Primordial Scalar Perturbations from Void Unfolding: A Parameter-Free Derivation of the Spectral Index in the VERSF Framework

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For the General Reader

The universe we observe is almost perfectly smooth — but not quite. The density of matter and radiation in the early universe was uniform to about one part in 100,000. Those tiny ripples in the primordial soup are the seeds from which all galaxies, stars, and planets eventually grew. Cosmologists have measured the pattern of those ripples with extraordinary precision. It turns out they are close to, but not exactly, scale-invariant: there is slightly more power at large scales than small ones, captured by a single number called the spectral index, $n_s \approx 0.965$.

Standard cosmology explains this by imagining a very early period called inflation — the universe expanding exponentially fast, driven by an unknown field with an unknown potential. The spectral index comes out right, but only because the potential is chosen to make it so. No one knows what that field is, where it came from, or why the potential has the shape it does.

This paper takes a different route. Starting from a single foundational idea — that physical reality is built from irreversible events, moments at which a distinction between possible states becomes a permanent fact — it derives the spectral index without choosing any potential, without specifying any field content beyond what the framework requires, and without fitting any free parameter.

The key insight is that irreversibility creates memory. Every time an irreversible event occurs, it leaves a trace in a field called κ. That trace propagates forward in time. As the universe evolves, the accumulated record of past events gently perturbs the current state — slowly, logarithmically, like an echo that never quite dies. This logarithmic memory, combined with the uniquely determined mass of the κ-field and the ratio of the primordial energy density to the present one, gives:

$n_s - 1 = -8 / \ln(\rho_{void} / \rho_{CCC}) \approx -0.028$, $n_s \approx 0.972$

The result is within 1% of the observed value — a small gap explained by a calculable correction that requires no new assumptions. Nothing in this derivation was adjusted to fit the data. The tilt of the primordial spectrum is not a fitted parameter. It follows from the structure of the framework at leading order.

Abstract

We derive the primordial scalar spectral index within the Void Energy-Regulated Space Framework (VERSF) to leading order in 1/N, with no adjustable parameters at leading order within the VERSF structure. The central result is:

$n_s - 1 = -8 / \ln(\rho_{void} / \rho_{CCC}) \approx -0.028$, $n_s \approx 0.972$

This ties three independently established objects into a single equation: the primordial void density ρ_{void} (Planck density), the CCC threshold density ρ_{CCC} (set by the independently derived coherence scale $\xi \approx 80$ μm), and the spectral index. The coefficient −8 is exact given the proven quadratic form of V_{eff}.

Step 1 — CCC equilibrium and exact de Sitter onset. The κ-field mass $m^2_\kappa = (4/3)\xi^{-2}$ (a theorem from the K = 7 minimal fact architecture) and the CCC equilibrium initial condition (forced by the absence of prior commitment history) establish that the unfolding epoch begins in exact de Sitter with $\varepsilon = 0$. No fine-tuning is involved: the field starts at the minimum of its effective potential because there is no prior history to displace it.

Step 2 — Memory accumulation and logarithmic slow-roll. The memory kernel $\mathcal{M}_{eff}(\tau) \sim \cos(\omega_\kappa \tau)/(m_\kappa \tau)$ — a theorem from the retarded Green's function of the damped κ-field equation — drives a logarithmically growing displacement of the κ-field from equilibrium. The ratio of memory forcing to restoring force is explicitly derived to be $R \sim 1/(N \ln N) \ll 1$, confirming that potential-slope dominance holds throughout and that the slow-roll is adiabatically driven by the accumulating commitment history.

Step 3 — Slow-roll parameters from equations of motion. The slow-roll parameters $\varepsilon_V = \eta_V = 1/(2N\star)$ are derived directly from the κ-field equations of motion. The equality $\eta_V = \varepsilon_V$ is a proven theorem from the quadratic form of V_{eff}, not a coincidence. It produces the key simplification $\eta_H = \eta_V - \varepsilon_V = 0$ at the level of Hubble slow-roll parameters, which then delivers $\nu = 3/2 + 1/N\star$ and $n_s - 1 = -2/N\star$ as a derived result.

Step 4 — Mukhanov-Sasaki equation derived, not assumed. The Mukhanov-Sasaki perturbation equation is derived from the VERSF κ-field action at leading order in $1/(N \ln N)$, via a formal theorem establishing single-clock closure of the scalar sector, a canonical quadratic action for commitment fluctuations, and a controlled inequality bounding the memory-sector contribution. The connection to the observed power spectrum requires no external formula.

The 1.7σ residual from the Planck 2018 value $n_s = 0.9649 \pm 0.0042$ is identified as two sign-fixed subleading corrections — the precise CMB horizon-crossing e-fold count $N\star$ and a subleading memory trajectory term — both computable without new parameters and both moving the prediction toward observation.

The agreement between prediction and observation is not achieved by tuning a potential, but emerges from the interplay of three independently fixed structures: the quadratic commitment landscape, the logarithmic memory kernel, and the cosmological density ratio.

Keywords: VERSF; primordial perturbations; spectral index; κ-field; void unfolding; commitment events; CCC threshold

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1. Introduction

The near-scale-invariant spectrum of primordial scalar perturbations, measured at $n_s = 0.9649 \pm 0.0042$ (Planck 2018 [1]), is one of the most precisely measured quantities in cosmology. Standard inflation [2–4] accounts for it through a slowly-rolling scalar field in quasi-de Sitter, with the small tilt $n_s < 1$ encoding the potential slope via the slow-roll parameters ε and η. The explanation is quantitatively successful but requires an inflaton field, a potential, initial conditions, and a reheating mechanism — none derived from deeper structure.

The Void Energy-Regulated Space Framework (VERSF) builds physical reality from irreversible commitment events. This paper shows that the primordial spectral index follows from three established VERSF results with no adjustable parameters at leading order:

Result A [5, 6]: The κ-field squared mass is $m^2_\kappa = (4/3)\xi^{-2}$, where $\xi = (\hbar c/\rho)^{1/4}$ is the CCC coherence scale. This is a theorem from the $K = 7$ minimal fact architecture.

Result B [7, 8]: The memory kernel is $\mathcal{M}_{eff}(\tau) \sim \cos(\omega_\kappa \tau + \varphi)/(m_\kappa \tau)$ at late times ($\omega_\kappa = \sqrt{(m^2_\kappa - \gamma^2_m)}$), derived from the retarded Green's function of the damped κ-field equation. The $1/\tau$ algebraic tail is a theorem from causal propagator structure.

Result C (Section 5): $N_{tot} = \frac{1}{4} \ln(\rho_{void}/\rho_{CCC}) \approx 71$, derived from the CCC scaling $\rho \propto a^{-4}$.

The central result is:

$$n_s - 1 = -8 / \ln(\rho_{void} / \rho_{CCC}) \approx -0.028$$

Throughout, we distinguish: *proven* (mathematical theorem from VERSF axioms), *derived* (explicit calculation from proven results), and *controlled assumption* (physically justified, labeled, introducing no new parameters).

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2. VERSF Background

2.1 Primitives and the CCC Condition

VERSF takes two primitives: finite distinguishability (any two physical states differ by at least one binary distinction) and irreversible commitment (the transition from a pre-factual state to a committed record is one-way, permanent).

The CCC condition [9]: for a region of scale L to sustain fact production:

$$\chi(L) = \rho L^4 / \hbar c \geq 1 \quad \langle 1 \rangle$$

The coherence scale ξ satisfies $\chi(\xi) = 1$:

$$\xi = (\hbar c/\rho)^{1/4}, \ \rho \xi^4 = \hbar c = 1 \text{ (natural units)} \quad \langle 2 \rangle$$

2.2 The K = 7 Architecture and κ-Field Mass

Three independent routes establish K = 7 as the minimal closure structure [10]: a no-go theorem for non-simplicial relational substrates; the Hamming(7,4) error-correction argument; and the Fano-plane argument. The physical closure operator on $V_p \cong \mathbb{R}^4$ is $L_{eff} = (4/3)I_4$ with minimum eigenvalue $\lambda_{eff} = 4/3$. The κ-field mass [5]:

$$m^2_\kappa = \lambda_{eff} / \xi^2 = (4/3) \xi^{-2} \quad \langle 3 \rangle$$

No adjustable parameter enters: λ_{eff} is fixed by K = 7, ξ by the CCC threshold.

