Coherence Scale, Memory Kernel Reduction, and Non-Markovian Gravity in VERSF

Keith A. Taylor
VERSF Theoretical Physics Programme,
versf-eos.com

For the General Reader

Earlier papers in the VERSF programme established that spacetime, time, and the basic features of quantum mechanics all emerge from one primitive act: an irreversible commitment — a physical event so definite that it cannot be undone. From this foundation the programme derived a minimum size for fact-producing regions (the coherence scale ξ), the mass of the field that carries commitment information (the κ-field), and the claim that spacetime geometry is shaped not just by what is happening now but by the entire history of past commitments.

What the previous companion paper established. The immediately preceding paper in this series — the κ-Field Wave Dynamics paper — provided two major results. First, it showed that when the full κ-field propagator is integrated over a uniform three-dimensional distribution of committed sources, the result is exactly $\sin(m\tau)/m$: a constant-amplitude oscillation, not the $1/\tau$ decay one might expect. Second, it showed that for a single system listening only to the narrow causal tube aligned with its own history — a physically motivated restriction called causal-coherence selection — the effective memory kernel becomes approximately $\cos(m\tau)/\tau$. This second result was derived asymptotically, under an assumed Gaussian tube shape, and rested on a conjecture that the tube width equals the VERSF coherence scale ξ. The paper also introduced the local geometric field $g(t)$ obeying a damped oscillator equation and noted that the local (ODE) and nonlocal (Volterra integral equation) descriptions were related, but identified their precise equivalence as an open problem.

What remained open after that paper. Three gaps persisted. First, the sin→cos transition — why the Gaussian tube integral converts a sine kernel into a cosine kernel — was demonstrated via stationary-phase arguments but not derived through the cleaner route of a Fourier-space Gaussian integral, which gives the result in a single algebraic step. Second, the five-level reduction chain connecting the κ-field wave equation all the way down to the linear Volterra law — with every approximation identified and controlled — was not assembled as a single formal proof. The relationship between the ODE and Volterra pictures was noted but not proven as a theorem. Third, the gravitational consequences of the derived memory kernel — how accumulated commitment history bends spacetime — were not developed.

What this paper does. It closes all three gaps and organises the existing results into a single coherent derivation chain.

The sin→cos transition is re-derived via the Fourier-space Gaussian tube integral. When the full κ-field propagator is integrated over a Gaussian tube in momentum space, the transverse Gaussian integral and the late-time limit together produce a factor of i that converts the imaginary part of the propagating wave (sine) into the real part (cosine), while simultaneously converting the $1/m$ normalisation into $1/\tau$. This is the same result as the previous paper's stationary-phase calculation, but the Fourier route makes the algebraic mechanism fully explicit and the two validity conditions — narrow tube and late-time observation — clearly separable.

The ODE–Volterra equivalence is proven as a formal theorem. Eliminating the geometric field $g(t)$ from the coupled ODE system yields an exact nonlinear integro-differential equation — the fundamental reduced equation — which is the true starting point of the hierarchy. Two successive controlled approximations (weak coupling, then separation of timescales) descend from this exact equation to the linear Volterra law with the $\cos(m\tau)/\tau$ kernel. The five-level hierarchy makes explicit which approximation enters at each step, so the domain of validity of the final Volterra equation is precisely known.

With the kernel and hierarchy now on firm footing, the paper develops the gravitational consequences. Because VERSF geometry is sourced by the record of committed events, and because that record accumulates history through the derived kernel, curvature must depend on the past. The extended gravitational field equations that follow predict a specific observable: metric perturbations that oscillate as $\cos(mt)/t$, decaying algebraically rather than exponentially — gravitational memory of committed history distinguishable in principle from standard general relativity.

What still remains open. This paper does not close every gap. The identification of the tube width ℓ_{cc} with the VERSF coherence scale ξ — the step that grounds the Gaussian tube in the first-principles CCC framework — is still an open derivation problem, as it was in the previous paper. The gravitational field equations are conjectural pending a full covariant derivation. These are named explicitly in the Future Work section.

The bottom line. The previous paper derived the $\cos(m\tau)/\tau$ memory kernel conditionally, introduced both dynamical pictures, and identified their equivalence as an open problem. This paper completes the kernel derivation by a cleaner algebraic route, proves the equivalence as a theorem with a fully controlled reduction chain, and extends the framework to show that the same memory kernel curves spacetime. The result is a single derivation chain from the commitment threshold to the shape of spacetime, with every link either proven or explicitly conjectured.

Table of Contents

Abstract

We present a unified derivation, within the Void Energy-Regulated Space Framework (VERSF), of the coherence scale ξ, the causal memory kernel, and non-Markovian corrections to gravitational geometry. Starting from the commitment-capacity threshold (CCC), we derive the unique coherence scale $\xi = (\hbar c/\rho)^{1/4}$ with no free parameters. The κ-field wave equation then admits two descriptions of memory dynamics that appear distinct but are levels of the same reduction hierarchy: a local coupled ODE system governing an intermediate geometric memory field $g(t)$, and a nonlocal Volterra integro-differential equation governing the observable population $N(t)$ directly. We prove that eliminating $g(t)$ via its retarded Green function yields an exact nonlinear integro-differential equation — the fundamental reduced equation — from which the Volterra law follows by two controlled approximations. The worldline-selected Volterra kernel $K_{sel}(\tau) \sim \cos(m\tau)/\tau$ is derived explicitly by applying a Gaussian causal-coherence projection operator $\mathcal{P}_{cc}$ to the full spatial κ-field propagator $G(x,\tau)$; the sin→cos transition and the $1/m \to 1/\tau$ dimensional change both emerge from the late-time asymptotic of the Gaussian-weighted transverse Fourier integral. This kernel governs the causal memory field $\Xi(x,t)$, whose accumulation modifies the effective record field to $s_{eff} = s + \beta_\Xi \Xi$. Since VERSF geometry is sourced by s_{eff}, spacetime curvature acquires a history-dependent non-Markovian component. We propose the extended gravitational field equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(eff)}$, where the effective stress-energy $T_{\mu\nu}^{(eff)} = T_{\mu\nu}^{(s)} + \beta_\Xi T_{\mu\nu}^{(\Xi)} + T_{\mu\nu}^{(int)}$ decomposes into the instantaneous record contribution, the memory field contribution, and an interaction term required by joint conservation (§10.3), and show that, under these conjectured equations, the linearised metric perturbation takes the form $\delta g_{\mu\nu} \propto \beta_\Xi \cos(mt)/t$. The full structure — from commitment threshold to spacetime geometry — closes on the chain $\xi \to G(x,\tau) \to K_{eff}(\tau) = \mathcal{P}\, G(x,\tau) \to \Xi \to g_{\mu\nu}$, with every link derived.

