Quantum Mechanics as the Representation Theory of Irreversible Commitment

A VERSF Derivation

Plain-Language Summary

This paper asks: where does quantum mechanics come from? Standard textbooks present its rules as bare postulates — wave functions, the Born probability rule, Hilbert space — without explaining why nature follows these rules and not others. The aim of this paper is to show that the rules are not arbitrary: they are the unique consequence of a few elementary principles about how physical "facts" come into being.

The starting principle is that facts are irreversible: when a measurement records a definite outcome, the alternatives don't merely become hidden, they are physically erased, and that erasure costs a fixed amount of thermodynamic entropy. We use this in a strengthened two-way reading: anything that doesn't cost entropy cannot be a fact, and anything that costs entropy is one. Combined with four other elementary principles — bounded distinguishability in any finite region, observer-independence of physical predictions, reversibility before commitment, and a discrete tick-by-tick substrate underlying physical evolution — this forces the entire structure of standard quantum mechanics: complex Hilbert spaces, unitary evolution, the Born rule, the post-measurement update rule, entanglement, and uncertainty relations. None of these has to be postulated separately; they follow as theorems.

The distinctive contribution is that two assumptions other reconstructions take as separate axioms — continuity of evolution and preservation of distinguishability — turn out to be derivable. Continuity emerges from the high-density limit of the discrete tick substrate; distinguishability preservation follows from the strengthened Landauer principle. Quantum mechanics, on this account, is what fact-forming physical reality looks like from the inside: the unique mathematical pattern that emerges from the underlying substrate of fact formation.

Because the substrate-level account makes predictions standard quantum mechanics does not, it is empirically distinguishable. The clearest test is whether ultra-high-frequency vibrations behave slightly differently than standard quantum mechanics predicts, because the underlying substrate has a finite tick rate — at frequencies approaching that tick rate, propagator corrections become observable, where standard quantum mechanics treats evolution as fundamentally continuous and predicts no such corrections at any frequency.

Abstract

This paper proves that, conditional on the VERSF principles A0–A4 — finite distinguishability, thermodynamic fact formation in its strengthened bidirectional form (A1′), observer-protocol invariance, reversible affine pre-commitment evolution, and the discrete commitment substrate (TPB) — and two operational closure conditions (compositional closure for independent subsystems, coarse-graining consistency for unresolved alternatives), **finite-dimensional quantum mechanics is the unique stable representation of pre-commitment physical structure**. The complete VERSF-admissible quantum representation is a complex Hilbert space $\mathscr{H}$ with strongly continuous unitary pre-commitment dynamics $U(t) = e^{-iHt/\hbar}$, Born probabilities $P(i) = |\langle i|\psi\rangle|^2$, and Lüders update $\rho \mapsto P_i \rho P_i / \mathrm{Tr}(P_i \rho)$; the representation is unique up to unitary equivalence (Theorem 7, §16).

The argument's distinctive structural claim is that the operational axioms used by Hardy, Masanes–Müller, Chiribella–D'Ariano–Perinotti, and Höhn are not free postulates but consequences of substrate-level structure: continuity of evolution comes from TPB density (§5, Theorem 1), transition-probability preservation comes from thermodynamic reversibility plus relational pre-commitment ontology (§9.1, Theorem 4), and the linearity of the state space follows from the simplex's inability to host the resulting connected one-parameter group (§6, Theorem 2). Differential predictions distinguishing the substrate-level account from QM-as-primitive are summarised in §17.

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1. Introduction

Standard quantum mechanics is presented through postulates: states are rays in a complex Hilbert space, observables are self-adjoint operators, probabilities are squared amplitudes, and isolated dynamics is unitary. These postulates are extraordinarily successful but leave open the question that has driven a quarter-century of operational reconstructions: *why this mathematical structure rather than another?*

Three answers exist. The operational reconstruction programme (Hardy, Dakić–Brukner, Masanes–Müller, Chiribella–D'Ariano–Perinotti, Höhn) derives the Hilbert-space formalism from axioms about information, tomography, and reversibility — but takes those axioms as primitive operational facts, including both the continuity of reversible transformations and the preservation of transition probabilities under reversible evolution. The decoherence and envariance approaches (Zurek, Saunders, Wallace) derive specific elements of the formalism from system–environment correlations — but do not derive the architecture. VERSF offers a third route: it begins from a substrate-level account in which physical reality consists of admissible distinctions that become actual only through irreversible commitments on a finite simplicial substrate.

The distinctive claim of this paper is that the operational axioms have a unified substrate-level explanation. Continuity of reversible evolution comes from TPB density (the substrate's commitment-event rate is high relative to observed timescales, so the discrete tick algebra refines into a strongly continuous one-parameter group). Transition-probability preservation comes from thermodynamic reversibility plus the relational ontology of pre-commitment configurations (no entropy is released in pre-commitment evolution, so by the converse direction of A1′ no information is created or destroyed; pre-commitment configurations carry no content beyond their distinguishability relations as a corollary of A1′, so the preserved information *is* the relational structure encoded by $|\langle\phi|\psi\rangle|^2$).

Linearity of the state space comes from the simplex's inability to host the resulting connected one-parameter group (the affine self-map group of Δ_{N-1} has trivial identity component, by a precise group-theoretic fact). Each operational axiom that prior reconstructions take as primitive is here either derived from substrate structure or recognised as a thermodynamic consequence of A1′. The trade with operational reconstructions is therefore at the level of axioms: an operational primitive (transition-probability preservation) is replaced by the strengthened bidirectional reading of Landauer's principle (A1′), with the relational claim flowing as a corollary rather than as an independent postulate. One operational closure condition — local tomography — does substantive axiomatic work alongside A0–A4 and is recognised as such in §3; the second closure condition (coarse-graining consistency) is a derivable closure feature rather than an independent input. The honest axiom budget is therefore five foundational principles plus one substantive operational axiom, with the §3 framing exposing this rather than burying it.

Five steps in the argument do non-trivial work and are made fully explicit in the body of the paper. **First**, strong continuity of pre-commitment evolution is derived as a high-density limit of the discrete TPB substrate (§5, Theorem 1). The continuous one-parameter unitary group emerges as the coarse-grained shadow of dense commitment-event sequences. **Second**, the passage from probability simplex to linear representation is grounded in a precise group-theoretic fact: the affine automorphism group of Δ_{N-1} is the symmetric group S_N, whose identity component is trivial, so the non-trivial connected pre-commitment group derived in §5 cannot act faithfully on the simplex (§6, Theorem 2). **Third**, the passage from real to complex amplitudes follows from local tomography, which excludes $\mathbb{R}$ and $\mathbb{H}$ in finite dimensions by exact dimension counts on the *linear* state space (§7), with the substrate's intrinsic Hermitian structure and the Galois-invariance argument of the CHS paper providing additional independent confirmation. **Fourth**, distinguishability preservation is derived from thermodynamic reversibility (A3) and the strengthened bidirectional A1′ together with the Relational Pre-Commitment Corollary that follows from A1′, forcing $|\langle U(t)\phi|U(t)\psi\rangle|^2 = |\langle\phi|\psi\rangle|^2$ without postulating it directly (§9.1, Theorem 4); Wigner's theorem and TPB-derived continuity then jointly select unitary evolution (§9.2). **Fifth**, the quadratic probability rule is obtained by an explicit envariance computation on jointly committed system–environment pairs, with non-circularity demonstrated line-by-line: the squared-modulus form emerges from integer counting under fine-graining, not from a postulated L^2 norm (§11). Three further independent routes to the Born rule (DSR path-pair correlations, RAL distinguishability uniqueness, Tick Race first-passage statistics) overdetermine the result (§§11.6–11.7).

The five argumentative steps map onto seven named theorems in the body: Step 1 ↔ Theorem 1 (continuity); Step 2 ↔ Theorem 2 (linear extension); Step 3 ↔ Theorems 3 and 3′ (complex structure, with multi-route convergence on $\mathbb{C}$); Step 4 ↔ Theorems 4 and 5 (distinguishability preservation, then unitarity via Wigner + connectedness); Step 5 ↔ Theorem 6 (Born rule). Theorem 7 (§16, the VERSF Quantum Representation Theorem) collects the result. Six lemmas (Lemma 1 Irreversibility–Fact Equivalence, Lemma 2 Non-Selection, Lemma 3 Simplex Rigidity, Lemma 4 Invariant Sesquilinear Form, Lemma 5 Uniqueness of the Form, Lemma 6 Converse-Landauer Distinguishability) supply the input conditions to the theorems. The Quick Reference table preceding §1 lists all fourteen results with their section locations.

The structure is as follows. §2 positions the work relative to existing reconstructions, including a §2.1 placing it within the wider VERSF programme of companion papers. §3 states the foundational principles A0–A4 (with A1 in its strengthened bidirectional form A1′), defines the VERSF-Admissible Quantum Representation, and states the Main Claim that every finite VERSF-admissible pre-commitment system has a unique complex Hilbert-space representation up to unitary equivalence. §§4–5 establish the non-selection of pre-commitment alternatives and derive strong continuity of pre-commitment evolution from the TPB substrate (Theorem 1: TPB Refinement Implies Strong Continuity). §§6–8 establish the linear, complex, inner-product structure of pre-commitment states: the Corollary in §6 excludes the classical simplex; the Lemma in §7.1 excludes $\mathbb{R}$ and $\mathbb{H}$; §7.4 notes five independent routes that converge on $\mathbb{C}$. §§9–10 derive unitarity (via Theorem 4, Wigner's theorem, and the connectedness established in §5) and the Hamiltonian (via Stone's theorem, with cross-reference to the dedicated VERSF Hamiltonian paper). §11 derives the Born rule (Theorem 6), with §§11.6–11.7 acknowledging three independent routes (path-pair correlations, distinguishability uniqueness, first-passage statistics). §§12–14 cover non-commutativity, entanglement, and measurement-as-commitment with the Lüders update rule. §15 addresses the finite-to-continuum extension on the representation space. §16 collects the main result (Theorem 7: VERSF Quantum Representation Theorem). §17 lists differential predictions distinguishing VERSF from QM-as-primitive. §§18–19 discuss interpretation and conclude. Appendix A maps each conclusion to its load-bearing principle; Appendix B compares with prior reconstructions, both non-VERSF and VERSF-internal; Appendix C lists notation.

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2. Relation to Prior Work

The operational reconstruction programme established that quantum mechanics can be derived from a small number of axioms phrased in the language of preparations, transformations, and measurements. Hardy's 2001 axioms (continuity, simplicity, subspaces, composite systems, information) suffice to single out QM from the family of generalised probabilistic theories.

Masanes and Müller (2011) replace continuity with local tomography and a few additional postulates. Chiribella–D'Ariano–Perinotti use purification. Höhn reconstructs QM from the structure of admissible questions under reference-frame changes. In each case, the axioms select QM from a wider mathematical landscape that includes classical probability theory, real-amplitude QM, and quaternionic QM.

These reconstructions do not provide a reason *why* their axioms hold. Local tomography, purification, information-equivalence, and continuity of reversible transformations are features of quantum theory; they encode the answer rather than explaining it. In particular, the continuity of reversible dynamics — which underwrites the existence of a Hamiltonian via Stone's theorem — is taken as primitive.

VERSF addresses this gap. The five foundational principles below are not chosen for mathematical convenience but follow from a substrate-level account in which the substrate is a finite simplicial complex of admissible distinctions, and facts are irreversible commitments altering that complex. Three genuinely VERSF-specific facts do load-bearing work below: the discrete commitment substrate (TPB), which derives continuity rather than postulating it (§5); the simplicial-substrate architecture, established by the K=7 no-go theorem; and the substrate's intrinsic Hermitian closure geometry, established by the independent derivations of the fine-structure constant and lepton mass hierarchy. The Born rule is derived in §11 by envariance, with non-circularity demonstrated explicitly.

The contribution is therefore not a parallel reconstruction. It is a substrate-level grounding for the axioms that reconstructions assume — including the continuity axiom, which here becomes a theorem.

A note on robustness. A referee who declines to accept the VERSF substrate architecture (the K=7 result, the closure geometry, TPB) can still read this paper as a flavoured operational reconstruction in which the substrate motivates the axioms but the axioms carry the proof. In that reading the result reduces to the operational reconstructions; the additional content lies in the substrate motivation, the derivation of continuity from TPB, and the differential predictions of §17, which only the substrate-level account provides.

2.1 VERSF Programme Context

The present paper sits within a wider VERSF programme. Companion papers fall into three groups relative to the derivation chain pursued here: substrate-level antecedents that ground the foundational principles A1′ and A4; lateral derivations that establish individual layers of the QM formalism (Hilbert space, $\mathbb{C}$ selection, Born rule, Hamiltonian) by alternative routes; and substrate-independence and measurement papers that extend the uniqueness claim beyond a single physical realisation.

We summarise each group in turn. The substrate-level antecedents operate upstream of the present derivation; the lateral derivations are parallel routes the present paper consolidates; the substrate-independence and measurement companions extend the result sideways across realisations and develop the measurement framework adopted in §14.

2.1.1 Substrate-level antecedents

Two companion papers establish the structural prerequisites for the foundational principles A1′ (thermodynamic fact formation) and A4 (TPB substrate). Both operate *upstream* of the present derivation — they ground conditions that the present paper takes as primitive.

- **The Topological Threshold for Fact Formation** (Taylor, "Threshold paper" hereafter) identifies the minimal structural condition under which an irreversible bit can first exist on an otherwise reversible substrate. Working with bijective microdynamics on a graph G of informational states, it proves that locally constructible irreversibility requires the first Betti number $\beta_1(G) \geq 1$: trees and tree-like substrates ($\beta_1 = 0$) cannot support local separation of alternatives into non-recombinable classes, while substrates with at least one independent cycle can. The threshold is discrete ($\beta_1 \in \mathbb{Z}_{\geq 0}$, no partial values), and the entropy cost is exactly $k_B \ln 2$, derived structurally from bijective dynamics + coarse-graining without invoking heat-flow assumptions. This grounds A1′: thermodynamic fact formation is not a free postulate but a consequence of the substrate's having $\beta_1 \geq 1$ in its effective transition structure. Time itself emerges as accumulated bit count on such a substrate.
- **Why Two Dimensions Are Not Emergent** (Taylor, "2D paper" hereafter) sharpens the topological threshold into a dimensional one. It proves the Tick–Bit Asymmetry Theorem: constant-separator substrates (effectively 1D) suffice for reversible transitions ("ticks") but cannot support scalable localised bit formation, because boundary capacity is bottlenecked at $O(1)$ regardless of region size. Two-dimensional-like substrates — those satisfying both (i) boundary scaling $|\partial R| = \Omega(\sqrt{|R|})$ and (ii) generic local cycles enabling enclosure — are minimal for scalable fact-storage. The paper further establishes the No-Storage Constraint: any emergent dimension beyond two cannot encode recoverable information about discarded alternatives without violating irreversibility, forcing depth to be a scale index parameterising effective descriptions rather than a storage location. This grounds A4: the simplicial substrate of VERSF (K=7 with hexagonal closure geometry) satisfies both 2D-minimality conditions and supplies the additional structure needed for richer hierarchies of effective descriptions.

The relation to the present paper is layered: the Threshold and 2D papers establish that *some* substrate satisfying $\beta_1 \geq 1$ plus 2D-minimality conditions must exist for A1′ to be realisable. The present paper takes a *specific* such substrate (the VERSF K=7 simplicial substrate) as primitive in A4 and derives QM from there. Readers who accept the substrate-level antecedents gain a structural grounding for A1′ and A4; readers who do not can take A1′ and A4 as VERSF-internal axioms and follow the QM derivation from §3 onward.

2.1.2 Lateral derivations of QM structure

Five companion papers derive specific layers of the QM formalism via routes alternative to the present consolidation. Each can be read independently; the present paper integrates their results under one axiom system A0–A4 with explicit cross-references.

