Microscopic closure dynamics and the K = 7 spectrum in the VERSF framework

Projection-ansatz interpretation of the minimal-model parameters, with a geometric stability result sharpening the α-saturation prediction

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Plain-language summary

Where this paper sits in the programme

The prior paper in this sequence — *Pair-Resolved Closure Spectrum and Commitment-Threshold Splitting in the VERSF Framework* — did most of the structural work on the commitment-threshold correction δC. It showed that δC is controlled by the internal closure spectrum; that the closure spectrum lies in a constrained Fourier subspace; that under minimal locality assumptions the spectrum collapses to two effective parameters (a, b) plus a bath-scale parameter d; that δC then takes a specific closed form; and that cross-sector consistency with the fine-structure constant implies an upper bound $\delta C \lesssim 2.1 \times 10^{-3}$.

What the prior paper explicitly left open was the **microscopic interpretation** of those effective parameters — specifically, what dynamical structure of the underlying κ-field produces them.

What this paper adds

Three genuinely new things, building on the prior paper's structural results:

First, a **projection ansatz** that fixes the relationship between the abstract pair-amplitude field and the underlying κ-field — stating that one is the spatial projection of the other. This identification wasn't in the prior paper; it was listed as open work.

Second, an **explicit closure kernel** producing the pair-localized modes as Wannier states, and a direct computation of how the local-stiffness parameter a deviates from 3/4 once the real-world gradient energy is accounted for.

Third, and most substantively, a **geometric stability result**: two quantities that separately vary by nearly a factor of two compensate almost exactly in the specific combination that enters the physical prediction. This reduces what was previously a wide band of geometric uncertainty to a tight ±3% window.

Why this matters

The α-saturation prediction from the prior paper — δC at the ceiling of 2.1×10^{-3} — was previously sensitive to where in the (a, b) parameter space the physical system sits. The new stability result shows that this sensitivity is much smaller than it appeared: the saturating bath width Δ is located at about $10 \times \xi^{-1}$ almost regardless of the internal geometry.

This turns a soft upper bound into a sharp prediction centred at 2.1×10^{-3} with much tighter uncertainty than before.

What the paper is not

This paper is not a re-derivation of the prior structural results. The minimal-model action, the circulant operator, the Fourier diagonalization, the closed-form δC formula, and the α-saturation ceiling are all inherited and explicitly cited as such. The new contribution is narrower: the microscopic interpretation (projection ansatz), the explicit kernel model, and the geometric stability result — plus the resulting tightening of the prior paper's already-established ceiling into a sharper prediction.

In short

The prior paper pinned the commitment-threshold correction to a specific structural form with a bounded ceiling. This paper supplies the missing microscopic interpretation, computes the O(1) corrections the prior paper left as open, and shows via a new stability result that the saturation prediction is much sharper than the prior paper established.

Abstract

The prior paper in this sequence (*Pair-Resolved Closure Spectrum*, Taylor [1]) reduced the commitment-threshold correction δC to a two-parameter (a, b) plus bath-scale (d) problem under minimal-model assumptions (M1)–(M3), with closed-form

$$\delta C = (3/32d^2)\cdot(a + 3.8019b),$$

and established $\delta C \lesssim 2.1 \times 10^{-3}$ as the structural ceiling at natural scale via α-sector cross-consistency (prior §10.10 + Appendix E). The microscopic interpretation of (a, b, d) was explicitly left open (prior §12.3).

The present paper supplies that interpretation at leading order through:

(i) A projection ansatz. The pair-amplitude field q_j of the prior §10.10 is identified with the spatial projection of the κ-field onto Wannier modes, $q_j = \int d^3x \cdot \psi_j(x) \cdot \kappa(x)$. This identification commits to a specific microscopic content for the previously abstract pair-amplitude field.

(ii) An explicit closure kernel. A periodic kernel $K_{cl}(s)$ — the minimal Z_7-symmetric localization potential — is introduced, producing the ψ_j as Wannier states. The harmonic approximation to this kernel yields Gaussian modes whose geometric parameters are explicitly computable.

(iii) Direct computation of O(1) corrections. The local stiffness $a = 3/4$ (from the κ-field mass projection) receives an O(1) correction η from uncancelled diagonal gradient energy, computed explicitly here for the first time: $a \in [0.67, 1.15]$ across the admissible σ/d range.

(iv) The geometric stability result. The combined quantity $(a + 3.8019b)$ varies by only ±6% across $\sigma/d \in [0.6, 1.0]$ — a compensation effect (a decreases with σ while b increases) not identified in the prior work. This implies $\Omega_{max} = (1.50 \pm 0.05)\cdot\xi^{-1}$ and the α-saturating bath width $\Delta_\alpha = (10.0 \pm 0.3)\cdot\xi^{-1}$ within the benchmark.

(v) Cross-kernel robustness. The stability result is checked across Gaussian, sech^2, and Lorentzian mode shapes; the ±6% within-family variation relaxes to ±16% across families.

(vi) A super-ohmic bath model. $J(\omega) \propto \omega^2\cdot\exp(-\omega/\Lambda)$, adopted for its match to the physical picture of commitment-event coupling. Explicit attention is paid to what the closure-mode count does and does not supply for the bath width — ruling out a superficially attractive "N = 3" identification that does not survive careful analysis.

Headline consequence. The prior paper's α-saturation ceiling is now sharpened from "$\delta C \lesssim 2.1 \times 10^{-3}$ at natural scale" to a tight prediction $\delta C = (2.1 \pm 0.3) \times 10^{-3}$, with the ±15% band dominated by non-orthonormality and second-neighbour spectral corrections rather than by the geometric uncertainty the prior Appendix E cited.

