The Spectral Density of the Commitment-Event Bath in the VERSF Framework

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For the General Reader

Physics has long sought a single, simple starting point from which the entire structure of the universe can be derived. The VERSF framework proposes that this starting point is the irreversible *commitment event* — a fundamental moment at which a physical possibility becomes a physical fact, permanently and without reversal. Everything we observe, from the expansion of the universe to the behaviour of particles, is proposed to emerge from the accumulated weight of such events.

This paper asks a specific question: how does a sequence of these microscopic, irreversible events collectively produce the smooth, slowly expanding early universe we infer from observations of the cosmic microwave background? The answer turns on a concept borrowed from the physics of open quantum systems — the *spectral density* of the commitment-event bath. Think of the bath as the sum total of all the microscopic commitment events occurring around any given point in space, constantly jostling a field called the κ-field (kappa-field) the way a crowded room of people jostles anyone trying to walk through it.

The central result is that this jostling has a very specific statistical character: it is *white noise* at low frequencies — all low-pitched sounds are equally loud. This flatness, which we derive rather than assume, has a remarkable consequence: the κ-field drifts *logarithmically* with the expansion of the universe. As the universe doubles in size the κ-field displacement grows by a fixed amount; as it doubles again, by the same fixed amount. This slow, steady growth produces the nearly flat spectrum of primordial density fluctuations seen in the CMB — the seeds from which all galaxies grew.

The paper derives this chain step by step, from the geometry of the K=7 structure through the statistics of commitment events to an explicit prediction for the CMB spectral index. Along the way several results are established that go beyond what was previously derived: a law-of-large-numbers argument showing that when commitment events are averaged over many spatial regions their collective bath noise becomes smooth and well-behaved; an explicit formula for the bath strength in terms of four physical quantities characterising individual commitment events (their amplitude, their spatial uniformity, their sensitivity to local density fluctuations, and the mean number of events per region); and a stability analysis showing that the bath remains well-behaved unless the commitment dynamics undergo a phase transition — for which there is no apparent mechanism in the K=7 architecture.

The final output is a concrete, testable prediction: the slope of the CMB power spectrum is given by $n_s = 1 - 2/N★$ plus a small correction whose size is controlled by the same event-statistics quantities. For the expected number of inflationary e-folds ($N★ \approx 55$), the leading term gives $n_s \approx 0.964$, in good agreement with the measured value of 0.9649 ± 0.0042. The remaining open problem has narrowed to determining five numbers — characterising single-event strength, uniformity, disorder sensitivity, and inter-cell coupling — from the underlying K=7 nonlinear dynamics. The structural derivation is complete; what remains is parameter matching.

Table of Contents

Abstract

We derive the spectral density of the commitment-event bath that drives κ-field dynamics in the VERSF framework. Starting from the K=7 minimal fact architecture, we show that the bath consists of seven oscillatory modes fixed by the K=7 simplex geometry — one uniform mode that decouples from the κ-field by symmetry, and six coupled non-zero modes that carry the bath dynamics. Poisson-distributed commitment events provide stochastic forcing; the Caldeira-Leggett open-systems formalism converts this into an effective spectral density $J(\omega)$ seen by the κ-field.

The response-corrected spectral density is:

$$\mathbf{J(\omega) = \Gamma\, |\hat{f}(\omega)|^2\, W\_K7(\omega),\quad W\_K7(\omega) = \Sigma_a\, |g_a|^2\, \sigma_a^2\, |\chi_a(\omega)|^2}$$

where $\chi_a(\omega) = (\omega_a^2 - \omega^2 - i\varepsilon\omega)^{-1}$ is the mode response function. We show that under coarse-graining over orientations and weak disorder, $W\_K7(\omega)$ is asymptotically constant in the infrared:

$$\mathbf{\bar{W}\_K7(\omega) = \bar{W}\_K7(0) + O(\omega^2)}$$

This is a conditional theorem, valid under mild self-averaging assumptions (SA1–SA4): that the local response-weighted spectral weights have finite mean and variance, short-range spatial correlations, and smooth local response functions. Under these conditions the coarse-grained bath is self-averaging with variance suppressed as V^{-1}, and the mean is smooth and positive near $\omega = 0$. The result generically emerges in the infrared limit and places the κ-field in the $\alpha = 1$ (white-noise) universality class in the infrared limit.

We derive the resulting Caldeira-Leggett memory kernel, solve the κ-field equation in the slow-roll regime, and demonstrate that $\delta s(t) \sim A \ln(Ht)$. A parameter unification result follows: all bath-induced cosmological parameters (λ_m, β, γ_m, C) are controlled by the single infrared spectral weight $\bar{W}\_K7(0)$, up to cosmological background quantities. We close with an explicit dimensional consistency check and a statement of the one remaining open calculation.

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1. Introduction and Motivation

1.1 The Role of the Bath in VERSF

The companion paper (Taylor, VERSF Friedmann Paper) derived the VERSF Friedmann equation and identified two energy sectors controlling primordial dynamics: a void carrier bath ($\rho\_void \propto a^{-4}$) and a κ-field displacement sector ($\rho\_sr \propto (\ln N)^2$). The crossover between them generates a quasi-de Sitter phase producing a red-tilted, nearly scale-invariant primordial spectrum.

Both sectors are controlled by the spectral density $J(\omega)$ of the commitment-event bath. The Friedmann paper used $J(\omega)$ as an input. This paper derives it.

Three questions organise the paper:

1. **What is $J(\omega)$?** We derive its form from the K=7 mode structure and Poisson event statistics, taking care to distinguish the forcing spectrum from the response-corrected spectral density.
2. **Is $J(\omega)$ flat at low frequencies?** We establish the conditions under which $\bar{W}$_K7(ω) is asymptotically constant in the infrared and state these conditions as a conditional theorem.
3. **What does this imply for κ-field dynamics?** We derive the memory kernel and solve the equation of motion explicitly in the slow-roll regime.

1.2 Structure of This Paper

Section 2 establishes the K=7 microscopic model. Section 3 introduces Poisson event forcing and its statistical properties. Section 4 derives $J(\omega)$ correctly, including the response function, and establishes the infrared flatness as a conditional theorem with an explicit coarse-graining formula. Section 5 derives the Caldeira-Leggett memory kernel and its $\alpha = 1$ decay. Section 6 solves the equation of motion and derives $\delta s \sim A \ln(Ht)$. Section 7 states the parameter unification result. Section 8 performs the dimensional consistency check. Section 9 states the epistemic status of each result.

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2. The K=7 Microscopic Bath Model

2.1 Local Variables and Hamiltonian

Each K=7 cell has seven vertices carrying local commitment-density deviations $q_i(t)$, $i = 1, \ldots, 7$. The dynamics are governed by the quadratic Hamiltonian:

$$\mathbf{H = \tfrac{1}{2} \Sigma_i \dot{q}_i^2 + \tfrac{1}{2} \Sigma_{ij} q_i L_{ij} q_j}$$

The matrix **L** is the graph Laplacian of the complete graph K_7. Its entries are fixed exactly by the K=7 geometry: $L_{ii} = 6$ (vertex degree) and $L_{ij} = -1$ for $i \neq j$. This is not a model parameter; it is determined by the K=7 no-go theorem.

2.2 Normal Mode Decomposition

Diagonalise **L**:

$$\mathbf{L\, u^{(a)} = \lambda_a\, u^{(a)},\ a = 0, 1, \ldots, 6}$$

For K_7 the eigenvalue spectrum is known exactly:

- **a = 0:** $\lambda_0 = 0$ (uniform mode)
- **a = 1, …, 6:** $\lambda_a = 7$ (sixfold-degenerate)

Expand vertex variables in the eigenbasis: $\mathbf{q_i = \Sigma_a\, u_i^{(a)}\, Q_a(t)}$. The Hamiltonian decouples:

$$\mathbf{H = \tfrac{1}{2}\, \Sigma_a\, (\dot{Q}_a^2 + \omega_a^2\, Q_a^2),\ \omega_a^2 = \omega_0^2 + \lambda_a}$$

The zero mode oscillates at ω_0; the six non-zero modes oscillate at $\omega_1^2 = \omega_0^2 + 7$. The physical frequency ω_0 is set by the CCC threshold condition at the Planck scale.

2.3 κ-Field Coupling

The κ-field couples to the collective variable:

$$\mathbf{\delta s(t) = \Sigma_i\, c_i\, q_i(t) = \Sigma_a\, g_a\, Q_a(t),\ g_a = \Sigma_i\, c_i\, u_i^{(a)}}$$

The coupling vector **c** is fixed by the D_{5h} symmetry of the pentagonal bipyramid realisation of K_7. The coupling of the κ-field to the bath collective variable is mediated by a dimensionful coupling constant g_κ, so the interaction Hamiltonian is:

$$\mathbf{H_{int} = -g_\kappa\, \delta s \cdot \delta s_{bath}}$$

with $[g_\kappa] = M^3$ (required for H_{int} to have dimension M in natural units with $[\delta s] = M$ and $[\delta s_{bath}] = M^{-1}$, the dimension of a bath oscillator coordinate). The coupling constant g_κ is fixed by the VERSF action normalisation and will be specified in the companion normalisation paper (in preparation). All results in the present paper are insensitive to the numerical value of g_κ: it appears only through $\gamma_m \equiv g_\kappa^2\, \Gamma|\hat{f}(0)|^2 \bar{W}_{K7}(0)$, which is treated as a single free parameter until $\bar{W}_{K7}(0)$ is computed from the K=7 eigensystem.

When $\Sigma_i\, c_i = 0$ — which holds when the κ-field displacement tracks deviations from the mean density — the zero mode decouples ($g_0 = 0$) and the coupling is exclusively to the six degenerate non-zero modes.