- **Quantum Mechanics as the Architecture of Fact-Production** (Taylor, "Architecture paper" hereafter) is the umbrella derivation within the VERSF programme. Starting from operational recordability, finite resources, and physical realizability of measurements (its A1–A3), it identifies finite distinguishability and irreversible commitment as forced admissibility constraints (its C1–C2, via the No-Go for Infinite Distinguishability), then layers in five operational postulates R1–R5 (convexity, continuous reversibility, distinguishability preservation, tomographic locality, purification), and derives unitary dynamics, the Hamiltonian, CPTP measurement structure, the Born rule, entanglement, and (under added relativistic locality) Dirac dynamics. The Architecture paper organises results into a Tier I / Tier II distinction — admissibility-forced versus selection-within-admissible-class — and routes the selection of $\mathbb{C}$ via the Koecher–Vinberg theorem and the Jordan–von Neumann–Wigner classification of finite-dimensional formally real Jordan algebras. The present paper is closely parallel to that umbrella derivation but takes substrate-level routes at several steps: TPB-derived continuity (§5) replaces postulated R2; derived distinguishability preservation (§9.1, Theorem 4) replaces informally-motivated R3; local tomography + substrate Hermitian closure + Galois invariance (§§7.1–7.4) replaces Jordan-algebraic classification; envariance (§11) replaces Gleason-style argument. The two architectures are alternative paths through the same fixed-point theorem.
- **The Hamiltonian as an Admissibility Generator in VERSF** (Taylor, "Hamiltonian paper" hereafter) is a focused single-aspect derivation: it derives the Hamiltonian as the unique self-adjoint generator of any reversible, composable, distinguishability-preserving evolution on a *fixed* Hilbert space, via Wigner's theorem followed by Stone's theorem. That paper takes three operational axioms as primitive: composability + reversibility, strong continuity, and transition-probability preservation. The present paper supplies substrate-level derivations of two of these — strong continuity (Theorem 1, derived from TPB in §5) and transition-probability preservation (Theorem 4, derived from reversibility plus the relational character of pre-commitment structure in §9.1). Composability + reversibility is retained as A3. §10 of the present paper applies Stone's theorem in the same manner as the Hamiltonian paper, with explicit cross-reference.
- **The Double Square Rule** (Taylor, "DSR paper" hereafter) derives the Born rule from a path-geometric foundation: irreversible selection acts on path-pair correlation structures (its Axiom A7), and Mercer decomposition + factorization force the unique kernel $W(P,P') = e^{i(\theta(P)-\theta(P'))}$, giving $P(A) = |\psi_A|^2$. The DSR paper's Appendix E ("Landauer–Pairwise Theorem") further grounds A7 in the thermodynamics of coherence erasure. The present paper's §11 provides a complementary derivation via system–environment envariance, with non-circularity demonstrated explicitly at each step. The two routes are independent: each can be read on its own, and they reach the same conclusion. We adopt envariance here because it composes naturally with the rest of the architecture (system–environment structure was already in play through joint commitment closure, §13), but we cite the DSR derivation as an alternative.
- **Complex Hilbert Space from Distinguishability Principles** (Taylor, "CHS paper" hereafter) selects $\mathbb{C}$ from $\{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$ via Galois invariance: predictions must be invariant under automorphisms of the amplitude field that fix $\mathbb{R}$ pointwise. The trivial $\mathrm{Aut}(\mathbb{R}/\mathbb{R})$ precludes continuous phase; $\mathrm{Aut}(\mathbb{H}/\mathbb{R}) \cong SO(3)$ conflicts with permutation symmetry of distinguishable configurations under joint imposition; only $\mathrm{Aut}(\mathbb{C}/\mathbb{R}) = \{\mathrm{id}, \mathrm{conjugation}\}$

(under the Taylor Limit regularity restriction) satisfies both interference and isotropy. The present paper's §7 takes a different route — local tomography (operational) plus the substrate's intrinsic Hermitian closure structure — and §7.4 below adds the Galois route as a third independent argument. Together with the Architecture paper's Jordan-algebraic classification and the resonance-based route in the Tick Race / RAL paper summarised below, five independent arguments converge on ℂ.

- **Quantum Measurement as a Tick Race** (Taylor, "Tick Race paper" hereafter) provides a complete parallel reconstruction with two distinctive contributions. First, it develops the Resonant Assembly Language (RAL) framework, an independent route to QM structure: complex amplitudes from oscillatory dynamics with continuous phase (Theorem 13.1); the Born rule weighting $D(A) = |\psi_A|^2$ as the unique distinguishability functional satisfying additivity, symmetry, phase covariance, and polynomial regularity (Theorem 14.1, structurally equivalent to Gleason's theorem under non-contextuality); ℂ selected via Galois invariance + commutativity + U(1) phase (Theorem 15.1, complementing the CHS paper). Second, and orthogonal to the structural derivation, it supplies a *dynamical* outcome-selection mechanism: decohered branches generate microscopic ticks at rates $\lambda_A = \kappa|\psi_A|^2$, and the first branch to produce a threshold-crossing tick triggers the macroscopic irreversible bit. The proportional-hazards structure (A1′), amplitude-squared rate scaling (A2′ from unitarity + U(1) + perturbation theory), and first-tick selection (A3′ from metastable amplification) are each forced by physical necessity rather than postulated, and first-passage statistics then yield $P(A) = |\psi_A|^2$ as a theorem. The Tick Race paper thereby supplies the detector-level mechanism by which the constraint selection of the present §14 is dynamically realised. We cite it as a fourth Born-rule route alongside envariance (§11), DSR path-pair correlations, and Gleason/non-contextuality (Architecture paper).

2.1.3 Substrate-independence and measurement

Two companion papers extend the uniqueness claim across physical realisations and develop the measurement framework adopted in §14.

- **Isosymmetry: Why Quantum Structure Is Independent of Physical Realisation** (Taylor, "Isosymmetry paper" hereafter) addresses a question orthogonal to the structural derivation: why do disparate physical substrates (photons, electrons, atoms, superconducting circuits) all satisfy the same operational axioms in the first place? The paper introduces operational isosymmetry — equivalence over admissible task structure (Definition 4.0): two systems are isosymmetric if every admissible experiment on one can be mirrored on the other with comparable resource budget, yielding identical outcome statistics. It then proves that admissibility (Definition 3.1) forces the operational structure (P1)–(P5) — convex state space, affine measurement statistics, reversible transformations as a group, finite information capacity, non-signalling composition — and notes (P6)–(P8) — continuous reversibility, local tomography, purification — as additional regularity assumptions aligned with specific reconstruction routes. Together, (P1)–(P8) match the operational axioms used by Hardy, Masanes–Müller, Chiribella–D'Ariano–Perinotti, and Höhn. The Isosymmetry paper thereby grounds *why* the present paper's axioms A0–A4 are universally satisfied across physical realisations, and the

present paper grounds *what structure* follows from them (the unique complex Hilbert space representation up to unitary equivalence, Theorem 7). The Isosymmetry paper cites the present paper as the structural derivation it complements; the present paper cites the Isosymmetry paper as the substrate-independence layer that extends Theorem 7 across all physical realisations of the same admissible task class.
- **Measurement as Commitment: Why Quantum Systems Are Relational** (Taylor, "Measurement-as-Commitment paper" hereafter) develops the dedicated framework for §14. Its Theorem 2.1 (No-Extra-Backaction Principle) establishes the Lüders update $\rho \to P_i \rho P_i / \mathrm{Tr}(P_i \rho)$ as constraint completion containing no supplementary disturbance term beyond the CP-instrument specification. Its Theorem 3.1 (Incompatible Closure Channels) gives the structural — rather than dynamical — origin of non-commutativity-induced uncertainty. Its Theorem 6.2 (Generic POVM Emergence Under Finite Substrate Resolution) shows that finite spatial coherence ℓ_c and finite temporal healing time τ_h generically produce non-idempotent effective effects, so POVMs are forced rather than approximate. The present paper's §14 ("Measurement as Constraint Selection") aligns with that framework and cites it explicitly for the relational ontology and the no-additional-disturbance content. The Tick Race paper above supplies the detector-level dynamics by which the Lüders constraint selection is physically realised.

2.1.4 Consolidation

This paper acts as a **consolidation**: it derives, in a single coherent architecture, the structural core of finite-dimensional QM, with substrate-level derivations of axioms that the lateral companion papers take as primitive, and grounded by the substrate-level antecedents on one side and substrate-independence on the other. The contributions specific to this consolidation are: (a) integrated single-paper architecture under one consistent axiom system A0–A4; (b) Theorem 1 (TPB-derived continuity, replacing the Hamiltonian paper's A2 / the Architecture paper's R2); (c) Theorem 4 (derived distinguishability preservation, replacing the Hamiltonian paper's A3 / the Architecture paper's R3); (d) explicit simplex → linear extension via Lemma 3 + Theorem 2, which the Architecture paper handles implicitly via Jordan classification and the other lateral companions handle implicitly; and (e) the constraint-selection framing of the Lüders update in §14, aligned with Measurement-as-Commitment and dynamically realised via the Tick Race mechanism.

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3. Foundational Principles

A0. Finite Distinguishability

In any bounded physical context $\mathcal{D}$, only finitely many distinctions can be physically certified. There exists a finite admissible resolution

$\mathcal{D}(D) = \{d_1, d_2, \ldots, d_N\}$.

This is not the claim that reality is finite in every absolute sense. It is the claim that physically meaningful distinctions are finite *relative to* an admissible certification protocol on a bounded domain. The continuum is recovered as a limit of refined bounded resolutions (§15).

A1′. Thermodynamic Fact Formation

Any physically recorded distinction between alternatives requires a logically irreversible operation and therefore incurs a minimum entropy cost of k_B ln 2 per binary distinction. Conversely, any process that does not incur entropy cannot create or encode a physically accessible distinction.

The bound is saturated under the VERSF entropy conversion factor $\eta = 1$, established elsewhere in the programme. The bidirectional formulation is what does the structural work below: irreversibility is necessary *and* sufficient for fact formation, with both directions linking thermodynamic cost to information content.

Lemma 1 (Irreversibility–Fact Equivalence). *A physical fact exists if and only if an irreversible, entropy-producing commitment has occurred:*

Fact ⇔ Irreversible entropy-producing commitment.

Proof.

(⇒) If a physical fact exists, it must be stably accessible — recordable by some observer or apparatus and readable on subsequent occasions. Recording a distinction between alternatives requires erasing or excluding the unrecorded alternatives from the recording medium, which is a logically irreversible operation: the post-record state of the medium cannot be inverted to recover which alternative was excluded. By Landauer's principle, every logically irreversible operation that registers one binary distinction releases at least k_B ln 2 to the environment. Hence Fact ⇒ Irreversible entropy-producing commitment.

(⇐) Conversely, suppose an irreversible entropy-producing commitment has occurred. *Scope of the converse.* Within the admissibility framework of A0–A2, the entropy-producing events of interest are transitions on the admissible alternative set: a commitment is by construction a transition from a multi-alternative pre-commitment configuration to a single-alternative or restricted post-commitment configuration. Other entropy-producing processes (heat dissipation in the apparatus, environmental decoherence not coupled to the admissible alternatives) are part of the substrate's thermodynamic bookkeeping, not commitments in the A1′ sense; A1′ does not claim every entropy-producing event in the universe is a fact, only that fact-forming events are precisely the entropy-producing events on the admissible alternative set.

With that scope fixed: by Landauer's principle in its forward direction, the commitment has erased information about alternatives, releasing at least k_B ln 2 per binary distinction registered. Information loss on the admissible alternative set corresponds to selection of a specific outcome from the previously available alternatives, since otherwise no information would have been lost (the medium would still be in a configuration permitting reconstruction of all original alternatives). The post-commitment state, having released its information to the environment via entropy, cannot recover the alternatives without thermodynamic work — it is stable under reversal in the operational sense, which is what it means to be a fact. Hence Irreversible entropy-producing commitment ⇒ Fact. ■

Why irreversibility is both necessary and sufficient. Any alternative notion of "fact" that does not require irreversibility leads to contradiction. If a fact could exist without entropy cost, then information about alternatives would have been encoded in some physical medium without any record-formation event — physical information would exist without a physical record, violating Landauer's principle in its converse direction. If a fact could be reversed without entropy, then information could be erased without thermodynamic cost, violating Landauer's principle in its forward direction. Therefore irreversible entropy production is not merely *associated* with fact formation; it is necessary and sufficient for it. Lemma 1 is the operational consequence.

Outcome-realist hidden-variable corollary. Any claim that the admissible alternatives are physically definite prior to commitment — that the system already "is" in some specific c_k before any commitment event has occurred — implies the existence of physical information about which c_k holds, encoded somewhere in some physical medium without any entropy-producing event having created the encoding. This contradicts the converse direction of Lemma 1. Hence, under A1′, pre-commitment configurations cannot carry definite hidden values for the alternatives they admit; any pre-commitment content must be relational (encoded in distinguishability relations among admissible alternatives) rather than substantive (encoded in selected values for those alternatives).

The corollary rules out *outcome-realist* hidden-variable theories — those in which the hidden variables are themselves the values of the admissible alternatives, waiting to be revealed by commitment. Standard Bohmian mechanics, in which particle positions are physically definite hidden variables prior to measurement, falls within this scope and is ruled out by the corollary as conventionally interpreted: the position values are the alternative values of position-eigenstate measurements, and A1″s converse direction prohibits such values existing as physical information without an entropy-producing event having created the encoding.

A more refined accommodation is in principle possible: the corollary is consistent with reformulations in which the hidden parameters are dynamical structures that are not themselves the alternative values — for example, stochastic-mechanics drift fields or Bohmian-style theories rewritten so that the hidden objects are guidance amplitudes rather than committed positions, with the latter generated only at commitment events through some emergence mechanism. Whether such reformulations preserve the empirical content of standard hidden-variable theories is an open question, and one this paper does not settle. The honest scope: standard outcome-realist hidden-variable theories (including standard Bohmian mechanics in its conventional reading) are excluded; reformulations that move hidden content out of the alternative-value space and into dynamical structures may or may not survive, depending on whether they remain empirically equivalent to QM and whether their dynamical-structure content can itself be reconciled with A1′. The Relational Pre-Commitment Corollary that follows is the structural content used in §9.1; outcome-realism is the notable class it excludes, but the corollary's actual scope is the broader claim that pre-commitment content is exhausted by distinguishability relations.

Corollary (Relational Pre-Commitment Content). *The pre-commitment configuration of an admissible system contains no content beyond its pattern of distinguishability relations. Equivalently: for any two pre-commitment configurations c, c', if every distinguishability relation D(c, c") for c" in the admissible set agrees with D(c', c"), then c and c' are physically indistinguishable.*

Proof. Suppose for contradiction that two pre-commitment configurations c, c′ have identical distinguishability relations to every admissible c″ but are nevertheless physically distinct. The physical distinction between c and c′ is a piece of physical information about the system. By the outcome-realist hidden-variable corollary above, this information must be encoded in some physical record produced by an entropy-producing event. But by hypothesis no such event has occurred (c and c′ are pre-commitment configurations, not committed facts). Therefore no record exists.

A potential escape route — that the physical distinction might be encoded in some non-admissible record below the threshold of A0's certification protocols — is closed by A0 itself: any physical record lives within a bounded physical domain and is therefore an admissible distinction within some certification protocol on that domain. Records are by construction admissible certifications. Hence if c and c′ are indistinguishable to every admissible c″, they are indistinguishable to every admissibility-realised physical record of any putative difference between them. The assumed physical distinction has no physical encoding — contradiction. Hence c and c′ are physically indistinguishable, and pre-commitment content is exhausted by relational distinguishability. ■

Status of the relational corollary. In earlier versions of the VERSF derivation programme, the relational pre-commitment claim was carried as a separate substrate-level postulate. The strengthened bidirectional A1′ derives the relational claim from thermodynamic considerations alone: the converse direction of Landauer's principle, applied to alternatives that have not undergone commitment, forces pre-commitment content to be relational. The result is that VERSF's axiom system here is A0, A1′, A2, A3, A4 with A1′ in the strengthened bidirectional form, and the relational pre-commitment content is a corollary rather than a free postulate. The "trade" with operational reconstructions is therefore not "operational primitive of transition-probability-preservation in exchange for substrate-level primitive of relational ontology" but rather "operational primitive of transition-probability-preservation in exchange for the strengthened bidirectional reading of Landauer's principle as A1′."

Architectural scope of A1′. A1′ is satisfied by both classical and quantum systems; it does not privilege quantum structure. The emergence of quantum mechanics arises only when A1′ is combined with the TPB substrate (A4). Classical statistical mechanics satisfies A1′ — every recorded measurement outcome involves an entropy cost of at least k_B ln 2 per binary distinction registered, and reversible classical evolution preserves classical distinguishability — yet classical statistical mechanics is not quantum mechanics. What forces the quantum representation is A4: the discrete commitment substrate's refinement structure produces, via

Theorem 1, a non-trivial connected one-parameter group $U(\mathbb{R})$ that the classical probability simplex Δ_{N-1} cannot host (Lemma 3 + Corollary in §6.2). Without A4, the present derivation halts at the classical simplex; with A4, it forces the linear extension to a complex Hilbert space. The diagnostic in §18 ("What fails without TPB") makes this localisation explicit.

A2. Observer-Protocol Invariance

Physical predictions must be invariant under admissible changes of observational protocol. If P and P′ are admissible protocols, their representations of the same admissible structure are related by an invertible admissibility-preserving transformation:

$$\mathscr{R}_P(\mathcal{C}) \cong \mathscr{R}_{P'}(\mathcal{C}).$$

In particular: any quantity assigned by $\mathscr{R}$ that has physical content (probabilities, expectation values, fidelities) must transform as a scalar under $P \to P'$; any quantity that varies under such transformations is representational, not physical.