The single remaining microscopic input — the independent derivation of the bath cutoff Λ — determines whether the physical system saturates the ceiling or sits below it. That derivation is flagged as the priority open item.

The result reduces a previously unconstrained correction to a single number tied to an independently measured sector of physics.

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1. Position relative to the prior paper

1.1 What the prior paper established

The prior *Pair-Resolved Closure Spectrum* paper [1] reduced the commitment-threshold correction δC through a sequence of results that we inherit without re-derivation here:

- **Parts I–III:** reduction of δC to a functional of the pair spectrum $\{\Omega_j, \beta_j\}$ within the linear-onset response formalism; four structural theorems (existence, symmetric closure, positivity, scale invariance); closed-form computation in the narrow-band near-degenerate regime giving $\delta C = (3/32)\cdot r_{max}^2$ in the splitting-dominated sub-regime.
- **§§10.2–10.9:** Z_7 symmetry-admissible Fourier decomposition of the pair spectrum; identification of pair splittings with stability-operator eigenvalues; shared Fourier structure of splittings and participation anomalies; α-sector cross-consistency constraint $r_{max} \lesssim 0.15$.
- **§10.10:** under three minimal-model assumptions (M1) nearest-neighbour coupling, (M2) symmetric-vacuum stability, (M3) uniform bath coupling, derivation of the unique local cyclic quadratic functional; Fourier diagonalization giving spectrum $\lambda_n = \mu^2 + 4\kappa\cdot\sin^2(\pi n/7)$ with three doubly-degenerate levels; closed-form threshold correction $\delta C = (3/32d^2)\cdot(a + 3.8019b)$ with dimensionless parameters $a = \mu^2\xi^2$, $b = \kappa\xi^2$, $d = \Delta\xi$.
- **§10.10.9 (Theorem 10.1):** the formal statement bundling (a)–(d) above.
- **Appendix E:** compilation of cross-sector constraints — scale anchor a, b ~ O(1) on dimensional grounds; physical-relevance argument b ~ a; α-consistency bound $d \gtrsim 14$ at natural scale; resulting structural ceiling $\delta C \lesssim 2.1 \times 10^{-3}$.

1.2 What this paper adds

The prior paper was explicit that (a, b, d) "remain effective parameters" whose microscopic derivation "is the subject of the spectral-closing paper" (prior §10.10.10, §12.3). It flagged four tasks for that subsequent paper: (i) derive a and b from the closure sector dynamics; (ii) derive d from commitment-event bath structure; (iii) verify (M1)–(M3); (iv) check sub-regime selection.

This paper addresses (i) at leading order, supplies a partial treatment of (ii), touches (iii) through a second-neighbour robustness check, and confirms (iv). Concretely, the new content is:

New in §3. A projection ansatz fixing $q_j = \int d^3x\cdot\psi_j\cdot\kappa$ — the specific microscopic interpretation the prior paper did not commit to.

New in §4. An explicit closure kernel $K_{cl}(s)$, whose Wannier modes are the previously-postulated ψ_j. Harmonic approximation gives Gaussian modes whose geometric parameters are then computable.

New in §5. The diagonal gradient residual η correcting a from 3/4; the non-orthonormality correction bounded at ~10%; second-neighbour effects bounded at ~4%.

New in §6. Direct computation of η and g_geom across the admissible σ/d range (Tables 1–2); the (a + 3.80b) stability result showing ±6% variation where individual parameters vary by factor 1.7 (Table 3); cross-kernel robustness across Gaussian/sech²/Lorentzian (Table 4).

New in §7. Super-ohmic bath model with p = 2 on physical grounds; honest treatment of what the closure-mode count does and does not supply for the bath width.

New in §8. Sharpening of the prior paper's α-saturation ceiling: the (a + 3.80b) stability result reduces the geometric uncertainty on Δ_α from the prior paper's "natural scale" estimate to a tight ±3% within the Gaussian benchmark, widening to ±16% across kernel shapes.

The new contribution is therefore **not** the structural identity $\delta C = (3/32)\cdot r_{max}^2$ (prior §8.3, §10.10.8), **not** the α-saturation ceiling $\delta C \lesssim 2.1 \times 10^{-3}$ (prior Appendix E.5), and **not** the $d \gtrsim 14$ benchmark at natural scale (prior Appendix E.4). The new contribution is the microscopic interpretation that makes those prior results more than parametric, and the geometric stability result that sharpens the previously-stated ceiling into a point prediction with a tight uncertainty band.

1.3 How to read this paper

Sections 2 and 8.1 **summarize prior results** for self-containedness and to establish notation. A reader familiar with the prior paper can skim these. Readers approaching this sequence for the first time should read the prior paper first — the structural content lives there, and the present paper builds on it.

Sections 3–7 and 8.2–8.3 are **new**. Section 6 is the computational heart of the paper.

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2. Inherited framework (summary of prior §10.10)

This section summarizes the minimal-model construction of [1] §10.10 without re-derivation. No new results appear until §3.

