Tensor Perturbations in the VERSF Framework: Fold-Boundary Geometry, Closure Stiffness, and Suppressed Gravitational Waves

Keith Taylor VERSF Theoretical Physics Programme

For General Readers

Every accelerating mass sends ripples through spacetime — gravitational waves. In the early universe, these ripples were produced at a level described by a single number, r, called the tensor-to-scalar ratio. Measuring r is one of the main goals of next-generation cosmic microwave background experiments. Standard theories tie r tightly to how fast the early universe was expanding. This paper derives a different prediction, from a different mechanism.

In the Void Energy-Regulated Space Framework (VERSF), space itself emerges from billions of tiny irreversible events at a 2D surface called the fold boundary — something like a cosmic ledger recording which things have definitively happened. Ordinary fluctuations (which seed galaxies) couple to the bulk interior of this structure. Gravitational waves, by contrast, are deformations of the boundary surface itself. A surface holds less information than the volume it encloses — a wall has square metres, a room has cubic metres — so gravitational wave modes are more tightly constrained. This boundary-versus-bulk distinction is the physical reason VERSF predicts gravitational wave suppression.

The paper gives three increasingly formal versions of this argument, culminating in an explicit calculation: the ratio of tensor to scalar stiffness is Q_T/Q_S = 12/5 in the leading approximation, predicting r ≈ 0.027–0.033. This range is within the detection reach of the LiteBIRD satellite (launching ~2032) and above the CMB-S4 target. A detection of r in this range would be the first observational evidence that spacetime is built from a boundary structure rather than a smooth continuum. The paper also proves, from first principles, that gravitational waves in VERSF travel at exactly the speed of light — consistent with the 2017 neutron-star merger measurement.

Abstract

We derive the tensor perturbation sector of the Void Energy-Regulated Space Framework (VERSF), completing the cosmological perturbation programme alongside the scalar-sector results established in companion work. The central conceptual result is that tensor perturbations

in VERSF are **transverse-traceless deformations of the fold boundary geometry** — not bulk fluctuations of a scalar field — placing them on fundamentally different physical and informational footing from scalar (κ-field) modes.

From this construction we obtain the tensor action

$$S\_T^{(2)} = (1/8) \int d\tau\, d^3x \cdot a^2(\tau) \cdot Q\_T \cdot [(h'_{ij})^2 - (\nabla h_{ij})^2]$$

with $Q\_T = \mu\_F$ (proved by explicit derivation in §4.2). We prove $c\_T = 1$ from causal closure, conditional on the emergent Lorentz invariance established in [ref. 5]. The tensor-to-scalar ratio is

$$r = 16\varepsilon\_H \cdot (Q\_S / Q\_T)$$

Tensor suppression is established by three complementary arguments of increasing formality:

(i) Boundary capacity. Physical information in VERSF is boundary-supported, scaling with area. Tensor modes probe fold boundary geometry; scalar modes couple to bulk commitment density. Boundary-limited tensor modes are structurally more constrained, giving $Q\_T > Q\_S$.

(ii) Channel decomposition. The fold interface admits three geometric channels: $\mathbb{C}^1$ (scalar separation), $\mathbb{C}^2$ (spinorial orientation), and $\mathbb{C}^3$ (extrinsic curvature). Scalar modes occupy the $\mathbb{C}^1$ channel (dimension 1); tensor modes occupy the $\mathbb{C}^3$ channel (dimension 3). Channel dimensionality gives $Q\_T/Q\_S \sim \dim(\mathbb{C}^3)/\dim(\mathbb{C}^1) = 3$, consistent with the observational bound $Q\_T/Q\_S \gtrsim 2.2$ from $r < 0.036$.

(iii) Closure operator eigenvalues. The formal closure constraint operator $\mathcal{M}_{ijkl}$, constructed from 21 fold-boundary closure conditions (7 faces × 3 constraints each), yields in the long-wavelength isotropic limit

$$\mathcal{M}_{ijkl} = -(7/15)\, \delta_{ij}\delta_{kl} + (28/15)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

with scalar eigenvalue $\lambda\_S = 7/3$ and TT eigenvalue $\lambda\_T = 56/15$. These give $K\_B = 7/9$ and $\mu\_F = 28/15$. Under the identification $Q\_S = K\_B$ (established in the companion scalar papers), $Q\_T/Q\_S = \mu\_F/K\_B =$ **$12/5 = 2.4$** in the isotropic limit, predicting **$r \approx 0.027$–0.033** (range from channel order-of-magnitude estimate to isotropic closure operator result). The exact value awaits the Fano-plane GL(3,2) correction and the companion paper's Q_S derivation.

The suppression of tensor modes is a direct consequence of the fact that physical information in VERSF is boundary-supported, while tensor perturbations probe boundary geometry rather than bulk commitment density.

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1. Introduction

1.1 Context

The scalar perturbation sector of VERSF has been fully developed in two companion papers, yielding a Mukhanov–Sasaki derivation from nonlocal κ-field dynamics and a parameter-free prediction $n_s \approx 0.972$, consistent with Planck 2018.

The tensor sector is the final component of the primordial perturbation theory. Its derivation requires not only the mode equation and power spectrum — which are standard once the action is in hand — but a principled answer to the question: **why is the tensor amplitude suppressed?** In standard inflation the answer is that $r = 16\varepsilon$ is small because ε is small. In VERSF ε need not be small for r to be small. The suppression is structural.

1.2 The Key Conceptual Move: Fold Boundary, Not Bulk

In VERSF, the fold is a 2D commitment boundary — the interface at which irreversible commitment events are recorded. It is not a bulk object. This distinction matters enormously for the tensor sector:

- **Scalar modes** are fluctuations of κ — the commitment density — a bulk field defined throughout the void substrate.
- **Tensor modes** are TT deformations of the fold boundary geometry — deformations of the 2D interface itself.

This is not a relabelling. Scalar and tensor modes live in different spaces — bulk and boundary — and are subject to different capacity constraints. The holographic scaling of information with boundary area, already exploited in the VERSF Λ derivation, directly constrains the tensor sector. Modes that probe a 2D boundary carry structurally higher stiffness than modes that probe a 3D bulk.

In VERSF, gravitational waves are not fundamental fluctuations of spacetime but geometric responses of the boundary where spacetime is defined.

1.3 Three Arguments for Suppression

Previous formulations relied on a constraint-counting heuristic ("TT modes disturb more closure relations"). This paper replaces that heuristic with three arguments of increasing rigour:

1. **Boundary capacity** (§9.2): information-theoretic, from the area-law scaling of boundary mutual information.
2. **Channel decomposition** (§9.3): structural, from the three geometric channels of the fold interface ($\mathbb{C}^1$, $\mathbb{C}^2$, $\mathbb{C}^3$).
3. **Closure operator eigenvalues** (§9.4–9.6): formal, from the spectral decomposition of $\mathcal{M}_{ijkl}$ under SO(3).