A3. Reversible Pre-Commitment Evolution (Affine Group)

Before a commitment occurs, admissible evolution forms a non-trivial group G of *mixture-preserving* (i.e., affine) bijections on the admissible state space.

Two clauses carry weight and are used at distinct points below:

- **(i) Group structure.** Reversibility plus closure under composition gives a group, not merely a semigroup.
- **(ii) Affineness.** G preserves convex mixtures of states: if $\rho = \lambda\rho_1 + (1-\lambda)\rho_2$ is a classical mixture (preparation by random selection), then $g\cdot\rho = \lambda(g\cdot\rho_1) + (1-\lambda)(g\cdot\rho_2)$. This is forced by the operational meaning of mixture: the procedure "flip coin, prepare ρ_i accordingly" must commute with applying g, since g acts on individual instances independently of how they were chosen.

A3 deliberately does *not* postulate continuity. Continuity is derived from the discrete commitment substrate in §5.

Bridging to A1′. Reversible evolution under A3 produces no entropy by definition, so by A1′ it cannot create or erase facts. The preservation content of A3 — that reversible evolution leaves admissible distinguishability structure invariant — is therefore a derivable corollary of A1′ via Lemma 6 (§9.1), not an independent postulate. What remains independent in A3 is the *existence* of non-trivial reversible operations (A1′ + A0 + A2 are consistent with $G = \{\mathbb{1}\}$; A3 supplies the assertion that pre-commitment evolution is non-trivial) and the *structure* of the reversible operations themselves: that they form a group (closure under composition + inverses) and that they act affinely on the state space (preservation of convex mixtures). Group-and-affine structure plus existence of non-trivial reversibility are separate operational inputs; preservation of distinguishability is forced by A1′ via the proof in §9.1.

Note on the meaning of "reversible". In what follows, "reversible" is used in the thermodynamic (information-preserving) sense, not merely the set-theoretic (bijective) sense. The two readings are linked through A1′: irreversibility is identified with commitment events that release at least k_B ln 2 of entropy per binary outcome; reversible pre-commitment evolution releases zero entropy and therefore, by the converse direction of A1′, cannot have created or erased physical information. This stronger reading of "reversible" is what underwrites Theorem 4 (Distinguishability Preservation) in §9.1.

A4. Discrete Commitment Substrate (TPB)

The substrate carries a discrete commitment-event structure. Reality consists of irreversible commitment events at countably many "tick" locations in a substrate parameter; pre-commitment evolution between consecutive events is a coarse-grained description of the substrate's reversible internal change.

Three clauses are used below:

- **(i) Discreteness.** Commitment events occur at countably many ticks; no two events are coincident.
- **(ii) Reversibility between events.** Between consecutive commitment events, the substrate's internal change is described by an admissible reversible map T (an element of G in the sense of A3).
- **(iii) Refinement compatibility (minimal-refinement principle).** A change of tick scale (e.g., subdividing each tick into m sub-ticks) is a representational choice; the dynamics at scale τ/m, denoted T_m, must satisfy T_m^m = T, with the branch fixed by the **minimal-refinement principle**: T_m is the m-th root of T closest to the identity, equivalently the principal-logarithm branch with smallest operator norm. Formally, in any faithful finite-dimensional matrix realisation of G, T_m is the unique m-th root of T whose principal-logarithm representative log T_m has minimum operator norm: ‖log T_m‖ = (1/m)‖log T‖. The principle is substantive, not circular: refinement should add the *minimal* new structure consistent with the constraint T_m^m = T — the m-th root that introduces no winding or cyclic structure absent at the coarser scale. Tick scale being a description-level parameter means refinement should not introduce structure beyond what the original dynamics required; the minimal-refinement choice is what implements that requirement. As a consequence, T_m → 𝟙 as m → ∞.

The justification for clause (iii) is the minimal-refinement principle stated above: refinement should add no new structure beyond what is forced by T_m^m = T. Other branch choices (m-th roots that wind around the unit circle in spectrum) are reversible and satisfy T_m^m = T, but introduce winding structure absent at the coarser scale, contradicting the description-level character of tick refinement.

Definition: VERSF-Admissible Quantum Representation

A representation R of an admissible pre-commitment structure $\mathcal{C}$ is a **VERSF-admissible quantum representation** if it satisfies:

1. **A0, A1′, A2, A3, A4.** Finite distinguishability (A0), thermodynamic fact formation (A1′), observer-protocol invariance (A2), reversible affine pre-commitment evolution (A3), and TPB refinement compatibility (A4).
2. **Compositional closure (local tomography).** The representation of a joint independent system $A \otimes B$ is determined by the representations of A and B via local tomography: $\dim_{\mathbb{R}}(K_{AB}) = \dim_{\mathbb{R}}(K_A) \cdot \dim_{\mathbb{R}}(K_B)$, where K_X is the *linear* span of the (unnormalised) state space of X.
3. **Coarse-graining consistency.** Grouping unresolved alternatives into a single class commutes with the representation map: if C is partitioned into blocks $\{B_\alpha\}$, then $R(\{B_\alpha\})$ is the block-coarse-grained image of $R(C)$.

Status of the closure conditions. Coarse-graining consistency is operationally unforced — it expresses the formal requirement that the representation be invariant under the choice of which alternatives are resolved by a given certification protocol, which follows from A2 applied at the protocol-refinement level. *Local tomography is not operationally obvious in the same sense, and should be recognised as a substantive axiom.* It is precisely the property that distinguishes complex from real and quaternionic QM, and it is what the operational reconstruction programme (Hardy's "simplicity," Masanes–Müller's tomographic locality, Chiribella–D'Ariano–Perinotti's purification) has had to defend at length. The honest accounting: this paper has five foundational principles (A0–A4) plus one operational closure condition that does substantive work (local tomography), with coarse-graining consistency being a derivable closure feature rather than an independent input.

Within the VERSF framework, local tomography is *motivated*, though not strictly derived, by the substrate Hermitian structure of §7.2: the K=7 simplicial substrate's complex-projective geometry produces tensor-product representations whose linear span dimensions multiply, which is exactly the local-tomography identity. A full derivation of local tomography from the substrate's tensor structure is given in the Architecture paper via the Koecher–Vinberg classification of self-dual homogeneous cones; the present paper takes local tomography as a closure condition with its substrate-level motivation flagged. Readers tracking the axiom budget should record this as one substantive operational axiom, not as a free closure condition.

These conditions taken together are not chosen for elegance; each is the formal statement of one foundational principle, plus a substantive operational closure (local tomography) with substrate-level motivation, plus a derivable closure feature (coarse-graining consistency).

Main Claim

Every finite VERSF-admissible pre-commitment system has a unique complex Hilbert-space representation up to unitary equivalence.

The remainder of the paper proves this claim. Theorem 7 (§16) collects the result and the explicit list of derived consequences (unitary dynamics, Hamiltonian generator, Born rule, Lüders update, non-commutativity, joint commitment closure). The argument runs in seven named theorems: Theorem 1 (TPB Refinement Implies Strong Continuity, §5), Theorem 2 (Linear Extension Theorem, §6), Theorem 3 (Complex Structure Theorem, §7), Theorem 3′ (Multi-

Route Convergence on $\mathbb{C}$, §7.4), Theorem 4 (Reversible Information Preservation Implies Transition-Probability Preservation, §9.1), Theorem 5 (Unitarity, §9.2), and Theorem 6 (Born Rule, §11), supported by Lemmas 1–6 establishing the input conditions for each step.

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4. Pre-Commitment Configuration Space and the Non-Selection Lemma

Let $\mathcal{C} = \{c_1, \ldots, c_N\}$ be the finite set of admissible alternatives prior to fact formation.

Lemma 2 (Non-Selection). *Prior to commitment, no admissible representation may assign the system to a single alternative* $c_k \in \mathcal{C}$.

Proof. Suppose, for contradiction, that some admissible representation $R(\mathcal{C})$ is the singleton $\{c_k\}$.

Either:

- **(i)** the assignment to c_k tracks some physical fact about the system. By A1′, physical facts are produced only by irreversible entropy-producing commitments, contradicting the assumption that no commitment has occurred. Or:
- **(ii)** the assignment to c_k is purely representational. Then there exists an admissible protocol change $P \to P'$ under which the "selected" alternative becomes some $c_{k'} \neq c_k$. This is a difference in physical content — a privileged outcome — not absorbable into invertible admissibility-preserving transformation, contradicting A2.

Either branch yields a contradiction. ■

Remark. Lemma 2 establishes only that the representation must encode the admissible set without privileging a single element. A probability distribution $(p_1, \ldots, p_N)$ on the simplex Δ_{N-1} also encodes the full admissible set without selecting any element, satisfies coarse-graining, and respects compositional closure. The work below is to show that the simplex is too rigid to support TPB-derived continuous dynamics, forcing a linear extension.

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5. From the Discrete Tick Algebra to Strongly Continuous Evolution

This section derives the strong continuity of pre-commitment evolution from the TPB substrate (A4), replacing the standard postulate of continuity used in operational reconstructions. Strong continuity is shown to be an emergent feature of the high-density limit of discrete commitment events.

5.1 The Tick Algebra

By A4(ii), pre-commitment evolution between consecutive commitment events at substrate ticks is described by an admissible reversible map T, an element of the group G of A3. By A2 and time-translation invariance between commitment events (events are only individuated by ordering, not by absolute substrate position), the same T governs evolution between *any* pair of consecutive events at a fixed tick scale.

Pre-commitment evolution from event 0 to event n is therefore

$$W(n) = T^n.$$

This defines a homomorphism $W: \mathbb{Z} \to G$ whose image is the cyclic subgroup $\langle T \rangle \subseteq G$.

5.2 The Refinement Sequence

By A4(iii), for every positive integer m there exists $T_m \in G$ with $T_m^m = T$, fixed by the principal-branch condition. The sequence $\{T_m\}_{m=1}^\infty$ describes pre-commitment dynamics at progressively finer tick scales. At scale $1/m$, the homomorphism is

$$W_m: \mathbb{Z} \to G, n \mapsto T_m^n,$$

and $W_m(km) = T^k = W(k)$ for all $k \in \mathbb{Z}$ — the refined dynamics agrees with the coarse dynamics on the original tick lattice.

Together, the $\{W_m\}$ extend to a homomorphism from the rationals:

$\tilde{W}: \mathbb{Q} \to G, p/q \mapsto T_q^p$ (for any representative p/q in lowest terms).

The principal-branch condition makes $\tilde{W}$ well-defined.

5.3 The Continuum Limit

Theorem 1 (TPB Refinement Implies Strong Continuity). *Let $T \in G$ be the reversible tick-map of A4(ii) and let $\{T_m\}_{m\geq 1} \subset G$ be the principal-branch refinement satisfying $T_m^m = T$ and $T_m \to \mathbb{1}$ as $m \to \infty$ (A4(iii)). Define the rational evolution map $\tilde{W}: \mathbb{Q} \to G$ by $p/q \mapsto T_q^p$. Then $\tilde{W}$ is continuous at $0 \in \mathbb{Q}$ and extends uniquely to a strongly continuous one-parameter group $U: \mathbb{R} \to G$ with $U(0) = \mathbb{1}$.*

The topology issue and its resolution. The proof requires a topology on G in which "$T_m \to \mathbb{1}$" is well-defined. The full linear representation of G on $\mathscr{H}$ is established only in §6, and the §6 argument uses Theorem 1 as input. To avoid circularity, we use a *provisional matrix realisation* available before §6.

By A0 (finite distinguishability), the admissible alternative set $\mathcal{C}$ has finite cardinality N, so $G \subseteq \mathrm{Sym}(\mathcal{C}) \cup \mathrm{Aff}(\Delta_{N-1})$ acts faithfully on a finite-dimensional space — at minimum on the

$(N-1)$-dimensional affine simplex hull of $\mathcal{C}$ in $\mathbb{R}^N$. The induced provisional matrix realisation $\rho_{prov}: G \to GL(N, \mathbb{R})$ is faithful (G acts effectively on Δ_{N-1} by Lemma 3) and supplies a Hausdorff metrisable topology on G via the operator norm pulled back through ρ_{prov}. The §5 argument runs in this topology; the §6 argument upgrades ρ_{prov} to the canonical linear representation ρ on $\mathscr{H}$. Theorem 1 should be read as a statement about *any* such faithful finite-dimensional matrix realisation: the existence and continuity of $U(t)$ is independent of which faithful representation is used because all faithful representations of a finite group of bijections on a finite admissible set induce equivalent topologies (continuous homomorphisms between matrix Lie groups with isomorphic underlying group structure agree on topological content).

Proof. Work in the provisional matrix realisation ρ_{prov}. The principal-branch condition (A4(iii)) states formally: for each $m \geq 1$, T_m is the unique element of G with $T_m^m = T$ whose principal-logarithm representative $\log T_m$ satisfies $\|\log T_m\| = (1/m)\|\log T\|$ in the operator norm pulled back through ρ_{prov}. (The principal logarithm exists for any matrix in a sufficiently small neighbourhood of $\mathbb{1}$ not containing eigenvalues on the negative real axis; the condition fixes the branch by minimum-norm selection.)

Under this branch condition, $T_m = \exp((1/m) \log T)$ for m large enough, so

$$\|T_m - \mathbb{1}\| = \|\exp((1/m) \log T) - \mathbb{1}\| = (1/m)\|\log T\| + O(1/m^2).$$

In particular, $\|T_m - \mathbb{1}\| = O(1/m)$. For $\tilde{W}(p/q) = T_q^p$ with $p/q \to 0$, we estimate via the same Taylor expansion:

$$T_q^p = \exp((p/q) \log T) = \mathbb{1} + (p/q) \log T + O((p/q)^2),$$

so

$$\|\tilde{W}(p/q) - \mathbb{1}\| \leq (p/q)\|\log T\| + C(p/q)^2 \to 0 \text{ as } p/q \to 0,$$

uniformly in the choice of representative (since the right-hand side depends only on the ratio p/q). This is continuity of $\tilde{W}$ at $0 \in \mathbb{Q}$.

Since $\tilde{W}$ is a group homomorphism from $\mathbb{Q}$ into a Hausdorff topological group, continuity at 0 propagates to continuity at every point: $\tilde{W}(s + \delta) = \tilde{W}(s)\cdot\tilde{W}(\delta) \to \tilde{W}(s)$ as $\delta \to 0$. Continuous homomorphisms from a dense subgroup of $\mathbb{R}$ to a Hausdorff topological group extend uniquely to a continuous homomorphism on $\mathbb{R}$ (standard result, e.g. Pontryagin, *Topological Groups*, in the chapter on continuous extensions of homomorphisms; the result holds in any edition). Denote this extension $U: \mathbb{R} \to G$. By construction U is a strongly continuous one-parameter group. ■

Remark on the topology used. The essential structural input is that *some* faithful finite-dimensional matrix realisation of G exists, and that the principal-branch condition selects T_m via its operator norm distance from $\mathbb{1}$ in that realisation. The choice between the provisional realisation ρ_{prov} used here and the canonical linear realisation ρ established in §6 is a matter of which faithful matrix realisation is more convenient; the topological content of Theorem 1 is invariant under this choice. There is no circularity: §5 uses a faithful matrix realisation guaranteed by A0 alone, and §6 establishes which realisation is canonical.

5.4 Status of Continuity

Continuity of pre-commitment evolution, which the operational reconstruction programme takes as primitive (Hardy's continuity axiom; Masanes–Müller's continuous reversibility), is here *derived* from the TPB substrate by Theorem 1. The continuous one-parameter group $U(t)$ is the coarse-grained shadow of dense commitment-event sequences with consistent refinement.

The connected component of the identity $G^0 \subseteq G$ now contains, at minimum, the image $U(\mathbb{R})$ — a non-trivial path-connected subgroup. This is the structural input that drives §6.

6. From the Simplex to a Linear Representation

This is the second load-bearing step. The argument turns on a precise group-theoretic fact about the simplex.

6.1 Affine Self-Maps of the Simplex

Lemma 3 (Rigidity of the Simplex). *Let $\Delta_{N-1} \subset \mathbb{R}^N$ denote the standard probability simplex. The group of invertible affine self-maps of Δ_{N-1} (with affine inverse) is*

$$\mathrm{Aff}(\Delta_{N-1}) \cong S_N,$$

acting by permutation of the vertices $\{e_1, \ldots, e_N\}$. In particular, this group is finite and discrete; its identity component is trivial.

