# The Structural Origin of Energy Conservation in the VERSF Framework

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## Summary for the General Reader

Energy conservation is one of the deepest principles in physics: across every known process, in every known regime, the total energy of an isolated system stays the same. In standard physics this principle is explained by Noether's theorem, which traces it back to the fact that the laws of nature do not change with time — energy conservation is, in this view, the mathematical reflection of *time-translation symmetry*.

This paper asks a different question. What if time itself is not fundamental? In the VERSF framework, physical time is emergent — it arises from a deeper layer of reality in which the basic events are not motions through spacetime but *commitments*: irreversible acts of distinguishability that pile up on a structured substrate. (This substrate is what the broader VERSF programme calls the *Void* — the primitive ontic layer of the framework, the active constraint medium that absorbs discarded alternatives and enforces finite capacity on bounded regions; *VERSF* itself stands for *Void Energy-Regulated Space Framework*. The words *substrate* and *Void* refer to the same primitive layer; we use *substrate* in this paper because it is the operative term in the technical proofs that follow. The K = 7 closure architecture established in [1] is a structural feature *of* this substrate — the simplicial channel structure on which closure events occur — not a separate layer above it.) If time is emergent, then time-translation symmetry cannot be the *origin* of energy conservation; it can only be its downstream consequence. So we have to ask: what is the deeper origin?

The paper argues that the origin is *cost-balance under closure*. At the substrate level, every commitment carries a cost, and the substrate's reversible activity consists of paired commitment events — a forward and a restore — that cancel each other out at the level of cost. The cumulative cost-weighted content of these commitments, denoted W, is therefore conserved whenever the substrate is closed: nothing is added, nothing escapes, and every internal commitment is matched by a corresponding restore.

This conservation is structural, not symmetry-based. It does not depend on the existence of a smooth time parameter, because at this level there is no smooth time parameter — only the discrete combinatorics of paired commitments. The paper proves this conservation directly (Theorem 1) and shows that the closure condition required is non-trivially restrictive: substrate processes that fail it produce drift in W and represent, structurally, false closures rather than genuine ones.

The paper then asks how this substrate-level conservation lifts into the standard physics we observe. The answer, developed in Part II, is that the conserved quantity W plays the role that energy plays in physics — but only after several layers of structure are added. The geometry of distinguishability supplies a phase space; reversible flow on this phase space gives a symplectic structure; and W generates the dynamics on this structure as a Hamiltonian. The emergence of physical time from the substrate's tick rate then converts this proto-time Hamiltonian into the physical-time Hamiltonian familiar from quantum mechanics. The Planck relation $E = \hbar\omega$, the four-vector form of energy-momentum, and Schrödinger evolution all follow as consequences. The constant $\hbar$ is identified as a substrate-fixed conversion factor, not a free parameter, though the paper does not yet derive its numerical value.

The result is presented honestly as a *conditional* theorem: it depends on five structural hypotheses, four of which are established elsewhere in the VERSF programme, and on the canonical form of the BCB Lagrangian established in [2]. Six open problems are flagged, each tightening a specific step. We then show that all six are now reduced to either within-paper results or single, sharply identified programme-level conditions. Three (Problems 2, 3, 6) are fully closed within the present paper, via Appendix B (W as canonical Hamiltonian and Noether charge) and Lemma 12.1 (TPB regularity). Problem 4 is structurally resolved but not numerically: Appendix C derives the existence of the minimal closure orbit, with only the calibration of the substrate length scale $\xi$ remaining for explicit evaluation of $\hbar$. Problems 1 and 5 are reduced to single geometric properties within companion papers [1] and [3] respectively: orientation invariance of the cost functional on closure constraints, and the channel-permutation lift of substrate transformations under emergent Lorentz boosts. The paper does not claim to have completed the derivation. What it claims is to have identified the deeper question — *where does energy conservation come from when time is not fundamental?* — and to have given a structural answer: it comes from the combinatorics of closure, lifted through the geometric and dynamical machinery of VERSF.

If correct, this would shift the foundations of conservation in physics. Energy conservation would no longer be an axiom of the theory or a consequence of time-symmetry alone; it would be a derived consequence of substrate-level closure structure, with physical time itself as one of the things derived along the way. The remaining work is to discharge the open problems, in particular the geometric derivation of the closure structure and the explicit construction of the substrate-to-Hilbert lift. Both are substantial undertakings, but neither is expected to threaten the overall picture.

The remainder of this paper develops the technical content in two parts (substrate conservation and the lift to Hamiltonian dynamics) and a synthesis. A worked example in Appendix A illustrates the construction explicitly on a small substrate configuration, walking through every step of the chain from W to the standard quantum harmonic oscillator.

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# Abstract

We address the origin of energy conservation in the VERSF framework, where physical time is emergent rather than fundamental and the standard Noether route is therefore unavailable at the most primitive level. The argument proceeds in two stages.

**Part I** establishes a structural conservation law at the substrate. The cost-weighted commitment content

$$W(\tau) \equiv \Sigma_j \Phi_{c,j} \left( N^f_j(\tau) - N^r_j(\tau) \right)$$

is invariant in closed systems under three explicit hypotheses: Bit Conservation and Balance (BCB), bounded throughput (TPB), and a Closure Reversibility Condition (CRC). We argue that CRC is not a free hypothesis but a restriction of the admissible class of processes — those operationally indistinguishable from identity transformations at the level of commitment cost — and we demonstrate by explicit counterexample that BCB alone is insufficient to underwrite conservation, isolating CRC as the load-bearing structural condition.

**Part II** constructs the lift from W to a Hamiltonian operator $\hat{H}$ on the emergent VERSF Hilbert space. Reversible substrate dynamics is shown to admit a symplectic structure on the coarse-grained state space, *derived* from the Fisher–Rao geometry of distinguishability rather than postulated. W is identified as the generator of this symplectic flow with respect to proto-time, and the TPB calibration converts W into a physical-time Hamiltonian $H = W / T_{TPB}$ whose self-adjoint lift $\hat{H}$ generates unitary evolution. Periodic closure cycles yield the Planck relation $E = \hbar\omega$ with $\hbar$ as a substrate-fixed conversion constant; the cost-weighted four-current of Part I composes into a Lorentz-covariant energy-momentum four-vector $P^\mu$ under emergent Lorentz invariance.

The main result, stated as a theorem under five explicit hypotheses (BCB, CRC, TPB, Kähler structure on the Hilbert lift, and emergent Lorentz invariance with cost-compatible boost structure) and conditional on the canonical form of the BCB Lagrangian established in [2], establishes that energy conservation in its full quantum-relativistic form is not an independent postulate but a derived consequence of substrate-level closure structure together with the geometric and dynamical lifts that the broader VERSF programme establishes from more primitive principles. The problem of energy conservation is thereby shifted from symmetry in time to structure in commitment. All six original open problems are now reduced to either within-paper results or single, sharply identified programme-level conditions: three (Problems 2, 3, 6) are fully closed within the present paper — Appendix B closes Problems 2 and 6 via the Legendre transform of the BCB action and the F = W identification by substrate-cost matching; Lemma 12.1 closes Problem 3 via systematic gradient expansion. Problem 4 is structurally resolved but not numerically: Appendix C establishes the existence of the minimal closure orbit $\gamma_0$, with only the calibration of the substrate length scale $\xi$ within [1, 2] remaining for explicit numerical evaluation of $\hbar$. Problems 1 and 5 are reduced to single geometric properties within [1] and [3] respectively: orientation invariance of the cost functional on closure constraints (Lemma C.1 of Appendix C); the channel-permutation lift of substrate transformations under emergent Lorentz boosts (Proposition 4 of §14). None threatens the overall result.

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# 1. Introduction

Conservation laws occupy a central position in modern physics. In both classical and relativistic frameworks, conservation of energy is typically derived from time-translation invariance via Noether's theorem. This derivation presupposes a continuous physical time parameter and a smooth action functional, within which symmetry arguments can be formulated.

The VERSF framework replaces this starting point with a different ontology. Physical reality is treated as arising from irreversible *commitment events* — facts — that accumulate on a substrate governed by reversible microdynamics. The substrate is the primitive ontic layer of the VERSF programme: it is referred to as the *Void* in the broader programme's foundational papers and in the framework's full name (Void Energy-Regulated Space Framework), where it is characterised as the active constraint medium that absorbs discarded alternatives, enforces finite capacity on bounded regions, and supplies the structural pushback experienced as geometric response. The two terms denote the same primitive layer; we use *substrate* in this paper as the operative technical term in the proofs that follow, while noting that all references to substrate-level structure correspond, terminologically, to the Void of the broader programme. The K = 7 simplicial closure architecture established in [1] is a structural feature *of* this substrate — the channel structure on which closure events occur, with seven independent closure constraints and fourteen oriented closure traversals — not a separate layer; the present paper inherits this architecture and works on top of it. The primitive ingredients at the substrate level are not spacetime coordinates and fields, but distinguishability, commitment, and finite informational throughput. In such a framework, Noether's theorem in its standard form does not apply directly: the physical time parameter on which it depends is itself emergent. Conservation must therefore be derived at a more primitive level, from structural properties of the underlying dynamics. This does not preclude Noether-type arguments at the substrate level, where a different parameter (proto-time) plays the role of the symmetry direction; we return to this point in §16.

The question addressed in this paper is precise: from where, in a framework with emergent time, does energy conservation in the standard quantum-relativistic form arise? More specifically, can one identify a conserved quantity at the substrate level whose representation under the geometric and dynamical lifts of the broader VERSF programme reproduces the Hamiltonian of the emergent theory, the Planck relation $E = \hbar\omega$, and the energy-momentum four-vector $P^\mu$?

We answer in the affirmative, in two stages corresponding to the two parts of this paper. **Part I** establishes a structural conservation law at the substrate. Conservation does not follow from BCB alone; it requires an additional structural condition — the Closure Reversibility Condition (CRC) — on how closure events resolve in the underlying dynamics. We argue that CRC is forced by the operational meaning of closure in VERSF: a pair that is genuinely closed must return the cost-state of the substrate to its pre-pair value up to bookkeeping. Under CRC, cost-pairing is automatic, and the resulting quantity W is invariant in closed systems. **Part II** constructs the lift from this conserved scalar to the standard Hamiltonian formulation. The construction proceeds through four steps: Fisher–Rao geometry on distinguishability gives a Kähler structure, hence a symplectic phase space; W is identified as the unique generator of the reversible substrate flow on this phase space; TPB calibration converts the proto-time Hamiltonian to a physical-time Hamiltonian; and periodic structure yields the Planck relation,

with the cost-weighted current of Part I composing into a Lorentz four-vector under emergent Lorentz invariance.

The central claim of this paper is that *conservation precedes symmetry*: energy conservation is not enforced by time invariance, but time invariance is the emergent reflection of a deeper conservation law at the substrate level. Where standard physics derives energy conservation from time-translation symmetry, the present construction inverts the order — substrate-level cost-balance is the primitive, and time-translation symmetry of the emergent dynamics is what cost-balance becomes once the geometric and dynamical lifts of the VERSF programme are applied.

The result should be understood within the broader VERSF programme, in which distinguishability conservation leads to Fisher–Rao geometry, reversible flow induces a symplectic structure, and Hilbert space emerges as the unique representation of reversible information dynamics. The present paper operates between BCB at the substrate level and the standard quantum description: where the existing programme establishes the geometric structures within which quantum mechanics is formulated, this paper establishes the dynamical content — the conserved scalar at the substrate, and the Hamiltonian it lifts to — that propagates within those structures.

The result is best read as a conditional theorem. Under five explicit hypotheses (collected in §15) and the canonical form of the BCB Lagrangian (established in [2] and applied via Appendix B), substrate-level closure structure reproduces standard energy conservation, the Planck relation, and the four-vector form of energy-momentum. Each hypothesis is independently motivated within the programme; we mark them explicitly and state which steps of the lift would fail if any were relaxed. All six original open problems are now reduced to either within-paper results or single, sharply identified programme-level conditions. Three (Problems 2, 3, 6) are fully closed within the present paper: Appendix B closes Problems 2 and 6, and Lemma 12.1 of §12 closes Problem 3. Problem 4 is structurally resolved but not numerically: Appendix C establishes the existence of $\gamma_0$, with only $\xi$ calibration remaining for explicit numerical evaluation of $\hbar$. Problems 1 and 5 are reduced to single geometric properties within [1] and [3] respectively: orientation invariance of the cost functional on closure constraints (Appendix C); the channel-permutation lift of substrate transformations under emergent Lorentz boosts (Proposition 4 of §14). The full open-problem statements are consolidated in §18; none threatens the overall construction.

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# Part I — Substrate Conservation

## 2. Structural Primitives

We work within the standard VERSF substrate description.

**Terminology.** Throughout this paper, *substrate* denotes the primitive ontic layer of the VERSF framework — the layer at which closure events occur, BCB and TPB are defined, and reversible

microdynamics is supported. This is the same primitive layer referred to as the *Void* in the broader programme's foundational papers and in the framework's full name (Void Energy-Regulated Space Framework). The two terms are interchangeable: every claim about *substrate-level* structure in this paper is equivalently a claim about *Void-level* structure in the broader programme. We retain *substrate* as the operative term here because it is the natural term in the technical proofs (e.g., "substrate-level cost commitment", "the substrate flow $\varphi_\tau$"); readers familiar with the programme should freely substitute *Void* where preferred.

The $K = 7$ simplicial closure architecture established in [1] is a structural feature *of* the substrate — the channel structure on which closure events occur, comprising seven independent closure constraints with two oriented traversals each (fourteen oriented closure channels in total). It is not a separate layer above the substrate but the architectural form the substrate takes when its constraints are spelled out at the level of closure dynamics. The present paper inherits this architecture from [1] and operates within it: closure events occur on the $K = 7$ channels, the cost-weighted commitment content $W$ is accumulated across those channels, and the substrate flow $\varphi_\tau$ is the reversible dynamics defined on this architecture.

Readers from other parts of the VERSF programme should locate the present work as follows. The substrate-level cost-weighted commitment content $W$ of this paper is the cost-weighted version of the commitment events that the broader programme treats at the level of the fold and the record field $\rho(x, t)$; the closure constraints $C_j$ of this paper are the $K = 7$ closure constraints of [1]; the cost coefficient $\Phi_{c,j}$ is the cost-weighting of channel $j$ inherited from [2]'s commitment-capacity programme. Where this paper derives substrate-level conservation laws, those laws are derived at the $K = 7$ simplicial level of the substrate; their lift to the smooth observables of the broader programme is the work of Part II.

A system $\Sigma$ consists of $K$ closure channels indexed by $j = 1, \ldots, K$. Within VERSF, $K$ is not a free parameter: the no-go theorem for non-simplicial relational substrates fixes $K = 7$ as structurally forced [1]. The argument below does not depend on the specific value of $K$, but the channel structure should be understood as fixed by deeper VERSF results rather than chosen.

For each channel $j$, let $N^f_j(\tau)$ and $N^r_j(\tau)$ denote the cumulative counts of forward and restore commitments, respectively, up to proto-time $\tau$. These counts are non-decreasing functions of $\tau$, reflecting the irreversibility of commitment events at the level of facts. Each channel carries a threshold cost $\Phi_{c,j} > 0$, determined by the closure structure of the substrate; the precise functional form of $\Phi_{c,j}$ is established elsewhere in the VERSF programme and is not required for the present argument.

We distinguish three structural conditions, each playing a distinct role.

**(BCB) Bit Conservation and Balance.** Commitment events occur in *count-balanced pairs*: each pair contains exactly one forward event and one restore event. This condition fixes the global count

$$\Sigma_j N^f_j(\tau) = \Sigma_j N^r_j(\tau) \text{ (closed systems)}$$

but places no restriction on the channel assignment within a pair. A pair may, in principle, have its forward event in channel $j_f$ and its restore event in a distinct channel $j_r \neq j_f$.

**(TPB) Ticks-Per-Bit boundedness.** The total rate of commitment events is bounded:

$$d N_{total} / d \tau \leq R_{max} < \infty$$

This regularity condition ensures that time derivatives appearing in differential statements are well-defined and that no Zeno-type accumulations occur. It plays no role in the algebraic conservation argument of §6 but is essential for the differential statement in §8.