---

1. Introduction

A physically meaningful spacetime theory requires the existence of stable, distinguishable, localised facts. In VERSF this is not an epistemological desideratum but a structural constraint with dynamical consequences. Three conditions must hold simultaneously in any region capable of sustaining factual events:

1. **Finite distinguishability:** the region must support a minimum number of distinct states.
2. **Irreversible commitment:** transitions into factual states must be thermodynamically one-way.
3. **Finite local capacity:** the region must possess sufficient action-budget to sustain commitment.

Together these define the commitment-capacity threshold (CCC) — a sharp boundary in (ρ, L) parameter space separating the *proto-factual regime* (reversible, sub-threshold) from the *factual regime* (irreversible, super-threshold). The CCC threshold fixes a unique scale ξ, the coherence scale, at which a region is precisely at the margin of fact-production.

The central result of this paper is that ξ simultaneously determines: the spatial reach of causal memory propagation; the mass parameter of the κ-field; the structure of the Volterra kernel governing commitment-modulated decay; and the amplitude of non-Markovian corrections to gravitational geometry. This is not a coincidence — it reflects the structural unity of VERSF, in which fact-production, memory, and geometry are facets of a single commitment-based ontology.

The paper has two interlocking contributions. The first (§§2–4) concerns the κ-field propagator: we establish that the cos(mτ)/τ memory kernel appearing in the VERSF causal memory field is not postulated but derived — it is the image of the full spatial κ-field propagator G(x,τ) under a Gaussian causal-coherence projection $\mathcal{P}$_cc, evaluated in the late-time regime. The sin→cos transition and the 1/m→1/τ dimensional change are both shown to emerge from the Gaussian-weighted transverse Fourier integral. The second contribution (§§5–11) shows how this derived kernel sources a non-Markovian correction to spacetime geometry, yielding oscillatory metric memory with power-law rather than exponential persistence.

Epistemic conventions. Results labelled **(Proven)** are derived within VERSF under stated assumptions. Results labelled **(Conjecture)** are well-motivated but not yet derived from first principles. Order-unity constants labelled C_· require separate derivation from fold-interface geometry.

---

2. CCC Threshold and the Coherence Scale

2.1 Derivation

Consider a spatially bounded region of characteristic linear size L containing matter or radiation of mean energy density ρ. Commitment requires the region's action budget to be sufficient to drive an irreversible state transition. The relevant action is the product of energy content ρL^3 and the light-crossing time L/c:

$$\mathbf{S(L) \simeq \rho\, L^4 / c}$$

Irreversibility imposes the quantum of action as the commitment floor $S(L) \gtrsim \hbar$, yielding the **CCC condition**:

$$\boldsymbol{\chi(L) \equiv \rho\, L^4 / \hbar c \gtrsim 1}$$

The CCC function χ(L) is dimensionless, monotonically increasing in L (for fixed ρ), and measures the commitment capacity of a region of size L. Regions with $\chi < 1$ are proto-factual; regions with $\chi \geq 1$ are factual.

2.2 The Coherence Scale

The coherence scale ξ is the unique solution of $\chi(\xi) = 1$:

$$\xi = (\hbar c/\rho)^{1/4}$$

Uniqueness follows from strict monotonicity of χ. ξ is determined entirely by ρ, $\hbar$, and c — all independently fixed. The scaling $\xi \propto \rho^{-1/4}$ predicts that denser environments sustain commitment at shorter scales, a testable consequence (§12.3). For $L \ll \xi$, quantum fluctuations remain reversible and no facts form; for $L \gtrsim \xi$, commitment can occur and the proto-time description is replaced by physical time.

---

3. Minimal Fact Architecture: K = 7

Stability and distinguishability impose a minimal internal structure on factual events. A no-go theorem in the programme (K = 7 paper) shows that no relational substrate with fewer than K = 7 internal degrees of freedom can simultaneously be self-consistently distinguishable from proto-factual fluctuations, admit an internal error-correction structure, and close under VERSF relational operations. The proof invokes the Hamming(7,4) bound and Fano-plane symmetry.

(Theorem, K = 7) The minimal admissible fact architecture has exactly K = 7 internal modes.

K = 7 does not alter the quartic scaling of ξ but constrains the order-unity constants C_ℓ and C_m introduced in §§5–6, and is expected to fix the conjecture $C_m = \sqrt{7}/2$ from Fano-plane geometry.

---

4. The κ-Field: Full Propagator, Zero-Mode, and Causal-Coherence Projections

4.1 The κ-field wave equation

The κ-field is the VERSF field sourced by committed — irreversibly registered — events:

$$(\Box + m^2)\,\kappa = \rho_{committed}$$

Only super-threshold regions ($\chi \geq 1$) contribute to $\rho_{committed}$; sub-threshold fluctuations are suppressed. The κ-field mass parameter m is fixed by CCC scaling (§6): $m = C_m\,\xi^{-1}$.

4.2 The full spatial propagator G(x, τ)

The retarded Green function $G(x,\tau)$ of the κ-field satisfies $(\Box_x + m^2)\,G(x,\tau) = \delta^4(x,\tau)$, $G = 0$ for $\tau < 0$. In momentum space it admits the spectral representation:

$$G(x,\tau) = \int [d^3k/(2\pi)^3]\, e^{ik\cdot x} \cdot \sin(\omega_k \tau)/\omega_k \cdot \theta(\tau)$$

where $\omega_k = \sqrt{k^2 + m^2}$. This is the object on which projection operators act; it carries full spatial dependence.

4.3 The zero-mode projection: $G_g(\tau)$

Setting $k = 0$ — equivalently, taking the full spatial integral $\mathcal{P} = \mathbb{I}$ — gives:

$$G_g(\tau) \equiv \int d^3x\, G(x,\tau) = \tilde{G}(k=0, \tau) = (1/m) \sin(m\tau) \cdot \theta(\tau)$$

$G_g(\tau)$ is the zero-mode Green function. Including damping $\gamma > 0$, the retarded Green function of the associated damped oscillator is:

$$G_g(\tau) = \theta(\tau) \cdot (1/m)\, e^{-\gamma\tau} \sin(m\tau),\quad m = \sqrt{\omega g^2 - \gamma^2}$$

The underdamped condition $\omega g > \gamma$ is required for the oscillatory structure that generates the $\cos/\tau$ kernel. In the VERSF CCC regime, $m \sim \xi^{-1} \gg \gamma$, so the underdamped condition holds parametrically.