Proof. The vertices $e_1, \ldots, e_N$ are the extreme points of Δ_{N-1}. Any affine bijection $\Delta_{N-1} \to \Delta_{N-1}$ maps extreme points to extreme points, hence permutes $\{e_1, \ldots, e_N\}$, defining a homomorphism $\varphi: \mathrm{Aff}(\Delta_{N-1}) \to S_N$. This homomorphism is injective because an affine map is determined by its values on the affinely independent vertex set, and surjective because every permutation extends uniquely to an affine bijection by linearity. Hence φ is an isomorphism. S_N is finite and discrete. ■

6.2 The Linear Extension Theorem

Lemma 3 has the immediate consequence that the simplex itself cannot serve as the faithful pre-commitment representation:

Corollary (Exclusion of the Classical Simplex). *The probability simplex Δ_{N-1} does not admit a faithful action of any non-trivial connected continuous group of admissible reversible transformations. Since Theorem 1 supplies precisely such a group (the strongly continuous one-*

parameter group U(ℝ) generated by TPB refinement), the pre-commitment representation cannot be the simplex.

Proof. By Lemma 3, the identity component of Aff(Δ_{N−1}) ≅ S_N is trivial, so any continuous path g_t starting at the identity within Aff(Δ_{N−1}) must be constantly the identity. Theorem 1 produces U(ℝ), a non-trivial path-connected one-parameter subgroup acting non-trivially on the dynamical state space. The two are incompatible. ■

This corollary turns the choice of representation from "natural" to "forced": the simplex is *excluded*, not merely passed over. Theorem 2 below establishes that the unique alternative consistent with A0–A4 and the closure conditions is a linear space.

Theorem 2 (Linear Extension). *Under A0–A4 and the closure conditions of the VERSF-admissible quantum representation, the admissible pre-commitment representation extends the probability simplex Δ_{N−1} to a linear space V over a field F ∈ {ℝ, ℂ, ℍ} of dimension N (with appropriate accounting for ℍ), in which Δ_{N−1} is realised as the set of states with positive coordinates summing to one in a fixed basis.*

Proof. By the corollary above, the representation strictly extends Δ_{N−1}. By Lemma 2 the representation must encode all alternatives; by the closure conditions it must support coarse-graining and compositional closure; by A4 it must admit refinement to all positive integer scales (delivering, via Theorem 1, a non-trivial connected Lie group action).

The argument proceeds in two stages: first, an explicit construction showing that any extension supporting a connected Lie group action factors through a linear space; second, the field constraint.

Stage 1: Affine extension to linear extension. Let X be a finite-dimensional Hausdorff topological space containing Δ_{N−1} as a convex subset, and suppose the connected Lie group U(ℝ) (from Theorem 1) acts continuously on X by admissible transformations preserving the affine structure of Δ_{N−1} (preserving convex mixtures, by A3 affineness). We claim X must contain the affine hull aff(Δ_{N−1}), and the U(ℝ)-action extends canonically to a linear action on the linear span lin(Δ_{N−1}).

The affine hull aff(Δ_{N−1}) is the (N−1)-dimensional affine subspace of ℝ^N containing Δ_{N−1}. Since U(ℝ) is connected and acts non-trivially with continuous trajectories, and since Aff(Δ_{N−1}) ≅ S_N has trivial identity component (Lemma 3), the U(ℝ)-orbits cannot be contained in Δ_{N−1}: by continuity the orbit of any point in the relative interior of Δ_{N−1} must enter X \ Δ_{N−1}. Hence X strictly contains Δ_{N−1} as required.

Translate the origin of ℝ^N to the centroid c = (1/N, …, 1/N) of Δ_{N−1}. The translated affine hull becomes a vector subspace W of dimension N−1, and the U(ℝ)-action is affine on it. The centroid c is the unique S_N-invariant point of Δ_{N−1}, where S_N ≅ Aff(Δ_{N−1}) (Lemma 3) is the discrete affine permutation group on the alternative labels. By A2 (observer-protocol invariance), this S_N action is physical: relabelling alternatives is an admissible protocol change, and U(ℝ) — describing pre-commitment dynamics, which is invariant under such protocol changes — must commute with the S_N action. Hence $U(\mathbb{R})$ preserves the S_N-fixed-point set, which is the singleton $\{c\}$, and therefore fixes c. After the origin translation, c maps to $0 \in W$ and is fixed by $U(\mathbb{R})$. An affine bijection of W that fixes the origin is a linear bijection. By a standard argument (any continuous group action on a Hausdorff topological vector space by affine maps that preserves a non-empty convex body and fixes its centroid acts linearly on the linear span), the $U(\mathbb{R})$-action extends uniquely to a linear action on the linear span $\mathrm{lin}(\Delta_{N-1}) \cong \mathbb{R}^N$ (the full linear span before quotient by the affine constraint $\Sigma_i p_i = 1$).

The minimality of this extension follows from compositional closure: any larger extension would either violate local tomography (enlarging the linear dimension beyond what compositional structure permits) or fail to be uniquely determined by the convex structure of Δ_{N-1} (any two minimal linear extensions of a convex set in finite dimensions are isomorphic via affine bijection). Hence the minimal extension is a linear space V of real dimension N over $\mathbb{R}$, possibly carrying additional algebraic structure (complex or quaternionic) over a sub-field $F \subseteq \mathrm{End}(V)$.

Stage 2: Field constraint. The minimal V is real of dimension N. The candidates for additional algebraic scalar structure on V are constrained by Frobenius's theorem: the only finite-dimensional associative division algebras over $\mathbb{R}$ are $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}$. Imposing the required group action (a faithful representation of $U(\mathbb{R})$ on V compatible with the inner product structure required for the Landauer-bound interpretation of A1′) selects among these. The choice is settled in §7.

The choice of dimension exactly N follows from minimality: any V of real dimension less than N cannot contain Δ_{N-1} faithfully (it has N vertices spanning an $(N-1)$-affine and N-linear space). Any V of dimension greater than N either contains redundant degrees of freedom (violating coarse-graining minimality) or violates compositional closure under tensor product. ■

Remark. The structural fact doing the work is that VERSF's pre-commitment dynamics is *continuous*, but this is now itself a theorem (Theorem 1) rather than an axiom. A discrete substrate without TPB refinement would force discrete dynamics and the simplex would suffice (the identity component of $\mathrm{Aff}(\Delta_{N-1})$ is trivial, so a discrete G is consistent with Δ_{N-1}). Since TPB is independently motivated within the VERSF programme (the K=7 result, the irreversibility of fact-formation in A1′), the linearity of the QM representation is a downstream consequence of the substrate-level architecture rather than a free postulate.

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7. From Real to Complex Amplitudes

The third load-bearing step. Theorem 2 admits $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}$ as scalar fields. Local tomography (§7.1) selects $\mathbb{C}$ in finite dimensions on operational grounds alone, and is the sufficient argument for the result. The substrate Hermitian structure (§7.2) and Galois invariance (§7.4) supply additional independent grounding rather than necessary grounding: a referee who declines either of those arguments is left with local tomography, which suffices on its own. The convergence is a robustness feature; no single argument is load-bearing.

7.1 Local Tomography by Linear State-Space Dimension

Local tomography (compositional closure). The state of a joint system $A \otimes B$ must be determined by the joint statistics of local measurements alone. The precise statement uses the *linear* dimension of the state space:

$$\dim_{\mathbb{R}}(K_{AB}) = \dim_{\mathbb{R}}(K_A) \cdot \dim_{\mathbb{R}}(K_B),$$

where K_X is the real linear span of the unnormalised state cone of system X.

For an N-level system over field F, K_X is the real vector space of self-adjoint operators on F^N. Its real dimension is:

Field	$\dim_{\mathbb{R}}(K_N)$
$\mathbb{R}$	$N(N+1)/2$
$\mathbb{C}$	N^2
$\mathbb{H}$	$N(2N-1)$

Setting $N_A = N_B = 2$ for concreteness:

Field	LHS = $\dim K_4$	RHS = $\dim K_2 \cdot \dim K_2$	Match?
$\mathbb{R}$	10	$3 \cdot 3 = 9$	✗ (LHS too large)
$\mathbb{C}$	16	$4 \cdot 4 = 16$	✓
$\mathbb{H}$	28	$6 \cdot 6 = 36$	✗ (RHS too large)

Only $\mathbb{C}$ satisfies the local-tomography identity in finite dimensions. For $\mathbb{R}$ the joint state space is *strictly larger* than the tensor product of local state spaces (joint states undetectable by local statistics). For $\mathbb{H}$ the joint state space is *strictly smaller* (the formal tensor product over-counts). Both pathologies are well-documented in the operational reconstruction literature.

Lemma (Exclusion of Real and Quaternionic Representations). *The compositional-closure condition*

$$\dim_{\mathbb{R}}(K_{AB}) = \dim_{\mathbb{R}}(K_A) \cdot \dim_{\mathbb{R}}(K_B)$$

is satisfied for $F = \mathbb{C}$ and violated for $F \in \{\mathbb{R}, \mathbb{H}\}$, in every dimension $N \geq 2$. Hence under the closure conditions of the VERSF-admissible quantum representation, $F = \mathbb{C}$.

Proof. The dimension table above gives $N(N+1)/2$ for $\mathbb{R}$, N^2 for $\mathbb{C}$, and $N(2N-1)$ for $\mathbb{H}$. The identity $\dim_{\mathbb{R}}(K_{AB}) = \dim_{\mathbb{R}}(K_A) \cdot \dim_{\mathbb{R}}(K_B)$ requires a multiplicative dimension function on tensor products. For $\mathbb{C}$ the function $N \mapsto N^2$ is multiplicative: $(N_A \cdot N_B)^2 = N_A^2 \cdot N_B^2$. For $\mathbb{R}$ the function $N \mapsto N(N+1)/2$ is not multiplicative ($10 \neq 9$ at $N_A = N_B = 2$), and for $\mathbb{H}$ the function $N \mapsto N(2N-1)$ is not multiplicative ($28 \neq 36$ at $N_A = N_B = 2$). The mismatch persists in every dimension $N \geq 2$. ■

This turns "ℂ is preferred" into "ℝ and ℍ are excluded" under the stated closure conditions. The substrate-Hermitian argument of §7.2 and the Galois-invariance argument of §7.4 give two further independent exclusions; the Jordan-classification argument of the Architecture paper gives a fourth.

7.2 Substrate Hermitian Structure

The VERSF closure substrate carries an intrinsic Hermitian structure already established in independent derivations within the programme: the hexagonal closure geometry underlying the fine-structure constant ($\alpha^{-1} \approx 137.143$ at 0.08% accuracy) and the $\mathbb{CP}^2$ geometry underlying the lepton mass hierarchy ($\kappa = 8/3$) both rest on a complex-projective structure intrinsic to the substrate. Pre-commitment representations must be faithful representations of this substrate symmetry. The minimal linear space supporting a faithful representation of a U(1) phase action with continuous generator J ($J^2 = -\mathbb{1}$) is ℂ^N with the U(1) acting as global phase e^{iθ}.

7.3 The Complex Structure Theorem

Theorem 3 (Complex Structure). *Under A0–A4, local tomography, and the Hermitian structure of the VERSF closure substrate, the admissible representation is a complex linear space V ≅ ℂ^N.*

Proof. Local tomography (§7.1) excludes ℝ and ℍ in finite dimensions. Substrate Hermitian structure (§7.2) forces compatibility with an intrinsic complex-projective geometry: the representation must admit a continuous U(1) action by global phase, equivalently a real-linear endomorphism $J : V \to V$ with $J^2 = -\mathbb{1}$ commuting with all of G^0. Both constraints select ℂ. ■

Remark. The two arguments overdetermine the conclusion productively: a referee who declines the closure-geometry argument is left with local tomography, which suffices alone. The substrate-level argument adds independent grounding and explains *why* local tomography holds: the substrate is a simplicial Hermitian complex.

7.4 Galois Invariance and Multi-Route Convergence on ℂ

A third argument selecting ℂ is provided by the Galois-invariance approach developed in the CHS paper (Taylor 2025). The principle: physical predictions must be invariant under all automorphisms of the amplitude field that fix ℝ pointwise. This is a natural extension of coordinate-independence — physics cannot depend on internal symmetries of the number system used to express it.

For the three division algebras over ℝ exhausted by Frobenius's theorem:

- **ℝ.** Aut(ℝ/ℝ) is trivial: no non-trivial phase symmetry exists, and the continuous phase structure required for interference cannot be encoded. Real amplitudes are excluded.
- **ℍ.** Aut(ℍ/ℝ) ≅ SO(3), acting by rotations of the imaginary subspace {bi + cj + dk}. For symmetric distinguishability metrics where multiple configurations are equally distinguishable, isotropy under permutations S_N conflicts with SO(3) Galois invariance:

jointly imposed (e.g., on three symmetric configurations with amplitudes (i, j, k)), they force the probability functional to depend only on $\Sigma_i |\psi_i|^2$, destroying phase-sensitive interference. Quaternionic amplitudes are excluded.

- **ℂ.** Aut(ℂ/ℝ) = {id, conjugation} under the regularity constraint excluding wild discontinuous automorphisms (which require the axiom of choice and have no physical content). Conjugation commutes with all permutations, so isotropy and Galois invariance are mutually consistent. ℂ is uniquely selected.

Theorem 3′ (Multi-Route Convergence). *Five independent arguments select ℂ as the amplitude field for the VERSF representation:*

1. *Local tomography (§7.1): operational, requiring* dim_ℝ(K_AB) = dim_ℝ(K_A) · dim_ℝ(K_B).
2. *Substrate Hermitian structure (§7.2): substrate-level, requiring faithful representation of the closure-geometry U(1) action.*
3. *Galois invariance (CHS paper, summarised in §7.4): algebraic, requiring invariance under Aut(F/ℝ).*
4. *Jordan-algebraic classification (Architecture paper): the Koecher–Vinberg theorem identifies the operational state cone with the cone of squares in a formally real Jordan algebra; the Jordan–von Neumann–Wigner classification then excludes real algebras (failing local tomography and continuous transitivity), quaternionic algebras (failing local tomography), the exceptional Albert algebra (lacking consistent tensor structure), and spin factors (reducing to the lower-dimensional cases), leaving Hermitian matrices over ℂ.*
5. *Resonance with continuous phase (Tick Race / RAL paper, §13): physical, requiring that any representation of systems with oscillatory dynamics, continuous phase, and superposition must be naturally isomorphic to ℂ. The two-dimensional state space carrying U(1) rotational structure with closure under addition is uniquely the complex numbers; ℝ lacks phase and ℍ lacks commutativity of superposition.*

Each argument is independently sufficient. Their joint application overdetermines the conclusion.

Remark on independence. The five arguments invoke different structural inputs — composite-system dimension counting (operational), intrinsic substrate symmetry (substrate-level), field automorphisms (algebraic), Jordan-algebraic classification of self-dual homogeneous cones (representation-theoretic), and the physical structure of oscillatory systems with continuous phase (resonance-theoretic) — and none reduces to another. A referee skeptical of any single route is left with four others. Within the VERSF programme, the substrate-Hermitian argument is the most foundational (it grounds *why* local tomography, Galois invariance, the Jordan structure, and resonance hold downstream); local tomography is the most operational (matching standard reconstructions); Galois invariance is the most algebraic (matching the field-selection literature of Hardy, Stueckelberg, Adler); Jordan classification is the most architectural (matching the umbrella derivation of the Architecture paper); and the resonance argument is the most physical (grounded directly in the kinematics of oscillation). The five perspectives are in cross-validating tension only in the sense that any failure of $\mathbb{C}$ as the right field would have to defeat all five simultaneously.

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8. The Inner Product

For the pre-commitment representation $V \cong \mathbb{C}^N$ to support TPB-derived continuous dynamics that is reversible and probability-preserving, V must carry an additional geometric structure.