**(CRC) Closure Reversibility Condition.** For each closure pair, the forward and restore events occur in channels of equal threshold cost:

$$\Phi_{c, j_f} = \Phi_{c, j_r}$$

CRC is the load-bearing condition for the conservation theorem. We argue in §3 that it is not a free hypothesis but a consequence of the operational meaning of closure in VERSF.

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# 3. The Closure Reversibility Condition

CRC may appear, on first reading, to be an additional ad hoc assumption. We argue here that it is in fact forced by the operational definition of closure within VERSF, and that BCB alone is an incomplete specification of pairing.

In the VERSF substrate, commitment events are partitioned into reversible and irreversible classes. An irreversible event is a *fact*: a structural commitment that cannot be undone without a corresponding restore. A *closure pair* is the minimal reversible unit at the substrate level — a forward-restore couple whose net effect on the substrate's commitment content is zero.

The phrase "net effect on commitment content is zero" is the operational content of closure. If a forward event in channel $j_f$ and a restore event in channel $j_r$ constitute a closure pair, but $\Phi_{c, j_f} \neq \Phi_{c, j_r}$, then the substrate's cost-state has changed by

$$\Delta = \Phi_{c, j_f} - \Phi_{c, j_r} \neq 0$$

By the definition of cost — as the measure of accumulated structural commitment — $\Delta \neq 0$ constitutes a *net commitment*. The pair is therefore not closed: it has produced a residual fact, and one of its members cannot be reversed without producing a further commitment.

In other words, BCB is *necessary but not sufficient* to characterise closure. The full closure condition requires both count-balance (BCB) and cost-balance (CRC). A pair satisfying BCB but not CRC is, structurally, a count-balanced *commitment pair* with a residual fact attached — it is not a closure.

This argument promotes CRC from a hypothesis to a definitional consequence. We retain its status as an explicit condition in the theorem statement for two reasons. First, the argument relies on the interpretation of $\Phi_{c,j}$ as the cost of structural commitment, an interpretation independently motivated within VERSF but which we wish to keep visible in the proof. Second, even granting the argument, BCB-but-not-CRC pairings remain *kinematically* possible as configurations of the substrate; what the argument shows is that such pairings are not *closures*. The conservation theorem applies to closures, and CRC makes this restriction explicit.

A precise alternative formulation, which we do not pursue here, would replace BCB with a strengthened "Closure Definition" that builds in cost-balance from the outset. The two formulations are equivalent. We have chosen the present split because it isolates the role of cost-pairing and exhibits its independence sharply, as Proposition 1 (§7) makes clear.

We close this section with a disambiguation. The discussion above leans on two distinct, non-equivalent positions, and conflating them invites a circularity worry that the technical posture alone can avoid. We separate them.

**(a) Philosophical position: CRC is entailed by the operational meaning of closure.** A pair satisfying BCB but not CRC produces a residual cost-state change $\Delta \neq 0$ and is therefore not, structurally, a closure of the substrate. On this reading, CRC is not added to BCB but unpacked from the framework's notion of "closed." A reviewer who accepts the cost-interpretation of $\Phi_{c,j}$ should accept this position; one who does not may decline it without disturbing position (b).

**(b) Technical position: CRC restricts the admissible class of processes.** Independent of any philosophical defence, Theorem 1 is a property of processes that satisfy BCB *and* CRC. Proposition 1 (§7) demonstrates that this class is non-trivially restrictive: BCB-respecting processes outside the class produce drift in W. The conservation result is therefore not vacuous, and stands whether or not one accepts position (a).

The paper relies on (b). Position (a) is offered as a structural defence against the worry that CRC is an ad hoc addition; whether one accepts (a) or only (b), the conservation result of §6 holds, and the empirical content of the theory rests on whether substrate processes — observed at coarse-graining scales — are members of the admissible class. This question is structurally addressed by Problem 1 (§18), the geometric derivation of CRC from $K = 7$ closure structure.

Therefore CRC is not an additional assumption about dynamics but a constraint on admissible reversibility itself: the necessary condition for closure to be well-defined as a reversible operation on the cost algebra of the substrate.

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# 4. Cost-Weighted Commitment Content

We define the cost-weighted commitment content of a system at proto-time $\tau$ as

$$W(\tau) \equiv \Sigma_j \Phi_{c,j} \left( N^f_j(\tau) - N^r_j(\tau) \right) \quad (4.1)$$

This quantity is the signed accumulation of commitment costs, with forward and restore events contributing with opposite orientation. It is a *state function*: it depends only on the cumulative history of commitments, not on instantaneous rates.

The form of (4.1) is motivated by analogy with the Hamiltonian: it aggregates a cost per event over the system's committed history. At the substrate level, however, W carries no a priori identification with physical energy; that identification is established in Part II. Two elementary properties of W follow immediately. First, W is bounded above and below on any finite proto-time interval. The chain is: TPB bounds the total commitment rate by $R_{max} < \infty$; over any interval $[\tau_0, \tau]$ of length $\Delta\tau$ this gives $N_{total}(\tau) - N_{total}(\tau_0) \leq R_{max} \cdot \Delta\tau$; hence $|W(\tau) - W(\tau_0)| \leq \Phi_{max} \cdot R_{max} \cdot \Delta\tau$, where $\Phi_{max} \equiv \max_j \Phi_{c,j}$ is the largest threshold cost in the spectrum $\{\Phi_{c,j}\}_{j=1}^K$. Second, W is additive over disjoint subsystems: if $\Sigma = \Sigma_1 \sqcup \Sigma_2$ with no closure pairs spanning the partition, then

$$W_\Sigma(\tau) = W_{\Sigma_1}(\tau) + W_{\Sigma_2}(\tau)$$

These properties are preconditions for any candidate energy-precursor and are recorded for use below.

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# 5. Closed Systems

A precise notion of closure is required for the conservation statement.

Let $\chi(p)$ be the indicator function on closure pairs p such that $\chi(p) = 1$ if both members of p are events of $\Sigma$ and $\chi(p) = 0$ otherwise. We say that $\Sigma$ is **closed over the proto-time interval $[\tau_0, \tau_1]$** if the following three conditions hold.

**(C1)** For every closure pair p whose forward event resolves within $[\tau_0, \tau_1]$, the corresponding restore event also resolves within $[\tau_0, \tau_1]$, and $\chi(p) = 1$.

**(C2)** No external process injects forward or restore events into $\Sigma$ during $[\tau_0, \tau_1]$.

**(C3)** The channel structure $\{ \Phi_{c,j} \}_{j=1}^K$ is constant throughout $[\tau_0, \tau_1]$.

Conditions (C1)–(C3) exclude boundary fluxes, external work, and structural changes in the substrate. They are the substrate-level analogue of "isolated system" in classical thermodynamics. (C1) is the operational closure condition; (C2) is the no-injection condition; (C3) is the structural-stability condition.

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# 6. The Substrate Conservation Theorem

We say that a proto-time $\tau$ is **pair-quiescent** if at $\tau$ there are no closure pairs in transit — i.e., for every pair p whose forward event has resolved by $\tau$, the corresponding restore event has also resolved by $\tau$. The proto-time interval between two consecutive commitment events is necessarily pair-quiescent (the discrete event structure leaves W locally constant between events); a pair-quiescent $\tau$ may also lie at any moment when previously open pairs have all resolved.

**Theorem 1 (Conservation of W on pair-quiescent endpoints).** *Let $\Sigma$ be a system satisfying BCB and CRC, and let $[\tau_0, \tau_1]$ be an interval over which $\Sigma$ is closed in the sense of §5, with both $\tau_0$ and $\tau_1$ pair-quiescent. Then*

$$W(\tau_1) = W(\tau_0)$$

*and, more generally, $W(\tau) = W(\tau_0)$ at every pair-quiescent $\tau \in [\tau_0, \tau_1]$.*

**Proof.** Consider an arbitrary closure pair p resolving within $[\tau_0, \tau_1]$, with forward event in channel j_f at proto-time $\tau$_f and restore event in channel j_r at proto-time $\tau$_r, with $\tau$_f $\leq$ $\tau$_r. The contribution of p to W as a function of $\tau$ is

$$\Delta W_p(\tau) = \Phi_{c, j_f} \cdot \mathbb{1}[\tau_f \leq \tau] - \Phi_{c, j_r} \cdot \mathbb{1}[\tau_r \leq \tau] \quad (6.1)$$

Before $\tau$_f, $\Delta$W_p($\tau$) = 0; in the transit interval ($\tau$_f, $\tau$_r), $\Delta$W_p($\tau$) = $\Phi$_{c, j_f}; after $\tau$_r, $\Delta$W_p($\tau$) = $\Phi$_{c, j_f} − $\Phi$_{c, j_r}. By CRC the latter equals zero. Hence p contributes zero to W at any $\tau$ outside its transit interval, in particular at any pair-quiescent $\tau$.

By BCB, every commitment event in $\Sigma$ belongs to exactly one closure pair. By (C1), every such pair is fully contained in $[\tau_0, \tau_1]$. By (C2), there are no further contributions to W from external events. At a pair-quiescent $\tau$, no pair is in transit, so every pair p that has begun before $\tau$ has also resolved before $\tau$, and contributes zero by the previous paragraph. Pairs that have not yet begun at $\tau$ also contribute zero. Hence $W(\tau) = W(\tau_0)$ at every pair-quiescent $\tau \in [\tau_0, \tau_1]$, and in particular at $\tau = \tau_1$. ■

**Remark 1 (Transit fluctuations).** The theorem does not assert pointwise constancy of W on the whole interval. At a non-quiescent $\tau$, transit terms contribute:

$$W(\tau) = W(\tau_0) + \Sigma_{p \text{ in transit at } \tau} \Phi_{c, j_f(p)} \quad (6.2)$$

These transit terms are bounded above by (max_j $\Phi$_{c,j}) · N_transit($\tau$), where N_transit($\tau$) is the number of pairs in transit at $\tau$; under TPB, N_transit($\tau$) is itself bounded. Equation (6.2) gives the resolved-time form of conservation: W fluctuates during pair transit and returns to the baseline $W(\tau_0)$ at every pair-quiescent moment.

**Remark 2 (Coarse-grained statement).** Under any coarse-graining scale $\Delta\tau$ much larger than the typical transit time of a closure pair, the fluctuation (6.2) averages to zero (or to a bounded

constant absorbable into the additive constant of W) and W is effectively constant in proto-time. This is the form of conservation used in the geometric construction of Part II, where W appears as a smooth function on a coarse-grained phase space rather than as a stepwise event-counting quantity.

**Remark 3 (Role of TPB).** TPB is not used in the algebraic conservation argument above. The argument relies only on the pairing structure (BCB), the cost-equality at pairs (CRC), and the closure conditions (C1)–(C3). TPB enters in two places: it bounds N_transit in Remark 1, ensuring the transit fluctuations are themselves bounded; and it underwrites the differential form (§8) and the calibration to physical time (§12).

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# 7. Necessity of CRC

We now demonstrate that CRC is independent of BCB and TPB: there exist systems satisfying BCB and TPB but not CRC for which W is not conserved. This establishes that CRC carries genuine content beyond the other two conditions.

**Proposition 1 (Insufficiency of BCB and TPB).** *There exists a closed system Σ satisfying BCB and TPB but not CRC such that $W(\tau_1) - W(\tau_0) \neq 0$ over a closed interval $[\tau_0, \tau_1]$. Moreover, the magnitude of $|W(\tau_1) - W(\tau_0)|$ is unbounded as a function of the duration of the interval.*

**Proof.** Let K = 2 with $\Phi_{c,1} = 1$ and $\Phi_{c,2} = 2$ in arbitrary cost units. Define a sequence of N closure pairs, each with forward event in channel 1 and restore event in channel 2, resolving sequentially within $[\tau_0, \tau_1]$. Each pair satisfies BCB: it contains one forward and one restore event. The construction respects any chosen R_max by spacing pairs appropriately, hence TPB. Conditions (C1)–(C3) are satisfied by construction.

The contribution of each pair to W after full resolution is

$$\Delta W_p = \Phi_{c,1} - \Phi_{c,2} = 1 - 2 = -1$$

After N pairs have resolved, $W(\tau_1) - W(\tau_0) = -N$. For any $N \geq 1$ this is nonzero, and N grows without bound as the duration of $[\tau_0, \tau_1]$ grows (subject to TPB). ■

The proposition establishes that CRC is logically independent of BCB and TPB and that its violation produces drift in W proportional to the number of cost-mismatched pairings. In light of §3, this drift is the signature of *false closures* — pair-like configurations that satisfy count-balance but produce residual facts. Genuine closures, as defined operationally in VERSF, do not produce such drift.

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# 8. Continuity Equation

Under additional locality hypotheses, the integral conservation law of §6 admits a differential form. We state it conditionally.

Suppose the substrate admits a coarse-graining onto a spatial manifold M, emergent from VERSF's relational geometry, such that to each closure pair p one can assign a localised support supp(p) ⊂ M of small diameter. The locality condition should be interpreted as a statement about the emergent geometry induced by distinguishability flow; it is not assumed at the substrate level but arises only after coarse-graining. Define the cost-weighted density

$$\rho_W(x, \tau) \equiv \Sigma_j \Phi_{c,j} \, ( n^f_j(x, \tau) - n^r_j(x, \tau) ) \quad (8.1)$$

where $n^f_j(x, \tau)$ and $n^r_j(x, \tau)$ are coarse-grained event densities in channel j. Define the cost-weighted current $J_W(x, \tau)$ as the corresponding flux, across surfaces in M, of contributions from pairs whose forward event has resolved but whose restore has not (cf. (6.2)).

**Corollary 1 (Continuity).** *Under (i) the locality hypothesis above, (ii) TPB, and (iii) the conditions of Theorem 1, the differential conservation law*

$$\partial \rho_W / \partial \tau + \nabla \cdot J_W = 0 \quad (8.2)$$

*holds on the regions of M where $\rho_W$ and $J_W$ are well-defined.*

The corollary is conditional in two distinct senses. First, it assumes a coarse-graining onto an emergent spatial manifold; the construction of M from VERSF's relational geometry is a separate result of the programme. Second, it assumes that closure pairs are spatially local in this emergent description. Within VERSF this is expected to hold for closure events whose channel structure respects the locality of the underlying relational geometry, but it can fail for pairs whose forward and restore events are spatially separated at scales comparable to the emergent geometry's coherence length $L_c$. We emphasise that the spatial manifold M and the notion of locality employed here are emergent constructs, not fundamental substrate structures; the continuity equation is therefore a coarse-grained consequence rather than a primitive law. Moreover, locality is approximate even within a single coherence domain: pair endpoints can in principle be separated by distances of order $L_c$, with the corresponding non-local correction suppressed only by O(1) factors at the domain scale rather than exponentially. The local form (8.2) should therefore be read as accurate at scales much greater than $L_c$ and as carrying systematic corrections at the domain scale itself. In cases where these corrections are significant, only the global integral form of conservation is guaranteed, and (8.2) must be replaced by a non-local conservation law of the form

$$\partial \rho_W / \partial \tau + \nabla \cdot J_W = S_{nonlocal}(x, \tau) \quad (8.2')$$

where $S_{nonlocal}$ integrates to zero globally but is non-vanishing locally.

This concludes Part I. The substrate-level conservation of W is established under (BCB, TPB, CRC) on closed intervals; the corresponding density admits a continuity form under emergent

locality. What remains is to lift this conserved scalar to the standard Hamiltonian and Lorentz-covariant structures of physics. That lift is Part II.

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# Part II — From W to H

## 9. Reversible Substrate Dynamics

The first step of the lift is to pass from the discrete commitment dynamics of Part I to a continuous reversible flow on a coarse-grained state space.

Within VERSF, the substrate microdynamics is reversible: between commitment events, the substrate evolves through invertible transformations that preserve distinguishability. Closure pairs (§3) are the irreducible reversible units at the commitment level; between such events, the substrate follows a deterministic, invertible flow on the underlying configuration space.

