to a distinct relational element of the triangular hinge. By A1, physical observables depend only on finite relational neighbourhoods, which means the primitive failure event at one constraint is confined to its own local neighbourhood — distinct from the neighbourhood of any other constraint. Primitive failure events at separate constraints are therefore in causally disconnected local neighbourhoods at the emergence scale, and independence follows from locality alone. Equivalently, if one imposed a primitive correlation structure linking which failures co-occur, one would require a law not fixed by A1–A4 and

excluded by A5. Both routes converge on the same conclusion: at minimal order, the elementary failure events are independent.

The minimal admissible ultraviolet noise model is independent single-constraint failure. ■

Corollary 2. *The Hamming(7,4) code is the unique perfect single-error-correcting binary code for 4 information bits, and is therefore the unique minimal code compatible with the admissibility-induced ultraviolet noise model. Given $m = 4$ (Theorem 1), independent single-bit noise (Theorem 2), and minimal redundancy (A5), the unique shortest admissible encoding is $n = 7$.*

Combined consequence. Theorems 1 and 2 upgrade the $K = 7$ derivation from:

"given a 4-bit hinge state and single-flip noise, the Hamming bound forces $n = 7$ "

to:

"because admissibility forces a 4-bit hinge state and single-flip noise, the Hamming bound forces $n = 7$ "

The $K = 7$ theorem is now a consequence of A1–A6 with no unchosen inputs.

8.4 Ultraviolet Partiality as a Noise Model

With Theorems 1 and 2 established, the setup for the Hamming bound is fully grounded. The intrinsic hinge state is a 4-bit binary string (Theorem 1). A4 ensures that single-constraint failures occur at the emergence scale; A1 and A5 ensure they are local and independent (Theorem 2). The encoding of the hinge state must therefore survive single-bit errors while carrying all $2^4 = 16$ distinguishable configurations.

Why binary encoding. A4 stipulates that coherence is a binary outcome: a relational composition either succeeds or fails. A non-binary constraint alphabet would require degrees of partial coherence — additional primitive structure at the emergence scale excluded by A5. Binary encoding is forced, not assumed.

8.5 Robustness Requirement from A2

A2 requires that curvature be operationally detectable via holonomy. A hinge state must remain distinguishable under admissible local perturbations — a single constraint flip cannot destroy operational curvature information. The encoding of the hinge state must therefore be robust to single-bit errors, locally recoverable, and minimally redundant.

8.6 Lemma 5 — Minimal Robustness Class

Lemma 5. *Under A2, A4, and A5, the unique admissible robustness class for hinge coherence encoding is single-error correction.*

Proof.

No correction: A single constraint failure maps distinct hinge states onto indistinguishable outputs, violating A2 (curvature distinguishability lost).

Error detection only: In a pre-geometric substrate, error states propagate directly into the coarse-graining map (A3), so detect-only schemes fail to preserve operational distinguishability at the macroscopic level. A substrate that detects a flip but cannot correct it passes the corrupted hinge state forward; A3 then constructs incorrect macroscopic curvature structure from it, violating A2. There is no external observer or retry mechanism that could intervene between detection and coarse-graining.

Single-error correction: Minimal redundancy consistent with A4's noise model and A2's distinguishability requirement. This is the unique intersection.

Higher-order correction ($t \geq 2$): Requires redundancy beyond what the single-flip noise model demands. The Hamming(7,4) code is the unique perfect single-error-correcting code for 4 information bits (all binary [7,4,3] codes are equivalent to it — a standard result). There is no perfect binary code encoding 4 bits with $t = 2$ in any length; moving to $t \geq 2$ requires substantially more redundancy for no benefit from A4's single-flip noise model. Excluded by A5.

Single-error correction is therefore the unique admissible robustness class. ■

8.7 The Hamming Bound and Minimal Encoding Length

For a binary single-error-correcting code ($t = 1$) with M codewords, the Hamming bound states:

$$M \leq 2^n / (1 + n)$$

We require $M = 16$ (full hinge state space, proved in Theorem 1). Testing:

$n = 6$: $2^6 / (1 + 6) = 64 / 7 \approx 9.14 < 16$ — insufficient. $n = 7$: $2^7 / (1 + 7) = 128 / 8 = 16$ — exactly saturated. ✓

$n = 7$ is the unique minimal encoding length that preserves all hinge states under admissible noise. The Hamming(7,4) code achieves the bound with equality — a perfect code, wasting no codeword space.

