Spin as a Double-Cover Representation of Distinguishability Isometries in the VERSF Framework

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Plain Language Summary

When you turn yourself around once — through a full 360° rotation — you end up exactly where you started. This seems obvious. But quantum theory tells us that for many of the particles that make up ordinary matter, including electrons, a 360° rotation does *not* leave them unchanged: it flips a hidden quantum phase to its opposite. To restore the original state you have to rotate them *twice*, through 720°. This strange behaviour is what physicists call *spin-½*, and it underlies most of the matter that makes up you, this page, and the planet beneath you.

Standard quantum theory describes this accurately but does not really *explain* it. Spin enters the theory as an extra postulate, alongside other unexplained rules like the Born probability law and the use of complex numbers in the wave function.

This paper, working within the **Void Energy-Regulated Space Framework (VERSF)** — an attempt to derive quantum theory from a deeper principle about how distinguishable patterns relate to one another — shows that spin-½ is not an extra postulate. Once you accept that physical states are best understood as patterns of distinguishability rather than as objects with properties, the mathematics forces the existence of states that flip sign under one full rotation and recover only after two. There is no consistent way to write the theory down without them.

The result does *not* claim that every physical particle must be spin-½ — only that the *kind of state* required for spin-½ behaviour is unavoidable in the underlying mathematical framework. Whether nature actually fills that slot with real particles is a separate empirical question, settled by the Standard Model, not by this paper. We give an explicit calculation showing the 720° periodicity, point to the 1975 neutron-interferometry experiments that observed it directly, and outline (without claiming to prove) that the same logic might also account for the carriers of forces and even gravity.

The result is part of a broader VERSF programme aiming to derive each of the apparently unrelated postulates of quantum theory — complex amplitudes, the Born rule, unitary dynamics, and now half-integer spin — from a single underlying principle: that physical reality is built out of *relational closure* between distinguishable patterns.

Abstract

We give a reconstruction-level derivation of the structural necessity of half-integer spin representations within the admissible symmetry structure of the **Void Energy-Regulated Space Framework (VERSF)**. Building on prior results — in which complex Hilbert space, the Born rule, and unitary dynamics are derived from the geometry of distinguishable configurations — we show that (i) given a three-dimensional locally isotropic closure substrate (imported from the K = 7 simplicial closure result of VERSF–KSEVEN), the connected isometry group is SO(3), (ii) phase redundancy intrinsic to relational closure forces symmetries to act *projectively* on rays, and (iii) by Wigner's theorem and Bargmann's theorem on central extensions, the most general such projective representation lifts to a true linear representation of the universal cover **SU(2) → SO(3)**, with integer-spin representations descending to genuine SO(3) representations and half-integer-spin representations only projective on SO(3). The kernel $\mathbb{Z}_2 = \{+I, -I\}$ of the covering map is the source of the 4π periodicity. The minimal nontrivial irreducible representation of SU(2) is two-dimensional, yielding the spin-½ representation. We give an explicit worked example in Pauli-matrix form, computing U(2π) = −I and U(4π) = +I directly, and outline a *possible* pathway by which the same closure-representation logic may extend to spin-1 (gauge) and spin-2 (gravitational) sectors. We are careful to distinguish what is *derived* in this paper (the necessity of SU(2) as the linear lift of the projective SO(3) action, the structural availability of the spin-½ representation, the 4π periodicity) from what is *imported* ($\mathbb{C}$-amplitudes, $\mathscr{H}$, unitarity, three-dimensional substrate). The result of §4 is conditional; §§5–9 are binding for any framework satisfying that conditional.

Contents

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1. Introduction

Quantum theory attributes intrinsic angular momentum — *spin* — to elementary systems, with half-integer spin exhibiting the well-known property that a 2π rotation reverses the sign of the state vector while a 4π rotation restores it:

$$\psi(\theta + 2\pi) = -\psi(\theta),\ \psi(\theta + 4\pi) = +\psi(\theta).$$

The textbook explanation is representation-theoretic: spin-½ corresponds to the fundamental representation of SU(2), the universal double cover of SO(3). But this *describes* the structure rather than *explaining* it. Three questions remain unanswered in the standard treatment:

1. **Why does Hilbert space exist at all?**
2. **Why is the relevant rotation group SO(3) rather than something else?**
3. **Why do physical states transform under SU(2) rather than directly under SO(3)?**

The VERSF programme has previously addressed (1): complex amplitudes and $\mathscr{H}$ are derived from the distinguishability substrate together with admissibility axioms. This paper addresses (2) and (3). The argument proceeds as a chain of forced consequences from a small set of clearly-stated assumptions, summarised in §2.