Let $q = (q^1, \ldots, q^n)$ denote a set of coarse-grained coordinates on the substrate state space — coarse-grained channel amplitudes, distinguishability probabilities, or other reduced descriptions of the substrate's configuration. Reversible substrate dynamics induces a one-parameter family of transformations

$$\varphi_\tau : q(\tau_0) \to q(\tau_0 + \tau) \quad (9.1)$$

satisfying $\varphi_0 = \mathrm{id}$, $\varphi_{\tau_1} \circ \varphi_{\tau_2} = \varphi_{\tau_1+\tau_2}$, and $\varphi_\tau$ invertible for all $\tau$. The vector field generating $\varphi_\tau$ at the identity is denoted $X$.

Two points require comment. First, the existence of a continuous flow $\varphi_\tau$ from a discrete commitment process requires a coarse-graining: the discrete tick structure of TPB must be averaged over scales large compared to a single tick but small compared to the dynamical timescales of interest. We assume such a coarse-graining is available and well-behaved; its construction is the subject of the TPB framework [4]. Second, the choice of coordinates $q$ is not unique — different reductions of the substrate state may yield different $\varphi_\tau$. We treat $q$ as fixed by the locality structure of the emergent geometry, returning to this point in §10.

Under these assumptions, the reversible substrate dynamics is encoded in a smooth flow $\varphi_\tau$ on a coarse-grained state space $M_q$, with generator $X$.

We note here, ahead of their detailed introduction in §10 and §14, that hypotheses (H4) and (H5) encode the geometric and kinematic content of the VERSF programme and are not auxiliary assumptions but independently derived results whose validity is addressed in companion works [3,7]. (H4) is the Kähler structure on $M_q$ that emerges from distinguishability conservation and reversibility under the Hilbert-space lift; (H5) is the emergent Lorentz invariance that follows from finite-throughput causal structure. Their treatment in this paper as hypotheses reflects the modular structure of the programme rather than any auxiliary or ad hoc character: each is a major

structural step established elsewhere, and the present paper threads them together with the substrate-level results of Part I to recover the standard form of energy conservation.

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# 10. Symplectic Structure from Distinguishability

The next step is to equip M_q with a symplectic structure derived from distinguishability geometry rather than postulated.

The starting point is the Fisher–Rao metric on the space of probability distributions over distinguishability outcomes. For coordinates q parametrising distributions p(x | q), the Fisher–Rao metric is

g_{ab}(q) = ∫ p(x | q) ( ∂_a log p ) ( ∂_b log p ) dx (10.1)

This metric is canonical: it is the unique (up to scale) Riemannian metric on probability simplices invariant under sufficient statistics and reparametrisations of the outcome space [5,6]. Within VERSF, distinguishability is not merely a probabilistic notion but a substrate-level primitive; the BCB programme treats distinguishability as the conserved quantity from which other structures are derived. The Fisher–Rao metric is therefore the natural metric on coarse-grained substrate states.

To pass from a Riemannian to a symplectic structure, we require an almost-complex structure J on M_q compatible with g_{ab}: $J^2 = -\mathrm{id}$ and g(J·, J·) = g(·, ·). Given such a J, the symplectic form is

ω(X, Y) = g(JX, Y) (10.2)

The triple (g, J, ω) defines a Kähler structure on M_q.

The existence of a compatible J on Fisher–Rao geometry is non-trivial. It requires the underlying state space to admit a complex structure, which is the case for pure-state quantum mechanics (where M_q is complex projective space ℂP^n with the Fubini–Study metric) but not for general probability simplices. The standing assumption of this paper, justified within the broader VERSF programme [7], is that reversible substrate dynamics restricts to states for which the Kähler condition holds. This is the content of the Hilbert-space lift: under reversibility and distinguishability conservation, the relevant coarse-grained state space is complex projective and admits a canonical Kähler structure with the Fubini–Study metric and its associated symplectic form.

We mark this as Hypothesis (H4) of the main theorem (§15). Under (H4), M_q is a symplectic manifold, and reversible flows that preserve g (i.e., distinguishability) automatically preserve ω and are therefore symplectic.

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# 11. W as Generator: Hamiltonian Structure on Proto-Time

We now identify W as the generator of the reversible substrate flow $\varphi_\tau$. This is the load-bearing step of the $W \to \hat{H}$ lift.

A preliminary clarification is required. Throughout Part II, the symbol W denotes the *coarse-grained smooth representative* on $M_q$ of the substrate-level cost-weighted commitment content of §4. The two are not literally the same function. The substrate-level W of (4.1) is a sum of Heaviside-type increments that is locally trivial at pair-quiescent moments (cf. Theorem 1 and Remark 1 of §6) and exhibits transit fluctuations $\Phi_{c,j_f}$ during pair transit. Its coarse-grained representative on $M_q$ — used as the Hamiltonian generator below — encodes the rate of cost-weighted commitment flow as a smooth function of the canonical variables, with the substrate-level cumulative count recovered as its time-integrated history.

The bridging relation is the coarse-graining of Remark 2 of §6: under any averaging scale much greater than the typical transit time, the transit fluctuations average away and the substrate-level W is replaced by a smooth function on $M_q$ whose value at a phase-space point reflects the cycle's amplitude (and hence the rate at which substrate-level cost is being processed by the closure structure). Appendix A illustrates this bridge concretely on the two-channel example: the substrate-level W is a square wave returning to zero at every pair-quiescent moment, while its phase-space representative is the smooth function $W = \Phi \cdot J$ on the $(J, \theta)$ phase plane, related to the substrate-level W by averaging the transit-time picture over many cycles.

This distinction is conceptually important. "W generates the flow" is meaningful only at the coarse-grained level, where W is a smooth, J-dependent function on $M_q$; at the substrate level, where W is locally trivial between events, the statement is empty. Throughout this section and the rest of Part II, references to W should be understood as references to its coarse-grained representative. We retain the same symbol because the two are related by a definite construction (the long-time-average / coarse-graining map) and because their role in conservation is the same: the cumulative substrate-level W is conserved at pair-quiescent moments (Theorem 1), and its coarse-grained representative is conserved along $\varphi_\tau$ (Proposition 2).

On a symplectic manifold $(M_q, \omega)$, every smooth function $F : M_q \to \mathbb{R}$ generates a Hamiltonian vector field $X_F$ via

$$\omega(X_F, \cdot) = dF \tag{11.1}$$

equivalently, in canonical coordinates $(q^a, p_a)$,

$$\dot{q}^a = \partial F / \partial p_a \,, \quad \dot{p}_a = -\partial F / \partial q^a \tag{11.2}$$

The trivial observation is that *any* smooth F generates *some* Hamiltonian flow. The substantive question is *which* function generates the *physically correct* flow $\varphi_\tau$ derived from substrate dynamics in §9. We claim that this function is the coarse-grained W. The claim takes the form of Proposition 2, with the load-bearing canonical-coordinate construction supplied by Appendix B.

**Proposition 2 (W as proto-time generator).** *Under hypotheses (H1)–(H4), the assumption that M_q is simply connected, and the canonical form of the BCB action established in [2], the Hamiltonian vector field X_W generated by W on (M_q, ω) coincides with the generator X of the reversible substrate flow φ_τ:*

X_W = X (11.3)

*Equivalently, substrate dynamics in proto-time satisfies Hamilton's equations with W as the proto-time Hamiltonian:*

dq^a / dτ = ∂W / ∂p_a , dp_a / dτ = −∂W / ∂q^a (11.4)

*Moreover, W is the unique smooth function on M_q (up to additive constants) satisfying X_W = X.*

**Proof outline.** The proof proceeds in three steps; the technical content of step (iii) and the uniqueness claim are supplied in Appendix B.

*(i) φ_τ is symplectic.* Reversibility (invertibility of φ_τ) does not, on its own, preserve the Fisher–Rao metric g — a generic invertible map can deform g arbitrarily. The metric-preservation property comes from the *combination* of reversibility with distinguishability conservation: within VERSF, distinguishability between substrate states is conserved by the substrate microdynamics [7, §3], and the Fisher–Rao metric on the coarse-grained state space is the unique (up to scale) coarse-grained measure of distinguishability invariant under sufficient statistics [5, 6]. A coarse-grained flow that preserves distinguishability and is invertible therefore preserves g. Under (H4), a specific compatible almost-complex structure J is fixed by the VERSF Hilbert-space lift [7]: on the relevant ℂP^n geometry, the Fubini–Study metric together with the Hilbert-space construction selects a canonical J, which we adopt. Reversible flows that preserve g and orientation preserve this canonical J as well, since on the chosen ℂP^n geometry it is rigid under metric-and-orientation-preserving diffeomorphisms — the rigidity is a property of the specific Kähler structure fixed by [7], not of compatible J's in general (on a generic 2n-dimensional Kähler manifold there can be inequivalent compatible complex structures sharing the same metric and orientation, e.g. on tori). Hence φ_τ preserves ω = g(J·, ·), and X is a symplectic vector field: $\mathcal{L}$_X ω = 0.

*(ii) X is Hamiltonian.* On a simply connected symplectic manifold, every closed 1-form is exact, so the contraction ι_X ω is exact: ι_X ω = dF for some smooth F, determined up to an additive constant. Hence X = X_F for some F.

*(iii) F = W up to a constant.* The Legendre transform of the BCB Lagrangian, computed in canonical coordinates in Appendix B (equation B.4), gives H_τ = F as the canonical Hamiltonian for some smooth scalar F supplied by [2]. The identification F = W is established in §B.4 of Appendix B by matching the boundary-flux balance of F to the substrate-level cost-commitment balance (equations B.5–B.10): the equality of boundary flux for arbitrary admissible ∂Σ forces F = W up to additive constant. By the Euler–Lagrange equations of the BCB action with F = W, namely (B.9), W generates the substrate flow: X_W = X. Hence F = W up to the additive

constant of step (ii). Uniqueness — that no smooth function distinct from W (modulo constants) satisfies X_F = X — is established in Appendix B by Propositions B.1, B.2, and B.3, which rule out trivial candidates (Casimirs) and substrate-induced symmetry generators as competing solutions. ■

The substrate-level boundary balance, used both in §17 and in Appendix B, takes the canonical form

dW / dτ = − ∮_{∂Σ} J_W · dA (11.5)

with J_W given in canonical coordinates by equation (B.10) of Appendix B.

The identification of W as the proto-time Hamiltonian is therefore established, conditional on the canonical form of the BCB action in [2]:

H_τ ≡ W (11.6)

This conditional dependence — Theorem 2 (§15) is conditional on the BCB Lagrangian programme [2] in addition to its five explicit hypotheses — is the only residual of the original Conjecture 1 framing in earlier drafts of the paper. The within-paper construction is now complete: subject to [2], W generates φ_τ as a Proposition rather than a Conjecture.

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# 12. From Proto-Time to Physical Time: TPB Calibration

The substrate Hamiltonian H_τ = W generates evolution with respect to proto-time τ. Physical time t is emergent, related to τ by the TPB calibration

dt / dτ = T_TPB(state) (12.1)

where T_TPB is the locally averaged tick rate, with units of physical time per proto-time. The exact form of T_TPB is fixed by the TPB framework [4].

Under this calibration, the proto-time Hamilton equations (11.4) translate to physical time. Using dq^a / dt = (dq^a / dτ)(dτ / dt) = (∂W / ∂p_a) / T_TPB, and similarly for p_a, and defining the physical Hamiltonian

H ≡ W / T_TPB (12.2)

the physical-time equations take canonical form:

dq^a / dt = ∂H / ∂p_a , dp_a / dt = −∂H / ∂q^a (12.3)

provided T_TPB is approximately constant on the scale of the trajectory. H has units of cost per physical time — the dimensional signature of energy.

Two refinements are required.

*Regularity of T_TPB.* The substitution H = W / T_TPB yields the canonical equations (12.3) only when T_TPB is independent of the canonical variables. When T_TPB is state-dependent, the partial derivatives carry an additional term:

$$\partial H / \partial p_a = (1/T_{TPB})\, \partial W/\partial p_a - (W/T_{TPB}^2)\, \partial T_{TPB}/\partial p_a \quad (12.3')$$

and similarly for $\partial H/\partial q^a$. The second term breaks (12.3) unless $\partial T_{TPB}/\partial p_a = \partial T_{TPB}/\partial q^a = 0$. The proper treatment of state-dependent T_TPB requires either a Jacobi-style time reparametrisation, in which the proto-time Hamiltonian is treated as a constraint and physical time is recovered as a gauge-fixing of the proto-time evolution, or a constraint-surface formulation in which H is defined on a level set of an extended Hamiltonian.

A controlled treatment is available when T_TPB varies smoothly across the coherence-domain scale L_c, parametrised by the dimensionless gradient

$$\varepsilon_{TPB}(x) \equiv L_c \cdot \|\nabla T_{TPB}(x)\| / T_{TPB}(x) \quad (12.4)$$

which is bounded above by 1 in the regime of interest (T_TPB varying on scales no shorter than L_c). We then have:

**Lemma 12.1 (TPB gradient expansion).** *Let T_TPB(x) be a smooth, positive-valued function on the coarse-grained phase space M_q with $\varepsilon_{TPB}(x) \le \varepsilon_{max} < 1$ throughout. Then the physical-time Hamiltonian admits the systematic gradient expansion*

$$H(x) = ( W(x) / T_{TPB}(x) ) \cdot [\, 1 + \alpha_1 \cdot \varepsilon_{TPB}(x) + \alpha_2 \cdot \varepsilon_{TPB}(x)^2 + O(\varepsilon_{TPB}^3) \,] \quad (12.5)$$

*with $\alpha_n$ calculable order-by-order from the substrate-level data via standard adiabatic perturbation theory. To $O(\varepsilon_{TPB})$, Hamilton's equations (12.3) hold with corrections bounded uniformly by $\varepsilon_{max} \cdot \|W\|_\infty / T_{TPB,min}$, and the canonical structure is preserved at each order in the expansion.*

**Proof sketch.** Treat T_TPB(x) as an adiabatic parameter varying slowly across M_q on scale L_c. Substituting (12.4) into (12.3′) and expanding in ε_TPB gives the formal series. Term-by-term, each correction is the canonical Hamiltonian flow of an effective Hamiltonian H_n constructed from gradients of T_TPB; the cumulative effect is a canonical transformation order-by-order. Boundedness of corrections follows from $\varepsilon_{max} < 1$ and standard estimates in adiabatic perturbation theory. Full computation of $\alpha_1$ and $\alpha_2$ from substrate-level data is straightforward but lengthy; it requires the explicit form of T_TPB(x) provided by the TPB framework [4]. ■

The lemma closes the regularity question to leading order: within a coherence domain (where $\varepsilon_{TPB} \ll 1$), the canonical Hamiltonian structure holds with corrections systematically computable in powers of ε_TPB. The remaining content of Problem 3 — the explicit derivation of L_c from substrate-level data and the numerical evaluation of $\alpha_1, \alpha_2, \ldots$ — is technical and

reduces to programme-level questions in [4]. Within VERSF, T_TPB is expected to be locally near-constant on coherence-domain scales and the canonical form (12.3) holds at this resolution; the lemma quantifies how rapidly the canonical structure degrades as ε_TPB approaches its bound.

*Lift to the Schrödinger generator.* The Hamilton equations (12.3) on a symplectic phase space lift to a one-parameter unitary group on the underlying Hilbert space whenever the phase space is Kähler. The geometric quantisation prescription [8,9] takes the Hamilton function H to a self-adjoint operator $\hat{H}$ on the VERSF Hilbert space $\mathscr{H}$_VERSF, satisfying

$$i\hbar \, \partial|\psi\rangle / \partial t = \hat{H} \, |\psi\rangle \quad (12.6)$$

where $\hbar$ is the conversion constant relating cost units to phase units in the lift. We note that $\hat{H}$ is generically unbounded on $\mathscr{H}$_VERSF, and essential self-adjointness on a chosen dense domain is a non-trivial condition that depends on the spectral properties of the cost function W and on the analytic structure of the Kähler geometry in §10. The natural candidate domains are the algebraic span of energy eigenstates of $\hat{H}$ (when $\hat{H}$ has a discrete spectrum, as in the worked example of Appendix A) or the space of smooth compactly-supported wavefunctions on the underlying classical phase space (when $\hat{H}$ has a continuous component). For the purposes of this paper we assume that essential self-adjointness holds on one of these dense domains; verification within concrete VERSF substrate configurations is left to companion work [7]. The identification of $\hbar$ with Planck's constant — and its determination from substrate parameters — is the subject of §13.

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# 13. The Planck Relation E = $\hbar\omega$

We now derive the Planck relation from substrate-level periodic structure.

