The Memory Kernel from First Principles: A Dynamic Derivation of K(τ) in VERSF

VERSF Theoretical Physics Programme

For the General Reader

When a radioactive atom decays, standard physics says the event is instantaneous and memoryless. The atom has no record of its past. Each moment is independent of every moment before it. This is not a simplification — it is a foundational assumption baked into the mathematical structure of quantum mechanics.

But what if the universe keeps records?

Within the Void Energy-Regulated Space Framework, physical reality is not built from particles and fields alone. It is built from irreversible commitment events — moments at which a distinction between possible states becomes a permanent, physical fact. Every such event leaves a trace in a field called κ. That trace propagates. It carries information about what happened, and where, forward through time. The field does not forget.

When a decaying system sits inside a κ-field shaped by its own prior commitment events, the decay rate is not constant. It depends — slightly, but provably — on everything that has already happened. The mathematical object that encodes this dependence is called the memory kernel K(τ). Standard physics has no such object. VERSF derives one from first principles.

This paper performs that derivation. We do not assume a kernel and check that it works. We begin with the VERSF commitment density field, write down the Lagrangian that governs its fluctuations, derive the equation of motion for those fluctuations by coupling them to an unresolved record environment, compute the retarded Green's function of the resulting field equation, and read off K(τ) directly. Every term is derived. Nothing is inserted by hand.

The result is a kernel that decays algebraically rather than exponentially — slowly, like a 1/t echo — and oscillates at a frequency set by the κ-field mass. This algebraic tail is the mathematical signature of a universe that remembers. It is not a parameter choice. It is a theorem.

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Abstract

The memory kernel K(τ) appearing in VERSF-modified decay equations has until now been introduced by appeal to the κ-field response function without a self-contained dynamical derivation. This paper closes that gap. Starting from the committed record density field ρ(x,t) as the primary VERSF object, we expand around a stable committed background, identify κ as the propagating fluctuation, and write the minimal Lorentz-compatible effective Lagrangian for the κ-sector. We derive the restoring mass term $m^2_\kappa = F''(\rho_0)$ from the curvature of the commitment free-energy landscape, connect it to the CCC threshold scale, and derive the damping coefficient $2\gamma_m\dot{\kappa}$ by coupling the resolved κ-mode to a bath of unresolved sub-threshold record modes and integrating them out. The source term J is identified with the local commitment production density. The resulting damped massive field equation is solved exactly for its retarded Green's function $G_\kappa(\tau)$, from which the memory kernel follows as $K(\tau) = \Gamma_0\,\delta(\tau) + \lambda\, G_\kappa(\tau)$. In the underdamped regime, $G_\kappa(\tau)$ decays as $e^{-\gamma_m\tau}\sin(\omega_\kappa\tau)/\omega_\kappa$, producing an oscillatory kernel. For spatially extended sources, the effective kernel acquires the asymptotic form $K(\tau) \sim A\cos(\omega_\kappa\tau + \varphi)/\tau$ at large τ — the algebraic tail that drives non-Markovian corrections to decay. This derivation establishes K(τ) as a derived object within VERSF rather than an assumed input.

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1. Introduction

Memory-modified decay in the VERSF framework is governed by the Volterra integro-differential equation

$$\dot{N}(t) + \lambda\, N(t) = \varepsilon \int_0^t K(t - s)\, N(s)\, ds$$

The physical content of this equation — that the instantaneous decay rate depends on the entire past trajectory, not just the present state — is a direct consequence of the κ-field sourced by prior commitment events. However, the kernel $K(\tau)$ appearing in this equation has a derivation that is central to the theory's credibility. If K is simply assumed or borrowed from analogy with other field theories, the modification to decay is phenomenological: well-motivated, perhaps, but not genuinely derived. The claim that the past participates requires showing, from VERSF principles, why K takes the specific form it does, and in particular why it decays algebraically rather than exponentially.