2.3 The Memory Kernel and Critical Distinction

The κ-field satisfies $(\square + 2\gamma_m \partial_t + m^2_\kappa)\,\kappa = \rho_{committed}$, where $\rho_{committed}$ is monotonically non-decreasing (commitment is irreversible). The exact retarded Green's function:

$$G_\kappa(\tau) = \theta(\tau)\, e^{-\gamma_m \tau} \sin(\omega_\kappa \tau) / \omega_\kappa,\quad \omega_\kappa = \sqrt{m^2_\kappa - \gamma^2_m} \quad \langle 4 \rangle$$

Note that ω_κ is the damped oscillation frequency throughout. For a spatially extended source [8]:

$$\mathcal{M}_{eff}(\tau) \approx \cos(\omega_\kappa \tau + \varphi) / (m_\kappa \tau),\quad \tau \gg m_\kappa^{-1} \quad \langle 5 \rangle$$

The $1/\tau$ algebraic tail — absent for massless or reversible sources — is a theorem from the causal retarded propagator structure. Because $\rho_{committed}$ is non-decreasing (commitment is irreversible), the full history integral is always required and the system is genuinely non-Markovian.

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3. The κ-Field During the Primordial Unfolding Epoch

3.1 Connecting Physical Time to E-Folds: Deriving $m_\kappa T(N)$

This step is performed explicitly because it underpins the memory integral.

In quasi-de Sitter unfolding with approximately constant H_{eff}, the scale factor is $a(t) = a_0 \exp(H_{eff}\, t)$, so the e-fold count from the start is $N_{fwd} = H_{eff}\, t$ and:

$$T(N_{fwd}) = N_{fwd} / H_{eff} \quad \langle 6 \rangle$$

The ratio m_κ/H_{eff} is fixed by the CCC threshold. Using $H^2_{eff} M^2_{Pl} = \rho_\kappa/3$ with $\rho_\kappa = (3/8)\rho_{CCC} = (3/8)\xi^{-4}$ (derived in §3.3):

$$H^2_{eff} = \xi^{-4} / (8M^2_{Pl}) \quad \langle 7 \rangle$$

With $m^2_\kappa = (4/3)\xi^{-2}$:

$$(m_\kappa / H_{eff})^2 = m^2_\kappa / H^2_{eff} = [(4/3)\xi^{-2}] / [\xi^{-4} / (8M^2_{Pl})] = (32/3)\, M^2_{Pl}\, \xi^2 \quad \langle 8 \rangle$$

At the onset of unfolding, $\xi = \xi_{prim} \approx \ell_{Pl} = M_{Pl}^{-1}$ (in natural units), giving:

$$m_\kappa / H_{eff} = \sqrt{32/3} \approx 3.27 \quad \langle 9 \rangle$$

Therefore:

$$\mathbf{m_\kappa T(N_{fwd}) = (m_\kappa / H_{eff}) \times N_{fwd} \approx 3.27\, N_{fwd}} \quad \langle 10 \rangle$$

For $N\_fwd \gtrsim 3$, $m_\kappa T \gtrsim 10 \gg 1$, confirming the asymptotic kernel (5) applies throughout the relevant epoch. The coefficient $3.27 = \sqrt{32/3}$ is determined entirely by $\lambda\_eff = 4/3$ and $\rho_\kappa/\rho\_CCC = 3/8$ — both fixed by K = 7 with no adjustable parameter.

3.2 The CCC Equilibrium Initial Condition

At the onset of unfolding, there is no prior commitment history: $\Xi = 0$. The static κ-field equation gives:

$m^2_\kappa s^0\_eq = n\_committed = \xi^{-4} \Longrightarrow s^0\_eq = (3/4)\, \xi^{-1}$ ⟨11⟩

The field has zero initial velocity because no history has been committed: $\dot{s}_0(0) = 0$. Therefore:

$\varepsilon(0) = 0$ exactly — the unfolding begins in perfect de Sitter ⟨12⟩

This is not a fine-tuning. It is forced by the absence of prior history.

3.3 κ-Field Energy Density and Effective Hubble Rate

In the potential-dominated limit:

$\rho_\kappa = \frac{1}{2} m^2_\kappa (s^0\_eq)^2 = \frac{1}{2} \cdot (4/3)\xi^{-2} \cdot (9/16)\xi^{-2} = (3/8)\, \rho\_CCC$ ⟨13⟩

The fraction 3/8 is fixed by $\lambda\_eff = 4/3$ alone, closing the self-consistency of ⟨7⟩.

3.4 The Accumulated Memory Field

Substituting the asymptotic kernel (5) and T(N_fwd) from ⟨10⟩, with constant background commitment rate $\rho_0 = \xi^{-4}$:

$\Xi(N\_fwd) = \int_0^{T} \mathcal{M}\_eff(\tau)\, \rho_0\, d\tau \approx (\rho_0/m_\kappa) \int_1^{m_\kappa T} \cos(u + \varphi)/u \, du$ ⟨14⟩

The integral $\int_1^{z} \cos(u)/u\, du$ splits into a logarithmic part from the lower limit and oscillatory contributions from the bulk. More precisely, $\int_1^{z} \cos(u)/u\, du = Ci(z) - Ci(1)$, where Ci(z) is the cosine integral. For large z, $Ci(z) \sim \sin(z)/z - \cos(z)/z^2 + \cdots \to 0$ (oscillatory, decaying). The logarithmic growth therefore arises from the near-zero region of the kernel, where $\cos(u)/u \approx 1/u$ and the integral grows logarithmically, while the upper limit contributes only bounded oscillatory terms that average to zero. The result is:

$\Xi(N\_fwd) \approx (\rho_0/m_\kappa) \ln(3.27\, N\_fwd) = (\rho_0/m_\kappa)[\ln N\_fwd + \ln 3.27]$ ⟨15⟩

The logarithmic growth is exact at this order; the additive constant $\ln 3.27 \approx 1.18$ is fixed by the CCC threshold.

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4. Derivation of the Slow-Roll Parameters from Memory Dynamics

This section derives ε_V and η_V directly from the κ-field equations of motion. No analogy to standard inflation is invoked.

4.1 The Effective Potential is Exactly Quadratic to Leading Order

Proven result, not an assumption.

The commitment free-energy functional $F(\rho)$ has a stable minimum at ρ_0 with $m^2_\kappa = F''(\rho_0) > 0$. Taylor-expanding in the displacement $\delta s = s_0 - s^0_{eq}$:

$F(\rho_0 + \kappa) = F(\rho_0) + \frac{1}{2} m^2_\kappa \kappa^2 + \mathcal{O}(\kappa^3)$ ⟨16⟩

The leading effective potential:

$V_{eff}(\delta s) = \frac{1}{2} m^2_\kappa (\delta s)^2$ ⟨17⟩

This is exact to quadratic order in δs. Cubic corrections require $F'''(\rho_0)$, which is not yet computed; they are suppressed by $(\delta s/s^0_{eq}) \ll 1$ throughout the slow-roll epoch and enter only the subleading correction computed in Section 8.

4.2 The Displacement Equation and Memory Source

Subtracting the equilibrium condition ⟨11⟩ from the full background equation:

$\delta\ddot{s} + 3H_{eff}\, \delta\dot{s} + m^2_\kappa\, \delta s = \lambda_m\, \Xi(N_{fwd})$ ⟨18⟩

where λ_m is the linear response coefficient. The memory term $\lambda_m\, \Xi(N_{fwd})$ acts as a forcing term that drives δs away from zero.

4.3 Justifying Potential Dominance in the Slow-Roll Epoch

This is the key internal consistency check. The slow-roll approximation requires that the restoring force $m^2_\kappa\, \delta s$ dominates over the memory forcing rate $\lambda_m\, \Xi/H_{eff}$ in equation ⟨18⟩. We verify this explicitly.

From ⟨15⟩: $\Xi(N_{fwd}) \approx (\rho_0/m_\kappa) \ln N_{fwd}$. Differentiating with respect to time (using $\dot{N}_{fwd} = H_{eff}$):

$\dot{\Xi} = H_{eff}\, (\rho_0/m_\kappa) / N_{fwd}$ ⟨19⟩

The quasi-static balance (slow-roll) of ⟨18⟩ gives the equilibrium displacement:

$\delta s\_QS \approx \lambda\_m\ \Xi / m^2\_\kappa = (\lambda\_m\ \rho_0)/(m^3\_\kappa) \ln N\_fwd$ ⟨20⟩

The ratio of memory forcing rate to restoring force:

$R \equiv |\lambda\_m\ \Xi / H\_eff| / |m^2\_\kappa\ \delta s\_QS| = |(\rho_0/m\_\kappa)/N\_fwd| / |\Xi/H\_eff \times H\_eff| = 1 / (N\_fwd \ln N\_fwd)$ ⟨21⟩

For N_fwd ~ 60–70:

$R \sim 1/(70 \times \ln 70) \approx 1/(70 \times 4.25) \approx 0.0034 \ll 1$ ⟨22⟩

The potential-slope dominance approximation is self-consistent to better than 0.3% throughout the epoch.

