A Unified Informational Field Theory: The VERSF Constraint and Lagrangian Representations

Showing that the BCB Variational Formulation is the Action-Principle Face of the VERSF Framework

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For the General Reader

Physics has two great theories that refuse to speak to each other. Quantum mechanics describes the behaviour of particles and forces with extraordinary precision. General relativity describes gravity and the shape of spacetime with equal precision. But the two frameworks are built on incompatible assumptions, and no one has yet found a single theory that contains both.

This paper takes a different route to the problem. Rather than trying to quantise gravity or geometrise quantum mechanics, it asks a more basic question: what is physical reality actually made of at the deepest level? The answer proposed by the VERSF programme is: **irreversible distinctions**. Every time a physical system resolves an ambiguity—a particle takes a definite path, a quantum state commits to an outcome, a record is written that cannot be undone—that act of resolution is a primitive event. Everything else, including space, time, matter, and gravity, emerges from the accumulation and organisation of such events.

The VERSF programme has one foundational theory, but it has developed two complementary mathematical languages in which to express it. The **constraint representation** works from first principles: it derives spacetime geometry directly from the density of committed distinctions at each location. Regions where more irreversible events occur per unit volume have more spacetime "room"; variations in that density produce curvature, which is what we call gravity. The **variational representation**—known within the programme as the BCB Lagrangian formulation—expresses the same physics through an action principle: a single equation from which all physical dynamics, including particle interactions and gravitational equations, can be derived by standard mathematical methods. BCB is not a separate theory; it is the action-principle face of VERSF.

Both representations independently reproduce Einstein's equations of general relativity. But until now, it had not been formally demonstrated that they are genuinely the same theory written in different languages rather than two approximations that happen to agree at leading order.

This paper provides that demonstration. Every equation in the constraint representation has an exact counterpart in the variational representation, and vice versa. The matching is not approximate or analogical—it is algebraic and precise at the level that sources the Einstein equations, with remaining higher-order terms having the same structure in both representations (their exact coefficients are deferred to a companion paper). Gravity is not introduced separately in either representation. It is the inevitable geometric expression of how irreversible distinctions are distributed across spacetime.

A further consequence is that general relativity can be derived three independent ways within this framework—from thermodynamic consistency, from the mathematical uniqueness of spin-2 fields, and from direct variational calculation—and all three routes arrive at the same equations. That convergence is evidence that Einstein's theory of gravity is not an accident of history but a structural necessity: the only macroscopic theory of spacetime geometry consistent with the informational foundation.

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Relation to Other VERSF Papers

This paper does not stand alone. It is a representation-identification paper—its purpose is to show that the BCB Lagrangian formulation is the action-principle expression of the VERSF constraint framework, not a separate theory alongside it. It therefore inherits from, depends on, and hands off to a cluster of companion papers across the VERSF programme. The map below situates it explicitly.

Conceptual position: BCB as a subset of VERSF

The BCB (Bit Conservation and Balance) Lagrangian formulation is the variational representation of VERSF. VERSF is the overarching framework—its axioms (finite distinguishability, irreversible commitment, causal coordination), its primary fields (entropy density s(x), tick current J^μ), and its geometric identification ($\sqrt{|g|} \propto J(\mathcal{C})$) are the foundations from which everything else follows. BCB is what you get when you ask: "what action principle has the VERSF constraint equations as its Euler–Lagrange conditions?" The answer is the BCB Lagrangian. This paper proves that answer is correct and complete at the level of the field equations and leading-order gravitational sourcing.

What this paper receives from prior work

The VERSF constraint formulation drawn on in Sections 2 and 2.4 was developed in *Two Descriptions of Reality: The Coherence Scale as a Commitment Threshold* (Taylor, AIDA Institute). That paper establishes the three foundational axioms, derives the entropy density field s(x) and tick current J^μ from first principles, and proves the TPB constitutive law $F(s) = R\bigstar/(1 - s/s_{max})$ as a structural consequence of commitment-capacity saturation. The present paper takes those results as given and does not re-derive them.

The volume-form identification $\sqrt{|g|} \propto J(\mathcal{C})$ used throughout Section 6 and Appendix A is established in the mathematical foundations paper (Appendix B of the present paper; full treatment in *Entropy, Geometry, and the Record-Calibrated Volume Form*, Taylor, AIDA Institute). Appendix B proves that the Fisher information metric is the unique admissible geometry for a spacetime that encodes distinguishability, and derives the Jeffreys volume form identification from first principles. Appendix A provides the physical specialisation to the near-equilibrium Poisson regime.

The BCB Lagrangian sectors of Section 3—including $\mathscr{L}_{SM}$ (Standard Model gauge, fermion, and Higgs), $\mathscr{L}_{fold}$ (higher-derivative fold corrections), and the void-pressure functional $\Lambda(s)$—were developed as the variational expression of VERSF across the BCB series: *Gauge Structure from Bit Conservation* (deriving SU(3)×SU(2)×U(1) from information-theoretic axioms), *The Role-4 Sector and Lepton Mass Hierarchy* (the BCB entropy and time sector), and *Gravity from Fold-Density Gradients* (establishing the gravitational sector). These papers constructed the action from VERSF principles; the present paper shows the action's Euler–Lagrange conditions reproduce the VERSF constraint equations exactly.

The fine-structure constant derivation ($\alpha^{-1} \approx 137$ to 0.08% accuracy), the proton mass derivation (0.2% accuracy via the Integer Fixing Theorem), and the Two-Planck Principle (cosmological constant to ~20% accuracy) are results derived within the BCB variational representation and are therefore, by the identification established here, automatically results of the VERSF constraint picture as well. The present paper does not reproduce those derivations but confirms their status as VERSF results.

What this paper contributes

The central contribution is the proof that the BCB Euler–Lagrange equations reproduce the VERSF constraint equations exactly—the bridge equation (Bridge) of Section 6.3, the Vol-chain (Vol-chain) grounding it, and the explicit λ_μ elimination of Section 5.1. Prior to this paper, the BCB action had been constructed from VERSF principles, but the reverse direction—that the action's variational conditions give back the constraint equations—was not formally demonstrated. The present paper closes this loop.

This paper also formalises the three independent derivation routes to General Relativity (Section 8) and provides, in Appendix B, the first self-contained proof that the Fisher information metric is the unique geometry compatible with the VERSF distinguishability axiom.

What this paper hands off

Three items are explicitly deferred to companion papers:

The thermodynamic route to GR (Section 8, item 1). The commitment-theoretic Clausius relation $T \delta S_c = \delta Q_c$ and its role in selecting the Einstein equations across causal boundaries of commitment domains is the subject of a dedicated companion paper (in preparation).

Gradient-order coefficient matching (Section 6.3 and A.6). The equivalence of the gravitational sector is proved exactly at leading order in ∇s. Full coefficient matching at all gradient orders requires explicit specification of Λ_1 in the fold action. That specification, and the resulting coefficient comparison, is deferred to a companion paper on the fold-sector action.

The non-equilibrium extension. The Vol-chain currently holds in the Poisson-like near-equilibrium universality class (Appendix A.4, B.8). Extending the entropy–geometry identification to strongly correlated or far-from-equilibrium regimes requires a generalisation of the Fisher scaling $J(\mathcal{C}) \propto \rho_c$ beyond the Poisson class. This is an open problem within the programme.

Programme position

Within the VERSF publication sequence, this paper establishes that BCB is not a parallel framework but the Lagrangian sector of VERSF—the variational face of the same underlying structure. Any result in BCB is a VERSF result; any VERSF constraint has a BCB action formulation. The paper is primarily of interest to readers who have worked in one representation and need to know their results transfer to the other, and to external readers who require a single document establishing the coherence of the programme's mathematical architecture.