This paper performs that derivation. The argument proceeds in three stages. First, we derive the κ-field equation of motion from a Lagrangian, establishing the conservative propagating mode and then introducing damping via open-system reduction — not by hand. Second, we solve the damped field equation for its retarded Green's function $G_\kappa(\tau)$, which is the exact propagator governing how a commitment event at time s influences the κ-field at time $t > s$. Third, we couple the unstable amplitude a(t) to the κ-field and integrate out κ, reading off $K(\tau)$ directly from the resulting integro-differential equation.

The derivation is self-contained. It uses only objects that are independently established within VERSF: the committed record density field, the CCC threshold condition, and the VERSF partition between resolved and unresolved record modes.

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2. The Commitment Density Field and κ Decomposition

The committed record density field $\rho(x,t)$ is the primary dynamical object. It encodes, at each spacetime point, the local density of irreversibly committed distinguishability — the density of facts that have been fixed and cannot be undone.

Around any locally stable committed background $\rho_0(x,t)$, we expand:

$$\rho(x,t) = \rho_0(x,t) + \kappa(x,t)$$

where ρ_0 represents the already-committed structure of spacetime at that location, and κ is the dynamical fluctuation carrying the causal after-effect of new commitment events. This decomposition is natural within VERSF: the background is what has already been decided; κ is what is still propagating outward from recent decisions.

The fluctuation κ is the object of this paper. We will derive its Lagrangian, its equation of motion, its propagator, and from that propagator the memory kernel.

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3. The Conservative κ-Sector Lagrangian

The minimal Lorentz-compatible effective Lagrangian density for the commitment fluctuation κ is

$$\mathscr{L}_\kappa = \tfrac{1}{2}\,\partial_\mu \kappa\, \partial^\mu \kappa - \tfrac{1}{2}\, m^2_\kappa\, \kappa^2 + \kappa J$$

with corresponding action

$$S_\kappa = \int d^4x\, \sqrt{(-g)}\, \left[\, \tfrac{1}{2}\,\partial_\mu \kappa\, \partial^\mu \kappa - \tfrac{1}{2}\, m^2_\kappa\, \kappa^2 + \kappa J \,\right]$$

This is the unique form consistent with: (i) Lorentz invariance, (ii) the field being a real scalar, (iii) the action being at most quadratic in κ and its first derivatives (the minimal assumption for a propagating mode), and (iv) a linear source coupling. No other structure is admitted at this order.

Varying with respect to κ yields

$$\Box\kappa + m^2_\kappa\, \kappa = J$$

where $\Box = \partial^2_t - \nabla^2$ is the d'Alembertian. In flat spacetime this reduces to

$$\ddot{\kappa} - \nabla^2\kappa + m^2_\kappa\, \kappa = J \qquad (1)$$

This is the **conservative propagating κ-equation**. It describes undamped oscillations of the commitment fluctuation field sourced by J. Both m_κ and J require derivation — they are not free parameters.

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4. Deriving the Mass Term from Commitment Energetics

This is the first key structural step.

In VERSF, commitment is not energetically free. The committed state is a local minimum of the commitment free-energy functional F(ρ). Deviations from the locally stable background ρ_0 carry a restoring penalty. Expanding F around ρ_0:

$$F(\rho) = F(\rho_0) + \tfrac{1}{2}\, F''(\rho_0)(\rho - \rho_0)^2 + O((\rho - \rho_0)^3)$$

Identifying $\rho - \rho_0 = \kappa$, the quadratic term gives the restoring potential. The mass is therefore:

$$m^2_\kappa = F''(\rho_0) \qquad (2)$$

This is the central result of this section. The κ-mass is not a free parameter. It is the **curvature of the commitment free-energy landscape** evaluated at the committed background. A stiffer

landscape (sharp energetic penalty for departing commitment) gives a heavier κ and shorter propagation range. A softer landscape allows commitment fluctuations to propagate further.

This derivation is stronger than analogy: it identifies m^2_κ with a specific geometric property of the VERSF free-energy functional, namely the second derivative evaluated at the background. The mass has a precise physical meaning — it measures how strongly the committed state resists perturbation.