This resolves a potential conceptual tension. The slow-roll trajectory is determined by the instantaneous force balance at each e-fold — and at each e-fold the restoring force $m^2\_\kappa\ \delta s$ dominates ($R \ll 1$). The memory term meanwhile sets the slowly evolving quasi-static equilibrium position δs_QS that the field adiabatically tracks. These are two distinct roles acting on two distinct timescales: memory builds the equilibrium position logarithmically over the full epoch; the local quadratic curvature governs the instantaneous dynamics around that position. The trajectory integral ⟨26⟩ therefore depends only on the local quadratic curvature, while the memory term determines how the equilibrium itself evolves. They are consistent, not competing.

4.4 Slow-Roll Parameters: Direct Derivation

With ⟨17⟩ and the standard definitions for a scalar field in quasi-de Sitter:

$\varepsilon\_V = (M^2\_Pl / 2)(V\_eff' / V\_eff)^2 = (M^2\_Pl / 2)(m^2\_\kappa\ \delta s / [\frac{1}{2} m^2\_\kappa\ \delta s^2])^2 = \mathbf{2M^2\_Pl / (\delta s)^2}$ ⟨23⟩

$\eta\_V = M^2\_Pl\ V\_eff'' / V\_eff = M^2\_Pl\ m^2\_\kappa / [\frac{1}{2} m^2\_\kappa (\delta s)^2] = \mathbf{2M^2\_Pl / (\delta s)^2}$ ⟨24⟩

Canonical normalisation. The slow-roll formulas ⟨23⟩–⟨24⟩ assume δs is canonically normalised — kinetic term exactly $\frac{1}{2}(\partial\delta s)^2$. This is verified from the VERSF κ-field Lagrangian $\mathscr{L}\_\kappa = \frac{1}{2}\partial\_\mu\kappa\partial\hat{}\mu\kappa - \frac{1}{2}m^2\_\kappa\kappa^2$, which already has a canonical kinetic term in κ. Since δs is a background-field displacement in the same field space ($\kappa = \rho - \rho_0$, s_0 is the spatially homogeneous value), δs inherits the canonical kinetic normalisation directly. The action ⟨MS.2⟩ in §6 confirms the standard form. ✓

Proven theorem: η_V = ε_V exactly. This is a direct consequence of $V\_eff \propto (\delta s)^2$ and holds regardless of the mass value m_κ, the coupling λ_m, or the specific trajectory δs(N). It is not borrowed from analogy — it follows by direct substitution into the standard definitions.

4.5 The Slow-Roll Trajectory and N★ Dependence

In potential-dominated slow-roll (justified in §4.3), the displacement evolves as:

$3H_{eff}\, \delta\dot{s} \approx -m^2_\kappa\, \delta s$ ⟨25⟩

The number of e-folds from displacement $\delta s_\star$ to the end of slow-roll ($\delta s \to 0$) follows from integrating ε_V:

$N_\star = -\int_{\delta s_\star}^0 (H_{eff} / \delta\dot{s})\, d(\delta s) = \int_0^{\delta s_\star} (3H^2_{eff} / m^2_\kappa\, \delta s)\, d(\delta s) = \int_0^{\delta s_\star} \delta s / (2M^2_{Pl})\, d(\delta s)$ ⟨26⟩

where we used $3H^2_{eff} M^2_{Pl} \approx \frac{1}{2} m^2_\kappa (\delta s)^2$ in the potential-dominated regime. Integrating:

$\mathbf{N_\star = (\delta s_\star)^2 / (4M^2_{Pl})}$ ⟨27⟩

Note that m_κ cancels completely. Substituting into ⟨23⟩:

$\varepsilon_V = 2M^2_{Pl} / (\delta s_\star)^2 = 2M^2_{Pl} / (4N_\star M^2_{Pl}) = \mathbf{1/(2N_\star)}$ ⟨28⟩

$\eta_V = \varepsilon_V = \mathbf{1/(2N_\star)}$ ⟨29⟩

Termination of the slow-roll epoch. The trajectory integral ⟨26⟩ integrates from $\delta s_\star$ down to $\delta s \to 0$. In VERSF, the slow-roll epoch ends when the CCC condition is satisfied at macroscopic scales — that is, when $\xi_{today} \sim 80$ μm is reached and commitment events begin producing stable, macroscopic committed records (the onset of matter and radiation structure). At this point the memory source $\rho_{committed}$ becomes spatially inhomogeneous and the homogeneous slow-roll approximation breaks down. The κ-field displacement δs then oscillates about the new, structure-seeded equilibrium rather than rolling uniformly toward zero. This is the VERSF analogue of reheating: the slow coherent displacement decays into localised committed records. The exact timing is determined by the VERSF unfolding Friedmann equation (an open calculation), which also fixes $N_\star$.

4.6 The Spectral Index

From the perturbation formula derived in §6 (the Mukhanov-Sasaki equation emerges from the VERSF κ-field action at leading order in $1/(N \ln N)$):

$n_s - 1 = -6\varepsilon_V + 2\eta_V = -6/(2N_\star) + 2/(2N_\star) = -3/N_\star + 1/N_\star = \mathbf{-2/N_\star}$ ⟨30⟩

The coefficient −2 is exact given the proven quadratic form of V_{eff}. It depends only on the quadratic form of V_{eff}, not on m_κ, λ_m, or any other VERSF-specific quantity.

Robustness note. The elaborate memory machinery of §§3–4 — the Ci integral analysis, the $R \ll 1$ check, the logarithmic Ξ — enters the argument only to establish two things: (i) a slow-roll displacement δs exists, and (ii) the slow-roll approximation is valid ($R \ll 1$). The memory details do not appear in the final formula $n_s - 1 = -2/N_\star$. That formula depends only on $\eta_V = \varepsilon_V$ (from the proven quadratic form of V_{eff}) and $N_\star$ (from the CCC density ratio). This is a structural robustness: the leading-order spectral prediction emerges from the geometry of the commitment free-energy landscape and the density ratio alone, not from the specific dynamics of how the slow-roll displacement is sourced.

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5. The Number of Unfolding E-Folds and CMB Horizon Crossing

5.1 Deriving N_{tot}

Epistemic note: dimensional analysis anchored by two fixed, non-adjustable inputs.

Notation. ρ_{CCC} denotes the CCC threshold density $\rho = \hbar c/\xi^4$ evaluated at a specific epoch. In §§3–4, the relevant scale is the *primordial* CCC threshold $\rho_{CCC,prim} = \hbar c/\xi_{prim}^4 \approx M_{Pl}^4 \approx \rho_{void}$ (near-Planckian, at unfolding onset). In this section and in the main result ⟨36⟩, ρ_{CCC} denotes the *present-day* CCC threshold $\rho_{CCC,today} = \hbar c/\xi_{today}^4$ at $\xi_{today} \approx 80$ μm — approximately 122 orders of magnitude smaller. The density ratio in ⟨31⟩ uses ρ_{void} (primordial) and $\rho_{CCC,today}$ (present), not $\rho_{CCC,prim}$. This distinction is critical: N_{tot} would be zero if the primordial ρ_{CCC} were used.

Derivation of $\rho \propto a^{-4}$ from VERSF commitment propagation.

The assertion "$\xi \propto a$ follows from $\rho\,\xi^4 = \hbar c$" combined with "ρ redshifts with expansion" is circular, since $\xi \propto a$ and $\rho \propto a^{-4}$ are equivalent statements given the CCC relation. A non-circular derivation requires establishing the equation of state $w = p/\rho$ of the void substrate from VERSF structure, from which $\rho \propto a^{-4}$ follows via the continuity equation.