Structural remark on the isolated-cell bath. A consequence of the exact K_7 eigenvalue degeneracy is that the six coupled non-zero modes all lie at the same bare frequency $\omega_1 = \sqrt{(\omega_0^2 + 7)}$. The response-weighted spectral function of a single isolated cell is therefore not a broad bath in the usual Caldeira-Leggett sense; it is a single resonance profile multiplied by the total overlap weight:

$$\mathbf{W_{K7}^{(single)}(\omega) = |\chi_1(\omega)|^2\, \Sigma_{a=1}^{6}\, |g_a|^2\, \sigma_a^2}$$

This is a single Lorentzian peak at ω_1, not a distribution of frequencies. The effective infrared smoothness therefore does not arise at the level of an isolated cell. It must emerge through coarse-graining. Section 4.3 shows that under self-averaging assumptions this coarse-grained quantity converges to a smooth mean — but to make that concrete, we give a minimal disorder model showing the mechanism.

Minimal disorder model. Suppose each cell r experiences a small random perturbation δ_a,r to its mode frequency squared:

$$\omega_a,r^2 = \omega_1^2 + \delta_a,r,\ \delta_a,r \sim \text{i.i.d. with } \mathbb{E}[\delta_a,r] = 0,\ \mathrm{Var}(\delta_a,r) = \sigma\_\delta^2$$

The local response function becomes:

$$|\chi_a,r(\omega)|^2 = (\omega_1^2 + \delta_a,r - \omega^2)^{-2}$$

The coarse-grained spectral weight (averaging over N cells, all with the same overlap weights for simplicity) is:

$$\bar{W}\_N(\omega) = (1/N)\ \Sigma_r\ |\chi_1,r(\omega)|^2\ G,\ G = \Sigma_{a=1}^6\ |g_a|^2\ \sigma_a^2$$

For $\omega \ll \omega_1$, expand: $|\chi_a,r(\omega)|^2 \approx (\omega_1^2 + \delta_a,r)^{-2}\ (1 + 4\omega^2/(\omega_1^2 + \delta_a,r) + \ldots)$. The mean over disorder realisations is:

$$\mathbb{E}[|\chi_a,r(\omega)|^2] \approx \mathbb{E}[(\omega_1^2 + \delta_a)^{-2}]\ (1 + O(\omega^2))$$

Since δ_a has zero mean and finite variance, $\mathbb{E}[(\omega_1^2 + \delta_a)^{-2}] = \omega_1^{-4}\ (1 + 3\sigma\_\delta^2/\omega_1^4 + \ldots)$ — finite, positive, and independent of ω at leading order. The averaged spectral weight $\bar{W}\_N(0)$ is therefore positive and finite for any $\sigma\_\delta > 0$, and the distribution of response amplitudes across cells is continuous rather than a delta spike. This is the mechanism: disorder lifts the exact degeneracy cell-by-cell, and averaging over many cells produces a smooth, positive infrared spectral function. The self-averaging proposition in Section 4.3 then guarantees that fluctuations about this mean are suppressed as $N^{-1/2}$.

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3. Stochastic Forcing from Commitment Events

3.1 Event Structure

Commitment events arrive at times $\{t_n\}$ and force the vertex variables:

$$J_i(t) = \Sigma_n\ \eta_i^{(n)}\ f(t - t_n)$$

where f(t) is the single-event temporal profile (smooth, finite duration $\sim m\_\kappa^{-1}$) and $\eta_i^{(n)}$ encodes vertex excitation for event n. Projecting onto mode a:

$$\xi_a(t) = \Sigma_i u_i^{(a)} J_i(t) = \Sigma_n \zeta_a^{(n)} f(t - t_n),\ \zeta_a^{(n)} = \Sigma_i u_i^{(a)} \eta_i^{(n)}$$

3.2 Statistical Assumptions

P1 — Poisson arrivals: Events occur as a Poisson process with rate Γ.

Justification: Commitment events are irreversible and non-anticipatory by definition. This is the defining property of a Poisson process; inter-event correlations would require proto-causal structure inconsistent with CCC axioms.

P2 — Event independence: $\langle \eta_i^{(n)} \eta_j^{(m)} \rangle = 0$ for $n \neq m$. Isotropy implies $\langle \zeta_a \zeta_\beta \rangle = \sigma_a^2 \delta_{a\beta}$.

Justification: The irreversibility of commitment prevents information about past event geometry from influencing future events within the bath.

Under P1 and P2, the Wiener-Khinchin theorem applied to the Poisson process gives the forcing spectral matrix:

$$S_{a\beta}\text{^\{forcing\}}(\omega) = \Gamma \sigma_a^2 |\hat{f}(\omega)|^2 \delta_{a\beta}$$

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4. Derivation of the Spectral Density

4.1 Forcing Spectrum vs Response: The Distinction

A critical distinction must be maintained. The forcing spectral density S^{forcing}(ω) characterises the statistical properties of the stochastic drive $\xi_a(t)$. The bath spectral density $J(\omega)$ seen by the κ-field must account for how the bath modes respond to that forcing. Confusing the two — equating $J(\omega)$ with S^{forcing}(ω) — would be physically incorrect.

The response of mode a to forcing $\xi_a(\omega)$ in frequency space is:

$$\tilde{Q}_a(\omega) = \chi_a(\omega)\, \tilde{\xi}_a(\omega)$$

where the retarded response function is:

$$\chi_a(\omega) = (\omega_a^2 - \omega^2 - i\varepsilon\omega)^{-1}$$

The imaginary part $-i\varepsilon\omega$ provides the infinitesimal damping required for causal propagation ($\varepsilon \to 0^+$ taken after integration). The spectral density seen by the κ-field is therefore:

$$J(\omega) = \Sigma_a |g_a|^2 \langle |\tilde{Q}_a(\omega)|^2 \rangle\text{\_noise}$$

Substituting $\tilde{Q}_a = \chi_a \tilde{\xi}_a$:

$$\langle|\tilde{Q}_a(\omega)|^2\rangle\_\mathrm{noise} = |\chi_a(\omega)|^2 \langle|\tilde{\xi}_a(\omega)|^2\rangle = |\chi_a(\omega)|^2\, \Gamma\, \sigma_a^2\, |\hat{f}(\omega)|^2$$

Therefore:

$$J(\omega) = \Gamma\, |\hat{f}(\omega)|^2\, \Sigma_a\, |g_a|^2\, \sigma_a^2\, |\chi_a(\omega)|^2$$

4.2 The K=7 Spectral Weight (Response-Corrected)

Define the response-corrected K=7 spectral weight:

$$W\_K7(\omega) = \Sigma_a\, |g_a|^2\, \sigma_a^2\, |\chi_a(\omega)|^2$$

So:

$$J(\omega) = \Gamma\, |\hat{f}(\omega)|^2\, W\_K7(\omega)$$

The three factors encode distinct physical layers:

Factor	Physical origin	Status
Γ	Mean commitment rate per coherence volume	From CCC threshold; Γ ~ H
$|\hat{f}(\omega)|^2$	Single-event power spectrum	Fixed by event profile shape
W_K7(ω)	Response-weighted K=7 mode structure	Fixed by K=7 geometry and **c**

At low frequencies $\omega \ll \omega_a$, the response function simplifies:

$$|\chi_a(\omega)|^2 = (\omega_a^2 - \omega^2)^{-2} + O(\varepsilon^2) \to \omega_a^{-4} + 4\omega^2/\omega_a^6 + \ldots\ (\omega \ll \omega_a)$$

So W_K7(ω) is analytic in ω at ω = 0 for a single isolated cell, with:

$$W\_K7(0) = \Sigma_a\, |g_a|^2\, \sigma_a^2\, \omega_a^{-4}$$

This is finite and positive provided the mode frequencies ω_a are non-zero — which holds for all non-zero modes (a = 1, …, 6). Since all coupled modes satisfy $\omega_a \geq \omega_1 = \sqrt{(\omega_0^2 + 7)} > 0$, the response function $|\chi_a(\omega)|^2$ remains finite as $\omega \to 0$, ensuring W_K7(0) is well-defined with no infrared divergence. The zero mode decouples ($g_0 = 0$) by the symmetry argument of Section 2.3 and does not contribute.

4.3 Infrared Flatness: Self-Averaging of the Coarse-Grained Bath

In a single isolated cell, W_K7(ω) is a single Lorentzian resonance at ω_1 (as established in Section 2.3) — sharply structured, not a smooth infrared continuum. The relevant observable, however, is not the single-cell spectral function but the coarse-grained average over the many K=7 cells within a cosmological volume. We now show that this coarse-grained quantity is self-averaging: its fluctuations about the mean shrink with the averaging volume, and the mean is smooth and positive at low observation frequency. This replaces the earlier C1 regularity assertion with a mechanistic argument.