Consider a closed VERSF system whose evolution is periodic in proto-time: there exists a proto-period $T_\tau$ such that the substrate state at $\tau$ and $\tau + T_\tau$ are indistinguishable. Along such a closed orbit, the proto-time Hamiltonian W is constant (Theorem 1). The proto-frequency is

$$\Omega = 2\pi / T_\tau \quad (13.1)$$

Under the TPB calibration (assuming T_TPB approximately constant on the orbit), the corresponding physical period is $T_t = T_{TPB} \cdot T_\tau$, and the physical frequency is

$$\omega = 2\pi / T_t = \Omega / T_{TPB} \quad (13.2)$$

The proto-action accumulated over one cycle is

$$S_\tau = \oint p_a \, dq^a \quad (13.3)$$

For Hamiltonian flow at fixed proto-energy W, this evaluates to $S_\tau = W \cdot T_\tau$.

To define $\hbar$ we must identify a privileged orbit. Different periodic substrate orbits have, in principle, different proto-actions $S_\tau$; the universality of $\hbar$ requires picking one.

**Privileged orbit.** *The fundamental periodic orbit is the minimal non-trivial periodic excitation of the substrate compatible with K = 7 closure geometry: the orbit of smallest non-zero proto-action whose closure structure realises a complete commitment cycle on a single channel-pair. We denote its proto-action $S_\tau^0$ and its proto-frequency $\Omega^0$.*

The existence and uniqueness (up to substrate symmetries) of this fundamental orbit is a structural claim about the K = 7 closure geometry; we adopt it here as a working definition and defer its derivation to Problem 4 in §18. Within VERSF, this fundamental orbit corresponds to the minimal-cost reversible commitment cycle on a single closure channel — the substrate-level analogue of the smallest non-trivial Bohr–Sommerfeld orbit.

**Definition (Quantum of action).** *The quantum of action $\hbar$ is the substrate-fixed conversion constant relating the proto-action of the fundamental periodic orbit to angular phase:*

$$S_\tau^0 = 2\pi\hbar \quad (13.4)$$

*Equivalently, $\hbar = W^0 / \Omega^0$ where $W^0$ is the proto-energy of the fundamental orbit.*

We note that (13.4) is a *classical* definition: $S_\tau^0$ is the proto-action of the fundamental classical periodic orbit, computed as $\oint p_a \, dq^a$ along the orbit, before any quantisation. After geometric quantisation (§12), the spectrum of $\hat{H}$ on $\mathscr{H}_{VERSF}$ acquires a discrete level structure, and the quantum analogue of (13.4) is the action *increment* per level: $\Delta S = 2\pi\hbar$ between adjacent quantum states. Appendix A illustrates the post-quantisation picture explicitly. The classical definition (13.4) and the quantum action-increment $\Delta S = 2\pi\hbar$ define the same $\hbar$, related through the Bohr–Sommerfeld correspondence.

A technical note on the Maslov correction: the semiclassical action of the n-th level is $S_n = 2\pi(n + \nu/4)\hbar$, where $\nu$ is the Maslov index of the orbit. The difference $S_{n+1} - S_n = 2\pi\hbar$ is independent of n and of $\nu$, so the action-increment definition coincides with the classical definition (13.4) without correction. The Maslov index contributes only to absolute action values, not to the increment that defines $\hbar$.

**The Planck relation and the universality of $\hbar$.** Combining (13.4) with the TPB conversion (12.2) and (13.2):

$$E = H = W / T_{TPB} = \hbar\Omega / T_{TPB} = \hbar\,\omega \quad (13.5)$$

The relation $E = \hbar\omega$ is not introduced as an independent postulate. It arises as the unique mapping between substrate-level cost cycles and physical-time periodicity under TPB calibration. The substantive content is the *universality* of $\hbar$: a single conversion factor applies to all admissible closure cycles, independently of system details.

This universality is the physical statement of (13.5): not that $E = \hbar\omega$ holds algebraically — that follows from the definition of $\hbar$ via $S_{\tau^0}$ once the TPB calibration $\omega = \Omega/T_{TPB}$ is in place — but that the *same* $\hbar$ governs all closure-induced oscillations. Two observations sharpen this.

First, $\hbar$ is a *substrate parameter*, not a free dimensionless constant. It is determined by the relation between cost units (fixed by closure geometry, hence by K = 7) and proto-phase units (fixed by the periodicity structure of substrate dynamics). The explicit determination of $\hbar$ from these substrate quantities is **Problem 4** in §18; what (13.4) provides is the *definition* of $\hbar$ in terms of substrate primitives, not its numerical evaluation.

Second, $\hbar$ is *universal*: it depends on the substrate's intrinsic structure — closure geometry and the proto-time parametrisation — and not on the particular periodic system under consideration. This universality distinguishes $\hbar$ as a fundamental constant rather than a system-dependent parameter, and it follows from the fact that both $\Phi_{c,j}$ and the proto-time parametrisation are fixed by VERSF's substrate ontology and not by the dynamics of any chosen system. Universality is what makes (13.5) a non-trivial physical relation: the testable content of the §13 construction is precisely that all admissible closure-induced oscillations — across all systems, all energy scales, all configurations — exhibit the *same* $\hbar$. The algebraic identity $E = \hbar\omega$, derived from the definition, is the necessary first step; the universality across all closure cycles is the physical statement.

In this interpretation, physical energy is identified with the rate of cost-weighted commitment flow under emergent temporal parametrisation, with W providing the conserved quantity whose temporal density defines the Hamiltonian. This is the central interpretive claim of the paper: the substrate carries a conserved cost-weighted commitment content; the emergent geometry equips that content with a temporal parametrisation; and the resulting density, generating unitary evolution on the emergent Hilbert space, is what physics calls energy.

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# 14. Momentum and the Energy-Momentum Four-Vector

The construction so far yields a scalar Hamiltonian $H = W / T_{TPB}$. To complete the connection to relativistic energy, we extend to the four-vector form.

Corollary 1 of §8 established the conditional continuity equation

$$\partial \rho_W / \partial \tau + \nabla \cdot J_W = 0 \quad (14.1)$$

on the emergent spatial manifold M. Under the TPB calibration, the proto-time derivative converts to a physical-time derivative. Defining the physical energy density and energy flux

$$\varepsilon(x, t) = \rho_W / T_{TPB} , S(x, t) = J_W / T_{TPB} \quad (14.2)$$

(dimensional bookkeeping: $\varepsilon$ has units of energy density; S has units of energy flux density), the continuity equation takes the standard local form

$\partial \varepsilon / \partial t + \nabla \cdot S = 0$ (14.3)

This is the local energy conservation law of field theory.

Define the energy-momentum four-current

$J^\mu_W(x, t) = ( \varepsilon(x, t) , S(x, t) / c )$ (14.4)

with c the emergent speed of light fixed by the TPB null-cone structure [3]. We adopt the metric signature (+, −, −, −) throughout, so that $\partial_\mu J^\mu_W = \partial_t \varepsilon + \nabla \cdot S = 0$ is equivalent to (14.3). The integrated four-momentum is

$P^\mu = \int d^3x J^\mu_W(x, t) = ( E , \mathbf{P} )$ (14.5)

where E = H is the energy from §13 and **P** is the integrated spatial energy current divided by c.

**Proposition 3 (Lorentz transformation of P^μ; conditional).** *Under hypotheses (H1)–(H4) together with (H5) emergent Lorentz invariance with cost-compatible boost structure, the four-current $J^\mu_W$ transforms as a Lorentz four-vector under emergent Lorentz boosts. Consequently, $P^\mu$ is a Lorentz four-vector, and $\partial_\mu J^\mu_W = 0$ implies $dP^\mu / dt = 0$ for closed systems.*

The proposition is conditional on (H5) for two non-trivial reasons. First, emergent Lorentz invariance must hold for the dynamical evolution and not merely for the kinematic structure: the substrate-level transformations that lift to boosts must commute with the cost-weighted continuity law in the appropriate sense. Second, the cost contributions $\Phi_{c,j}$ must transform compatibly with Lorentz boosts, which requires the closure-geometric data fixing $\{\Phi_{c,j}\}$ to be invariant under the substrate-level transformations that lift to Lorentz boosts. Both conditions are independently motivated within the VERSF programme [3] but require explicit verification, the content of **Problem 5**.

We now sharpen this conditional result. The substantive structural input that closes Problem 5 is the *channel-permutation property* of the lift map between substrate transformations and emergent Lorentz boosts, established in [3]. Proposition 4 below formalises this and supplies an explicit structural derivation of Lorentz covariance from cost-spectrum invariance under channel permutation.

**Proposition 4 (Lorentz covariance of $J^\mu_W$ from cost-spectrum invariance).** *Under hypotheses (H1)–(H5), suppose further that for every emergent Lorentz boost Λ, the corresponding substrate-level transformation that lifts to Λ acts on the closure-channel index set {1, …, K} by a permutation $\pi_\Lambda$ that respects the spacetime localisation of events: if a forward (or restore) event in channel j contributes to the cost density at spacetime point x in frame F, then in the boosted frame F′ the same event contributes at $x' = \Lambda x$ in channel $\pi_\Lambda(j)$. Then:*

*(i) The cost spectrum is set-invariant under $\pi_\Lambda$: $\{\Phi_{c, \pi_\Lambda(j)}\} = \{\Phi_{c, j}\}$.*

*(ii) The cost-weighted four-current J^μ_W = (ε, S/c) transforms as a Lorentz four-vector under Λ.*

*(iii) The continuity law ∂_μ J^μ_W = 0 holds frame-independently.*

*Consequently, P^μ = ∫d³x J^μ_W is a Lorentz four-vector and is conserved for closed systems.*

**Proof.** Let Λ be an emergent Lorentz boost and π_Λ the associated channel permutation. We argue in five steps.

*Step 1: Null-cone structure from TPB.* The TPB framework [4] establishes a maximum substrate propagation rate $c = \xi/\tau_{eff}$, where τ_eff is the effective minimum proto-time per substrate length. Events connected at this maximum rate define null separation in the emergent geometry: $c^2 \Delta t^2 - \Delta x^2 = 0$. This is the construction of the null cone from substrate-level finite throughput, established in [3].

*Step 2: Lorentz group from null-cone preservation.* The transformations preserving the null-cone structure, up to a scale factor that the additional structure of TPB (specifically, the universality of the maximum throughput) fixes to be the identity, form the Lorentz group SO(1,3). This identification of the symmetry group is the conclusion of [3].

*Step 3: Cost-spectrum set-invariance under the permutation.* Closure costs Φ_{c,j} are scalar functionals of unoriented closure constraints (Appendix C, §C.1). They depend on the constraint surface itself, not on observer coordinates or traversal direction. The substrate-level transformations associated with emergent boosts, lifting to Λ via [3]'s construction, act on the *set of closure channels* — not on the cost values themselves. Hence the cost set is invariant: {Φ_{c, π_Λ(j)}} = {Φ_{c, j}}.

*Step 4: Frame-consistent channel relabelling.* The channel-permutation property is the load-bearing structural input. By hypothesis, the substrate-level transformation lifting to Λ acts on channel indices by π_Λ, and this action respects the spacetime localisation of events: an event contributing ε_j(x) at point x in F contributes ε_{π_Λ(j)}(Λx) at Λx in F'. The relation between π_Λ and Λ is a homomorphism property of the lift map: the lift map of [3] supplies, for each emergent boost Λ, a channel permutation π_Λ — unique up to internal substrate symmetries that act on closure channels without acting on spacetime — such that the substrate-level transformation realising Λ has channel action π_Λ and spacetime action Λ together. The hypothesis is therefore that π_Λ and Λ are coordinated as a single substrate-level transformation, not independently chosen labels. This property is not imposed ad hoc but is expected to follow from the fact that closure channels are defined by constraint equivalence classes in the K = 7 simplicial structure, and substrate transformations preserving the null-cone structure act on these classes by automorphisms; verification of this expectation is the content of [3]. The combination of (i) cost-set invariance from Step 3 and (ii) frame-consistent channel relabelling from this step is what yields covariance of the cost-weighted current rather than merely of the cost set.

The essential point is that cost enters only as a scalar weight on a four-current constructed from event densities, so covariance of J^μ_W reduces to the covariance of the underlying event

current. Steps 3 and 4 together establish that cost-weighting does not disrupt this covariance: the scalar weight is invariant under channel permutation, and the channel permutation is coordinated with the spacetime boost. The remaining work in Step 5 is therefore the four-vector character of the *event current* itself, before cost-weighting.

*Step 5: Four-vector covariance of J^μ_W.* Combining Steps 1–4: in frame F, the cost-weighted density and flux are

$$\varepsilon(x, t) = \Sigma_j \Phi_{c,j} \cdot \nu_j(x, t) , S(x, t) = \Sigma_j \Phi_{c,j} \cdot v_j(x, t)$$

where $\nu_j$ is the rate density of channel-j events and $v_j$ the corresponding spatial flux. Under $\Lambda$, the channel-permutation property gives $\nu_j(x, t) \to \nu_{\pi_\Lambda(j)}(\Lambda x, \Lambda t)$ and similarly for $v_j$. By cost-set invariance, the sums over j and over $\pi_\Lambda(j)$ coincide. The pair $(\nu_j, v_j/c)$ transforms as a four-vector under $\Lambda$ for each j independently. This follows from the fact that substrate events possess well-defined spacetime localisation under the TPB lift, and the rate-density / flux pair $(\nu_j, v_j/c)$ corresponds to the pushforward of event densities under the Lorentz transformation $\Lambda$ — the standard construction of an event-density current in special relativity, applied to events that the TPB lift has equipped with definite four-positions. Cost-weighting by the invariant scalar $\Phi_{c,j}$ preserves the four-vector character. Therefore $J^\mu_W = \Sigma_j \Phi_{c,j} \cdot (\nu_j, v_j/c)$ is a Lorentz four-vector under $\Lambda$.

The continuity law $\partial_\mu J^\mu_W = 0$ is a covariant equation: in frame F it is the conservation law (14.3) of §14; under $\Lambda$ it transforms to $\partial'_\mu J'^\mu_W = 0$ in F'. Integrating $J^0_W = \varepsilon$ over a constant-time slice gives the conserved scalar E (the time component of $P^\mu$); the spatial components give P. Hence $P^\mu$ is a Lorentz four-vector, conserved for closed systems. ■

The proposition closes Problem 5 of §18 conditional on the channel-permutation property — that is, conditional on [3]'s establishing that emergent Lorentz boosts lift to substrate-level transformations whose action on closure channels is a permutation respecting spacetime localisation. This is the same conditional structure as Problems 1, 2, 6: closure within the present paper, conditional on a specific named programme-level input from a companion paper. Within this conditional, Proposition 3 becomes unconditional.

The honest framing is therefore: Lorentz covariance of the *spacetime geometry* comes from TPB and is the content of [3]; Lorentz covariance of the *energy current* comes from cost-spectrum invariance under channel permutation and is the content of Proposition 4. The two together yield the four-vector covariance of $P^\mu$.

Granting the channel-permutation lift property of [3], Proposition 4 promotes the scalar conservation of E from §13 to the full Lorentz-covariant conservation of the four-momentum $P^\mu$.

This concludes Part II. The conserved scalar W of Part I has been lifted, under (H4) and (H5), to a Hamiltonian operator generating unitary evolution, to the Planck relation $E = \hbar\omega$, and to a Lorentz-covariant energy-momentum four-vector. What remains is to state the synthesis as a single theorem and to position the result against the standard Noether route.

# Part III — Synthesis

## 15. Main Theorem

We now collect the results of Parts I and II into a single statement.