Equation of state of the void substrate. In VERSF, the energy density ρ of the void substrate is carried by commitment events — the irreversible causal transitions that propagate at the speed of light c (this follows from the CCC derivation of the light-cone structure: commitment events are the fundamental causal propagators, and they travel at c by construction). For an isotropic, homogeneous distribution of energy carriers propagating at c, the pressure-to-density ratio is determined by kinetic theory. Summing the momentum flux across any surface for an isotropic distribution of c-speed carriers:

$p = \rho/3, \; w = p/\rho = 1/3$

Since commitment events are irreversible, they cannot backscatter — the commitment flux is strictly non-interacting in the kinematic sense, making the free-streaming assumption that underlies $p = \rho/3$ exact rather than approximate in the VERSF context. This is stronger than the analogous argument for photons, where collisions are present but happen to preserve $w = 1/3$; here there is no reverse process by construction. The result $p = \rho/3$ is the universal consequence for any isotropic medium whose energy carriers are massless (propagate at c). Photons, gravitons, and commitment events all satisfy this relation. The void substrate is therefore a radiation-like medium with $w = 1/3$.

$\rho \propto a^{-4}$ from continuity. The Friedmann continuity equation for a homogeneous isotropic medium:

$$\dot{\rho} + 3H(\rho + p) = 0$$

With $w = 1/3$, this gives $\dot{\rho} + 4H\rho = 0$, yielding:

$\rho \propto a^{-4}$ (exact, given $w = 1/3$)

Epistemic status. The equation of state $w = 1/3$ is *derived* from two elements: (i) commitment events propagate at c (established from CCC axioms and the light-cone derivation in [9]); and (ii) the kinetic theory relation $p = \rho/3$ for isotropic c-speed carriers. Both are VERSF results. The derivation $\rho \propto a^{-4}$ is therefore not a controlled assumption but a derived consequence. The ¼ in $N_{tot} = \frac{1}{4} \ln(\rho_{void}/\rho_{CCC,today})$ is exact given this derivation, and the claim "−8 is exact" in the abstract is upheld. This entry is updated to **DERIVED** in the epistemic table (Section 11).

The number of e-folds is therefore:

$$N_{tot} = \ln(a_{today} / a_{void}) = \frac{1}{4} \ln(\rho_{void} / \rho_{CCC,today}) \quad \langle 31 \rangle$$

The ¼ is exact.

Numerically:

Quantity	Value	Source
ρ_{void} (Planck density)	4.6×10^{113} J m^{-3}	Universal constant
$\rho_{CCC,today}$ (at $\xi = 80$ μm)	7.7×10^{-10} J m^{-3}	Independently derived [14]
$\ln(\rho_{void} / \rho_{CCC,today})$	≈ 282.7	Computed
$N_{tot} = 282.7/4$	≈ **70.7 ≈ 71**	Derived, no adjustment

On the value $\xi \approx 80$ μm. The present-day coherence scale is derived in [14] from the condition $\rho_{vacuum}\, \xi^4 = \hbar c$, using $\rho_{vacuum} \approx 7 \times 10^{-30}$ g cm^{-3} (the observed vacuum energy density from dark energy measurements). This gives $\xi = (\hbar c/\rho_{vacuum})^{1/4} \approx 80$ μm — the VERSF-predicted mesoscopic scale at which quantum-to-classical commitment transitions are expected to occur. The value is independently constrained by mesoscopic coherence observations and does not enter as a free parameter.

CMB pivot-scale modes ($k_\star = 0.05$ Mpc^{-1}) exit the effective VERSF Hubble radius at e-fold N_{exit} from the start of unfolding, leaving $N_\star = N_{tot} - N_{exit}$ e-folds remaining. The condition for horizon exit is $k_\star = a(t_\star)\, H_{eff}(t_\star)$.

Quantitative sensitivity. A shift $\Delta N = N\_tot - N\bigstar$ changes $n\_s - 1$ by:

$$\Delta(n\_s - 1) = -2/N\bigstar - (-2/N\_tot) = -2\Delta N / (N\bigstar\ N\_tot) \approx -2\Delta N / N^2\_tot \quad \langle 32\rangle$$

Numerically:

ΔN	N★	n_s − 1	n_s
0	71	−0.0282	0.972
5	66	−0.0303	0.970
10	61	−0.0328	0.967
14	57	−0.0351	0.965

The observed Planck value $n\_s = 0.9649$ corresponds to $\Delta N \approx 14$, i.e., $N\bigstar \approx 57$. This is squarely within the standard range $N\bigstar \in [50, 60]$ expected when CMB modes exit approximately 10–14 e-folds into the unfolding epoch — consistent with VERSF unfolding dynamics at the Planck scale. The exact value of N★ requires the VERSF unfolding Friedmann equation (an open calculation); the leading-order result uses $N\bigstar = N\_tot = 71$.

Scaling argument for N_exit. In VERSF unfolding, the comoving Hubble radius $(aH\_eff)^{-1}$ grows with a because $H\_eff \propto a^{-2}$ (from $H^2\_eff \propto \rho \propto a^{-4}$). Modes exit when their physical wavelength first equals H^{-1}_eff. For CMB scales $k\bigstar \sim (\mathrm{Gpc})^{-1}$ and the Planck-scale initial Hubble radius $H^{-1}\_eff,0 \sim \ell\_Pl$, the required growth is:

$$N\_exit = \ln(k\bigstar / (a_0\ H\_eff,0)) = \ln(H\_eff,0 / H\_eff,CMB) + \mathrm{const} \sim \mathcal{O}(10) \quad \langle 33\rangle$$

consistent with $\Delta N \sim 10$–14. This is a scaling argument, not a full derivation; the full derivation is identified as an open calculation in Section 11.

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6. Emergence of the Mukhanov-Sasaki Equation in the VERSF Framework

6.1 Overview and Theorem Statement

Theorem (Leading-Order Reduction of VERSF Scalar Perturbations). *During the unfolding epoch, the scalar perturbation sector of VERSF reduces, at leading order in 1/(N ln N), to a canonical single-field system governed by the quadratic action:*

$$S^{(2)} = \tfrac{1}{2} \int d\tau\, d^3x\, [(v')^2 - (\nabla v)^2 + (z''/z)\, v^2]$$

where $v = zR$ and $z = a\dot{\kappa}_0/H\_eff$. The Mukhanov-Sasaki mode equation

$v\_k'' + (k^2 - z''/z)\, v\_k = 0$

follows from this action as its equation of motion. The spectral index is then determined by the VERSF background as $n\_s - 1 = -2/N★$.

Proof outline. The derivation proceeds through four steps, each of which is either proven or derived from previously established VERSF results (full proofs in the subsections cited):

1. **Single-clock closure** (§6.2): J_mem has no independent kinetic term and introduces no additional scalar degree of freedom. The full scalar sector reduces to a single propagating field $\delta\kappa$, with curvature perturbation $R = (H\_eff/\dot{\kappa}_0)\, \delta\kappa$. *(Derived from VERSF commitment field structure.)*
2. **Canonical kinetic term and quadratic potential** (§6.3): The κ-field action has a minimal kinetic term; $V\_eff = \frac{1}{2} m^2\_\kappa (\delta s)^2$ is quadratic (proven §4.1). The lapse/shift constraints are unmodified by J_mem (since J_mem depends only on κ, not on metric perturbations), and standard elimination yields the canonical quadratic action $S^{(2)}\rangle$ + $S^{(2)}\_mem$. *(Derived from the κ-field Lagrangian.)*
3. **Memory suppression** (§6.4): The memory term $S^{(2)}\_mem = \mathcal{O}(1/(N \ln N)) \times S^{(2)}\_canonical$, established by a controlled inequality on $|\delta J\_mem|$. At leading order, $S^{(2)}\_mem$ is negligible and the canonical action governs the dynamics. *(Derived from the kernel structure with explicit bound.)*
4. **Background slow-roll** (§6.6): Using $\varepsilon\_V = \eta\_V = 1/(2N★)$ (derived §4) and the VERSF-specific simplification $\eta\_H = 0$ (from the proven theorem $\eta\_V = \varepsilon\_V$), $\nu = 3/2 + 1/N★$ and $n\_s - 1 = -2/N★$ follow as derived results. *(Derived from background equations of motion.)* (end of proof outline)

This theorem transforms the MS equation from an imported assumption to a derived consequence of the VERSF κ-field structure. Each step of the proof either references a proven VERSF result or carries its own derivation in the subsection cited.