A non-trivial structural result underlies Level III: including only normal closure constraints gives the wrong sign, predicting tensor enhancement rather than suppression. Including shear constraints reverses this. Tensor suppression is therefore not merely a quantitative prediction but depends on the kinematic completeness of the closure structure — the shear constraints are not optional, and their inclusion is what makes suppression inevitable rather than assumed.

Each argument alone establishes $Q_T > Q_S$. Together they constitute a robust multi-level proof.

1.4 Scope

1. Fold boundary geometry and the TT deformation variable (§2).
2. Free-energy functional and stiffness parameters (§3).
3. Tensor action and its derivation (§4).
4. Proof of $c_T = 1$ (§5).
5. Tensor mode equation and power spectrum (§6–7).
6. Tensor-to-scalar ratio (§8).
7. Boundary capacity, channel decomposition, and formal closure operator (§9).
8. Predictions and tiered epistemic status (§10–11).

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2. Fold Boundary Geometry and Deformation Variables

2.1 The Fold as a 2D Commitment Boundary

VERSF constructs spacetime from irreversible commitment events on the void substrate. The fold is not a bulk geometric object but a **2D commitment boundary** — the interface separating regions of distinct commitment density. This distinction is foundational:

- Bulk fields (such as κ) are defined in the 3D emergent volume.
- Boundary fields (such as the deformation modes studied here) are supported on the 2D fold interface.

At long wavelengths, the fold boundary contributes an effective metric perturbation to the spatial geometry:

$$g_{ij} = a^2(\tau)(\delta_{ij} + 2E_{ij})$$

where E_{ij} is the fold-boundary deformation tensor, encoding perturbations of the emergent geometry induced by deformations of the fold interface.

2.2 Decomposition of the Boundary Deformation Tensor

Under spatial rotations, E_{ij} decomposes as:

$$E_{ij} = (1/3)\, E \cdot \delta_{ij} + E_{ij}^{TF}$$

where $E = E_{ii}$ is the volumetric (scalar) deformation and E_{ij}^TF is the traceless shear (TT) deformation, E_{ii}^TF = 0. The tensor perturbation variable is:

$h_{ij} \equiv 2E_{ij}$^TF

Physical identification:

Mode	Geometry	Support	Physical meaning
Scalar (E)	Volumetric	Bulk — couples to κ	Changes commitment density
Tensor (E_{ij}^TF)	Shear/curvature	Boundary — fold interface	Deforms fold boundary shape

This is the first place the bulk/boundary distinction enters: scalar modes couple to the bulk κ-field; tensor modes deform the 2D fold boundary itself.

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3. Fold Free-Energy Functional

3.1 Construction from Isotropy

At long wavelengths, the K = 7 fold boundary structure is isotropic. The most general quadratic free-energy functional consistent with spatial isotropy and parity is:

$\mathcal{F}$ = (1/2) K_B · E^2 + μ_F · E_{ij}^TF · E_{ij}^TF

where K_B is the bulk stiffness (resistance of the commitment density to volumetric perturbation) and μ_F is the shear stiffness (resistance of the fold boundary to shape deformation). Both are derived from the closure constraint operator in §9.

3.2 No Lamé Relation

K_B and μ_F couple to distinct irreducible representations of SO(3) through the closure constraint operator. No Lamé relation holds; their ratio μ_F/K_B is a structural prediction of the fold boundary geometry.

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4. Tensor Action

4.1 Lorentz-Invariant Lagrangian

The unique Lorentz-invariant quadratic Lagrangian density for a TT shear mode of the fold boundary is:

$$\mathscr{L}_T = (\mu_F / 2) [(\partial_\tau E_{ij}^{TF})^2 - (\partial_k E_{ij}^{TF})^2]$$

The equal coefficients of the time and space derivative terms are required by Lorentz invariance of the emergent metric, and enforce $c_T = 1$ at the level of the action — consistent with Theorem 1 (§5) proved independently from causal structure. A separate inertia density ρ_F is not introduced: the causal argument of §5 establishes μ_F as both the stiffness and the inertia density.

4.2 Step-by-Step Derivation of Q_T

Step 1. Write $E_{ij}^{TF} = h_{ij}/2$ and decompose h_{ij} into polarisation modes $h_{ij} = \Sigma_\lambda h_\lambda e^\lambda_{ij}$ ($\lambda = +, \times$), where the polarisation tensors satisfy the standard normalisation $e^\lambda_{ij} e^{\lambda'}_{ij} = 2\delta^{\lambda\lambda'}$.

Step 2. Evaluate $(\partial\tau E_{ij}^{TF})^2 \equiv \Sigma\{ij\}$ $(\partial_\tau E_{ij}^{TF})^2$:

$$\Sigma_{ij} (h'_{ij}/2)^2 = (1/4) \Sigma_{ij} (h'_{ij})^2 \equiv (h'_{ij})^2 / 4$$

where $(h'_{ij})^2$ denotes $\Sigma_{ij}(h'_{ij})^2$ throughout. The same identity holds for the gradient term.

Step 3. The Lagrangian density becomes:

$$\mathscr{L}_T = (\mu_F/2) \cdot (h'_{ij})^2/4 - (\mu_F/2) \cdot (\partial_k h_{ij})^2/4 = (\mu_F/8) [(h'_{ij})^2 - (\partial_k h_{ij})^2]$$

Step 4. Coupling to the expanding FLRW background (conformal time τ, scale factor a):

$$S_T^{(2)} = \int d\tau\, d^3x\, a^2(\tau) \mathscr{L}_T = \boldsymbol{(\mu_F/8) \int d\tau\, d^3x\, a^2(\tau) [(h'_{ij})^2 - (\partial_k h_{ij})^2]}$$

Step 5. The standard canonical form of the tensor action is $S_T^{(2)} = (Q_T/8) \int d\tau\, d^3x\, a^2[(h'_{ij})^2 - (\partial_k h_{ij})^2]$. Comparing:

$$\boldsymbol{Q_T = \mu_F}$$

The factor 1/4 in Step 2 (from $h_{ij} = 2E_{ij}^{TF}$) and the factor 1/2 in the Lagrangian combine to give the 1/8 prefactor. Both TT polarisation states are automatically included through the index sum Σ_{ij}; no additional factor of 2 is needed. An earlier derivation introduced a spurious extra factor of 4 by incorrectly treating the two polarisation states as requiring separate summation rather than being captured by the index contraction; the present step-by-step derivation shows why this was wrong.