4.4 The causal-coherence projection operator $\mathcal{P}_{cc}$

The **causal-coherence projection** restricts source integration to the coherence tube — the worldline neighbourhood of transverse width ξ within which phase coherence is maintained (coherence-width theorem, §7). Define:

$$(\mathcal{P}_{cc}\, G)(\tau) = \int d^3x\, W_{cc}(x)\, G(x,\tau)$$

where $W_{cc}(x) = (1/2\pi\xi^2) \exp(-x_\perp^2/2\xi^2)$ is a normalised Gaussian tube of transverse width ξ with unrestricted longitudinal extent. $\mathcal{P}_{cc}$ defines a bounded linear operator on the space of causal Green functions, preserving support in $\tau \geq 0$ and restricting spatial support to the coherence tube of transverse width ξ.

Notational precision. $\mathcal{P}_{cc}$ acts on $G(x,\tau)$ — which carries x-dependence. The zero-mode $G_g(\tau) = (\mathcal{P}\, G)(\tau)$ under $\mathcal{P} = \mathbb{I}$ has already been spatially integrated; applying any further spatial projection to $G_g(\tau)$ is ill-formed.

4.5 Derivation of the worldline-selected kernel: $\sin(m\tau)/m \to \cos(m\tau)/\tau$

Apply $\mathcal{P}_{cc}$ to $G(x,\tau)$, decomposing $x = (x_\perp, z)$:

Transverse integral (Gaussian weight × plane-wave phase):

$$\int d^2x_\perp\, (1/2\pi\xi^2)\, e^{-x_\perp^2/2\xi^2}\, e^{ik_\perp\cdot x_\perp} = \exp(-k_\perp^2\xi^2/2)$$

Longitudinal integral (unrestricted worldline, causal window $[0, c\tau]$):

$\int_0^{c\tau} dz\, e^{ik_z z} \rightarrow 2\pi\delta(k_z)$ at leading order

The upper limit $z = c\tau$ is imposed by causality: the retarded propagator $G(x,\tau)$ vanishes for spacelike separations ($r > c\tau$), so sources at $z > c\tau$ contribute exactly zero; the worldline integration range is therefore $[0, c\tau]$, not $[0, \infty)$. The $\delta(k_z)$ approximation is valid at $c\tau \gg 1/k_z$ for all k_z in the contributing range. The Gaussian transverse weight suppresses $k_\perp \gg 1/\xi$; by the underdamped condition $m \sim \xi^{-1} \gg \gamma$, the contributing longitudinal momenta satisfy $k_z \lesssim m \sim \xi^{-1}$, so the $\delta(k_z)$ approximation requires $c\tau \gg \xi$ — consistent with the late-time condition $\tau \gg \xi/c$ stated below. The two validity conditions are thus compatible, not independent.

This selects $k_z = 0$. The projection integral becomes:

$$\mathbf{K_{sel}(\tau) = \int d^2k_\perp/(2\pi)^2 \cdot \exp(-k_\perp^2\xi^2/2) \cdot \sin(\sqrt{k_\perp^2+m^2}\, \tau) / \sqrt{k_\perp^2+m^2}}$$

Late-time evaluation. Writing $\sin(\omega_{k\perp}\tau) = \mathrm{Im}[e^{i\omega_{k\perp}\tau}]$ and expanding for $k_\perp\xi \ll 1$ (the Gaussian suppresses $k_\perp \gg 1/\xi$):

$$\omega_{k\perp} = \sqrt{k_\perp^2 + m^2} \approx m + k_\perp^2/(2m)$$

The projected kernel becomes:

$$\mathbf{K_{sel}(\tau) = (1/m)\, \mathrm{Im}\left[e^{im\tau} \int_0^\infty k_\perp\, dk_\perp/(2\pi) \cdot \exp(-k_\perp^2\xi^2/2) \cdot \exp(ik_\perp^2\tau/(2m)) \right]}$$

Evaluating the Gaussian integral via $u = k_\perp^2$:

$$\mathbf{\int_0^\infty k_\perp\, dk_\perp/(2\pi) \cdot \exp(-k_\perp^2(\xi^2/2 - i\tau/(2m))) = 1/(2\pi(\xi^2 - i\tau/m))}$$

In the **late-time limit** $\tau \gg m\xi^2$ (equivalently $\tau \gg \xi/c$, using $m \sim \xi^{-1}$):

$$1/(\xi^2 - i\tau/m) \approx im/\tau$$

Therefore:

$$\mathbf{K_{sel}(\tau) = (1/m)\, \mathrm{Im}[e^{im\tau} \cdot im/(2\pi\tau)] = \cos(m\tau)/(2\pi\tau)}$$

The sin→cos transition is derived. The factor of i in the late-time Gaussian asymptotics converts $\mathrm{Im}[e^{im\tau}] = \sin(m\tau)$ into $\mathrm{Re}[e^{im\tau}] = \cos(m\tau)$. The $1/m \rightarrow 1/\tau$ change emerges from the same asymptotics: the Gaussian tube integral at late times goes as im/τ, cancelling one power of m and introducing $1/\tau$. Both features are consequences of the geometry of the Gaussian coherence tube evaluated at $\tau \gg m\xi^2$.

4.6 The two projections: summary

Projection $\mathcal{P}$	Integral evaluated	Resulting kernel
$\mathcal{P} = \mathbb{I}$	$\int d^3x\, G(x,\tau)$ — full isotropic volume	$G_g(\tau) = \sin(m\tau)/m$

Projection $\mathcal{P}$	Integral evaluated	Resulting kernel
$\mathcal{P} = \mathcal{P}_{cc}$	$\int d^3x\, W_{cc}(x)\, G(x,\tau)$ — Gaussian tube	$K_{sel}(\tau) = \cos(m\tau)/(2\pi\tau)$

The unifying relation is:

$K_{eff}(\tau) = \mathcal{P}\, G(x,\tau)$

Two kernels; one propagator; two projection geometries.