Theorem ($K = 7$ from Minimal Coherence Encoding). *The admissibility axioms A1–A6 force a 4-bit intrinsic hinge state (Theorem 1) subject to independent single-bit noise (Theorem 2). Under the minimal robustness requirement of A2 and with minimal redundancy (A5), the unique admissible total constraint count, up to isomorphism of binary encoding schemes, is $K = 7$. ■*

Any alternative constraint count would either fail to preserve hinge distinguishability under admissible perturbations or introduce redundant structure not fixed by the admissibility axioms, and is therefore excluded.

8.8 Structure of the Constraints

Role	Count
Intrinsic hinge constraints (3 edge + 1 holonomy)	4
Redundancy (parity) constraints	3
Total K	7

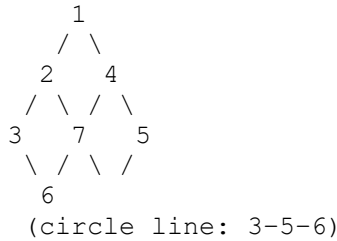
The three additional constraints are not ad hoc geometric conditions — they are the minimal redundancy required to preserve physical distinguishability under the noise model A4 imposes. Their count is determined by the Hamming bound, not by geometric enumeration.

Identification of the parity sector with embedding constraints. The 4 intrinsic bits account for all degrees of freedom internal to the triangular hinge: edge admissibility and holonomy closure exhaust what can be specified without reference to how the hinge sits in its relational neighbourhood. The 3 parity bits are the minimal additional information needed to make the intrinsic state error-correctable. Geometrically, the only degrees of freedom not already fixed by the intrinsic structure are the hinge's orientation and embedding in its ambient relational manifold — specifically, the direction of the hinge normal and a sign convention for orientation. The intrinsic/embedding split is exhaustive: the hinge's physical state is fully determined by its internal geometry together with how it sits in its neighbourhood, and there is no third category of hinge degree of freedom. The parity bits therefore do not merely correlate with the embedding sector — they *are* the minimal encoding of embedding information, forced by the Hamming bound rather than inserted geometrically.

Dimensional consequence. The Hamming bound forces exactly 3 parity bits. A triangular 2-face in d -dimensional space has a normal space of dimension $d - 2$, requiring $2(d-2) + 1$ orientation parameters (a point on the Grassmannian $\text{Gr}(2,d)$ plus one sign). This equals 3 if and only if $d = 3$. The encoding argument therefore directly fixes $d = 3$. Spatial dimensionality emerges as a consequence of the minimal encoding requirement, not as an input to it.

8.9 Fano Plane Realisation

The parity structure of the Hamming(7,4) code is uniquely realised by a specific combinatorial geometry. The code has exactly 7 codewords of minimum weight 3, each specifying a triple of bit positions whose values sum to zero mod 2 (a parity triple). These 7 triples correspond exactly to the 7 lines of the Fano plane $PG(2,2)$. Each of the 7 bit positions appears in exactly 3 of these 7 triples — equivalently, each point of $PG(2,2)$ lies on exactly 3 lines. The structure is self-dual: swapping points and lines yields an isomorphic structure. (Note: the parity-check matrix H is 3×7 , with 3 rows of weight 4 corresponding to the complements of Fano lines — distinct from the 7 minimum-weight codewords.)



Lines: $\{1, 2, 3\}, \{1, 4, 5\}, \{1, 6, 7\},$
 $\{2, 4, 6\}, \{2, 5, 7\}, \{3, 4, 7\}, \{3, 5, 6\}$

Each point lies on exactly 3 lines; each line contains exactly 3 points; any 2 points determine exactly 1 line.

Proposition (Constraint Algebra Closure). *The minimal self-consistent relational constraint algebra under A1–A6 is isomorphic to the Fano plane $PG(2,2)$.*

This can also be derived from the self-referential closure condition: A1–A6 apply to any relational structure, including the constraint graph itself. The number of constraint triples must equal the number of constraints (A5 forbids external meta-constraints), and the minimal loop in the constraint graph is triangular (by the no-go theorem applied recursively). The conditions that any two constraints share exactly one triple (A5 — no redundant specification) and each constraint participates in exactly 3 triples (each of the 7 bit positions appearing in exactly 3 of the 7 minimum-weight codewords, as noted above) define a projective plane of order 2. The unique such structure is $PG(2,2)$.