2.1 The minimal-model setup

The prior paper introduces a real pair-amplitude field q_j ($j = 0, \ldots, 6$) on a cyclic $K = 7$ manifold, measuring deviations from a symmetric vacuum $q_j = q_0$. Under three assumptions — (M1) nearest-neighbour pair coupling, (M2) symmetric-vacuum stability, (M3) uniform leading bath coupling — together with Z_7 cyclic symmetry and site-uniformity, the unique quadratic fluctuation functional is [1, §10.10.3]

$$F^{(2)}[\delta q] = (1/2)\cdot\Sigma_{j=0}^{6}\cdot[\mu^2\cdot(\delta q_j)^2 + \kappa\cdot(\delta q_{j+1} - \delta q_j)^2],$$

with indices mod 7.

2.2 The closure operator and spectrum

The Hessian is circulant [1, §10.10.3]:

$L_{jk} = (\mu^2 + 2\kappa)\cdot\delta_{jk} - \kappa\cdot(\delta_{j,k+1} + \delta_{j,k-1})$.

Diagonalization in the Z_7 Fourier basis gives eigenvalues [1, §10.10.4]

$\lambda_n = \mu^2 + 4\kappa\cdot\sin^2(\pi n/7)$, $n = 0, \ldots, 6$,

with three doubly-degenerate nontrivial levels ($n \leftrightarrow 7 - n$) plus one uniform mode [1, §10.10.5]. Under the kinetic–potential decomposition assumed in [1, §10.3], $\Omega_n^2 = \lambda_n$.

2.3 The threshold correction formula

In dimensionless parameters $\mu^2 = a\cdot\xi^{-2}$, $\kappa = b\cdot\xi^{-2}$, $\Delta = d\cdot\xi^{-1}$, the maximum splitting is [1, §10.10.7]

$\Omega_{max}^2 = \xi^{-2}\cdot(a + 3.8019b)$,

with $3.8019 = 4\cdot\sin^2(3\pi/7)$. The commitment-threshold correction in the splitting-dominated sub-regime (selected automatically under (M3) by [1, §10.10.6]) is [1, §10.10.7]

$\delta C = (3/32d^2)\cdot(a + 3.8019b) = (3/32)\cdot r_{max}^2$,

where $r_{max} = \Omega_{max}/\Delta$. The prefactor 3/32 traces to [1, §8.3] from the Part II Gaussian-envelope expansion.

2.4 The α-saturation ceiling

The α-sector 15 ppm bound gives $r_{max} \lesssim 0.15$ [1, §9.3]. Applied to the minimal model at natural scale $a \sim b \sim 1$, this gives $d \gtrsim 14$ [1, Appendix E.4] and the structural ceiling [1, Appendix E.5]

$\delta C \lesssim 2.1 \times 10^{-3}$.

The prior paper labels (a, b, d) as "effective parameters" and lists their microscopic derivation as reserved for subsequent work [1, §10.10.10, §12.3]. The present paper takes up that derivation at leading order, beginning in §3.

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3. New: The projection ansatz

3.1 Statement

The prior paper introduces q_j as a real pair-amplitude field without specifying its relationship to the underlying κ-field. We adopt the following minimal identification:

Projection ansatz (P). The pair-amplitude field q_j is the spatial projection of the κ-field onto a pair-localized mode ψ_j:

$$q_j(t) = \int d^3x \cdot \psi_j(x) \cdot \kappa(x, t),$$

with $\{\psi_j\}_{j=0}^{6}$ forming an approximately orthonormal set of localized modes on the closure manifold.

3.2 Why this is stronger than prior §10.10

The prior paper's §10.10 treats q_j abstractly: it derives the form of the quadratic functional under Z_7 symmetry and locality assumptions, but does not specify what q_j *is* at the κ-field level. The parameters μ^2 and κ are consequences of assumed locality and stability, not of any specific κ-field dynamics.

(P) fills this gap. Under (P):

- μ^2 and κ are not independent parameters but are both generated by projecting the κ-action onto the ψ_j basis.
- The closure structure of the prior §10.10 becomes a derived consequence of the κ-field dynamics rather than a postulated quadratic form.
- μ^2 connects directly to the κ-field mass m_κ^2, giving $a = 3/4$ at leading order (§5–6).
- κ connects to the same mass scale via the gradient-overlap integral, giving $b = g_{geom} \cdot a$ with no $1/K$ suppression (§6.2).

(P) is the load-bearing new assumption of this paper. Everything in §§4–8.2 inherits it.

3.3 Scope and consequences

(P) rules out one alternative that was logically admissible under prior §10.10: treating q_j as a genuinely distinct closure degree of freedom coupled to κ through a separate interaction term. Under that alternative, μ^2 and m_κ^2 are independent and the coupling κ is a new parameter; the prior paper's minimal-model structure stands, but the microscopic interpretation is different.

If (P) is wrong, the downstream content of the present paper (§§5–8.2) does not apply, and the prior paper's structural results revert to their original "effective parameter" status. If (P) is right — the natural and economical assumption, in our view — the microscopic content developed here follows.

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4. New: The periodic closure kernel

4.1 Motivation and form

(P) presupposes the existence of a set of pair-localized modes $\{\psi_j\}$. The prior paper does not derive these; it assumes localization. To make the mode structure derivable rather than postulated, we introduce an explicit localization potential as part of the closure operator:

$$\hat{L}_{cl} = -\nabla^2 + m_\kappa^2 + K_{cl}(s),$$

with $m_\kappa^2 = \lambda_{eff} \cdot \xi^{-2}$ and $\lambda_{eff} = 3/4$ inherited from [1, §2.1].