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2. What Is Assumed and What Is Derived

To avoid the common pitfall of importing classical structure through the back door, we state the input/output ledger explicitly.

2.1 Imported from prior VERSF reconstruction (assumptions for *this* paper)

Imported result	Source
A distinguishability substrate Λ with metric d preserving an interference-compatible algebra	VERSF–CHS
$\mathbb{C}$-valued amplitudes (the field is forced; $\mathbb{R}$ and $\mathbb{H}$ are excluded)	VERSF–CHS
Complex Hilbert space $\mathscr{H}$ as the state space	VERSF–CHS
Reversible dynamics are unitary on $\mathscr{H}$	VERSF–CHS
Relational closure ontology (states are closure patterns, not object-properties)	VERSF–FSN
Simplicial K = 7 closure structure of the substrate	VERSF–KSEVEN

2.2 Derived in this paper

Derived result	Section
The continuous spatial isometry group is SO(3)	§4
Phase redundancy ⇒ physical state space is $\mathbb{P}(\mathscr{H})$	§5
Symmetries act projectively (Wigner)	§6
Mandatory lift to SU(2) (Bargmann + $\pi_1(SO(3)) = \mathbb{Z}_2$)	§7
Minimal nontrivial closure representation is dim 2 (spin-½ structurally available)	§8
Explicit $U(2\pi) = -I$, $U(4\pi) = +I$ via Pauli matrices	§9

The six imported items in §2.1 are *all* of the structural inputs. No spin operator, no SU(2), no rotation group, no 4π periodicity, and no spinor space is imported. Each appears as output.

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3. Distinguishability Geometry and the State Space $\mathscr{H}$

Let Λ be the relational substrate, equipped with metric

$d : \Lambda \times \Lambda \longrightarrow \mathbb{R}_+$

encoding pairwise distinguishability. Reversible physical transformations preserve d:

$g : \Lambda \longrightarrow \Lambda$, $d(g\cdot\lambda_i, g\cdot\lambda_j) = d(\lambda_i, \lambda_j)$.

The collection of such maps forms a group **G** — the isometry group of (Λ, d). Under the imported reconstruction, the state space carrying linear superpositions of distinguishable configurations is the complex Hilbert space $\mathscr{H}$, and reversible dynamics descend to a unitary representation

$U : G \longrightarrow \mathcal{U}(\mathscr{H})$, $g \mapsto U(g)$, $U(g)^\dagger U(g) = I$.

This is where the present paper begins.

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4. The Spatial Isometry Group Is SO(3): A Conditional Result

A referee-level concern about any reconstruction is whether classical spatial structure has been smuggled in. We address this by separating the section into two distinct claims: a *dependency* (substrate dimensionality, imported) and a *theorem* (the symmetry group, derived in this paper conditional on the dependency).

4.1 Conditional input: substrate dimensionality

We take as input the result of [VERSF–KSEVEN] that the closure-consistent VERSF substrate is **three-dimensional**; the derivation of this fact from K = 7 simplicial closure is given in that companion paper.

We denote the resulting locally Euclidean closure-neighbourhood by $\mathbb{E}_3$.

4.2 Definition: closure isotropy

Before invoking isotropy we make its content precise.

Definition (Closure Isotropy). A distinguishability substrate Λ is *closure-isotropic* if the automorphism group of its closure neighbourhoods acts transitively on the set of directions defined by closure relations. Equivalently: there exists no closure-invariant feature distinguishing one direction in $\mathbb{E}_3$ from another.

This definition does the work the word "isotropy" was previously asked to do tacitly. It states what is required of the substrate (transitive action on closure-defined directions) and what is *not* required (no metric on the directions need be assumed; the metric is inherited from d).

4.3 Theorem: the connected isometry group is SO(3)

We now prove the only claim §4 contributes to this paper.