Setup. Let $W\_r(\omega)$ denote the local response-weighted spectral contribution from spatial region r:

$$\mathbf{W\_r(\omega) = \Sigma_a |g_a,r|^2 \sigma_a,r^2 |\chi_a,r(\omega)|^2}$$

where the subscript r indicates that the mode frequencies, overlaps, and variances may vary from cell to cell due to disorder and orientation. Define the coarse-grained average over N regions:

$$\mathbf{\bar{W}\_N(\omega) = (1/N)\ \Sigma_{r=1}^N\ W\_r(\omega)}$$

and its continuum counterpart over a spatial volume V:

$$\mathbf{\bar{W}\_V(\omega) = (1/V) \int\_V\ d^3x\ W(x, \omega)}$$

Proposition (Self-Averaging of the Coarse-Grained Bath). *Assume:*

(SA1) Finite mean: $\mathbb{E}[|W\_r(\omega)|] < \infty$

(SA2) Finite variance: $Var(W\_r(\omega)) < \infty$

(SA3) Short-range spatial correlations: $\Sigma_s\ |Cov(W\_r(\omega), W_s(\omega))| < \infty$*, equivalently the spatial covariance function* $C\_\omega(r) = Cov(W(x,\omega), W(x+r,\omega))$ *is integrable.*

(SA4) Local smoothness: each $W\_r(\omega)$ *is* C^2 *in* ω *near* $\omega = 0$*, uniformly in r.*

Then as $N \to \infty$ *(equivalently* $V \to \infty$*):*

(i) $\bar{W}\_N(\omega) \to \mu(\omega) := \mathbb{E}[W\_r(\omega)]$ *in probability for each fixed* ω*.*

(ii) $Var(\bar{W}\_N(\omega)) = O(N^{-1})$*; rms fluctuation* $\delta\bar{W}\_N(\omega) \sim N^{-1}/^2$*.*

(iii) The mean $\mu(\omega)$ *is* C^2 *near* $\omega = 0$*, with* $\mu(0) = \mathbb{E}[W\_r(0)] > 0$*.*

(iv) Continuum limit: $Var(\bar{W}\_V(\omega)) \sim V^{-1}$*; rms* $\sim V^{-1}/^2$*.*

Proof. (i) follows from the law of large numbers applied to the sequence $\{W\_r(\omega)\}$, which has finite mean by SA1. (ii) follows from:

$$\mathbf{Var(\bar{W}\_N(\omega)) = (1/N^2)\ \Sigma_{r,s}\ Cov(W\_r(\omega), W_s(\omega))}$$

Under SA3 the double sum is bounded by $N \times \Sigma_s|Cov(W\_r, W_s)| < \infty$, giving $Var(\bar{W}\_N) = O(N^{-1})$. The continuum version (iv) follows identically with the replacement $N \to V/\xi^3$ where ξ is the spatial correlation length. (iii) follows from: $\mu(\omega) = \mathbb{E}[W\_r(\omega)]$ is the expectation of a C^2

function (by SA4), which inherits C^2 regularity under the expectation when SA2 holds; positivity $\mu(0) > 0$ holds provided $\mathbb{E}[W_r(0)] > 0$, i.e. no systematic cancellation drives the mean to zero. ■

Physical interpretation. Each K=7 cell is sharply structured — a single Lorentzian at ω_1. But the κ-field couples to the bath averaged over many cells across the cosmological horizon volume. Cell-to-cell variation in $\omega_1(\Delta)$, in the coupling weights $\{|g_a,r|^2\}$, and in the orientation of each cell relative to the κ-field mode contribute independent random fluctuations to $W_r(\omega)$. Under SA3 (short-range correlations — cells separated by more than a correlation length ξ contribute independently), these fluctuations cancel in the average at rate $N^{-1/2}$. The observable bath is therefore not the single-cell resonance but a smooth, self-averaged quantity whose rms variation about the mean is suppressed by the square root of the number of contributing regions.

Status of assumptions SA1–SA4. These are mild conditions:

- **SA1** and **SA2** require finite mean and variance of the local spectral weight. Since all mode frequencies $\omega_a,r > 0$ (gapped spectrum), each $|\chi_a,r(0)|^2 = \omega_a,r^{-4}$ is bounded, and σ_a,r^2 is finite by the physical admissibility of commitment events. SA1–SA2 are expected to hold.
- **SA3** requires that the spatial covariance $C(r) = \mathrm{Cov}(W(x,0), W(x+r,0))$ be integrable: $\int d^3r\, |C(r)| < \infty$. This condition is now **derived** from a minimal model of inter-cell coupling, rather than assumed.

  Derivation of SA3 from a gapped inter-cell bath. Define the local zero-frequency spectral weight for cell r:

  $$\mathbf{W_r(0) = \Sigma_{a=1}^{6} |g_a,r|^2 \sigma_a,r^2 \omega_a,r^{-4}}$$

  and its fluctuation $\delta Wr := W_r(0) - \langle W(0)\rangle$. Assume neighbouring K=7 cells are weakly coupled, so that δWr has the quadratic effective Hamiltonian:

  $$\mathbf{H_{eff} = \tfrac{1}{2} \Sigma_r \mu^2 u_r^2 + (\kappa/2) \Sigma_{\langle r,r'\rangle} (u_r - u_r')^2}$$

  where $u_r \equiv \delta Wr$, $\mu^2 > 0$ is the local restoring curvature set by the nonzero K=7 gap, and $\kappa > 0$ is the inter-cell coupling strength. Fourier transforming on a cubic lattice and expanding at small k:

  $$\mathbf{H_{eff} \approx \tfrac{1}{2} \int d^3k/(2\pi)^3 (\mu^2 + \kappa a^2k^2) |u_k|^2}$$

  The two-point covariance of a Gaussian field is the inverse kernel:

  $$\mathbf{\langle u_k u_{-k}\rangle = (\mu^2 + \kappa a^2k^2)^{-1}}$$

  Transforming to real space gives the Yukawa form:

  $$\mathbf{C(r) \propto e^{-r/\xi} / r, \quad \xi = \sqrt{\kappa} \cdot a / \mu}$$

Since $C(r) \sim e^{-r/\xi}/r$, the covariance is absolutely integrable in 3D: $\int d^3r\, |C(r)| < \infty$. **SA3 is therefore derived** under the gapped inter-cell model.

Derivation of $\mu^2 > 0$ from the K=7 gap. SA3 requires only $\mu^2 > 0$, which we now derive in the minimal quadratic inter-cell model. Let x_r denote the coarse-grained local control variable that shifts the non-zero K=7 mode frequency: $\omega_{1,r}^2 = \omega_1^2 + \alpha x_r$, where $\omega_1^2 = \omega_0^2 + 7 > 0$ and α is the coupling coefficient. The local spectral weight is then:

$$\mathbf{W_r(0; x_r) = \sigma_r^2 (\omega_1^2 + \alpha x_r)^{-2}}$$

Its second derivative at the homogeneous background $x_r = 0$ is:

$$\mathbf{d^2W_r/dx_r^2|_{x_r=0} = 6\alpha^2\sigma^2\omega_1^{-8} > 0}$$

since $\sigma^2 > 0$ and $\omega_1^2 > 0$. The local spectral-weight fluctuation is therefore a *convex* function of x_r — an exact consequence of the K=7 gap.

Now express the coarse-grained curvature μ_0^2 in terms of the local free-energy curvature m_x^2 of the control variable. If $F_{loc}(x_r) = F_0 + \frac{1}{2} m_x^2 x_r^2 + \ldots$ and $u_r \approx (dW/dx)x_r$ to leading order, then:

$$\mathbf{\mu_0^2 = m_x^2 / (dW_r/dx_r)^2 = m_x^2\, \omega_1^{12} / (4\alpha^2\sigma^4)}$$

The sign of μ_0^2 is determined entirely by the sign of m_x^2. Since x_r perturbs the same local sector whose isolated-cell non-zero modes have gap $\omega_1^2 = \omega_0^2 + 7$, the local free-energy curvature inherits that gap: $m_x^2 \sim \omega_1^2 > 0$ to leading order. Therefore:

$$\mathbf{\mu_0^2 > 0}$$

With gradient-type inter-cell coupling $(u_r - u_{r'})^2$, the full Fourier kernel is $\mu_0^2 + \kappa a^2 k^2$. At $k = 0$ this equals μ_0^2 — nearest-neighbour gradient coupling raises higher-k modes but does not reduce the zero-momentum mass. Therefore:

$$\mathbf{\mu_{eff}^2 = \mu_0^2 > 0 \ \text{(minimal quadratic inter-cell model)}}$$

and SA3 is derived, not assumed, in this model. The Yukawa covariance $C(r) \propto e^{-r/\xi}/r$ with $\xi = \sqrt{\kappa} \cdot a/\mu$ follows immediately, and $\int d^3r|C(r)| < \infty$.

What remains open. The derivation above holds in the minimal quadratic model. In general, nonlinear inter-cell interactions contribute a zero-momentum self-energy $\Sigma(0)$ to the effective mass:

$$\mathbf{\mu_{eff}^2 = \mu_0^2 + \Sigma(0)}$$

SA3 requires $\mu_{eff}^2 > 0$. The leading nonlinear corrections arise from cubic (g_3) and quartic (g_4) terms in the effective Hamiltonian density $H[u] = \frac{1}{2}\mu_0^2 u^2 + \frac{1}{2}\kappa a^2(\nabla u)^2 +$

$(g_3/3!)u^3 + (g_4/4!)u^4 + \ldots$. These give one-loop self-energy corrections that are explicitly computable:

Quartic tadpole ($g_4 > 0$ stabilises): $\Sigma_4(0) = (g_4/4\pi^2\kappa a^2)\,[\Lambda - (\mu_0/\sqrt{(\kappa a^2)})\,\arctan(\sqrt{(\kappa a^2)}\Lambda/\mu_0)] > 0$

Cubic one-loop (g_3 can destabilise): $\Sigma_3(0) = -(g_3^2/8\pi^2(\kappa a^2)^{3/2}\mu_0)\,[\arctan(\sqrt{(\kappa a^2)}\Lambda/\mu_0) - \sqrt{(\kappa a^2)}\Lambda\mu_0/(\mu_0^2 + \kappa a^2\Lambda^2)] \leq 0$

where Λ is the UV momentum cutoff (set by the inverse lattice spacing a^{-1}). The quartic correction is strictly positive for $g_4 > 0$ — self-interaction stabilises. The cubic correction is negative and can destabilise if g_3^2/μ_0 is large. The net stability condition is:

$\mu_0^2 + \Sigma_4(0) + \Sigma_3(0) > 0$

In the natural regime where the fluctuation theory is approximately symmetric about the mean ($g_3 \approx 0$) and $g_4 > 0$, the corrections are stabilising and SA3 is robust. The worst-case scenario requires a large cubic asymmetry $|g_3|^2 > (\text{constant}) \times \mu_0^3/\kappa a^2$, with no evident mechanism in the K=7 architecture. The determination of g_3 and g_4 from the K=7 nonlinear commitment dynamics is the final open item of the programme; it is identified in Section 9.2.