8.1 Existence of an Invariant Form

Lemma 4 (Invariant Sesquilinear Form). *Assume that G^0 admits a non-trivial maximal compact subgroup $K \subset G^0$ acting on V (a substrate input established in the Closure-Geometry and the K=7 Architecture paper (Taylor 2025) from the closure-geometry symmetry analysis of the VERSF programme; the K=7 simplicial substrate's symmetries are compact Lie groups, so a non-trivial K exists). Then the action of G^0 on V preserves a non-zero positive-definite Hermitian sesquilinear form $\langle\cdot,\cdot\rangle : V \times V \to \mathbb{C}$.*

Proof. By Theorems 2 and 3 plus Theorem 1, G^0 acts continuously on $V \cong \mathbb{C}^N$ and contains the strongly continuous one-parameter group $U(\mathbb{R})$. Under the standing compactness hypothesis on K, the action of K on V is a continuous representation of a compact group. By the Weyl integration trick, K preserves a positive-definite Hermitian form: starting from any positive-definite Hermitian form h_0 on $\mathbb{C}^N$, define

$$\langle v, w\rangle := \int_K h_0(k\cdot v, k\cdot w)\, d\mu(k),$$

where μ is normalised Haar measure on K. This integral converges, is sesquilinear, K-invariant by construction, and positive-definite. The full G^0 then preserves $\langle\cdot,\cdot\rangle$ provided the non-compact directions of G^0 act compatibly with K-averaging — which holds when G^0 is the unitary-extension closure of K, the canonical case for the VERSF substrate. ■

Remark on the compactness hypothesis. The compactness of $K \subset G^0$ is a substrate-level input, not a free assumption. It is established within the wider VERSF programme by the closure-geometry symmetry analysis: the symmetries of the K=7 simplicial substrate are compact Lie groups (U(N) and its subgroups), and the no-go theorem for non-simplicial relational substrates (companion paper, "No-Go Theorem for Non-Simplicial Relational Substrates") rules out non-compact alternatives. Readers who suspend judgement on the substrate may read Lemma 4 conditionally on the compactness of K; readers who accept the substrate-level argument may treat the hypothesis as discharged.

8.2 Uniqueness up to Scale

Lemma 5 (Uniqueness of the Form). *If G^0 acts irreducibly on $V \cong \mathbb{C}^N$, the invariant positive-definite Hermitian form is unique up to positive real scale.*

Proof. Schur's lemma. If $\langle\cdot,\cdot\rangle_1$ and $\langle\cdot,\cdot\rangle_2$ are both G^0-invariant positive-definite Hermitian forms, the operator $A = (\text{form}_1) \circ (\text{form}_2)^{-1}$ is positive, self-adjoint, and G^0-equivariant. By Schur, $A = \lambda\mathbb{1}$ for some $\lambda > 0$. ■

8.3 Hilbert Structure

Completion under the norm $\|\psi\| := \sqrt{\langle\psi|\psi\rangle}$ yields a finite-dimensional Hilbert space $\mathscr{H}$.

This section establishes that the natural geometric object on the representation space is a Hermitian sesquilinear form, unique up to positive scale. It does *not* establish that probabilities equal $\langle\psi|\psi\rangle$. The Born rule — the identification of probabilities with norm-squares — is the content of §11 and is not assumed here.

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9. Distinguishability Preservation, Wigner's Theorem, and Unitary Evolution

This section derives unitarity from three arguments: (i) distinguishability preservation, itself derived from reversibility plus the relational character of pre-commitment structure (Theorem 4); (ii) Wigner's theorem, applied to the resulting transition-probability-preserving evolution; (iii) continuity-induced selection of unitaries over antiunitaries.

9.1 Distinguishability Preservation as a Derived Principle

Operational reconstructions typically take preservation of transition probabilities $|\langle\phi|\psi\rangle|^2$ under reversible evolution as primitive — either directly, or as part of a "continuous reversibility" axiom. In VERSF, this principle is *derived* from more fundamental commitments: A3 (reversibility) and A1′ (the strengthened bidirectional Landauer principle, which entails the Relational Pre-Commitment Corollary as a derived consequence rather than a separate postulate).

Lemma 6 (Converse-Landauer Distinguishability Lemma). *Let T be an admissibility-preserving transformation on the pre-commitment state space that releases zero entropy (i.e. is reversible in the strong thermodynamic sense of A3). Then for any pair of admissibly distinguishable pre-commitment configurations ϕ, ψ, the images $T\phi$ and $T\psi$ are admissibly distinguishable to the same degree.*

Note on "degree of admissible distinguishability". The lemma is stated at a level of abstraction that admits two readings, both of which the proof secures. At the discrete substrate level (before Hilbert-space structure is in play), admissible distinguishability is integer-valued: the count of admissible certifications that distinguish ϕ from ψ within the certification protocol's resolution. At the continuum level after Theorem 1 + Theorems 2–3 establish the Hilbert-space representation, this integer count refines to the continuous transition-probability quantity $|\langle\phi|\psi\rangle|^2$ which §9.1(iii) identifies as the unique ray-level relational invariant. Lemma 6 asserts preservation in both readings: zero-entropy maps preserve the integer count of distinguishing certifications (substrate-level reading) and, in the continuum limit, preserve the continuous transition-probability quantity (Hilbert-space reading). The §9.1(iii) uniqueness argument supplies the connection between the two readings; Lemma 6 establishes preservation under either.

Proof. By A1′, every act of forming a fact — including any act that records information about admissible pre-commitment configurations — releases at least k_B ln 2 of entropy per binary distinction registered (forward direction), and conversely any process that does not incur entropy cannot create or encode a physically accessible distinction (converse direction). By A3, T releases zero entropy. We show that T cannot have changed the degree of admissible distinguishability between any pair (ϕ, ψ).

Suppose for contradiction that T strictly *decreased* admissible distinguishability between ϕ and ψ. Then physical information about which alternative the system instantiated has been erased — fewer admissible certifications can now distinguish $T\phi$ from $T\psi$ than originally distinguished ϕ from ψ. By the forward direction of A1′, information erasure requires entropy production of at least k_B ln 2 per binary distinction erased. But T releases zero entropy, contradicting A3.

Suppose instead that T strictly *increased* admissible distinguishability between ϕ and ψ. Then physical information has been created — more admissible certifications can now distinguish $T\phi$ from $T\psi$ than originally. By the converse direction of A1′, no zero-entropy process can create a physically accessible distinction. Again contradicting A3.

Hence the degree of admissible distinguishability between ϕ and ψ is invariant under T. ■

Theorem 4 (Reversible Information Preservation Implies Transition-Probability Preservation). *Let U(t) be a reversible pre-commitment transformation under A3, governing pre-commitment evolution between commitment events under A1′. Then U(t) preserves the ray-level transition-probability quantity:*

$$|\langle U(t)\phi \mid U(t)\psi\rangle|^2 = |\langle\phi|\psi\rangle|^2 \text{ for all } \phi, \psi \in \mathscr{H} \text{ and all } t \in \mathbb{R}.$$

Proof. The argument has three parts.

(i) Information conservation under reversibility (A3 + A1′). By Lemma 6, U(t) preserves the degree of admissible distinguishability between any pair of pre-commitment configurations. The lemma converts the reversibility hypothesis on U(t) (zero entropy release, A3) into invariance of admissible distinguishability via the converse direction of A1′.

(ii) Identification of pre-commitment information with ray-level relational content (Relational Pre-Commitment Corollary). By the Relational Pre-Commitment Corollary derived in §3 from A1′, the pre-commitment configuration of an admissible system contains no content beyond its pattern of distinguishability relations. The "degree of admissible distinguishability" preserved by Lemma 6 is therefore exhaustive: it captures everything physical about the pre-commitment configuration. There is no residual non-relational information that U(t) might or might not preserve, because by the corollary there is no such information.

The role of the corollary here is essential. Without it, the converse-Landauer argument of (i) preserves *something* — admissible distinguishability — but leaves open the possibility that pre-commitment configurations carry additional non-relational content that U(t) might alter without contradicting A3 + A1′. The corollary closes this gap by deriving from A1′ that no such additional content exists: pre-commitment configurations cannot carry physical information beyond their distinguishability relations, because any such information would have to have been created by an entropy-producing event (which would make it post-commitment, not pre-commitment) and stored in some physical medium (which by hypothesis no such event has produced).

The trade with operational reconstructions is therefore at the level of axioms rather than postulates: the strengthened bidirectional A1′ does the work that operational reconstructions assign to a separate transition-probability-preservation axiom. The relational content of pre-commitment configurations is a thermodynamic consequence rather than a free metaphysical posit.

(iii) Uniqueness of the ray-level relational quantity. On a complex Hilbert space (Theorems 2–3, Lemmas 4–5), the only structures available to encode pairwise relational content between configurations are functionals built from the inner product $\langle \cdot | \cdot \rangle$ — which is essentially unique by Lemma 5. Phase-invariance is forced by the ray-level character of the relational content (rays, not vectors, carry physical content; A2 invariance under phase rotations $|\phi\rangle \to e^{i\alpha}|\phi\rangle$ on each factor independently).

Any continuous phase-invariant scalar function $f(\phi, \psi)$ on pairs of rays factors through the three quantities

$$\langle\phi|\phi\rangle, \langle\psi|\psi\rangle, |\langle\phi|\psi\rangle|^2,$$

since these are a complete set of phase-invariant polynomial scalars in the components of ϕ and ψ (every phase-invariant polynomial in (ϕ_i, ϕ_i^*, ψ_j, ψ_j^*) reduces to products of these by elementary invariant theory). With normalisation $\langle\phi|\phi\rangle = \langle\psi|\psi\rangle = 1$ fixed (a representational choice without physical content), the unique non-trivial ray-level relational invariant is $|\langle\phi|\psi\rangle|^2$. Hence the relational content preserved by U(t) is exactly $|\langle\phi|\psi\rangle|^2$.

Combining (i)–(iii): U(t) preserves physical pre-commitment information (Lemma 6); by the Relational Pre-Commitment Corollary, that information is exhausted by ray-level relational content; the unique ray-level relational invariant on a complex Hilbert space with normalisation fixed is $|\langle\phi|\psi\rangle|^2$. Hence U(t) preserves $|\langle\phi|\psi\rangle|^2$ for all ϕ, ψ. ■

Status. Distinguishability preservation is the operational input that the operational reconstruction programme typically takes as primitive — postulated as A3 in the Hamiltonian paper, as R3 in the Architecture paper, with informal motivation only. Here it is *derived* as a consequence of reversibility (A3), the strengthened bidirectional Landauer principle (A1′), and the relational pre-commitment content that follows as a corollary from A1′. The substantive VERSF content is in (ii): the relational character of pre-commitment configurations is a thermodynamic consequence of A1″s converse direction, not a separate postulate. The trade with operational reconstructions is at the level of axioms: the operational primitive of transition-probability preservation is replaced by the strengthened bidirectional reading of Landauer's principle as A1′, with the relational pre-commitment claim flowing as a corollary rather than as an independent postulate.

9.2 Wigner's Theorem and the Exclusion of Antiunitaries

By Theorem 4, U(t) preserves transition probabilities $|\langle\phi|\psi\rangle|^2$ for all pairs of rays. Wigner's theorem (1931) then gives that each U(t) is implemented (uniquely up to phase) by a unitary or antiunitary operator on $\mathscr{H}$. The set U(N) ∪ V(N) of such Wigner symmetries is $\mathbb{Z}/2\mathbb{Z}$-graded by unitary/antiunitary parity, and the two cosets are disconnected. By Theorem 1, the family {U(t)} is strongly continuous and contains $\mathbb{1} = U(0) \in U(N)$, so the entire family lies in the unitary component.

Theorem 5 (Unitarity). *Pre-commitment evolution is implemented by a strongly continuous one-parameter unitary group* $\{U(t)\} \subset U(\mathscr{H})$. ■

The full Wigner + antiunitary-exclusion derivation is given in the Hamiltonian paper (§5) and the Architecture paper (§5.3). The architectural difference here is the upstream chain: Theorem 4 (distinguishability preservation) is itself derived (§9.1) rather than postulated, and the connectedness through $\mathbb{1}$ that excludes antiunitaries is supplied by Theorem 1 (TPB-derived continuity, §5) rather than by a postulated continuity axiom. Three derivation steps replace what reconstructions typically take as a single primitive.

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10. Stone's Theorem and the Hamiltonian

Stone's theorem. *Let* $\{U(t)\}_{t\in\mathbb{R}}$ *be a strongly continuous one-parameter group of unitary operators on a Hilbert space* $\mathscr{H}$ *— that is,* $U(t)U(s) = U(t+s)$ *for all* $t, s \in \mathbb{R}$, $U(0) = \mathbb{1}$, *each* $U(t)$ *is unitary on* $\mathscr{H}$, *and* $t \mapsto U(t)v$ *is norm-continuous for every* $v \in \mathscr{H}$. *Then there exists a unique densely-defined self-adjoint operator H on* $\mathscr{H}$ *(the infinitesimal generator) such that*

$U(t) = \exp(-iHt/\hbar)$ *for all* $t \in \mathbb{R}$,

equivalently $i\hbar\, \partial_t|\psi\rangle = H|\psi\rangle$ *on Dom(H).* (Stone, 1932; standard reference: Reed & Simon, *Methods of Modern Mathematical Physics* I, Theorem VIII.8.)

By Theorem 5, {U(t)} as derived from A0–A4 satisfies all three hypotheses of Stone's theorem (one-parameter group property by construction, unitarity by Theorem 5, strong continuity by Theorem 1). Hence Stone's theorem applies and yields a unique self-adjoint H with

$U(t) = \exp(-iHt/\hbar)$, $i\hbar\, \partial|\psi\rangle/\partial t = H|\psi\rangle$.

The constant $\hbar$ enters as the dimensional rate constant of pre-commitment phase evolution; its specific value is not derived here.

The Hamiltonian is therefore not a postulate but the unique self-adjoint generator forced by Theorem 1 (continuity from TPB), Theorem 5 (unitarity from Theorem 4 + Wigner + connectedness through $\mathbb{1}$), and Stone's theorem applied to the resulting one-parameter unitary group. The Stone application, the GKSL extension to irreversible Markovian dynamics, the VERSF reading of H as a "reconfiguration generator" rather than a substantive energy operator, and the recovery of energy as Noether charge under time-translation symmetry are developed in the Hamiltonian paper (Taylor); the present paper differs only by deriving the inputs that paper postulates.

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11. The Born Rule from Commitment Envariance

The fourth load-bearing step. We obtain $P_i = |a_i|^2$ without assuming the L^2 norm in advance.

11.1 Setup

§§7–8 have established a Hilbert space $\mathcal{H}$ with a Hermitian sesquilinear form, unique up to scale. The associated quantity $\langle\psi|\psi\rangle$ is intrinsically quadratic in amplitudes by sesquilinearity. The question is whether the probability of commitment to outcome i equals $\langle\psi|P_i|\psi\rangle = |a_i|^2$ where P_i is the projector onto $|i\rangle$.

Let

$|\psi\rangle = \Sigma_i a_i |i\rangle \in \mathcal{H}_S$

be a pre-commitment system state. By A2 and §13, there exist admissible joint states of system and environment of Schmidt form

$|\Psi\rangle_{SE} = \Sigma_i a_i |i\rangle_S \otimes |e_i\rangle_E,$

with $\{|e_i\rangle_E\}$ *orthonormal. Commitment probabilities are functions p:* $\mathcal{H}_{SE} \to [0,1]^N$ satisfying:

- **(P1) State-determined.** p depends only on $|\Psi\rangle_{SE}$ (up to global phase).
- **(P2) Normalised.** $\Sigma_i p_i(|\Psi\rangle_{SE}) = 1$.
- **(P3) No-signalling.** Operations acting only on E do not change p_i for outcomes on S. (This is a direct consequence of A2.)

11.2 Step 1 — Equal Amplitudes

Claim. *If* $|a_1| = |a_2| = \cdots = |a_N|$, *then* $P_i = 1/N$ *for all i.*

Proof. Choose phases by basis adjustment so $a_1 = \cdots = a_N = 1/\sqrt{N}$. Pick (i, j). Define swap unitaries:

$\sigma_S^{(ij)} : |i\rangle_S \leftrightarrow |j\rangle_S$ (others fixed), $\sigma_E^{(ij)} : |e_i\rangle_E \leftrightarrow |e_j\rangle_E$ (others fixed).

Compute $\sigma_S^{(ij)}|\Psi\rangle$:

$\sigma_S^{(ij)}|\Psi\rangle = (1/\sqrt{N})[\, \Sigma_{k\neq i,j}\, |k\rangle|e_k\rangle + |j\rangle|e_i\rangle + |i\rangle|e_j\rangle \,]$.

Then $\sigma_E^{(ij)}$:

$\sigma_E^{(ij)}\, \sigma_S^{(ij)}|\Psi\rangle = (1/\sqrt{N})[\, \Sigma_{k\neq i,j}\, |k\rangle|e_k\rangle + |j\rangle|e_j\rangle + |i\rangle|e_i\rangle \,] = |\Psi\rangle$.

Three equalities:

- **(P1):** $p_i(\sigma_E^{(ij)}\, \sigma_S^{(ij)}|\Psi\rangle) = p_i(|\Psi\rangle)$.
- **(P3):** $p_i(\sigma_E^{(ij)}\, \sigma_S^{(ij)}|\Psi\rangle) = p_i(\sigma_S^{(ij)}|\Psi\rangle)$ (σ_E acts only on E).
- **Direct computation:** $\sigma_S^{(ij)}|\Psi\rangle$ swaps system labels i and j, so $p_i(\sigma_S^{(ij)}|\Psi\rangle) = p_j(|\Psi\rangle)$.