Theorem 4.1. *Let $\mathbb{E}_3$ be a three-dimensional locally Euclidean closure neighbourhood that is closure-isotropic in the sense of §4.2. Then the connected component of the orientation-preserving distinguishability isometry group is SO(3).*

Proof sketch. The full isometry group of a three-dimensional locally Euclidean space is

$$\mathrm{Iso}(\mathbb{E}_3) = \mathbb{R}^3 \rtimes O(3).$$

We first separate translations from rotations. **Translations are factored out because closure patterns are invariant under global relabelling**: the relational substrate has no preferred origin, so a uniform shift of the substrate labels carries any closure pattern to a pattern with identical closure structure. Translations are therefore *automorphisms of substrate labelling rather than of closure structure*, and are removed at the outset. The same is not true for rotations, which can in principle act non-trivially on the closure relations defining the pattern; whether they do is precisely the question closure isotropy resolves.

The point-stabiliser of the remaining symmetry is therefore O(3). Its connected component containing the identity is

$$O(3)_0 = SO(3),$$

a fact that holds independently of any property of the substrate. Closure isotropy enters at a separate step: it ensures that this connected component acts as the *full* symmetry group of closure neighbourhoods rather than as some proper subgroup of it. Without isotropy, the action on directions could fail to be transitive, and the effective symmetry would reduce to a strict subgroup of SO(3); the §4.2 definition rules this out by stipulation. ■

4.4 Status of this section

The result of §4 is conditional: *given* a three-dimensional locally isotropic closure substrate, the connected isometry group is SO(3).

Crucially, however, **the spin derivation that follows is logically independent of how $\mathbb{E}_3$ and SO(3) are obtained.** Sections 5–9 establish that for *any* physical framework whose continuous spatial isometry group is SO(3) and whose state space is a complex Hilbert space modulo global phase, half-integer spin representations are forced into the representation theory. A reader unconvinced by §4.1 may therefore take SO(3) as an external input and still find §§5–9 fully binding. The architectural separation between dependency (§4.1) and theorem (§4.3) is what makes the §§5–9 derivation portable to any framework satisfying the §4.1 input.

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5. Relational Closure and the Projective State Space

In the VERSF account, a physical state encodes a *closure pattern* — a relational structure capable of producing facts under interaction. Because no closure event can register the global phase of an amplitude, two vectors related by a global phase represent the same physical state:

$|\psi\rangle \sim e^{i\theta} |\psi\rangle, \theta \in \mathbb{R}$.

This equivalence is not a convention. In a relational ontology, anything that no closure event can register is, by definition, not part of the physical state. Global phase is exactly such a quantity.

The physical state space is therefore the quotient

$\mathbb{P}(\mathscr{H}) = (\mathscr{H} \setminus \{0\}) / \sim$,

i.e. the set of *rays* (one-dimensional subspaces of $\mathscr{H}$). A physical state is a ray, not a vector. This is the geometric object on which symmetries must act.

5.1 Definition: closure representation

The phrase *closure representation* will recur throughout the remainder of the paper. We define it precisely.

Definition (Closure Representation). A *closure representation* of a distinguishability isometry group G is a unitary representation of G on a complex Hilbert space $\mathscr{H}$ that (i) preserves the transition-probability metric on $\mathbb{P}(\mathscr{H})$, and (ii) is compatible with the relational closure equivalence $|\psi\rangle \sim e^{i\theta}|\psi\rangle$ — i.e. descends to a well-defined action on ray space.

Within VERSF, every physical sector that transforms under G must furnish a closure representation in this sense. The remainder of the paper analyses what such representations must look like when G = SO(3).

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6. Symmetries Act Projectively: Wigner's Theorem

A symmetry of physical states is a bijection of $\mathbb{P}(\mathscr{H})$ preserving the only relational invariant that survives the quotient. Because relational closure registers exactly the transition probabilities between rays — and registers nothing else — the **transition probability**

$P(\psi, \varphi) = |\langle\psi \mid \varphi\rangle|^2 / (\langle\psi \mid \psi\rangle\langle\varphi \mid \varphi\rangle)$

is the unique invariant that any closure-respecting symmetry must preserve.

Wigner's theorem (1931). Every bijection of $\mathbb{P}(\mathscr{H})$ preserving P is induced by a map U on $\mathscr{H}$ that is either unitary or anti-unitary, and U is unique up to a phase.

