Derivation of the κ-Field Mass from Minimal Fact Architecture in the VERSF Framework

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For the General Reader

Physics has long assumed that the fundamental constants of nature — the masses of particles, the strengths of forces — must be measured before they can be used. They appear in our equations as inputs, not outputs. This paper challenges that assumption for one specific quantity: the mass of the κ-field, a propagating degree of freedom that arises within the Void Energy–Regulated Space Framework (VERSF).

VERSF begins from a single primitive idea: physical reality is built from *irreversible commitment events* — moments at which something genuinely, permanently happens. Before such an event, a system exists in a pre-factual state of possibility; afterwards, a definite record exists that cannot be undone. Time, space, and the laws of physics all emerge from the statistical structure of these events.

Two earlier results in the programme established that fact-producing events must occur within a minimum region of space (set by the *coherence scale* ξ), and that the internal structure of any stable fact requires exactly seven mutually consistent constraints — a number forced by the mathematics of distinguishability, encoded in a beautiful combinatorial object called the Fano plane.

What this paper shows is that once those two results are in hand, the mass of the κ-field — the field that carries the memory of past commitment events through spacetime — is no longer a free parameter. It is *calculated*, not measured. The derivation proceeds in three clean steps: the coherence scale fixes the units; the seven-constraint architecture determines an operator whose spectrum encodes the dynamics; and the smallest non-trivial eigenvalue of that operator, after projection to the four physical degrees of freedom of a minimal fact, gives the mass directly.

The result is $\mathbf{m = \sqrt{(4/3)} \cdot \xi^{-1}}$, a number of order one in natural units, with no fine-tuning and no external input. It enters observable predictions — oscillation frequencies, late-time decay corrections, gravitational memory signatures — in ways that are, in principle, measurable. The κ-field mass is not an assumption about nature. It is a consequence of the internal logic of fact formation.

Abstract

We derive the mass scale of the κ-field within the Void Energy–Regulated Space Framework (VERSF) as a structural consequence of the minimal fact architecture. Building on two previously established results — (i) the coherence scale ξ = (ℏc/ρ)^(1/4) from the Causal–Coherence Compatibility (CCC) condition, and (ii) the necessity of a K = 7 constraint structure for stable fact formation — we prove that the κ-field mass is uniquely constrained to the form

$$\mathbf{m = C_m \, \xi^{-1}}$$

where C_m is a dimensionless spectral invariant of the minimal closure operator. We construct this operator explicitly from the Fano-plane realisation of the K = 7 architecture, compute its spectrum, and obtain the primary value $C_m = \sqrt{(4/3)} \approx 1.155$, together with an alternative value arising from the raw (uniform-mode-inclusive) closure space. We propagate these results through the κ-field Green's function and memory kernel, deriving quantitative predictions for oscillation frequencies and late-time decay behaviour. The central result is that C_m is not a free parameter: it is fully determined by the internal combinatorial structure of fact formation, with direct and measurable consequences for experimental signatures of VERSF memory effects.

Contents

- For the General Reader
- Abstract
- 1. Introduction
- 2. Structural Constraints on the κ-Field
  - 2.1 Dimensional Closure
  - 2.2 Causal Closure
- 3. The Closure Operator and Spectral Determination of C_m
  - 3.1 The Fano-Plane Realisation of K = 7
  - 3.2 The Raw Closure Operator
  - 3.3 Projection to Physical State Space
- 4. Mass Spectrum
  - 4.1 Alternative Representations
- 5. Memory Kernel and Observable Consequences
  - 5.1 Kernel Structure
  - 5.2 Oscillation Frequency
  - 5.3 Late-Time Decay Correction
  - 5.4 Sensitivity of Observables to C_m
  - 5.5 Summary of Quantitative Predictions
- 6. Physical Implications of the κ-Field Mass Scale
  - 6.1 Emergent Memory Frequency
  - 6.2 Regime Structure and Breakdown of Markovian Dynamics
  - 6.3 Quantitative Modification of Decay Laws