Combining: $p_i(|\Psi\rangle) = p_j(|\Psi\rangle)$ for all (i,j). With (P2): $p_i = 1/N$. ■

Non-circularity. The argument uses only (P1)–(P3) and unitarity. No norm exponent is assumed.

11.3 Step 2 — Rational Squared Moduli

Claim. *If* $|a_i|^2 = m_i/M$ *with* $m_i, M \in \mathbb{Z}_{>0}$ *and* $\Sigma_i m_i = M$, *then* $P_i = m_i/M$.

Proof. Embed $\mathscr{H}_S$ in a larger Hilbert space $\tilde{\mathscr{H}}_S$ of dimension M by fine-graining each $|i\rangle$ into m_i orthogonal microstates:

$|i\rangle = (1/\sqrt{m_i})\, \Sigma_{k=1}^{m_i} |i, k\rangle$.

This map $\iota: \mathscr{H}_S \to \tilde{\mathscr{H}}_S$, defined on basis vectors by $|i\rangle \mapsto (1/\sqrt{m_i})\, \Sigma_k |i, k\rangle$ and extended linearly, is an isometry: $\langle\iota(|i\rangle), \iota(|j\rangle)\rangle = (1/\sqrt{m_i})(1/\sqrt{m_j})\, \Sigma_{k,k'} \langle i,k|j,k'\rangle = \delta_{ij}$, so inner products are preserved. The image $\iota(\mathscr{H}_S)$ is the subspace of $\tilde{\mathscr{H}}_S$ spanned by uniform superpositions over each fine-grained block, and the original pre-commitment dynamics + commitment structure on $\mathscr{H}_S$ is implemented by the corresponding restricted structure on this subspace. Probabilities transfer because P_i in $\mathscr{H}_S$ and $\Sigma_k P_{(i,k)}$ in $\tilde{\mathscr{H}}_S$ are computed by physically equivalent commitment events on ι-related states.

In the fine-grained representation:

$|\psi\rangle = (1/\sqrt{M})\, \Sigma_{(i,k)} |i,k\rangle$,

so each microstate has equal amplitude $1/\sqrt{M}$ across all M microstates. By Step 1, each microstate has probability 1/M. By coarse-graining consistency:

$P_i = \Sigma_{k=1}^{m_i} P_{(i,k)} = m_i/M = |a_i|^2$. ■

Non-circularity. The squared-modulus form $|a_i|^2$ appeared from the integer ratio m_i/M produced by fine-graining: $\sqrt{(m_i/M)} \cdot \sqrt{(m_i/M)} = m_i/M$ when computing the contribution of m_i microstates of equal amplitude $1/\sqrt{M}$. The exponent 2 was *not* postulated.

11.4 Step 3 — General Amplitudes

The set of states with rational $|a_i|^2$ is dense in projective Hilbert space. We need to show that the rule $P_i = |a_i|^2$ extends uniquely from this dense set to all amplitudes. The required regularity input is *continuity of commitment probabilities in the pre-commitment state*: small changes in $|\psi\rangle$ produce small changes in $(P_1, \ldots, P_N)$.

This continuity is not a free assumption; it follows from Theorem 1 plus the unitary action on states. Concretely: Theorem 1 establishes that pre-commitment evolution $U(t)$ is strongly continuous, so the orbit $\{U(t)|\psi\rangle\}$*$\{t \in \mathbb{R}\}$ is a continuous trajectory in $\mathcal{H}$. By A2, commitment probabilities are protocol-invariant scalars, which in particular means they are scalars of the system state (not of any auxiliary parameterisation). For any continuous family of states $\{|\psi_t\rangle\}$, the commitment probability vector $(P_1(|\psi_t\rangle), \ldots, P_N(|\psi_t\rangle))$ inherits continuity from the continuity of $|\psi_t\rangle$ itself: if probabilities were discontinuous in $|\psi\rangle$, then either there would exist some $|\psi_0\rangle$ at which arbitrarily small perturbations produce $O(1)$ changes in commitment statistics — which violates A2 read in its continuous form (admissible protocol changes parameterised continuously by ε produce representations $\mathcal{R}$*$\{P(\varepsilon)\}(\mathcal{C})$ and statistics that depend continuously on ε; an infinitesimal protocol-noise perturbation should produce an infinitesimal statistical perturbation) — or there would exist a non-trivial reversible action of $U(\mathbb{R})$ under which the commitment probabilities discontinuously change, which contradicts Theorem 1's strong continuity of the unitary action. Hence $(P_1(|\psi\rangle), \ldots, P_N(|\psi\rangle))$ is continuous on the projective Hilbert space.

Since the rule $P_i = |a_i|^2$ is continuous on the dense rational-amplitude set, the unique continuous extension to all amplitudes coincides with the formula $P_i = |\langle i|\psi\rangle|^2$ on the full state space. This completes the extension of Step 2's rational-amplitude result.

11.5 The Born Rule

Theorem 6 (Born Rule). *For any pre-commitment state* $|\psi\rangle = \Sigma_i a_i|i\rangle \in \mathcal{H}$ *with normalisation* $\langle\psi|\psi\rangle = 1$,

$P_i = |\langle i|\psi\rangle|^2 = |a_i|^2$. ■

The Schlosshauer–Fine (2005) circularity critique of envariance is addressed by the explicit non-circularity at each step above. The Hilbert norm $\langle\psi|\psi\rangle = 1$ is a *consequence* of Theorem 6 plus probability normalisation; the scale freedom of Lemma 5 is fixed by the unique form yielding normalised probabilities.

11.6 An Independent Path-Geometric Route

The DSR paper (Taylor, "The Double Square Rule") establishes Theorem 6 via a different route: irreversible selection acts on path-pair correlation structures (its Axiom A7), and Mercer decomposition + factorization force the unique kernel

$$W(P, P') = e^{i(\theta(P) - \theta(P'))},$$

yielding $P(A) = |\Sigma_{P \in R_A} e^{i\theta(P)}|^2 = |\psi_A|^2$. The DSR paper's Appendix E ("Landauer–Pairwise Theorem") further derives A7 itself from the thermodynamics of coherence erasure, showing that entropy export under measurement is second-order in coherences and hence bilinear in path contributions.

The two routes — envariance here, path-pair correlation kernels in DSR — are independent and complementary:

- **Envariance** (this paper) operates at the Hilbert-space level, using system-environment swap unitaries and coarse-graining consistency. It is closer to the operational reconstruction tradition (Zurek, Saunders, Wallace) and composes naturally with the joint-commitment-closure structure of §13.
- **Path-pair correlation** (DSR) operates at the path-geometric level, before Hilbert space is fully assembled. It is closer to the path-integral and informational-geometry traditions and provides a microphysical grounding via Landauer's principle.

Both arrive at the same conclusion. A reader skeptical of the envariance argument's premises (e.g., (P3) no-signalling, or the Schmidt decomposition's availability prior to the Born rule) can substitute the DSR derivation; conversely, a reader skeptical of the path-pair foundations can substitute the envariance route. The convergence of independent routes overdetermines the Born rule in the same way that §7's five-route convergence overdetermines the choice of $\mathbb{C}$.

11.7 First-Passage and Distinguishability-Uniqueness Routes

The Tick Race / RAL paper (Taylor, "Quantum Measurement as a Tick Race") establishes Theorem 6 by two further routes — one *structural*, one *dynamical* — that together complete a four-way overdetermination of the Born rule.

Structural route: distinguishability uniqueness. The RAL framework (Tick Race paper §14) seeks a non-negative functional D on outcomes satisfying (i) additivity for distinguishable outcomes, (ii) permutation symmetry over microstates, (iii) phase covariance with interference, and (iv) polynomial dependence on amplitudes. Its Theorem 14.1 establishes that the unique such functional is

$$D(A) = c \cdot |\psi_A|^2,$$

with the normalisation $c > 0$ fixed by $\Sigma_B P(B) = 1$, giving $P(A) = |\psi_A|^2$. The argument runs: condition (iv) restricts D to polynomial form; reality of D plus phase covariance under global $U(1)$ forces equal powers of a_i and a_j^*; permutation symmetry forces the coefficients to be uniform; the lowest-order term satisfying additivity is the bilinear $|\psi_A|^2 = \Sigma_{i,j \in A} a_i a_j^*$;

higher-order terms violate additivity. This is structurally equivalent to Gleason's theorem under non-contextuality, with the polynomial-regularity condition (iv) playing the role of the GPT-literature smoothness assumption that excludes pathological non-measurable functionals.

Dynamical route: first-passage statistics. Beyond the structural derivation, the Tick Race paper supplies a *physical mechanism* by which the Born rule is realised at the detector level. After decoherence, each branch A generates microscopic threshold-approaching events ("ticks") at hazard rate λ_A, and the first branch to produce a threshold-crossing tick triggers the irreversible macroscopic record (the "bit"). Three constraints (A1′)–(A3′) are individually forced by physical necessity rather than postulated:

- **(A1′) Proportional hazards.** The detector Hamiltonian cannot condition its response on which decohered branch it is coupled to; hence hazard functions share the same time-shape $h_0(t)$, differing only by branch-dependent scale factors λ_A.
- **(A2′) Amplitude-squared rate scaling.** Transition rates must be non-negative, gauge-invariant, U(1)-covariant quadratic functionals of complex amplitudes (Schrödinger linearity + perturbation theory). The unique such functional is $\lambda_A \propto |\psi_A|^2$ (Fermi's golden rule applied to the measurement interaction).
- **(A3′) First-tick selection.** Single-quantum-sensitive detectors are necessarily metastable amplifying systems near an instability threshold (avalanche photodiodes, photomultipliers, superconducting nanowires, Geiger–Müller tubes). For such systems, the first microscopic event crossing the barrier triggers deterministic macroscopic relaxation; subsequent events occur after the macrostate is fixed.

Under (A1′)–(A3′), competing-risk first-passage statistics yield

$$P(A) = \lambda_A / \Sigma_B \lambda_B = \kappa|\psi_A|^2 / \Sigma_B \kappa|\psi_B|^2 = |\psi_A|^2,$$

as a theorem of competing exponentials. The Tick Race paper's Lemma 2.2 establishes uniqueness of this assignment given symmetry, homogeneity, continuity, and normalisation; its Section 2.7 robustness analysis shows that violating any of (A1′)–(A3′) produces calculable, generically non-zero deviations from Born statistics — Tick Race is a *robust fixed point* in the space of detector-layer outcome-selection models.

Relation to envariance and DSR. The four Born-rule routes — envariance (§§11.1–11.5), DSR path-pair correlations (§11.6), distinguishability uniqueness (Tick Race §14), and first-passage statistics (Tick Race §2) — operate at different levels of the architecture:

- *Envariance* is symmetry-based, operating on Hilbert-space states under system-environment swap unitaries and coarse-graining.
- *DSR* is path-geometric, operating on path-pair correlation kernels prior to full Hilbert-space assembly.
- *Distinguishability uniqueness* (RAL) is functional-analytic, operating on the space of admissible outcome functionals.
- *First-passage* (Tick Race) is dynamical, operating on the temporal evolution of competing detector microprocesses.

The four routes are independent and complementary. Their convergence overdetermines the Born rule: a reader rejecting any one is left with three others. The first-passage route additionally supplies the **detector-level dynamical mechanism** by which the Born rule probabilities arise from underlying deterministic microdynamics — a layer absent from envariance, DSR, and the structural distinguishability argument, all of which establish *what* the probabilities are without specifying *how* they emerge in real apparatus. For the present paper's §14 framing of measurement as constraint selection, the Tick Race mechanism supplies the physical realisation: the constraint selected by the irreversible commitment event is the one whose corresponding branch produced the first threshold-crossing tick.

The Tick Race paper also predicts a **falsifiable deviation**: detectors engineered to require $k > 1$ *genuinely independent* trigger events (rather than amplification-chain stages of a single initiating event) should follow Gamma(k, λ_A) waiting-time statistics and yield P(A) deviating from $|\psi\_A|^2$ in a calculable way. For two branches with $\lambda_1 = 2\lambda_2$, the prediction is $P_1 = 0.667$ ($k = 1$) versus $P_1 = 0.741$ ($k = 2$), distinguishable at 3σ with ~1,600 trials. No such experiment has yet been performed; existing single-photon detectors are $k = 1$ by design.

The principal experimental difficulty in testing this prediction lies not in the statistics but in establishing that $k > 1$ trigger events are *genuinely independent* rather than constituting amplification-chain stages of a single initiating event. Most multi-stage detectors are amplification-chained: a single primary event triggers cascading secondary events, and the apparent "$k > 1$ trigger" is statistically equivalent to a $k = 1$ detector with a longer dead time. Distinguishing genuine independence from chained amplification requires architectural arguments about the initiating event distribution, not merely counting trigger stages. The VERSF Tick Race paper discusses candidate experimental architectures (independently-pumped multi-mode systems, separated-cavity coincidence schemes) where the genuine-independence assumption is structurally defensible; engineering such systems is a non-trivial open problem, and the prediction's testability depends on solving it.

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12. Non-Commutativity and Uncertainty

12.1 Non-Commutativity from Incompatible Commitment Protocols

Let A and B be admissible fact-forming protocols. If executing A alters the admissible distinction-structure available to B — i.e., if the post-A admissible substrate differs from the pre-A substrate in a way affecting B's admissible outcomes — then operation order matters:

$AB|\psi\rangle \neq BA|\psi\rangle$, $[A, B] \neq 0$.

Non-commutativity is not a passive feature of operators on a pre-existing space. It reflects the *substrate-altering* character of irreversible commitments.

12.2 Uncertainty Relations

Given Hilbert structure and self-adjoint observables, the **Robertson inequality**

$$\Delta A \cdot \Delta B \geq (1/2)|\langle[A, B]\rangle|$$

follows from Cauchy–Schwarz on the inner product of §8. For canonical pairs with $[X, P] = i\hbar$:

$$\Delta x \cdot \Delta p \geq \hbar/2.$$

In VERSF terms: incompatible commitment protocols cannot be jointly executed beyond the finite admissible resolution of A0. Uncertainty is the quantitative expression of substrate-level finiteness applied to incompatible fact-forming structures.

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13. Entanglement from Joint Commitment Closure

A naïve compositional rule would give $\mathcal{C}_{AB} = \mathcal{C}_A \times \mathcal{C}_B$. But if the fact-forming constraints of A and B are *jointly closed* — the admissible commitment structure belongs to the joint system rather than to either subsystem — then

$$|\Psi\rangle_{AB} = \Sigma_{ij}\, a_{ij}\, |i\rangle_A |j\rangle_B$$

with $a_{ij} \neq \alpha_i\beta_j$ in general.

Entanglement is the structural signature of joint commitment closure: the admissible fact-structure cannot be factored across the subsystem boundary. The non-locality of measurement correlations is then not action at a distance but the consequence of a pre-existing joint commitment structure that has not yet committed. The Schmidt-decomposition theorem on Hilbert spaces, used in §11, is itself a consequence of joint commitment closure given the structure of §§6–8.

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14. Measurement as Constraint Selection: The Lüders Update

Measurement is treated here as irreversible commitment with explicit state-update rule.

14.1 The Commitment Map

A measurement is a fact-forming transition (A1′) implemented through interaction with a certifying apparatus or observer-protocol whose admissible distinction set is a basis $\{|i\rangle\}$, equivalently a complete set of projectors $\{P_i = |i\rangle\langle i|\}$. By Theorem 6, outcome i is selected with probability

$P_i = \langle\psi|P_i|\psi\rangle$ (pure state) $P_i = \mathrm{Tr}(P_i\,\rho)$ (mixed state).

14.2 The Lüders Rule

After commitment to outcome i, the post-commitment state is, for an arbitrary initial density matrix ρ:

$\rho \longmapsto \rho'_i = (P_i\,\rho\,P_i) / \mathrm{Tr}(P_i\,\rho)$.

For a pure pre-commitment state $\rho = |\psi\rangle\langle\psi|$:

$\rho'_i = (P_i\,|\psi\rangle\langle\psi|\,P_i) / |\langle i|\psi\rangle|^2 = |i\rangle\langle i|$.

The post-commitment state is the rank-1 projector onto $|i\rangle$.

14.3 Constraint Selection, Not Physical Collapse

The Lüders update is a *constraint selection*: the post-commitment state encodes the new admissible structure given fact F_i. Specifically:

- Pre-commitment, the admissible alternatives are $\mathcal{C} = \{c_1, \ldots, c_N\}$.
- Post-commitment with outcome i, the admissible alternatives are $\mathcal{C} \cap$ (subspace consistent with F_i) $= \{c_i\}$.
- The state ρ'_i encodes this restricted admissible set.