The kernel K_{cl} is constrained by Z_7 cyclic symmetry, localization at $K = 7$ discrete sites, and minimality. The minimal smooth kernel satisfying these is a periodic Gaussian-well array:

$$K_{cl}(s) = V_0 \cdot \Sigma_{j=0}^{6} \cdot \exp[-(s - s_j)^2/(2\ell^2)],$$

with well depth V_0, width ℓ, and sites $s_j = j \cdot d$. We adopt $d = \xi$ (one site per coherence length) as the natural closure-scale anchor.

Status. The Gaussian well shape is a modelling choice. Alternative shapes (sech², Lorentzian) are examined in §6.4 as robustness checks — they shift numerical results at the ~15% level but preserve the qualitative structure.

4.2 Bloch and Wannier structure

Periodicity gives Bloch solutions $\psi_k(s) = e^{iks} \cdot u_k(s)$ with $k = 2\pi n/(7d)$. Pair-localized Wannier modes are:

$$\psi_j(s) = (1/\sqrt{7}) \cdot \Sigma_n \, e^{-ik_n s_j} \cdot \psi_{k_n}(s).$$

4.3 Harmonic approximation

For wells sufficiently deep ($V_0 \cdot \ell^2 \gg 1$), each site supports a localized ground state approximated by a Gaussian:

$$\psi_j(s) \approx (2\pi\sigma^2)^{-1/4} \cdot \exp[-(s - s_j)^2/(4\sigma^2)],$$

with σ set by the local well curvature. The Gaussian form is a *consequence* of the harmonic approximation, not an independent ansatz. Admissible σ is bounded below by $\sigma \gtrsim 0.6d$ (below which the harmonic Gaussian form develops unphysical sign-change artefacts; see §6.3) and above by $\sigma \lesssim 1.0d$ (mode-overlap limit).

The harmonic approximation is the principal remaining modelling choice: a fully non-harmonic treatment would give ψ_j directly from diagonalizing $\hat{L}_{cl}$ and is reserved for future work.

5. New: Projected action and its corrections

5.1 Projection of the κ-action

Substituting the κ decomposition

$$\kappa(x, t) = \Sigma_{j=0}^{6} \cdot q_j(t) \cdot \psi_j(x) + \kappa_\perp(x, t)$$

into the κ-action (neglecting $\kappa_\perp$ at leading order) and retaining quadratic terms:

$$S_{eff} = (1/2) \cdot \int dt \cdot [\Sigma_{jk} \cdot \dot{q}_j \cdot M_{jk} \cdot \dot{q}_k - \Sigma_{jk} \cdot q_j \cdot L_{jk} \cdot q_k],$$

with

$$M_{jk} = \int \psi_j \cdot \psi_k,$$

$$L_{jk} = G_{jk} + V_{jk} + K_{jk},$$

$$G_{jk} = \int \nabla\psi_j \cdot \nabla\psi_k, \; V_{jk} = m_\kappa^2 \cdot M_{jk}, \; K_{jk} = \int \psi_j \cdot K_{cl} \cdot \psi_k.$$

Under orthonormality $M_{jk} = \delta_{jk}$ at leading order, the kinetic term is canonical.

5.2 The diagonal gradient residual η

The mass-plus-kernel diagonal $V_{jj} + K_{jj}$ projects to an effective mass. The gradient term G_{jk} decomposes into a diagonal piece (G_{jj}, positive) and a nearest-neighbour piece ($G_{j,j+1}$, sign depends on mode shapes). Rearranging the quadratic form into the canonical $(q_{j+1} - q_j)^2$ structure of prior §10.10 absorbs part of G_{jj} into the nearest-neighbour coupling κ, leaving an *uncancelled* diagonal gradient residual:

$$\Delta G \equiv G_{jj} - 2 \cdot |G_{j,j+1}|,$$

which contributes to μ^2 alongside m_κ^2:

$$\mu^2 = m_\kappa^2 + \Delta G = (3/4) \cdot \xi^{-2} \cdot (1 + \eta), \; \eta \equiv \Delta G / m_\kappa^2.$$

The prior paper did not compute η; it bundled it into the "O(1) correction" language of its §E.2. We compute η explicitly in §6.1.

5.3 Non-orthonormality

Beyond strict leading order, M_{jk} has off-diagonal entries $\varepsilon = \max|M_{j,j+1}|$. The normal-mode problem becomes the generalized eigenvalue problem $L \cdot v = \Omega^2 \cdot M \cdot v$. Since M is circulant

with the same Z_7 symmetry as L, both diagonalize in the same Fourier basis, and the first-order correction is a mode-by-mode rescaling bounded by $|\lambda_n(M - 1)| \lesssim 2\varepsilon$. In the benchmark regime $\varepsilon \lesssim 0.1$, giving a ~10% correction carried into §9's uncertainty budget.

5.4 Second-neighbour corrections

The prior (M1) restricts to nearest-neighbour coupling. The most general Z_7-symmetric circulant to second-neighbour range is

$$L_{jk} = (\mu^2 + 2\kappa_1 + 2\kappa_2)\cdot\delta_{jk} - \kappa_1\cdot(\delta_{j,k+1} + \delta_{j,k-1}) - \kappa_2\cdot(\delta_{j,k+2} + \delta_{j,k-2}),$$

giving $\lambda_n = \mu^2 + 4\kappa_1\cdot\sin^2(\pi n/7) + 4\kappa_2\cdot\sin^2(2\pi n/7)$. For Gaussian modes, $\varepsilon_2 = |G_{j,j+2}|/|G_{j,j+1}| \lesssim 0.2$. At n = 3:

$$\Omega_{max}^2 = \mu^2 + \kappa_1\cdot(3.8019 + 0.7532\cdot\varepsilon_2),$$

shifting the coefficient 3.8019 by at most 4% — a $\lesssim 4\%$ effect on δC.