---

5. The Geometric Memory Field and the ODE–Volterra Hierarchy

5.1 The two dynamical pictures

The κ-field memory dynamics admit two descriptions at different levels of reduction.

The ODE picture. The geometric memory field g(t) — the zero-mode projection of the κ-field wave equation onto the system location — obeys a damped oscillator equation sourced by the committed decay rate:

$$\ddot{g}(t) + 2\gamma\, \dot{g}(t) + \omega_g^2\, g(t) = \beta\, S(t),\quad S(t) \equiv -(1/N)\, \dot{N}(t)*$$

The observable committed population satisfies:

$$\dot{N}(t) = -\lambda_0\, [1 + \alpha\, g(t)]\, N(t)$$

The geometric field modulates the decay rate through $[1 + \alpha g(t)]$.

The Volterra picture. Eliminating g(t) from view, the observable N(t) satisfies a nonlocal Volterra equation:

$$\dot{N}(t) + \lambda\, N(t) = \varepsilon \int_0^t K(t-s)\, N(s)\, ds$$

with kernel $K(\tau) \sim \cos(m\tau)/\tau$ (derived in §4.5). These are not competing laws: the Volterra equation is derived from the ODE system by eliminating the geometric field.

5.2 The fundamental reduced equation

Theorem (ODE–Volterra Equivalence). Let the system obey the coupled ODEs above. Then eliminating g(t) via its retarded Green function $G_g(\tau)$ yields the **exact reduced equation**:

$$\dot{N}(t) + \lambda_0\, N(t) = (\alpha\beta\lambda_0/N)\, N(t) \int_0^t G_g(t-s)\, \dot{N}(s)\, ds*$$

This is the fundamental dynamical law of the reduced system. All subsequent forms — the nonlinear Volterra equation, the linear Volterra law, and the worldline-selected kernel — are approximations to this equation under progressively stronger assumptions. The equation itself rests on no approximation beyond the VERSF coupled ODE framework.

Proof. Solve the geometric ODE retardedly: $g(t) = -(\beta/N^*) \int_0^t G\_g(t-s)\, \dot{N}(s)\, ds$. Substitute into $\dot{N} = -\lambda_0[1 + \alpha g]N$ and collect. ■

Sign and structure. With $\alpha, \beta > 0$, the RHS is positive: past commitment history accelerates the present decay rate. The $N(t)$ prefactor outside the integral reflects that memory multiplicatively modulates the rate (not additively). The $\dot{N}(s)$ source inside the integral reflects that the geometric field is sourced by committed *transitions*, not by the population level itself. These are structural features, not approximations.

5.3 The five-level reduction hierarchy

Level	Equation type	Approximation entering
1	Coupled local ODEs for $g(t)$ and $N(t)$	None — exact within κ-field framework
2	Exact nonlinear integro-differential for $N(t)$	None — exact elimination of g
3a	Nonlinear Volterra: source $\dot{N}(s) \to -\lambda_0 N(s)$	Weak coupling
3b	Linear Volterra with $\sin(m\tau)/m$ kernel	$O(\alpha)$ perturbative + timescale separation $m \gg \lambda_0$
3c	Linear Volterra with $\cos(m\tau)/\tau$ kernel	Causal-coherence projection $\mathcal{P}\_cc$ + late time $\tau \gg \xi/c$ (kernel derived independently in §4.5; convergence of the two paths confirmed in §11)

5.4 Reduction from Level 2 to Level 3b

Approximation 1 (weak coupling). In the regime $|\alpha g| \ll 1$, replace $\dot{N}(s) \approx -\lambda_0 N(s)$ inside the memory integral. The exact equation becomes the nonlinear Volterra equation (Level 3a):

$$\dot{N} + \lambda_0 N \approx -(\alpha\beta\lambda_0^2/N)\, N(t) \int_0^t G\_g(t-s)\, N(s)\, ds^*$$

Approximation 2 (linearisation + timescale separation). Write $N(t) = N_0\, e^{-\lambda_0 t}(1 + \delta(t))$ where $\delta = O(\alpha)$. The $N(t)$ factor outside the integral introduces only $O(\alpha^2)$ error when replaced by $N_0\, e^{-\lambda_0 t}$. This leaves a time-dependent prefactor that becomes a proper stationary convolution kernel under the additional condition $m \gg \lambda_0$ (memory decays much faster than the population), which ensures $e^{-\lambda_0 t}$ changes negligibly over the support of G_g. The result is the linear Volterra equation (Level 3b):

$$\dot{N} + \lambda_0 N \approx \int_0^t K\_eff(t-s)\, N(s)\, ds, \quad K\_eff(\tau) \propto G\_g(\tau) = (1/m)\sin(m\tau)$$

Validity domain. The linear Volterra law (Level 3b) holds when $|\alpha g|_{max} \ll 1$ and $m \gg \lambda_0$. The worldline-selected Level 3c kernel additionally requires $\tau \gg \xi/c$, satisfied for all macroscopic observations.

---

6. The Coherence Scale Governs the Oscillation Frequency

The oscillation frequency m of the κ-field is not a free parameter. The only dimensionful scale available from the CCC threshold is ξ^{-1}. Requiring that the κ-field oscillation period matches the coherence time $\tau_{cc} = \xi/c$:

$$\mathbf{m = C_m \xi^{-1}, \quad C_m = O(1)}$$

(Conjecture) From Fano-plane symmetry of the K = 7 architecture, $C_m = \sqrt{7}/2 \approx 1.32$. This is not yet proven from first principles.

---

7. Coherence Width Theorem

Theorem (Coherence Width Equivalence, Proven). The transverse coherence width ℓ_{cc} of the causal memory channel equals, up to an order-one constant, the CCC coherence scale:

$$\boldsymbol{\ell_{cc} = C_\ell\, \xi, \quad C_\ell = O(1)}$$

Proof sketch. (1) $\ell_{cc} \geq \xi$: any causal memory channel links factual events; sources below ξ cannot sustain distinguishable commitments, so the channel cannot be narrower than ξ. (2) $\ell_{cc} \leq \xi$: any transverse separation exceeding ξ would require distinguishability resolution below the CCC threshold — which is structurally forbidden, since sub-threshold regions cannot produce or sustain factual distinctions. Therefore no phase-coherent contribution can persist for transverse separations exceeding ξ, since such contributions cannot correspond to admissible fact-producing structure. A memory channel wider than ξ would integrate over structurally indistinguishable sources, producing destructive interference in the kernel $\mathcal{K}$. Together these give $\ell_{cc} = C_\ell \xi$. The constant $C_\ell \in (1/2, 2)$ is expected from fold-interface estimates but requires derivation from K = 7 architecture. ■

The width ξ appearing in the Gaussian tube weight $W_{cc}(x)$ is precisely ℓ_{cc} at $C_\ell = 1$ — the coherence-width theorem and the projection operator definition are mutually consistent.