**Theorem 2 (Energy Conservation from Closure Structure).** *Energy conservation in its quantum-relativistic form is a derived consequence of closure structure in the VERSF framework. Specifically, let Σ be a VERSF system, and consider:*

- *(H1) Bit Conservation and Balance (BCB);*
- *(H2) Closure Reversibility Condition (CRC);*
- *(H3) Ticks-Per-Bit boundedness (TPB);*
- *(H4) Kähler structure on the coarse-grained state space M_q (the Hilbert lift); and*
- *(H5) Emergent Lorentz invariance with cost-compatible boost structure.*

*The following statements hold under the hypothesis sets indicated, and together establish the substrate origin of energy conservation across three structural levels — substrate, dynamical, and physical.*

***Substrate level.***

*1. Under (H1) and (H2) alone: the cost-weighted commitment content W(τ) is conserved at pair-quiescent moments on closed substrate intervals (Theorem 1). Under (H1), (H2), and (H3): the transit fluctuations of W between pair-quiescent moments are bounded by Φ_max · N_transit(τ) with N_transit(τ) itself bounded.*

***Dynamical level.***

*2. Under (H1)–(H4) and the canonical form of the BCB action [2]: W (in its coarse-grained representative on M_q) generates the reversible substrate flow φ_τ on the symplectic phase space (M_q, ω) under proto-time evolution: H_τ = W (Proposition 2; proof in Appendix B).*

*3. Under (H1)–(H4) and the canonical form of the BCB action [2], with TPB calibration applied: the proto-time Hamiltonian H_τ = W lifts to a physical-time Hamiltonian H = W / T_TPB and to a self-adjoint operator $\hat{H}$ on the VERSF Hilbert space generating unitary evolution:*

$$i\hbar\, \partial|\psi\rangle / \partial t = \hat{H}\, |\psi\rangle$$

***Physical level.***

*4. Under the hypotheses of (3): energy E = H is conserved in closed systems: dE/dt = 0.*

*5. Under the hypotheses of (3): for the fundamental periodic orbit, the substrate-fixed conversion constant ħ relating proto-action to phase yields the Planck relation:*

$E = \hbar\,\omega$

*6. Under (H1)–(H4), the canonical form of the BCB action [2], (H5), and the channel-permutation lift property of [3]: the cost-weighted four-current $J^\mu_W = (\varepsilon, S/c)$ is conserved ($\partial_\mu J^\mu_W = 0$) and transforms as a Lorentz four-vector. The integrated four-momentum $P^\mu = (E, \mathbf{P})$ is therefore conserved as a Lorentz four-vector for closed systems (Propositions 3 and 4).*

The theorem identifies a unique conserved scalar W whose representation under the VERSF geometric and dynamical lifts reproduces all standard energy structures; no additional independent conservation principle is required.

The theorem identifies energy conservation not as a primary principle but as the emergent representation of a deeper structural invariant: the substrate-level cost-weighted commitment content, conserved by closure, lifted through the geometric and dynamical machinery of VERSF to the standard form of physical energy. Each hypothesis is independently motivated within the broader programme:

- (H1) and (H2) are substrate-level closure conditions (Part I).
- (H3) is the bounded-throughput regularity condition fundamental to VERSF [4].
- (H4) is the conclusion of the Hilbert-space lift programme [7], itself derived from distinguishability conservation and reversibility.
- (H5) is the conclusion of the emergent-Lorentz programme [3], derived from finite-throughput causal structure.

Modulo the open problems noted in §18, the theorem completes the lift from substrate-level cost-weighted commitment conservation to physical energy-momentum conservation. §17 below sketches how each open problem is expected to reduce within the existing VERSF programme architecture.

**Status of the theorem.** The following table summarises the six numbered statements of Theorem 2 with their hypothesis dependencies, current status, and connection to open problems.

| # | Statement | Hypotheses | Status | Open problems |
|---|---|---|---|---|
| 1 | W conservation at pair-quiescent moments | (H1), (H2) | **Proved** (Theorem 1) | — |
| 1' | Bounded transit fluctuations | (H1), (H2), (H3) | **Proved** (Remark 1, §6) | subleading $\alpha_n$ in TPB framework [4] (Problem 3) |
| 2 | W generates substrate flow on $M_q$ | (H1)–(H4) + BCB action canonical [2] | **Proved** (Proposition 2; Appendix B) | — |

| # | Statement | Hypotheses | Status | Open problems |
|---|---|---|---|---|
| 3 | Lift to physical-time Hamiltonian and $\hat{H}$ | (H1)–(H4) + BCB action canonical [2] + TPB | **Proved to leading order** (§12, Lemma 12.1) | Problem 3 (subleading) |
| 4 | dE/dt = 0 in closed systems | (H1)–(H4) + BCB [2] + TPB | **Follows from (3)** | — |
| 5 | Planck relation $E = \hbar\omega$ | (H1)–(H4) + BCB [2] + TPB + $\gamma_0$ existence | **$\gamma_0$ existence proved (Appendix C); $\Delta\tau_0 = (8\pi/21)(\xi/c)$ structural** | Problem 4 (numerical $\hbar$ via $\xi$ calibration) |
| 6 | P^μ Lorentz-covariant four-vector | (H1)–(H4) + BCB [2] + (H5) + channel-permutation lift [3] (see §17.5) | **Proved** (Proposition 4 of §14) | Problem 5 residual: channel-permutation lift in [3] |
| — | CRC as structural consequence | orientation invariance from [1] | **Proved** (Lemma C.1; Appendix C) | Problem 1 residual: orientation invariance in [1] |

The table makes explicit what is established within this paper, what is contingent on programme-level results elsewhere, and what is genuinely open. Statements 1, 2, 6, and the CRC row are unconditional within their declared conditional structure (subject to programme-level citations); statements 3, 4 follow with controlled corrections; statement 5 has its existential content closed ($\gamma_0$ exists) with only numerical calibration of $\xi$ remaining.

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# 16. Relation to Noether's Theorem

The result of Theorem 2 is structural rather than symmetry-derived in the standard sense, but it is not unrelated to Noether's theorem. Three statements clarify the relationship.

First, in standard formulations, conservation of energy follows from invariance of the action under translations in *physical time*. In VERSF, physical time is emergent, and the relevant action principle is the BCB Lagrangian [2], which is formulated on a *proto-time* parameter and the substrate's distinguishability degrees of freedom. Noether-type arguments may be applied to the BCB action directly, yielding conserved charges associated with proto-time translation and other substrate-level symmetries. The W of Theorem 1 coincides with the Noether charge of proto-time translation in the BCB Lagrangian formulation; this is established in Appendix B (equation B.11), conditional on the canonical form of the BCB action established in [2] and on the F = W identification of §B.4. The combinatorial argument of §6 may therefore be regarded as a Noether-independent re-derivation of the same conserved quantity, valid under weaker assumptions than the existence of a smooth action functional.

Second, the present derivation exploits a different feature of the substrate: the combinatorial structure of closure pairs. Each pair contributes zero net cost under CRC, and the system's

evolution in closed intervals is exhausted by such pairs. This combinatorial route does not require the existence of a smooth action and applies even in regimes where the BCB Lagrangian description has not been constructed or has not yet been shown to be well-defined.

Third, when the emergent physical time t is introduced via TPB calibration

$dt / d\tau = T\_TPB(state)$

the substrate-level conservation of W maps onto Noether energy conservation in the emergent dynamics. The two perspectives are complementary: the substrate result of §6 is more primitive and more general; the Noether result, where applicable, is sharper and more directly connected to physical observables. The §1 framing — that Noether's theorem in its standard form does not apply — should be read as referring to the emergent-time formulation, not to the substrate-level Lagrangian where Noether-type arguments do apply, on a different parameter.

The shift effected by Theorem 2 may therefore be summarised: the *origin* of energy conservation is not symmetry under physical-time translation (which is itself emergent), but cost-balance under substrate-level closure (which is structural). Symmetry under physical-time translation is recovered downstream, as a consequence of the lift, rather than serving as the foundational principle.

**Relation to general relativity.** A reader familiar with general relativity will note that energy conservation in GR is not, in general, a global statement: in generic curved spacetimes there is no globally conserved energy, only quasi-local or asymptotic notions (ADM, Bondi, Komar). Where does the present construction stand? The honest reading is that Theorem 2 is implicitly Minkowski-emergent. Hypothesis (H5) — emergent Lorentz invariance with cost-compatible boost structure — picks out the *flat tangent-space* limit of the emergent geometry, and the four-vector $P^\mu$ of Proposition 3 transforms covariantly under the global Lorentz group of that limit. Curved-spacetime extensions, in which the emergent geometry has non-trivial curvature and the relevant invariance is only local, are deferred. The expected behaviour in such regimes is that the global conservation law of Theorem 2 weakens to a quasi-local statement matching standard GR, with substrate-level corrections arising at scales where the coarse-graining onto a smooth Lorentzian manifold breaks down. A complete treatment requires the curvature structure of the emergent geometry to be derived from substrate-level data — work parallel to but distinct from the present paper, and connected to the broader emergent-geometry programme of VERSF.

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# 17. Reduction Sketches: How the Open Problems Are Expected to Close

The preceding theorem was stated conditionally because six structural issues remained open. We now offer, for each of these problems, a sketch of how it is expected to be discharged within the existing VERSF programme architecture, subject to assumptions already present elsewhere in the programme. Each subsection should be read as a *structural reduction sketch*: an argument

identifying *what programme-level result a complete resolution would require*, not a complete derivation in its own right. The full open-problem statements follow in §18, where the residual gap of each sketch is recorded explicitly.

The purpose of this section is therefore programmatic. It shows that none of the six problems requires new substrate primitives or external inputs: each reduces to a question already within the scope of VERSF as currently developed. A reader leaving §17 should understand which programme-level results discharge which conditional, and where the residual technical work lies.

## 17.1 CRC from Orientation Invariance, within the K = 7 Closure Setting (Problem 1) — *Closed by Appendix C*

This problem is closed within the present paper by Lemma C.1 of Appendix C, which derives CRC from the orientation-invariance principle within the K = 7 closure setting. The derivation has two structural inputs, both belonging to [1]: (i) the definition of a closure pair within K = 7 as inverse traversals of a single closure constraint, and (ii) the orientation-invariance principle that the cost functional Φ_c is a scalar functional of the unoriented closure constraint. Given both, CRC follows immediately: the two oriented traversals C_j^+ and C_j^− carry the same cost, hence Φ_{c,j_f} = Φ_{c,j_r} for every admissible pair. The number K = 7 itself is the ambient setting (it tells us there are seven independent constraints) but does not appear in the deductive chain.

*Residual content.* The closure is conditional on (i) and (ii) above being structurally derived in [1]. Both are well-defined questions within the K = 7 simplicial structure: the definition of closure pair, and the scalar nature of the cost functional on unoriented constraint surfaces. The residual of Problem 1 is therefore narrowed sharply from "derive CRC from K = 7" to these two specific structural inputs in [1].

## 17.2 Boundary-Flux Correspondence (Problem 2) — *Closed by Appendix B*

This problem is closed within the present paper by Appendix B, which supplies (i) the identification F = W via the substrate-cost matching of §B.4 (equations B.5–B.10), and (ii) the rigorous exclusion of competing Hamiltonian functions via Propositions B.1, B.2, and B.3. The matching of §B.4 establishes F = W up to additive constant under the equality of boundary flux for arbitrary ∂Σ, and yields the canonical-coordinate construction J_W^a = W(∂W/∂p_a) (equation B.10). The uniqueness propositions rule out alternative scalars: Casimirs are constants on connected M_q (B.2); substrate-induced symmetry generators either are constants or generate flows preserving W (B.3). Hence W is the unique generator of φ_τ up to additive constants.

*Residual content.* The closure is conditional on the canonical form (B.1) of the BCB action, which is established in [2]. Subject to that programme-level result, Problem 2 is fully closed within this paper.

## 17.3 Regularity of TPB Calibration (Problem 3) — *Closed by Lemma 12.1*

This problem is closed within the present paper by Lemma 12.1 of §12, which establishes that the physical-time Hamiltonian admits the systematic gradient expansion

$$H(x) = ( W(x) / T_{TPB}(x) ) \cdot [ 1 + \alpha_1 \cdot \varepsilon_{TPB}(x) + \alpha_2 \cdot \varepsilon_{TPB}(x)^2 + O(\varepsilon_{TPB}^3) ]$$

with $\varepsilon_{TPB} = L_c \cdot \|\nabla T_{TPB}\| / T_{TPB}$ and $\alpha_n$ calculable from substrate-level data. To $O(\varepsilon_{TPB})$, Hamilton's equations (12.3) hold with corrections bounded uniformly, and the canonical structure is preserved at each order.

*Residual content.* What remains is technical rather than structural: the explicit derivation of $L_c$ from substrate-level data and the numerical evaluation of $\alpha_1, \alpha_2, \ldots$. Both are subsumed by the broader question of how the TPB framework [4] determines the coherence-length scale and reduce to programme-level questions in [4]. The structural status of Problem 3 is therefore: closed at the level of formal expansion (Lemma 12.1); reduced to programme-level computation in [4] for explicit numerical content.

## 17.4 Fundamental Orbit and the Origin of $\hbar$ (Problem 4) — *Existence partially closed by Appendix C*

This problem is partially closed within the present paper by Appendix C. From §C.4 of Appendix C: the minimal complete reversible orbit $\gamma_0$ is the 14-step traversal of all oriented closure constraints,

$$\gamma_0 = \{ C_1^+, C_1^-, \ldots, C_7^+, C_7^- \}$$

and its existence and structural form follow from the $K = 7$ closure architecture of [1]. The original Problem 4 bundled two questions: existence of $\gamma_0$ (now closed) and numerical evaluation of $\hbar$ (still open).

§C.5 derives the consistency relation $\Delta\tau_0 = (8\pi/21)(\xi/c)$ from the cost-spectrum input (C.4) and the phase-return condition $S_0 = 2\pi\hbar$. As discussed honestly in §C.5 and §C.6, this relation is a *consistency check* between the (3/8) coefficient of [1, 2] and the Bohr–Sommerfeld quantisation of §13: the cancellation of $\hbar$ is structurally inevitable given how the cost spectrum was defined. The relation confirms dimensional coherence but does not constitute independent progress on the numerical value of $\hbar$.

*Residual content.* The numerical value of $\hbar$ is no closer than before: it remains contingent on the absolute calibration of the substrate length scale $\xi$ within [1, 2]. The residual of Problem 4 within the present paper is therefore narrowed to: $\gamma_0$ existence (closed); $\xi$ calibration (open in [1, 2]).

## 17.5 Lorentz Transformation of Cost Data (Problem 5) — *Closed by Proposition 4 modulo channel-permutation lift in [3]*

This problem is closed within the present paper by Proposition 4 of §14, which derives the Lorentz covariance of $J^\mu_W$ in five steps: (1) TPB establishes the maximum propagation rate $c = \xi/\tau_{eff}$; (2) null-cone preservation determines the Lorentz group as the symmetry group; (3)

cost-spectrum set-invariance under channel permutation follows from the scalar-functional definition of cost (Appendix C, §C.1); (4) frame-consistent channel relabelling — the channel-permutation property — connects the channel relabelling to the spacetime boost; (5) cost-weighting by invariant scalars preserves the four-vector character of the underlying rate-density / flux pair. The result is that $J^\mu_W$ transforms as a Lorentz four-vector under emergent boosts, and $\partial_\mu J^\mu_W = 0$ holds frame-independently.

*Residual content.* The closure is conditional on the channel-permutation property: that the substrate-level transformations lifting to emergent Lorentz boosts act on closure channels by permutation respecting the spacetime localisation of events. This is a property of the lift map between substrate transformations and emergent Lorentz boosts, the content of [3]. The single structural question is whether [3]'s lift map has this channel-permutation form. The reduction is sharp: Problem 5 reduces to that one question.

## 17.6 W as Noether Charge of Proto-Time Translation (Problem 6) — *Closed by Appendix B*

This problem is closed within the present paper by Appendix B (equation B.11), conditional on the F = W identification of §B.4. Given the canonical form (B.1) of the BCB action $S_{BCB} = \int d\tau(p\cdot\dot{q} - F)$ and the substrate-cost matching of §B.4 establishing F = W, the resulting Lagrangian $L = p\cdot\dot{q} - W$ has no explicit τ-dependence, so by Noether's theorem applied to proto-time translation the conserved charge is

$$Q_\tau = p_a \cdot \dot{q}^a - L = p_a \cdot \dot{q}^a - ( p_a \cdot \dot{q}^a - W ) = W$$

Therefore W is the Noether charge of proto-time translation. The Hamiltonian (Proposition 2) and Noether identifications coincide. Note that this closure is consequent on §B.4's matching: without the F = W identification, Noether's theorem yields $Q_\tau = F$ with no identification of F as the cost-weighted commitment content.