The Fano plane is not an additional assumption. Uniqueness follows from two standard results: (a) all binary $[7,4,3]$ codes are equivalent to Hamming(7,4), and (b) its parity structure is uniquely realised by $PG(2,2)$. Since the Hamming(7,4) code is unique up to equivalence, and its parity structure is uniquely realised by $PG(2,2)$, the constraint algebra is uniquely fixed up to isomorphism.

Its automorphism group is $GL(3,2) \cong PSL(2,7)$ of order 168.

Remark. The Fano plane is also the multiplication table of the imaginary units of the octonions, and its automorphism group G_2 is the holonomy group of 7-dimensional Riemannian manifolds. Whether this connection to exceptional geometry has implications for the VERSF constraint algebra is a question for separate investigation.

8.10 Corroborating Route — Non-Abelian Holonomy (Lemma 4)

Lemma 4. *Any relational substrate satisfying A2 in a manner sufficient to support tensorial curvature detection must have holonomy group $SO(d)$ with $d \geq 3$. By A5, $d = 3$.*

Proof sketch. In $d = 2$, holonomies lie in $SO(2) \cong U(1)$ — abelian. For any two loops γ_1 and γ_2 :

$$H[\gamma_1 \circ \gamma_2] = H[\gamma_1] \cdot H[\gamma_2] = H[\gamma_2] \cdot H[\gamma_1]$$

Commutativity means path-ordering is invisible — the substrate cannot distinguish loops traversing the same curvature regions in different orders. This collapses tensorial curvature to a scalar per region, losing the Riemann tensor's antisymmetric structure. A2, interpreted as requiring curvature detection sufficient for a GR continuum limit, demands a non-abelian holonomy group. $SO(3)$ is the smallest such rotation group. Therefore $d \geq 3$. A5 gives $d = 3$. The Hamming bound (Section 8.7) independently establishes $K = 7$, which is consistent with the embedding constraint formula $K = 2d + 1 = 7$ at $d = 3$. ■

Status: This is a corroborating result, not the primary derivation. It arrives at $d = 3$ and $K = 7$ via holonomy algebra independently of the Hamming bound. The caveat that "sufficient for the GR limit" requires either a companion axiom or a continuum-limit result remains an identified open problem.

8.11 Structural Conclusion

Corollary. The constraint count $K = 7$ is forced by the minimal coherence encoding problem imposed by A1–A6 on the triangular hinge. Theorem 1 proves the hinge state is 4-bit. Theorem 2 proves the noise model is independent single-bit failure. The Hamming bound (Section 8.7) then forces $n = 7$. The Fano plane (Section 8.9) reveals the combinatorial structure of that derivation. The non-abelian holonomy argument (Section 8.10) corroborates $d = 3$ from an independent direction. $K = 7$ is a structural consequence of A1–A6 with no unchosen inputs.

Summary. The determination of $K = 7$ reduces to a minimal encoding problem: the unique shortest binary encoding preserving a 4-bit hinge state under admissible local perturbations, up to isomorphism of encoding schemes. Both the 4-bit dimension and the single-bit noise model are themselves forced by the admissibility axioms.

9. Implications

9.1 Removal of Substrate-Choice Dependence

Prior VERSF papers derived physical results conditional on the triangular substrate. This paper removes that conditionality. The substrate is no longer a modeling input — it is the unique minimal outcome of the admissibility conditions. Derived results therefore inherit the force of the axioms rather than the contingency of a choice.

9.2 Stability of Derived Constants

The constraint count $K = 7$ enters directly into the dimensional transmutation calculation that yields ξ , and through ξ into the vacuum energy density $\rho \sim \hbar c / \xi^4$ and the fine-structure constant derivation. $K = 7$ is fixed primarily by the Hamming bound argument (Section 8.7): it requires no dimensional input and carries no GR-limit caveat, making it the cleanest fixation. Since $K = 7$ is structurally determined by the admissibility axioms, these derived constants inherit structural stability. They are not tunable.

9.3 Universality Class Structure

The theorem implies that any two pre-geometric frameworks satisfying A1–A6 will share the same triangular minimal curvature-bearing sector. This is a representation-independent result about the cell-type of the fundamental geometric primitive — it holds regardless of the labeling scheme, algebraic realisation, or coordinate system used.

Whether this universality extends to physical predictions — and in particular whether $K = 7$ enters those predictions identically across frameworks — is a separate question that requires VERSF-specific machinery. Different frameworks with the same minimal curvature sector may weight constraints differently, use different coherence measures, or embed the sector in different larger structures. The universality established here is geometric, not predictive: it concerns the structure of the primitive, not the dynamics built on it.