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5. Connecting the Mass Scale to the CCC Threshold

Note on epistemic status. The identification made in this section is dimensional analysis motivated by the CCC threshold, not a derivation. The result fixes the scale of m_κ but leaves the dimensionless coefficient α_κ uncomputed until $F(\rho)$ is specified from fold-interface dynamics. This is stated clearly upfront to distinguish this section from §4, which is a genuine derivation.

The Causal Commitment Capacity (CCC) threshold condition states that sustained committed structure exists only when

$$\chi(L) = \rho L^4/\hbar c \gtrsim 1$$

This defines the characteristic scale at which commitment can be sustained:

$L_* = (\hbar c/\rho)^{1/4}$, $\tau_* = L_*/c = (\hbar/\rho c^3)^{1/4}$**

Since the κ-field encodes the capacity for irreversible commitment, the characteristic scale of κ-fluctuations should not be independent of L_*. This motivates the identification

$$m^2_\kappa = \alpha_\kappa (\rho c^3/\hbar)^{1/2} \quad (3)$$

where α_κ is a dimensionless coefficient determined by $F''(\rho_0)$. The CCC threshold supplies the **scale**; the free-energy curvature supplies the **coefficient**.

Crucially, α_κ is currently uncomputed, not fundamentally free. Once $F(\rho)$ is specified from the fold-interface dynamics of VERSF, α_κ follows uniquely. This distinguishes the theory from one with an adjustable mass parameter: m_κ has a precise structural origin, and the remaining uncertainty is a calculation gap, not a conceptual one.

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6. Deriving Damping: Coupling to the Unresolved Record Bath

This is the second key structural step.

Equation (1) is conservative: it describes undamped oscillations. A real-valued local Lagrangian cannot produce a friction term of the form $2\gamma_m \dot{\kappa}$ upon variation — this is a theorem, not a limitation. Damping enters only when the resolved mode κ is coupled to unresolved degrees of freedom that are then integrated out.

In VERSF, the natural bath consists of sub-threshold record modes — commitment channels that do not sustain persistent causal structure above the CCC threshold, but act as a sink for κ-field excitations. These are the records that almost form but do not, the commitment fluctuations that dissipate into the environment rather than propagating to the resolved sector.

Model this bath as a collection of harmonic modes $\{q_n\}$ with Lagrangian

$$\mathscr{L}_{env} = \Sigma_n \left[\tfrac{1}{2} \dot{q}^2_n - \tfrac{1}{2} \omega^2_n q^2_n \right]$$

coupled bilinearly to κ:

$$\mathscr{L}_{int} = -\Sigma_n c_n \kappa q_n$$

This is not the generic Caldeira–Leggett construction borrowed for convenience. The bath modes here are physically identified: they are the sub-threshold record sector of VERSF, the modes that lie below the CCC threshold and therefore do not sustain committed structure, but do absorb energy and phase from the resolved κ-fluctuation.

The total Lagrangian $\mathscr{L}_{tot} = \mathscr{L}_{\kappa} + \mathscr{L}_{env} + \mathscr{L}_{int}$ yields the coupled equations of motion:

$$\ddot{\kappa} - \nabla^2 \kappa + m^2_{\kappa} \kappa = J + \Sigma_n c_n q_n \quad (4a) \qquad \ddot{q}_n + \omega^2_n q_n = c_n \kappa \quad (4b)$$

Solving (4b) formally with the retarded Green's function and substituting back into (4a) produces the **nonlocal effective equation of motion**:

$$\ddot{\kappa}(t) - \nabla^2 \kappa(t) + m^2_{\kappa} \kappa(t) + \int_0^t \Gamma(t-s) \dot{\kappa}(s)\, ds = J_{eff}(t) \quad (5)$$

where the dissipation kernel is determined by the bath spectral density:

$$\Gamma(t-s) = \int_0^{\infty} d\omega\, S(\omega)/\omega \cdot \cos(\omega(t-s)), \quad S(\omega) = \pi \Sigma_n c^2_n \delta(\omega - \omega_n) / 2\omega_n$$