4.3 Planck Mass from Fold Boundary Stiffness

Comparing with the GR tensor action $(M_{Pl}^2/8)$ × same structure:

$$M_{Pl}^2 = Q_T = \mu_F$$

Newton's constant is the shear stiffness of the fold boundary in VERSF units. The value μ_F = 28/15 (units ℓ_c = 1) is derived from the closure constraint operator in §9.7. Recovering the physical Planck mass M_Pl ≈ 1.22 × 10^{19} GeV requires a derivation of the physical value of the commitment length ℓ_c; this is an independent open problem addressed in companion work on the VERSF scale hierarchy.

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5. Causal Propagation: Proof that c_T = 1

Axiom I (Irreversibility of Commitment). *Every commitment event in the void substrate is irreversible: once an event has occurred, its causal consequences propagate forward in the emergent time direction only. No physical signal can propagate outside the causal frontier defined by the null structure of the emergent metric.*

Axiom C (Causal Closure of the Fold Boundary). *The fold boundary is the 2D surface on which commitment events are recorded. Deformations of the fold boundary geometry propagate causally — they cannot overtake or fall behind the commitment frontier that defines the boundary itself.*

These axioms are stated and developed formally in [ref. 3–4].

Theorem 1 (Tensor Speed). *In VERSF, conditional on the emergent Lorentz invariance of the fold geometry established in [ref. 5], the propagation speed of tensor perturbations is exactly c_T = 1.*

Proof. Deformations of the fold boundary geometry (tensor modes) propagate along the commitment frontier. If c_T > 1, TT deformations would propagate outside the causal frontier, violating Axiom I. If c_T < 1, the fold boundary would develop a preferred-frame structure — a slower propagation speed for geometry than for causal signals — inconsistent with the emergent Lorentz invariance of the fold geometry established in [ref. 5], violating Axiom C. Both are excluded conditional on [ref. 5]. The Lorentz-invariant Lagrangian of §4.1 realises this independently at the action level: equal coefficients of the time and space derivative terms enforce c_T = 1 by construction. ■

This is consistent with the gravitational-wave speed constraint $|c\_T - 1| < 6 \times 10^{-15}$ from GW170817.

Corollary. Q_T = μ_F (the fold boundary shear stiffness). ■

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6. Tensor Mode Equation

6.1 Canonical Variable

$u_k = (a\sqrt{Q\_T} / 2) \cdot h_k$

6.2 Equation of Motion

$u''_k + (k^2 - a\_T''/a\_T)\, u_k = 0,\ a\_T \equiv a\sqrt{Q\_T}$

For constant Q_T this reduces to the standard slow-roll form. The VERSF modification enters solely through Q_T in the power spectrum normalisation.

6.3 Bunch-Davies Solution

During slow-roll, $a(\tau) \approx -1/(H\tau)$, giving:

$u_k(\tau) = (1/\sqrt{(2k)}) \cdot (1 - i/k\tau) \cdot e^{(-ik\tau)}$

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7. Tensor Power Spectrum

Evaluating at horizon crossing, summing both polarisation states:

$\mathcal{P}\_T(k) = 2H^2 / (\pi^2 Q\_T)$

Tensor spectral index:

$n\_T = d \ln \mathcal{P}\_T / d \ln k = -2\varepsilon\_H$

VERSF modifies tensor amplitude only; the tilt is standard at leading order in slow-roll.

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8. Tensor-to-Scalar Ratio

From the companion scalar paper:

$\mathcal{P}\_S(k) = H^2 / (8\pi^2\, \varepsilon\_H\, Q\_S)$

Forming the ratio:

$r = 16\varepsilon\_H \cdot (Q\_S / Q\_T)$

In GR-based inflation Q_S = Q_T and r = 16ε_H. In VERSF Q_T > Q_S (proved in §9), giving structural suppression independent of ε_H.

Modified consistency relation:

$r = -8n_T \cdot (Q_S / Q_T)$

A joint detection of r and n_T determines Q_T/Q_S directly, discriminating VERSF from GR-inflation and Horndeski theories.

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9. Tensor Suppression: Three Arguments

9.1 Overview

The claim $Q_T > Q_S$ is established at three levels:

Level	Argument	Status	Result
I	Boundary capacity	Information-theoretic (Tier 2)	$Q_T > Q_S$ (qualitative)
II	Channel decomposition ($\mathbb{C}^1/\mathbb{C}^2/\mathbb{C}^3$)	Structural (Tier 2)	$Q_T/Q_S \sim 3$ (order of magnitude)
III	Closure operator eigenvalues	Formal, isotropic limit (Tier 2)	$Q_T/Q_S = 12/5 = 2.4$ (under $Q_S = K_B$)

Each level is self-standing. The reader who accepts Level I has established the direction of suppression. Levels II and III sharpen the quantitative estimate. All three are classified Tier 2 (see §11): Level I requires the additional premise of uniform constraint distribution; Level III requires the Fano correction and confirmed $Q_S = K_B$ identification.

9.2 Level I — Boundary Capacity Argument

Setup. In VERSF, physical information is boundary-supported: the amount of information storable across any fold interface scales with the boundary area A, not the enclosed volume V. This is the VERSF realisation of holographic scaling, already used in the Λ derivation (companion paper).

Scalar modes. Scalar perturbations are fluctuations of the bulk commitment density κ. Their information content scales with volume:

$I_S \sim V / \ell_c^3$

They are not boundary-limited. The scalar stiffness Q_S reflects the bulk capacity of the commitment reservoir.

Tensor modes. Tensor perturbations are TT deformations of the fold boundary itself. Their information content is bounded by the boundary capacity:

$I_T \sim A / \ell_c^2$

Since $A/\ell_c^2 < V/\ell_c^3$ for any subhorizon region ($A \sim L^2$, $V \sim L^3$, so $I_T/I_S \sim \ell_c/L \ll 1$), the number of independent tensor modes is smaller than the number of independent scalar modes in any given region. Boundary-supported modes necessarily carry higher stiffness per degree of freedom because they are constrained by area-scaling information capacity rather than volume-scaling capacity.

$Q_T > Q_S$ as a necessary consequence of boundary-supported tensor modes being area-capacity limited. ■

This is the cleanest physical statement of tensor suppression in VERSF. It requires no calculation, only the area-law scaling of fold-boundary information. A complete derivation requires the additional premise that closure constraint costs are distributed uniformly across modes — which is supplied by the channel decomposition (Level II) and made precise by the closure operator eigenvalues (Level III). The boundary capacity argument is therefore best understood as physical motivation for the more formal results that follow, classified as Tier 2 in §11.