- **SA4** requires each local $W_r(\omega)$ to be smooth in ω near zero. Since $|\chi_{a,r}(\omega)|^2 = (\omega_{a,r}^2 - \omega^2)^{-2}$ is analytic at $\omega = 0$ for all $\omega_{a,r} > 0$, SA4 holds for each realisation, uniformly in r (since $\omega_{a,r}$ is bounded below by $\omega_1 - \delta$ for any finite disorder δ).

SA1–SA2 and SA4 hold analytically from the gapped spectrum. SA3 is derived in the minimal quadratic inter-cell model: $\mu_0^2 > 0$ from the K=7 gap, Yukawa covariance, integrable. **The infrared flatness result is therefore derived in the minimal model.** The sole residual caveat is whether nonlinear inter-cell corrections $\Sigma(0)$ are large enough to cancel the bare gap μ_0^2; this is identified as an open question in Section 9.2 but has no evident mechanism in the K=7 framework.

Consequent statement of C1. Condition C1 is now not an assumption about smoothness but a consequence of statistical physics:

(C1, revised) The local spectral weights $\{W_r(\omega)\}$ satisfy SA1–SA4. Under these conditions, condition C1 is satisfied: the coarse-grained bath $\bar{W}_V(\omega)$ is self-averaging, its variance shrinks as V^{-1}, and its mean $\mu(\omega) = \mathbb{E}[W_r(\omega)]$ is C^2 near $\omega = 0$ with $\mu(0) > 0$.

The key logical step is: smoothness of $\bar{W}_V(\omega)$ follows not from an assertion that the coarse-grained function happens to be smooth, but from the fact that (i) each local $W_r(\omega)$ is C^2 in ω for every realisation r (SA4, which holds analytically from the gapped spectrum), and (ii) the law of large numbers guarantees that $\bar{W}_V$ converges to $\mathbb{E}[W_r]$, which inherits C^2 regularity from each term. The role of SA3 is to ensure convergence is fast enough (variance $O(V^{-1})$); without SA3, convergence still holds but may be slow. Smoothness of the coarse-grained bath is therefore a theorem of statistical mechanics under SA1–SA4, not a phenomenological assumption.

Conditions C2–C3 (bounded overlap weights; smooth response functions) are retained as before and are expected to hold for the reasons given in the previous version.

Remark on robustness. The $O(\omega^2)$ correction to flatness is parametrically controlled: deviations from $\mu(0)$ at frequency ω are of order $(\omega/\omega_a)^2 \sim (H/m_\kappa)^2 \ll 1$ in the cosmologically relevant regime. The self-averaging fluctuations satisfy $\mathrm{Var}(\bar{W}_V) \sim V^{-1}$, rms $\sim V^{-1/2}$, additionally suppressed by the large number of contributing cells. The infrared flatness is therefore doubly protected: by the Taylor smoothness of each local response function, and by the law-of-large-numbers averaging over many cells.

4.4 Single-Event Profile Contribution

Step 1 of the flatness argument concerns the single-event power spectrum $|\hat{f}(\omega)|^2$. For any smooth impulse with non-zero total area ($\int f\,dt \neq 0$):

$$|\hat{f}(\omega)|^2 = |\hat{f}(0)|^2 + O(\omega^2/m_\kappa^2)$$

This follows from Taylor expansion. The non-zero total area condition corresponds to the physical requirement that each commitment event produces net entropy — a necessary condition in the VERSF ontology for the event to count as a commitment.

Combined with the conditional infrared flatness of $\bar{W}_{K7}(\omega)$, the zero-frequency spectral density of the bath is:

$$J(0) = \Gamma\, |\hat{f}(0)|^2\, \bar{W}_{K7}(0)$$

with $[J(0)] = M^{-3}$. The effective amplitude entering the κ-field equation of motion incorporates the bath coupling constant g_κ introduced in Section 2.3:

$$\gamma_m \equiv g_\kappa^2\, \Gamma\, |\hat{f}(0)|^2\, \bar{W}_{K7}(0)$$

$[\gamma_m] = M^6 \times M \times 1 \times M^{-4} = M^3$, matching the dimension of each kinetic term in the equation of motion (Section 8). Throughout the paper γ_m appears in the dimensionless observable combinations γ_m/H^3 and $\beta = \gamma_m/H^3_{\text{end}}$.

The infrared spectral density is asymptotically constant, generically emerging in the limit $\omega \ll m_\kappa$ under self-averaging conditions SA1–SA4:

$$J(\omega) \approx J(0) + O(\omega^2/m_\kappa^2)$$

4.5 Explicit Computation of $\bar{W}_{K7}(0)$

The spectral weight $\bar{W}\_{K7}(0)$ was previously identified as an open numerical calculation. We now derive it explicitly using the projector identity of the K=7 eigensystem and the disorder model of Section 2.3.

Step 1 — Projector identity. The six non-zero modes of K_7 form an orthonormal basis of the subspace orthogonal to the uniform mode. Their outer product satisfies the projector identity:

$$\boldsymbol{\Sigma_{a=1}^{6} u_i^{(a)} u_j^{(a)} = \delta_{ij} - 1/7}$$

This follows from completeness: the full resolution of identity is $\Sigma_{a=0}^{6} u_i^{(a)} u_j^{(a)} = \delta_{ij}$, and the zero-mode contribution is $u_i^{(0)} u_j^{(0)} = 1/7$ for all i, j.

Step 2 — Collapse of the overlap sum. Since $g_a = \Sigma_i c_i u_i^{(a)}$:

$$\boldsymbol{\Sigma_{a=1}^{6} |g_a|^2 = \Sigma_{ij} c_i c_j (\delta_{ij} - 1/7) = \Sigma_i c_i^2 - (1/7)(\Sigma_i c_i)^2}$$

The κ-field symmetry condition $\Sigma_i c_i = 0$ (Section 2.3) eliminates the second term exactly:

$$\boldsymbol{\Sigma_{a=1}^{6} |g_a|^2 = \|c\|^2}$$

This is an exact result from the K=7 projector, requiring no approximation.

Step 3 — Isolated-cell result. For the exact isolated K=7 cell all six non-zero modes are degenerate at $\omega_1^2 = \omega_0^2 + 7$. With isotropic event variance $\sigma_a^2 = \sigma^2$ (equal for all six modes by the sixfold symmetry):

$$\boldsymbol{W\_{K7}(0) = \sigma^2\|c\|^2 / (\omega_0^2 + 7)^2}$$

For the normalized D_{5h} coupling vector with $\Sigma_i c_i = 0$ and $\|c\|^2 = 1$ (six equatorial entries $a = \pm 1/\sqrt{42}$, one axial entry $b = \mp 6/\sqrt{42}$, satisfying $6a^2 + b^2 = 1$ and $6a + b = 0$):

$$\boldsymbol{W\_{K7}(0) = \sigma^2 / (\omega_0^2 + 7)^2}$$

This is the exact isolated-cell spectral weight. It is finite and positive, confirming $\bar{W}\_{K7}(0) > 0$ for the isolated cell.

Step 4 — Leading disorder correction. Including the weak disorder model $\omega_{a,r}^2 = \omega_1^2 + \delta_{a,r}$ with $\mathbb{E}[\delta_{a,r}] = 0$ and $\mathrm{Var}(\delta_{a,r}) = \sigma\_\delta^2$:

$$\boldsymbol{\bar{W}\_{K7}(0) = \|c\|^2 \sigma^2 \mathbb{E}[(\omega_1^2 + \delta)^{-2}]}$$

Expanding for weak disorder:

$$\boldsymbol{(\omega_1^2 + \delta)^{-2} = \omega_1^{-4} (1 - 2\delta/\omega_1^2 + 3\delta^2/\omega_1^4 + O(\delta^3))}$$

Taking the expectation with $\mathbb{E}[\delta] = 0$:

$$\mathbb{E}[(\omega_1^2 + \delta)^{-2}] = \omega_1^{-4} (1 + 3\sigma\_\delta^2/\omega_1^4 + O(\sigma\_\delta^3))$$

Therefore, with $\omega_1^2 = \omega_0^2 + 7$ and $\|c\|^2 = 1$:

$$\bar{W}\_K7(0) = \sigma^2/(\omega_0^2 + 7)^2 \cdot (1 + 3\sigma\_\delta^2/(\omega_0^2 + 7)^2 + O(\sigma\_\delta^3))$$

Step 5 — Derivation of σ^2 from vertex-level event statistics. The modal variance $\sigma_a^2 = \langle\zeta_a^2\rangle$ can be computed explicitly from the most general permutation-symmetric covariance of single-event vertex excitations on K_7. Since all seven vertices are equivalent, the unique such covariance is:

$$\langle\eta_i \eta_j\rangle = \nu^2 [(1-\rho)\delta_{ij} + \rho]$$

where ν^2 is the per-vertex event variance and ρ is the common-mode correlation between vertices. Substituting into $\sigma_a^2 = \Sigma_{ij} u_i^{(a)} u_j^{(a)} \langle\eta_i \eta_j\rangle$:

$$\sigma_a^2 = \nu^2(1-\rho) \Sigma_i (u_i^{(a)})^2 + \nu^2\rho (\Sigma_i u_i^{(a)})^2$$

For every non-zero mode: $\Sigma_i (u_i^{(a)})^2 = 1$ (normalisation) and $\Sigma_i u_i^{(a)} = 0$ (orthogonality to the uniform mode). The second term vanishes exactly, giving:

$$\sigma^2 \equiv \sigma_a^2 = \nu^2(1-\rho) \text{ for all } a = 1, \ldots, 6$$

Isotropy across the six non-zero modes is therefore not an additional assumption — it follows automatically from the permutation symmetry of K_7 and the orthogonality of non-zero modes to the uniform mode. The physical interpretation is sharp: events that are perfectly common-mode across all vertices ($\rho = 1$) contribute zero variance to the non-zero modes and hence decouple entirely from the κ-field bath. Only the non-uniform component of each event drives the bath.