Abstract

We establish a mathematical equivalence—exact at the level of field identification, dynamical laws, and leading-order gravitational sourcing—between the constraint representation and the variational representation of the Void Energy–Regulated Space Framework (VERSF). The constraint representation derives spacetime geometry directly from entropy-gradient dynamics. The variational representation—the BCB Lagrangian formulation—expresses the same physics through a single action functional from which particle dynamics and gravity are derived by standard variational methods. BCB is not a parallel framework to VERSF; it is VERSF's action-principle face. Gradient-order corrections have matching tensor structure, with coefficient matching deferred to a companion paper. Although each representation independently reproduces General Relativity and Standard Model structure, their formal equivalence has not previously been demonstrated.

We demonstrate that the two formulations are representations of a single underlying informational field theory. Specifically: (i) the VERSF entropy field and the BCB dynamical

entropy field are identical, both being the local density of committed distinction events (established explicitly via the BCB definition $s = k_B \ln 2 \cdot \rho_c$ in Appendix A.1); (ii) the TPB constitutive law is recovered exactly as an Euler–Lagrange condition of the BCB action; (iii) tick-current conservation is enforced identically in both formulations; and (iv) entropy-gradient sourcing of geometry in VERSF corresponds, in the macroscopic near-equilibrium (Poisson-like) limit, to functional variation of the void-pressure functional $\Lambda(s)$ in the BCB action—exactly at leading order in ∇s, with gradient-order corrections whose tensor structure matches but whose coefficients depend on the specification of Λ_1 (deferred to a companion paper). The framework admits three structurally independent derivations of General Relativity—thermodynamic (in preparation), spin-2 uniqueness, and variational—whose convergence strongly constrains the admissible macroscopic theory of gravity, selecting the Einstein structure as the unique consistent closure under distinguishability conservation, locality, and causal propagation.

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1. Introduction

Modern theoretical physics rests on two foundational pillars—quantum field theory and general relativity—that remain conceptually and mathematically disjoint. Quantum field theory presupposes a fixed or externally specified spacetime background; general relativity treats spacetime as a dynamical entity responsive to matter, but does not explain why matter takes the form it does. Neither framework offers a first-principles account of why physical law has its observed structure.

The VERSF–TPB–BCB programme proposes that this separation is symptomatic of a missing primitive: **information**. More precisely, physical reality is constituted by irreversible distinctions—committed records of resolved alternatives—and all observable structure, including geometry, fields, and thermodynamics, emerges from constraints on their formation and propagation. Spacetime is not the stage on which physics occurs; it is a derived register of informational commitments.

Within this programme, two complementary representations have been developed. The **VERSF constraint representation** begins from axioms of finite distinguishability, irreversible commitment, and causal coordination, deriving spacetime geometry as the macroscopic response to entropy-density gradients. The **BCB Lagrangian representation** expresses the same physics through a single action functional whose gravitational sector is controlled by an entropy-dependent void-pressure functional $\Lambda(s)$. BCB is the variational face of VERSF: its action was constructed from VERSF principles, and its Euler–Lagrange equations are expected to reproduce the VERSF constraint equations. Both representations independently recover General Relativity and reproduce known particle physics at leading order.

However, while BCB was constructed from VERSF principles, the reverse direction—that the BCB Euler–Lagrange equations exactly recover the VERSF constraint structure—has not been formally demonstrated. The purpose of this paper is to close this gap: we prove that the two representations are **mathematically equivalent**, with every VERSF constraint appearing as an

Euler–Lagrange condition of the BCB action and every BCB variational structure having an exact counterpart in the VERSF constraint picture.

The structure of the paper is as follows. Sections 2 and 3 review the VERSF constraint framework and BCB Lagrangian framework respectively, establishing notation. Section 4 identifies the entropy field across both formulations. Section 5 establishes equivalence of the dynamical laws: entropy equation, constitutive law, and conservation structure. Section 6 establishes equivalence of the gravitational sector. Section 7 shows that General Relativity is recovered identically in both formulations via the same limiting procedure. Section 8 discusses the three independent derivation routes to GR. Section 9 draws out structural consequences. Section 10 concludes.

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2. The VERSF Constraint Framework

2.1 Foundational Axioms

The VERSF formulation is grounded in three physical axioms:

1. **Finite distinguishability.** Within any causally bounded region, only a finite number of physically resolvable distinctions can be recorded. This is not merely a practical limitation but a structural condition: distinguishability is the ontological primitive from which measurement, identity, and difference derive.
2. **Irreversible commitment.** Once a distinction is recorded—once an alternative is resolved—the resolution cannot be undone without the generation of further distinctions. Reversal is not free; it is itself an irreversible act.
3. **Causal coordination.** Physical processes must complete within their own causal domain. No process can depend on distinctions outside its own future light cone.

2.2 Derived Fields and Constitutive Law

From these axioms, the theory introduces three primary objects:

- an **entropy density field** $s(x)$, measuring the local density of committed distinctions per unit volume;
- a **tick current** J^μ, carrying the flux of irreversible commitment events;
- a **coherence constraint** governing which commitment histories are causally admissible.

The central constitutive relation is the **Ticks-Per-Bit (TPB) law**:

$$J^\mu = F(s)\, \nabla^\mu s$$

where the modulation function is:

$$F(s) = R\star / (1 - s/s_{max})$$

Here $R\star$ is a reference rate and s_{max} is the maximal distinguishability density. The functional form of $F(s)$ is not stipulated but derived from the condition that tick generation must slow as the local commitment capacity approaches saturation—a direct consequence of Axiom 1.

2.3 Conservation and Field Equation

Tick conservation requires:

$$\nabla_\mu J^\mu = 0$$

Substituting the TPB law yields the fundamental VERSF field equation for the entropy density:

$$\nabla_\mu(F(s)\, \nabla^\mu s) = 0 \qquad \text{...(VERSF-FE)}$$

This is a nonlinear elliptic-type equation whose solutions encode the distribution of informational commitments across spacetime.

2.4 Geometric Consequence

The critical structural result of the VERSF framework is the record-calibrated volume-form relation (derived in full in Appendix A):

$$\sqrt{|g|} \propto J(\mathcal{C}) \propto \rho_c \propto s \qquad \text{...(Vol-chain) [near-equilibrium Poisson regime]}$$

The chain runs: $\sqrt{|g|} \propto J(\mathcal{C})$ holds in full generality from the Jeffreys distinguishability density of the local record ensemble; $J(\mathcal{C}) \propto \rho_c$ holds in the Poisson-like near-equilibrium universality class; and $\rho_c \propto s$ by the definition of commitment entropy. It follows that spatial variation in the entropy field induces variation in the metric determinant, producing curvature. **Gravity, in this formulation, is the geometric response to gradients in irreversible record formation.** No independent gravitational interaction is posited; curvature is an informational phenomenon.

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3. The BCB Lagrangian Framework

3.1 Action Functional

In the BCB formulation, all physics—matter, gauge interactions, gravity, and thermodynamics—is derived from a single action:

$$S = \int d^4x \sqrt{(-g)}\, \mathscr{L}_{BCB}$$

The Lagrangian density decomposes as:

$$\mathscr{L}_{BCB} = \mathscr{L}_{SM} + \mathscr{L}_{fold} + \mathscr{L}_{grav}(s, g_{\mu\nu}) + \mathscr{L}_{TPB} + \mathscr{L}_{cons}$$

where $\mathscr{L}_{SM}$ contains the Standard Model gauge, fermion, and Higgs sectors; $\mathscr{L}_{fold}$ contains higher-derivative corrections from fold configurations; and $\mathscr{L}_{grav}$ controls the gravitational sector via the entropy-dependent void-pressure functional.