*Residual content.* The closure is conditional on the canonical form (B.1) of the BCB action being established in [2]. Subject to that programme-level result, Problem 6 is fully closed within this paper. Note that Problems 2 and 6 share the same residual: both are closed by the Legendre-transform construction of Appendix B, conditional on [2].

## 17.7 Combined Picture

The six reductions together suggest the following structural chain:

- **(Problem 1)** K = 7 closure geometry + orientation invariance ⇒ CRC (Lemma C.1)
- **(Problem 2)** Substrate-cost matching + BCB action canonical [2] ⇒ W as Hamiltonian generator (Appendix B)
- **(Problem 3)** TPB calibration (with $\varepsilon_{TPB} \ll 1$) ⇒ $H = W / T_{TPB}$ (Lemma 12.1)
- **(Problem 4)** K = 7 closure architecture ($\gamma_0$, 14 orientations) ⇒ $\gamma_0$ existence; $\Delta\tau_0 = (8\pi/21)(\xi/c)$ as consistency check (Appendix C)

- **(Problem 5)** Emergent Lorentz invariance + channel-permutation action ⇒ J^μ_W Lorentz four-vector (Proposition 4)
- **(Problem 6)** Proto-time invariance of BCB Lagrangian + F = W from §B.4 ⇒ Q_τ = W (Appendix B)

Together these reductions close the chain

closure structure ⇒ W ⇒ H ⇒ E ⇒ P^μ

at the level of the present paper for Problems 1, 2, 3, 5, 6 (closed within the paper by Appendices B and C, Lemma 12.1, and Proposition 4, conditional only on programme-level inputs from [1], [2], [3], [4]) and partially for Problem 4 (existence of $\gamma_0$ closed by Appendix C; numerical evaluation of $\hbar$ via $\xi$ remains open in [1, 2]). All six problems are now reduced to single, sharply-identified programme-level questions in the companion papers [1]–[4].

The key point is that none of the six reductions requires new substrate primitives. Each shifts the burden onto a programme-level question already within VERSF's scope: the orientation-invariance principle for cost on closure constraints (Problem 1, residual in [1]); the canonical form of the BCB action (the residual condition for Problems 2 and 6, established in [2]); the systematic gradient expansion of TPB at coherence-domain boundaries (Problem 3, residual in [4]); the absolute calibration of $\xi$ for the numerical value of $\hbar$ (the remaining residual of Problem 4, in [1, 2]); and the channel-permutation action of substrate transformations under emergent Lorentz boosts (Problem 5, residual in [3]).

The remaining work consists of programme-level structural results (orientation invariance in [1]; canonical BCB action in [2]; channel-permutation lift in [3]; numerical $\alpha_n$ in [4]) and one outstanding numerical calibration: $\xi$ within [1, 2]. With these programme-level inputs verified and $\xi$ calibrated, the conditional theorem of §15 becomes a structural one and the structural origin of energy conservation is fully established, with remaining work confined to explicit geometric realisation and numerical calibration. The within-paper construction supplied by Parts I, II, and Appendices A–C is complete.

The full open-problem statements, retained for reference and including the residual content of each reduction, follow in §18.

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# 18. Open Problems

The six original open problems are listed here in their full form. All six are now reduced to either within-paper results or single, sharply identified programme-level conditions. Three (Problems 2, 3, 6) are fully closed within the present paper; Problem 4 is structurally resolved but not numerically; Problems 1 and 5 are reduced to single geometric properties within [1] and [3] respectively. Each problem is stated below together with its current status and the residual content.

**Problem 1 (Geometric derivation of CRC).** §3 argued that CRC is a consequence of the operational meaning of closure within VERSF. *Status: closed within the present paper by Appendix C (Lemma C.1), conditional on the orientation-invariance principle (§C.1) being structurally derived in [1].* The principle states that the cost functional Φ_c is a scalar functional of the unoriented closure constraint, so the two oriented traversals C_j^+ and C_j^− carry the same cost; CRC follows immediately. The residual of Problem 1 is sharply narrowed from "derive CRC from K = 7" to the single structural question of whether the cost functional is scalar on unoriented constraint surfaces — answerable within the simplicial structure of [1].

**Problem 2 (Boundary-flux correspondence; canonical-coordinate construction).** *Status: closed within the present paper by Appendix B (substrate-cost matching of §B.4 with equations B.5–B.10, and uniqueness via Propositions B.1–B.3), conditional on the canonical form (B.1) of the BCB action being established in [2].* The identification F = W is established by the boundary-flux matching of §B.4; the canonical-coordinate construction of J_W is given by (B.10); the uniqueness of W as generator (up to additive constants) is established by Propositions B.1–B.3, ruling out Casimirs and substrate-induced symmetry generators as competing solutions. The original Conjecture 1 of earlier drafts is now Proposition 2 of §11.

**Problem 3 (Regularity of TPB calibration).** *Status: closed within the present paper to formal expansion order by Lemma 12.1 of §12, which establishes the systematic gradient expansion $H = (W/T_{TPB}) \cdot [1 + \alpha_1 \varepsilon_{TPB} + \alpha_2 \varepsilon_{TPB}^2 + \ldots]$. Residual: explicit derivation of L_c from substrate-level data and numerical evaluation of the α_n, both belonging to the TPB framework [4].*

**Problem 4 (Determination of $\hbar$ from substrate parameters; existence of the fundamental orbit).** *Status: partially closed within the present paper by Appendix C.* The existence and structural form of $\gamma_0$ as the 14-step minimal complete orbit (§C.4) are established, conditional on the K = 7 closure architecture of [1]. The consistency relation $\Delta\tau_0 = (8\pi/21)(\xi/c)$ (equation C.11) confirms dimensional coherence between the (3/8) coefficient of [1, 2] and the phase-return condition of §13 but does not constitute independent progress on the numerical value of $\hbar$. *Residual: numerical evaluation of ħ via absolute calibration of the substrate length scale ξ in [1, 2].*

**Problem 5 (Lorentz transformation of cost data).** *Status: closed within the present paper by Proposition 4 of §14, conditional on the channel-permutation lift property of [3].* The proof structures Lorentz covariance of J^μ_W in five steps: TPB null-cone construction; Lorentz group from null-cone preservation; cost-spectrum set-invariance under channel permutation; frame-consistent channel relabelling (the load-bearing structural input from [3]); and four-vector covariance from cost-weighting invariant scalars. *Residual: showing that substrate-level transformations lifting to emergent Lorentz boosts act on the channel index set by permutation respecting spacetime localisation of events — a single structural question about the lift map of [3].*

**Problem 6 (W as Noether charge of proto-time translation).** *Status: closed within the present paper by Appendix B (equation B.11), conditional on the canonical form (B.1) of the BCB action being established in [2] and on the F = W identification of §B.4.* W is identified as the conserved

charge of proto-time translation in the BCB Lagrangian; the Hamiltonian (Proposition 2) and Noether identifications coincide.

**Summary of status.** All six original open problems are now reduced to either within-paper results or single, sharply identified programme-level conditions. Three (Problems 2, 3, 6) are fully closed within the present paper, conditional only on programme-level citations to [2] and [4]. Problem 4 is structurally resolved but not numerically: $\gamma_0$ existence is established by Appendix C, with only the calibration of $\xi$ within [1, 2] remaining for explicit numerical evaluation of $\hbar$. Problems 1 and 5 are reduced to single geometric properties within [1] and [3] respectively: orientation invariance of the cost functional on closure constraints, and the channel-permutation lift of substrate transformations under emergent Lorentz boosts. None of the residuals threatens the overall construction; each is a sharp programme-level question with an identified target paper.

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# 19. Conclusion

We have established the structural origin of energy conservation in the VERSF framework. The argument proceeds in two stages, presented as Parts I and II of this paper.

Part I established a structural conservation law at the substrate level. The cost-weighted commitment content

$$W(\tau) \equiv \Sigma_j \Phi_{c,j} \left( N^f_j(\tau) - N^r_j(\tau) \right)$$

is invariant in closed systems under BCB, TPB, and CRC. We argued that CRC is not a free hypothesis but a restriction of the admissible class of processes — those operationally indistinguishable from identity transformations at the level of commitment cost — and we showed by explicit counterexample (Proposition 1) that BCB alone is insufficient: cost-mismatched pairings produce unbounded drift in W and represent, structurally, false closures rather than genuine ones. Appendix C strengthens this further by deriving CRC geometrically from the K = 7 closure architecture under the orientation-invariance principle (Lemma C.1), reducing the residual content of CRC to a single structural question in [1].

Part II constructed the lift from W to the standard Hamiltonian formulation. Reversible substrate dynamics admits a symplectic structure derived from the Fisher–Rao geometry of distinguishability; W generates the corresponding flow with respect to proto-time (Proposition 2; proof in Appendix B via Legendre transform of the BCB action); TPB calibration converts W to a physical-time Hamiltonian H whose self-adjoint lift $\hat{H}$ generates unitary evolution, with the state-dependent regularity controlled by Lemma 12.1; periodic closure structure yields the Planck relation $E = \hbar\omega$ with $\hbar$ as a substrate-fixed conversion constant; and the cost-weighted current of Part I composes into a Lorentz-covariant energy-momentum four-vector under emergent Lorentz invariance (Propositions 3 and 4, with Proposition 4 closing Problem 5 modulo the channel-permutation lift property of [3]). The synthesis is Theorem 2, conditional on the five hypotheses (H1)–(H5) and on the canonical form of the BCB action established in [2].

The significance of the result is twofold. First, it provides a substrate-level account of *what energy is*: the rate of cost-weighted commitment flow under emergent temporal parametrisation. Second, it shows that the standard relations of physics — Hamilton's equations, the Schrödinger equation, the Planck relation $E = \hbar\omega$, and four-momentum conservation — are not independent inputs but consequences of a single underlying conservation law together with the geometric and dynamical structures that the broader VERSF programme derives from more primitive principles.

The present work establishes that energy conservation is not a fundamental axiom but a derived consequence of closure structure at the substrate level. Once the geometric and dynamical lifts of the VERSF programme are applied — Fisher–Rao geometry on distinguishability, the Hilbert-space lift on simply-connected reversible flows, the TPB calibration of emergent time, and emergent Lorentz invariance from finite-throughput causal structure — the conserved quantity W reproduces the Hamiltonian, the Schrödinger generator, the Planck relation, and the energy-momentum four-vector of standard physics. All six original open problems are now reduced to either within-paper results or single, sharply identified programme-level conditions. Three (Problems 2, 3, 6) are fully closed within the present paper: the Legendre-transform construction of Appendix B closes Problems 2 and 6 via the $F = W$ identification by substrate-cost matching; the gradient expansion of Lemma 12.1 closes Problem 3. Problem 4 is structurally resolved but not numerically: Appendix C establishes the existence of $\gamma_0$ with only $\xi$ calibration remaining. Problems 1 and 5 are reduced to single geometric properties within [1] and [3] respectively: orientation invariance of the cost functional on closure constraints (Lemma C.1); the channel-permutation lift of substrate transformations under emergent Lorentz boosts (Proposition 4). With these programme-level inputs verified and $\xi$ calibrated, the conditional theorem of §15 becomes a structural one and the structural origin of energy conservation is fully established, with remaining work confined to explicit geometric realisation and numerical calibration.

The remaining open problems concern the explicit geometric realisation of structures already required by the framework, not the existence of the conservation law itself.

The deeper claim is the one announced in the introduction: that conservation precedes symmetry. In a framework where physical time is emergent, time-translation invariance cannot be the source of energy conservation; it is, rather, what cost-balance under closure becomes when the substrate is lifted to the smooth dynamical regime where Noether's theorem applies. The standard derivation runs from symmetry to conservation; the VERSF derivation runs from closure structure to conservation, with symmetry recovered as a downstream consequence. This is the inversion of order that the paper has aimed to make precise.

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The author thanks the AIDA Institute for hosting the VERSF Theoretical Physics Programme. This paper draws on results and conventions established elsewhere in the programme, in particular the $K = 7$ no-go theorem [1] (used in Appendix C for the closure architecture), the BCB Lagrangian formulation [2] (used in Appendix B for the canonical action and in Appendix

C for the cost-spectrum coefficient), the emergent-Lorentz invariance results [3] (used in Proposition 4 of §14 for the channel-permutation lift property and the null-cone construction), the TPB framework [4], and the Hilbert-space lift programme [7].

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# Appendix A — Worked Example: Two-Level System with a Single Closure Pair

We illustrate the $W \to H_\tau \to \hat{H} \to E = \hbar\omega$ chain on the simplest non-trivial substrate configuration: a system with two equal-cost closure channels and one closure pair cycling between them. This example is not intended to model any particular physical system; it is a minimal walk-through showing how each ingredient of the construction acts when the substrate is small enough to compute explicitly.

**Substrate setup.** Let $K = 2$ with $\Phi_{c,1} = \Phi_{c,2} = \Phi$. Consider a system in which a single closure pair cycles indefinitely: the forward event resolves in channel 1 at proto-times $\tau = 0, T_\tau, 2T_\tau, \ldots$, and the corresponding restore event resolves in channel 2 at proto-times $\tau = T_\tau/2, 3T_\tau/2, \ldots$. (After each restore, the next forward begins; this is the minimal periodic substrate orbit.) BCB is satisfied: each cycle contains one forward and one restore. CRC is satisfied: $\Phi_{c,1} = \Phi_{c,2} = \Phi$. TPB is satisfied with $R_{max} = 2/T_\tau$.

**Part I: W conservation (§6).** Apply the W definition (4.1):

$$W(\tau) = \Phi \cdot ( N^f_1(\tau) - N^r_1(\tau) ) + \Phi \cdot ( N^f_2(\tau) - N^r_2(\tau) ) = \Phi \cdot N^f_1(\tau) - \Phi \cdot N^r_2(\tau)$$

(since $N^r_1 = N^f_2 = 0$ by construction). At the start of each cycle (just before $\tau = 0, T_\tau, 2T_\tau, \ldots$), $N^f_1 = N^r_2 = n$ for some integer $n$, and $W = 0$. At each forward event, $N^f_1$ increments and $W$ jumps to $\Phi$. At each restore event, $N^r_2$ increments and $W$ returns to 0. So $W(\tau)$ is a square-wave: it equals 0 at every pair-quiescent $\tau$ and $\Phi$ during transit. Theorem 1 holds: $W$ is conserved at pair-quiescent moments. The transit fluctuation $\Phi$ is bounded by $\Phi_{max} \cdot 1$ (one pair in transit), in agreement with Remark 1.

**Part II: phase space and $H_\tau$ (§§9–11).** The coarse-grained state of this system can be parametrised by a single phase variable $\theta \in [0, 2\pi)$ tracking the position within the cycle, with conjugate "occupation" variable $J$. At the substrate level, $\theta$ encodes how far into the current commitment-restore cycle the system has progressed; $J$ encodes the cycle's amplitude. Reversible evolution between events is represented as continuous rotation through $\theta$, with one full cycle per proto-period $T_\tau$.

The coarse-grained state space is a two-dimensional phase plane $(J, \theta)$ with symplectic form $\omega = dJ \wedge d\theta$. The Fisher–Rao metric on this space, derived from the distinguishability between substrate states at different $(J, \theta)$, is $g = dJ^2 + J^2\, d\theta^2$ (Euclidean in suitable coordinates) [7, Appendix]. This phase space is Kähler, so (H4) holds.

**The two senses of W on this example.** This is where the example exposes the structure described in §11. The substrate-level W of (4.1), here a square wave returning to 0 at every pair-quiescent moment with transit fluctuation Φ, is locally trivial in the phase variable θ: it cannot drive any flow on (J, θ) because between events it is constant. Its coarse-grained representative on (J, θ) — the smooth function used as Hamiltonian generator — is *not* the cumulative-count function but rather $W_{phase}(J, \theta) = \Phi \cdot J$, encoding the rate at which substrate-level cost-weighted commitment is being processed by the closure cycle (hence its proportionality to the cycle's amplitude J).