Step 6 — Derivation of σ_δ^2 from Poisson commitment disorder. The disorder variance $\sigma\_\delta^2 = \mathrm{Var}(\delta_r)$ arises from cell-to-cell fluctuations in commitment-event density. Let N_r be the number of commitment events in cell r during a coarse-graining interval τ_c, with $N_r \sim \mathrm{Poisson}(\bar{N})$ and $\bar{N} = \Gamma\tau\_c$. Define the fractional fluctuation $\Delta_r := (N_r - \bar{N})/\bar{N}$, giving $\langle\Delta_r\rangle = 0$ and $\mathrm{Var}(\Delta_r) = 1/\bar{N}$ (Poisson shot noise). The local mode frequency shifts linearly with fractional excess density:

$$\delta_r = \lambda\_\delta\, \omega_1^2\, \Delta_r$$

where λ_δ is the dimensionless sensitivity of the local K=7 gap to density disorder. Therefore:

$$\sigma\_\delta^2 = \mathrm{Var}(\delta_r) = \lambda\_\delta^2\, \omega_1^4\, \mathrm{Var}(\Delta_r) = \lambda\_\delta^2\, (\omega_0^2 + 7)^2 / \bar{N}$$

The disorder correction to $\bar{W}\_K7(0)$ then simplifies to:

$$3\sigma\_\delta^2/(\omega_0^2 + 7)^2 = 3\lambda\_\delta^2/\bar{N}$$

Complete explicit formula. Substituting $\sigma^2 = \nu^2(1-\rho)$ and $\sigma_\delta^2 = \lambda_\delta^2(\omega_0^2 + 7)^2/\bar{N}$ into the structural result of Step 4:

$$\bar{W}_{K7}(0) = \nu^2(1-\rho)/(\omega_0^2 + 7)^2 \cdot (1 + 3\lambda_\delta^2/\bar{N} + O(\lambda_\delta^3))$$

where $\bar{N} = \Gamma\tau_c$ is the mean number of commitment events per cell per coarse-graining time. This is the complete leading-order formula for the infrared spectral weight, expressed in terms of four physical parameters: the per-vertex event variance ν^2, the common-mode fraction ρ, the gap sensitivity λ_δ, and the mean event count $\bar{N}$.

Physical interpretation: the bath strength increases with ν^2 (stronger events), decreases as $\rho \to 1$ (more common-mode events decouple), and the disorder correction $3\lambda_\delta^2/\bar{N}$ is suppressed by large $\bar{N}$ — more events per cell average out disorder. All four parameters are in principle computable from the K=7 nonlinear commitment dynamics; their determination is identified as the final numerical open item in Section 9.2.

5.1 Caldeira-Leggett Effective Kernel

Integrating out the bath in the Caldeira-Leggett framework produces the effective κ-field equation of motion (derived fully in the companion Friedmann paper):

$$(\partial_t^2 + 3H\,\partial_t + m_\kappa^2)\,\delta s + g_\kappa^2 \int_0^t \mathcal{M}_{\mathrm{eff}}(t - t')\,\delta s(t')\,dt' = \xi_{\mathrm{noise}}(t)$$

where the memory kernel is:

$$\mathcal{M}_{\mathrm{eff}}(\tau) = (2/\pi) \int_{\omega_{IR}}^{\omega_{UV}} [J(\omega)/\omega] \cos(\omega\tau)\,d\omega$$

with $\omega_{IR} = H$ (physical IR cutoff; commitment events occur at rate $\Gamma \sim H$) and $\omega_{UV} = m_\kappa$ (physical UV cutoff; the κ-field mass screens sub-m_κ contributions). The g_κ^2 prefactor is kept explicit here for dimensional transparency; in subsequent expressions $\gamma_m \equiv g_\kappa^2 J(0)$ absorbs this factor, so results written in terms of γ_m already carry the correct dimension M^3 (as verified in Section 8).

5.2 Evaluating the Kernel

Inserting $J(\omega) = \gamma_m$ (asymptotically flat, Section 4.4):

$$\mathcal{M}_{\mathrm{eff}}(\tau) = (2\gamma_m/\pi) \int_H^{m_\kappa} (1/\omega) \cos(\omega\tau)\,d\omega = (2\gamma_m/\pi)\,[\mathrm{Ci}(m_\kappa\tau) - \mathrm{Ci}(H\tau)]$$

Both cosine integrals admit the large-argument approximation $\mathrm{Ci}(x\tau) \sim \cos(x\tau)/(x\tau)$. In the regime $m_\kappa^{-1} \ll \tau \ll H^{-1}$, the kernel is dominated by the m_κ oscillatory envelope; at later times ($\tau \sim H^{-1}$), both contributions decay but retain the τ^{-1} envelope:

$$\mathcal{M}_{\mathrm{eff}}(\tau) \sim (2\gamma_m / \pi m_\kappa) \cdot \cos(m_\kappa\tau) / \tau \quad (m_\kappa^{-1} \ll \tau)$$

The envelope decays as τ^{-1} — a power-law with exponent $\alpha = 1$.

5.3 Why $\alpha = 1$ Is the Unique Consistent Value

The decay exponent α of the memory kernel is read off from the integrand of $\mathcal{M}_{\mathrm{eff}}(\tau) = (2/\pi) \int [J(\omega)/\omega] \cos(\omega\tau)\, d\omega$. The large-$\tau$ envelope of this integral is controlled by the low-frequency behaviour of $J(\omega)/\omega$:

- For $\mathbf{J(\omega) \propto \omega^s}$ ($s > 0$), the integrand $J(\omega)/\omega \propto \omega^{s-1}$ vanishes at $\omega = 0$ for $s > 1$ and diverges for $s < 1$. The Riemann-Lebesgue lemma gives $\mathcal{M}_{\mathrm{eff}}(\tau) \to 0$ as $\tau \to \infty$ for $s > 1$ (super-Ohmic); for $s < 1$ (sub-Ohmic) the IR-divergent integrand produces slower-than-τ^{-1} decay.
- For $\mathbf{J(\omega) \propto \omega}$ (Ohmic, $s = 1$), $J(\omega)/\omega = \text{const}$, so $\mathcal{M}_{\mathrm{eff}}(\tau) \propto \int \cos(\omega\tau)\, d\omega \to \sin(\omega_c \tau)/\tau$ — a τ^{-1} envelope, $\alpha = 1$.
- For $\mathbf{J(\omega) = \gamma_m}$ (white-noise, VERSF), $J(\omega)/\omega \propto 1/\omega$ is IR-divergent but regulated by $\omega_{IR} = H$; the result from Section 5.2 is $\mathcal{M}_{\mathrm{eff}}(\tau) \propto \cos(m_\kappa\tau)/\tau$ — also a τ^{-1} envelope, $\alpha = 1$.

Both Ohmic and white-noise baths give $\alpha = 1$ via this route. The sub-Ohmic case gives $\alpha < 1$ (slower decay); super-Ohmic gives faster decay. The table summarises:

Bath class	**$J(\omega \to 0)$**	**$\mathcal{M}_{\mathrm{eff}}(\tau)$ envelope**	**α**	**δs growth**
Sub-Ohmic	$\propto \omega^s$, $s < 1$	slower than τ^{-1}	< 1	Power-law/exponential
Ohmic	$\propto \omega$	$\sin(\omega_c \tau)/\tau$	1	Logarithmic
VERSF (white-noise)	**$= \gamma_m$ (flat)**	**$\cos(m_\kappa\tau)/\tau$**	**1**	**Logarithmic**
Super-Ohmic	$\propto \omega^s$, $s > 1$	faster than τ^{-1}	> 1	Saturates

The Ohmic and VERSF cases both give logarithmic displacement growth, but are observationally distinguishable in principle: the VERSF cosine kernel carries the oscillation frequency m_κ, whereas the Ohmic sine kernel carries ω_c. If m_κ is independently constrained (e.g. from the σ_τ/σ_{opt} bench-top prediction), the kernel form is in principle falsifiable.

The sub-Ohmic case is excluded on observational grounds. For $\alpha < 1$ the memory kernel decays slower than τ^{-1}, producing a displacement that grows faster than logarithmically — specifically as a power law t^α with $\alpha < 1$, or faster. In the κ-field context this generates a scalar power spectrum $P(k) \propto k^{ns}$ with $n_s > 1$ (blue tilt), since a steeper displacement growth amplifies small-scale modes relative to large scales. Planck measures $n_s = 0.9649 \pm 0.0042$ (red tilt), so any $\alpha < 1$ bath is observationally excluded. The super-Ohmic case saturates displacement growth and fails to sustain sufficient e-folds for the observed horizon. The $\alpha = 1$ white-noise result is the unique universality class consistent with a red-tilted nearly scale-invariant spectrum.

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6. Logarithmic Displacement Growth: Explicit Derivation

6.1 The Equation of Motion in the Slow-Roll Regime

The full κ-field equation is:

$$\delta\ddot{s} + 3H\,\delta\dot{s} + \int_0^t [\cos(m_\kappa(t - t')) / (t - t')]\,\delta s(t')\,dt' = \xi(t)$$

where we have substituted the derived form of $\mathcal{M}_{\text{eff}}(\tau)$ and absorbed the amplitude $(2\gamma_m/\pi m_\kappa)$ into the coefficient of the memory integral.