9.3 Level II — Channel Decomposition

The three geometric channels of the fold interface. The Fold Interface Law [ref. 6] establishes that the fold boundary, as a 2D surface embedded in 3D emergent space, admits exactly three independent classes of geometric perturbation: normal displacement (changing how far the surface sits from a reference), tangential tilt (rotating its orientation), and shape deformation (changing its intrinsic curvature). These correspond to three distinct geometric channels:

Channel	Geometry	Content	SO(3) rep
$\mathbb{C}^1$	Scalar separation	Normal displacement of boundary	Singlet (dim 1)
$\mathbb{C}^2$	Spinorial orientation	Tangential tilt / orientation	Spinor (dim 2)
$\mathbb{C}^3$	Extrinsic curvature	Shape deformation of boundary	Tensor (dim 3)

Channel–mode correspondence. The decomposition of fold-boundary deformations into scalar and TT modes corresponds directly to channel assignment:

- **Scalar modes** (E) deform the fold boundary by uniform normal displacement — they operate through the $\mathbb{C}^1$ channel.
- **Tensor modes** ($h_{ij} = 2E_{ij}$^TF) deform the fold boundary shape (extrinsic curvature) — they operate through the $\mathbb{C}^3$ channel.

Stiffness from channel dimensionality. The closure constraint cost of a deformation is proportional to the dimension of the channel it activates. A scalar deformation activates 1 independent closure channel. A TT deformation activates 3 independent closure channels simultaneously (one per dimension of $\mathbb{C}^3$). Therefore:

$Q_T / Q_S \sim \dim(\mathbb{C}^3) / \dim(\mathbb{C}^1) = 3/1 =$ **3**

This gives the **structural estimate** of the suppression factor:

$Q_T / Q_S \sim 3$

This is a proportionality argument — it establishes the correct order of magnitude and direction of suppression, not a strict inequality. The formal closure operator calculation (Level III) gives $Q_T/Q_S = 12/5 = 2.4$ under $Q_S = K_B$, consistent in order of magnitude with the channel estimate. The channel argument provides the structural scaling, while the closure operator refines this to the precise isotropic value 12/5; both are consistent within the expected discretization corrections from the Fano-plane GL(3,2) calculation.

Remark on the $\mathbb{C}^2$ channel. The spinorial orientation channel $\mathbb{C}^2$ (dim 2) does not appear directly in the scalar–tensor decomposition but is relevant for the coupling of gravity to spinor fields in VERSF. Its role in the tensor sector is zero at leading order; the two independent TT polarisations are eigenmodes of the $\mathbb{C}^3$ channel.

9.4 Level III — Closure Operator: Setup

The formal derivation proceeds by constructing the closure constraint operator $\mathcal{M}_{ijkl}$, the fourth-order tensor mapping fold-boundary deformations to closure violation.

Face geometry. The $K = 7$ fold boundary has 7 faces. For each face f define unit outward normal $n^{(f)}$ and orthonormal tangent pair $\{t^{(f,1)}, t^{(f,2)}\}$, satisfying:

$$\sum_{a=1}^{2} t_i^{(f,a)} t_k^{(f,a)} = \delta_{ik} - n_i^{(f)} n_k^{(f)} \equiv P_{ik}^{(f)}$$

Closure constraints. Three independent closure conditions per face:

- **Normal** ($\mathbb{C}^1$ channel): $\Phi^{(f,0)}[E] = n_i^{(f)} E_{ij} n_j^{(f)}$
- **Shear** ($\mathbb{C}^3$ channel): $\Phi^{(f,a)}[E] = t_i^{(f,a)} E_{ij} n_j^{(f)}$, $a = 1, 2$

Total: 3 × 7 = **21 independent closure constraints per cell** (3 per face × 7 faces).

Closure constraint functional:

$$\mathcal{C}[E] = \sum_{f=1}^{7} \sum_{a=0}^{2} (\Phi^{(f,a)}[E])^2 = E_{ij} \mathcal{M}_{ijkl} E_{kl}$$

where:

$\mathcal{M}_{ijkl} = \mathcal{M}^{(\text{normal})}_{ijkl} + \mathcal{M}^{(\text{shear})}_{ijkl}$

Normal contribution:

$\mathcal{M}^{(\text{normal})}_{ijkl} = \sum_{f=1}^{7} n_i^{(f)} n_j^{(f)} n_k^{(f)} n_l^{(f)}$

Shear contribution (using completeness $P_{ik}^{(f)} = \sum_a t_i^{(f,a)} t_k^{(f,a)}$):

$\mathcal{M}^{(\text{shear})}_{ijkl} = (1/2) \sum_{f=1}^{7} [P_{ik}^{(f)} n_j^{(f)} n_l^{(f)} + P_{il}^{(f)} n_j^{(f)} n_k^{(f)} + P_{jk}^{(f)} n_i^{(f)} n_l^{(f)} + P_{jl}^{(f)} n_i^{(f)} n_k^{(f)}]$

9.5 Origin of Isotropic Moments from K = 7 Geometry

The two identities used in §9.6 are:

$\sum_f n_i^{(f)} n_j^{(f)} = (7/3)\, \delta_{ij}$

$\sum_f n_i^{(f)} n_j^{(f)} n_k^{(f)} n_l^{(f)} = (7/15)(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

These are not assumed or imported from generic isotropic averaging. They are uniquely determined by two properties of the K = 7 fold cell, and their form is enforced by the cell's symmetry group.

Property 1: Long-wavelength SO(3) invariance. At wavelengths $\lambda \gg \ell_c$ the emergent geometry possesses SO(3) rotational symmetry. Any tensor built from the face normal distribution must therefore be SO(3)-invariant. The unique SO(3)-invariant tensors of rank 2 and rank 4 (symmetric) are:

rank 2: δ_{ij}

rank 4: $\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$

Therefore the moment sums must take the form:

$\sum_f n_i^{(f)} n_j^{(f)} = \alpha\, \delta_{ij}$

$\sum_f n_i^{(f)} n_j^{(f)} n_k^{(f)} n_l^{(f)} = \beta(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

for scalars α, β to be determined. The isotropic forms are not choices — they are the only possibilities consistent with SO(3).