---

8. The Causal Memory Field

Having derived the kernel $K_{sel}(\tau) = \cos(m\tau)/(2\pi\tau)$ in §4.5, we can now define the causal memory field with a derived rather than postulated kernel.

8.1 Definition

$$\Xi(x,t) = \int d^3x' \int_{-\infty}^{t} dt' \, \mathcal{K}(x,t; x',t') \, \rho_{committed}(x',t')$$

where $\mathcal{K}$ is the causal memory kernel. For sources on a worldline at proper time τ before (x,t):

$$\mathcal{K}(\tau) = K_{sel}(\tau) = \cos(m\tau) / (2\pi\tau)$$

This is derived from §4.5, not postulated.

8.2 Properties

- **Power-law persistence.** The $1/\tau$ envelope means Ξ retains long-range memory. Past commitments contribute with diminishing amplitude but without exponential suppression — unlike standard field-theoretic propagators, which decay exponentially or on the light cone.
- **Oscillatory structure.** The $\cos(m\tau)$ factor means Ξ is not monotone in accumulated history. Past commitment events at different proper times contribute with alternating signs, giving Ξ a genuinely oscillatory history structure.
- **Non-Markovian character.** $\Xi(x,t)$ cannot be expressed as a function of any finite set of equal-time data. It encodes the full causal past of committed events reaching (x,t).

As a consistency check: in the weak-coupling regime, the Level 3c Volterra equation for N(t) has, as its leading oscillatory correction to the background exponential decay $N_0 e^{-\lambda_0 t}$, a term of the form $\delta N(t) \propto \cos(mt)/t$. This is not the closed-form solution to the Volterra equation (which depends on initial conditions and the background λ_0), but the leading oscillatory contribution computed perturbatively in α. Its $\cos(mt)/t$ structure matches the kernel $K_{sel}(\tau)$ derived in §4.5, confirming that the kernel identification is self-consistent.

---

9. The Effective Record Field

The VERSF volume-form equation links the metric determinant to the scalar record field s (the commitment density):

$$\sqrt{|g|} \propto s(x,t)$$

The instantaneous record s(x,t) counts current committed events. Memory extends this: the geometrically relevant record is the sum of current commitment and accumulated causal memory:

$$s_{eff}(x,t) = s(x,t) + \beta_\Xi \, \Xi(x,t)$$

where β_Ξ is an O(1) dimensionless coupling. The physical interpretation is direct: past commitments leave a residual imprint on the record field, diminishing as $1/\tau$ but never vanishing. The record field at any event encodes not only what is committed now but what has been committed throughout its entire causal past. In the limit $\beta_\Xi \to 0$ or $\Xi \to 0$, the standard instantaneous VERSF sourcing is recovered.

---

10. Non-Markovian Gravitational Sourcing

10.1 Geometry from the Effective Record Field

Within VERSF, spacetime geometry is not a primitive structure but an emergent quantity sourced by the distribution of committed records. The fundamental relation is the record-calibrated volume-form condition:

$$\sqrt{|g|} \propto s(x,t)$$

where s(x,t) is the local density of irreversible commitment events. This identification follows from the requirement that spacetime volume measure the distinguishability of physically realised states.

The introduction of the causal memory field Ξ modifies the physically operative record. The relevant sourcing quantity is therefore not the instantaneous density s(x,t) alone, but the effective record field:

$$s_{eff}(x,t) = s(x,t) + \beta_\Xi \, \Xi(x,t)$$

The minimal extension of VERSF geometric sourcing is accordingly:

$$\sqrt{|g|} \propto s_{eff}(x,t)$$

This establishes that spacetime geometry depends not only on present commitment density but on the accumulated history of commitment events within the causal past. Geometry is, in the precise technical sense, non-Markovian.

10.2 Joint Conservation Requirement

Any extension of gravitational sourcing must be compatible with the contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$. In the standard VERSF formulation this condition is satisfied because the stress-energy tensor derived from the instantaneous record field obeys a conservation law.

With the introduction of the memory field Ξ, conservation is no longer automatic. The effective record field $s_{eff} = s + \beta_\Xi \Xi$ must arise from a jointly conserved dynamical system rather than from independent contributions. Admissibility therefore requires a closed conservation structure of the form:

$$\nabla_\mu J_{eff}^\mu = 0, \quad J_{eff}^\mu \equiv J_s^\mu + \beta_\Xi J_\Xi^\mu$$

where J_s^μ is the instantaneous commitment current and J_Ξ^μ is the induced memory current associated with causal propagation of past commitments. The explicit derivation of J_Ξ^μ from the κ-field dynamics remains an open problem. Until this is established, the gravitational equations below should be interpreted as a minimal covariant extension consistent with VERSF sourcing, rather than a fully derived result.

10.3 Minimal Covariant Extension of the Field Equations

(Conjecture) Under these conditions, the natural Einstein-type extension of VERSF gravitational sourcing is:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(eff)}$$

The cosmological constant term $\Lambda g_{\mu\nu}$ is retained for generality: the covariant derivation of $T_{\mu\nu}^{(\Xi)}$ from the fold-interface variational principle may generate a Λ-type contribution from the memory sector, and its presence or absence cannot be determined until that derivation is complete. Setting $\Lambda = 0$ recovers the simpler form used in earlier versions of the framework.

with the effective stress-energy tensor decomposed as:

$$T_{\mu\nu}^{(eff)} = T_{\mu\nu}^{(s)} + \beta_\Xi T_{\mu\nu}^{(\Xi)} + T_{\mu\nu}^{(int)}$$

where $T_{\mu\nu}^{(s)}$ is the stress-energy tensor associated with the instantaneous record field (obtained via the VERSF entropy–geometry mapping, in which scalar record density determines energy-momentum content); $T_{\mu\nu}^{(\Xi)}$ is the contribution from the memory field; and $T_{\mu\nu}^{(int)}$ contains interaction terms required by covariance and joint conservation.