There is no "physical collapse" in the sense of a non-unitary dynamical process superimposed on the unitary one. Pre-commitment dynamics is unitary (§§9–10). Commitment is irreversible (A1′). The Lüders update is the representation-level consequence of the new admissible set — a change in *what is admissible to ask*, not a change in *what was there*.

The "two laws" of standard QM (unitary evolution and projection) are recognised as governing distinct phases — pre-commitment and at-commitment — rather than competing for the same phase. The minimum entropy released at commitment, at least $k_B \ln 2$ per binary outcome (saturated under $\eta = 1$), is the thermodynamic signature distinguishing the two regimes.

Cross-reference. The framework adopted here for measurement aligns with the dedicated VERSF companion paper, the Measurement-as-Commitment paper (Taylor, "Measurement as Commitment: Why Quantum Systems Are Relational"). That paper develops the relational ontology in fuller detail and proves three load-bearing results: (i) Theorem 2.1 (No-Extra-Backaction Principle), establishing that the Lüders update $\rho \to P_i\,\rho\,P_i / \mathrm{Tr}(P_i\,\rho)$ introduces no supplementary disturbance term beyond the CP-instrument specification; (ii) Theorem 3.1 (Incompatible Closure Channels), establishing that non-commutativity-induced uncertainty has structural rather than dynamical origin; and (iii) Theorem 6.2 (Generic POVM Emergence Under Finite Substrate Resolution), establishing that finite spatial coherence ℓ_c and finite temporal healing time τ_h generically produce non-idempotent effective effects, so POVMs are forced rather than approximate. The "constraint selection" framing of the present §14 is the same content viewed through the present axiom system; readers seeking the relational ontology, the substrate parameters $\{\ell_c, \tau_h, \mu_{eff}\}$, and the comparison with Copenhagen / many-worlds / Bohmian / QBist / transactional interpretations are referred there.

Detector-level dynamics. The Measurement-as-Commitment framework establishes *what* the post-commitment update is and *why* it is constraint completion rather than dynamical disturbance; the Tick Race paper (Taylor, "Quantum Measurement as a Tick Race") supplies the complementary *how*: a first-passage mechanism in which the constraint selected by a given commitment event is the one whose corresponding decohered branch produced the first threshold-crossing tick, with branch-specific tick rates $\lambda_A \propto |\psi_A|^2$ forced by unitarity, U(1) phase covariance, and Fermi's golden rule (§11.7). The Lüders update is then the representation-level encoding of the constraint selected by this dynamical race, and the Born probabilities arise as competing-exponential first-passage probabilities. This grounds the §14 framing in concrete detector physics without modifying its content.

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15. Scope: Finite to Continuum on the Representation Space

The arguments above establish the structural core of finite-dimensional quantum mechanics. The extension to $L^2(\mathbb{R})$ and continuous spectra requires an additional limiting argument on the *representation space* (not to be confused with the TPB continuum limit of §5, which is on the *time parameter*). The treatment given here is a sketch; a full treatment is deferred to a dedicated companion paper.

Sketch. The natural extension is via **inductive limit**. A nested sequence of admissible resolutions

$$\mathcal{D}_1 \subset \mathcal{D}_2 \subset \cdots$$

at increasingly fine certification protocols generates a sequence of finite-dimensional Hilbert spaces $\mathcal{H}_n$ with isometric inclusions $\iota_n : \mathcal{H}_n \to \mathcal{H}_{n+1}$ satisfying $\iota_n^* \iota_n = \mathbb{1}_{\mathcal{H}_n}$. The infinite-dimensional separable Hilbert space arises as the metric completion of the algebraic inductive limit:

$$\mathcal{H}_\infty = (\lim_{\to n} \mathcal{H}_n)^{\sim}.$$

Continuous spectra arise as limits of densifying discrete spectra. Self-adjointness in the limit follows from standard criteria (Reed–Simon I, §VIII; von Neumann's deficiency-index theorem for symmetric extensions).

Stone's theorem extends to this setting: the strongly continuous one-parameter unitary group on $\mathcal{H}_\infty$ has a unique densely-defined self-adjoint generator. The Schrödinger equation on $\mathcal{H}_\infty$ is the inductive-limit version of the finite-dimensional version derived in §10.

What a full treatment would establish. The sketch above relies on three points that require dedicated argument: (i) admissible certification-protocol refinements form a *directed system* in the appropriate categorical sense, with refinement maps that compose consistently under common refinement; (ii) the inductive limit of the Hilbert spaces commutes with the Hamiltonian flow, so that the §10 Hamiltonian extends to a self-adjoint operator on $\mathscr{H}_\infty$ via deficiency-index analysis on each $\mathscr{H}_n$ followed by limit; and (iii) the Born rule established in §11 transfers correctly to spectral measures under the deficiency-index extension, which requires checking that the envariance argument of §11 commutes with refinement (it does, since fine-graining is the special case of refinement that §11.3 already analyses, but the full argument over an arbitrary directed system needs explicit treatment).

These points are technical rather than structural — none of them threatens the finite-dimensional core derivation. The limit construction is the same for VERSF as for any operational reconstruction, with the substrate-level grounding inherited from the finite case.

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16. Main Result

Theorem 7 (VERSF Quantum Representation Theorem). *Let $\mathcal{C}$ be a finite admissible pre-commitment configuration of cardinality N. The only representation of $\mathcal{C}$ satisfying A0, A1', A2, A3, A4 and the closure conditions of the VERSF-admissible quantum representation (compositional closure, coarse-graining consistency) is a complex Hilbert space $\mathscr{H}$ of dimension N, with:*

1. *pre-commitment states* $|\psi\rangle \in \mathscr{H}$ *(Theorems 2, 3; Lemmas 4, 5),*
2. *strongly continuous pre-commitment dynamics* $U(t) = e^{-iHt/\hbar}$ *(Theorem 1; §10 via Stone),*
3. *distinguishability preservation* $|\langle U(t)\phi|U(t)\psi\rangle|^2 = |\langle\phi|\psi\rangle|^2$ *(Theorem 4),*
4. *unitarity* $U(t)^\dagger U(t) = \mathbb{1}$ *(Theorem 5),*
5. *Hamiltonian generator* $i\hbar\, \partial_t|\psi\rangle = H|\psi\rangle$ *(§10),*
6. *Born probabilities* $P(i) = |\langle i|\psi\rangle|^2$ *(Theorem 6),*
7. *Lüders update* $\rho \mapsto P_i \rho P_i / \mathrm{Tr}(P_i \rho)$ *(§14),*
8. *non-commutative observable structure for incompatible commitment protocols (§12),*
9. *joint commitment closure manifesting as entanglement (§13).*

The representation is unique up to unitary equivalence: any other VERSF-admissible quantum representation R' of $\mathcal{C}$ is related to $\mathscr{H}$ by a unitary map preserving inner product, dynamics, and commitment probabilities. Antiunitary alternatives are excluded by Theorem 5 (continuity through $\mathbb{1}$). The infinite-dimensional case follows by inductive limit (§15). ■

Three exclusion arguments turn each apparent choice into a forced consequence: the simplex (§6 corollary), $\mathbb{R}$ and $\mathbb{H}$ (§7.1 lemma), and antiunitary evolution (§9.2). The substrate-level grounding of the inputs — continuity from TPB in Theorem 1 and transition-probability preservation from reversibility in Theorem 4 — closes the remaining gap that operational reconstructions leave open.

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17. Differential Predictions

The reconstruction above recovers standard QM in all regimes where standard QM is tested. To distinguish VERSF as a substrate-level account from QM-as-primitive, one needs predictions that follow from the substrate structure but do not follow from the operational axioms alone. The VERSF programme provides several such predictions, established in companion papers and summarised here.

Tensor-to-scalar ratio. The K=7 substrate architecture together with the spectral density of the commitment-event bath J(ω) predicts $r \approx 0.027$–0.033 for primordial gravitational waves.

Fine-structure constant. The hexagonal closure geometry predicts $\alpha^{-1} \approx 137.143$ at approximately 0.08% accuracy.

CKM and PMNS mixing matrices. The substrate closure geometry predicts the structure of the quark and lepton mixing matrices; standard QM treats these as free phenomenological inputs.

Lepton mass hierarchy. The $\kappa = 8/3$ ratio derived from $\mathbb{CP}^2$ geometry constrains the mass hierarchy.

Two-Planck cosmological constant. The substrate accounts for the cosmological constant scale via a two-Planck mechanism that is outside the scope of QM-as-primitive.

Ultra-high-frequency dynamics. This is the most qualitatively distinctive prediction unique to the substrate-level account. The TPB-derived continuity of §5 holds only in the high-density limit, where the rational evolution map $\tilde{W}: \mathbb{Q} \to G$ is densely sampled relative to the observation timescale. Below the K=7 commitment-density scale, the discrete-tick origin of pre-commitment evolution becomes resolvable: departures from strict continuity manifest as suppression of high-frequency Fourier components of the propagator at frequencies approaching the substrate tick rate, and as discrete-step corrections to the Stone-derived Hamiltonian flow. The functional form of the suppression is fixed by the principal-branch refinement structure (A4(iii)): high-frequency modes feel the discreteness of the substrate's commitment-event lattice. Standard QM, treating evolution as fundamentally continuous, predicts no such suppression at any frequency.

Whether this is directly testable depends on which bath-cutoff regime the K=7 architecture realises, an input determined in the companion treatment. In the laboratory-bath regime the suppression is in principle accessible to optical-to-X-ray frequency-domain experiments; in the cosmological-bath regime the prediction is qualitative — there is some suppression at substrate-rate frequencies, but the onset frequency is far above any directly accessible laboratory regime.

The detailed calculation of the onset frequency from the K=7 commitment-event density and the spectral density J(ω) of the commitment-event bath is given in the companion paper on J(ω). Both regimes (laboratory and cosmological) generate the same qualitative signature — frequency-dependent suppression with the principal-branch functional form — at very different scales. A measured suppression scaling consistent with the K=7 commitment-density would constitute substrate-level evidence not derivable from any operational reconstruction. Conversely, a confirmed absence of suppression up to a definite frequency bound would constrain the substrate's tick rate from above. The structural prediction is given here; the quantitative onset frequency is left to the companion treatment because it depends on bath-cutoff inputs not derived in this paper.

In each case, the prediction follows from the same substrate structure that, in this paper, grounds the operational axioms. Successful tests provide differential evidence for the substrate-level account; failures constrain the substrate architecture without affecting the operational reconstruction.

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18. Discussion

The result should be read carefully.

It does **not** claim to derive all of quantum field theory, the Standard Model interaction structure, or the values of physical constants. Those are addressed by other VERSF papers using closure geometry, the K=7 architecture, and the spectral density of the commitment-event bath.

It **does** claim that the Hilbert-space machinery of ordinary quantum mechanics — including continuity of evolution and the existence of a Hamiltonian — is the unique stable representation, up to canonical isomorphism, of finite pre-commitment possibility once the VERSF principles are accepted. The axioms that Hardy, Masanes–Müller, Chiribella et al., and Höhn take as primitive — local tomography, continuity of reversible transformations, purification, information-equivalence — are not free in the VERSF framework. They follow from the substrate being a simplicial complex with TPB substructure, continuous internal symmetry, and a closure geometry already independently constrained by the fine-structure constant and mixing-matrix derivations.

Six structural reframings deserve emphasis.

Continuity is not a free axiom. The continuous one-parameter unitary group U(t), and consequently the Hamiltonian, emerge from the high-density limit of discrete TPB events (Theorem 1). The Schrödinger equation is not a postulate but a consequence of refined commitment-event sequences.

Transition-probability preservation is not a free axiom either. What operational reconstructions take as primitive — that reversible evolution preserves $|\langle\phi|\psi\rangle|^2$ — is here a theorem (Theorem 4). The argument runs: thermodynamic reversibility (A3) plus the strengthened bidirectional Landauer principle (A1′) entail information conservation via the converse-Landauer Lemma 6; the information content of pre-commitment structure is exhausted by ray-level relations by the relational pre-commitment corollary derived from A1′; the unique ray-level relational quantity is $|\langle\phi|\psi\rangle|^2$. The trade with operational reconstructions is now visible at the level of axioms: the operational primitive of transition-probability preservation is replaced by the strengthened bidirectional reading of Landauer's principle as A1′, with the relational pre-commitment claim derived rather than postulated. This is the conceptually deepest derivation in the paper, because it explains *why* Wigner's theorem is the right tool to apply.

Superposition is not mystical. It is the representational consequence of non-commitment: when no admissible protocol has yet selected an outcome, the representation cannot behave as though one had been selected.

The Born rule is not a free postulate. It is the unique probability assignment consistent with envariance, coarse-graining, and continuity. The exponent $p = 2$ is forced by integer counting under fine-graining (§11.3), not read off a postulated norm. Three further independent routes — path-pair correlation kernels (DSR paper, §11.6), distinguishability functional uniqueness under additivity + symmetry + phase covariance + polynomial regularity (Tick Race / RAL paper Theorem 14.1, §11.7), and first-passage statistics on metastable amplifying detectors with proportional hazards (Tick Race paper Theorem 2.1, §11.7) — all converge on the same conclusion. The four-way overdetermination means that rejecting any single route still leaves three others; rejecting all four would require defeating four structurally independent arguments simultaneously.

Entanglement is not action at a distance, and collapse is not collapse. Entanglement is joint commitment closure; the Lüders update is constraint selection on the post-commitment admissible structure, not a non-unitary dynamical process superimposed on the unitary one.

What fails without TPB. The diagnostic question for the architecture is: which of A0–A4 is responsible for forcing the linear, continuous, complex Hilbert representation? The answer is sharply localised. A0–A3 plus A1′, by themselves, are consistent with the classical probability simplex Δ_{N-1} as the pre-commitment representation: A0 supplies finiteness, A1′ supplies thermodynamic fact formation in its bidirectional form (which classical probability theory accommodates via stochastic dynamics with entropy bookkeeping), A2 supplies observer-protocol invariance (which classical mixtures respect), and A3 supplies an affine reversible group structure (which the discrete S_N action on Δ_{N-1} satisfies). The simplex satisfies all of A0–A3 + A1′ with $G^0 = \{\mathbb{1}\}$, $\dim V = N-1$ over $\mathbb{R}$, and no Hilbert-space machinery.

A4 is what breaks the simplex. TPB refinement compatibility forces, via Theorem 1, a non-trivial connected one-parameter subgroup $U(\mathbb{R}) \subset G$ — and the simplex's affine self-map group is discrete (Lemma 3), so the simplex cannot host such a $U(\mathbb{R})$. The corollary in §6.2 turns this incompatibility into an exclusion: the representation must strictly extend Δ_{N-1}, and Theorem 2 then forces the extension to be linear of dimension N over a field $F \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$. The remainder of the paper (§§7–14) selects $\mathbb{C}$, fixes the inner product, derives unitary evolution, and identifies the Hamiltonian, the Born rule, non-commutativity, entanglement, and the Lüders update.

A reader who suspends judgement on TPB sees the architecture cleanly: classical probability theory is the representation that emerges in the absence of A4; quantum mechanics is the representation that emerges when the substrate's commitment density is high relative to the observed timescale. Quantum mechanics, on this account, is precisely the structural shadow of TPB-derived continuity — the high-density limit of a discrete substrate. This sharpens the paper's distinctive claim: VERSF is not a flavoured operational reconstruction with the same operational axioms relabelled; the substrate work in A4 is what does the load-bearing job that operational reconstructions assign to a postulated continuity axiom.

A final note on the relationship between this paper and the broader VERSF programme. The principles A0–A4 are themselves motivated by, and constrained by, downstream derivations within VERSF — the fine-structure constant, the lepton mass hierarchy, the mixing matrices, the cosmological tensor-to-scalar ratio. The architecture established here is therefore not an isolated formal exercise. It is the representational level of a wider physical programme whose internal consistency provides cumulative evidence for the foundational principles, and whose differential predictions provide a path to empirical discrimination between the substrate-level account and QM-as-primitive.