The bridging relation between the two senses can be stated explicitly. Over a coarse-graining window much greater than $T_\tau$ but much smaller than the timescale on which J evolves, the substrate-level cumulative count records $\langle N^{f}_1 \rangle \approx J \cdot \Delta\tau / T_\tau$ forward events (hence $\Delta\tau / T_\tau \approx J \cdot \Delta\tau$ pair-completions per unit J·proto-time on suitable normalisation). The smooth $W_{phase} = \Phi \cdot J$ is the rate of cost-weighted commitment flow, equal to Φ times the rate of pair-completion, and recovers the substrate-level W of (4.1) by time-integration up to transit fluctuations. A units check confirms consistency: J carries units of action (it is the conjugate of the dimensionless angle θ); dividing by a proto-period of order $T_\tau$ in the rate sense gives $W_{phase} = \Phi \cdot J$ units of cost — the same units as the cumulative substrate-level W, as required for the two functions to share a name. The substrate-level W and the phase-space W are therefore different functions sharing a name: the first is a step-function of cumulative counts, the second is a smooth rate density on phase space, and the relation is exactly the long-time-average / coarse-graining map of §11.

**Hamilton's equations.** Under Proposition 2 — applied at the coarse-grained level — $H_\tau = W_{phase} = \Phi \cdot J$ on the (J, θ) phase plane, and

$$d\theta/d\tau = \partial W_{phase} / \partial J = \Phi \,, \quad dJ/d\tau = -\partial W_{phase} / \partial \theta = 0$$

This is exactly the desired flow: J is conserved (the cycle's amplitude is constant in proto-time, consistent with substrate-level W conservation under closure at every pair-quiescent moment), and θ advances at constant rate Φ in proto-time.

**Part II: physical time (§12).** Apply the TPB calibration with $T_{TPB}$ constant. Then $H = W / T_{TPB} = (\Phi/T_{TPB}) \cdot J$, and

$$d\theta/dt = \partial H/\partial J = \Phi/T_{TPB} \equiv \omega$$

The system rotates in physical time at angular frequency $\omega = \Phi/T_{TPB}$.

**Hilbert lift (§12).** The geometric quantisation of (J, θ) with symplectic form $dJ \wedge d\theta$ on a phase plane gives a Hilbert space spanned by states $|n\rangle$ for n = 0, 1, 2, … (the harmonic-oscillator quantisation), with $\hat{J}\,|n\rangle = (n + 1/2)\,\hbar\,|n\rangle$ for some normalisation constant $\hbar$. The Hamiltonian operator is

$$\hat{H} = \omega \cdot \hat{J} = \hbar\,\omega \cdot (\hat{N} + 1/2)$$

where $\hat{N}\,|n\rangle = n\,|n\rangle$. This is the standard quantum harmonic oscillator with frequency $\omega$.

**Planck relation (§13).** The fundamental excitation is the $n = 0 \to n = 1$ transition: the smallest non-trivial periodic excitation of the substrate. Its action increment is $\Delta S = 2\pi\hbar$ — the difference between adjacent quantised levels rather than the action of any single orbit (in semiclassical quantisation the action of the n-th orbit is $(n + 1/2) \cdot 2\pi\hbar$, with adjacent levels separated by $2\pi\hbar$). The Definition (13.4), read in this light, identifies $\hbar$ via this action increment per quantum: $\Delta S = 2\pi\hbar$ defines $\hbar$ as the substrate-fixed conversion constant relating proto-action to phase. With $\omega = \Phi/T\_TPB$ the physical frequency, the Planck relation $E = \hbar\omega$ records the energy of the fundamental transition:

$$E = \hbar\,\omega = \hbar \cdot (\Phi/T\_TPB)$$

The relation is, as §13 acknowledges, a definitional identity once $\hbar$ is defined via $S\_\tau^0$; the substantive content is that *the same* $\hbar$ governs all higher harmonics ($E\_n = \hbar\omega \cdot (n + 1/2)$) — a property here visible directly in the harmonic-oscillator spectrum.

**What this example does and does not do.** The example walks through every step of the $W \to H\_\tau \to \hat{H} \to E = \hbar\omega$ chain on a single substrate configuration, demonstrating concretely how the abstract construction acts. It does *not* establish that this configuration realises any particular physical system, nor does it determine $\hbar$ from substrate parameters (Problem 4): the value of $\hbar$ is read off from the geometric quantisation, not derived from $K = 7$ closure structure. What it shows is the internal consistency of the construction: the conserved scalar W of Part I lifts, on a tractable example, to the standard quantum harmonic oscillator with $E = \hbar\omega$, in agreement with Theorem 2.

**Comparison of the two senses of $\hbar$.** As a sanity check, we verify that the proto-action definition (13.4) and the action-increment definition of Appendix A yield the same $\hbar$ on this example. The classical fundamental orbit has proto-action

$$S\_\tau^0 = \oint p\,dq = \oint J\,d\theta = 2\pi \cdot J^0$$

where $J^0$ is the value of J on the fundamental orbit. Definition (13.4) gives $\hbar = S\_\tau^0 / 2\pi = J^0$. After geometric quantisation, the spectrum of $\hat{J}$ is $\{(n + ½)\,\hbar\_q\}\_\{n=0,1,2,\ldots\}$ for some quantum normalisation constant $\hbar\_q$, and the action increment between adjacent levels is $\Delta S = 2\pi\,\hbar\_q$. The mapping between the classical and quantum pictures is via the *transition* $n = 0 \to n = 1$, not the $n = 0$ state itself: semiclassically, the $n = 0$ ground state already carries an absolute action $(1/2)\cdot 2\pi\hbar$ from the Maslov correction, so the classical fundamental orbit cannot be identified with $n = 0$. What the classical fundamental orbit *does* identify with is the minimal non-trivial excitation, i.e. the action *gap* between adjacent quantum levels: $S\_\tau^0 = \Delta S = 2\pi\hbar\_q$, which gives $J^0 = \hbar\_q$. Hence $\hbar\_q = \hbar$. The classical proto-action definition and the quantum action-increment definition pick out the same constant, as expected from the Bohr–Sommerfeld correspondence — confirming on this example what §13 asserts in general.

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# Appendix B — Legendre Transform of the BCB Lagrangian: Closure of Conjecture 1

This appendix supplies the canonical-coordinate construction underwriting Proposition 2 of §11. Working in the BCB Lagrangian formulation [2], we (i) compute the Legendre transform yielding W as the canonical Hamiltonian, (ii) apply Noether's theorem to proto-time translation yielding W as the conserved charge, and (iii) prove uniqueness of W as the generator of the substrate flow up to additive constants. Together these promote Conjecture 1 of earlier drafts to Proposition 2 in the present paper, modulo the canonical form of the BCB action which is established in [2].

## B.1 The BCB Action in Canonical Form

We adopt from the BCB Lagrangian programme [2] that the substrate action, expressed in canonical coordinates $(q^a, p_a)$ on the coarse-grained phase space $M_q$, takes the Hamiltonian first-order form

$$S_{BCB}[q, p] = \int d\tau \, ( p_a \dot{q}^a - F(q, p) ) \quad \text{(B.1)}$$

where $F(q, p)$ is a smooth scalar on $M_q$ whose specific identification is the work of this appendix. Three remarks on the status of (B.1) and the division of work between [2] and the present paper.

First, what [2] supplies is the *form* of the action: that substrate dynamics, reduced to the coarse-grained phase space $M_q$, admits a Hamiltonian first-order action of the type (B.1) with $q^a$ parametrising distinguishability rays, $p_a$ their conjugate momenta in the symplectic structure of (H4), and $F$ a smooth scalar generating the dynamics. The derivation of this form from substrate-level distinguishability conservation and reversibility is the content of [2].

Second, what [2] does *not* supply, and what this appendix establishes, is the identification $F = W$: that the scalar generating substrate dynamics in (B.1) is precisely the coarse-grained cost-weighted commitment content of §4. This identification is the load-bearing claim of Conjecture 1 (now Proposition 2), and it requires both the boundary-flux matching (Appendix B equations B.4–B.7) and the uniqueness arguments of Propositions B.1–B.3. Without these, the canonical form (B.1) is consistent with any number of candidate scalars $F$; the appendix's work is to demonstrate that $F = W$ is the unique choice fixed by substrate-level cost commitment.

Third, the form (B.1) is the *generic* Hamiltonian first-order action: any system whose phase space is symplectic and whose dynamics is generated by a single smooth scalar admits this form. The non-trivial content of [2] is therefore that substrate dynamics admits *some* such Hamiltonian first-order action, with the specific scalar to be identified separately. The non-trivial content of this appendix is the identification of that scalar as W. The two together close Problems 2 and 6.

With this caveat acknowledged, the construction below is conditional on [2] only at the level of the canonical form (B.1); the identification F = W and its associated uniqueness are work done within the present paper.

## B.2 Euler–Lagrange and Hamilton's Equations

Variation of (B.1) with respect to q^a and p_a yields the Euler–Lagrange equations

$$d/d\tau \left( \partial L / \partial \dot{q}^a \right) = \partial L / \partial q^a \quad \text{(B.2)}$$

with $L(q, p, \dot{q}) = p_a \dot{q}^a - F(q, p)$. Computing each piece:

$$\partial L / \partial \dot{q}^a = p_a , \quad \partial L / \partial q^a = -\partial F / \partial q^a$$

so (B.2) gives $\dot{p}_a = -\partial F / \partial q^a$. Variation with respect to p_a gives

$$\partial L / \partial p_a = \dot{q}^a - \partial F / \partial p_a = 0$$

i.e., $\dot{q}^a = \partial F / \partial p_a$. These are Hamilton's equations with F as Hamiltonian:

$$\dot{q}^a = \partial F / \partial p_a , \quad \dot{p}_a = -\partial F / \partial q^a \quad \text{(B.3)}$$

## B.3 Legendre Transform

The Legendre transform of L with respect to $\dot{q}^a$ defines the canonical Hamiltonian:

$$H_\tau(q, p) \equiv p_a \dot{q}^a - L = p_a \dot{q}^a - \left( p_a \dot{q}^a - F \right) = F(q, p) \quad \text{(B.4)}$$

This identity is exact: the canonical Hamiltonian generated by the Legendre transform of the BCB action is precisely the scalar F appearing in (B.1). The remaining work is the identification of F with the cost-weighted commitment content W of §4.

## B.4 Identification F = W via Substrate-Cost Matching

We now show that F = W up to an additive constant.

The substrate-level rate of cost-weighted commitment along $\varphi_\tau$ is governed by the boundary-flux balance of §11 (equation 11.5):

$$dW / d\tau = - \oint_{\partial \Sigma} J_W \cdot dA \quad \text{(B.5)}$$

with J_W the cost-weighted current of §8. Within the BCB Lagrangian formulation, the rate of change of F along $\varphi_\tau$ is determined by Hamilton's equations (B.3):

$$dF / d\tau = \left( \partial F / \partial q^a \right) \dot{q}^a + \left( \partial F / \partial p_a \right) \dot{p}_a = \left( \partial F / \partial q^a \right)\left( \partial F / \partial p_a \right) + \left( \partial F / \partial p_a \right)\left( -\partial F / \partial q^a \right) = 0 \quad \text{(B.6)}$$

So F is conserved along φ_τ on closed regions. For an open region Σ, F changes only via boundary flux. By the divergence theorem applied on the canonical phase-space measure, the rate of change of the integrated F is

d/dτ ∫Σ *F dq^n dp_n* = − ∮{∂Σ} J_F · dA (B.7)

where J_F is the canonical Hamiltonian current associated with F under the flow it generates,

J_F^a = F · ( ∂F / ∂p_a ) (B.8)

The matching condition is that the BCB action describes the same substrate dynamics that generates the cost-weighted boundary flux of §11. That is, the φ_τ generated by F via (B.3) must coincide with the substrate flow φ_τ underlying (B.5), and the boundary-flux balance for F (equation B.7) must coincide with the boundary-flux balance for W (equation B.5) for *every* admissible boundary configuration ∂Σ.

The condition ∮{∂Σ} *J_F · dA* = ∮{∂Σ} J_W · dA holding for all admissible ∂Σ forces J_F = J_W as fields (modulo divergence-free additions, which integrate to zero on every closed boundary and therefore do not affect the matching). From J_F^a = F(∂F/∂p_a) = J_W^a, taking F nontrivially dependent on p_a, this implies F = W + c for some constant c, with the constant absorbable into the action's additive freedom.

This identification — F = W up to additive constant — is the load-bearing step closing Problem 2 within the present paper. The work is genuinely done here: [2] supplies (B.1) with F a generic scalar; the matching to substrate-level cost commitment (B.5) supplies F = W. The two together give Hamilton's equations with W as Hamiltonian, and (B.3) becomes

q̇^a = ∂W / ∂p_a , ṗ_a = −∂W / ∂q^a (B.9)

which is the canonical-coordinate form of the substrate flow φ_τ. The cost-weighted current J_W in canonical coordinates is given by

J_W^a = W · ( ∂W / ∂p_a ) (B.10)

closing the canonical-coordinate construction gap of Problem 2.

## B.5 Uniqueness of W as Generator

We now prove that W is the unique Hamiltonian function (up to additive constants) generating the substrate flow on (M_q, ω). The matching argument of §B.4 fixes F = W under boundary-flux equality for arbitrary ∂Σ; the propositions below rule out alternative scalars that might satisfy the matching trivially.

**Proposition B.1 (Uniqueness of the Generator).** *On a connected symplectic manifold (M_q, ω) with M_q simply connected, let X be a smooth vector field. If X = X_W for the W identified in*

*§B.4, then any smooth function $G : M_q \to \mathbb{R}$ satisfying $X_G = X$ must satisfy $G = W + c$ for some constant $c \in \mathbb{R}$.*

**Proof.** Suppose $X_G = X = X_W$. Then $\omega(X_G - X_W, \cdot) = 0$, equivalently $d(G - W) = 0$ by the definition of Hamiltonian vector field via $\iota_X \omega = dH$. A closed 0-form is a locally constant function. On a connected manifold, a locally constant function is globally constant, so $G - W = c$ for some $c \in \mathbb{R}$. ■

**Proposition B.2 (Exclusion of Casimirs).** *On $(M_q, \omega)$ under (H4), let $C : M_q \to \mathbb{R}$ be a Casimir of the symplectic structure (i.e., a function satisfying $X_C = 0$ identically). Then $C$ is constant on connected components of $M_q$.*

**Proof.** $X_C = 0$ means $\iota_{X_C} \omega = dC = 0$, so $C$ is locally constant. Connectedness of $M_q$ (which follows from simple-connectedness under (H4)) gives global constancy. ■

**Proposition B.3 (Exclusion of Substrate-Induced Symmetry Generators).** *Let $G : M_q \to \mathbb{R}$ be the Hamiltonian function generating an internal substrate-induced symmetry — a transformation that preserves the cost-weighted commitment content of every closure pair. Then either (i) $X_G = 0$ (and $G$ is constant by Proposition B.2), or (ii) $X_G \neq 0$ and $G$ generates a flow that does not match the substrate-level boundary flux of $W$; hence $X_G \neq X$.*

**Proof.** A substrate-induced symmetry preserving cost-weighted commitment content of every closure pair acts trivially on $W$: $G \mapsto \{G, W\} = X_G[W] = 0$ by the symmetry property. If $X_G = 0$, the conclusion is immediate from Proposition B.2. If $X_G \neq 0$, then $G$ generates a non-trivial flow that preserves $W$ (by $\{G, W\} = 0$). The substrate flow $\varphi_\tau$, by contrast, *generates* changes in $W$ when integrated against open-system boundary conditions (cf. equation (11.5) of §11): $dW/d\tau = -\oint_{\partial\Sigma} J_W \cdot dA$ along $\varphi_\tau$ for non-vanishing boundary flux. A flow that preserves $W$ cannot equal $\varphi_\tau$ when $\varphi_\tau$ has non-trivial boundary flux. Hence $X_G \neq X$. ■

Together, Propositions B.1–B.3 establish that $W$ is the unique generator of the substrate flow $\varphi_\tau$ on $(M_q, \omega)$, up to additive constants:

- Any smooth function with the same Hamiltonian vector field as $W$ differs from $W$ by a constant (B.1).
- Casimirs of $\omega$ are constants and do not generate non-trivial flow (B.2).
- Substrate-induced symmetry generators either are constants or generate flows preserving $W$, hence cannot equal $\varphi_\tau$ (B.3).

This closes the second gap identified in §17.2: competing Hamiltonian functions are ruled out either by trivial action (Casimirs, Proposition B.2) or by inability to match the substrate boundary flux (substrate-induced symmetry generators, Proposition B.3).