Since transition probability is the only closure-invariant observable, any symmetry of the physical state space must preserve it. By Wigner's theorem, such a symmetry is induced by a unitary or anti-unitary map on $\mathscr{H}$ — and because that map is determined only up to a phase, the induced action on $\mathscr{H}$ is projective rather than linear.

For continuous symmetry groups connected to the identity — including SO(3) — the anti-unitary branch is excluded. The residual phase ambiguity, however, is irreducible:

$U(g_1)\, U(g_2) = e^{i\alpha(g_1, g_2)}\, U(g_1 g_2), \; \alpha : G \times G \longrightarrow \mathbb{R}.$

This is a **projective representation**: composition is preserved only up to a phase $\alpha(g_1, g_2)$, called a 2-cocycle. The cocycle is not an artefact of representation choice; it is a structural feature of how closure patterns transform on ray space.

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7. The Topology of SO(3) Forces a Double Cover

7.1 SO(3) is not simply connected

The rotation group has the topological property

$\pi_1(SO(3)) = \mathbb{Z}_2$.

A continuous loop of rotations corresponding to a 2π rotation about any axis is *not* contractible to a point in SO(3); the doubled loop (4π) is. This is the topological content of Dirac's belt trick.

7.2 The universal cover is SU(2)

The universal covering group of SO(3) is SU(2), the group of 2×2 unitary matrices of unit determinant. The covering map

$\pi : SU(2) \longrightarrow SO(3)$

is a 2-to-1 group homomorphism with kernel

$\ker(\pi) = \{+I, -I\} \cong \mathbb{Z}_2$.

7.3 Bargmann's theorem closes the lift

Bargmann's theorem (1954). Let G be a connected Lie group with Lie algebra $\mathfrak{g}$ satisfying $H^2(\mathfrak{g}, \mathbb{R}) = 0$. Then every projective unitary representation of G lifts to a true unitary representation of its universal cover $\tilde{G}$, and **the lift is unique up to homomorphisms into the centre of $\tilde{G}$**.

For G = SO(3) the universal cover is SU(2), whose centre is precisely $Z(SU(2)) = \{+I, -I\} \cong \mathbb{Z}_2$. The non-uniqueness of the lift is therefore exactly the $\mathbb{Z}_2$ ambiguity that distinguishes integer-spin from half-integer-spin representations — a feature, not a defect, of the construction.

The Lie algebra $\mathfrak{so}(3) = \mathfrak{su}(2)$ is semisimple; a direct calculation gives $H^2(\mathfrak{su}(2), \mathbb{R}) = 0$. Bargmann's hypothesis is satisfied.

Theorem (Mandatory Double Cover). *Within VERSF, the most general projective unitary representation of SO(3) on the physical state space requires lifting to a linear unitary representation of SU(2). The integer-spin representations form the subset that descends to genuine linear representations of SO(3); the half-integer-spin representations are faithful on SU(2) and only projective on SO(3).*

This formulation is precise: SO(3) does have linear unitary representations — they are exactly the integer-spin ones. What it lacks is a linear representation that captures the *full* projective structure forced by §6. The half-integer sector is accessible only via SU(2).

Why a strictly linear SO(3) representation is insufficient. A strictly linear representation of SO(3) eliminates the nontrivial $\mathbb{Z}_2$ topology of its fundamental group and therefore cannot capture the full closure-compatible symmetry structure. Concretely: a purely linear unitary representation of SO(3) on $\mathcal{H}$ would require $U(2\pi) = +I$ for every 2π rotation, since 2π is the identity in SO(3). But projective composition $U(g_1)U(g_2) = e^{i\alpha} U(g_1g_2)$ — forced by §6 — admits a nontrivial cocycle that takes the 2π loop to $-I$ rather than $+I$, and $\pi_1(SO(3)) = \mathbb{Z}_2$ is precisely what licenses this cocycle to be non-removable. Removing it would require simply-connectedness of SO(3), which fails. The −I element is therefore unavoidable in the general case, and **lifting to SU(2) is the only construction that captures all admissible closure representations**.

The 4π periodicity of the half-integer sector follows immediately: −I ∈ SU(2) covers the identity in SO(3), so a 2π rotation in physical space corresponds to multiplication by −1 on a half-integer-spin state, while a 4π rotation returns to +I.