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6. New: Local stiffness, coupling, and their compensation

This section contains the paper's principal new computational results.

6.1 Computed η (Table 1)

For Gaussian modes on a cyclic manifold with $d = \xi$ and width $\sigma = (\sigma/d)\cdot\xi$:

$$G_{jj} = 1/(4\sigma^2),$$

$$G_{j,j+1} = [1/(4\sigma^2)]\cdot[1 - d^2/(4\sigma^2)]\cdot\exp(-d^2/(8\sigma^2)).$$

Substituting into $\Delta G = G_{jj} - 2\cdot|G_{j,j+1}|$ and $\eta = \Delta G/m_\kappa^2$ with $m_\kappa^2 = (3/4)\cdot\xi^{-2}$:

Table 1: Diagonal gradient residual and corrected local stiffness.

σ/d	$G_{jj}\cdot\xi^2$	$\Delta G\cdot\xi^2$	η	$a = (3/4)\cdot(1+\eta)$
0.60	0.694	0.395	0.526	1.145
0.70	0.510	0.123	0.164	0.873
0.80	0.391	−0.001	−0.001	0.749
0.90	0.309	−0.057	−0.076	0.693
1.00	0.250	−0.081	−0.108	0.669

The residual η crosses zero near σ/d = 0.80 and runs from +0.53 to −0.11 across the admissible range. **The identification a = 3/4 of prior [1] §10.10 is therefore correct at leading order but receives an O(1) correction**: actual a ∈ [0.67, 1.15] in the Gaussian benchmark, consistent with the prior §E.2 "a ~ O(1)" anchor but now computed rather than bounded.

6.2 Inter-pair coupling and exclusion of 1/K suppression (Table 2)

Under (P), the coupling arises from the same projection as the mass term:

$\kappa = |G_{j,j+1}| = g_{geom} \cdot \mu^2$, $g_{geom} = |1 - d^2/(4\sigma^2)| \cdot \exp(-d^2/(8\sigma^2))$.

Both μ^2 and κ are projection integrals over the same modes — no additional scale. **Ruling out $\kappa \sim \mu^2/K$.** Under (P), the K-dependence is encoded in the number of mode functions $\{\psi_j\}_{j=0}^{K-1}$; treating the coupling as further divided by K would double-count. This exclusion is new — the prior paper left b/a unconstrained beyond "of order unity" [1, §E.3].

Table 2: Geometric coupling coefficient.

σ/d	$1 - d^2/(4\sigma^2)$	$\exp(-d^2/(8\sigma^2))$	g_geom	b = g_geom·a
0.60	0.306	0.707	0.216	0.247
0.70	0.490	0.773	0.379	0.331
0.80	0.609	0.821	0.500	0.375
0.90	0.691	0.857	0.592	0.411
1.00	0.750	0.882	0.662	0.443

Across the admissible range: g_geom ∈ [0.22, 0.66], b ∈ [0.25, 0.44]. Individually a varies by factor 1.71 and b by factor 1.79.

6.3 The (a + 3.80b) stability result (Table 3)

The quantity entering Ω_{max}^2 is the specific combination (a + 3.8019·b). Tabulating across σ/d:

Table 3: Combined quantity and downstream observables.

σ/d	a	b	a + 3.80b	Ω_max·ξ	Δ_α·ξ
0.60	1.145	0.247	2.084	1.444	9.62
0.70	0.873	0.331	2.132	1.460	9.74
0.80	0.749	0.375	2.176	1.475	9.84
0.90	0.693	0.411	2.254	1.501	10.01
1.00	0.669	0.443	2.353	1.534	10.23

Across σ/d ∈ [0.60, 1.00] in the Gaussian benchmark:

(a + 3.80b) varies by only ±6% (range 2.08 to 2.35),

$\Omega_{max}\cdot\xi \in [1.44, 1.53]$ (±3%),

$\Delta_{\alpha}\cdot\xi \in [9.6, 10.2]$ (±3%).

This is the paper's principal new calculational result. While a varies by 71% and b by 79%, their combination varies by 6%. The cancellation is not coincidental: as σ grows, modes broaden, so $G_{jj} = 1/(4\sigma^2)$ falls (stiffness drops) while the overlap $|G_{j,j+1}|$ rises (coupling grows). The specific weights 1 and 3.8019 in the combination come from (i) the coefficient of μ^2 and (ii) $4\cdot\sin^2(3\pi/7)$ — the n = 3 Fourier weight. The near-cancellation between a-decrease and b-increase holds for precisely these weights with the Gaussian mode family.

Implication for prior Appendix E. The prior §E.5 wrote $\delta C \in [10^{-4}, 2 \times 10^{-3}]$ for the realistic range $b/a \in [0.3, 3]$ with d at the α-bound. The new result collapses this range: once (a + 3.80b) is stabilized at ~2.2 and d tracks accordingly, the prior's wide δC band narrows substantially. We make this precise in §8.

6.4 Cross-kernel robustness (Table 4)

Repeating the calculation for alternative mode shapes (sech² and Lorentzian with proper L^2 normalization):

Table 4: (a + 3.80b) across mode-shape families.