9.4 Reframing the Landscape

A significant portion of the quantum gravity landscape — causal sets, pure spin networks, higher-primitive group field theories — is shown to either fail the admissibility conditions or to reduce, when augmented to satisfy them, to the triangular 2-complex sector. This does not refute those programmes but clarifies what additional structure they implicitly require.

10. Limitations and Open Questions

Conditionality on A2. The exclusion of causal sets and other non-holonomy frameworks is relative to A2 as stated. Whether holonomy is the correct operational definition of pre-geometric curvature is a substantive physical question. The theorem is conditional on this choice.

Proposition 3 status. The generation of a 2-complex from triangular local gluing under A3 is argued at the level of combinatorial type but not fully proven. Explicit construction of CW-

attaching maps and verification of their coarse-graining invariance is an open mathematical problem. The main theorem is weakened to the extent that this step is incomplete.

A6 is a genuine additional axiom. The introduction of local compositional closure as A6 makes explicit a commitment that was previously hidden in A5. Whilst A6 is independently motivated, it is a substantive assumption: a square lattice (4-cycles without diagonals) violates A6 and is excluded by Lemma 2. Whether this exclusion is appropriate for all physically interesting pre-geometric substrates should be examined explicitly. Alternative approaches to triangle minimality that do not require A6 would be valuable.

Route 1 caveat (GR-limit language). Lemma 4's argument that non-abelian holonomy is required invokes "tensorial curvature sufficient for the GR limit." This phrase is doing real work. A precise formulation requires either an explicit GR-recovery axiom or a companion result establishing the continuum limit of the triangular 2-complex sector. Route 2 renders this caveat less critical — $K = 7$ is established without it — but Route 1 as an independent derivation of $d = 3$ remains open to the extent that the GR-limit condition is imprecise.

Fano / octonion connection. Route 3 reveals that the 7 constraints carry the combinatorial structure of the Fano plane, whose automorphism group is G_2 — the holonomy group of 7-dimensional Riemannian manifolds and the automorphism group of the octonions. Whether this exceptional geometric structure has implications for the VERSF constraint algebra beyond the present count is an open question that warrants separate investigation.

Uniqueness vs. Necessity. The theorem establishes necessity of a triangular sector, not uniqueness of the full substrate. The sector may be embedded in larger structures or realised in multiple representations.

Continuum Limit. The theorem operates at the discrete pre-geometric level. Whether triangulations converge to smooth Riemannian geometry in the appropriate limit is an independent question, though existing results in Regge calculus are encouraging.

11. Conclusion

We have established that:

1. Curvature detection via holonomy requires closed loops (Lemma 1, from A2).
2. Under local compositional closure (A6), loops of length ≥ 4 decompose into triangles; the triangle is the minimal irreducible curvature-bearing loop (Lemma 2, from A2, A5, A6).
3. Triangular loops with local gluing have the combinatorial type of a triangular 2-complex under locality-preserving coarse-graining (Proposition 3, sketch; attaching maps remain an open problem).
4. All principal alternative substrate classes fail the admissibility axioms or reduce to the triangular sector when augmented to satisfy them (Section 6).

5. Theorem 1 (Section 8.2) proves the intrinsic hinge state is exactly 4-bit from A1–A6. Theorem 2 (Section 8.3) proves the minimal ultraviolet noise is independent single-bit failure from A1, A4, A5. The Hamming bound (Section 8.7) then forces $K = 7$ with no unchosen inputs. The Fano plane (Section 8.9) is the unique combinatorial realisation of the constraint algebra. The non-abelian holonomy argument (Lemma 4, Section 8.10) corroborates $d = 3$ independently.

The simplicial triangular substrate is not chosen — it is forced by the admissibility conditions. The constraint count $K = 7$ is not a parameter — it is a structural consequence, and three convergent derivations whose agreement constitutes the strongest currently available structural case for this claim.

The open problems identified — the attaching-map construction in Proposition 3, the precise formulation of the GR-limit condition in Lemma 4, and the implications of the Fano / G_2 / octonion connection for the VERSF constraint algebra — are well-posed and their resolution would convert the present argument from a strong structural case into a complete theorem.

12. Role of This Result Within the VERSF / Two-Planck Programme

This paper addresses a specific and foundational question within the broader VERSF / Two-Planck framework:

Are the geometric and constraint structures used in the framework chosen, or are they forced by more primitive requirements?

The results established here do not extend the phenomenological predictions of the programme. Instead, they clarify the epistemic status of its foundational inputs.