- 6.4 Sensitivity of Observables to the Spectral Invariant
- 6.5 Unified Role Across Dynamical Sectors
- 6.6 Interpretation as a Spectral Constant of Fact Architecture
- 6.7 Connection to Prior VERSF Results
- 6.8 Summary of Physical Implications
- 7. Conclusion
- Appendix: Spectral Computation
- References

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1. Introduction

The VERSF framework holds that physical reality is constituted by irreversible commitment events — discrete, causally ordered transitions in which a pre-factual distinguishable state becomes a committed record. Space, time, geometry, and field dynamics are all emergent from the statistical and topological structure of these events.

Two foundational results anchor the present analysis.

The coherence scale. The Causal–Coherence Compatibility condition establishes that fact production requires a minimum region of spacetime volume. The characteristic length scale emerging from this condition is

$\xi = (\hbar c/\rho)^{(1/4)}$

where ρ is the vacuum energy density (the VERSF void energy density), ℏ is the reduced Planck constant, and c is the speed of light. The scale ξ defines the minimal fact-producing region: no irreversible commitment event can be localised within a region smaller than ξ without violating CCC.

The minimal fact architecture. Separately, admissibility and stability constraints on distinguishable, persistent records force the minimal closure structure to carry exactly K = 7 constraints. This result — proven via a no-go theorem using the Hamming(7,4) code and the Fano plane — establishes that no stable fact can be formed from fewer than seven mutually consistent constraint relations.

While these results fix the existence scale and internal architecture of fact formation, the dynamical sector has retained an undetermined parameter: the mass m of the κ-field. The κ-field is the VERSF propagating degree of freedom sourced by committed events; its mass sets the characteristic frequency and decay timescale of memory effects. Prior work left m as a structural constant to be fixed by additional analysis.

This paper closes that gap. We demonstrate that:

1. The scaling $m \sim \xi^{-1}$ is forced by dimensional closure and the restriction to CCC variables alone.
2. The proportionality coefficient C_m is not a free parameter but is determined by the spectrum of the minimal closure operator constructed from the K = 7 architecture.
3. The resulting mass value propagates unambiguously into observable predictions via the κ-field memory kernel.

The κ-field mass is therefore a *derived* quantity, encoding in a single number the internal combinatorial structure of physical facts.

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2. Structural Constraints on the κ-Field

The κ-field satisfies the sourced Klein–Gordon equation

$$(\partial_t^2 - c^2\nabla^2 + m^2)\, \kappa = \mathcal{S}(x, t)$$

where $\mathcal{S}(x, t)$ is the commitment source density. The field κ carries the causal memory of committed events through spacetime.

2.1 Dimensional Closure

The mass parameter m must be constructible from the fundamental variables of the CCC condition alone. These are:

Quantity	Symbol	Dimension
Void energy density	ρ	$[E\, L^{-3}]$
Reduced Planck constant	$\hbar$	$[E\, T]$
Speed of light	c	$[L\, T^{-1}]$

The unique combination with dimension $[L^{-1}]$ (natural units, $\hbar = c = 1$: [mass]) is

$$(\rho/\hbar c)^{(1/4)} = \xi^{-1}$$

No other independent mass scale exists within the CCC sector. Any additional parameter would require an external input not derivable from the fact-formation conditions, violating the closure of the architecture. It follows immediately and without approximation that

$$\mathbf{m = C_m\, \xi^{-1}}$$

for some dimensionless C_m. The content of this paper is the determination of C_m from first principles.

2.2 Causal Closure

A second structural constraint fixes the propagation speed of the κ-field. Requiring that committed records define a *single* consistent causal order — no record may lie in the causal shadow of itself — forces the κ-field to propagate at

$$v_\kappa = c$$

This eliminates the possibility of an independent propagation scale. The dispersion relation is then exactly that of a relativistic massive field, with no Lorentz-violating corrections at leading order.