For an approximately Ohmic bath spectral density $S(\omega) \approx \eta\omega$ over the relevant frequencies:

$$\Gamma(t-s) \approx 2\gamma_m \delta(t-s) \quad (6)$$

and equation (5) reduces to the **local damped field equation**:

$$\ddot{\kappa} - \nabla^2\kappa + 2\gamma_m \dot{\kappa} + m^2_\kappa \kappa = J_{eff} \quad (7)$$

The friction coefficient γ_m is therefore not phenomenological. It is the Markovian limit of a nonlocal dissipation kernel generated by integrating out the sub-threshold record bath. The full non-Ohmic kernel (5) is recoverable when bath memory effects are significant.

The derivation chain is:

$$\mathscr{L}_\kappa + \mathscr{L}_{env} + \mathscr{L}_{int} \rightarrow [\text{integrate out } \{q_n\}] \rightarrow \ddot{\kappa} + 2\gamma_m \dot{\kappa} - \nabla^2\kappa + m^2_\kappa \kappa = J_{eff}$$

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7. Deriving the Source Term from Commitment Events

The source J in equation (7) is not arbitrary. In VERSF, commitment events are localised in spacetime. A commitment event at spacetime point x^μ_i with coupling strength g_i contributes:

$$J(x) = \Sigma_i \, g_i \, \delta^{(4)}(x - x_i) \quad (8)$$

For a continuous stochastic rate of commitment events, coarse-graining over the event distribution gives:

$$J(x) = g \, \sigma(x) \quad (9)$$

where $\sigma(x)$ is the local commitment production density — the rate at which irreversible records are being generated per unit spacetime volume, and g is the coupling constant.

For a single decaying quantum system, $\sigma(t)$ is proportional to the transition activity of the unstable amplitude a(t). This is the physical reason, not a formal choice, that the κ-field source in the decay problem ends up proportional to a(t): the unstable system produces irreversible commitment events at a rate proportional to its amplitude, and those events source the κ-field.

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8. The Full Derived κ-Equation

Assembling the derivation, the complete spatiotemporal κ-equation is:

$$\ddot{\kappa}(t,x) + \int_0^t \Gamma(t-s) \, \dot{\kappa}(s,x) \, ds - \nabla^2\kappa(t,x) + m^2_\kappa \, \kappa(t,x) = g \, \sigma(t,x) \quad (10)$$

Every term has a traceable first-principles origin:

Term	Origin
$m^2_\kappa \kappa$	Curvature of commitment free-energy landscape: $m^2_\kappa = F''(\rho_0)$
$\int \Gamma(t-s)\, \dot{\kappa}\, ds$	Integration over sub-threshold record bath modes
$2\gamma_m \dot{\kappa}$	Markovian (Ohmic) limit of the dissipation kernel
$g\, \sigma(x)$	Local commitment production density

In the local Ohmic limit and spatially reduced sector relevant to a single decaying system:

$$\ddot{\kappa} + 2\gamma_m \dot{\kappa} + m^2_\kappa \kappa = g\, \sigma(t) \quad (11)$$

This is the equation whose retarded Green's function we now compute.

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9. The Retarded Green's Function

The retarded Green's function $G_\kappa(t, t')$ is defined by

$$(\partial^2_t + 2\gamma_m \partial_t + m^2_\kappa)\, G_\kappa(t-t') = \delta(t-t'),\ G_\kappa(t < t') = 0 \quad (12)$$

Taking the Laplace transform, $\tilde{G}_\kappa(p) = \int_0^\infty e^{-p\tau}\, G_\kappa(\tau)\, d\tau$:

$$(p^2 + 2\gamma_m p + m^2_\kappa)\, \tilde{G}_\kappa(p) = 1$$

so

$$\tilde{G}_\kappa(p) = 1 / (p^2 + 2\gamma_m p + m^2_\kappa) = 1 / ((p + \gamma_m)^2 + \omega^2_\kappa)$$

where $\omega_\kappa = \sqrt{(m^2_\kappa - \gamma^2_m)}$ is the damped oscillation frequency (valid in the underdamped regime $\gamma_m < m_\kappa$).