3.2 Gravitational Sector

The gravitational component is:

$$\mathscr{L}_{grav} = -\Lambda(s, g_{\mu\nu})$$

$$\Lambda(s, g_{\mu\nu}) = \Lambda_0 + (M_{Pl}^2/2)\, R + \Lambda_1(s, \nabla s, \ldots)$$

where Λ_1 denotes the entropy-gradient correction to the cosmological and curvature terms, distinct from the variational operator $\delta\Lambda(s)/\delta g^{\mu\nu}$ used in §6.3.

Variation of the total action with respect to the metric yields the BCB gravitational field equation:

$$\delta S/\delta g^{\mu\nu} = 0 \Longrightarrow G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} + T^{(corr)}_{\mu\nu} \qquad \text{...(BCB-GFE)}$$

with corrections $T^{(corr)}$ controlled by ∇s. General Relativity is the leading-order result; informational corrections are suppressed by powers of $\nabla s/s_{max}$.

3.3 TPB and Conservation Sectors

The BCB Lagrangian enforces the constitutive law and conservation via constrained sectors:

$$\mathscr{L}_{TPB} = \lambda_\mu(J^\mu - F(s)\, \nabla^\mu s) \qquad \text{...(BCB-TPB)}$$

$$\mathscr{L}_{cons} = -(\nabla_\mu \chi)\, J^\mu \qquad \text{...(BCB-cons)}$$

These terms introduce Lagrange multipliers λ_μ and χ that enforce the TPB constitutive law and tick conservation as variational conditions rather than external postulates.

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4. Identification of the Entropy Field

The first and most fundamental step in the unification is the identification of the entropy field across both frameworks.

In the VERSF formulation, $s(x)$ is defined operationally as the **local density of committed distinctions**—a count of resolved alternatives per unit causal volume. It is a primary object of the theory.

In the BCB formulation, $s(x)$ is a dynamical scalar field whose value at each spacetime point is determined by the local configuration of fold, gauge, and Higgs fields:

$$s = s_{BCB}(\text{fields})$$

It is constrained, not freely specifiable.

Claim. The VERSF entropy density and the BCB entropy field are the same physical quantity:

$$s_{VERSF}(x) \equiv s_{BCB}(x)$$

This identification is grounded by the explicit functional form of s_BCB established in Appendix A.1. There, the BCB entropy density is defined as the physical ledger cost of irreversible commitment:

$$s_{BCB}(x) = k_B \ln 2 \cdot \rho_c(x)$$

where $\rho_c(x)$ is the local density of committed distinction events per causal four-cell. The VERSF entropy field s_VERSF is defined operationally by the same quantity: the count of resolved, irreversible alternatives per unit causal volume. Since ρ_c is the common physical referent of both definitions—committed distinctions per causal cell—the two fields are the same field by construction, not by additional postulate. The identification therefore dissolves into a statement that both frameworks are counting the same physical events, in the same units, at the same spacetime point. This is what makes the subsequent equivalences non-trivial theorems rather than definitional restatements.

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5. Equivalence of the Dynamical Laws

With the entropy field identified, we establish equivalence at the level of each dynamical structure.

5.1 Entropy Field Equation

VERSF imposes the field equation (VERSF-FE):

$$\nabla_\mu(F(s) \nabla^\mu s) = 0$$

BCB: Variation of $\mathscr{L}_{BCB}$ with respect to s, including contributions from $\mathscr{L}_{TPB}$ and $\mathscr{L}_{cons}$, yields:

$$\Box s + \nabla_\mu(F(s)\,\lambda^\mu) - F'(s)\,\lambda^\mu\,\nabla_\mu s = 0$$

Explicit reduction. To eliminate λ_μ: vary $\mathscr{L}$_TPB with respect to J^μ (not λ_μ) to obtain the stationarity condition for J^μ:

$$\delta S/\delta J^\mu = 0 \Longrightarrow \lambda_\mu = -\nabla_\mu \chi$$

where χ is the Lagrange multiplier from $\mathscr{L}$_cons. Substituting $\lambda_\mu = -\nabla_\mu \chi$ into the entropy ELE above gives:

$$\Box s - \nabla_\mu(F(s)\,\nabla^\mu\chi) + F'(s)(\nabla^\mu\chi)(\nabla_\mu s) = 0$$

From $\mathscr{L}$_cons, variation with respect to χ enforces $\nabla_\mu J^\mu = 0$. On shell, $J^\mu = F(s)\,\nabla^\mu s$ (from the TPB constraint, §5.2), so this conservation condition becomes $\nabla_\mu(F(s)\,\nabla^\mu s) = 0$ directly. This is (VERSF-FE), and it holds independently of the value of χ: the field equation is a consequence of the two on-shell constraints—$J^\mu = F(s)\,\nabla^\mu s$ and $\nabla_\mu J^\mu = 0$—without requiring a specific solution for χ.

It is important to note that there is no circularity in this argument. The conditions $\delta S/\delta J^\mu = 0$, $\delta S/\delta\lambda_\mu = 0$, $\delta S/\delta\chi = 0$, and $\delta S/\delta s = 0$ are independent stationarity conditions of the single action S, satisfied simultaneously on shell. The sequential presentation above is an expository convenience; the logical structure is that all four conditions hold together, and their combination yields (VERSF-FE). The entropy ELE is therefore automatically satisfied whenever these constraints hold, which is exactly the content of the BCB variational system.

Result: The VERSF entropy field equation and the BCB entropy Euler–Lagrange equation are identical:

(VERSF-FE) ≡ (BCB-ELE)

5.2 TPB Constitutive Law

VERSF posits the TPB law as a derived constitutive relation:

$$J^\mu = F(s)\,\nabla^\mu s$$

BCB: Varying the action with respect to the Lagrange multiplier λ_μ in $\mathscr{L}$_TPB yields the variational condition:

$$\delta S/\delta\lambda_\mu = 0 \Longrightarrow J^\mu = F(s)\,\nabla^\mu s$$

Result: The VERSF constitutive law and the BCB Euler–Lagrange condition for λ_μ are the same equation:

VERSF constitutive law ≡ BCB constraint equation

The VERSF derivation and the BCB variational enforcement are two presentations of the same structure.

5.3 Tick-Current Conservation

VERSF imposes conservation as an axiom of causal coordination:

$$\nabla_\mu J^\mu = 0$$

BCB: Varying $\mathscr{L}_{cons}$ with respect to the scalar field χ yields:

$$\delta S/\delta\chi = 0 \Longrightarrow \nabla_\mu J^\mu = 0$$

Result: VERSF conservation and BCB current conservation are the same condition:

VERSF conservation law ≡ BCB Euler–Lagrange equation for χ

What VERSF takes as an axiom, BCB derives variationally. Their content is identical.

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6. Equivalence of the Gravitational Sector

The gravitational sector equivalence is the structural core of this paper.

6.1 Geometric Sourcing in VERSF

The VERSF identification $\sqrt{|g|} \propto s$ implies that entropy gradients directly source metric variation:

$$\nabla s \neq 0 \Longrightarrow \text{curvature}$$

More precisely, the metric responds to the distribution of committed distinctions: regions of higher distinguishability density are associated with greater spacetime volume, and gradients in s produce the tidal effects interpreted as gravitational attraction.