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3. The Closure Operator and Spectral Determination of C_m

3.1 The Fano-Plane Realisation of K = 7

The K = 7 minimal architecture admits a canonical realisation in terms of the Fano plane PG(2, 2): the unique projective plane of order 2, with 7 points and 7 lines, each line containing exactly 3 points and each point lying on exactly 3 lines.

This structure is encoded in an incidence matrix $B \in \mathbb{R}^{7\times7}$, where $B_{ij} = 1$ if point j lies on line i, and 0 otherwise. The key algebraic property of B is

$$BB^T = 2I_7 + J_7$$

where I_7 is the 7×7 identity matrix and J_7 is the 7×7 all-ones matrix. This relation expresses the regularity of the Fano plane: any two distinct lines share exactly one point.

3.2 The Raw Closure Operator

Define the raw closure operator

$$L_0 = BB^T$$

Its spectrum is obtained by observing that J_7 has eigenvalues 7 (once) and 0 (six times), giving

$$\text{spec}(L_0) = \{9, 2, 2, 2, 2, 2, 2\}$$

The dominant eigenvalue λ = 9 corresponds to the uniform mode — the direction in which all constraints are simultaneously satisfied. This mode encodes global consistency rather than local distinguishability, and must be projected out.

Define the reduced operator

$L_1 = L_0 - (9/7)\, J_7$

on the orthogonal complement of the uniform mode. Here $(9/7)\, J_7 = 9 \cdot (1/7)J_7$ removes precisely the contribution of the uniform eigenprojection — the rank-1 projector onto the all-ones vector, scaled by its eigenvalue 9. The non-zero spectrum of L_1 on the resulting 6-dimensional complement V_6 is

$\lambda_{min}^{+}(L_1) = 2$

This is the lowest eigenvalue governing non-trivial constraint structure — the spectral gap that controls how the minimal architecture resists perturbations to its closure.

κ-mode selection. The κ-field corresponds to the lowest-energy non-uniform excitation of the closure operator. The uniform mode ($\lambda = 9$) encodes global consistency across all constraints simultaneously and is not physically propagating — it carries no local distinguishability content. Higher non-uniform modes are suppressed relative to the spectral gap. The mass term is therefore set by the minimal non-zero eigenvalue $\lambda_{min}^{+}(L_1) = 2$, and by no other mode. This removes the last implicit step in the spectral identification.

3.3 Projection to Physical State Space

The K = 7 closure architecture over-parametrises the physical state space, and the uniform mode has been excluded from the dynamical sector by construction. The relevant projection domain is therefore the 6-dimensional non-uniform subspace $V_6 \subset V_c$, not the full $\mathbb{R}^7$. A minimal fact carries 4 intrinsic distinguishable degrees of freedom. The projection from V_6 to physical state space is constrained by admissibility conditions, which we now establish as a formal result.

Theorem (Unique Isotropic Rank-4 Projection on the Non-Uniform Subspace V_6). *Let $V_6 \cong \mathbb{R}^6$ denote the non-uniform subspace of the K = 7 closure-constraint space — the orthogonal complement of the uniform mode — and let $V_p \cong \mathbb{R}^4$ denote the physical state space of intrinsic fact-carrying degrees of freedom. Let*

$P : V_6 \to V_p$

be the linear map projecting non-uniform closure-space perturbations onto physical perturbations. Assume:

1. **Minimal embedding.** The physical image of the closure structure has dimension exactly 4.
2. **Constraint isotropy.** No direction in V_6 is physically preferred.
3. **Norm preservation up to scale.** Physically distinguishable amplitudes are preserved by projection up to a single global normalisation factor.
4. **No redundant structure.** The projection introduces no additional anisotropic or mode-dependent weighting beyond that already present in the K = 7 architecture.

Then

$\mathbf{P^\top P = (2/3)\, I_6}$

Equivalently, the admissible projection is a tight frame embedding of rank 4 with uniform weight 2/3 on the non-uniform subspace. This is the unique admissible projection metric under Assumptions 1–4.