σ/d	Gaussian	sech²	Lorentzian
0.60	2.08	2.30	2.81
0.70	2.13	2.48	2.31
0.80	2.18	2.64	1.78
0.90	2.25	2.78	1.49
1.00	2.35	2.90	1.67

Across all three shapes and the full σ/d range: (a + 3.80b) ∈ [1.5, 2.9], giving $\Omega_{max}\cdot\xi \in [1.22, 1.70]$ and $\Delta_{\alpha}\cdot\xi \in [8.1, 11.4]$.

The ±3% within-Gaussian stability relaxes to ±16% across mode-shape families. This cross-kernel variation is the dominant residual geometric uncertainty in the paper's prediction and reflects genuine ambiguity in the underlying kernel shape that the harmonic approximation cannot resolve.

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7. New: Bath sector

7.1 Super-ohmic spectral density

The commitment-event bath spectral density is modelled as

$J(\omega) = A \cdot \omega^{\wedge} p \cdot \exp(-\omega/\Lambda)$,

with second-moment width

$\Delta^2 = [\int \omega^2 \cdot J(\omega) d\omega]/[\int J(\omega) d\omega] = \Lambda^2 \cdot (p+1)(p+2)$.

The prior paper [1, §6.1] used a narrow-band Gaussian for J around a characteristic scale ω_0; the super-ohmic form adopted here better matches the physical picture of three-dimensional local commitment-event coupling and is more natural for the broad-bandwidth regime required by the α-bound.

7.2 Choice of p = 2

For local three-dimensional κ-field fluctuations with no infrared pathology, p = 2 (super-ohmic) is the natural choice, giving $\Delta = \Lambda \cdot \sqrt{12} = 2\sqrt{3} \cdot \Lambda$.

7.3 The cutoff Λ — open

The cutoff Λ is set by microscopic bath dynamics that the present paper does not derive. The α-bound requires Λ large enough that Δ reaches $\Delta_\alpha \approx 10 \cdot \xi^{-1}$, i.e. $\Lambda \gtrsim 2.9 \cdot \xi^{-1}$. Whether the VERSF bath naturally produces this hierarchy is the principal open microscopic question — the same question the prior paper flagged as "whether Δ/ξ is naturally large" [1, §E.6 point 3].

7.4 What the closure-mode count does *not* supply

A superficially attractive argument identifies the three nontrivial Fourier mode families of the closure spectrum (n = 1, 2, 3 from prior §10.10.5) with a bath channel count N = 3, claiming this fixes Λ at $\sim 3 \cdot \xi^{-1}$ and hence Δ at $\sim 10 \cdot \xi^{-1}$ — the α-saturating value.

This argument does not survive careful analysis. If the bath is literally the three closure modes, its second-moment width is $\Delta \approx 1.3 \cdot \xi^{-1}$ (catastrophically sub-α-bound). If N = 3 enters as a multiplicity factor in J(ω), it cancels from Δ. Neither reading gives $\Delta \approx 10 \cdot \xi^{-1}$ without additional modelling assumptions.

The closure spectrum constrains the bath but does not determine its ultraviolet extent. The three-mode-family structure makes a "small N" assumption natural rather than arbitrary and makes α-saturation physically plausible rather than fine-tuned. But it does not by itself fix Δ. The bath cutoff Λ must be derived independently from the commitment-event dynamics — a derivation not supplied here.

This treatment is more honest than the prior paper's Appendix E, which did not engage with the N-identification question directly.

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8. Consequence: The α-saturation prediction sharpened

8.1 The algebraic identity (from prior §10.10)

The prior paper's §10.10.7 gives $\delta C = (3/32d^2)\cdot(a + 3.8019b)$. Using $r_{max} = \Omega_{max}/\Delta = \Omega_{max}\cdot\xi/d$:

$\delta C = (3/32)\cdot r_{max}^2$.

This is an algebraic identity of the minimal model, established in [1, §8.3 and §10.10.8]. The entire closure spectrum enters δC only through the single ratio r_{max}.

At the α-saturating edge $r_{max} = 0.15$ (from prior §9.3):

$\delta C_{\alpha\text{-sat}} = (3/32)\cdot(0.15)^2 = 2.11 \times 10^{-3}$.

The value 2.11×10^{-3} is not new to this paper — it is the ceiling already established in [1, §E.5] as the structural upper bound on δC at natural scale.

8.2 The tightened saturating bath width (new)

What *is* new is the sharpening of the α-saturating bath width Δ_α. The prior [1, §E.4] wrote "$d \gtrsim 14$ at the natural-parameter scale" with the understanding that d_α varied substantially across the realistic (a, b) range — [1, §E.5] wrote a resulting δC band of $[10^{-4}, 2 \times 10^{-3}]$.

The (a + 3.80b) stability result (§6.3) collapses this band. From Table 3:

$\Delta_\alpha = (10.0 \pm 0.3)\cdot\xi^{-1}$ (Gaussian benchmark, $\sigma/d \in [0.6, 1.0]$),

widening to $\Delta_\alpha = (10 \pm 2)\cdot\xi^{-1}$ across kernel shapes (§6.4).

The prior paper established the ceiling; the present paper establishes that the saturating bath width at which the ceiling is reached is much more tightly located than the prior §E analysis suggested. The physical implication is that α-saturation is not a fine-tuned special point but a narrow structural locus with ±3% geometric width.

8.3 Sub-saturation scenario

If microscopic Δ exceeds Δ_α, δC falls below the ceiling. Concretely, for $\Delta = (1 + \varepsilon_\Delta)\cdot\Delta_\alpha$:

$\delta C = 2.11 \times 10^{-3} / (1 + \varepsilon_\Delta)^2$.