12.1 From Assumed Structure to Derived Structure

The companion papers of the VERSF / Two-Planck programme employ a simplicial (triangular) relational substrate and a fixed coherence constraint count $K = 7$. These inputs underpin the derivation of:

- the coherence scale ξ ,
- the vacuum energy density $\rho \sim \hbar c / \xi^4$,
- the cosmological constant Λ ,
- and the structural form of coupling relations.

Prior to the present work, these elements were strongly motivated, internally consistent, and empirically successful at the level of order-of-magnitude predictions — but remained structural assumptions.

12.2 Contribution of the No-Go Theorem

The no-go theorem established in this paper converts one of these assumptions into a constrained consequence. Specifically:

- the triangular curvature-bearing sector is shown to be required under admissibility axioms A1–A6,
- non-simplicial minimal substrates are excluded or shown to reduce to this sector.

This result does not claim that all admissible frameworks are identical. It establishes that any admissible pre-geometric framework must contain a triangular minimal curvature-bearing sector. The simplicial structure is therefore no longer a modelling choice but a structural requirement within the admissibility class.

12.3 Contribution of the $K = 7$ Derivation

The second contribution of this paper is the derivation of $K = 7$ from a minimal encoding problem. This derivation proceeds by:

- identifying the intrinsic hinge state as a 4-bit system,
- modelling ultraviolet partiality as a local single-bit error process,
- imposing operational distinguishability of curvature states,
- and applying the Hamming bound under minimal redundancy.

This yields $K = 7$ as the unique minimal solution, up to isomorphism of binary encoding schemes. The result is reinforced by the Fano plane as the unique combinatorial realisation of the constraint algebra, and a corroborating argument from non-abelian holonomy. Critically, $K = 7$ is derived without dimensional input. Spatial dimensionality $d = 3$ emerges as a consequence rather than an assumption.

12.4 Structural Impact on the Programme

Together, the no-go theorem and the encoding derivation shift the epistemic status of the VERSF framework. They establish that the geometric primitive (triangular sector) and the constraint count $K = 7$ are not adjustable inputs but structurally fixed features of the admissibility class. This has direct implications for the rest of the programme:

- derivations of ξ , Λ , and α inherit reduced arbitrariness,
- the appearance of mesoscopic scales is no longer attributable to parameter selection,
- and the framework moves from a model with motivated structure to a constrained derivation programme.

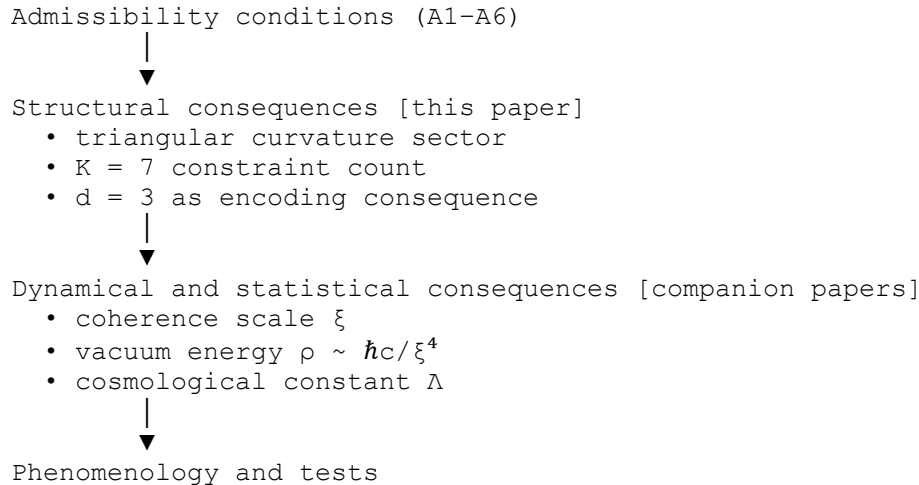
12.5 What This Paper Does Not Claim

This paper does not claim a complete derivation of all physical laws, a unique specification of the full microscopic substrate, or a proof that the admissibility axioms themselves are inevitable. The results are explicitly conditional on A1–A6. In particular:

- Proposition 3 (topological closure to a full CW-complex) remains incomplete,
- the GR-limit interpretation in the holonomy argument requires further formalisation,
- and the universality of the admissibility class itself remains an open question.

