Facts as Constraints: Constraint Closure and the Fixing of Physical Structure

A Meta-Theoretical Criterion for Structural Uniqueness in Physics

Abstract for General Readers

Physics is, at root, a system of rules. The Pauli exclusion principle is one — it tells us that no two electrons can occupy the same quantum state, and nothing in physics lies beneath it to explain why. It is simply a rule that the world obeys. The same is true of the speed-of-light limit, the uncertainty principle, and conservation of energy. These rules are not merely decorations added to deeper machinery; in many cases they are the machinery. And rules do not appear from nowhere. They are distilled from facts — from what the world has been observed to do and, equally importantly, what it has been observed never to do.

This paper asks a simple question that follows from taking that seriously. If physics is ultimately made of rules, and rules are made from facts, then what must the world be like for there to be facts at all? Not the specific facts we happen to observe, but facts of any kind: definite, comparable, persistent things that can be recorded and tested. The minimum conditions for such facts to exist turn out to be demanding. And when they are made precise and followed through, they do something unexpected: they stop being a filter on possible physical structures and start acting as a forge. Under strong enough conditions — a property we call *constraint closure* — the rules needed for facts to exist leave only one structure standing.

If this is right, the form of physical law is not a lucky choice among many. It is what remains when everything that could not support facts has been ruled out. The paper makes this idea precise, proves the key result as a formal proposition, and identifies the conditions under which such structural fixing occurs. The VERSF programme is presented as a candidate instance where these conditions appear to be realised.

Abstract

Most physical theories are built by positing a mathematical structure and then constraining it to reproduce observed facts. We identify and formalise an alternative: a *constraint-first* construction in which the minimal conditions required for facthood are imposed first, and admissible structure is derived from them.

We distinguish two paradigms of theory construction. In the structure-first (descriptive) paradigm, structural underdetermination is generic — multiple inequivalent frameworks reproduce the same observable content. In the constraint-first paradigm, propagated constraints can restrict the admissible structure space. We introduce **constraint closure** as a formal property of a constraint system: closure obtains when every admissible extension of the structure is either observably redundant or inconsistent with the constraints. We then prove a **Structural Fixing Proposition**: when a constraint set propagates to closure, the admissible structure space collapses to a single equivalence class.

As a case study (developed in a companion paper), the VERSF framework arises as a constraint-closed structure under a minimal fact-based requirement — operational distinguishability — from which persistence, comparability, determinacy, and non-contradiction follow as derived conditions. The central suggestion is that the structure of physical law may not be freely chosen but forced by the conditions required for facts to exist.

We situate this claim against existing constraint-first programmes — quantum reconstruction (Hardy; Chiribella–D'Ariano–Perinotti; Masanes–Müller), the Ehlers–Pirani–Schild derivation of Lorentzian geometry, causal set theory, and structural-realist philosophy — and identify constraint closure as the novel meta-theoretical criterion distinguishing the present approach.

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1. Introduction

Why does physics take the particular mathematical form that it does?

Most approaches begin by choosing a framework — a spacetime manifold, a Hilbert space, a Lagrangian, a higher-dimensional construction — and then investigating what follows. Consistency requirements and empirical data are applied afterward. The question "why this structure?" is typically answered by *empirical adequacy* rather than by *structural necessity*.

This paper identifies a different route. Instead of asking *what structure explains the facts we observe?*, we ask *what must be true of the world for facts to exist at all?*

This shift is not merely rhetorical. It changes not only how theories are evaluated, but what counts as an admissible starting point for physical reasoning, and how much freedom remains once those starting points are fixed. It changes the role of facts within theory construction. Rather than serving as data to be fitted, facts become constraints that restrict the space of admissible theories. The purpose of this paper is to formalise this distinction, to identify the conditions under which constraint-first construction can fix physical structure uniquely, and to state the result as a proposition with explicit hypotheses.

The central contribution of this paper is not a specific physical model, but a criterion for when physical structure is uniquely fixed by constraints.

We introduce two paradigms:

- **Structure-first (descriptive):** structure is assumed; constraints are applied afterward.
- **Constraint-first:** minimal conditions for facthood are imposed first; structure is derived.

Section 2 develops this contrast and gives a precise operational definition of *fact*. Section 3 distinguishes two kinds of non-uniqueness in structure-first physics, defines constraint propagation and constraint closure as formal notions, and proves the Structural Fixing Proposition. Section 4 (companion paper) shows that the VERSF framework arises as a constraint-closed structure under distinguishability.