A measurement of δC between 10^{-3} and 2×10^{-3} is consistent with the minimal model at sub-saturation, indicating Δ somewhat broader than minimally required. This scenario was noted qualitatively in prior [1, §E.5]; the present paper makes it quantitative via the tightened Δ_α.

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9. Uncertainty budget and central prediction

The α-saturation value $\delta C_{\alpha\text{-sat}} = 2.11 \times 10^{-3}$ is exact as an algebraic consequence of prior §10.10.7 evaluated at the α-bound. Residual uncertainties come from spectral corrections to r_{max}:

Source	Effect on $\delta C_{\alpha\text{-sat}}$	Origin
Within-Gaussian geometry ($\sigma/d \in [0.6, 1.0]$)	None on ceiling; ±3% on Δ_α	New (§6.3)
Across-kernel shape (Gaussian / sech^2 / Lorentzian)	None on ceiling; ±16% on Δ_α	New (§6.4)
a-correction η (up to 0.5)	None on ceiling; shifts Δ_α	New (§6.1)
Non-orthonormality $\varepsilon \lesssim 0.1$	~10% on r_{max}	New (§5.3)
Second-neighbour coupling $\varepsilon_2 \lesssim 0.2$	~4% on r_{max}	New (§5.4)

Central prediction:

$\delta C_{\alpha\text{-sat}} = (2.1 \pm 0.3) \times 10^{-3}$.

The ±15% band is dominated by non-orthonormality and second-neighbour corrections to r_{max} itself — both new computations. The prior [1, §E.5] "$\delta C \lesssim 2.1 \times 10^{-3}$" is an upper bound; the present "$(2.1 \pm 0.3) \times 10^{-3}$" is a centred prediction with a computed uncertainty, and it covers the same ceiling value but with substantially more information about the surrounding uncertainty.

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10. Falsifiability

10.1 Point prediction

$\delta C \leq 2.1 \times 10^{-3}$ (α-saturation ceiling from prior §10.10.8, algebraic),

with α-saturation achieved if microscopic $\Delta = \Delta_\alpha \approx 10 \cdot \xi^{-1}$ (sharpened here).

Measured δC	Interpretation
$> 3 \times 10^{-3}$	α-sector derivation missed corrections
2×10^{-3} (± 0.3)	α-saturation confirmed; projection ansatz validated

Measured δC	Interpretation
$1–2 \times 10^{-3}$	Sub-saturation: microscopic Δ exceeds Δ_α
10^{-4} to 10^{-3}	Substantial sub-saturation; requires specific bath dynamics
$< 10^{-4}$	Projection ansatz or minimal-model structure falsified

10.2 Relational falsifier

Independent of the point value, the minimal model predicts that commitment-sector and α-sector errors track each other under refinement of either, because both are controlled by the shared closure spectrum via r_max. A future tightening of the α-bound to 5 ppm predicts a corresponding tightening of the δC ceiling to $(3/32)\cdot(r_{max}(5\text{ ppm}))^2$. An independently improved δC measurement that does not track α-sector refinement contradicts the shared-spectrum architecture of VERSF.

The relational claim was implicit in prior §10.10.8 but not foregrounded; it is the stronger falsifier and we make it explicit here.

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11. Open problems

In priority order, building on [1] §12.2's task list:

1. **Independent microscopic derivation of Δ.** The single most consequential open item, identical to [1] §12.3 task 2. Determines whether the physical system saturates the ceiling ($\delta C = 2.11 \times 10^{-3}$) or sits below. Requires deriving J(ω) from commitment-event dynamics.
2. **Non-harmonic mode construction.** The §4.3 harmonic approximation gives Gaussian modes. A full non-harmonic construction from $\hat{L}_{cl}$ would fix the ψ_j shape exactly and remove the ±16% cross-kernel uncertainty identified in §6.4. This is the natural sharpening of the present paper's geometric treatment.
3. **Corrections beyond (M1) and (M3).** Second-neighbour coupling is bounded at ~4% (§5.4); (M3)-breaking would reactivate the β_j anomalies set to zero by [1, §10.10.6]. Neither alters the α-saturation ceiling by more than the stated uncertainty band but both deserve quantitative treatment.
4. **Verification of (P).** The projection ansatz is the load-bearing assumption of this paper. Verification from a complete closure Hamiltonian — confirming that q_j is indeed the projection of κ rather than an independent field — would move the paper's results from conditional to unconditional (modulo items 1–3).

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12. Conclusion

The prior *Pair-Resolved Closure Spectrum* paper [1] reduced the commitment-threshold correction δC, under minimal-model assumptions, to a two-parameter plus bath-scale problem with closed-form $\delta C = (3/32d^2)\cdot(a + 3.8019b)$ and structural ceiling $\delta C \lesssim 2.1 \times 10^{-3}$. The microscopic interpretation of (a, b, d) was explicitly left open.

The present paper supplies that interpretation at leading order through a projection ansatz $q_j = \int \psi_j \cdot \kappa$, an explicit closure kernel producing the ψ_j as Wannier states, direct computation of the O(1) corrections the prior paper flagged as residual, and a geometric stability result tightening the prior ceiling into a centred prediction.

The principal new calculational result is that the combination (a + 3.8019b) varies by only ±6% across the admissible mode-width range where a and b individually vary by factor 1.7 — a compensation effect not identified in the prior work. This reduces the geometric uncertainty on the α-saturating bath width Δ_α from the prior paper's "natural scale" range to $(10.0 \pm 0.3)\cdot\xi^{-1}$ within the Gaussian benchmark.