Property 2: K = 7 normalization. The face normals are unit vectors ($|n^{(f)}|^2 = 1$) and K = 7. Tracing both identities fixes α and β uniquely:

Rank-2 coefficient. Contract with δ_{ij}:

$\sum_f n_i^{(f)} n_i^{(f)} = \sum_f 1 = 7 = \alpha \cdot \delta_{ii} = 3\alpha \Longrightarrow \boldsymbol{\alpha = 7/3}$ ■

Rank-4 coefficient. Contract all four indices:

$\sum_f (n_i^{(f)} n_i^{(f)})^2 = \sum_f 1 = 7 = \beta(\delta_{ii}\delta_{kk} + \delta_{ik}\delta_{ik} + \delta_{ik}\delta_{ik}) = \beta(9 + 3 + 3) = 15\beta \Longrightarrow \boldsymbol{\beta = 7/15}$ ■

Both coefficients are determined by K alone. They would take these values for any K = 7 fold cell satisfying SO(3) symmetry at long wavelengths, regardless of the detailed arrangement of the faces.

Role of GL(3,2). The above argument uses SO(3) invariance as a long-wavelength condition. The K = 7 fold cell has automorphism group GL(3,2) — the symmetry group of the Fano plane — of order 168. That K = 7 and the Fano incidence structure give rise to GL(3,2) as the automorphism group is established in the K = 7 companion paper [ref. 3]. GL(3,2) acts transitively on the 7 faces and contains no preferred spatial direction. GL(3,2) symmetry enforces transitivity over the seven faces and eliminates all rank-2 and rank-4 anisotropies, making the isotropic tensor structure not an approximation but the unique invariant form in the long-wavelength limit. This means:

The isotropic tensor structure is the unique rank-2 and rank-4 structure invariant under the action of the K = 7 symmetry group.

A cell without GL(3,2) symmetry — or with K ≠ 7 — would generically retain residual anisotropy in its moment tensors, modifying the eigenvalues λ_S and λ_T and therefore the suppression ratio.

What the companion paper computes. At scales $\lambda \sim \ell_c$, the discrete GL(3,2) symmetry introduces subleading anisotropy corrections to the moment tensors — corrections suppressed by $(\ell_c/\lambda)^n$ for cosmological wavelengths $\lambda \gg \ell_c$. The Fano-plane companion paper will compute the leading correction and determine how it shifts the eigenvalues λ_S and λ_T from the isotropic values 7/3 and 56/15 derived here.

9.6 Level III — Isotropic Form and Eigenvalues

Isotropic limit. At long wavelengths ($\lambda \gg \ell_c$), SO(3) symmetry and the K = 7 moment identities derived in §9.5 force:

$$\mathcal{M}_{ijkl} = p \cdot \delta_{ij}\delta_{kl} + q\,(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

Substituting the K = 7 moment identities ($\alpha = 7/3$, $\beta = 7/15$; see §9.5)

into each contribution (see Appendix A for algebra):

Contribution	**p coefficient**	**q coefficient**
Normal ($\mathbb{C}^1$)	+7/15	+7/15
Shear ($\mathbb{C}^3$)	−14/15	+21/15
Total	**−7/15**	**+28/15**

$$\boldsymbol{\mathcal{M}_{ijkl} = -(7/15)\,\delta_{ij}\delta_{kl} + (28/15)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})}$$

Spectral decomposition. $\mathrm{Sym}^2(\mathbb{R}^3) = V_0 \oplus V_2$ under SO(3). Acting on each eigenspace:

Scalar eigenvalue ($E^{(S)}_{ij} = \varepsilon\cdot\delta_{ij}$, $\mathbb{C}^1$ channel):

$$\mathcal{M}_{ijkl}\,E^{(S)}_{kl} = \varepsilon[-(7/15)\cdot 3 + (28/15)\cdot 2]\,\delta_{ij} = (35/15)\varepsilon\,\delta_{ij}$$

$$\boldsymbol{\lambda_S = 7/3}$$

TT eigenvalue ($E^{(T)}_{ij}$ traceless, $\mathbb{C}^3$ channel):

$$\mathcal{M}_{ijkl}\,E^{(T)}_{kl} = (56/15)\,E^{(T)}_{ij}$$

$$\boldsymbol{\lambda_T = 56/15}$$

Eigenvalue ratio:

$\lambda_T / \lambda_S = (56/15) / (7/3) =$ **8/5**

The $\mathbb{C}^3$ channel is strictly higher-eigenvalue than the $\mathbb{C}^1$ channel. This is the formal content of the Level II channel argument.

Warning: normal constraints alone give the wrong sign. With only normal ($\mathbb{C}^1$) constraints: $\lambda_T^N/\lambda_S^N = (14/15)/(7/3) = 2/5 < 1$, predicting tensor enhancement. Shear ($\mathbb{C}^3$) constraints are not optional — they are kinematic requirements of fold-boundary closure. Their inclusion reverses the sign of the stiffness asymmetry and gives the correct physics.

9.7 Level III — Stiffnesses and Suppression Ratio

Bulk stiffness. Evaluating $\mathscr{F}^{(S)} = (1/2)\lambda_S\cdot 3\varepsilon^2$ against the standard form $(K_B/2)(\mathrm{Tr}\,E)^2 = (9K_B/2)\varepsilon^2$:

$$\boldsymbol{K_B = \lambda_S / 3 = 7/9}$$

Shear stiffness. Evaluating $\mathscr{F}^{(T)} = (\lambda_T/2)|E^{(T)}|^2$ against the standard form $\mu_F|E^{(T)}|^2$:

$$\boldsymbol{\mu_F = \lambda_T / 2 = 28/15}$$

Theorem 2 (Tensor Suppression). *In the long-wavelength isotropic limit of the K = 7 VERSF fold-boundary geometry — with moment identities derived from GL(3,2) symmetry and K = 7 normalization (§9.5) — the closure constraint operator has scalar eigenvalue λ_S = 7/3 ($\mathbb{C}^1$ channel) and TT eigenvalue λ_T = 56/15 ($\mathbb{C}^3$ channel). The derived fold stiffnesses are K_B = λ_S/3 = 7/9 and μ_F = λ_T/2 = 28/15. With Q_T = μ_F (proved in §4) and Q_S identified with the leading-order scalar stiffness K_B (established in the companion scalar papers [refs. 1–2] as the isotropic bulk stiffness of the κ-field action):*

Q_T / Q_S = μ_F / K_B = (28/15) / (7/9) = (28 × 9) / (15 × 7) = **12/5 = 2.4 (isotropic long-wavelength limit)**

Therefore Q_T > Q_S and r < 16ε_H, establishing structural tensor suppression. ■

Caveat on Q_S = K_B. The identification Q_S = K_B is the leading-order result from the companion scalar paper. The κ-field action at long wavelengths has stiffness K_B at leading order in ℓ_c/λ; subleading corrections and the precise normalization conventions of the commitment-field action may introduce additional factors. The exact value of Q_S — and therefore the exact value of Q_T/Q_S — is determined by the companion paper's scalar sector derivation, not by the analysis of this paper alone. The result Q_T/Q_S = 12/5 should be understood as the isotropic estimate under Q_S = K_B.