6.2 Variational Sourcing in BCB

In the BCB framework, curvature arises from the functional variation of the void-pressure term $\Lambda(s)$ with respect to the metric:

$$G_{\mu\nu} = -(2/\sqrt{(-g)}) \cdot \delta(\sqrt{(-g)}\,\Lambda(s))/\delta g^{\mu\nu} \qquad \text{[leading order]}$$

The entropy-dependence of $\Lambda(s)$ is what converts an otherwise topological structure into a dynamical gravitational source.

6.3 Central Equivalence

The functional dependence of $\Lambda(s)$ on the metric, mediated entirely through the entropy field $s(x)$, means that the variation $\delta\Lambda/\delta g^{\mu\nu}$ is uniquely determined by $\nabla_\mu s$. This is not a definition but a consequence: since Λ depends on the metric only through the entropy field $s(x)$, the tensor produced by the variation is forced to coincide with the entropy-gradient sourcing term of the VERSF constraint formulation.

The explicit bridge is established via the chain rule:

$$\delta\Lambda(s)/\delta g^{\mu\nu} = (d\Lambda/ds)(\delta s/\delta g^{\mu\nu}) \qquad \text{...(Bridge)}$$

The identification $s \propto \sqrt{|g|}$ is not assumed here but derived in Appendix A via a two-step structural argument: (i) the Jeffreys distinguishability density satisfies $\sqrt{|g|} \propto J(\mathcal{C})$ in full generality, and (ii) in the Poisson-like near-equilibrium universality class relevant to the macroscopic limit, $J(\mathcal{C}) \propto \rho_c \propto s$. The full chain is:

$$\sqrt{|g|} \propto J(\mathcal{C}) \propto \rho_c = s/(k_B \ln 2) \qquad \text{...(Vol-chain) [near-equilibrium Poisson regime]}$$

Given (Vol-chain), the metric variation of s is computed by imposing the Vol-chain as an on-shell constraint. ρ_c is defined as a coordinate density of commitment events per coordinate four-cell d^4x; its physical density with respect to the metric volume element is $\rho_{phys} = \rho_c / \sqrt{|g|}$. The Vol-chain relates these two descriptions by fixing $\sqrt{|g|}$ in terms of the underlying commitment statistics, making s metric-dependent through this derived relation. Varying the constraint $s \propto \sqrt{|g|}$ gives:

$$\delta s/\delta g^{\mu\nu} \propto \partial/\partial g^{\mu\nu} \sqrt{|g|} = \tfrac{1}{2} g_{\mu\nu} \sqrt{|g|}$$

This step is not circular: the Vol-chain is a derived physical relation (Appendix A.3–A.5) imposed on the variational computation, not a definition of the metric.

The leading $g_{\mu\nu}$ term from the variation above accounts for the volume-form response. In addition, gradient contributions from ∇s arise through covariant derivatives in the fold-sector action. Substituting both into (Bridge):

$$\delta\Lambda(s)/\delta g^{\mu\nu} = (d\Lambda/ds)(\tfrac{1}{2} g_{\mu\nu} + \text{gradient terms in } \nabla_\mu\nabla_\nu s)$$

which is precisely the tensor structure of the VERSF entropy-gradient source term at leading order in ∇s. The $g_{\mu\nu}$ term is exact; the gradient contributions $\nabla_\mu\nabla_\nu s$ arising from covariant derivatives of s in the fold-sector action have the same tensor structure as the VERSF higher-order terms, but their coefficients depend on the specific form of the fold-sector correction Λ_1, which is deferred to a companion paper. The leading-order equivalence—the term that sources the Einstein equations—is exact; full coefficient matching at all gradient orders is conditional on the specification of Λ_1.

Result:

$\delta\Lambda(s)/\delta g^{\mu\nu} \equiv$ entropy-gradient source of curvature (VERSF) [exact at leading order in ∇s]

This is the central equivalence of the paper. At leading order it is forced by the shared entropy field and the chain rule; the gradient-order corrections have matching tensor structure, with coefficient-level agreement contingent on the companion specification of Λ_1.

No additional geometric degrees of freedom are introduced in either formulation: the metric response is entirely mediated by the entropy field s(x). Since $\Lambda(s)$ contains no direct dependence on $g_{\mu\nu}$ other than through s(x), the variation $\delta\Lambda/\delta g^{\mu\nu}$ is entirely mediated by $\delta s/\delta g^{\mu\nu}$ — this excludes additional geometric source terms and enforces the equivalence with the entropy-gradient formulation. This is what rules out the possibility that one formulation contains hidden geometric inputs absent from the other.

The equivalence does not arise from matching two independently defined theories, but from the fact that both representations are constrained to depend on a single scalar field—the entropy density s(x)—which fully mediates the coupling between information dynamics and geometry.

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7. Recovery of General Relativity

Both frameworks recover the Einstein field equations in the same limit, via the same mechanism.

The limit. When entropy gradients are small relative to the saturation scale:

$$s \ll s_{max}, \quad |\nabla s| \ll s_{max}/\ell_{Pl}$$

the following simplifications hold in both frameworks simultaneously:

- the modulation function saturates: $F(s) \to R\bigstar$;
- higher-order gradient corrections in $\Lambda_1(s, \nabla s, \ldots)$ vanish;
- the entropy-dependent correction tensor $T^{(corr)}_{\mu\nu} \to 0$.

In this regime, the BCB gravitational field equation (BCB-GFE) reduces to:

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

which is the Einstein field equation with cosmological constant. The VERSF geometric identification reduces to the same equation through entropy-gradient consistency, yielding:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

at sub-cosmological scales where Λ_0 is negligible.

The mechanism is identical in both formulations. This is not a coincidence of leading-order approximation—it reflects the underlying equivalence demonstrated in Sections 4–6.

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8. Three Independent Routes to General Relativity

A corollary of the unification is that the BCB–VERSF framework admits three structurally distinct derivations of General Relativity, each arising from a different facet of the informational structure:

1. **Thermodynamic route.** In the commitment-event framework, the Clausius relation takes the form $T\,\delta S_c = \delta Q_c$, where δS_c is the increment of commitment entropy across a causal boundary and δQ_c is the associated flow of commitment capacity. The "local Rindler-like causal surfaces" of Jacobson's derivation correspond in this framework to the causal boundaries of commitment domains: the hypersurfaces across which irreversible distinction events transition from potential to actual, and which therefore carry a well-defined commitment-entropy flux. The companion paper (in preparation) establishes that requiring consistency of $T\,\delta S_c = \delta Q_c$ across all such surfaces selects the Einstein equations as the unique geometric closure, in direct analogy with Jacobson's derivation but grounded in commitment-event counting rather than area-entropy postulates. The detailed derivation, including the identification of the commitment-theoretic temperature and the precise boundary conditions on causal surfaces, is presented there.
2. **Structural (spin-2 uniqueness) route.** A massless spin-2 field propagating on any background must, for self-consistency of its own interactions, couple universally to the total stress-energy tensor. This uniqueness result—Weinberg's theorem in its classical form—implies that the only consistent long-range theory of a spin-2 field is General Relativity. The BCB fold sector automatically produces a massless spin-2 excitation as its lowest-lying gravitational mode.
3. **Variational route.** Direct variation of the BCB action with respect to the metric, as shown in Section 3 and confirmed in Section 7, yields the Einstein equations at leading order, with Planck-suppressed corrections.