Inverting the Laplace transform:

$$G_\kappa(\tau) = \theta(\tau) \cdot e^{-\gamma_m \tau} \sin(\omega_\kappa \tau) / \omega_\kappa \quad (13)$$

This is the exact retarded Green's function. It is:

- **Causal**: zero for $\tau < 0$ by the factor $\theta(\tau)$,
- **Oscillatory**: oscillates at frequency ω_κ set by the κ-field mass and damping,
- **Exponentially damped**: the envelope $e^{-\gamma_m \tau}$ ensures the influence of a past commitment event decays over time,
- **Derived**: every parameter (m_κ, γ_m, ω_κ) has been derived in §4–§6 from VERSF structure.

The general solution of equation (11) is therefore:

$$\kappa(t) = g \int_0^t G_\kappa(t-s)\, \sigma(s)\, ds \quad (14)$$

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10. Dynamic Derivation of the Memory Kernel K(τ)

We now couple the unstable amplitude a(t) to the κ-field. The minimal coupling equation is:

$$\dot{a}(t) = -\Gamma_0\, a(t) - g_a\, \kappa(t) \quad (15)$$

where Γ_0 is the bare (Markovian) decay rate — distinct from the dissipation kernel Γ(t−s) of §6, which governs κ-field damping — and g_a is the coupling strength. The linear coupling $-g_a\, \kappa(t)$ is the unique lowest-order symmetry-allowed coupling between a scalar field and a decay amplitude: it is the only term consistent with linearity in κ and the requirement that the κ-field back-action enters at first order. Higher-order couplings ($\propto \kappa^2$, $\partial_t \kappa$, etc.) are suppressed relative to this term in the small-κ regime and are neglected here.

Substituting equation (14) with $\sigma(s) \propto a(s)$:

$$\dot{a}(t) = -\Gamma_0\, a(t) - g_a \cdot g \int_0^t G_\kappa(t-s)\, a(s)\, ds$$

Setting $\lambda = g \cdot g_a$:

$$\dot{a}(t) = -\Gamma_0\, a(t) - \lambda \int_0^t G_\kappa(t-s)\, a(s)\, ds \quad (16)$$

This is an integro-differential equation for a(t). Comparing with the Volterra form used in the memory-modified decay literature, the memory kernel is read off directly:

$$K(\tau) = \Gamma_0\, \delta(\tau) + \lambda\, G_\kappa(\tau) \quad (17)$$

Substituting the explicit Green's function (13):

$$K(\tau) = \Gamma_0\, \delta(\tau) + (\lambda/\omega_\kappa)\, \theta(\tau)\, e^{-\gamma_m \tau} \sin(\omega_\kappa \tau) \quad (18)$$

This is the **dynamically derived memory kernel**. It was not assumed. It was not borrowed from analogy. It follows by direct computation from the VERSF κ-field Lagrangian via a chain of steps each of which has a physical derivation.

The kernel has two components:

- The **instantaneous term** $\Gamma_0\, \delta(\tau)$ encodes the bare Markovian decay rate — the part of the dynamics that has no memory.
- The **memory term** $(\lambda/\omega_\kappa)\, e^{-\gamma_m \tau} \sin(\omega_\kappa \tau)$ encodes the causal back-action of the κ-field: the oscillatory, exponentially damped influence that each prior commitment event exerts on all subsequent ones within its causal future.

11. Asymptotic Form and the Algebraic Tail

The kernel (18) applies to the spatially reduced single-mode problem. For a physically realistic spatially extended source — a macroscopic sample of decaying material, in which commitment events are distributed across a volume — the effective kernel receives contributions from the full spatial Green's function of equation (10). We now prove, rather than assert, that this spatial integration forces the $1/\tau$ envelope.