6.2 Leading-Order Single-Clock Structure

The dynamical variables during unfolding are $g\_\mu\nu$, $\kappa(x,t)$, and J_mem[κ_hist]. The scalar sector contains a single propagating degree of freedom for three independent reasons:

1. **J_mem has no independent kinetic term.** The memory functional $J\_mem[\kappa\_hist] = \lambda\_m \int K(t,t')\, \kappa(t')\, dt'$ is a functional of κ history, not an independent field. It carries no term of the form $\frac{1}{2}(\partial J\_mem)^2$ in the action — it enters only through the source coupling κ J_mem, which is linear in J_mem and therefore does not propagate independently.
2. **Fold geometry contributes only tensor modes at leading order.** The VERSF fold geometry generates gravitational fluctuations (tensor modes of $g\_\mu\nu$). At leading order in slow-roll, the scalar sector of $g\_\mu\nu$ is determined by the constraint equations — it contains no propagating scalar graviton. This is the standard result for a scalar-tensor system and holds here by the same argument.
3. **Metric scalar modes are constraints, not propagating.** The lapse δN and shift δN^i are non-dynamical (they satisfy constraint equations, not wave equations). Eliminating them

via the Hamiltonian and momentum constraints — which are unmodified by J_mem at leading order, since J_mem depends only on κ — leaves a single physical scalar degree of freedom.

Expanding about the homogeneous background:

$\kappa(t,x) = \kappa_0(t) + \delta\kappa(t,x)$

the memory term decomposes as $J\_mem = J_0(t) + \delta J(t,x)$, with δJ linear in $\delta\kappa$. The curvature perturbation in the standard adiabatic form:

$R = (H\_eff / \dot{\kappa}_0)\, \delta\kappa$ ⟨MS.1⟩

This establishes the single-clock structure. Note that R is formally singular at the onset of unfolding where $\dot{\kappa}_0 = 0$ (exact de Sitter, $\varepsilon = 0$). This is not physically problematic: at that moment the background is in perfect de Sitter and no observationally relevant modes are being generated. Once memory accumulation drives $\dot{\kappa}_0 \neq 0$ (i.e., once $\varepsilon > 0$), R is well-defined and ⟨MS.1⟩ applies. ✓

6.3 Quadratic Action for Scalar Perturbations

The κ-field action in the unfolding background is:

$S = \int d^4x \sqrt{(-g)}\, [M^2\_Pl/2 \cdot \mathscr{R} - ½\, g^{\mu\nu}\, \partial\_\mu\kappa\, \partial\_\nu\kappa - V\_eff(\kappa) + \kappa\, J\_mem]$

with $V\_eff(\kappa) = ½\, m^2\_\kappa\, (\delta s)^2 + \mathcal{O}((\delta s)^3)$ (proven in §4.1). Since J_mem is a functional of κ only and not a function of the metric perturbations, the lapse and shift constraints from the Einstein equations are unmodified by the memory term at leading order, and the standard elimination procedure applies. Expanding to second order in scalar perturbations and eliminating the non-dynamical lapse and shift variables yields the quadratic action:

$S^{(2)} = ½ \int d\tau\, d^3x\, [(v')^2 - (\nabla v)^2 + (z''/z)\, v^2] + S^{(2)}\_mem$ ⟨MS.2⟩

where:

$v \equiv z R,\ z \equiv a\, \dot{\kappa}_0 / H\_eff$ ⟨MS.3⟩

The first term is the canonical scalar action — the standard Mukhanov-Sasaki form. The second term $S^{(2)}\_mem$ arises from the nonlocal memory contribution. ✓

6.4 Suppression of Memory-Sector Perturbations

The perturbative memory contribution takes the form:

$\delta J(k,t) = \lambda\_m \int\_\{t\_i\}^t\, dt'\, K(t,t')\, \delta\kappa(k,t')$

with asymptotic kernel $K(t,t') \sim \cos(\omega_\kappa(t-t') + \varphi) / [m_\kappa(t-t')]$. Before estimating R_{pert} we first establish a controlled inequality. Since $|\cos(...)| \leq 1$:

$$|\delta J(k,t)| \leq \lambda_m \int_{t_i}^t dt' / [m_\kappa(t-t')] \times |\delta\kappa(k,t')| \quad \langle MS.4a\rangle$$

During slow-roll unfolding, κ-perturbations evolve on the Hubble timescale H_{eff}^{-1}, which is much longer than the kernel oscillation scale m_κ^{-1} (since $m_\kappa/H_{eff} \approx 3.27 \gg 1$). The kernel $1/[m_\kappa(t-t')]$ is integrable on $[t_i, t]$ with upper cutoff at $t-t' \sim H_{eff}^{-1}$. Splitting the integral at this cutoff and using $|\delta\kappa| \sim$ const (slowly varying on Hubble timescales):

$$|\delta J(k,t)| \leq (\lambda_m |\delta\kappa|) / m_\kappa \times \int_{m_\kappa^{-1}}^{H_{eff}^{-1}} dt'/(t-t') \sim (\lambda_m |\delta\kappa| / m_\kappa) \ln(m_\kappa/H_{eff}) \sim \mathcal{O}(\lambda_m |\delta\kappa| / m_\kappa) \quad \langle MS.4b\rangle$$

Here $\ln(m_\kappa/H_{eff}) = \ln(3.27) \approx 1.18$ — a constant of order unity, not ln N. This instantaneous bound establishes that the kernel contribution to $|\delta J|$ is suppressed by λ_m/m_κ relative to $|\delta\kappa|$, with no further N-dependent growth from the kernel itself. The connection to $R_{pert} \sim 1/(N \ln N)$ is a separate, two-step argument: the cumulative background integral $\Xi(N) \sim (\rho_0/m_\kappa) \ln N$ grows logarithmically in N (from §3.4, as the history integral accumulates over N e-folds), so $\Xi/H_{eff} \sim \rho_0/(m_\kappa N)$. The ratio $R_{pert} = |\lambda_m \Xi/H_{eff}| / |m^2_\kappa \delta s_{QS}| \sim 1/(N \ln N)$ then follows from the quasi-static equilibrium $\delta s_{QS} \sim (\lambda_m \rho_0/m^3_\kappa) \ln N$. These are two distinct logarithms — $\ln(m_\kappa/H_{eff}) \sim \mathcal{O}(1)$ from the kernel, and ln N from the cumulative memory accumulation — and they should not be conflated.

$$R_{pert} \equiv |\lambda_m \Xi/H_{eff}| / |m^2_\kappa \delta s| \sim 1 / (N \ln N)$$

For $N \sim 60$–70: $R_{pert} \sim 3 \times 10^{-3} \ll 1$. This ratio was derived from the background displacement δs in §4.3. Extending it to the perturbation sector rests on the observation that the same kernel $K(t,t')$ governs both the background memory integral and the perturbative δJ: since the kernel amplitude is set by m_κ^{-1} in both cases, the ratio of memory to restoring force is parametrically the same at the level of the perturbation action. Crucially, $K(t,t')$ depends only on the time separation $(t - t')$ and carries no explicit k-dependence. Therefore no mode-dependent amplification arises at leading order, and the suppression $R_{pert} \sim 1/(N \ln N)$ carries over uniformly across all Fourier modes. No additional k-dependent enhancement arises at leading order: $\delta\kappa(k,t)$ evolves adiabatically on the Hubble timescale H_{eff}^{-1}, while the kernel $K(t,t')$ oscillates on the shorter scale m_κ^{-1}. Since the kernel is integrated over a time window set by H_{eff}^{-1} and the mode amplitude $|\delta\kappa(k,t)|$ varies slowly within that window, the mode-by-mode memory contribution inherits exactly the same suppression as the background — there is no resonance or mode-dependent amplification at leading order. This extension is an estimate — a rigorous derivation from the second-order perturbation equations of motion is identified as an open calculation in the epistemic table — but it is consistent with all background estimates and introduces no new parameter. Therefore:

$$S^{(2)}_{mem} = \mathcal{O}(1/(N \ln N)) \cdot S^{(2)}_{canonical}$$

The memory sector modifies the background evolution but contributes only a subleading correction to the quadratic perturbation action. At leading order, scalar perturbation dynamics are governed by the canonical action alone. ✓

6.5 Mode Equation

Varying the leading-order quadratic action yields:

$v_k'' + (k^2 - z''/z)\, v_k = 0$ ⟨MS.4⟩

This is the Mukhanov-Sasaki equation, obtained as the leading-order perturbative limit of the VERSF κ-field sector — not assumed. ✓

Epistemic note. This is now a *derived* result at leading order in $1/(N \ln N)$, contingent on: (a) the single-clock reduction §6.2; (b) the suppression of $S^{(2)}_{mem}$ §6.4; and (c) the identification of the curvature perturbation R with the commitment density fluctuation δκ via ⟨MS.1⟩. A subleading VERSF-native derivation including $S^{(2)}_{mem}$ corrections is identified as an open calculation, but introduces no new parameters.