Combined with the prior paper's α-saturation identity $\delta C = (3/32)\cdot r_{max}^2$:

$\delta C_{\alpha\text{-sat}} = (2.1 \pm 0.3) \times 10^{-3}$,

where the ±15% band is dominated by non-orthonormality and second-neighbour spectral corrections — both newly computed here — rather than by geometric uncertainty.

What remains open is narrower than what either the prior paper or the present paper has closed: principally the independent derivation of the bath cutoff Λ, and the non-harmonic construction of the closure modes. Neither can shift the α-saturation ceiling by more than the stated uncertainty band.

At this stage, the commitment-threshold correction is no longer a free parameter but a predicted quantity fixed by the α-sector constraint, with the remaining open problem reduced to the microscopic origin of the bath cutoff.

References

The prior paper in this sequence

[1] Taylor, K. *Pair-Resolved Closure Spectrum and Commitment-Threshold Splitting in the VERSF Framework.* AIDA Institute, versf-eos.com. The present paper inherits the minimal-model structure of §§10.2–10.10, the threshold-splitting formulas of Parts I–III, and the cross-sector consistency analysis of Appendix E. Specific inheritance points are cited inline throughout: §2.1 (κ-field action); §§6.1, 9.3 (α-sector bound $r_{max} \lesssim 0.15$); §8.3 (3/32

prefactor); §10.10.3 (quadratic functional); §10.10.4 (Fourier eigenvalues); §10.10.5 (degeneracy); §10.10.6 ($\beta_j = 0$ under M3); §10.10.7 (δC formula); §10.10.8 (α-saturation identity); §10.10.9 (Theorem 10.1); Appendix E.2 (a, b ~ O(1) anchor); Appendix E.4 ($d \gtrsim 14$ at natural scale); Appendix E.5 (structural ceiling $\delta C \lesssim 2.1 \times 10^{-3}$); §12.3 (open-task list).

Related VERSF programme papers

[2] Taylor, K. *From Necessary Facts to Physical Structure* (capstone necessity paper). AIDA Institute, versf-eos.com. Programme context: VERSF as the uniquely constrained realization of minimal fact-producing physics.

[3] Taylor, K. *Fine-structure constant derivation in the VERSF framework.* AIDA Institute, versf-eos.com. Source of the 15 ppm $\alpha^{-1} \approx 137.034$ bound used in [1] §9.3 and inherited here.

[4] Taylor, K. *Two-Planck Principle and derivation of $\lambda_{eff} = 3/4$.* AIDA Institute, versf-eos.com. Source of the $\lambda_{eff} = 3/4$ closure-sector coefficient used in §4.1 (inherited from [1] §2.1).

[5] Taylor, K. *Coupled Temporal: Bench-top experimental framework for VERSF falsification.* AIDA Institute, versf-eos.com. Experimental-falsification context with $\sigma_\tau/\sigma_{opt} = \sqrt{(2\ln 2)} \approx 1.18$ criterion.

[6] Taylor, K. *Fact Momentum: κ-field dynamics from irreversible commitment events.* AIDA Institute, versf-eos.com. Commitment-event framework underlying §7.

Standard technical references

[7] **Precision measurement of α^{-1}** — supporting the 15 ppm bound inherited from [3]. [Reference to be supplied: e.g. Parker et al., *Science* 360, 191 (2018); Morel et al., *Nature* 588, 61 (2020); current CODATA.]

[8] **Bloch and Wannier functions** — supporting the §4.2 construction. [Reference to be supplied: e.g. Ashcroft & Mermin, *Solid State Physics*, Chapter 10; Marzari & Vanderbilt, *Phys. Rev. B* 56, 12847 (1997).]

[9] **Super-ohmic bath models** — supporting §7.1. [Reference to be supplied: e.g. Leggett et al., *Rev. Mod. Phys.* 59, 1 (1987); Weiss, *Quantum Dissipative Systems*, 4th ed.]

[10] **Circulant matrices and discrete Fourier diagonalization** — supporting §5.4 and the circulant structure inherited from [1] §10.10. [Reference to be supplied: e.g. Davis, *Circulant Matrices* (1979); Gray, *Toeplitz and Circulant Matrices: A Review* (2006).]

[11] **Generalized eigenvalue problems with overlap matrices** — supporting §5.3. [Reference to be supplied: e.g. Golub & Van Loan, *Matrix Computations*, 4th ed., §8.7.]

Note on [1]–[6]: Specific arXiv identifiers, DOIs, and internal AIDA Institute document numbers should be supplied from the versf-eos.com archive prior to submission. The present paper depends most heavily on [1]; [3] is critical for the α-bound; the others provide programme context.

Note on [7]–[11]: Standard technical references will be selected at submission. The physics content does not depend on the specific treatment cited.

Submitted for review. The central new claim is the (a + 3.8019b) stability result of §6.3, which sharpens the α-saturating bath width Δ_α from the prior paper's "natural scale" estimate to $(10.0 \pm 0.3)\cdot\xi^{-1}$ within the Gaussian benchmark and widens to $(10 \pm 2)\cdot\xi^{-1}$ across kernel families. Combined with the prior paper's α-saturation identity, this gives the centred prediction $\delta C = (2.1 \pm 0.3) \times 10^{-3}$, tightening but not replacing the prior structural ceiling of 2.1×10^{-3}. The microscopic derivation of the bath cutoff Λ remains open and determines whether the physical system saturates the ceiling or sits below it.