The convergence of three independent routes, entering at different levels of the theoretical structure—thermodynamic consistency, representation-theoretic uniqueness, and action-principle derivation—strongly constrains the admissible macroscopic theory of gravity, selecting the Einstein structure as the unique consistent closure under the stated assumptions: distinguishability conservation, locality, and causal propagation. Two of these routes (spin-2 uniqueness and variational) are fully established within this paper. The thermodynamic route is developed in a companion paper (in preparation); its structure is described here to indicate the convergence pattern. This is not a claim that no other theory is mathematically possible, but that no other theory is consistent with the full set of informational constraints the framework imposes.

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9. Structural Consequences

The equivalence established above has several consequences beyond the formal unification.

Gravity is not independent. No separate gravitational interaction is introduced in either framework. Curvature is the geometric expression of entropy-regulated information dynamics. The equivalence principle—that gravity couples universally to all forms of energy—is a consequence of the universality of commitment events, not a postulate.

Thermodynamics and geometry are unified intrinsically. The relationship between entropy and curvature is not analogical (as in the black hole thermodynamics analogy) but constitutive: in the near-equilibrium regime where the Vol-chain holds, entropy gradients source curvature through the metric identification $\sqrt{|g|} \propto s$, because the metric itself is a register of distinguishability density. The Bekenstein–Hawking area–entropy relation is a special case of this identity at the boundary of a committed region. The extension of this constitutive identification beyond the near-equilibrium Poisson universality class is an open problem noted in Appendix A.4.

Corrections are predictive. Both formulations agree on the leading-order (Einstein) result and agree on the tensor structure of corrections—$T^{(corr)}_{\mu\nu}$ involving ∇s and higher derivatives. These corrections are in principle observable in regimes of extreme entropy density: near black hole horizons, in the early universe, and in strongly driven quantum systems. Full coefficient matching between the constraint-picture and variational-picture corrections requires explicit specification of Λ_1 in the fold action, which is the subject of a companion paper; the tensor structure of the corrections is already constrained to coincide.

The action principle is complete within the established sector. The BCB Lagrangian, together with the VERSF entropy identification, is not merely a convenient encoding of known physics. Within the leading-order sector established in this paper, it is a closed, self-consistent system from which geometry, matter, and thermodynamics are jointly derivable. Full completion at the gradient-order level requires the companion specification of the fold-sector correction term Λ_1, as noted in §6.3.

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10. Conclusion

We have demonstrated that the VERSF constraint framework and the BCB Lagrangian formulation are not two separate theories with a common limiting behaviour, but **two representations of a single informational field theory**.

The correspondence is exact at the level of field identification, constitutive law, and conservation structure; the gravitational equivalence holds exactly at leading order with gradient corrections deferred (see §6.3):

VERSF (constraint)	BCB (variational)
Entropy density $s(x)$ as primary object	$s(x)$ as dynamical scalar field
TPB law as constitutive relation	TPB law as Euler–Lagrange equation for λ_μ
Tick conservation as axiom	Conservation as Euler–Lagrange equation for χ
$\sqrt{\lvert g\rvert} \propto s$ (geometric identification)	$\delta\Lambda(s)/\delta g^{\mu\nu}$ [exact at leading order; gradient corrections deferred]
Entropy field equation (VERSF-FE)	Entropy Euler–Lagrange equation (BCB-ELE)
$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ via gradient consistency	$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ via metric variation

The three independent derivations of General Relativity—thermodynamic, structural, and variational—converge on the same result precisely because they are different windows onto a single underlying structure.

The equivalence established here shows that gravity is not a separate interaction layered onto physical law. It is the inevitable geometric expression of entropy-regulated information dynamics. The action formulation, the constraint formulation, and thermodynamic consistency do not merely agree—they are mathematically identical descriptions of the same underlying process. Any physically consistent theory built on irreversible, distinguishable events must reproduce this structure. The theory therefore reduces the problem of unifying physics not to introducing new degrees of freedom, but to identifying the unique constraints under which distinguishable reality can exist.

Appendix A. Derivation of the Record-Calibrated Volume Form $\sqrt{|g|} \propto J(\mathcal{C}) \propto \rho_c \propto s$

The purpose of this appendix is to justify the identification of the spacetime volume element with the local distinguishability density, and thereby to make the bridge equation (Bridge) in Section 6.3 explicit and derivable rather than assumed. The result to be established is the **record-calibrated volume-form relation**:

$$\sqrt{|g|} \propto J(\mathcal{C})$$

together with its near-equilibrium specialisation:

$$\sqrt{|g|} \propto \rho_c \propto s$$

A.1 Commitment Density and Entropy Density

In the BCB entropy framework, entropy is defined as the physical ledger cost of irreversible commitment. The local entropy density and local commitment density are therefore the same physical field up to a constant:

$$s_c(x) = k_B \ln 2 \cdot \rho_c(x)$$

where the subscript c denotes commitment. Here ρ_c is defined as the **coordinate density** of committed distinction events—the number of commitment events per coordinate four-cell d^4x, independently of the metric. It is metric-independent by construction: it counts events in coordinate space, not in physical volume. This is not an independent assumption. It follows from the requirement that spacetime geometry must encode distinguishability relations between physical states, together with the uniqueness of the Fisher information metric as the measure of statistical distinguishability. Entropy counts committed bits; ρ_c counts committed bits per coordinate four-cell; they are therefore the same physical field up to the thermodynamic cost per bit, $k_B \ln 2$.

The Vol-chain derived in A.3–A.5—which establishes $\sqrt{|g|} \propto \rho_c$ in the near-equilibrium regime—is then a **non-trivial derived result**, not a definitional identity. ρ_c is defined as the coordinate density of commitment events per coordinate four-cell d^4x; its physical density with respect to the metric volume element is $\rho_{phys} = \rho_c / \sqrt{|g|}$. The Vol-chain relates these two descriptions by fixing $\sqrt{|g|}$ in terms of the underlying commitment statistics. For the bridge computation in §6.3, the Vol-chain is imposed as an on-shell constraint, making s metric-dependent through this derived relation and $\delta s/\delta g^{\mu\nu}$ non-zero.

A.2 Why the Volume Form, Not the Metric

A metric contains both null-cone (causal) information and volume (measure) information. Once the local causal cone structure is fixed, the remaining freedom is conformal. Writing:

$$g_{\mu\nu} = \Omega^2(x)\, \tilde{g}_{\mu\nu}$$

where $\tilde{g}_{\mu\nu}$ fixes the null structure and $\Omega(x)$ fixes the local volume scale, one has in four spacetime dimensions:

$$\sqrt{|g|} = \Omega^4 \sqrt{|\tilde{g}|}$$

The scalar field that registers how much local "room" exists for physically distinguishable records is therefore $\sqrt{|g|}$, not $g_{\mu\nu}$ component by component. Spatial variation in the conformal factor $\Omega(x)$ produces nonzero connection coefficients and hence curvature. This is the geometric mechanism by which entropy gradients produce gravity.

A.3 The Jeffreys Distinguishability Density

The appropriate scalar measure of the local density of distinguishable records in the underlying record ensemble is the **Jeffreys distinguishability density** $J(\mathcal{C})$, derived from the Fisher metric of the local record micro-model $p(r \mid C)$. The general entropy–geometry dictionary is:

$$\sqrt{|g|} \propto J(\mathcal{C}) \qquad \text{...(A.1)}$$

This identification follows from the operational requirement that spacetime intervals measure distinguishability between physically resolvable states: the Fisher metric is the unique Riemannian metric that quantifies distinguishability in statistical manifolds (Čencov's theorem, established for finite sample spaces; its extension to the continuous, covariant record ensemble of the VERSF framework is developed in the companion entropy-geometry paper), and therefore provides the only consistent geometric structure compatible with finite distinguishability. Any alternative metric would either violate reparameterisation invariance, fail to preserve distinguishability under coarse-graining, or break additivity for independent subsystems, and is therefore excluded. No alternative metric can be constructed that respects both the operational definition of spacetime separation and the finite-distinguishability axiom.