Proof. Let $G := P^\top P$, a symmetric positive semidefinite operator on V_6. By Assumption 2, G must commute with the full symmetry action of the $K = 7$ closure architecture on V_6. The symmetry group PGL(3,2) of the Fano plane acts on the full constraint space $\mathbb{R}^7$ with representation decomposing as trivial $\oplus$ V_6. On the non-uniform subspace V_6 this action is irreducible — V_6 admits no further invariant subspace decomposition under admissible perturbations. By Schur's lemma applied to this irreducible action, G must be a scalar multiple of the identity on V_6:

$G = \alpha I_6$, for some $\alpha > 0$.

By Assumption 1, the image of P has dimension 4, so $\mathrm{tr}(PP^\top) = 4$. By trace cyclicity,

$\mathrm{tr}(PP^\top) = \mathrm{tr}(P^\top P) = \mathrm{tr}(G) = 6\alpha$.

Therefore $6\alpha = 4$, giving $\alpha = 2/3$, and hence

$P^\top P = (2/3)\, I_6$. ■

Scope note: this theorem fixes the projection metric uniquely within the admissibility class relevant to the minimal K = 7 architecture; it does not yet derive the projection from a full microscopic fold-dynamical model.

The physical closure operator acts on Vp and is defined as

$L_{\mathrm{eff}} = P L_1 P^\top : V_p \to V_p$

where $P : V_6 \to V_p$ and $P^\top : V_p \to V_6$. This is the correct physical operator: it maps physical perturbations through the non-uniform closure structure and back. Since L_1 acts as $2 \cdot I_6$ on V_6, we have

$L_{\mathrm{eff}} = P \cdot (2\, I_6) \cdot P^\top = 2\, PP^\top$

The eigenvalue of $PP^\top$ on Vp is determined as follows. Since $P^\top P = (2/3)I_6$, the singular values of P all equal $\sqrt{(2/3)}$; therefore $PP^\top$ has the same non-zero eigenvalue 2/3 on the entire 4-dimensional image of P, giving $PP^\top = (2/3)I_4$ on Vp. Therefore

$\lambda_e ff = 2 \cdot (2/3) = 4/3$

This spectral invariant is *derived*, not assumed.

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4. Mass Spectrum

The κ-field mass arises from the identification of the closure operator eigenvalue with the squared mass in units of ξ^{-2}. This identification is transparent from the Klein–Gordon dispersion relation: the field equation $(p^2 + m^2)\tilde{\kappa} = 0$ in momentum space shows that m^2 is precisely the energy cost per unit ξ^{-2} of departing from the constraint surface — the same role played by $\lambda_e ff$ in the closure operator. Therefore

$m^2 = \lambda_e ff / \xi^2$

Substituting $\lambda_e ff = 4/3$:

$m^2 = (4/3)\ \xi^{-2}$

and therefore

$m = \sqrt{(4/3)}\ \xi^{-1} \approx 1.155\ \xi^{-1}$

The primary result is thus

$C_m = \sqrt{(4/3)} \approx 1.155$

This is the **spectral invariant of the minimal fact architecture**.

4.1 Alternative Representations

Different operator realisations yield distinct but related C_m values. These are not inconsistencies; they represent the range of admissible representations of the same underlying architecture.

Representation	Domain	C_m	C_m (numerical)
Physical projection on V_6 (primary)	$\mathbb{R}^6 \to \mathbb{R}^4$	$\sqrt{(4/3)}$	≈ 1.155
Raw closure space (uniform mode retained)	$\mathbb{R}^7 \to \mathbb{R}^4$	$\sqrt{(8/7)}$	≈ 1.069

The raw closure value $\sqrt{(8/7)}$ arises if the full $\mathbb{R}^7$ is used as the projection domain — i.e. the uniform mode is naively retained. The trace argument then runs over 7 dimensions giving $\alpha = 4/7$, and $\lambda_e ff = (4/7) \cdot 2 = 8/7$, hence $C_m = \sqrt{(8/7)}$. This is physically unmotivated: the uniform mode is not propagating and its inclusion incorrectly dilutes the projection weight. The primary value $\sqrt{(4/3)}$, derived from the 6-dimensional non-uniform domain, is the correct result.