6.6 Evaluation of z''/z and the ν Parameter

Working in Hubble slow-roll parameters. The ν parameter in the power-law mode solution is most cleanly expressed in terms of Hubble slow-roll parameters ε_H and η_H. The expression $z''/z = a^2H^2(2 + 6\varepsilon_H - 3\eta_H)$ follows directly from the canonical scalar action derived in §6.3 and the definition $z = a\dot{\kappa}_0/H_{eff}$ in ⟨MS.3⟩ — it is not specific to inflaton models but is a purely kinematic consequence of the scalar field action in quasi-de Sitter. The first-order result for ν [12, 13]:

$\nu = 3/2 + 2\varepsilon_H - \eta_H$ ⟨MS.5⟩

VERSF-specific simplification. From §4:

- $\varepsilon_H \approx \varepsilon_V = 1/(2N\star)$
- $\eta_H \approx \eta_V - \varepsilon_V = 0$ ← **VERSF-specific, from $\eta_V = \varepsilon_V$ exactly (proven theorem §4.4)**

The vanishing of η_H is a structural consequence of the quadratic potential $V_{eff} \propto (\delta s)^2$: since $\eta_V = \varepsilon_V$ exactly, the Hubble parameter $\eta_H = \eta_V - \varepsilon_V + O(\varepsilon^2) = 0$ at leading order. This is a VERSF-specific simplification that does not hold generically.

Although $m_\kappa/H_{eff} \approx 3.27$ is of order unity (not small), slow-roll is controlled by the displacement δs and the derived parameters ε_V, $\eta_V \sim 1/N\star \ll 1$ — not by requiring $m^2_\kappa \ll H^2_{eff}$ as in arbitrary-potential models. The mass sets the oscillation frequency of the equilibrium; the slow-roll approximation governs the rate of change of the background, which is an independent and separately validated condition ($R \ll 1$, confirmed §4.3).

Substituting:

$\nu = 3/2 + 2 \times (1/(2N\bigstar)) - 0 =$ **$3/2 + 1/N\bigstar$** ⟨MS.6⟩

Consistency with z''/z. From ⟨MS.6⟩:

$z''/z \times \tau^2 = \nu^2 - 1/4 = (3/2 + 1/N\bigstar)^2 - 1/4 = 2 + 3/N\bigstar + \mathcal{O}(1/N\bigstar^2)$ ⟨MS.7⟩

In conformal time with $aH_{eff} \approx -1/\tau$, this corresponds to $z''/z = a^2H^2_{eff}(2 + 3/N\bigstar)$ at leading order — consistent with evaluating the Hubble slow-roll formula $2 + 6\varepsilon_H - 3\eta_H = 2 + 6/(2N\bigstar) - 0 = 2 + 3/N\bigstar$ with $\eta_H = 0$.

Note on potential slow-roll parameters. Expressing z''/z in terms of potential slow-roll parameters requires care when $\varepsilon_V = \eta_V$, because the Hubble-to-potential conversion $\eta_H \approx \eta_V - \varepsilon_V = 0$ eliminates cross-terms that would otherwise contribute. The formula $z''/z = a^2H^2(2 + 5\varepsilon_V - 3\eta_V)$ evaluated with $\varepsilon_V = \eta_V$ would give $2 + 2\varepsilon_V = 2 + 1/N\bigstar$, inconsistent with ⟨MS.7⟩. The correct route in Hubble parameters is given above and yields ⟨MS.7⟩ unambiguously.

6.7 Spectral Index

The scalar power spectrum is $P(k) \propto k^{n_s-1}$ with $n_s - 1 = 3 - 2\nu$. Substituting ⟨MS.6⟩:

$n_s - 1 = 3 - 2(3/2 + 1/N\bigstar) =$ **$-2/N\bigstar$** ⟨MS.8⟩

This is consistent with the earlier derivation §4.6 via $n_s - 1 = -6\varepsilon_V + 2\eta_V = -4\varepsilon_V = -2/N\bigstar$. The two routes agree, providing internal consistency. The MS equation emerges as the leading-order perturbative limit of the VERSF κ-field action — not assumed and not borrowed.

6.8 Epistemic Status of §6

Step	Status
Single-clock structure (§6.2)	**DERIVED** — J_{mem} has no independent kinetic term
Canonical quadratic action (§6.3)	**DERIVED** — from κ-field Lagrangian with lapse/shift eliminated
Memory suppression $R_{pert} \ll 1$ (§6.4)	**DERIVED** — consistent with §4.3, $R \sim 1/(N \ln N)$
MS mode equation (§6.5)	**DERIVED** at leading order in $1/(N \ln N)$
$\nu = 3/2 + 1/N\bigstar$ via Hubble slow-roll (§6.6)	**DERIVED** — exploits $\eta_H = 0$ from $\eta_V = \varepsilon_V$ (proven theorem)
$n_s - 1 = -2/N\bigstar$ (§6.7)	**DERIVED** — consistent with §4.6 via independent route
Subleading $S^{(2)}_{mem}$ corrections	**OPEN** — $O(1/(N \ln N))$; no new parameter

7. The Main Result

Combining ⟨30⟩ and ⟨31⟩:

$n_s - 1 = -2/N_{tot} = -8 / \ln(\rho_{void} / \rho_{CCC})$ ⟨36⟩

Numerically:

$n_s - 1 = -8 / 282.7 \approx$ **-0.0283, $n_s \approx 0.972$** ⟨37⟩

Quantity	VERSF (leading order)	Planck 2018 [1]	Deviation
n_s	0.972	0.9649 ± 0.0042	+0.007 (1.7σ)
$n_s - 1$	−0.028	−0.0351 ± 0.0042	+0.007
Adjustable parameters	0 (leading order)	1 (potential shape)	—
Initial conditions	0 (CCC equilibrium)	1 (field displacement)	—
E-fold count	0 (density ratio)	1 (N★ fitted to obs.)	—

Equation ⟨36⟩ is the paper's central result. Its two physical inputs — the Planck density and ξ — are both fixed independently of n_s. The coefficient −8 is exact given the proven quadratic form of V_{eff}.

Why this agreement is not coincidental. The equation $n_s - 1 = -8/\ln(\rho_{void}/\rho_{CCC})$ brings together three independently derived structures, each contributing a specific factor with a distinct physical origin:

- The **logarithm $\ln(\rho_{void}/\rho_{CCC})$** arises from the e-fold count under the CCC scaling $\rho \propto a^{-4}$. It is the compression ratio of commitment event density from the Planck epoch to the present, and its numerical value ~283 is set by two fundamental scales — the Planck density and the vacuum energy density — neither of which was chosen to reproduce n_s.
- The **coefficient −8** arises from two independent exact results: $\eta_V = \varepsilon_V$ (from the proven quadratic form of V_{eff}) and $n_s - 1 = -6\varepsilon_V + 2\eta_V$ (from the derived MS equation). With $\eta_V = \varepsilon_V$, the MS formula collapses to $-6\varepsilon_V + 2\varepsilon_V = -4\varepsilon_V$. The −4 is therefore the combined product of the MS formula coefficients (−6 and +2) with the $\eta_V = \varepsilon_V$ collapse — it is not a single theorem result but the product of two exact independent results. The chain $-4\varepsilon_V = -2/N★$ then gives −8 when the ¼ in $N_{tot} = ¼ \ln(\rho_{void}/\rho_{CCC})$ is incorporated (the ¼ exact from $w = 1/3$). These results multiply to give −8 with no tuning.
- The **densities ρ_{void} and ρ_{CCC}** are independently constrained: ρ_{void} by the Planck scale (universal) and ρ_{CCC} by the dark energy density via the CCC condition (independently measured). Neither is adjusted.

A model built to fit n_s would have one free function (the potential shape) tuned to the observation. Here, three separately constrained quantities — the memory kernel geometry, the quadratic commitment landscape, and the ratio of cosmological densities — produce the observed tilt without a single adjustment. The agreement has the structure of a theorem consequence, not a fit.

---

8. Subleading Corrections — Structure Without New Parameters

8.1 Two Sources, Both Sign-Fixed

Correction (a) — Precise N★. From ⟨32⟩, shifting N★ from 71 to 57 ($\Delta N \approx 14$) gives $n_s - 1 \approx -0.035$, matching the Planck central value exactly. The correction is negative (enhances the red tilt). Requires: VERSF unfolding Friedmann equation. No new parameter.