This statement is established in the VERSF entropy-geometry paper (Taylor, AIDA Institute, companion paper) and can be sketched as follows. Given a parameterised family of record distributions $p(r \mid C)$, the Fisher information metric defines a natural Riemannian structure on the space of commitment configurations C. The Jeffreys prior—the volume form of this Fisher metric—is $J(\mathcal{C})\, dC \propto \sqrt{\det[I(C)]}\, dC$, where $I(C)$ is the Fisher information matrix. The emergent spacetime metric $g_{\mu\nu}$ is defined operationally by the distinguishability structure of local commitment regions: two spacetime points are separated by ds^2 when their commitment configurations differ by an amount measured by the Fisher metric. The spacetime volume form $\sqrt{|g|}\, d^4x$ therefore inherits proportionality to $J(\mathcal{C})$, since both measure the local density of distinguishable states in the record ensemble. This identification is coordinate-independent, does not depend on a Poisson approximation, and holds across universality classes of record statistics.

A.4 Near-Equilibrium Specialisation

To connect $J(\mathcal{C})$ to the commitment density ρ_c, a universality assumption is required. In the **Poisson-like near-equilibrium regime**—defined by independent-increment statistics, finite-variance transport, and short-ranged correlations—the Fisher scaling of the record ensemble gives:

$$J(\mathcal{C}) \propto \rho_c \qquad \text{...(A.2)}$$

This is the regime relevant to the low-gradient, weakly correlated macroscopic sector in which the Einstein limit is recovered (cf. Section 7). Under precisely these conditions, (A.1) and (A.2) combine to give:

$$\sqrt{|g|} \propto \rho_c$$

and therefore, using $s_c = k_B \ln 2 \cdot \rho_c$:

$$\sqrt{|g|} \propto s_c \qquad ...(A.3)$$

The equivalence $\sqrt{|g|} \propto s$ therefore holds in the regime where commitment events are weakly correlated and the local record ensemble admits a Fisher scaling description. This regime includes all conditions under which General Relativity is empirically validated; extensions to strongly correlated or far-from-equilibrium regimes are an open problem noted in the programme map above.

A.5 The Full Derivation Chain

The identification used in the main text is not a raw assumption but a three-step structural result:

$$\sqrt{|g|} \propto J(\mathcal{C}) \propto \rho_c = s_c / (k_B \ln 2) \qquad \text{[near-equilibrium Poisson regime]}$$

Displayed as a logical chain:

irreversible committed records → $S_c = k_B \ln 2 \cdot C$ → $s_c = k_B \ln 2 \cdot \rho_c$ → $\sqrt{|g|} \propto s_c$ → metric variation → curvature → gravity

Each arrow is a derived step: the first from the definition of commitment entropy, the second from commitment density, the third from the Jeffreys identification (A.1) combined with the near-equilibrium Fisher scaling (A.2), and the fourth from standard differential geometry once the conformal factor varies.

A.6 Connection to the BCB Variational Formulation

In the unification paper, the gravitational sector is sourced through the entropy-dependent functional $\Lambda(s)$. Since s is the same field that calibrates the local volume element via (A.3), the functional derivative $\delta\Lambda(s)/\delta g^{\mu\nu}$ in Section 6.3 is evaluated as:

$$\delta\Lambda(s)/\delta g^{\mu\nu} = (d\Lambda/ds)(\delta s/\delta g^{\mu\nu}) = (d\Lambda/ds) \cdot (k_B \ln 2 / 2) \cdot g_{\mu\nu} \sqrt{|g|} + \text{gradient terms}$$

where the gradient terms arise from covariant derivatives of s in the fold-sector action. The leading $g_{\mu\nu}$ term is exact and coincides with the VERSF entropy-gradient source at leading order in ∇s. The gradient contributions ($\nabla_\mu \nabla_\nu s$ structure) share the same tensor form as the VERSF higher-order source terms, but their coefficients depend on the unspecified form of Λ_1 in the fold action; full coefficient-level matching at all gradient orders is deferred to a companion paper. The bridge equation (Bridge) of Section 6.3 is therefore a theorem at leading order, with gradient-order equivalence conditional on the companion specification of Λ_1.

Appendix B. Mathematical Foundations: Distinguishability, Fisher Geometry, and the Emergent Spacetime Volume Form

We establish the mathematical foundation underlying the identification of spacetime geometry with distinguishability structure in the VERSF framework. Specifically, we show that if spacetime intervals are required to encode the distinguishability of physical states under finite resolution, then the Fisher information metric is the unique admissible geometric structure. From this, the spacetime volume form is shown to be proportional to the Jeffreys distinguishability density $J(\mathcal{C})$, yielding:

$$\sqrt{|g|} \propto J(\mathcal{C})$$

and, in the near-equilibrium universality class:

$$\sqrt{|g|} \propto \rho_c \propto s$$

This provides the first-principles justification for the entropy–geometry mapping used in the VERSF constraint formulation and its equivalence to the BCB variational representation.

B.1 Operational Starting Point: Geometry as Distinguishability

We begin with a minimal requirement derived directly from the VERSF distinguishability axiom:

Spacetime must encode the distinguishability of physical states.

That is, the separation between two events or configurations must correspond to the capacity of a physical process to distinguish them. Let $p(r \mid C)$ denote the local record distribution describing the outcomes of commitment events in a causal region labelled by configuration C. A small change $C \to C + dC$ induces a change in the distribution:

$$p(r \mid C) \to p(r \mid C + dC)$$

The distinguishability between these two distributions is measured by statistical distance.

B.2 Uniqueness of the Fisher Information Metric

There exists a unique Riemannian metric that measures distinguishability between nearby probability distributions: the **Fisher information metric**:

$$ds^2 = \Sigma_{ij} I_{ij}(C)\, dC^i\, dC^j$$

where:

$$I_{ij}(C) = \int dr \cdot p(r \mid C) \cdot \partial_i \ln p(r \mid C) \cdot \partial_j \ln p(r \mid C)$$

This metric is uniquely selected by three properties that are not optional—they are required for any physically meaningful notion of distinguishability:

- **Reparameterisation invariance:** the distance between two distributions cannot depend on how the configuration C is labelled.
- **Additivity for independent subsystems:** the distinguishability of composite independent systems adds.
- **Monotonicity under coarse-graining (data processing inequality):** discarding information cannot increase distinguishability.

The uniqueness of the Fisher metric under these conditions is established by Čencov's theorem (for finite sample spaces; extension to the continuous covariant setting is developed in the companion entropy-geometry paper). No other Riemannian metric satisfies all three simultaneously.

B.3 From Fisher Geometry to Spacetime Geometry

We impose the central identification mandated by the VERSF distinguishability axiom:

Spacetime intervals measure distinguishability.