Both values satisfy $C_m = O(1)$, confirming that the κ-field mass is naturally of order the inverse coherence scale with no fine-tuning.

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5. Memory Kernel and Observable Consequences

The κ-field mass enters observational physics through the memory kernel — the retarded Green's function evaluated along a committed worldline.

5.1 Kernel Structure

The worldline-selected κ-field kernel is

$$K(\tau) = \cos(m\tau) / (2\pi\tau)$$

where τ is proper time along the worldline. This form arises from the retarded massive propagator in 3+1 dimensions, restricted to the forward light cone along a committed worldline [T4]. In position space, the retarded Green's function for the massive Klein–Gordon equation evaluates on the light cone to a Bessel function $J_0(m\tau)$, which in the large-argument (long-proper-time) limit reduces to the cosine form above; the $1/\tau$ prefactor reflects the spreading of the causal cone in 3+1 dimensions. The cosine modulation encodes the oscillatory memory of past commitment events; the algebraic $1/\tau$ decay distinguishes the κ-field tail from standard exponential relaxation.

5.2 Oscillation Frequency

The characteristic oscillation frequency of memory effects is

$$f = C_m c / (2\pi \xi)$$

With the primary value $C_m = \sqrt{4/3}$, this gives

$$f \approx (1.155 c) / (2\pi \xi)$$

This is the frequency at which κ-field memory oscillates — a direct, parameter-free prediction of the VERSF architecture.

5.3 Late-Time Decay Correction

For a decay process with rate λ (the standard exponential contribution), the κ-field memory term modifies the late-time population as

$$N(t) \sim e^{-\lambda t} + \varepsilon \cdot \cos(mt + \varphi) / [(\lambda^2 + m^2)t]$$

The second term is the VERSF memory correction. Its amplitude is controlled by the combination ($\lambda^2 + m^2$), with $m = C_m \xi^{-1}$ — hence by C_m directly. The phase φ encodes the committed history of the system.

5.4 Sensitivity of Observables to C_m

Across the two admissible representations (Table in §4.1):

- Oscillation frequency varies by **~8%** (ratio $\sqrt{(4/3)}$ to $\sqrt{(8/7)} = \sqrt{(7/6)} \approx 1.080$)
- Late-time amplitude suppression varies by **~15%** (through the $(\lambda^2 + m^2)^{-1}$ factor in the mass-dominated regime, scaling as C_m^{-2}: ratio $(8/7)/(4/3) = 6/7 \approx 0.857$)

This sensitivity analysis yields two important conclusions:

1. C_m primarily controls *detectability* rather than *existence*: the memory effect is present for all admissible values.
2. A measurement of either f or the late-time amplitude directly determines C_m, and hence discriminates between representations.

5.5 Summary of Quantitative Predictions

Using the primary value $C_m = \sqrt{(4/3)}$:

Observable	VERSF Prediction
Oscillation frequency	$f = \sqrt{(4/3)} \cdot c / (2\pi\, \xi)$
Memory tail amplitude	$\propto (\lambda^2 + 4/(3\xi^2))^{-1}$
Late-time phase	$\varphi = \varphi_0 + m \cdot \Delta\tau$ per commitment

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6. Physical Implications of the κ-Field Mass Scale

The determination of the κ-field mass, $m = C_m \xi^{-1}$ with $C_m = \sqrt{(4/3)}$, has immediate and far-reaching physical consequences. Unlike conventional field theories in which mass parameters are empirically fitted, the κ-field mass is a derived quantity, fixed entirely by the structure of fact formation. This section outlines the principal implications.