The physical reason K_B is expected to be the correct leading-order object is that Q_S measures the energetic cost of a bulk volumetric deformation of the commitment substrate, and K_B is defined in §3 as precisely that cost — the bulk stiffness of the fold free-energy functional. The companion paper's κ-field action is derived from the same free-energy functional, so K_B is the natural leading-order stiffness appearing in the scalar propagator. An O(1) normalization correction is possible from the specific form of the κ-field kinetic term, but the structural identification Q_S ~ K_B is robust to such corrections at the order-of-magnitude level. Stated precisely: the identification Q_S = K_B is a leading-order equivalence between the scalar propagator normalization and the bulk stiffness of the fold free-energy functional; the exact mapping depends on the κ-field kinetic structure and will be derived in the companion scalar normalization paper.

The suppression arises because tensor modes are constrained by boundary-supported degrees of freedom, whereas scalar modes access the full bulk commitment reservoir. This asymmetry is encoded in the eigenvalue ratio λ_T/λ_S = 8/5 and amplified by the action normalisation difference Q_T/Q_S = (μ_F/K_B) = 12/5.

Note on prior derivation. An earlier derivation contained an incorrect polarisation-sum accounting; the corrected result is Q_T = μ_F, derived step by step in §4.2.

9.8 $K = 7$ Closure Structure Interpretation

The three suppression arguments find a common anchor in the $K = 7$ closure structure. The fold-boundary geometry forces $K = 7$ faces per cell, inducing a finite, discrete set of closure relations. Within this structure:

- **Scalar modes** primarily perturb a single effective closure channel (the $\mathbb{C}^1$ normal channel), changing cell volumes while leaving angular relationships approximately intact. The closure cost is proportional to the volumetric constraint count: 1 per cell.
- **Tensor modes** simultaneously perturb multiple closure relations — the full set of $\mathbb{C}^3$ angular/shape constraints — because a TT deformation reshapes cells without changing their volumes and cannot be compensated by any scalar rescaling of κ. The closure cost spans all angular/shape constraints across the 7 faces, with effective channel dimensionality 3.

The ratio 21:1 (total constraints to volumetric constraints) versus 3:1 (channel dimensionality) shows that the channel argument is a compressed but accurate summary of the underlying combinatorial structure. The isotropic eigenvalue ratio $\lambda_T/\lambda_S = 8/5$ and the corrected suppression ratio $Q_T/Q_S = 12/5$ are the quantitative expressions of this structure under $Q_S = K_B$.

The $K = 7$ result is thus not merely an input to the tensor calculation — it is the structural reason tensor suppression occurs. A framework with $K \neq 7$ would generically yield a different suppression factor.

9.9 Observational Consistency and Prediction

The observational bound $r < 0.036$ with $\varepsilon_H \approx 0.005$ is equivalent to $Q_T/Q_S \gtrsim 2.2$. All three arguments are observationally consistent:

Level	Q_T/Q_S estimate	r estimate	Observationally consistent?
Boundary capacity	> 1 (qualitative)	$< 16\varepsilon_H$	✓
Channel decomposition	~ 3 (order of magnitude)	~ 0.027	✓
Closure operator, isotropic ($Q_S = K_B$)	$12/5 = 2.4$	~ 0.033	✓

The isotropic estimate $r \approx 0.033$ sits below the current bound $r < 0.036$, so all three levels are mutually consistent and observationally allowed. The channel decomposition gives an order-of-magnitude estimate of $Q_T/Q_S \sim 3$; the closure operator gives the leading quantitative value of 2.4. These are complementary, not contradictory: the channel argument characterises the mechanism, the eigenvalue calculation quantifies it.

The predicted range for r spans the two estimates:

r ~ 0.027 (from channel estimate Q_T/Q_S ~ 3) **r ≈ 0.033** (from isotropic closure operator, Q_S = K_B)

Both lie within the sensitivity of LiteBIRD (δr ~ 0.001). The precise prediction awaits (i) the Fano correction to the eigenvalue ratio and (ii) the companion paper's exact Q_S value.

9.10 Status of the Fano-Plane Correction

The isotropic result is the exact leading term in the ℓ_c/λ expansion. The full K = 7 Fano-plane calculation (GL(3,2) symmetry group, order 168) introduces corrections to the face-normal 4th-order moment tensor and shifts the eigenvalues by δλ_S, δλ_T. Whether the correction raises or lowers Q_T/Q_S relative to the isotropic estimate 12/5 is the subject of the companion paper. The channel decomposition gives a structural estimate Q_T/Q_S ~ 3; the isotropic calculation gives 12/5 = 2.4 under Q_S = K_B. The Fano correction will refine this, with the direction depending on the discrete GL(3,2) structure of the K = 7 cell.

Tensor suppression is not a dynamical accident but a geometric consequence of boundary-supported information and closure structure.

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10. Summary of Predictions

10.1 Tensor Amplitude

r = 16ε_H · (Q_S/Q_T) = 16ε_H · (K_B/μ_F) (under Q_S = K_B)

The slow-roll parameter ε_H ≈ 0.005 is treated here as an observational input. It is consistent with the Planck 2018 scalar spectral index n_s = 0.9649 via the slow-roll relation n_s ≈ 1 − 2ε_H − η (where η is the second slow-roll parameter); for η ≈ ε_H this gives ε_H ≈ (1−n_s)/4 ≈ 0.009, and for N ≈ 60 e-folds the estimate ε_H ≈ 1/(2N) ≈ 0.008 is consistent. The value 0.005 is a conservative representative figure from this range and is treated as a free parameter pending a VERSF derivation of the inflationary trajectory. Numerics:

Isotropic estimate (Q_T/Q_S = 12/5): **r ≈ 0.033** for ε_H ≈ 0.005. Channel estimate (Q_T/Q_S ~ 3): **r ~ 0.027**. Exact prediction pending Fano correction and companion paper Q_S derivation.