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8. Spin-½ as the Minimal Nontrivial Closure Representation

The irreducible unitary representations of SU(2) are labelled by

$j \in \{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots\}$,

with representation space of dimension 2j + 1. Representations descend to genuine SO(3) representations precisely when $j \in \mathbb{Z}$; for half-integer j the representation is faithful on SU(2) and only projective on SO(3).

The trivial representation (j = 0) carries no rotational information. The smallest representation that genuinely uses the double-cover structure is

$j = \frac{1}{2}$, dim = 2.

A state in this representation is a **two-component spinor** $\chi \in \mathbb{C}^2$, transforming as

$\chi \longmapsto U(g)\,\chi$, $U(g) \in SU(2)$.

In VERSF terms, **spin-½ is the minimal nontrivial closure representation available within the framework**: the smallest dimensional closure representation that genuinely uses the double-cover structure forced by §§5–7. The result of this section is therefore a *structural availability* statement — it shows that the spin-½ representation must exist among the admissible closure representations of SO(3) — not a claim about which physical sectors must populate it.

The empirical realisation of particular representations is a separate physical question. Whether a given physical particle transforms as j = 0, ½, 1, … is determined by additional considerations (Standard Model field content, gauge structure, statistics) that lie outside the present derivation. What is established here is the *availability* and *non-removability* of the half-integer sector — that is, that any framework with the structural inputs of §2 must include j = ½ among its allowed closure representations.

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9. Worked Example: The Spinor Sign Flip

We make the abstract result concrete by writing down the explicit SU(2) action and computing the 4π periodicity directly.

9.1 The Pauli matrices

The generators of SU(2) are the three Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

They satisfy $[\sigma_j, \sigma_k] = 2i\, \varepsilon_{jkl}\, \sigma_l$, generating the Lie algebra $\mathfrak{su}(2)$.

9.2 The rotation operator

A rotation by angle θ about unit axis $\hat{\mathbf{n}} = (n_1, n_2, n_3)$ is represented on a spinor by

$U(\hat{\mathbf{n}}, \theta) = \exp(\, -i\, (\theta/2)\, \hat{\mathbf{n}}\cdot\boldsymbol{\sigma}\,)$,

where $\hat{\mathbf{n}}\cdot\boldsymbol{\sigma} = n_1\sigma_1 + n_2\sigma_2 + n_3\sigma_3$. Because $(\hat{\mathbf{n}}\cdot\boldsymbol{\sigma})^2 = I$ for any unit $\hat{\mathbf{n}}$, the exponential expands cleanly:

$U(\hat{\mathbf{n}}, \theta) = \cos(\theta/2)\, I - i \sin(\theta/2)\, (\hat{\mathbf{n}}\cdot\boldsymbol{\sigma})$.

The factor of θ/2 — half the geometric angle — is the explicit signature of the double cover.

9.3 Rotation about the z-axis

Take $\hat{\mathbf{n}} = \hat{\mathbf{z}}$, so $\hat{\mathbf{n}}\cdot\boldsymbol{\sigma} = \sigma_3$. Then:

$$U(\hat{z}, \theta) = \cos(\theta/2)\, I - i \sin(\theta/2)\, \sigma_3 = \begin{pmatrix} e^{(-i\theta/2)} & 0 \\ 0 & e^{(+i\theta/2)} \end{pmatrix}$$

Evaluate at the two key angles.

θ = 2π: cos(π) = −1, sin(π) = 0, so

$$U(\hat{z}, 2\pi) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I.$$

θ = 4π: cos(2π) = +1, sin(2π) = 0, so

$$U(\hat{z}, 4\pi) = \begin{pmatrix} +1 & 0 \\ 0 & +1 \end{pmatrix} = +I.$$

9.4 Action on a spinor

Let $\chi = (1, 0)^T$, an eigenstate of σ_3 with eigenvalue +1. Under rotation by θ about $\hat{\mathbf{z}}$:

$$\chi \;\mapsto\; U(\hat{z}, \theta)\,\chi \;=\; \begin{pmatrix} e^{(-i\theta/2)} \\ 0 \end{pmatrix}.$$

At **θ = 2π**: $\chi \mapsto (e^{(-i\pi)}, 0)^T = (-1, 0)^T = -\chi$. At **θ = 4π**: $\chi \mapsto (e^{(-i2\pi)}, 0)^T = (+1, 0)^T = +\chi$.

The spinor sign-flips after one full rotation and is restored only after two. The 4π periodicity is now explicit, computed, and concrete.