6.1 Emergent Memory Frequency

The κ-field mass defines a universal angular frequency

$$\omega = cm = C_m\, c / \xi$$

This frequency governs the oscillatory component of the κ-field memory kernel,

$K(\tau) = \cos(m\tau) / (2\pi\tau)$

which encodes the influence of past commitment events on present dynamics. The memory of the system is not merely persistent but dynamically structured: the past feeds forward into the present with a characteristic oscillation frequency fixed by the coherence scale. The existence of this frequency is not assumed but derived from the internal combinatorial structure of fact formation.

6.2 Regime Structure and Breakdown of Markovian Dynamics

The κ-field mass introduces a natural comparison scale between intrinsic system rates and memory effects. For a process characterised by a decay rate λ, the relative importance of κ-mediated memory is determined by the ratio

$m/\lambda = C_m / (\lambda\xi)$

Three regimes follow:

Markovian regime ($\lambda \gg m$). Memory effects are rapidly oscillatory and effectively average out, yielding standard exponential behaviour.

Transition regime ($\lambda \sim m$). Memory effects become dynamically relevant, producing measurable deviations from Markovian evolution.

Memory-dominated regime ($\lambda \ll m$). The algebraic tail dominates late-time behaviour, producing non-exponential decay.

This identifies a new structural boundary in physical dynamics: the onset of non-Markovian behaviour is not a phenomenological choice but is fixed by the coherence scale ξ and the spectral invariant C_m.

6.3 Quantitative Modification of Decay Laws

The late-time evolution of a system subject to κ-field memory takes the form

$N(t) \sim e^{(-\lambda t)} + \varepsilon \cdot \cos(mt + \varphi) / [(\lambda^2 + m^2)t]$

The κ-field mass enters both the oscillation frequency and the amplitude suppression factor. Substituting $m = C_m \xi^{-1}$:

correction $\propto \cos(C_m t/\xi + \varphi) / [(\lambda^2 + C_m^2/\xi^2)\, t]$

Thus the derived constant C_m directly controls the frequency of oscillatory deviations, the phase evolution, and the amplitude of the long-time tail. These effects persist parametrically at late times, in contrast to standard exponential decay.

6.4 Sensitivity of Observables to the Spectral Invariant

Because the κ-field mass enters observables only through the dimensionless combination C_m, experimental measurements of oscillation frequency or tail amplitude provide a direct probe of the internal structure of fact formation. Across the admissible range of representations, variations in C_m produce measurable, representation-discriminating shifts in oscillation frequency (~8%) and late-time amplitude suppression (~15%).

This implies that experimental detection of κ-field memory effects would not only validate the existence of the field but would directly constrain the underlying closure architecture. Detection of the oscillatory memory component would not merely confirm VERSF; it would *measure* the spectral structure of the minimal fact architecture.

6.5 Unified Role Across Dynamical Sectors

The κ-field mass is not an isolated parameter. It governs the oscillatory structure of the memory kernel, the temporal evolution of commitment-driven processes, and the frequency scale of gravitational memory effects of the form

$\delta g\mu\nu \sim \cos(mt) / t$

The appearance of the same mass scale across distinct physical sectors — quantum decay, thermodynamic commitment processes, and geometric memory — demonstrates that the κ-field provides a unifying dynamical bridge. A single derived number, $C_m = \sqrt{(4/3)} \approx 1.155$, spans all three.

6.6 Interpretation as a Spectral Constant of Fact Architecture

Unlike conventional constants of nature, C_m is not associated with interaction strength or symmetry breaking. It is a spectral invariant of the minimal closure operator:

$C_m = \sqrt{(\lambda_{min}^{+}(L_e ff))}$

This places C_m in a different conceptual class from coupling constants or mass parameters fitted to data. It is analogous to an eigenfrequency of a minimal structural unit — determined entirely by the internal geometry of fact formation, and not by any external physical input.

6.7 Connection to Prior VERSF Results

The result $m = C_m \xi^{-1}$ connects directly to several branches of the programme:

- The **Fact Momentum** paper (κ-field stress-energy tensor) requires a definite mass to compute the momentum carried by committed worldlines; this paper provides it.
- The **Coupled Temporal** paper's falsifiability criterion $\sigma\_\tau/\sigma\_opt = \sqrt{(2 \ln 2)} \approx 1.18$ is dimensionally consistent with a $C_m = O(1)$ mass scale.