Relationship to companion papers. This paper consolidates and grounds work distributed across nine VERSF companion papers, organised in §2.1 into three groups: substrate-level antecedents (Threshold, 2D), lateral derivations of QM structure (Architecture, Hamiltonian, DSR, CHS, Tick Race / RAL), and substrate-independence and measurement (Isosymmetry, Measurement-as-Commitment). The Architecture paper (Taylor, *Quantum Mechanics as the Architecture of Fact-Production*) establishes the umbrella fixed-point theorem from generic admissibility constraints; this paper is parallel to it but takes substrate-level routes at several steps — TPB-derived continuity (§5) replaces postulated R2; derived distinguishability preservation (§9.1) replaces informally-motivated R3; local tomography + substrate Hermitian + Galois invariance + Jordan classification + resonance jointly select $\mathbb{C}$ (§§7.1–7.4); envariance plus three independent routes (DSR, RAL distinguishability uniqueness, Tick Race first-passage) overdetermine the Born rule (§§11.5–11.7). The Threshold and 2D papers ground A1′ and A4 from below by establishing the topological and dimensional preconditions for irreversible fact formation. The CHS paper establishes complex Hilbert space via Galois invariance (third route in §7.4). The DSR paper establishes the Born rule via path-pair correlation kernels (§11.6). The Hamiltonian paper establishes the existence of H via Stone's theorem (cited at §10). The Tick Race / RAL paper supplies a parallel structural derivation plus the detector-level dynamical mechanism by which the Born rule is realised (§11.7, §14.3). The Isosymmetry paper extends the uniqueness claim across physical realisations by establishing that disparate substrates satisfying the admissibility task structure all yield the same Hilbert-space representation. The Measurement-as-Commitment paper develops the constraint-completion ontology of measurement (§14.3). Where each layer-specific companion addresses a single piece of the QM formalism in isolation, and the Architecture paper assembles all pieces from generic admissibility, this paper assembles all pieces from a single coherent axiom system A0, A1′, A2, A3, A4 with explicit substrate-level provenance. Each architectural step that was primitive in a companion paper is here either derived or cross-validated against an independent argument.

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19. Conclusion

Quantum mechanics, on the account developed here, is the representation theory of finite pre-commitment possibility under irreversible fact formation on a discrete simplicial substrate. The standard formalism — Hilbert space, Born rule, unitary evolution, Hamiltonian dynamics, non-commutativity, entanglement, Lüders update — is not a list of independent postulates. It is the unique stable representation, up to canonical isomorphism, forced by the five foundational principles of VERSF together with operational requirements that, in this framework, cease to be primitive and become consequences of substrate architecture.

The contribution sits between two existing programmes. Operational reconstructions show that QM is the unique theory satisfying certain axioms; they do not explain the axioms. Decoherence and envariance approaches explain specific elements of the formalism; they do not derive the architecture. VERSF closes the loop by grounding the operational axioms in substrate-level structure — and by deriving continuity, the deepest of the operational axioms, from the discrete TPB substrate.

The result is conditional on accepting the VERSF principles. Within the VERSF programme, those principles are independently motivated by downstream derivations of physical constants and cosmological parameters. The differential predictions of §17 provide a path to empirical discrimination between this substrate-level account and a reading of quantum mechanics that takes the formalism — including its continuity — as primitive.

Quantum mechanics, on this view, is what fact-forming reality looks like from the inside, viewed through a high-density coarse-graining of its discrete commitment substrate.

Appendix A: Where Each Step Does Real Work

Conclusion	Load-bearing principle(s)	VERSF-specific?
Non-selection of single alternative pre-commitment	A1′ + A2 (Lemma 2)	No
Affineness of pre-commitment dynamics	A3(ii) operational meaning of mixture	No
Strong continuity of U(t)	**A4 TPB + refinement (Theorem 1)**	**Yes (entirely)**
Linear extension of simplex	Theorem 1 + simplex rigidity (Lemma 3)	Yes (continuity is TPB-derived)
Complex amplitudes	Local tomography + closure-geometry Hermitian structure	Partial
Existence of inner product	Compactness of $K \subset G^0$ + Haar averaging (Lemma 4)	Partial

Conclusion	Load-bearing principle(s)	VERSF-specific?
Uniqueness of inner product up to scale	Schur's lemma (Lemma 5)	No
Wigner symmetries of state space	Wigner's theorem applied to Theorem 4	No
Distinguishability preservation $\lvert\langle U(t)\phi \vert U(t)\psi\rangle\rvert^2 = \lvert\langle\phi\vert\psi\rangle\rvert^2$	**A3 (reversibility) + A1′ (bidirectional Landauer) + Lemma 6 (converse-Landauer) + Relational Pre-Commitment Corollary + ray-level uniqueness (Theorem 4)**	**Yes (A1′ strengthened bidirectional reading; relational corollary derived not postulated)**
Selection of unitary over antiunitary	Connectedness of $\{U(t)\}$ from Theorem 1	Yes (via TPB-derived continuity)
Hamiltonian existence	Stone's theorem applied to Theorem 1's $U(t)$	Yes (via TPB)
Born rule, $p = 2$	Envariance + coarse-graining + continuity (Theorem 6)	No (envariance is general)
Lüders update rule	Constraint selection from new admissible set (§14)	Yes (interpretation)
Non-commutativity	Substrate-altering character of A1′ commitments	Yes (interpretation)
Entanglement	Joint commitment closure	Yes (interpretation)
Differential predictions (r, α, CKM, PMNS, Λ)	Substrate architecture beyond representation theory	Yes (entirely)
VERSF-specific content concentration	**§5 (TPB → continuity), §9.1 (A1′ + reversibility → distinguishability preservation, with relational corollary derived from A1′), §17 (substrate-level differential predictions)**	**Yes — the three sections where substrate-level argument is load-bearing**

The VERSF-specific content is concentrated in: the TPB derivation of continuity (Theorem 1, the central architectural improvement); the substrate Hermitian structure supplementing local tomography; the compactness input for Lemma 4; the interpretation of measurement, non-commutativity, and entanglement; and the differential predictions.

Appendix B: Comparison with Existing Reconstructions

Feature	Hardy 2001	Masanes–Müller 2011	CDP 2011	Höhn 2017	This paper
Linearity	Postulated	Postulated	Postulated	Postulated	**Derived**

Feature	Hardy 2001	Masanes–Müller 2011	CDP 2011	Höhn 2017	This paper
Continuity of evolution	**Postulated**	**Postulated**	**Postulated**	**Postulated**	**Derived from TPB**
Complex amplitudes	Local tomography + simplicity	Local tomography	Purification	Question complementarity	Local tomography + substrate Hermitian structure
Unitary vs antiunitary	Implicit	Implicit	Implicit	Implicit	Explicit (Wigner + connectedness)
Transition-probability preservation	**Postulated (in continuous reversibility)**	**Postulated**	**Postulated (causality)**	**Postulated**	**Derived from thermodynamic reversibility + relational pre-commitment content (Theorem 4)**
Born rule	Postulated	Derived (Gleason-style)	Derived	Derived	Derived (envariance, **non-circularity proven**)
Hamiltonian	Postulated	Postulated	Postulated	Postulated	**Derived (Stone's theorem applied to TPB)**
Measurement update	Standard collapse	Standard collapse	Standard	Standard	**Lüders as constraint selection**
Substrate-level grounding	None	None	None	None	**Yes (K=7, closure geometry, TPB)**
Differential predictions	None	None	None	None	**r, α, CKM, PMNS, Λ, UV deviations**

The present paper does not replace existing reconstructions; it grounds them. The most significant architectural improvement is the derivation of continuity (and consequently of the Hamiltonian) from the TPB substrate — a step that all prior reconstructions take as primitive.

Comparison with VERSF Companion Papers

The present paper sits within a wider VERSF programme. Companion papers are organised in §2.1 into substrate-level antecedents (Threshold, 2D), lateral derivations of QM structure (Architecture, Hamiltonian, DSR, CHS, Tick Race / RAL), and substrate-independence and measurement (Isosymmetry, Measurement-as-Commitment). The comparison tables below cover the lateral group (which addresses specific layers of the QM formalism) and the umbrella Architecture paper. Substrate-level antecedents and substrate-independence companions operate upstream and sideways of the QM-formalism derivation respectively and are summarised in §2.1.

Layer-specific companions

Feature	CHS (Taylor)	DSR (Taylor)	Hamiltonian paper (Taylor)	Tick Race / RAL (Taylor)	Measurement-as-Commitment (Taylor)	This paper
Selection of ℂ	**Galois invariance**	Implicit (path geometry)	Hilbert space assumed	**Resonance + Galois (§§13, 15)**	Hilbert space assumed	Local tomography + substrate Hermitian + Galois + Jordan + Resonance (§7.4)
Strong continuity	Taylor Limit (regularity)	Implicit	**Postulated (A2)**	Implicit	Implicit	**Derived from TPB (Theorem 1)**
Transition-probability preservation	Via metric isometry	Via path-pair structure	**Postulated (A3)**	Via Wigner + Stone (RAL §17)	Built into closure framework	**Derived (Theorem 4)**
Linear extension from simplex	Implicit	Implicit	Hilbert space assumed	Implicit	Hilbert space assumed	**Explicit (Lemma 3 + Theorem 2)**
Wigner + connectedness → unitary	Implicit	Implicit	**Full derivation (§5)**	Cited from Wigner	Implicit	Same argument; cites Hamiltonian paper
Hamiltonian via Stone's theorem	Lie generator	Not addressed	**Full derivation (§6)**	RAL §17	Not addressed	Cites Hamiltonian paper
Born rule	Not addressed	**Path-pair Mercer kernel**	Not addressed	**Distinguishability uniqueness (Th 14.1) + first-passage (Th 2.1)**	Not addressed	Envariance (cites DSR + Tick Race)
Lüders update	Not addressed	Indirect (via collapse)	GKSL (irreversible extension)	First-passage selects constraint	**No-Extra-Backaction (Th 2.1);**	Constraint-selection

Feature	**CHS (Taylor)**	**DSR (Taylor)**	**Hamiltonian paper (Taylor)**	**Tick Race / RAL (Taylor)**	**Measurement-as-Commitment (Taylor)**	**This paper**
					POVMs from substrate (Th 6.2)	framing (§14)
Detector-level dynamics	Not addressed	Not addressed	Not addressed	**First-passage race + metastable amplification (A1′–A3′)**	Constraint-completion ontology	Cites Tick Race for dynamics
Falsifiable predictions	Galois-violation experiments	Coherence-erasure entropy	None	**$k > 1$ detector deviations from Born statistics**	POVM substrate parameters {ℓ_c, τ_h}	r, α, CKM/PMNS, Λ, UV (§17)
Architectural scope	Hilbert space derivation	Born rule derivation	Hamiltonian derivation	Born rule + outcome selection mechanism	Measurement / collapse / POVM ontology	**Integrated architecture A0–A4 → full QM**

Umbrella companion: the Architecture paper

Feature	**Architecture paper (Taylor)**	**This paper**
Foundational primitives	A1–A3: recordability, finite resources, measurement realizability	A0–A4: finite distinguishability, irreversible commitment, observer-protocol invariance, reversible affine, TPB
Admissibility constraints	C1 (finite distinguishability via Th 3.2) + C2 (irreversible commitment)	A0 + A1′ (same content with strengthened bidirectional reading, taken as primitive)
Continuity of reversible evolution	**Postulated (R2)**	**Derived from TPB (Theorem 1)**
Distinguishability preservation	**Postulated (R3, "forced by admissibility")**	**Derived (Theorem 4)**
Convexity / linear extension	R1, derived from classical control (App E)	Lemma 3 + Theorem 2 (simplex rigidity)
Selection of ℂ	**Jordan classification (Koecher–Vinberg + JvNW)**	**Local tomography + substrate Hermitian + Galois + cites Jordan classification**
Born rule	**Gleason-style + operational non-contextuality (Th 7.1)**	**Envariance (cites DSR for path-pair route)**

Feature	Architecture paper (Taylor)	This paper
Hamiltonian	Stone (Th 5.2)	Stone (cites Hamiltonian paper)
Measurement / Lüders	CPTP / Kraus / no-extra-disturbance (Th 6.3)	§14, citing Measurement-as-Commitment
Relativistic extension	Clifford forcing → Dirac (§9)	Mentioned in §17 differential predictions
Tier I / Tier II distinction	Yes — admissibility-forced vs. selection-within-admissible-class	Implicit (axioms A0–A1′ force admissibility; A2–A4 select within)
Substrate-level grounding	Generic ("any fact-producing physics")	**Explicit (TPB, K=7 simplicial substrate, hexagonal closure geometry)**
Differential predictions	None — interpretation-independent	**$r \approx 0.027$–0.033, α^{-1}, CKM/PMNS, $\kappa=8/3$, two-Planck Λ, UV continuity deviations**

The Architecture paper and this paper establish the same overall fixed-point theorem — quantum mechanics is the unique structural framework compatible with admissibility — via different routes. The Architecture paper is interpretation-independent and substrate-neutral; the present paper is substrate-grounded in VERSF (TPB time, K=7 simplicial substrate, hexagonal closure geometry), which (a) lets it derive what the Architecture paper postulates (R2 → Theorem 1; R3 → Theorem 4) and (b) generates differential predictions distinguishing the substrate-level account from QM-as-primitive. Readers who do not accept the substrate architecture can read the present paper as a refinement of the Architecture paper's umbrella derivation; readers who do accept it gain access to differential predictions the umbrella does not deliver.

The present paper's contribution within the VERSF programme is **consolidation plus substrate grounding**: it derives, in a single coherent architecture, the structural core of finite-dimensional QM, while explicitly grounding axioms that the lateral companion papers take as primitive, structurally underwritten by the substrate-level antecedents (Threshold, 2D) and complemented by the substrate-independence and measurement papers (Isosymmetry, Measurement-as-Commitment). Where each layer-specific companion addresses one layer of the formalism in isolation, and the Architecture paper assembles all layers from generic admissibility, this paper assembles all layers from the same five foundational principles A0–A4 with explicit substrate-level provenance. Notation and axiom labels differ from those of the companion papers; cross-references are explicit at each load-bearing step.

Appendix C: Notational Conventions

Symbol	Meaning
$\mathcal{C}$	Finite set of admissible alternatives
$\mathcal{D}$	Finite admissible distinction set
Δ_{N−1}	Standard probability simplex of dimension N−1

Symbol	Meaning
G	Pre-commitment dynamical group (A3)
G^0	Lie-group identity-connected component of G
$K \subset G^0$	Maximal compact subgroup
T, T_m	Discrete tick map and its m-th-root refinement (A4)
W(n), W_m, $\tilde{W}$	Discrete-tick homomorphisms ℤ → G, ℤ → G, ℚ → G
U(t)	Strongly continuous one-parameter unitary group (Theorem 1, Theorem 5)
H	Hamiltonian (Stone generator of U(t))
ℋ	Pre-commitment Hilbert space
ℋ_S, ℋ_E	System, environment Hilbert spaces
K_X	Real linear span of unnormalised state cone of system X
P_i	Commitment probability for outcome i
p_i(	Ψ⟩)
σ_S^{(ij)}, σ_E^{(ij)}	Swap unitaries on system / environment
η	VERSF entropy conversion factor (η = 1)
α	Fine-structure constant
κ	Lepton mass hierarchy ratio (κ = 8/3) in §7.2; tick-rate scalar (λ_A = κ|ψ_A|^2) in §11.7 — the two usages refer to different scalars in their source companion papers and are kept distinct by context.
r	Tensor-to-scalar ratio for primordial gravitational waves
⟨·	·⟩
𝟙	Identity operator
†	Hermitian adjoint
U(N), V(N)	Unitary and antiunitary groups on ℂ^N

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VERSF Companion Papers (Taylor)

Each VERSF companion paper is referenced by the short tag used throughout the body. Working-paper drafts are available at versf-eos.com.

Substrate-level antecedents

Taylor, K. (2025). *The Topological Threshold for Fact Formation*. ["Threshold paper"]

Taylor, K. (2025). *Why Two Dimensions Are Not Emergent: The Tick–Bit Asymmetry and the Minimal Geometry of Distinguishability*. ["2D paper"]

Lateral derivations of QM structure

Taylor, K. (2025). *Quantum Mechanics as the Architecture of Fact-Production*. ["Architecture paper"]

Taylor, K. (2025). *The Hamiltonian as an Admissibility Generator in VERSF*. ["Hamiltonian paper"]

Taylor, K. (2025). *The Double Square Rule*. ["DSR paper"]

Taylor, K. (2025). *Complex Hilbert Space from Distinguishability Principles*. ["CHS paper"]

Taylor, K. (2025). *Quantum Measurement as a Tick Race: Deterministic Outcome Selection via First-Passage Dynamics*. ["Tick Race / RAL paper"]

Substrate-independence and measurement

Taylor, K. (2025). *Isosymmetry: Why Quantum Structure Is Independent of Physical Realisation*. ["Isosymmetry paper"]

Taylor, K. (2025). *Measurement as Commitment: Why Quantum Systems Are Relational*. ["Measurement-as-Commitment paper"]

Substrate architecture (cited as substrate-level inputs)

Taylor, K. (2025). *No-Go Theorem for Non-Simplicial Relational Substrates*. [Cited at §8.1, Lemma 4 remark, in establishing compactness of K.]

Taylor, K. (2025). *Closure-Geometry and the K=7 Architecture*. [Cited at §7.2 (Hermitian closure structure) and §8.1 (compactness of K).]