9.5 Empirical anchor

This sign-flip is not an interpretive curiosity. It was directly observed in the neutron-interferometry experiments of Rauch et al. (1975) and Werner et al. (1975), in which a neutron beam was split, with one arm passed through a magnetic field producing precisely a 2π precession. Recombination yielded destructive interference relative to the unrotated arm; doubling to a 4π precession restored constructive interference. The VERSF derivation reproduces this prediction without postulating spin as a primitive.

The empirical force of this is significant. Any framework — VERSF or otherwise — that fails to reproduce the SU(2) double-cover structure on rotation of fermionic states would be directly *falsified* by these experiments. The 4π periodicity is therefore not merely an elegant structural consequence of the reconstruction; it is a feature any candidate ontology of physics must reproduce on pain of conflict with measurement. The result of §§4–9 shows that VERSF passes this test by construction: nothing more than relational closure and the topology of the spatial isometry group is required to predict it.

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10. Interpretation: Why Half-Integer Representations Are Structurally Unavoidable

The result of §§5–9 is best stated as a structural-availability claim rather than as an existence puzzle. Conventionally, the question is asked as "why should anything sign-flip under 2π?" — a phrasing that invites a search for a *cause*. The VERSF analysis suggests the question admits a structural rather than causal answer: in any framework satisfying the inputs of §2, the half-integer sector cannot be removed *from the representation theory*. The relevant question is not why such representations exist; it is whether one could consistently exclude them, and the answer is that one cannot.

The structural force of the derivation can be summarised in three observations:

- Physical 3-space is itself emergent from the relational substrate (imported from VERSF–KSEVEN; see §4.1), so "rotation" is a closure-preserving relabelling, not an action on pre-given geometry.
- "State" is a ray in $\mathscr{H}$, not a vector, so any symmetry compatible with closure must act on rays.
- $\pi_1(SO(3)) = \mathbb{Z}_2$ then forces the projective action to admit a non-removable −I cocycle — and Bargmann's theorem tells us this cocycle is precisely the centre of the universal cover SU(2).

The doubled topology of SO(3) is therefore not a quirk of three-dimensional geometry but a structural feature of how closure patterns fit onto a manifold whose loop space has two homotopy classes. The 4π periodicity is the topological signature of relational closure under spatial reorientation — and **the half-integer sector is structurally available and irremovable from the representation theory** of any reconstruction of quantum theory whose state space is a complex Hilbert space modulo phase.

A careful caveat is in order. The claim above is about *availability*, not *population*. The argument establishes that the representation theory contains the half-integer sector and admits no consistent restriction to integer-spin-only representations. It does not establish that any particular physical theory must contain matter content transforming under that sector — a theory whose only fields were scalars would satisfy the §2 inputs and yet contain no half-integer states. The empirical question of which sectors are populated by physical particles is determined by additional inputs (Standard Model field content, gauge structure, statistics) beyond the scope of this paper. What is settled here is that the half-integer sector is a permanent feature of the admissible representation theory, not a feature one can choose to drop.

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11. Outlook: Toward Higher-Spin Closure Representations

The closure-representation principle naturally suggests an extension to j > ½. Each value of j corresponds to a distinct way that closure structure could transform consistently under the relational symmetry group. We outline — but do not claim to derive in this paper — a possible pathway:

Spin	Tentative closure interpretation	Standard physical role
0	Closure amplitude with no rotational structure	Order parameters, Higgs-like fields
½	Minimal nontrivial closure representation (this paper)	Matter spinors
1	Closure-preserving connection between rays	Gauge bosons (Yang–Mills)
2	Closure back-reaction on the substrate	Gravitational metric perturbations

The spin-1 case *suggests* that preserving the projective structure across distinguishable configurations requires a phase-transport connection — possibly the same object identified with gauge curvature in the BCB Lagrangian formulation of VERSF. The spin-2 case *suggests* that closure events back-react on the substrate that supports them, with a linearised representation that is symmetric and rank-2.

Taken together, these directions hint at a *possible* unified origin for spin, gauge structure, and gravitational coupling within the closure-representation framework. Whether such a unification is genuine — rather than merely suggestive — depends on companion derivations not provided here. The result of the present paper is the spin-½ derivation, full stop; the higher-spin and unification claims are explicitly conjectural.