In the slow-roll regime, the kinetic energy is small compared to the memory-driven displacement:

$$\delta\dot{s} \ll H\,\delta s \quad \text{(slow-roll condition)}$$

Under this condition the acceleration term $\delta\ddot{s}$ is also subdominant, and the equation reduces to:

$$3H\,\delta\dot{s} + \int_0^t [\cos(m_\kappa(t - t')) / (t - t')]\,\delta s(t')\,dt' \approx \xi(t)$$

6.2 Heuristic Asymptotic Form and Consistency with Logarithmic Growth

We do not attempt here to derive the numerical coefficient A from a full stochastic treatment of the forcing term. The replacement of $\xi(t)$ by an effective constant amplitude is only a heuristic device for identifying the asymptotic functional form of the growing solution; a proper derivation of A requires analysis of the variance of the Poisson-driven memory integral. The present section establishes the logarithmic functional form; Section 6.3 provides the more transparent microscopic argument.

Posit the ansatz $\delta s(t) \sim A \ln(Ht)$. Then $\delta\dot{s} \sim A/t$. The memory integral can be estimated by integration by parts: since $\ln(Ht')$ varies slowly on scales m_κ^{-1}, treating it as approximately constant over each oscillation cycle gives:

$$\int_0^t [\cos(m_\kappa(t-t'))/(t-t')]\,\ln(Ht')\,dt' = [\sin(m_\kappa(t-t'))/m_\kappa \cdot \ln(Ht')/(t-t')]_0^t + O(m_\kappa^{-1})$$

yielding suppression $O(1/m_\kappa)$ relative to the Hubble friction term, i.e. a contribution of order $A\cdot(H/m_\kappa)\cdot\ln(Ht) \ll A/t$. The slow-roll equation then reduces to:

$$3H \cdot (A/t) \sim \xi_{\text{eff}}(t)$$

where $\xi_{\text{eff}}(t)$ is the effective noise amplitude at time t. This equation is consistent with the ansatz only if $\xi_{\text{eff}}(t) \sim A/t$. This is a formal consistency condition on the ansatz, not a derivation: a stationary Poisson process has constant noise power spectral density, so the requirement $\xi_{\text{eff}} \sim 1/t$ cannot be justified from Poisson statistics alone at this level of argument. Section 6.2 should therefore be read strictly as a check that the logarithmic form is not self-contradictory within the equation of motion, not as a derivation of that form. The microscopic argument establishing logarithmic growth is given in Section 6.3.

$\delta s(t) \sim A \ln(Ht)$, where A is determined by the Poisson event variance (Section 6.3)

6.3 Microscopic Derivation from the Poisson Sum

The transparent microscopic derivation of the logarithmic form proceeds directly from the $1/\tau$ memory kernel and Poisson event statistics, without requiring the heuristic balance of Section 6.2.

Each commitment event at time t_n contributes to the current displacement through the memory kernel:

δs contribution from event n: $\sim K(t - t_n) \sim 1/(t - t_n)$

Events arrive as a Poisson process with rate $\Gamma \sim H$. The expected cumulative displacement is:

$\mathbb{E}[\delta s(t)] \propto \Gamma \int_{t_0}^{t} d\tau / \tau \sim \Gamma \ln(t/t_0) \sim H \ln(Ht)$

With $\Gamma \sim H$ and identifying $N = Ht$ as the e-fold count:

$\mathbb{E}[\delta s(t)] \propto \ln(Ht) \propto \ln N$

This derivation is clean and requires only:

1. The $1/\tau$ envelope of the memory kernel (established in Section 5.2 from the flat spectral density)
2. The Poisson statistics of commitment events (Assumption P1)
3. The Poisson rate $\Gamma \sim H$ (from the CCC threshold condition)

The logarithmic form is therefore a direct consequence of these three inputs. The coefficient of proportionality is not arbitrary — it is set by the Poisson event rate and the memory kernel amplitude — but its precise value is scheme-dependent until the full stochastic variance calculation is completed. To see why: the accumulated displacement involves the integral $\int_{t_0}^{t} K(\tau)/\tau \, d\tau$ where $K(\tau) \sim 2\gamma_m/(\pi m_\kappa)$ is the kernel envelope. This integral evaluates to $(2\gamma_m/\pi m_\kappa) \ln(t/t_0)$, so A depends explicitly on the lower cutoff t_0:

$A = (2g_\kappa^2 J(0) / \pi m_\kappa) \cdot \ln(t/t_0) \times$ (Poisson variance factor)

The logarithmic dependence on t/t_0 is the source of the logarithmic growth in δs — entirely consistent — but it means A is not a pure number; it depends on the initial time t_0 at which the quasi-static approximation becomes valid. This scheme dependence is resolved by the full stochastic variance calculation, which determines t_0 from the onset of the slow-roll regime. Once $\bar{W}_{K7}(0)$ is determined from the K=7 eigensystem and t_0 is fixed by the dynamics, A is predicted without free parameters. The present expression makes clear that A is computable and bounded; it does not yet constitute a numerical prediction.

7. Parameter Unification

7.1 Bath-Induced Parameters and Their Common Origin

With $\bar{W}_{K7}(0)$ now derived in explicit form (Section 4.5), all bath-induced cosmological parameters can be written out fully in terms of the five physical event-statistics inputs $\{v^2, \rho, \lambda_\delta, \bar{N}, g_\kappa\}$. Substituting $\bar{W}_{K7}(0) = v^2(1-\rho)/(\omega_0^2+7)^2 \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots)$:

γ_m — the bath spectral amplitude:

$$\gamma_m = g_\kappa^2\Gamma|\hat{f}(0)|^2 v^2(1-\rho) / (\omega_0^2+7)^2 \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots)$$

λ_m — the quasi-static response coefficient:

$$\lambda_m = N_{K7}\, v^2(1-\rho) / (\omega_0^2+7)^2 \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots)$$

where N_{K7} is the geometric normalisation from the K=7 eigenstructure. Note λ_m does not carry the $g_\kappa^2\Gamma$ prefactor — it is the purely bath-side geometric parameter.

β — the transition efficiency:

$$\beta = g_\kappa^2\Gamma|\hat{f}(0)|^2 v^2(1-\rho) / [H^3_{end}\, (\omega_0^2+7)^2] \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots)$$

Using $\Gamma \sim H$ near horizon crossing: $\beta \sim g_\kappa^2 H|\hat{f}(0)|^2 v^2(1-\rho) / [H^3_{end}(\omega_0^2+7)^2] \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots)$.

C — the subleading spectral correction:

C is proportional to γ_m through a positive coefficient χ_C from the perturbation mode equation:

$$C = \chi_C\, g_\kappa^2\Gamma|\hat{f}(0)|^2 v^2(1-\rho) / (\omega_0^2+7)^2 \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots)$$

The coefficient χ_C is positive (proved in the companion Friedmann paper) but requires the full perturbation spectrum calculation for its numerical value.

Physical interpretation of all four parameters: larger event strength v^2 raises all bath-induced parameters; more common-mode events ($\rho \to 1$) suppresses them; stronger gap sensitivity λ_δ increases the disorder correction; larger averaging count $\bar{N}$ suppresses it. All four parameters are controlled by the same five-input package — the unification is explicit.

7.2 Unification Statement

Result (Parameter Control). All bath-induced cosmological parameters λ_m, β, γ_m, and C are controlled by the single infrared spectral weight $\bar{W}_{K7}(0)$, up to cosmological background quantities (Γ, H_{end}).

This is the appropriate statement. It is weaker than claiming all parameters are uniquely predicted by $\bar{W}_{K7}(0)$ alone — β depends on H_{end}, which requires the background Friedmann solution — but it is defensible and non-trivial. It means that once $\bar{W}_{K7}(0)$ is computed, constraints on any one parameter from CMB data constrain the bath-induced portions of all others simultaneously.

7.3 Connection to Observable CMB Predictions

With all parameters now explicit, the spectral index prediction can be written out end-to-end. The logarithmic displacement growth $\delta s \sim A \ln N$ generates a scalar power spectrum with:

$$n_s = 1 - 2/N\bigstar + C/(N\bigstar \ln N\bigstar) + O(N\bigstar^{-2})$$

Substituting $C = \chi_C \gamma_m$ and the explicit γ_m formula:

$$n_s = 1 - 2/N\bigstar + \chi_C\, g_\kappa^2 \Gamma |\hat{f}(0)|^2 \nu^2 (1-\rho) / [N\bigstar \ln N\bigstar\, (\omega_0^2+7)^2] \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots) + O(N\bigstar^{-2})$$

This is the complete explicit VERSF prediction for the CMB spectral index, traced from K=7 geometry through bath statistics to observation.

The leading term $1 - 2/N\bigstar$ is the universal slow-roll result from logarithmic displacement growth. For $N\bigstar = 55$: $n_s \approx 1 - 2/55 \approx 0.9636$ — in agreement with the Planck central value $n_s = 0.9649 \pm 0.0042$.

The subleading correction is controlled by the bath parameter combination $g_\kappa^2 \Gamma |\hat{f}(0)|^2 \nu^2 (1-\rho)/(\omega_0^2+7)^2$, which equals γ_m (up to χ_C). Its sign is positive — bath events shift n_s upward from the pure slow-roll value. Its structure shows:

- stronger non-uniform events (larger ν^2, smaller ρ) increase n_s
- larger K=7 gap (larger ω_0^2+7) suppresses the correction
- stronger disorder sensitivity λ_δ enhances it; larger $\bar{N}$ suppresses it

Planck constraint as a bound on the event-statistics package. The correction must satisfy:

$$\chi_C\, g_\kappa^2 \Gamma |\hat{f}(0)|^2 \nu^2 (1-\rho) / [(55 \times \ln 55)(\omega_0^2+7)^2] \times (1 + 3\lambda_\delta^2/\bar{N}) \lesssim 0.004$$

This is a concrete observable upper bound on the combination $\nu^2(1-\rho)/(\omega_0^2+7)^2$ — the first direct constraint on K=7 event-statistics parameters from CMB data, expressible once χ_C, g_κ, and Γ are determined from the companion papers.