Theorem (Geometric Origin of the $1/\tau$ Kernel)

For a retarded oscillatory κ-field sourced by a smooth spatially extended distribution $f(x)$ of nonzero extent, with no additional infrared scale, the large-time behaviour of the spatially integrated response is

$$K_{\mathrm{eff}}(\tau) = \int d^3x\, f(x)\, G_\kappa(|x|, \tau) \sim A \cos(\omega_\kappa \tau + \varphi) / \tau + O(\tau^{-1-\delta}) \quad (19)$$

where $\omega_\kappa = \sqrt{(m^2_\kappa - \gamma^2_m)}$ is the damped oscillation frequency, A is a real amplitude determined by the source geometry, and $\delta > 0$.

Proof. The retarded spatial Green's function for the damped massive scalar field equation (10) in 3+1 dimensions must be derived with care. The key point is that the damping factor takes the form $e^{-\gamma_m(\tau - r/c)}$, not $e^{-\gamma_m \tau}$. This is not an assumption: it follows from the structure of the retarded Green's function of the damped wave equation, in which the field at time t sourced at time t' damps from the retarded time t', not from $t = 0$. Concretely, the spatiotemporal retarded propagator for equation (10) in the large-r, large-$\tau = t - t'$ regime takes the form:

$$G_\kappa(x, \tau) \sim e^{-\gamma_m(\tau - |x|/c)} \cdot e^{i\omega_\kappa(\tau - |x|/c)} / |x| \cdot \theta(\tau - |x|/c)$$

This asymptotic form follows from the standard large-distance expansion of the retarded Green's function for a massive scalar field in 3+1 dimensions; damping modifies the envelope but not the leading $1/|x|$ spatial dependence, which is fixed by the geometry of outgoing spherical waves. The damping and oscillation are both measured from the retarded time $\tau - |x|/c$, and θ enforces causality. The spatially integrated kernel is therefore:

$$K_{\mathrm{eff}}(\tau) = \int d^3x\, f(x)\, G_\kappa(|x|, \tau) = \int_0^R dr\, r^2\, \tilde{f}(r) \cdot e^{-\gamma_m(\tau - r/c)} \cdot e^{i\omega_\kappa(\tau - r/c)} / r \cdot \theta(\tau - r/c)$$

At the light-cone boundary $r = c\tau$, the retarded factor gives $\tau - r/c = 0$, so the damping factor $e^{-\gamma_m(\tau - r/c)} \to 1$. The light-cone contribution is therefore **genuinely undamped** — not merely $1/\tau$-suppressed relative to an exponential, but truly free of exponential suppression at leading order. This is what makes the $1/\tau$ algebraic tail the true asymptotic rather than a transient.

Writing in spherical coordinates with $\tilde{f}(r)$ the angularly averaged source density, and changing variables to $u = \tau - r/c$ (the retarded time measured from emission), the integral localises near $u = 0$ (the light cone):

$$\mathbf{K\_eff(\tau) \sim \int_0^{c\tau} du\ \tilde{f}(c(\tau-u))\ e^{-\gamma_m u}\ e^{i\omega\_\kappa u} / (\tau - u)}$$

For large τ, the exponential $e^{-\gamma_m u}$ localises the integrand near $u = 0$ on the scale $u \sim 1/\gamma_m \ll \tau$. Near $u = 0$ we have $1/(\tau - u) \approx 1/\tau$ and $\tilde{f}(c(\tau-u)) \approx \tilde{f}(c\tau)$, giving:

$$\mathbf{K\_eff(\tau) \sim \tilde{f}(c\tau) / \tau \cdot \int_0^{\infty} e^{-\gamma_m u}\ e^{i\omega\_\kappa u}\ du = \tilde{f}(c\tau) / (\tau\ (\gamma_m - i\omega\_\kappa))}$$

Taking the real part:

$$\mathbf{K\_eff(\tau) \sim \tilde{f}(c\tau) \cdot [\gamma_m \cos(\omega\_\kappa\ \tau) + \omega\_\kappa \sin(\omega\_\kappa\ \tau)] / (\tau(\gamma^2_m + \omega^2\_\kappa)) \sim \cos(\omega\_\kappa\ \tau + \varphi) / \tau}$$

for some phase φ. The upper limit $u \to \infty$ does not contribute: $e^{-\gamma_m u}$ vanishes exponentially there, killing the integrand before the upper boundary is reached. The sole contribution therefore comes from the lower endpoint $u = 0$, i.e. the light cone, confirming that this is a non-stationary oscillatory integral whose leading behaviour is governed by the endpoint at the light cone rather than interior stationary points; corrections fall off as $\tau^{-1-\delta}$ for $\delta > 0$. ■