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12. Discussion

12.1 What this derivation does *not* assume

To restate the input/output ledger of §2 in light of the full argument:

- No Hilbert space is postulated; $\mathscr{H}$ is a result of the prior reconstruction.
- No rotation group is postulated as a free-standing primitive; SO(3) is derived in §4 conditional on three-dimensional locally isotropic closure substrate (with three-dimensionality imported from VERSF–KSEVEN). The §§5–9 derivation is, however, fully binding for any framework satisfying that conditional.
- No spin operator or SU(2) is postulated; SU(2) appears as the universal cover required to capture the full projective representation theory of SO(3) (§7).
- No 4π periodicity is postulated; it is forced by $\pi_1(SO(3)) = \mathbb{Z}_2$ together with Bargmann's theorem.
- No spinor space is postulated; $\mathbb{C}^2$ appears as the minimal nontrivial irreducible representation of SU(2).

12.2 Relation to other reconstruction programmes

Approaches such as Hardy's five reasonable axioms, Masanes–Müller's information-theoretic reconstruction, and the categorical-quantum programme also derive Hilbert-space structure from operational primitives. VERSF differs in two ways. First, it grounds the operational primitives in a positive ontology — relational closure — rather than treating them as primitive operational facts. Second, **VERSF derives both the Hilbert-space structure and the rotational symmetry sector within a single framework, rather than treating spatial symmetry as an external classical input.** This is, to our knowledge, the principal structural differentiator between VERSF and existing reconstruction programmes.

12.3 What this paper does not address: spin–statistics

A natural follow-up question is whether the present derivation can be extended to obtain the spin–statistics theorem — the empirical fact that half-integer-spin systems obey Fermi–Dirac statistics and integer-spin systems obey Bose–Einstein statistics. **The connection between spin**

and statistics requires additional inputs (relativistic causal structure and microcausality, or equivalent locality conditions) and is not derived here. Establishing a closure-theoretic route to the spin–statistics theorem — for example, via the closure-event ordering that relational facts induce on spacelike-separated regions — is a natural next step but lies beyond the present paper.

12.4 What this paper is, and is not

This paper is **not** a theory of everything, a complete unification, or a derivation of the full Standard Model. It is a reconstruction-level result: the half-integer spin sector is shown to be a structurally available and irremovable feature of the representation theory entailed by the distinguishability-based reconstruction of quantum mechanics together with the topology of the spatial isometry group. The result places the half-integer sector alongside $\mathscr{H}$, the Born rule, and unitary dynamics on the list of features that earlier appeared as quantum postulates and now appear as derived consequences of distinguishability physics.

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13. Conclusion

We have shown that the half-integer spin sector is a *structurally available and irremovable* feature of the representation theory entailed by three independent inputs:

1. Physical states are equivalence classes of distinguishability amplitudes (relational closure ontology — imported from VERSF–FSN).
2. The continuous spatial isometry group is SO(3) — derived in §4 conditional on a three-dimensional locally isotropic closure substrate (a result imported from VERSF–KSEVEN).
3. Closure-respecting symmetries preserve transition probability on ray space (Wigner), and $\pi_1(SO(3)) = \mathbb{Z}_2$, so by Bargmann's theorem the most general projective unitary representation lifts to a true unitary representation of SU(2).

The minimal nontrivial closure representation is then the two-dimensional fundamental representation of SU(2) — spin-½ — and an explicit Pauli-matrix calculation confirms $U(2\pi) = -I$ and $U(4\pi) = +I$. The argument establishes structural availability and irremovability: the half-integer sector is a permanent feature of the admissible representation theory and cannot be excluded by any reconstruction satisfying inputs (1)–(3). Whether a given physical theory *populates* that sector with matter content is a separate empirical question, settled by inputs lying beyond the scope of this paper.

The derivation therefore shows not merely that spin-½ is compatible with quantum structure, but that it is unavoidable at the level of admissible representation theory.

The half-integer sector thus joins $\mathscr{H}$, the Born rule, and unitary dynamics on the list of derived structural features of distinguishability-based physics. Extensions to spin-1 (gauge) and spin-2 (gravitational) sectors, and to a closure-theoretic route to spin–statistics, are flagged as research directions rather than claims of this paper.