Complete microscopic-to-cosmological bridge:

Quantity	**Explicit formula**
$\bar{W}_{K7}(0)$	$\nu^2(1-\rho)/(\omega_0^2+7)^2 \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots)$

Quantity	Explicit formula
γ_m	$g_\kappa^2\Gamma\lvert\hat{f}(0)\rvert^2\nu^2(1-\rho)/(\omega_0^2+7)^2 \times (1 + 3\lambda_\delta^2/\bar{N} + \dots)$
λ_m	$N_{K7}\,\nu^2(1-\rho)/(\omega_0^2+7)^2 \times (1 + 3\lambda_\delta^2/\bar{N} + \dots)$
β	$\gamma_m/H^3_{\mathrm{end}}$
C	$\chi_C\,\gamma_m$
n_s	$1 - 2/N\bigstar + \chi_C\,\gamma_m/(N\bigstar \ln N\bigstar) + O(N\bigstar^{-2})$
Stability	$\mu_{\mathrm{eff}}^2 = \mu_0^2 + \Sigma(0) > 0$

Every entry traces back to five physical inputs: ν^2, ρ, λ_δ, $\bar{N}$ (event statistics) and g_κ (coupling). The framework has been converted from a structural existence claim to an explicit parameterised prediction.

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8. Dimensional Consistency

We verify that $J(\omega)$ has the correct dimensions throughout. Working in natural units with $\hbar = c = 1$, so [time] = [energy]$^{-1}$ = [mass]$^{-1}$.

Commitment rate Γ: Γ is the number of commitment events per coherence volume per unit time. **$[\Gamma] = T^{-1} = M$** (in natural units)

Single-event profile $f(t)$: $f(t)$ is normalised so that $\int f(t)\,dt$ is dimensionless (it encodes the fractional excitation per event). Then **$[f] = T^{-1} = M$**, so **$[\hat{f}(\omega)] = [\hat{f}(0)]$ = dimensionless**. Therefore **$[\lvert\hat{f}(0)\rvert^2] = 1$** (dimensionless).

Mode overlap g_a: $g_a = \Sigma_i c_i u_i^{(a)}$ where c_i and $u_i^{(a)}$ are dimensionless (eigenvectors normalised, coupling vector normalised). **$[g_a] = 1$** (dimensionless).

Modal variance σ_a^2: $\sigma_a^2 = \langle\zeta_a^2\rangle$ where $\zeta_a^{(n)} = \Sigma_i u_i^{(a)} \eta_i^{(n)}$ and $\eta_i^{(n)}$ encodes the fractional vertex excitation (dimensionless). **$[\sigma_a^2] = 1$** (dimensionless).

Response function $\chi_a(\omega)$: $\chi_a(\omega) = (\omega_a^2 - \omega^2 - i\varepsilon\omega)^{-1}$. **$[\chi_a] = M^{-2} = T^2$**.

K=7 spectral weight $W_{K7}(\omega)$: $W_{K7}(\omega) = \Sigma_a \lvert g_a\rvert^2 \sigma_a^2 \lvert\chi_a(\omega)\rvert^2$. **$[W_{K7}] = [\lvert\chi_a\rvert^2] = M^{-4} = T^4$**.

Spectral density $J(\omega)$: $J(\omega) = \Gamma\,\lvert\hat{f}(\omega)\rvert^2\,W_{K7}(\omega)$. **$[J] = T^{-1} \times 1 \times T^4 = T^3 = M^{-3}$**.

Verification via Caldeira-Leggett kernel: $\mathscr{M}_{\mathrm{eff}}(\tau) = (2/\pi) \int [J(\omega)/\omega] \cos(\omega\tau)\,d\omega$. $[J/\omega] = M^{-3}/M = M^{-4}$. Then $[\mathscr{M}_{\mathrm{eff}}] = [J/\omega] \times [d\omega] = M^{-4} \times M = \mathbf{M^{-3}}$.

Dimensional check of the equation of motion:

In 4D natural units ($\hbar = c = 1$), the κ-field is a standard scalar field whose action $\int d^4x \, \frac{1}{2}(\partial\delta s)^2$ is dimensionless. Since $[d^4x] = M^{-4}$ and $[(\partial\delta s)^2] = M^2 \times [\delta s]^2$, dimensionlessness requires **$[\delta s] = M$**. Each kinetic term in the equation of motion then has dimension M^3:

- **$(\partial^2_t)\delta s$:** $M^2 \times M = M^3$ ✓
- **$3H \, \partial_t \, \delta s$:** $M \times M \times M = M^3$ ✓
- **$m^2\_\kappa \, \delta s$:** $M^2 \times M = M^3$ ✓

The memory convolution term: $\int \mathcal{M}\_\mathrm{eff}(\tau) \, \delta s(t') \, dt'$ has dimensions $M^{-3} \times M \times M^{-1} =$ **M^{-3}**, which is inconsistent with M^3 as written. This is resolved by including the coupling constant g_κ (introduced in Section 2.3, $[g\_\kappa] = M^3$), which enters through the bath coupling Hamiltonian $H\_\mathrm{int} = -g\_\kappa \, \delta s \cdot \delta s\_\mathrm{bath}$. The correctly stated equation of motion is:

$$(\partial^2_t + 3H \, \partial_t + m^2\_\kappa) \, \delta s + g\_\kappa^2 \int \mathcal{M}\_\mathrm{eff}(\tau) \, \delta s(t') \, dt' = \xi(t)$$

With $\gamma_m \equiv g\_\kappa^2 \, \Gamma \, |\hat{f}(0)|^2 \, \bar{W}\_K7(0)$ carrying $[\gamma_m] = M^3$, the ratios γ_m/H^3 and $\beta = \gamma_m/H^3\_\mathrm{end}$ are dimensionless ✓. The numerical value of g_κ is fixed by the κ-field action normalisation and is the subject of the companion VERSF normalisation paper.

Summary table:

Quantity	**Dimensions (natural units)**
Γ	M ($= T^{-1}$)
$\lvert\hat{f}(0)\rvert^2$	1 (dimensionless)
$\lvert g_a\rvert^2 \, \sigma_a^2$	1 (dimensionless)
$\lvert\chi_a(\omega)\rvert^2$	M^{-4}
$W\_K7(\omega)$	M^{-4}
$J(\omega)$	M^{-3}
$J(0) = \Gamma \, \lvert\hat{f}(0)\rvert^2 \, \bar{W}\_K7(0)$	M^{-3}
$\gamma_m \equiv g\_\kappa^2 \, J(0)$	M^3
$\mathcal{M}\_\mathrm{eff}(\tau)$	M^{-3}
$g\_\kappa^2 \, \mathcal{M}\_\mathrm{eff}(\tau)$	M^3 (matches EOM terms)

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9. Epistemic Status

9.1 Status Table

Result	Status	Conditional on
K=7 Laplacian spectrum $\{\lambda_a\}$ exact	**PROVEN**	K=7 no-go theorem
Mode frequencies $\omega_a = \sqrt{(\omega_0^2 + \lambda_a)}$	**PROVEN**	above
Zero-mode decoupling $g_0 = 0$	**DERIVED**	$\Sigma_i c_i = 0$ (κ-field symmetry)
Response-corrected $J(\omega) = \Gamma\lvert\hat{f}\rvert^2 W\_K7(\omega)$	**DERIVED**	Caldeira-Leggett formalism, P1, P2
Single-event flatness $\lvert\hat{f}(\omega)\rvert^2 = \lvert\hat{f}(0)\rvert^2 + O(\omega^2)$	**DERIVED**	Smoothness and non-zero total area of f(t)
Coarse-grained flatness $\bar{W}\_K7(\omega) = \bar{W}\_K7(0) + O(\omega^2)$	**CONDITIONAL THEOREM**	SA1–SA4 self-averaging assumptions (see Section 4.3); C2, C3
Self-averaging variance suppression $\delta\bar{W}\_V \sim V^{-1/2}$	**DERIVED**	SA1–SA3 (law of large numbers under short-range correlations)
Infrared flatness $J(\omega \to 0) = \gamma_m$	**CONDITIONAL**	above
Memory kernel $\mathscr{M}\_eff \sim \cos(m\_\kappa\tau)/\tau$	**DERIVED**	flat J(ω)
$\alpha = 1$ universality class	**DERIVED**	white-noise bath, Poisson P1
Slow-roll regime: $\delta s \sim A \ln(Ht)$	**DERIVED**	$\alpha = 1$, slow-roll condition $\delta\dot{s} \ll H\delta s$
Parameter control by $\bar{W}\_K7(0)$	**DERIVED**	above
Dimensional consistency	**VERIFIED**	(Section 8)
$\bar{W}\_K7(0)$ complete formula	**DERIVED**	$\bar{W}\_K7(0) = \nu^2(1-\rho)/(\omega_0^2+7)^2 \cdot (1 + 3\lambda\_\delta^2/\bar{N} + \ldots)$; Section 4.5
$\sigma^2 = \nu^2(1-\rho)$ from vertex covariance	**DERIVED**	Permutation symmetry of K_7; orthogonality of non-zero modes to uniform mode
$\sigma\_\delta^2 = \lambda\_\delta^2(\omega_0^2+7)^2/\bar{N}$ from Poisson disorder	**DERIVED**	Poisson shot noise in commitment-event count
$\Sigma(0)$ one-loop expressions (quartic, cubic)	**DERIVED**	Standard EFT tadpole and bubble integrals; Section 4.3
Microscopic values ν^2, ρ, λ_δ, g_3, g_4	**OPEN**	Require K=7 nonlinear commitment dynamics; normal EFT parameter determination
SA3 from gapped inter-cell model	**DERIVED**	$C(r) \propto e^{-r/\xi}/r$, integrable; $\mu_0^2 > 0$ from K=7 gap and convexity argument
$\mu\_eff^2 = \mu_0^2 + \Sigma(0) > 0$ under nonlinear corrections	**OPEN (expected)**	$\Sigma(0)$ must not cancel bare gap; no evident mechanism for large negative $\Sigma(0)$ in K=7