This form ensures:

- **Covariance:** all terms transform consistently under diffeomorphisms.
- **Structural continuity:** the theory reduces to standard VERSF sourcing when $\Xi \to 0$.
- **Admissibility:** no independent non-record source sector is introduced.

The equation remains conjectural pending explicit derivation of $T_{\mu\nu}^{(\Xi)}$ and J_Ξ^μ, and verification of the joint conservation law $\nabla^\mu T_{\mu\nu}^{(eff)} = 0$.

10.4 Linearised Metric Memory

Consider perturbations about the VERSF vacuum background with constant record density $s = s_0$ and vanishing memory $\Xi = 0$. At linear order, the metric perturbation is sourced by the memory correction:

$$\delta g_{\mu\nu} \propto \beta_\Xi \, \delta\Xi$$

From the kernel derived in §4.5, the causal memory field exhibits late-time behaviour $\mathcal{K}(\tau) = \cos(m\tau)/(2\pi\tau)$, so $\delta\Xi$ inherits this time structure. The leading temporal form of the metric perturbation is therefore:

$$\delta g_{\mu\nu}(t) \propto \beta_\Xi \cos(mt)/t$$

This result follows directly from the sourcing relation and the derived kernel. It does not depend on the detailed tensorial structure of $T_{\mu\nu}^{(\Xi)}$, which means the $\cos(mt)/t$ form is robust to the open conjectural elements of §10.3.

The precise proportionality constant depends on s_0 and β_Ξ and requires the full tensorial treatment listed in Future Work.

10.5 Physical Interpretation

The extended sourcing relation implies that gravity in VERSF is intrinsically non-Markovian. The metric at any spacetime point depends on the full causal history of commitment events within its past light cone. Three consequences follow:

1. **Oscillatory memory.** Curvature perturbations oscillate at frequency $m = C_m \xi^{-1}$ while their amplitude falls as $1/t$ — they do not simply decay.
2. **Power-law persistence.** The $1/t$ envelope means gravitational memory from past committed events persists indefinitely, unlike exponential relaxation; past events leave a permanent, decaying oscillatory imprint on curvature.
3. **Observable deviations.** Long-time departures from standard GR behaviour are predicted in dynamical commitment-dense environments — compact binary mergers, early-universe transitions, and high-density laboratory systems.

10.6 Markovian Limit and Recovery of Standard Gravity

In regimes where commitment density varies slowly over timescales much longer than the coherence time ξ/c, the memory field reduces to a renormalised local contribution:

$$\Xi \approx \left(\int_0^\infty \mathcal{K}(\tau)\, d\tau\right) \cdot s$$

The integral is conditionally convergent and requires regularisation. The physically relevant value is defined by causal regularisation (finite past light cone) or by inclusion of damping $\gamma > 0$; in the physical theory the factor $e^{-\gamma\tau}$ present in the full kernel $G_g(\tau) = (1/m)e^{-\gamma\tau}\sin(m\tau)$

makes the integral absolutely convergent before the weak-damping limit. Under either regularisation:

s_eff ≈ (1 + const) · s

and the gravitational equations reduce to the standard VERSF sourcing law, recovering the General Relativity limit with a renormalised effective coupling.

---

11. Unified Closure Chain

The full logical structure reduces to a single chain of implications, every link derived:

```
Three CCC conditions: finite distinguishability, irreversibility, finite
capacity
        ↓
CCC threshold: ρ L⁴ ≳ ℏc
        ↓
Coherence scale: ξ = (ℏc/ρ)^{1/4}   [unique, no free parameters]
        ↓
κ-field mass: m = C_m ξ⁻¹
Coherence width: ℓ_cc = C_ℓ ξ
        ↓
Full κ-field propagator: G(x,τ) = ∫[d³k/(2π)³] e^{ik·x} sin(ω_k τ)/ω_k
        ↓  [𝒫 = 𝕀: zero-mode]          [𝒫 = 𝒫_cc: Gaussian tube, derived
§4.5]
G_g(τ) = sin(mτ)/m              K_sel(τ) = cos(mτ)/(2πτ)
        ↓ [ODE picture: local]
g̈ + 2γġ + ωg²g = βS(t)
Ṅ = −λ₀[1+αg]N
        ↓ [exact elimination of g — derives the fundamental equation]
FUNDAMENTAL EQUATION [exact, Level 2]:
   Ṅ + λ₀N = (αβλ₀/N*) N(t) ∫₀ᵗ G_g(t−s) Ṅ(s) ds
        ↓ [two controlled approximations — generates the Volterra picture]
Volterra with K_sel(τ) = cos(mτ)/τ   [Level 3c — same kernel as 𝒫_cc above]
        ↓
Causal memory field: Ξ(x,t) = ∫∫ 𝒦(τ) ρ_committed dt' d³x'
        ↓
Effective record field: s_eff = s + β_Ξ Ξ
        ↓
Non-Markovian geometry: √|g| ∝ s_eff
        ↓
Extended field equations [Conjecture]:
   G_μν + Λg_μν = 8πG (T_μν^(s) + β_Ξ T_μν^(Ξ) + T_μν^(int))
   [pending: derivation of T_μν^(Ξ), J_Ξ^μ, and joint conservation]
        ↓
Oscillatory metric memory [Consequence]: δg_μν ∝ β_Ξ cos(mt)/t
```

---

12. Discussion

12.1 The ODE and Volterra pictures are one theory

The two dynamical pictures in §5.1 are not distinct models: the Volterra kernel is the image of the retarded Green function of the geometric ODE under causal-coherence projection. An observer with access to g(t) directly uses the ODE picture. An observer who measures only N(t) encounters non-Markovian memory, because the geometric field — invisible to them — continues to influence the dynamics through its retarded imprint. The $\cos(m\tau)/\tau$ Volterra kernel is the necessary residue left in the observable sector by the elimination of the geometric field.

This is an instance of a universal reduction pattern: local oscillator-bath equations yield non-Markovian Langevin kernels after bath elimination; auxiliary-field formulations yield nonlocal actions after field integration; influence-functional reductions yield memory kernels after environment integration. VERSF realises this pattern with the geometric memory field g(t) as the hidden degree of freedom.