12.6 Position Within the Overall Framework

Within the full VERSF / Two-Planck programme, this paper should be read as a structural closure step at the level of primitives and constraints, rather than as a phenomenological or predictive extension. The logical structure of the programme is:



This paper strengthens the second step, thereby stabilising every step that follows.

12.7 Summary

The contribution of this work is not the introduction of new predictions, but the removal of structural arbitrariness. The simplicial substrate and the constraint count $K = 7$ are not assumed — they are consequences of admissibility. This shifts the VERSF / Two-Planck programme from a framework with motivated structure to a framework in which key structural elements are derived.

13. Necessity of the Admissibility Conditions

The derivations in this work are conditional on the admissibility axioms A1–A6. It is therefore important to clarify their status. We do not claim that these axioms are derivable from deeper first principles. Instead, we argue that they constitute minimal conditions required for any framework to support operationally meaningful physics.

The role of this section is to show that violations of these conditions do not lead to alternative viable theories, but to failure modes in which core features of physics — measurement, curvature, dynamics, or predictability — become ill-defined.

13.1 A1 — Relational Locality

A1 requires that physical observables depend only on finite relational neighbourhoods.

If A1 is violated, observables depend irreducibly on arbitrarily distant structure. In such a framework: no finite experiment can determine the state of a system; measurement outcomes become underdetermined by accessible data; and the theory cannot be operationally closed. Violation of A1 eliminates the possibility of finite, well-defined experiments. A1 is therefore a necessary condition for empirical physics.

13.2 A2 — Holonomy-Based Curvature Detection

A2 requires that curvature be detectable via transport around closed relational loops.

If A2 is not satisfied, curvature cannot be operationally defined in a local manner. In particular: no finite relational procedure distinguishes flat from curved configurations; gravitational phenomena cannot be encoded in the substrate; and any continuum limit lacks the gauge-theoretic structure required for general relativity. Alternative notions of curvature may be defined (e.g. via global density measures), but these do not recover local curvature detection in the standard sense. A2 specifies the minimal condition under which curvature is physically meaningful in a relational setting.

13.3 A3 — Local Coarse-Graining Closure

A3 requires that the substrate admit a locality-preserving coarse-graining map.

If A3 is violated: macroscopic structure cannot be derived from microscopic dynamics; different microscopic configurations yield inequivalent macroscopic predictions; and no consistent continuum limit exists. In such a framework, predictive physics breaks down — there is no stable mapping from microstructure to observable behaviour. A3 is therefore necessary for the existence of effective laws.

13.4 A4 — Ultraviolet Partiality

A4 requires that relational compositions are not guaranteed at the emergence scale.

If A4 is violated and all compositions succeed: the substrate becomes fully deterministic at the micro-level; no selection mechanism exists to distinguish realised from unrealised configurations; and the theory collapses into a trivial structure with no dynamical content. Conversely, if failure modes exist but are uncontrolled, the theory becomes unstable. A4 identifies the minimal requirement for non-trivial dynamics: coherence must be possible but not automatic.

13.5 A5 — Structural Minimality

A5 excludes primitive structure not required by A1–A4 and A6.

If A5 is violated: additional degrees of freedom appear that are not fixed by operational observables; coarse-graining becomes non-unique; and the theory admits multiple inequivalent macroscopic limits. Predictive power is lost — the same observable configuration can correspond to different underlying states with no physical criterion to distinguish them. A5 is required to ensure closure and uniqueness of the emergent theory.

13.6 A6 — Local Compositional Closure

A6 requires that locally composable relations are not structurally forbidden.

If A6 is violated: the existence of a relation between two elements depends on the absence of another relation elsewhere; hidden exclusion rules introduce non-local constraints; and locality (A1) becomes internally inconsistent. A6 makes explicit that relational availability is determined locally — whether a relation is realised is governed by A4, not by structural prohibition. Without A6, the substrate admits implicit non-local structure incompatible with a local physical theory.

13.7 Summary

The admissibility axioms A1–A6 are not arbitrary assumptions but minimal conditions for the existence of operational, predictive physics. Each axiom excludes a distinct failure mode:

Axiom	Failure if violated
A1	No finite measurement
A2	No operational curvature
A3	No continuum / no effective laws
A4	Trivial vacuum / no selectivity
A5	Non-uniqueness / loss of predictability
A6	Hidden non-locality

Taken together, A1–A6 define the class of relational frameworks in which physical observables are well-defined, curvature is detectable, dynamics are non-trivial, and macroscopic laws emerge consistently. The results of this paper should therefore be understood as applying not to a specific constructed model, but to any framework satisfying these minimal conditions.