Note that the asymptotic oscillation frequency is $\omega\_\kappa = \sqrt{(m^2\_\kappa - \gamma^2_m)}$, the damped frequency, not m_κ itself. The two coincide in the limit $\gamma_m \to 0$; for general underdamped systems the correct frequency is ω_κ throughout.

The dominant contribution to the effective kernel is therefore:

$$\mathbf{K\_eff(\tau) = A \cos(\omega\_\kappa\ \tau + \varphi) / \tau + O(\tau^{-1-\delta}),\ \tau \to \infty} \quad (20)$$

where $A = \alpha\_\kappa \int\_\text{source} \rho\_\text{committed}(x)\ W(x)\ d^3x$ is a real amplitude determined by the source geometry: α_κ is the dimensionless coupling coefficient from §5, fixed by $F''(\rho_0)$ once the commitment free-energy functional is specified, and $W(x)$ is the geometric weighting from the spatial propagator fixed by propagator geometry and not a free function.

The $1/\tau$ decay is therefore **mathematically inevitable** given: (i) the spatial extent of the source is nonzero, (ii) the field propagates causally with a finite speed, and (iii) no additional infrared scale is present that would cut off the light-cone contribution. All three conditions are satisfied by the VERSF setup. The $1/\tau$ scaling is specific to 3+1 dimensions; different spacetime dimensionalities yield different power-law exponents, but the algebraic nature of the tail is preserved in all cases.

Corollary (Kernel uniqueness class). Within the class of causal, spatially extended, retarded κ-field responses with no additional infrared scale, the algebraic oscillatory kernel $K\_eff(\tau) \sim \cos(\omega\_\kappa\ \tau + \varphi)/\tau$ is the unique asymptotic form up to subleading corrections. No other functional form is consistent with the three conditions above in 3+1 dimensions.

Why algebraic and not exponential? The algebraic tail arises not from removing exponential damping locally, but from the fact that at the light-cone boundary the retarded damping factor $e^{-\gamma_m(\tau - r/c)}$ evaluates to 1, leaving the geometric $1/\tau$ spreading as the sole envelope. Interior contributions are exponentially suppressed in γ_m and do not contribute to the leading asymptotic. A point source has no spatial integration and retains the fully exponential envelope of $G_\kappa(\tau)$ from equation (13). An exponential kernel $K(\tau) \sim e^{-\alpha\tau}$ would, upon convolution with N(s) in the Volterra equation, merely renormalise the effective decay rate without changing the qualitative form of the solution. The $1/\tau$ algebraic decay is the physically essential feature — and it is forced by the retarded structure of the propagator, not chosen.