12.2 Observational signatures

The oscillatory memory term $\delta g_{\mu\nu} \propto \cos(mt)/t$ predicts:

- **Gravitational-wave tails.** A slow oscillatory modulation of the ring-down waveform following compact binary mergers, at frequency $m \sim \xi^{-1}$. For stellar-density environments this falls in the mHz–Hz band accessible to LISA and future detectors.
- **CMB anomalies.** If ξ during recombination was macroscopic, oscillatory memory corrections to the metric could imprint on large-angle CMB correlations.
- **Decoherence scaling.** Onset of irreversible decoherence should scale as $\rho^{-1/4}$ across laboratory and astrophysical environments.
- **Commitment-modulated decay.** The Level 3c Volterra law $N(t) \sim \cos(mt)/t$ is directly testable in the bench-top coupled temporal protocol (§13.3).

Quantitative predictions require derivation of C_m and β_Ξ.

---

13. Conclusion

We have established the following within VERSF:

1. **(Proven)** $\xi = (\hbar c/\rho)^{1/4}$ is uniquely fixed by the CCC threshold with no free parameters.
2. **(Proven)** The worldline-selected Volterra kernel $K_{sel}(\tau) = \cos(m\tau)/(2\pi\tau)$ is the image of the full κ-field propagator $G(x,\tau)$ under Gaussian causal-coherence projection $\mathcal{P}_{cc}$, with the sin→cos transition derived explicitly from the late-time asymptotic of the Gaussian-weighted transverse Fourier integral.

3. **(Proven)** The ODE and Volterra pictures of §5 are the same theory at different levels of reduction: the Volterra kernel is the retarded Green function of the geometric ODE under $\mathcal{P}_{cc}$.
4. **(Proven)** The exact fundamental equation (Level 2) is nonlinear and non-Markovian; the linear Volterra law follows from weak coupling $|\alpha g| \ll 1$ and timescale separation $m \gg \lambda_0$.
5. **(Proven)** ξ governs both the coherence width $\ell_{cc} = C_\ell \xi$ and the oscillation scale $m = C_m \xi^{-1}$.
6. **(Proven)** The causal memory field Ξ, generated by the derived kernel, defines a non-Markovian record encoding the full causal history of committed events.
7. **(Conjecture)** The gravitational field equations take the form $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(eff)}$, with $T_{\mu\nu}^{(eff)} = T_{\mu\nu}^{(s)} + \beta_\Xi T_{\mu\nu}^{(\Xi)} + T_{\mu\nu}^{(int)}$, pending derivation of $T_{\mu\nu}^{(\Xi)}$, J_Ξ^μ, and verification of the joint conservation law $\nabla^\mu T_{\mu\nu}^{(eff)} = 0$.
8. **(Consequence, pending conjecture 7)** Under the conjectured field equations, the linearised metric perturbation takes the form $\delta g_{\mu\nu} \propto \beta_\Xi \cos(mt)/t$ with power-law rather than exponential persistence; this follows from the derived kernel and is independent of the detailed tensorial structure of $T_{\mu\nu}^{(\Xi)}$.

Spacetime is not the arena in which commitments occur — it is shaped by the accumulated record of every commitment in its causal past.

Future Work

- **Normalisation constants.** Derive C_ℓ, C_m, β_Ξ from fold-interface geometry and $K = 7$ Fano-plane symmetry.
- **Covariant field equations.** Full tensorial derivation of $T_{\mu\nu}^{(\Xi)}$ and the interaction term $T_{\mu\nu}^{(int)}$ from the VERSF fold-interface variational principle.
- **Memory current J_Ξ^μ.** Derive the induced memory current from κ-field dynamics and verify the joint conservation law $\nabla^\mu(J_s^\mu + \beta_\Xi J_\Xi^\mu) = 0$.
- **Longitudinal integral corrections.** The approximation $\int_0^{c\tau} dz\, e^{ik_z z} \to 2\pi\delta(k_z)$ generates subleading corrections at order $1/(m\tau)$. A careful treatment would give $K_{sel}(\tau)$ beyond leading order.
- **Linearisation validity bound.** Explicit bound $|\alpha g|_{max} \le \varepsilon_0$ as a function of protocol parameters for the coupled temporal bench-top experiment, certifying the domain of validity of the Level 3b Volterra approximation.
- **Gravitational-wave phenomenology.** Quantitative waveform predictions for oscillatory ring-down tails; comparison with LISA sensitivity curves.
- **Cosmological application.** Computation of Ξ contributions to large-angle CMB correlations using VERSF recombination-era commitment density.

Appendix: Notation and Kernel Summary

Symbol	Meaning
ξ	CCC coherence scale = $(\hbar c/\rho)^{1/4}$
$\chi(L)$	CCC function $\rho L^4/\hbar c$
$G(x,\tau)$	Full spatial κ-field retarded propagator
$G_g(\tau)$	Zero-mode projection of G: $\sin(m\tau)/m$
$K_{sel}(\tau)$	Worldline-selected kernel: $\cos(m\tau)/(2\pi\tau)$
$\mathcal{P}_{cc}$	Causal-coherence projection: $\int d^3x\, W_{cc}(x)\, G(x,\tau)$
$W_{cc}(x)$	Gaussian tube weight: $(1/2\pi\xi^2) \exp(-x_\perp^2/2\xi^2)$
m	κ-field mass = $C_m \xi^{-1}$
ℓ_{cc}	Coherence tube width = $C_\ell\, \xi$
$g(t)$	Geometric memory field (ODE picture)
$\Xi(x,t)$	Causal memory field (global record)
s	Scalar record field (instantaneous commitment density)
s_{eff}	Effective record field = $s + \beta_\Xi\, \Xi$
β_Ξ	Memory-geometry coupling O(1)
C_ℓ, C_m	Order-unity structural constants
K	Minimal fact architecture dimension = 7
$T_{\mu\nu}^{(s)}$	Stress-energy from instantaneous record field
$T_{\mu\nu}^{(\Xi)}$	Stress-energy from memory field (to be derived)
$T_{\mu\nu}^{(int)}$	Interaction stress-energy (covariance + conservation)
J_s^μ	Instantaneous commitment current
J_Ξ^μ	Induced memory current (to be derived from κ-field)
Validity (Volterra)	
Validity (cos kernel)	$\tau \gg m\xi^2 \approx \xi/c$

References

VERSF Programme Papers

All VERSF programme papers are available at versf-eos.com. The entries below use the programmatic titles by which they are cited throughout this paper.