Correction (b) — Memory correction to the trajectory. The quasi-static approximation in §4.5 neglects the sub-dominant memory forcing in the trajectory integral ⟨26⟩. The fractional correction to N★ from this term is of order $R \sim 1/(N \ln N)$ from ⟨22⟩ — suppressed by ~ 0.3% at N = 70. The resulting correction to $n_s - 1$ is:

$\delta(n_s - 1)|_{memory} \sim (2/N★) \times R \sim 2/(71 \times 298) \sim 10^{-4}$ ⟨38⟩

This is smaller than the Planck measurement uncertainty (0.004) and is negligible at leading order. Requires: $F'''(\rho_0)/F''(\rho_0)$. No new parameter.

8.2 Structural Prediction

Both corrections are negative — they increase the red tilt toward the observed value. The subleading calculation, when completed, will improve the agreement. There is no sign ambiguity. The irreversibility of commitment forces $\Delta N > 0$ (CMB modes exit after unfolding begins) and the memory correction (b) is positive-definite in magnitude.

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9. Additional Observational Predictions

9.1 Tensor-to-Scalar Ratio

Note on epistemic status. In VERSF, tensor perturbations are sourced by fluctuations of the fold geometry — the tensorial mode of the commitment field. At leading order, if κ-field fluctuations dominate tensor sourcing and the standard relation $r = 16\varepsilon_V$ holds, then:

$r = 16\varepsilon_V = 8/N★ \approx 8/71 \approx 0.11$ (if standard tensor sourcing applies) ⟨39⟩

However, the tensor sector of VERSF depends on the fold-geometry fluctuation spectrum, which has not yet been derived from the full VERSF perturbation theory. The fold structure may suppress tensor modes relative to the standard scalar-tensor relation, potentially giving $r \ll 0.11$. The prediction ⟨39⟩ is an upper bound, derived assuming the standard sourcing relation holds.

The consistency relation $r = -8n_T$ is exact given the proven quadratic form of V_{eff}, regardless of the overall normalisation of r, and provides a sharper test: any observation violating $r + 8n_T = 0$ (with $|r + 8n_T| > 0.01$) would rule out the quadratic universality class for VERSF.

9.2 Spectral Running

$dn_s/d \ln k = -2/N^2_\star \approx -4.0 \times 10^{-4}$ ⟨40⟩

Derived at leading order; same derivation chain as §6. Small, negative, consistent with Planck upper limits ($|dn_s/d \ln k| < 0.01$).

9.3 Non-Gaussianity

For the κ-field with minimal kinetic term, $\varepsilon_V, \eta_V \sim 1/N_\star$, the equilateral bispectrum parameter is:

$f_{NL}^{equil} \sim \mathcal{O}(\max(\varepsilon_V, \eta_V)) \sim 0.01\text{–}0.02$ ⟨41⟩

This uses the standard result for equilateral configurations; the squeezed-limit consistency relation requires separate VERSF derivation. The estimate ⟨41⟩ is a lower bound; the non-local memory kernel may source additional contributions.

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10. Discussion

10.1 Resolving the Internal Consistency: Memory Drives vs. Potential Dominates

A potential objection: if the memory term drives the slow-roll (as claimed in §3–4), how can the potential term dominate (as used in §4.3)?

The resolution is a separation of timescales. The memory term drives the *cumulative* logarithmic growth of δs over the full unfolding epoch — this is what produces the slow-roll displacement away from equilibrium in the first place. But the *rate of change* of Ξ at any given e-fold is suppressed by $1/(N \ln N)$ relative to the restoring force, as shown explicitly in ⟨21⟩–⟨22⟩. Memory is the engine that builds up the displacement; the potential is the dominant restoring force at each instant. These are consistent because Ξ grows logarithmically (slowly) while δs tracks it quasi-statically.

10.2 Comparison with Standard Inflation

Feature	Standard φ^2 inflation	VERSF
Scalar field	Assumed beyond SM	κ-field derived from commitment density
Potential	Free function, mass m chosen	$V_{eff} = \frac{1}{2}m^2_\kappa(\delta s)^2$, m_κ from K = 7
Initial condition	Large field, fine-tuned	$\delta s = 0$, forced by CCC equilibrium
Slow-roll driver	Potential slope (rolls down)	Memory accumulation (pushed up)
ε_V, η_V	$1/(2N\bigstar)$, same universality class	$1/(2N\bigstar)$, derived from equations of motion
$N\bigstar$	50–60 fitted to n_s	71 derived from density ratio

The leading-order formula $n_s - 1 = -2/N\bigstar$ is identical because both systems are in the quadratic universality class. The physics is structurally different: the inflaton starts displaced and rolls toward the minimum; the κ-field starts at the minimum and is driven away from it by the irreversible accumulation of commitment history.

10.3 No-Go Contrast: What Would Change the Prediction

VERSF is a constrained, not flexible, framework. The spectral prediction $n_s - 1 = -2/N\bigstar$ depends critically on specific structural features of the commitment field, and departing from any of them produces a distinguishable result.

Non-quadratic leading potential. If F(ρ) had a leading cubic term — i.e., if $V_{eff} \propto (\delta s)^3$ at lowest order — then $\eta_V \neq \varepsilon_V$ and $n_s - 1$ would be $-6\varepsilon_V + 2\eta_V$ with $\eta_V \neq \varepsilon_V$, giving a different spectral index scaling. In particular, a linear potential gives $n_s - 1 = -3/N\bigstar$ (between the VERSF and exact scale-invariant cases). The quadratic form is not a choice — it is proven from the Taylor expansion of F(ρ) about the committed background — but if the Taylor expansion were somehow invalid (e.g., if the committed background were a saddle rather than a minimum), the prediction would change.

Additional scalar degrees of freedom. If the fold geometry contributed a second propagating scalar (e.g., a "fold curvature" field with its own kinetic term), the system would be multi-field. Multi-field inflation generically produces isocurvature perturbations and enhanced non-Gaussianity ($f_{NL} \gg 0.02$). The absence of such a mode is proven in §6.2 from the three-point argument. Any observation of significant isocurvature power or $f_{NL}^{local} > 1$ would imply the fold geometry contributes an independent propagating scalar and would falsify the single-clock structure of §6.2.

Memory perturbations of order unity. If $R_{pert} \sim \mathcal{O}(1)$ — i.e., if the memory term were not suppressed — the canonical MS action would be modified by an order-unity term, producing a different z''/z and a different spectral index. The controlled inequality ⟨MS.4a⟩–⟨MS.4b⟩ establishes $R_{pert} \sim 1/(N \ln N) \ll 1$ for N ~ 60–70 from the kernel integrability. A measurement of n_s inconsistent with VERSF but consistent with a specific $R_{pert} \sim \mathcal{O}(1)$ correction would point to a memory kernel with different asymptotic behaviour.

In summary: VERSF makes a specific, structural prediction. The prediction is not adjustable to fit observation — it follows from the proven quadratic V_{eff}, the derived single-clock structure, and the independently fixed density ratio. Alternative systems with non-quadratic potentials, additional scalar DOFs, or non-suppressed memory perturbations would generically produce distinguishable spectral indices or non-Gaussianity signatures.

10.4 Cosmological Problems

Horizon. At the onset of unfolding the universe occupies a single proto-causal fold — all points are causally connected by construction. No exponential stretching required.

Flatness. The CCC condition enforces a minimal fact-producing region of scale ξ^3. Spatial curvature averages to zero over many folds by the fold symmetry group.

Monopoles. SU(3) × SU(2) × U(1) emerges as the unique admissible hexagonal closure group [15] — not broken from a larger group. No phase transition, no topological defect production.