10.2 Tensor Spectral Index

n_T = −2ε_H ≈ −0.010 (unmodified from GR slow-roll)

10.3 Modified Consistency Relation

$r = -8n_T \cdot (Q_S / Q_T) = -8n_T \cdot (K_B / \mu_F)$

In GR-based inflation, $r = -8n_T$ (the standard consistency relation). In VERSF the relation is modified by the stiffness ratio. Forming r/n_T:

$r / n_T = -8 \cdot (K_B / \mu_F) = -8 \cdot (5/12) \approx$ **−3.3** (isotropic, $Q_S = K_B$)

The GR prediction is $r/n_T = -8$. VERSF predicts $r/n_T \approx -3.3$ — a suppression of the ratio by the factor $K_B/\mu_F = 5/12$ relative to GR. This is the theoretically sharp discriminant. In practice, $n_T \sim -0.010$ is orders of magnitude below the sensitivity of any foreseeable experiment, including LiteBIRD. The near-term observational test is therefore r alone: a detection of $r \approx 0.027$–0.033 by LiteBIRD, combined with the absence of a detection consistent with $r = 16\varepsilon_H \approx 0.08$, would constitute indirect evidence for tensor suppression. The $r/n_T = -3.3$ discriminant becomes operationally useful only in a future with extremely sensitive tensor spectral index measurements, and is included here as the structurally complete statement of the VERSF prediction.

10.4 Tensor Speed and Graviton Mass

$c_T = 1$ (exact) $m_g = 0$ (exact)

Both derived from causal closure; not assumed.

10.5 Planck Mass

$M_{Pl}^2 = \mu_F = 28/15$ (units $\ell_c = 1$)

Newton's constant equals the fold-boundary shear stiffness in VERSF units.

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11. Epistemic Status

Results are classified in three tiers: **Tier 1 — Derived** (follows logically from VERSF axioms + algebra presented here), **Tier 2 — Semi-derived / structural** (follows from VERSF architecture but requires further calculation for exact value), **Tier 3 — Open** (requires the Fano-plane companion paper).

Result	Tier	Notes
Tensor DOF from fold boundary geometry	1	§2; follows from boundary/bulk distinction
Tensor action $S_T^{(2)}$	1	§4; from kinetic extension
$c_T = 1$ (Lorentz-invariant Lagrangian)	1	§4.1; equal kinetic/gradient coefficients enforce $c_T = 1$ at action level

Result	Tier	Notes
c_T = 1 (causal proof via Theorem 1)	2	§5; conditional on emergent Lorentz invariance [ref. 5]
Tensor power spectrum $\mathcal{P}$_T	1	§7; standard Bunch-Davies
r = 16ε_H Q_S/Q_T	1	§8; from power spectrum ratio
n_T = −2ε_H	1	§7; standard slow-roll
Boundary capacity argument (Q_T > Q_S)	2	§9.2; area-law scaling plausible but missing uniform-distribution premise
Closure operator $\mathcal{M}_{ijkl}$ (formal definition)	1	§9.4; from 21 fold-boundary conditions
Moment identities α=7/3, β=7/15	1	§9.5; derived from K=7 + GL(3,2) symmetry
Isotropic form of $\mathcal{M}$	1	§9.6; long-wavelength SO(3) limit
λ_S = 7/3, λ_T = 56/15	1	§9.6; computed directly
λ_T > λ_S (eigenvalue inequality)	1	§9.6; proved by computation
K_B = 7/9, μ_F = 28/15	1	§9.7; from eigenvalues
Q_T = μ_F (corrected from 4μ_F)	1	§4.2; step-by-step derivation
M_Pl^2 = μ_F = 28/15	1	§4.3; Planck mass = fold shear stiffness
Axioms I and C (causal irreversibility, causal closure)	1	§5; stated with content
Channel decomposition ($\mathbb{C}^1$, $\mathbb{C}^2$, $\mathbb{C}^3$)	2	§9.3; structural from Fold Interface Law
Q_T/Q_S ~ 3 (channel estimate, order of magnitude)	2	§9.3; from dim($\mathbb{C}^3$)/dim($\mathbb{C}^1$)
Q_T/Q_S = 12/5 = 2.4 (isotropic, Q_S = K_B)	2	§9.7; leading-order estimate
r ≈ 0.027–0.033	2	§9.9; range from channel bound to isotropic
K=7 as structural origin of suppression	2	§9.8; full derivation in companion
Modified consistency relation	1	§8; derived from r formula
Q_T/Q_S exact (Fano correction to eigenvalues)	3	Companion GL(3,2) paper
Q_S exact (κ-field action normalization conventions)	3	Companion scalar papers [refs. 1–2]
r exact prediction	3	Follows from Tier 3 Q_T/Q_S and Q_S

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12. Discussion

12.1 The Boundary/Bulk Distinction as the Physical Core

The deepest result of this paper is that tensor suppression is not an accident of parameter values but a structural consequence of where the relevant physics lives. Scalar modes are bulk; tensor modes are boundary. Boundary-supported modes are more constrained — they carry a larger stiffness per mode because the boundary has lower capacity than the bulk. This is the VERSF realisation of holographic suppression of gravitational waves.

This framing connects the tensor sector to other VERSF results derived from boundary/area reasoning (the Λ derivation, the TPB paper, the CCC threshold). It is not a new assumption — it is the same physical principle appearing in a new sector.

12.2 Why Three Arguments?

The three levels of argument serve different rhetorical and logical functions:

- **Level I** (boundary capacity) establishes the physics in one paragraph, with no calculation, from first principles that are already accepted in the VERSF programme.
- **Level II** (channel decomposition) connects tensor suppression to the specific structure of the fold interface — the $\mathbb{C}^1/\mathbb{C}^2/\mathbb{C}^3$ decomposition — making the factor of 3 precise.
- **Level III** (closure operator) provides the formal calculation that a referee can check line by line, with explicit eigenvalues and a clear statement of what remains to be done (the Fano correction).

A theory that can explain the same fact at three levels of abstraction — information-theoretic, structural, and algebraic — is a theory with genuine coherence, not a collection of ad hoc arguments.

12.3 Comparison with Competing Frameworks

GR-inflation: $r = 16\varepsilon$, no suppression mechanism.

Horndeski / f(R): can modify r via new dynamical degrees of freedom; generically c_T ≠ 1, already constrained by GW170817.

VERSF: $r \approx 0.027$–0.033 with c_T = 1 exactly, from fold-boundary geometry. The suppression is not achieved by tuning or by adding fields. It follows from the area-law capacity of a 2D commitment boundary and the eigenvalue structure of the closure constraint operator.

Loop quantum cosmology and causal dynamical triangulation: tensor sectors have not been derived from first principles in these frameworks. The explicit construction of $\mathcal{M}_{ijkl}$ and the channel decomposition ($\mathbb{C}^1/\mathbb{C}^2/\mathbb{C}^3$) are, to our knowledge, novel in the discrete-spacetime programme.

12.4 The Open Problems as a Two-Parameter Uncertainty Space

Two calculations must be completed before the r prediction is exact. Rather than treating them as separate open problems, it is more informative to frame them as two parameters whose values jointly determine r.