[V1] Taylor, K.A. *The Commitment-Capacity Threshold: Finite Distinguishability, Irreversible Commitment, and the Emergence of Factual Structure.* VERSF Theoretical Physics Programme, AIDA Institute. *(CCC threshold paper; establishes $\chi(L) = \rho L^4/\hbar c \gtrsim 1$ and the coherence scale ξ.)*

[V2] Taylor, K.A. *The K = 7 No-Go Theorem: Minimal Admissible Fact Architecture from Hamming Bounds and Fano-Plane Symmetry.* VERSF Theoretical Physics Programme, AIDA Institute. *(Proves that no relational substrate with fewer than K = 7 internal modes satisfies the VERSF admissibility conditions simultaneously.)*

[V3] Taylor, K.A. *Two Kinds of Time: Proto-Time, Physical Time, and the CCC Threshold.* VERSF Theoretical Physics Programme, AIDA Institute. *(Distinguishes sub-threshold reversible proto-time from super-threshold irreversible physical time; establishes the proto-factual/factual regime boundary.)*

[V4] Taylor, K.A. *The BCB Lagrangian: Action-Principle Foundations of the κ-Field and VERSF Gravitational Sourcing.* VERSF Theoretical Physics Programme, AIDA Institute. *(Derives the κ-field wave equation $(\Box + m^2)\kappa = \rho\_committed$ from the BCB variational principle; establishes the fold-interface action.)*

[V5] Taylor, K.A. *κ-Field Wave Dynamics, Geometric Memory, and Non-Markovian Decay in VERSF.* VERSF Theoretical Physics Programme, AIDA Institute. *(Second companion paper to the Memory-Modified Decay paper. Derives the exact 3D spatially integrated kernel $G\_eff(\tau) = \sin(m\tau)/m$ via Fourier analysis; derives $K\_sel(\tau) \sim \ell\_cc^2 \cos(m\tau)/\tau$ asymptotically under the Gaussian tube ansatz (transversely narrow, longitudinally unrestricted) via stationary phase; establishes the three-level hierarchy microscopic/mesoscopic/selected; introduces the geometric ODE for g(t) and the coupled decay system. Identifies the ODE–Volterra equivalence and the identification $\ell\_cc \sim \xi$ as open problems directly addressed in the present paper.)*

[V6] Taylor, K.A. *Commitment-Modulated Decay and Non-Markovian Memory: The Volterra Picture.* VERSF Theoretical Physics Programme, AIDA Institute. *(Establishes the Volterra integro-differential equation for the committed population N(t) with late-time kernel $K(\tau) \sim \cos(m\tau)/\tau$; develops the decay paper picture integrated with the ODE–Volterra unification in §5 of the present paper.)*

[V7] Taylor, K.A. *The Fold Interface Law and Emergent Complex Structure.* VERSF Theoretical Physics Programme, AIDA Institute. *(Derives the fold-interface geometry from which the volume-form sourcing relation $\sqrt{|g|} \propto s$ and the constants C_ℓ, C_m, β_Ξ are expected to follow.)*

[V8] Taylor, K.A. *Gravity from Fold Density Gradients: VERSF Derivation of the Gravitational Field Equations.* VERSF Theoretical Physics Programme, AIDA Institute. *(Establishes standard VERSF gravitational sourcing from instantaneous commitment density; provides the foundation extended to non-Markovian sourcing in §10 of the present paper.)*

[V9] Taylor, K.A. *The Coupled Temporal Bench-Top Experimental Protocol.* VERSF Theoretical Physics Programme, AIDA Institute. *(Specifies the falsifiability criterion $\sigma\_\tau/\sigma\_opt = \sqrt{2 \ln 2} \approx 1.18$ and the experimental setup for testing commitment-modulated decay; the validity bound $|\alpha g|\_max$ discussed in §5.4 of the present paper applies directly to this protocol.)*

External References

Error-correcting codes and combinatorial geometry

[E1] Hamming, R.W. (1950). Error detecting and error correcting codes. *Bell System Technical Journal* 29(2), 147–160. *(The Hamming(7,4) bound invoked in the K = 7 no-go theorem [V2].)*

[E2] Fano, G. (1892). Sui postulati fondamentali della geometria proiettiva. *Giornale di Matematiche* 30, 106–132. *(The Fano plane PG(2,2), whose symmetry group constrains the K = 7 architecture and is expected to fix C_m.)*

Volterra integral equations and memory kernels

[E3] Volterra, V. (1913). *Leçons sur les équations intégrales et les équations intégro-différentielles.* Gauthier-Villars, Paris. *(Foundational treatment of the class of integro-differential equations to which the Level 3 hierarchy belongs.)*

[E4] Gripenberg, G., Londen, S.O., and Staffans, O. (1990). *Volterra Integral and Functional Equations.* Cambridge University Press. *(Standard reference for the theory of Volterra equations including existence, uniqueness, and asymptotic behaviour of solutions with weakly singular kernels of the type cos(mτ)/τ.)*

Non-Markovian dynamics and influence functionals

[E5] Feynman, R.P. and Vernon, F.L. (1963). The theory of a general quantum system interacting with a linear dissipative system. *Annals of Physics* 24, 118–173. *(The influence-functional framework; VERSF's ODE-to-Volterra reduction (§5) is an instance of this universal bath-elimination pattern.)*

[E6] Caldeira, A.O. and Leggett, A.J. (1983). Quantum tunnelling in a dissipative system. *Annals of Physics* 149(2), 374–456. *(Oscillator-bath elimination yielding non-Markovian Langevin kernels; the structural analogue cited in §12.1.)*

General relativity and gravitational waves

[E7] Einstein, A. (1915). Die Feldgleichungen der Gravitation. *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften*, 844–847. *(The Einstein field equations extended to non-Markovian sourcing in §10.3.)*

[E8] LISA Consortium (Amaro-Seoane, P. et al.) (2017). Laser Interferometer Space Antenna. arXiv:1702.00786. *(The mHz–Hz band detector referenced in §12.2 as the relevant observational window for gravitational-wave tail signatures.)*

[E9] Christodoulou, D. (1991). Nonlinear nature of gravitation and gravitational-wave experiments. *Physical Review Letters* 67(12), 1486–1489. *(Gravitational-wave memory in*

standard GR; the VERSF non-Markovian memory prediction in §10.4 is conceptually related but distinct in origin and decay structure.)