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Appendix A: Substrate Realization of SU(2) in the Hexagonal Closure Framework

The main argument of this paper establishes that SU(2) acts on physical states by a representation-theoretic route: phase redundancy (§5) forces symmetries to act projectively on rays (§6), and the topology $\pi_1(SO(3)) = \mathbb{Z}_2$ together with Bargmann's theorem forces the projective action to lift to SU(2) (§7). This argument tells us *why* the spatial isometry group must be represented via SU(2), but it does not tell us *what* the SU(2) acts on at the substrate level. This appendix closes that loop by summarizing the complementary derivation given in companion work [VERSF–HEX, Appendix D], which identifies the substrate-level realization of SU(2) within the hexagonal closure structure.

A.1 The triangular orientation field

The companion derivation begins from the observation that each committed hexagonal cell of the K = 7 closure substrate (cf. §4.1 and [VERSF–KSEVEN]) contains three orientation-opposed triangle pairs, defining a local internal orientation state. This state is naturally represented by a unit vector field

$n(x) \in S^2 \cong \mathbb{CP}^1$

where x labels coarse-grained substrate position. The field n(x) is *internal* (not spatial): its magnitude is fixed by closure, and only its orientation carries physical information.

A.2 Gauge redundancy from closure

Closure of the hexagonal cell fixes all internal degrees of freedom up to a local reorientation of the triangular pairs:

$n(x) \sim g(x) \cdot n(x), g(x) \in SO(3)$.

This redundancy is not a symmetry of dynamics but a *redundancy of description*: closure eliminates absolute orientation information, so physical observables depend only on relative orientations. Promoting this local redundancy to a gauge structure and applying standard sigma-model coarse-graining yields, by the locality + isotropy + gauge-invariance arguments of [VERSF–HEX, Appendix D], a Yang–Mills sector with structure group SO(3).

A.3 The mandatory lift to SU(2)

The substrate-level group obtained directly from the orientation sigma model is SO(3), not SU(2). However, the physically relevant excitations of the substrate include localized spinorial defects (cf. §8 of the present paper), which require the double-cover representation. Equivalently: spin-½ representations cannot exist on a strictly SO(3) bundle, since $\pi_1(SO(3)) = \mathbb{Z}_2$ obstructs a globally consistent assignment of two-component spinors. The lift

$SU(2) \to SO(3)$

is therefore mandatory at the substrate level for exactly the same topological reason it is mandatory at the representation-theoretic level (§7.3 of this paper). The two derivations identify the *same* obstruction — the non-removable $\mathbb{Z}_2$ in $\pi_1(SO(3))$ — and force the *same* resolution — passage to the simply connected double cover.

A.4 Convergence with the main argument

This yields a two-route convergence on the SU(2) structure:

Route	Starting point	Argument	Endpoint
§§5–9 (representation-theoretic)	Phase redundancy ⇒ projective rays	Wigner + Bargmann + $\pi_1(SO(3)) = \mathbb{Z}_2$	SU(2) acts on physical states
[VERSF–HEX, Appendix D] (substrate-dynamical)	Triangular orientation field n $\in S^2$	Closure-induced gauge redundancy + SO(3) → SU(2) lift for spinorial defects	SU(2) acts on substrate orientation states

The two routes are logically independent — the main paper makes no use of the hexagonal substrate at all, and the companion derivation makes no use of Wigner or Bargmann — yet both arrive at SU(2) via the same topological obstruction. Within the VERSF programme, the spinor χ

$\in \mathbb{C}^2$ of §8 acquires a concrete substrate interpretation: it is the spinorial lift of the orientation state $n(x) \in S^2$ of the triangular subsector, with the lift forced by exactly the topology that the representation-theoretic argument identifies.

A.5 Status of this appendix

This appendix is a cross-paper bridge, not an independent derivation. The substrate-level result it summarizes — the emergence of SU(2) Yang–Mills from the triangular orientation field of the hexagonal closure substrate — is established as a conditional theorem in [VERSF–HEX, Appendix D, Theorem D.1] under the assumptions (H1′) orientation closure, (H2′) local redundancy, (H3′) locality and isotropy, and (H4′) coarse-graining. The full proof, including the sigma-model construction and the Yang–Mills uniqueness argument, is given there. The point of including this appendix in the present paper is to make visible the convergence between the two routes and to give the abstract spinor $\chi \in \mathbb{C}^2$ of §8 a concrete substrate referent within the broader VERSF programme.