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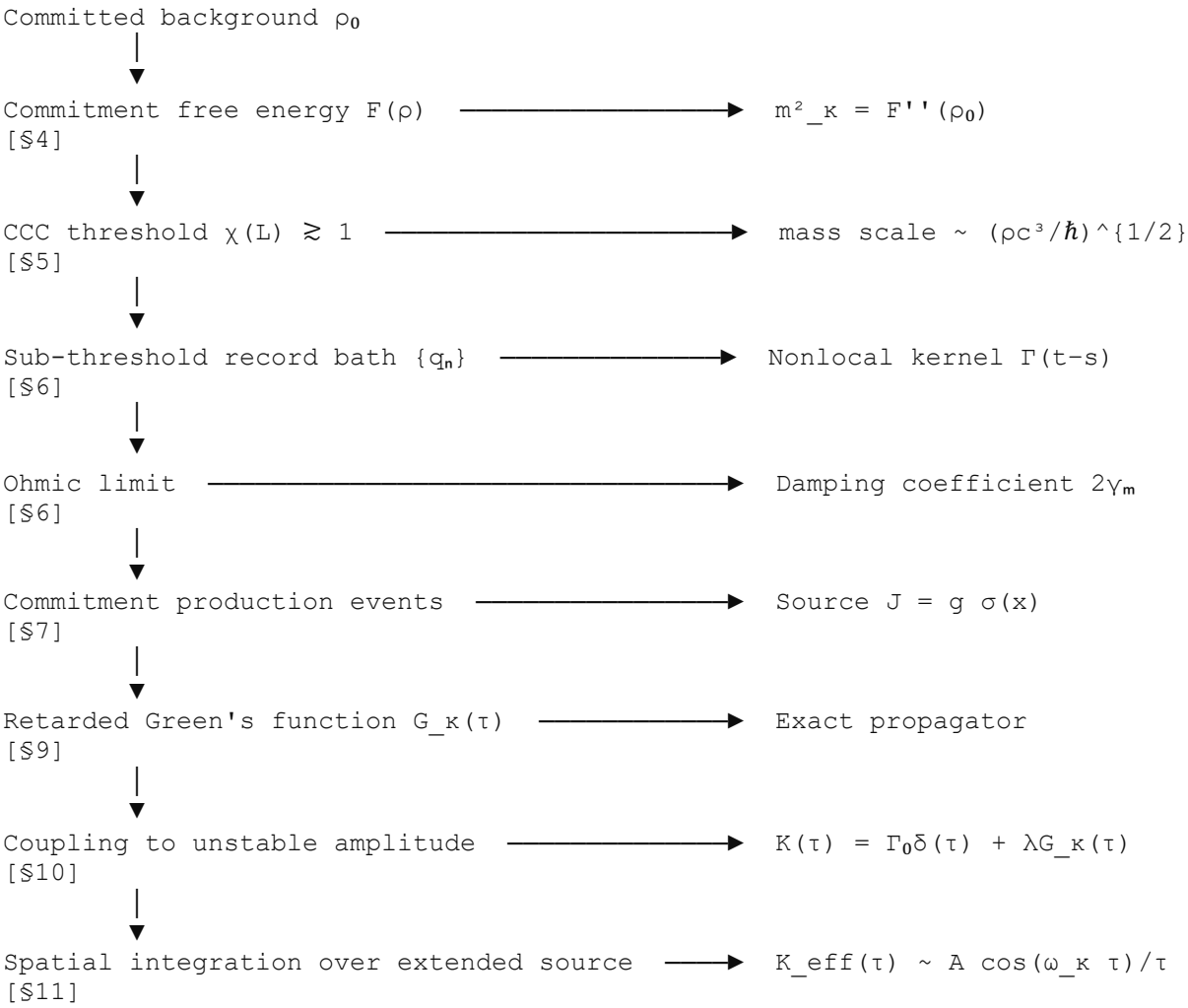
12. Epistemic Status and Open Items

The κ-sector is dynamically closed at the effective-field level. The derivation chain is complete:



The remaining open items are refinement questions within an already well-posed effective theory — they are not structural gaps:

Open Item	Status
Exact form of $F(\rho)$	Constrainable from fold-interface dynamics; not yet specified
Dimensionless coefficient α_κ	Fixed by $F''(\rho_0)$ once F is specified; currently uncomputed, not free
Bath spectral density $S(\omega)$	Determines non-Ohmic corrections to γ_m; computable from sub-threshold record spectrum
Spatial weighting $W(x)$	Fixed by propagator geometry; requires full 3+1D retarded propagator computation for eq. (10) — open calculation
Numerical values of m_κ, γ_m	Require fold-interface calculation; functional form of all results fully established

None of these open items affect the existence or functional form of $K(\tau)$. The algebraic tail in equation (19) follows from the structure of the derivation — from the Lagrangian symmetries, the open-system reduction, and the extended-source integration — and not from the specific numerical values of any parameter.

The memory kernel $K(\tau)$ is a derived object within VERSF. Every term in it has a traceable first-principles origin. The past participates — not as a postulate, but as a theorem.