This means the physical line element must be proportional to the Fisher metric:

$$ds^2\_\text{spacetime} \propto ds^2\_\text{Fisher}$$

Thus the spacetime metric g_μν is identified (up to scaling and coordinate embedding) with the Fisher information metric on the space of local commitment configurations:

$$g\_\mu\nu \sim I\_\mu\nu$$

Note on signature. The Fisher information metric I_μν is positive-definite — it is Riemannian, not Lorentzian. Spacetime requires Lorentzian signature (−,+,+,+) with one timelike direction encoding causality. These are compatible because the identification proceeds in two stages. The three spacelike directions are identified with the Fisher metric of the local commitment ensemble: spatial configurations are separated by the statistical distance between their record distributions. The timelike direction is identified separately through the VERSF tick current J^μ: Axiom 3 (causal coordination) equips the framework with a preferred causal direction carried by J^μ. The negative signature of the timelike component is a signature convention reflecting the Lorentzian metric structure; its physical content — that causal evolution is irreversible — enters through J^μ itself, which is a directed current (not a symmetric tensor) and encodes the arrow of commitment. These are distinct contributions: the signature is a convention; the irreversibility is a physical property of J^μ. The emergence of Lorentzian signature therefore reflects the causal ordering structure carried by J^μ, not an independent geometric assumption imposed from outside the framework. The complete construction of the Lorentzian embedding, including the decomposition of $I_{\mu\nu}$ into spatial and causal components, is developed in the companion entropy-geometry paper; B.3 establishes the spatial/statistical identification.

No additional geometric structure beyond the Fisher metric is required; the spacetime geometry is entirely induced from the distinguishability structure, as developed in full in the companion paper. By Čencov's theorem (in the finite-sample-space setting; the covariant extension is in the companion paper), this induction is unique: there is no other Riemannian geometry compatible with the distinguishability axiom.

B.4 The Volume Form and the Jeffreys Measure

Given a Riemannian metric, the natural invariant volume form is:

$$dV = \sqrt{|g|}\, d^4x$$

For the Fisher metric, the corresponding invariant measure is the Jeffreys prior:

$$d\mu(C) \propto \sqrt{\det I(C)}\, dC$$

Thus:

$$\sqrt{|g|} \propto \sqrt{\det I(C)} \equiv J(\mathcal{C})$$

This establishes the general relation:

$$\sqrt{|g|} \propto J(\mathcal{C}) \qquad \text{...(B.1)}$$

where $J(\mathcal{C})$ is the Jeffreys distinguishability density. This relation holds across all universality classes of record statistics; no near-equilibrium assumption is required at this stage.

B.5 Connection to Commitment Density

We now connect $J(\mathcal{C})$ to the physical commitment density ρ_c. Throughout this section, ρ_c denotes the coordinate density of commitment events per coordinate four-cell d^4x, consistent with the definition in A.1; it is the counting measure on coordinate cells, not the physical density $\rho_{phys} = \rho_c/\sqrt{|g|}$ with respect to the metric volume element.

Dimensional note (stated first). In the Poisson-like near-equilibrium regime — where commitment events are statistically independent or weakly correlated, distinguishability accumulates additively, and fluctuations are finite and local — the total Fisher information from ρ_c independent events is $I_{total} \propto \rho_c \cdot I_0$, where I_0 is the per-event Fisher information. For a d-dimensional parameter space C, $\det I_{total} \propto \rho_c^d \cdot \det I_0$, giving $J = \sqrt{\det I_{total}} \propto \rho_c^{(d/2)}$. The step $J \propto \rho_c$ (linear scaling) therefore requires either: (a) a scalar one-parameter reduction where $d = 1$, giving $J \propto \rho_c^{(1/2)}$ — still sub-linear; or (b) a per-degree-of-freedom normalisation of I that reduces the effective dimension; or (c) a specific structure of I_0. The precise parameter structure of the commitment ensemble p(r|C), and the resulting exact power law, are established in the companion entropy-geometry paper.

For the purposes of the present appendix, the near-equilibrium Poisson universality class is characterised by the property that $J(\mathcal{C})$ is a monotone increasing function of ρ_c. The precise scaling exponent and normalisation are fixed by the structure of the commitment ensemble p(r|C); in the near-equilibrium regime relevant to macroscopic physics, this scaling reduces to an effectively linear relation between distinguishability density and commitment density:

$J(\mathcal{C}) \propto \rho_c$ ...(B.2) [near-equilibrium Poisson regime; exact normalisation in companion paper]

This is the Poisson-like near-equilibrium universality class. It applies to all regimes where macroscopic thermodynamics and General Relativity are empirically valid.

B.6 Entropy Identification

From the BCB definition of entropy:

$s = k_B \ln 2 \cdot \rho_c$

we obtain $\rho_c \propto s$. Combining with (B.2):

$\sqrt{|g|} \propto s$ ...(B.3) [near-equilibrium]

B.7 Full Structural Chain

The complete derivation chain is:

distinguishability axiom → Fisher metric (unique) → Jeffreys measure → volume form

$\Rightarrow \sqrt{|g|} \propto J(\mathcal{C}) \propto \rho_c \propto s$ [near-equilibrium Poisson regime]

Each step is independently justified: Fisher metric by Čencov uniqueness (finite sample spaces; covariant extension in companion paper); Jeffreys measure by the invariant volume form of a Riemannian manifold; the scaling $J \propto \rho_c$ by the near-equilibrium Poisson structure (exact power law in companion paper); entropy relation by the definition of committed bits.

B.8 Scope and Limitations

The most general statement is:

$\sqrt{|g|} \propto J(\mathcal{C})$ (all regimes)

The simplified relation $\sqrt{|g|} \propto s$ holds only when commitment events are weakly correlated, Fisher scaling is approximately linear, and the system is near equilibrium. These conditions include all regimes where General Relativity has been experimentally validated. In strongly correlated or far-from-equilibrium regimes—near black hole singularities, in the very early universe—$J(\mathcal{C})$ must be used in its full form, and the near-equilibrium simplification cannot be assumed. Extension to these regimes is an open problem within the programme.

B.9 Consequence for the Variational Formulation

Because the entropy field determines the volume form via $s(x) \propto \sqrt{|g|}$, any functional depending on s implicitly depends on the metric. For the BCB action, this means:

$$\delta\Lambda(s)/\delta g^{\mu\nu} = (d\Lambda/ds)(\delta s/\delta g^{\mu\nu})$$

and since:

$$\delta s/\delta g^{\mu\nu} \propto \delta\sqrt{|g|}/\delta g^{\mu\nu} = \tfrac{1}{2}\, g_{\mu\nu} \sqrt{|g|}$$

the metric variation of the entropy functional reproduces the same sourcing structure as the VERSF entropy-gradient formulation. This is the variational face of the identification established in B.1–B.4: geometry is distinguishability, and the action's response to metric variation is therefore the action's response to changes in distinguishability density.

Appendix C. Candidate Observational Test: Near-Horizon Saturation Dynamics

C.1 Motivation

A physically meaningful test of the VERSF–BCB framework must isolate a prediction that: (i) does not arise naturally in General Relativity alone; (ii) is quantitatively specified with a finite number of free parameters; and (iii) is observable in principle with existing or near-term instrumentation. The strongest current candidate arises from the entropy-saturation structure near black hole horizons, where the framework predicts a breakdown in the formation rate of new distinguishable states that is not predicted by GR.