1.1 Relation to prior constraint-first programmes

Constraint-first theory construction is not itself new. The present paper's contribution lies in isolating a specific meta-property — *constraint closure* — that distinguishes constraint sets capable of fixing structure from those that merely restrict it.

Existing constraint-first programmes include:

- **Quantum reconstruction.** Hardy (2001) derived finite-dimensional quantum mechanics from five operational axioms; Chiribella, D'Ariano and Perinotti (2011) from informational principles; Masanes and Müller (2011) from measurement-theoretic axioms. These programmes recover Hilbert-space QM from constraints on operational structure.
- **Spacetime reconstruction.** Ehlers, Pirani and Schild (1972) derived Lorentzian geometry from projective and conformal structure induced by light-ray and free-fall congruences. Causal set theory (Sorkin and collaborators) derives spacetime from discrete causal order. Relational mechanics (Rovelli, Barbour) treats relational structure as primitive.
- **Philosophical antecedents.** Transcendental arguments (Kant, Strawson) ask what conditions must hold for experience; structural realism (Worrall, Ladyman, French) treats structure rather than substance as ontologically fundamental; operationalism (Bridgman) ties concepts to measurement procedures.

What is shared across these programmes is the direction of inference — from operational or experiential conditions to structural consequences. What is new in the present paper is the meta-level observation that a constraint set can propagate strongly enough that *no further structural freedom is available*. Hardy's axioms constrain QM; they do not force classical statistics, continuum spacetime, or a specific dynamical law. The Ehlers–Pirani–Schild derivation constrains spacetime geometry; it does not constrain matter content. In each case, constraint sets leave residual structural freedom elsewhere in the theory.

The Structural Fixing Proposition identifies the additional property — closure — needed for a constraint set to eliminate such residual freedom.

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2. Two Paradigms of Theory Construction

2.1 The structure-first paradigm

In most of modern physics the procedure is:

1. Posit a mathematical structure (spacetime manifold, Hilbert space, field content and Lagrangian, higher-dimensional construction).
2. Impose constraints (consistency, symmetry, renormalisability, empirical adequacy).
3. Compare predictions with observed facts.

Facts are external to the structure. The theory is evaluated by how well it reproduces them. We refer to this as the **structure-first paradigm**: *physics describes observed facts.*

2.2 The constraint-first paradigm

In the alternative, facts are not inputs but structural requirements. We begin by asking: *what conditions must be satisfied for a framework to produce stable, testable facts?*

Throughout this paper, we adopt the following operational definition:

Definition (Fact). A *fact* is a committed, irreversible record sufficient to ground a determinate operational distinction. Facts are the elementary bearers of observational content — the targets of comparison, the referents of measurement, and the units of empirical adequacy.

This definition is operational and theory-neutral: it specifies the minimal properties required for empirical content without presupposing any particular dynamical or geometric framework.

This definition is also VERSF-continuous: in the underlying ontology, facts are identified with *commitment-events* — irreversible transitions from indeterminacy to determinacy (Taylor, *Bit-Tick ontology*; *Fact-Momentum*).

A framework unable to support such records cannot produce testable outcomes and cannot function as a physical theory. We refer to this as the **constraint-first paradigm**: *facts constrain the admissible structure of physics.*

2.3 Minimal conditions for facthood

We adopt **operational distinguishability** as the primitive condition:

Primitive (Distinguishability). For any two candidate facts f_1, f_2, there exists an operational procedure that returns *same* or *different* and whose outcome is itself a fact.

The other conditions commonly invoked for facthood are not independent primitives but derived consequences.

Derivation (sketch).

- *Determinacy* follows because distinguishability requires each fact to have a definite value under the discriminating procedure; indeterminate facts cannot be compared.
- *Comparability* is built into the procedure: distinguishability *is* pairwise comparison.
- *Persistence* follows because the discriminating outcome is itself a fact; it must survive long enough to be itself distinguished, which forces a non-zero persistence interval.
- *Non-contradiction* follows because repeated application of a distinguishing procedure to the same pair must return the same result — otherwise the outcome fails to be a fact by the same criterion.

Full derivations are given in the companion distinguishability-primitive paper. The point here is that the constraint set *may be reduced to* a single primitive condition, not a disjoint list. The reduction is not assumed but argued for via the derivations sketched below and detailed in the companion work.

2.4 Logical direction of derivation

Paradigm	Logical direction
Structure-first	Structure → predictions → comparison with facts
Constraint-first	Conditions for facts → admissibility → structure

In the structure-first paradigm, constraints act as a filter on pre-existing structure. In the constraint-first paradigm, constraints act as a generator: they select the structure space itself.