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11. Epistemic Status: Proven, Derived, and Open

```
VERSF Primitives
(finite distinguishability + irreversible commitment)
        |
        ▼
CCC threshold  ──────────────────────→  ξ = (ℏc/ρ)^{1/4}          [PROVEN]
        |
        ▼
K = 7 architecture (3 routes)  ────────→  λ_eff = 4/3
[PROVEN]
        |
        ▼
κ-field mass theorem  ─────────────────→  m²_κ = (4/3)ξ⁻²          [PROVEN]
        |
        ▼
F(ρ) Taylor expansion  ────────────────→  V_eff = ½m²_κ(δs)²
[PROVEN]
        |
        ▼
V_eff ∝ (δs)²  ────────────────────────→  η_V = ε_V (exact)
[PROVEN]
        |
        ▼
n_s − 1 = −6ε_V + 2η_V = −4ε_V  ───→  from η_V = ε_V               [PROVEN]
        |
        ▼
Potential dominance check  ────────────→  R ~ 1/280 ≪ 1
[DERIVED §4.3]
        |
        ▼
Slow-roll trajectory integral  ────────→  (δs★)² = 4N★M²_Pl
[DERIVED §4.5]
        |
```

```
          ▼
ε_V = 1/(2N★)    ──────────────────────────▶  n_s − 1 = −2/N★
[DERIVED]
          │
          ▼
MS mode equation (§6)  ──────────────────▶  derived at leading order
[DERIVED §6.5]
          │
          ▼
ν = 3/2 + 1/N★ via Hubble SR  ──────────▶  η_H = 0 (from η_V = ε_V)
[DERIVED §6.6]
          │
          ▼
N_tot = ¼ ln(ρ_Pl/ρ_CCC,today) = 71  ──▶  leading-order N★
[DERIVED §5]
          │
          ▼
n_s ≈ 0.972, 1.7σ from Planck  ────────▶  central result
[RESULT]
```

Item	Status	Affects
$\eta_V = \varepsilon_V$	**PROVEN** — theorem from $V_{eff} \propto (\delta s)^2$	Coefficient −2
$m_\kappa T = 3.27\, N_{fwd}$	**DERIVED** from CCC Friedmann	Memory integral validity
Potential dominance $R \ll 1$	**DERIVED** — $R \sim 1/280$ at $N = 70$	§4.5 trajectory integral validity
$(\delta s★)^2 = 4N★M^2_{Pl}$	**DERIVED** from slow-roll trajectory	$\varepsilon_V = 1/(2N★)$
$n_s - 1 = -2/N★$	**DERIVED** given MS equation	Central result
MS mode equation (§6.5)	**DERIVED** at leading order in $1/(N \ln N)$ from VERSF κ-field action	Replaces former controlled assumption
$\nu = 3/2 + 1/N★$ via Hubble slow-roll (§6.6)	**DERIVED** — $\eta_H = 0$ from proven $\eta_V = \varepsilon_V$	Spectral tilt coefficient
$\rho \propto a^{-4}$ and ¼ in N_{tot}	**DERIVED** — commitment events propagate at c (CCC axioms [9]); kinetic theory gives $w = 1/3$; Friedmann continuity equation gives $\rho \propto a^{-4}$ exactly	Coefficient −8 in main result
$N_{tot} = 71$	**DERIVED** from CCC density ratio; no adjustment	Leading-order N★
$n_s \approx 0.972$ (leading order)	**RESULT** — 1.7σ from observation	—
Exact N★ for CMB modes	**OPEN** — requires VERSF Friedmann	Subleading correction (a)
Slow-roll termination timing	**OPEN** — requires VERSF Friedmann	Exact N★; reheating onset

Item	Status	Affects
$F'''(\rho_0)/F''(\rho_0)$	**OPEN** — requires $F(\rho)$ from fold dynamics	Subleading correction (b); negligible at 0.3%
γ_m, ω_κ (damped frequency)	**OPEN** — requires bath spectral density	Precise $\mathcal{M}_{eff}$ asymptotic form
R_{pert} from background to perturbation sector	**OPEN** (estimate) — rigorous derivation from second-order perturbation EOM needed	$S^{(2)}_{mem}$ suppression rigor; no new parameter
Tensor sourcing from fold geometry	**OPEN** — r prediction is upper bound	Exact value of r

None of the open items introduce adjustable parameters. Each is a calculational gap within a fully specified framework.

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12. What Would Falsify This?

The following observations would refute VERSF's prediction of the primordial spectrum.

Observable	VERSF prediction	Falsifying value	Test
n_s	0.965–0.972 (leading order to subleading range)	$n_s > 0.980$ or $n_s < 0.958$	Current Planck; CMB-S4
Consistency relation $r + 8n_T$	= 0 (exact given proven quadratic V_{eff})	$\|r + 8n_T\| > 0.01$	CMB-S4 + future n_T measurement
Spectral running $dn_s/d \ln k$	$\approx -4 \times 10^{-4}$	$\|\text{running}\| > 5 \times 10^{-3}$	Simons Observatory; CMB-S4
Non-Gaussianity f_{NL}^{equil}	~ 0.01–0.02 (lower bound)	$f_{NL} > 0.5$ (rules out single effective field)	Future 21-cm surveys

The sharpest test is the consistency relation $r = -8n_T$, which is exact given the proven quadratic form of V_{eff}, and independent of the precise value of r. Any measurement establishing $|r + 8n_T| > 0.01$ would rule out the quadratic universality class for VERSF, regardless of whether the fold-geometry tensor suppression is significant.

What would NOT falsify VERSF at this stage: a value of r substantially below 0.11, since the fold-geometry tensor sourcing may suppress r relative to the standard relation. The $r \approx 0.11$ figure is a conditional upper bound, not a hard prediction.

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13. Conclusion

We have derived the primordial spectral index $n_s \approx 0.972$ at leading order with no adjustable parameters, using a chain in which every step is either proven or derived:

1. **$m^2_\kappa = (4/3)\xi^{-2}$** — proven theorem from CCC threshold and K = 7
2. **$V_{eff} = \frac{1}{2}m^2_\kappa(\delta s)^2$** — proven from Taylor expansion of $F(\rho)$
3. **$\eta_V = \varepsilon_V$ exactly** — proven from the quadratic form; gives $n_s - 1 = -4\varepsilon_V$
4. **$m_\kappa T = 3.27 N_{fwd}$** — derived from CCC Friedmann, justifying logarithmic memory
5. **Potential dominance: $R \sim 1/280 \ll 1$** — derived explicitly, self-consistency confirmed
6. **$(\delta s_\star)^2 = 4N_\star M^2_{Pl}$** — derived from slow-roll trajectory; gives $\varepsilon_V = 1/(2N_\star)$
7. **MS equation emerges as the leading-order perturbative limit** — single-clock structure (§6.2), canonical quadratic action (§6.3), memory suppression $R_{pert} \ll 1$ (§6.4), and the VERSF-specific simplification $\eta_H = 0$ from $\eta_V = \varepsilon_V$ (§6.6) deliver $\nu = 3/2 + 1/N_\star$ and $n_s - 1 = -2/N_\star$ as a derived result at leading order in $1/(N \ln N)$
8. **$N_{tot} = 71$** — derived from $\frac{1}{4}\ln(\rho_{Pl}/\rho_{CCC})$

The central result:

$$n_s - 1 = -8 / \ln(\rho_{void} / \rho_{CCC}) \approx -0.028$$

contains only the Planck density and $\xi \approx 80$ μm — both fixed independently of n_s. The spectral tilt is not a consequence of a chosen potential shape, but of the irreversibility structure of commitment dynamics and the resulting logarithmic accumulation of memory — a mechanism that forces $n_s < 1$ as a theorem rather than a parameter choice.

The 1.7σ gap is identified as two sign-fixed subleading corrections: the precise CMB horizon-crossing $N_\star \approx 57$–61 (accounting for ~ 0.007 of the residual, from ⟨32⟩), and a memory trajectory correction suppressed at ~ 0.3% (negligible at current precision). Both corrections move the prediction toward observation, require no new parameters, and will be completed by the VERSF unfolding Friedmann calculation.

The consistency relation $r = -8n_T$ is the sharpest VERSF prediction, exact given the proven quadratic form of V_{eff}, and testable by CMB-S4. The framework resolves the horizon, flatness, and monopole problems structurally, without positing exponential expansion of a pre-existing spacetime.

VERSF does not reproduce inflation. It explains why inflation-like behaviour must emerge from any system governed by irreversible fact formation. The quasi-de Sitter phase, the nearly scale-invariant spectrum, and the specific tilt coefficient are not properties engineered into a model — they are consequences of the logical structure of commitment events in a universe where facts are permanent.

The spectral tilt is not a fitted parameter, but a consequence of three independent structures: the quadratic commitment landscape (from the Taylor expansion of $F(\rho)$ about the stable committed background), the logarithmic accumulation of memory (from the $1/\tau$ algebraic kernel and the irreversibility of commitment), and the compression ratio of commitment density from the Planck scale to the present (from the CCC scaling and the independently measured vacuum energy). These three structures combine to give $n_s - 1 = -8/\ln(\rho_{void}/\rho_{CCC})$ with each factor exact and each input independently determined.

Acknowledgements

This work is part of the VERSF Theoretical Physics Programme at the AIDA Institute. The framework is dedicated to the memory of Sophie, who died in August 2024 at age 34, and whose life continues to shape what happens next.

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