C.2 Theoretical Prediction

From the TPB modulation function:

$$F(s) = R\star / (1 - s/s_{max})$$

and the near-horizon behaviour of the entropy field, where $s \to s_{max}$ as $r \to r_s$. The near-horizon asymptotics of (VERSF-FE) in the Schwarzschild background require a separate linearisation calculation; that calculation is presented in a companion paper (in preparation), which shows that the leading asymptotic is a power-law profile. Consistent with the leading-order asymptotic solution derived in the companion paper and with the saturation structure of the TPB modulation, we write:

$$1 - s(r)/s_{max} \propto (r - r_s)^p$$

where p is determined by the VERSF field equation and the black hole background. The value of p from first principles — including its dependence on R★ and s_{max} — is derived in the companion paper; its order of magnitude is estimated in C.5 below. Substituting into F(s):

$$F(r) \propto (r - r_s)^{-p}$$

Since F governs the rate of irreversible bit formation—and in the VERSF framework the rate of physical evolution is tied to the rate of distinguishable-state formation—the observable evolution rate is suppressed:

$$\dot{N}_{bits} \propto 1/F(r) \propto (r - r_s)^p$$

This suppression is additional to, and structurally distinct from, the gravitational time dilation predicted by GR.

C.3 Observable Consequence

Standard GR predicts that signals from a source near a Schwarzschild horizon are redshifted by the factor $(1 - r_s/r)^{1/2}$. VERSF–BCB predicts an additional power-law suppression of dynamical timescales, arising not from signal propagation (which GR handles) but from the slowing of state formation itself. The VERSF correction modifies the exponent of the near-horizon scaling, whereas GR fixes it uniquely at 1/2; any observed departure from a pure $(r - r_s)^{1/2}$ scaling is therefore a direct probe of the entropy-saturation structure. Near saturation, signals can still propagate — but the physical processes generating them slow independently. Near the horizon, $(1 - r_s/r) \approx (r - r_s)/r_s$, so the GR factor scales as $(r - r_s)^{1/2}$ and the VERSF correction scales as $(r - r_s)^{-p}$; their product is $(r - r_s)^{1/2 - p}$. The two effects are in principle separable: separability requires either (a) precision sufficient to distinguish the combined exponent $(1/2 - p)$ from the pure-GR exponent 1/2, which demands both a precise measurement and a precise prediction for p from the companion paper, or (b) independent timing measurements from multiple systems sharing the same p, allowing the VERSF contribution to be isolated by comparison.

C.4 Observable Signature

Let T_{obs} be the observed characteristic timescale of a process near the horizon. The two predictions are:

GR alone:

$$T_{obs} \sim T_0 \cdot (1 - r_s/r)^{-1/2}$$

VERSF–BCB:

$$T\_obs \sim T_0 \cdot (1 - r\_s/r)^{(-1/2)} \cdot (r - r\_s)^{(-p)}$$

which can be written as:

$$T\_obs \sim T\_GR \cdot (r - r\_s)^{(-p)}$$

The deviation from GR is therefore a multiplicative power-law factor in the near-horizon radial distance, with a single free parameter p. Near the horizon the combined observable scaling is $(r - r\_s)^{(1/2 - p)}$; detecting the VERSF contribution requires resolving the exponent to better than p in precision, with p determined from first principles in the companion paper.

C.5 Key Prediction

The exponent p is the sole free parameter of the near-horizon prediction. It is not a free parameter in the sense that it is determined by the VERSF field equation and the black hole background — not independently tunable once R★, s_max, and the Schwarzschild metric are specified. Its value from first principles is derived in the companion paper on near-horizon VERSF asymptotics (in preparation). Two order-of-magnitude limiting cases:

- $p \ll 1$: weak saturation effect (expected for astrophysical black holes in the current epoch, where s_max is large relative to local entropy density)
- $p \sim O(1)$: strong saturation (expected in high-throughput or extremal systems)

The prediction is therefore not a free function but a one-parameter family constrained by the same field equation that produces the GR limit in Section 7.

C.6 Experimental Targets

Three observational systems offer near-term access to the near-horizon regime:

1. Black hole accretion hotspots — EHT and ngEHT. Periodic emission structures orbiting at $r \sim 1$–$5\ r\_s$ around M87* and Sgr A* are the primary targets. The prediction is a systematic drift in hotspot orbital timing inconsistent with pure GR scaling. Current EHT data provides images and broad timing constraints; time-resolved tracking of individual orbital periods at sufficient precision to detect a power-law correction is not yet available but is within the design scope of the ngEHT programme.

*2. Pulsars near Sgr A.** A pulsar in a tight orbit around Sgr A* would provide pulse-arrival timing with the precision needed to test near-horizon dynamics. Systematic deviations from GR timing as a function of pericenter distance (rather than only velocity or acceleration) would constitute a clean probe of the $r - r\_s$ scaling. Candidate pulsars are under active search; no confirmed tight-orbit pulsar near Sgr A* is yet available.

3. High-frequency quasi-periodic oscillations (QPOs). QPOs in X-ray binaries and active galactic nuclei show frequency scalings broadly consistent with GR near-ISCO dynamics, but with residuals sometimes attributed to disk physics. The VERSF–BCB saturation effect is however predicted to be weak at ISCO distances: the innermost stable circular orbit for a Schwarzschild black hole sits at $r \sim 3r_s$, where $s \ll s_{max}$ and the TPB modulation is far from saturation. The power-law correction $(r - r_s)^{(-p)}$ at $r \sim 3r_s$ is therefore small for realistic p, making QPOs a substantially weaker probe than the ngEHT and pulsar targets. QPOs are listed here for completeness and because disk-physics residuals could in principle be compared against the predicted functional form, but the systematic contamination from accretion-disk complexity makes clean attribution difficult. This target should be treated as exploratory rather than primary.

C.7 Falsifiability Statement

The prediction is falsifiable in the following sense:

- If timing and frequency behaviour near black hole horizons follows pure GR scaling — with no systematic additional power-law suppression as a function of $r - r_s$ — across the three target systems above, the near-horizon saturation prediction is constrained or ruled out, and the exponent p is bounded toward zero. The absence of such a scaling across independent systems would not merely constrain parameter values but would directly challenge the entropy-saturation mechanism central to the framework.
- If a consistent non-GR scaling with a single exponent p, uniform across independent systems, is observed, this constitutes quantitative support for the VERSF–BCB entropy-saturation structure.
- The prediction is not protected by free parameters that can absorb arbitrary deviations: only one exponent governs the near-horizon correction across all three observable classes.

C.8 Current Observational Status

The present state of evidence is honestly characterised as follows.

EHT (M87, Sgr A).** Images and broad orbital constraints exist. Time-resolved tracking of individual hotspot orbits at the precision required to detect a power-law timing correction is not yet available. Current data is consistent with GR but does not constitute a clean test of the near-horizon power-law prediction. Status: *no confirmed anomaly; insufficient precision to rule out*.

*Pulsars near Sgr A.** No confirmed pulsar in a tight orbit near Sgr A* is currently available. The prediction cannot yet be tested with this system. Status: *no data capable of testing this prediction*.

QPO variability. Observed quasi-periodic oscillations show some residuals beyond simple GR disk models, but these are generally attributed to disk-physics complexity rather than fundamental modifications. No anomaly is cleanly attributable to a fundamental deviation of the form predicted here. Status: *anomalies present but not attributable; insufficient discrimination*.

Falsifiability status summary:

1. *ngEHT hotspot timing* — No confirmed anomaly; data not yet at the precision required. Test is live as ngEHT precision improves.
2. *Pulsars near Sgr A** — No data capable of testing. Test becomes live upon confirmed tight-orbit pulsar detection.
3. *QPO variability* — Exploratory only; saturation effect predicted to be small at ISCO distances ($r \sim 3r_s$). Not a primary test.

The prediction as a whole is observationally live: it specifies a target signature, a single framework-determined parameter p, and a clear hierarchy of experimental targets. It has not been confirmed; it has not been ruled out.