2.5 The self-reference question

The constraint-first paradigm is itself articulated in some meta-language with some meta-structure. Must that meta-structure satisfy the constraints? The paradigm does not collapse into regress because meta-level consistency requires only that the act of articulating the constraints be itself a fact-generating process — which is assured by the operational nature of the distinguishability primitive. This is the same move made implicitly in the commitment-event ontology: structure is not presupposed at the meta-level but emerges with the first act of discrimination. We flag the point here; it is treated formally in the No-Go Theorem for Non-Simplicial Relational Substrates.

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3. Structural Degeneracy, Constraint Propagation, and Closure

3.1 Two kinds of non-uniqueness in structure-first physics

Structure-first formulations exhibit two distinct forms of non-uniqueness, which must be separated to avoid conflation.

(i) Formulation-equivalence. Hilbert-space, path-integral, and algebraic formulations of quantum mechanics are isomorphic as structures and identical in observable content. This is not degeneracy but change of coordinates, and it does not support the thesis of this paper.

(ii) Structural degeneracy proper. Genuinely inequivalent structures reproducing the same empirical content. Examples:

- String-theory landscape: many inequivalent vacua consistent with low-energy observables.
- Effective field theories: distinct UV completions matched to the same IR content.
- Modified gravity: families of Lagrangians reproducing the same cosmological phenomenology.
- Foundational interpretations with empirical equivalence (e.g., Bohmian mechanics vs. standard QM at the predictive level).

Only (ii) motivates the present paper. In (ii), the mapping from structure to facts is strictly many-to-one, and no amount of additional empirical data within the matched regime can discriminate the structures.

3.2 Constraint propagation

In the constraint-first paradigm, constraints on facthood do not act independently; they propagate. We present the propagation chain schematically; each step corresponds to a formally established result in the VERSF programme. We give each step in the form *premise ⇒ consequence*, with a one-line justification. Full proofs appear in the cited VERSF papers.

Step 1. Distinguishability + persistence ⇒ *ordering. Justification:* If facts persist and are pairwise distinguishable, the relation "f_1 is distinguished as prior to f_2" is asymmetric and transitive on the relevant subset, yielding a partial order (Bit-Tick ontology, §3).

Step 2. Ordering + irreversibility ⇒ *temporal structure. Justification:* Irreversibility selects a preferred direction on the partial order; the resulting directed order supports a scalar time-function up to monotonic reparametrisation (proto-time paper; discrete Noether / emergent time paper).

Step 3. Temporal structure + locality ⇒ *topological constraints. Justification:* Locality requires that distinguishing procedures access only bounded neighbourhoods of the record. Combined with the directed temporal structure, this forces a causal topology — neighbourhoods must be compatible with the ordering (No-Go Theorem for Non-Simplicial Relational Substrates, §§4–5).

Step 4. Causal topology + minimal-fact closure ⇒ *interface structure. Justification:* The minimal-fact count $K = 7$ (proved in the No-Go Theorem) forces the underlying 2-complex to be simplicial with triangular faces; interfaces between committed and uncommitted regions inherit this structure.

Step 5. Interface structure + distinguishability closure ⇒ *algebraic structure. Justification:* Consistent composition of interface transitions induces a groupoid, and under the projective irreducibility condition (PGL(3,2) on V_6), the algebraic structure is fixed up to isomorphism (κ-field mass derivation; fine-structure derivation).

Each arrow is a theorem in the cited literature. The chain is presented here in compressed form to make the propagation visible; this paper does not re-prove the individual steps.

3.3 Constraint closure

Let $\mathcal{C}$ be a category whose objects are candidate structures and whose morphisms are structure-preserving maps. Let $\mathbf{C} = \{C_1, ..., C_n\}$ be a constraint set, and let $\mathcal{C}\_\mathbf{C} \subseteq \mathcal{C}$ be the full subcategory of objects satisfying every $C \in \mathbf{C}$.

Definition (Observable equivalence). Objects $X, Y \in \mathcal{C}$ are *observably equivalent*, written $X \sim Y$, iff they induce the same fact-structure — i.e., the same set of committed records together with the same relations among them.

Definition (Constraint closure). The constraint set $\mathbf{C}$ is *closed* on $\mathcal{C}$ iff for every $X \in \mathcal{C}\_\mathbf{C}$ and every extension $X \hookrightarrow X'$ within $\mathcal{C}$, either: (i) $X' \in \mathcal{C}\_\mathbf{C}$ and $X' \sim X$ (the extension is observably redundant), or (ii) $X' \notin \mathcal{C}\_\mathbf{C}$ (the extension violates the constraints).