9.2 The Two Remaining Open Items

Two items are open:

Open 1 — Microscopic values of ν^2, ρ, λ_δ, g_3, g_4. Section 4.5 now gives $\bar{W}\_K7(0)$ in fully explicit form:

$$\bar{W}\_K7(0) = \nu^2(1-\rho)/(\omega_0^2 + 7)^2 \cdot (1 + 3\lambda\_\delta^2/\bar{N} + \ldots)$$

where $\sigma^2 = \nu^2(1-\rho)$ (derived from the permutation-symmetric vertex covariance), $\sigma\_\delta^2 = \lambda\_\delta^2(\omega_0^2+7)^2/\bar{N}$ (derived from Poisson shot noise), and the structural K=7 dependence is exact. The remaining open quantities are the four EFT parameters ν^2, ρ, λ_δ (entering $\bar{W}\_K7(0)$) and g_3, g_4 (entering the one-loop stability correction $\Sigma(0)$). These are normal effective-field-theory inputs computable from the K=7 nonlinear commitment dynamics. The structural problem is solved; what remains is parameter determination.

Open 2 — Nonlinear self-energy correction $\Sigma(0)$. SA3 is now derived in the minimal quadratic inter-cell model: the K=7 gap gives $\mu_0^2 > 0$, gradient coupling leaves the k=0 mass unchanged, and the Yukawa covariance is integrable. The sole residual question is whether nonlinear inter-cell interactions generate a self-energy correction $\Sigma(0)$ satisfying $\mu\_eff^2 = \mu_0^2 + \Sigma(0) > 0$, or whether a large negative $\Sigma(0)$ could destabilise the bath. Since $\mu_0^2 \sim \omega_1^{12}/(4\alpha^2\sigma^4)$ with $\omega_1^2 = \omega_0^2 + 7$ providing a substantial bare gap, a destabilising correction would require a nonlinear effect of comparable magnitude with no evident mechanism in the K=7 architecture. This is an open question but not a gap in the framework — it is the standard perturbative stability condition for any effective field theory with a positive bare mass.

9.3 Falsifiability

A theoretical framework earns scientific standing by specifying how it can be wrong. The VERSF spectral density framework fails — and should be abandoned or substantially revised — under any of the following conditions:

F1 — SA1–SA4 fail. If the local spectral weights $W\_r(\omega)$ have infinite variance, or if spatial correlations in the commitment bath are long-range (non-integrable covariance), self-averaging breaks down. The coarse-grained bath would then not converge to a smooth infrared limit, $J(\omega)$ would not be flat, and the $\alpha = 1$ universality class would not be reached. This is testable in principle by computing the spatial correlation function of the commitment density from the full K=7 nonlinear calculus.

F2 — $\bar{W}\_K7(0) = 0$. If the disorder and orientational average produces systematic cancellations in $\mathbb{E}[W\_r(0)] = 0$, then $\gamma_m = 0$, the bath decouples from the κ-field at low frequencies, and no logarithmic displacement growth occurs. This would eliminate the quasi-de Sitter phase entirely. The minimal disorder model of Section 2.3 shows this does not happen for generic disorder distributions, but it is possible for fine-tuned coupling geometries.

F3 — $\alpha \neq 1$ from the full K=7 dynamics. If the nonlinear K=7 calculus produces a bath spectral density that is sub-Ohmic ($J(\omega) \propto \omega^s$, $s < 1$) rather than white-noise, then $\alpha < 1$ and the displacement grows faster than logarithmically. As shown in Section 5.3, this produces a blue-tilted scalar spectrum incompatible with Planck $n_s = 0.9649$. Any derivation of the K=7 bath spectral density that gives $s < 1$ falsifies the present framework.

F4 — CMB spectral index incompatible with $1 - 2/N★$. If future CMB measurements establish n_s significantly above 0.97 or significantly below 0.96 (ruling out all $N★ \in [50, 60]$), the logarithmic displacement growth prediction is falsified. The leading-order VERSF prediction $n_s = 1 - 2/N★ \approx 0.964$ sits comfortably within current Planck uncertainties but would be testable by future experiments (LiteBIRD, CMB-S4) with $\sigma(n_s) \sim 0.002$.

F5 — $\sigma\_\tau/\sigma\_opt \neq \sqrt{(2 \ln 2)}$ in bench-top experiment. The companion bench-top paper (Coupled Temporal) predicts a specific ratio of temporal to optimal spreads in a three-channel emitter geometry. If this prediction fails, m_κ is not the relevant frequency scale for the commitment-event bath, undermining the UV cutoff identification ω_UV = m_κ used throughout this paper.

The framework is therefore falsifiable on multiple independent grounds — by structural calculation (F1, F2), by indirect CMB measurement (F3, F4), and by direct laboratory test (F5). This layered falsifiability structure is a feature, not a vulnerability.

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10. Summary

The main results of this paper are:

(1) Response-corrected spectral density. The spectral density of the commitment-event bath, correctly accounting for mode response, is:

$$J(\omega) = \Gamma\, |\hat{f}(\omega)|^2\, W\_K7(\omega), \quad W\_K7(\omega) = \Sigma_a |g_a|^2 \sigma_a^2 |\chi_a(\omega)|^2$$

This is derived from K=7 mode structure, Poisson event statistics, and the Caldeira-Leggett formalism.

(2) Conditional infrared flatness. Under coarse-graining satisfying self-averaging conditions SA1–SA4:

$$\bar{W}\_K7(\omega) = \bar{W}\_K7(0) + O(\omega^2), \quad J(\omega) \to \gamma_m = g\_\kappa^2\, \Gamma\, |\hat{f}(0)|^2\, \bar{W}\_K7(0)$$

This is a conditional theorem. The infrared flatness emerges generically in the limit ω ≪ m_κ; it is not unconditionally proven.

(3) Memory kernel. The flat spectral density gives:

$$\mathscr{M}_{\mathrm{eff}}(\tau) \sim (2\gamma_m/\pi m_\kappa) \cdot \cos(m_\kappa\tau)/\tau \quad (\tau \gg m_\kappa^{-1})$$

The $\alpha = 1$ decay is the unique value consistent with red-tilted perturbation spectra.

(4) Logarithmic growth. The κ-field equation in the slow-roll regime ($\delta\dot{s} \ll H\delta s$) has the asymptotic growing solution:

$$\delta s(t) \sim A \ln(Ht)$$

This is the microscopic VERSF origin of slow-roll — not a potential choice but a consequence of Poisson commitment-event statistics acting through an $\alpha = 1$ memory kernel.

(5) Parameter control. All bath-induced cosmological parameters λ_m, β, γ_m, C are controlled by the single infrared spectral weight $\bar{W}_{K7}(0)$, up to cosmological background quantities.

(5a) Complete microscopic-to-cosmological bridge (Sections 4.5, 7.1, 7.3). $\bar{W}_{K7}(0)$, γ_m, λ_m, β, C, and n_s are all derived in explicit form in terms of five physical inputs $\{\nu^2, \rho, \lambda_\delta, \bar{N}, g_\kappa\}$. The complete spectral index prediction is:

$$n_s = 1 - 2/N_\star + \chi_C\, g_\kappa^2 \Gamma |\hat{f}(0)|^2 \nu^2 (1-\rho) \,/\, [N_\star \ln N_\star\, (\omega_0^2+7)^2] \times (1 + 3\lambda_\delta^2/\bar{N} + \ldots)$$

For $N_\star = 55$ the leading term gives $n_s \approx 0.964$, consistent with Planck. The subleading correction provides a constraint on the event-statistics combination $\nu^2(1-\rho)/(\omega_0^2+7)^2$ once χ_C and g_κ are determined.

(5b) SA3 derived in the minimal quadratic inter-cell model (Section 4.3). The K=7 gap gives $\mu_0^2 > 0$ via the convexity argument: $d^2W/dx^2 = 6\alpha^2\sigma^2\omega_1^{-8} > 0$. Gradient-type inter-cell coupling leaves the k=0 mass unchanged at μ_0^2. The Yukawa covariance $C(r) \propto e^{-r/\xi}/r$ is integrable in 3D, so SA3 holds. The sole residual open question is whether nonlinear corrections $\Sigma(0)$ destabilise the bath; no such mechanism is apparent in the K=7 architecture.

(6) Dimensional consistency verified. In natural units with $[\delta s] = M$ (standard 4D scalar field), each kinetic term in the κ-field equation of motion has dimension M^3. The spectral density has $[J(\omega)] = M^{-3}$, the memory kernel has $[\mathscr{M}_{\mathrm{eff}}(\tau)] = M^{-3}$, and the bath coupling constant g_κ (introduced in Section 2.3) carries $[g_\kappa] = M^3$. The memory convolution term $g_\kappa^2 \int \mathscr{M}_{\mathrm{eff}}\, \delta s\, dt'$ has dimension $M^6 \times M^{-3} \times M \times M^{-1} = M^3$ ✓. With $\gamma_m \equiv g_\kappa^2\, \Gamma\, |\hat{f}(0)|^2\, \bar{W}_{K7}(0)$ carrying $[\gamma_m] = M^3$, the ratios γ_m/H^3 and $\beta = \gamma_m/H^3_{\mathrm{end}}$ are dimensionless ✓.