- The **BCB Lagrangian unification** requires a fixed mass to complete the κ-field kinetic sector; the present derivation closes that gap.
- The **non-arbitrariness** of C_m — determined by three nested results (CCC scaling → K = 7 architecture → spectral determination) with no free choices at any step — means the mass is a theorem of the architecture, not a parameter of it.

The κ-field mass derivation is therefore not an isolated result but a keystone connecting the structural and dynamical branches of the programme.

6.8 Summary of Physical Implications

The determination of the κ-field mass yields the following key consequences:

Consequence	Expression
Universal memory frequency	$\omega = C_m\, c / \xi$
Markovian boundary	$\lambda \sim C_m / \xi$
Decay law correction	$\varepsilon \cdot \cos(C_m\, t/\xi + \varphi) / [(\lambda^2 + C_m^2/\xi^2)\, t]$
Gravitational memory scale	$\delta g_{\mu\nu} \sim \cos(C_m\, ct/\xi) / t$
Spectral interpretation	$C_m = \sqrt{(\lambda_{min}^+(L_eff))} = \sqrt{(4/3)}$

The κ-field mass converts the VERSF framework from a structural description into a predictive dynamical system.

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7. Conclusion

We have established the following chain of results:

Theorem (κ-Field Mass). *Under the CCC condition and the K = 7 minimal fact architecture, the κ-field mass is given by*

$m = C_m\, \xi^{-1}$, with $C_m = \sqrt{(4/3)} \approx 1.155$

where $\xi = (\hbar c/\rho)^{(1/4)}$ is the coherence scale, and C_m is the square root of the minimum positive eigenvalue of the physical closure operator $L_{eff} = PL_1P^T$ on Vp, with P projecting from the 6-dimensional non-uniform subspace V_6.

This result is:

- **structurally forced** by dimensional closure over CCC variables,
- **spectrally determined** by the K = 7 Fano-plane architecture,
- **physically meaningful** through its direct entry into the memory kernel and observable predictions.

The κ-field mass is not an independent parameter of the VERSF framework. It is a derived quantity encoding, in a single real number, the internal combinatorial structure of irreversible fact formation.

Appendix: Spectral Computation

Fano incidence matrix B (rows = lines, columns = points):

```
B = [ 1 1 0 1 0 0 0 ]
    [ 0 1 1 0 1 0 0 ]
    [ 0 0 1 1 0 1 0 ]
    [ 0 0 0 1 1 0 1 ]
    [ 1 0 0 0 1 1 0 ]
    [ 0 1 0 0 0 1 1 ]
    [ 1 0 1 0 0 0 1 ]
```

Verification of BB^⊤: The diagonal entry $(BB^\top)_{ii}$ = |{points on line i}| = 3 for all i, contributing $3I_7$. The off-diagonal entry $(BB^\top)_{ij}$, $i \neq j$, = |{points shared by lines i and j}| = 1 for all distinct i, j, contributing $J_7 - I_7$. Combining: $BB^\top = 3I_7 + (J_7 - I_7) = 2I_7 + J_7$. ✓

Eigenvalues of L_0 = BB^⊤: $L_0 = 2I_7 + J_7$. Eigenvalues of J_7 are 7 (×1) and 0 (×6). Therefore eigenvalues of L_0 are 2 + 7 = 9 (×1) and 2 + 0 = 2 (×6). ✓

Projected eigenvalue: Projection domain is V_6 (6-dimensional non-uniform subspace). Projection theorem gives $P^\top P = (2/3)I_6$ on V_6, equivalently $PP^\top = (2/3)I_4$ on Vp. Physical operator: $L_{e}ff = PL_1P^\top = 2PP^\top = (4/3)I_4$. ∴ $\lambda_{e}ff = 4/3$ and $C_m = \sqrt{(4/3)} \approx 1.155$. ✓

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