Under closure, every extension either adds no observable content or fails admissibility. Structural freedom within the admissible subcategory collapses to observable equivalence.

3.4 Structural Fixing

We assume the relevant classes of physical theories admit standard composability operations (including coproduct-like constructions) sufficient to formulate the argument below.

Proposition (Structural Fixing). *Let* $\mathbf{C}$ *be a constraint set closed on* $\mathcal{C}$, *and let* $\mathcal{C}\_\mathbf{C}$ *be non-empty and non-trivial. Then, under standard composability assumptions on* $\mathcal{C}$, $\mathcal{C}\_\mathbf{C}/\sim$ *contains exactly one isomorphism class.*

Argument (under composability assumptions). Suppose for contradiction that $X, Y \in \mathcal{C}\_\mathbf{C}$ with $X \nsim Y$. Consider the coproduct (disjoint union) $X \sqcup Y$ within $\mathcal{C}$. If $X \sqcup Y \in \mathcal{C}\_\mathbf{C}$, then by closure either $X \sqcup Y \sim X$ or $X \sqcup Y \sim Y$; in either case the other factor contributes no observable content, contradicting $X \nsim Y$. If $X \sqcup Y \notin \mathcal{C}\_\mathbf{C}$, then at least one of X, Y fails to admit coproduct-compatible extension within $\mathcal{C}\_\mathbf{C}$, contradicting admissibility. Either horn contradicts the premises; hence $X \sim Y$. ■

The required composability conditions — existence of coproducts and closure of $\mathcal{C}\_\mathbf{C}$ under the relevant extensions — hold for the classes of physical theories considered in §4.

Remarks.

1. The proposition does not assert *existence* of an admissible structure — $\mathcal{C}$_**C** may be empty — only *uniqueness up to observable equivalence* when it is non-empty.
2. "Non-trivial" excludes the degenerate case in which $\mathcal{C}$_**C** contains only the terminal object (no facts at all).
3. The proposition is the meta-theoretical core of the paper. Its content is: *if* a constraint set propagates to closure and admits a non-trivial solution, *then* the solution is unique.

3.5 Epistemic status of the claim

The claim of this paper is conditional, not absolute. We do not assert that constraint closure obtains for every set of plausible constraints on facthood, nor that the VERSF framework uniquely fixes all of physics in a single stroke. The argument is:

- **Proven:** the Structural Fixing Proposition, given its categorical hypotheses.
- **Conditional:** that the VERSF constraint set propagates to closure — this depends on the cited chain of VERSF papers holding up under scrutiny.
- **Conjectural:** that distinguishability is *the* unique minimal primitive capable of generating a closed constraint set in physically relevant categories.

The first is a theorem. The second is a programme. The third is the open problem.

The framework does not assume that closure must occur; it identifies the structural consequences if it does.

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4. Case Study: VERSF as a Constraint-Closed Structure

In the VERSF case, closure is realised through the joint action of *compatibility* (enforcing cross-sector consistency), *minimality* (eliminating redundant degrees of freedom), and *substrate rigidity* (forbidding non-trivial deformations). Together these eliminate both redundant and inconsistent extensions — which is precisely the content of constraint closure in the sense of §3.3.

[The companion paper develops the VERSF case in full. In outline: distinguishability is adopted as primitive; the propagation chain of §3.2 is traced through the specific VERSF constructions (minimal-fact count K = 7, simplicial triangular 2-complex, κ-field on V_6, commitment-event dynamics); constraint closure is verified step-by-step; the Structural Fixing Proposition then delivers uniqueness of the resulting structure up to observable equivalence. The empirical consequences — the derived values of α^{-1}, C = 3/8, $m_\kappa^2 = (3/4)\xi^{-2}$, and the tensor-to-scalar ratio $r \approx 0.008$ — are not inputs to the closure argument but outputs from the fixed structure.]*

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5. Conclusion

Physics has traditionally proceeded by proposing structures and testing them against observation. This paper identifies a complementary possibility: that sufficiently strong constraints, imposed prior to any structural assumption, may determine the structure uniquely.

When closure is reached, structure is no longer chosen — it is forced.

If the conditions required for facts propagate to closure, then the form of physical law is not contingent. It is the unique structure that remains once all non-fact-supporting possibilities are eliminated. The present paper identifies the criterion for this transition; whether and where it is realised is a question for detailed physical analysis, for which the VERSF programme provides a concrete candidate.

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