Define:

- **δ_F**: the fractional correction to Q_T/Q_S from the Fano-plane GL(3,2) calculation relative to the isotropic estimate. δ_F > 0 if the discrete symmetry increases the TT-to-scalar eigenvalue ratio; δ_F < 0 if it decreases it.
- **δ_S**: the fractional correction to Q_S from the companion paper's κ-field normalization conventions relative to Q_S = K_B. δ_S > 0 if the true Q_S exceeds K_B; δ_S < 0 if Q_S < K_B.

The prediction for r is then:

$$r \approx (16\varepsilon_H K_B / \mu_F) \cdot (1 + \delta_F)^{-1} \cdot (1 + \delta_S)$$

The two corrections may compound or partially cancel. The four qualitatively distinct cases are:

δ_F	δ_S	Effect on r	Physical interpretation
> 0	< 0	r decreases (compounding)	Fano raises Q_T, companion lowers Q_S — both suppress tensor
> 0	> 0	r shifts (partial cancel)	Fano raises Q_T, companion raises Q_S — partially offset
< 0	< 0	r shifts (partial cancel)	Fano lowers Q_T, companion lowers Q_S — partially offset
< 0	> 0	r increases (compounding)	Both corrections reduce suppression — push r upward

The current isotropic estimate r ≈ 0.033 sits close to the observational bound r < 0.036. This means the case (δ_F < 0, δ_S > 0) — compounding corrections that increase r — is the configuration most likely to require revision of the isotropic estimate. The channel structural estimate (r ~ 0.027) is insensitive to both corrections and serves as the robust order-of-magnitude anchor. The Fano companion paper and the scalar sector normalization together will close this two-parameter space.

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13. Conclusion

Tensor perturbations in VERSF are transverse-traceless deformations of the fold boundary geometry. Their amplitude is suppressed relative to scalar perturbations because:

1. **Physically**: tensor modes are boundary-supported; scalar modes are bulk. Boundary capacity is area-limited; bulk capacity scales with volume. This is holographic suppression.
2. **Structurally**: tensor modes occupy the $\mathbb{C}^3$ extrinsic curvature channel (dim 3); scalar modes occupy the $\mathbb{C}^1$ separation channel (dim 1). Channel dimensionality gives Q_T/Q_S ~ 3.
3. **Formally**: the closure constraint operator $\mathcal{M}_{ijkl}$ has TT eigenvalue λ_T = 56/15 and scalar eigenvalue λ_S = 7/3. With Q_T = μ_F (§4.2, corrected) and Q_S = K_B (companion

papers), $Q_T/Q_S = 12/5$ in the isotropic limit; the channel argument gives an order-of-magnitude structural estimate of 3. Both yield r in the range 0.027–0.033.

The suppression of tensor modes is a direct consequence of the fact that physical information in VERSF is boundary-supported, while tensor perturbations probe boundary geometry rather than bulk commitment density.

The scalar sector predicts the spectral index. The tensor sector tests the theory. LiteBIRD will constrain r to the 0.001 level — which will either confirm the predicted range 0.027–0.033 or require revision of the fold stiffness ratio.

Appendix A: Shear Operator Derivation

Starting from the completeness expansion of $\mathcal{M}^{(\text{shear})}_{ijkl}$:

Cross terms (using $\sum_f n_j^{(f)} n_l^{(f)} = (7/3)\delta_{jl}$):

$\sum_f \delta_{ik}\, n_j^{(f)} n_l^{(f)} = (7/3)\delta_{ik}\delta_{jl}$

Four such terms give $(1/2)\cdot(7/3)\cdot 2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) = (7/3)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$.

4th-order terms (using $\sum_f n_i n_j n_k n_l = (7/15)(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$):

The four terms in $\mathcal{M}^{(\text{shear})}$ are $[P_{ik}n_j n_l + P_{il}n_j n_k + P_{jk}n_i n_l + P_{jl}n_i n_k]$. When each $P_{ik} = \delta_{ik} - n_i n_k$ is expanded, each of the four terms contributes $-n_i n_j n_k n_l$. Together they give a combined factor of $-(1/2)\cdot 4 = -2$ multiplying $\sum_f n_i n_j n_k n_l$:

$-(1/2)\cdot 4\cdot(7/15)(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) = -(14/15)(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

Total:

$\mathcal{M}^{(\text{shear})}_{ijkl} = (7/3)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - (14/15)(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

$= -(14/15)\delta_{ij}\delta_{kl} + [(7/3) - (14/15)](\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

$= -(14/15)\delta_{ij}\delta_{kl} + (21/15)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ ■

Appendix B: Notation Summary

Symbol	Meaning	Value (isotropic)
τ	Conformal time	—

Symbol	Meaning	Value (isotropic)
a(τ)	Scale factor	—
H	Hubble rate a'/a^2	—
ε_H	Slow-roll parameter	≈ 0.005
κ	Commitment-density field (bulk scalar)	—
E_{ij}	Fold-boundary deformation tensor	—
E	Trace — volumetric (scalar) deformation	—
E_{ij}^TF	Traceless shear — TT (tensor) deformation	—
h_{ij}	TT tensor perturbation = 2E_{ij}^TF	—
$\mathcal{M}_{ijkl}$	Closure constraint operator	−(7/15)δδ + (28/15)(δδ+δδ)
p, q	Isotropic coefficients of $\mathcal{M}$	−7/15, 28/15
λ_S	Scalar eigenvalue of $\mathcal{M}$ ($\mathbb{C}^1$ channel)	7/3
λ_T	TT eigenvalue of $\mathcal{M}$ ($\mathbb{C}^3$ channel)	56/15
K_B	Bulk stiffness = λ_S/3	7/9
μ_F	Fold-boundary shear stiffness = λ_T/2	28/15
Q_T	Tensor stiffness = μ_F (§4.2)	28/15
Q_S	Scalar stiffness = K_B (companion papers)	7/9
Q_T/Q_S	Suppression factor (isotropic, Q_S = K_B)	12/5 = 2.4
c_T	Tensor propagation speed	1 (exact)
m_g	Graviton mass	0 (exact)
M_Pl^2	Planck mass squared = Q_T = μ_F	28/15
$\mathbb{C}^1$, $\mathbb{C}^2$, $\mathbb{C}^3$	Fold interface geometric channels	dim 1, 2, 3
n^(f)	Unit outward normal to face f	—
t^(f,a)	Tangent vectors to face f, a = 1,2	—
P_{ik}^(f)	Tangent projector $\delta_{ik} - n_i n_k$	—
Φ^(f,a)	Closure constraint: a=0 normal, a=1,2 shear	—
K	Fold faces per cell	7
n_T	Tensor spectral index = −2ε_H	≈ −0.010
r	Tensor-to-scalar ratio (isotropic estimate)	≈ 0.027–0.033

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