## B.6 Noether Charge of Proto-Time Translation

With F = W established by §B.4, the BCB Lagrangian becomes L = p_a q̇^a − W(q, p), with no explicit dependence on proto-time τ. By Noether's theorem applied to the infinitesimal proto-time translation τ ↦ τ + ε, the conserved charge is

Q_τ = p_a ( ∂L / ∂q̇^a ) − L = p_a q̇^a − ( p_a q̇^a − W ) = W (B.11)

with d Q_τ / dτ = 0 along solutions of (B.9). This identifies W as the Noether charge associated with proto-time translation invariance of the BCB action — closing Problem 6 of §18.

The Hamiltonian and Noether identifications coincide: Q_τ = H_τ = W. This is not an accident; for any time-translation-invariant Lagrangian system, the Noether charge of time translation equals the Legendre-transform Hamiltonian. The two identifications are dual perspectives on the same conserved scalar.

Note that Problem 6's closure depends on F = W, which was established in §B.4 by the substrate-cost matching argument. Without that matching — i.e., if one had only the canonical form (B.1) with F a generic scalar — Noether's theorem would yield Q_τ = F, with no identification of F with the cost-weighted commitment content. The Noether identification is therefore *consequent* on §B.4's matching, not independent of it.

## B.7 Status

This appendix has supplied the four pieces of work identified as missing from Conjecture 1 in §11 and Problem 2 in §18:

1. **Identification F = W via substrate-cost matching.** §B.4 (equations B.5–B.10).
2. **Canonical-coordinate construction of J_W.** Equation (B.10), derived in §B.4.
3. **Uniqueness of W as generator.** Propositions B.1, B.2, B.3 of §B.5.
4. **Identification of W as Noether charge.** Equation (B.11), §B.6.

What [2] supplies is the canonical form (B.1) of the BCB action with a generic smooth scalar F generating substrate dynamics on M_q. What this appendix supplies is the identification F = W via the matching to substrate-level cost commitment, the uniqueness of this identification, the canonical-coordinate form of J_W, and the consequent identification of W as Noether charge of proto-time translation. Subject to [2]'s establishment of (B.1) as the canonical form, Conjecture 1 of §11 is upgraded to **Proposition 2** in the present paper, and Problems 2 and 6 of §18 are closed.

The overall picture, combined with the closures of Problems 1 and 4 (partial) supplied by Appendix C and the closure of Problem 5 supplied by Proposition 4 of §14 (modulo the channel-permutation lift property of [3]), is summarised in §15's status table and §17.7's synthesis: all six original open problems are now reduced to either within-paper results or single, sharply identified programme-level conditions — with Problem 4's residual being numerical (ξ calibration) and the remaining structural conditionals being in [1], [2], [3], [4].

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# Appendix C — The K = 7 Cost Spectrum and Minimal Closure Orbit

This appendix supplies the geometric closure of CRC and the structural derivation of the minimal closure orbit $\gamma_0$, drawing on the K = 7 closure structure established in [1]. We adopt one structural principle — that the cost functional $\Phi_c$ is a scalar functional of the unoriented closure constraint surface — and from this principle derive (i) CRC as a geometric lemma; (ii) the form of the cost spectrum on the K = 7 substrate; (iii) the minimal complete reversible orbit $\gamma_0$ as the 14-step traversal; and (iv) the structural closure-tick relation $\Delta\tau_0 = (8\pi/21)(\xi/c)$.

The honest scope is stated upfront: this appendix closes Problem 1 conditional on the orientation-invariance principle below, and partially closes Problem 4 by establishing $\gamma_0$'s existence and the $\Delta\tau_0$ relation. It does *not* numerically evaluate $\hbar$ — that quantity cancels in the $\Delta\tau_0$ derivation and remains conditional on the absolute calibration of the substrate length scale $\xi$.

## C.1 The Orientation-Invariance Principle

In the K = 7 substrate, a closure pair is not a pairing between two arbitrary channels; it is an oriented traversal and inverse traversal of the same closure constraint. Each of the seven independent closure constraints $C_j$ (j = 1, …, 7) admits two oriented traversals,

$C_j^+ , C_j^-$

corresponding to forward and restore directions. The total number of oriented channels is therefore 2K = 14, not K = 7.

**Principle (Orientation invariance of cost).** *The cost functional $\Phi_c$ is a scalar functional of the unoriented closure constraint:*

$\Phi_c = \Phi_c(C_j) , \text{ not } \Phi_c(C_j^\pm)$

*The orientations $C_j^+$ and $C_j^-$ describe traversal direction, not distinct constraint surfaces.*

This principle is the load-bearing input of the appendix. It is structurally motivated by the K = 7 simplicial geometry [1]: closure constraints are the intrinsic geometric objects of the substrate; orientations are labels assigned to traversals. A scalar cost functional defined on the constraint surface itself depends on the surface, not on the direction in which it is traversed. The principle is structurally reasonable but is *not* derived from substrate primitives within the present paper; its structural derivation belongs to [1]. We adopt it here as a working hypothesis with this status flagged.

## C.2 CRC as a Geometric Lemma

**Lemma C.1 (CRC from orientation invariance, within the K = 7 closure setting).** *Within the K = 7 closure structure (where every closure pair is the inverse traversal of a single closure constraint), and under the orientation-invariance principle, the threshold cost satisfies*

$$\Phi_c(C_j^+) = \Phi_c(C_j^-) = \Phi_c(C_j) \quad \text{(C.1)}$$

*for every closure constraint $C_j$. Consequently, every admissible closure pair, comprising a forward traversal $C_{j_f}^+$ and a corresponding restore traversal $C_{j_r}^-$, satisfies*

$$\Phi_{c, j_f} = \Phi_{c, j_r} \quad \text{(C.2)}$$

*Therefore CRC is a geometric consequence of orientation invariance within the K = 7 closure setting.*

**Proof.** A genuine restore event is, by definition of closure pair within K = 7, the inverse traversal of the same constraint surface as the forward event: $j_f = j_r$ as constraints, with opposite orientations. By the orientation-invariance principle, $\Phi_c(C_{j_f}^+) = \Phi_c(C_{j_r}^-)$. Hence $\Phi_{c, j_f} = \Phi_{c, j_r}$ for every admissible pair. ■

The lemma's load-bearing inputs are: (i) the definition of closure pair within the K = 7 closure structure as inverse traversals of a single constraint (a property of [1]); and (ii) the orientation-invariance principle of §C.1. The number K = 7 itself does not appear in the deductive chain; its role is to supply the ambient closure structure within which "closure pair" is defined. Lemma C.1 closes Problem 1 of §18 conditional on these two structural inputs, both of which belong to [1]. The residual is sharply identified: it is whether [1]'s closure structure together with orientation invariance jointly hold — a single structural question rather than the broader open question of "deriving CRC from K = 7."

## C.3 Symmetric Cost Spectrum

In the fully symmetric K = 7 closure state, all seven independent constraints are equivalent, and the cost spectrum reduces to a single value:

$$\Phi_{c,1} = \Phi_{c,2} = \ldots = \Phi_{c,7} = \Phi_0 \quad \text{(C.3)}$$

The VERSF commitment-threshold programme [1, 2] establishes the symmetric value as

$$\Phi_0 = (3/8) \cdot ( \hbar\, c / \xi ) \quad \text{(C.4)}$$

where $\xi$ is the substrate length scale fixed by the K = 7 simplicial geometry and (3/8) is the dimensionless commitment-threshold coefficient. We adopt (C.4) as input from [1, 2]; its derivation belongs to those programme papers and is not reproduced here.

For the pair-resolved corrected spectrum, allowing small closure-splitting corrections,

$$\Phi_{c,j}^{\pm} = ( \hbar\, c / \xi ) \cdot ( 3/8 + \delta_{C_j} ) \quad \text{(C.5)}$$

with orientation invariance preserved ($\delta_{C_j^+} = \delta_{C_j^-} = \delta_{C_j}$) and the symmetry-centring condition

$$\Sigma_{j=1}^{7} \delta_{C_j} = 0 \quad \text{(C.6)}$$

so that the (3/8) coefficient is preserved as the mean closure cost. The corrections $\{\delta_{C_j}\}$ encode how breaking of the full K = 7 symmetry distributes across the seven constraints; in the fully symmetric case all $\delta_{C_j} = 0$ and (C.5) reduces to (C.4).

The full cost spectrum is therefore

$$\mathrm{Spec}(\Phi_c) = \{ ( \hbar\, c / \xi ) \cdot ( 3/8 + \delta_{C_j} ) , j = 1, \ldots, 7 \} \quad \text{(C.7)}$$

with each value appearing twice under orientation ($j^+, j^-$) and the corrections satisfying the centring (C.6).

## C.4 The Minimal Closure Orbit

The minimal complete reversible closure orbit $\gamma_0$ is the shortest cycle that visits every independent closure constraint once in each orientation:

$$\gamma_0 = \{ C_1^+, C_1^-, C_2^+, C_2^-, \ldots, C_7^+, C_7^- \} \quad \text{(C.8)}$$

This orbit has 2K = 14 oriented traversal steps. Two structural points should be flagged.

First, *existence* of $\gamma_0$ follows from the K = 7 closure architecture: the set of fourteen oriented traversals exhausts the substrate's reversible step structure. Second, *uniqueness up to substrate symmetries* follows from the symmetry of the K = 7 simplicial structure under permutation of the seven constraints: any 14-step orbit visiting each oriented constraint once is related to (C.8) by a symmetry of the substrate, and yields the same total proto-action under (C.5) with the centring (C.6).

## C.5 The Structural Closure-Tick Relation

If each oriented traversal in $\gamma_0$ takes proto-time $\Delta\tau_0$ (the symmetric step duration), the total proto-action accumulated along $\gamma_0$ is

$$S_0 = \Sigma_{j=1}^{7} ( \Phi_{c,j}^+ + \Phi_{c,j}^- ) \cdot \Delta\tau_0 = 2 \cdot \Sigma_{j=1}^{7} \Phi_{c,j} \cdot \Delta\tau_0 \text{ (using orientation invariance)} = 2 \cdot 7 \cdot \Phi_0 \cdot \Delta\tau_0 \text{ (using the centring (C.6))} = 14 \cdot \Phi_0 \cdot \Delta\tau_0 \quad \text{(C.9)}$$

Imposing the fundamental phase-return condition $S_0 = 2\pi\hbar$ (the Bohr–Sommerfeld quantisation condition for the minimal orbit, established in §13):

$$14 \cdot \Phi_0 \cdot \Delta\tau_0 = 2\pi\, \hbar \quad \text{(C.10)}$$

Substituting (C.4) for $\Phi_0$:

$14 \cdot ( 3/8 ) \cdot ( \hbar \, c / \xi ) \cdot \Delta\tau_0 = 2\pi \, \hbar$

The factor of $\hbar$ cancels:

$( 42 / 8 ) \cdot ( c / \xi ) \cdot \Delta\tau_0 = 2\pi$

equivalently

$( 21 / 4 ) \cdot ( c / \xi ) \cdot \Delta\tau_0 = 2\pi$

Solving for $\Delta\tau_0$:

**$\Delta\tau_0 = ( 8\pi / 21 ) \cdot ( \xi / c )$** (C.11)

Numerically, $8\pi / 21 \approx 1.1968$. The minimal closure tick $\Delta\tau_0$ is therefore close to the causal crossing time $\xi/c$ (the time taken for a signal moving at the emergent speed of light to traverse one substrate length scale), with a factor $(8\pi/21) \approx 1.20$ representing the structural correction from the K = 7, 14-orientation closure architecture.

**Status of (C.11) as a consistency check.** It is important to be precise about what (C.11) does and does not establish. The cancellation of $\hbar$ between the cost-spectrum input (C.4) and the phase-return condition $S_0 = 2\pi\hbar$ is structurally inevitable: (C.4) was defined to carry exactly one power of $\hbar$ in its coefficient, and the phase-return condition carries one power of $\hbar$ on the right-hand side. The derivation therefore confirms *dimensional consistency* between the (3/8) coefficient established in [1, 2] and the Bohr–Sommerfeld quantisation condition of §13, but it does not constitute *independent progress* on the numerical value of $\hbar$. The closure tick $\Delta\tau_0$ is fixed in terms of $\xi/c$ by structural inputs — the (3/8) coefficient and the 14-orientation orbit count — both of which come from outside the present derivation.

The honest content of §C.5 is therefore: (i) the existence and structural form of $\gamma_0$ (the 14-step orbit) is established (real new content within the present paper, conditional on [1]); (ii) the consistency relation (C.11) between $\Delta\tau_0$, $\xi$, and c follows from the cost-spectrum input from [1, 2] together with the phase-return condition. The numerical value of $\hbar$ is no closer than before: it remains contingent on the absolute calibration of $\xi$ via the (3/8) coefficient in [1, 2], which is the original residual of Problem 4.

## C.6 Status

This appendix has supplied:

1. **Orientation invariance as the load-bearing principle for CRC.** §C.1.
2. **CRC as geometric lemma.** Lemma C.1 of §C.2 (within the K = 7 closure setting).
3. **The symmetric K = 7 cost spectrum.** Equation (C.4), cited from [1, 2].
4. **The minimal closure orbit $\gamma_0$.** §C.4: existence and structural form, conditional on the K = 7 closure architecture of [1].

5. **A consistency relation between $\Delta\tau_0$, $\xi$, and c.** Equation (C.11), confirming dimensional consistency between the (3/8) coefficient of [1, 2] and the phase-return condition of §13.

The honest scope of these closures is the following. **Problem 1 is closed within the present paper by Lemma C.1, conditional on the orientation-invariance principle of §C.1 being structurally derived in [1].** This narrows the residual of Problem 1 from "derive CRC from K = 7" to a single-line question: is the cost functional scalar on unoriented constraint surfaces? The reduction is sharp.

**Problem 4 is partially closed.** The existence and structural form of the minimal complete reversible orbit $\gamma_0$ as the 14-step traversal of all oriented constraints (§C.4) is established within the present paper, conditional on the K = 7 closure architecture of [1]. This is the closure of the *existence* part of Problem 4. However, the consistency relation (C.11) does not constitute independent progress on the numerical value of $\hbar$: $\hbar$ cancels by dimensional consistency between (C.4) and the phase-return condition. Numerical evaluation of $\hbar$ remains contingent on the absolute calibration of $\xi$ via the (3/8) coefficient in [1, 2] — the original residual of Problem 4. The residual content of Problem 4 within the present paper is therefore narrowed to: the absolute calibration of the substrate length scale $\xi$ (or equivalently of the action unit) within [1, 2].

A combined characterisation: the K = 7 closure architecture, under the orientation-invariance principle, closes the geometric content of CRC and the existence of $\gamma_0$; the consistency relation (C.11) confirms the dimensional coherence of the cost spectrum and the phase-return condition; but the absolute action unit $\hbar$ is not determined within the present paper, and remains contingent on [1, 2]'s calibration of $\xi$. This is the honest position the appendix establishes.

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# References


[1] Taylor, K. *A No-Go Theorem for Non-Simplicial Relational Substrates — K = 7.* VERSF Programme, AIDA Institute.

[2] Taylor, K. *Bit Conservation and Balance: A Lagrangian Formulation of VERSF.* VERSF Programme, AIDA Institute.

[3] Taylor, K. *Two Kinds of Time: Proto-Time, Physical Time, and Emergent Lorentz Invariance in VERSF.* VERSF Programme, AIDA Institute.

[4] Taylor, K. *The Ticks-Per-Bit Framework: Bounded Throughput and the Emergence of Physical Time.* VERSF Programme, AIDA Institute.

[5] Cencov, N.N. *Statistical Decision Rules and Optimal Inference.* AMS Translations of Mathematical Monographs, 1982.

[6] Amari, S. *Information Geometry and Its Applications.* Springer, 2016.

[7] Taylor, K. *Hilbert-Space Lift from Distinguishability and Reversibility in VERSF.* VERSF Programme, AIDA Institute.

[8] Souriau, J.-M. *Structure of Dynamical Systems: A Symplectic View of Physics.* Birkhäuser, 1997.

[9] Woodhouse, N.M.J. *Geometric Quantization.* Oxford University Press, 1992.