

The Unique Scalar Closure of the VERSF Record Current

Conservation, Dynamics, and the Constitutive Link Between Them

Keith Taylor VERSF Theoretical Physics Programme

Plain-Language Summary

Physics is full of equations — for fields, particles, forces, motion. But the equations themselves are largely silent on what their symbols ultimately refer to. A wave function is the central object of quantum mechanics, but textbooks rarely commit to what a wave function is *of*. A spacetime metric describes geometry, but does not say what the geometry is built from. Modern physics has remarkable predictive power and a corresponding reticence about its own subject matter.

The VERSF programme proposes a definite answer. The fundamental events of the world, on this view, are *irreversible commitments* — moments in which one possibility among many gets locked in as a fact, leaving a stable record. Quantum amplitudes describe the situation *before* such a commitment. Classical facts are what remain *after*. Entropy counts how many commitments have happened. Gravity is what happens when committed structure clusters densely. In short: physical reality is the accumulation and propagation of facts, and everything else is downstream of that.

Once this picture is accepted, a sharp question arises: *what is the law governing how facts form, propagate, and interact?* This paper gives a precise partial answer.

We define a mathematical object — the **record current** — that bookkeeps where facts exist and how they flow. We then prove two things. First, that under four minimal assumptions about what facts are, this current must satisfy a particular conservation equation. There is no other option at this level: the equation is *forced*. Second, we ask what the simplest possible equation of motion for the underlying field is. Under five further explicit assumptions, we prove that the answer is *unique*. Just one equation works. Any alternative either smuggles in extra structure, breaks a basic symmetry of nature, or violates stability.

The result is a clean mathematical skeleton on which the rest of the VERSF programme can hang. We are equally explicit about what the paper does *not* do. The simplest framework — called *scalar closure* — is not enough for everything. Gravity, in particular, requires a richer mathematical structure that companion papers in the programme supply. What the present paper establishes is the floor: the kinematic and dynamical law that any complete VERSF theory must rest upon.

Abstract

The VERSF (Void Energy-Regulated Space Framework) programme identifies physical reality with the accumulation of irreversible commitment events — transitions from admissible alternatives to stable, observer-invariant records. Closing this ontology into a working physics requires three distinct ingredients: (i) a **kinematic identity** stating what is conserved as facts accumulate, (ii) a **constitutive relation** specifying how committed structure flows, and (iii) a **dynamical principle** selecting which histories actually realise.

This paper provides (i) cleanly and (iii) at scalar order. We define the record current $C^u = (\rho_c, J_c)$ and prove that, under four minimal axioms — finite distinguishability, irreversibility, additivity, and local coupling — the unique scalar first-order local identity it satisfies is the continuity equation

$$\partial_\mu C^\mu(x) = s_c(x).$$

We then ask what minimal closure produces dynamics. Under five further assumptions — local field representation, linearity, Lorentz scalarity, second-order truncation, and stability — we derive the unique equation of motion within the minimal scalar closure class defined by these assumptions:

$$(\square + m^2) \Phi(x) = \sigma(x).$$

We are explicit that this is *one closure choice* among possible closures, that it is independent of (not derived from) the conservation identity, and that the relationship between the two equations is fixed by a constitutive map $J_c = J_c[\Phi, \sigma]$ which must be specified additionally. We give the constitutive map under the natural identifications, exhibit the corrected memory kernel for the propagating sector, run the Ignatowski compatibility argument with Lorentz invariance, and provide forward references for gravity, the Born rule, and entropy to the companion papers in the VERSF corpus.

The paper completes the VERSF dynamical programme **at scalar order** and identifies precisely what tensorial extension is needed for the full theory.

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1. Introduction

1.1 Position of this paper

Modern physics provides equations for fields and particles, but is largely silent on what these quantities fundamentally represent. The VERSF programme replaces this silence with a concrete commitment: the fundamental events of physics are *irreversible commitments* — transitions resolving genuinely distinguishable alternatives into stable record. Quantum amplitudes describe the pre-commitment regime; classical facts are post-commitment residues; entropy counts accumulation; gravitational response tracks gradients in committed density.

A long-standing question in the programme has been: *what is the dynamical law governing the formation, propagation, and interaction of facts?* This question decomposes into three sharply distinct sub-questions:

1. **Kinematic.** What invariants does the bookkeeping of irreversible facts force?
2. **Constitutive.** How does committed structure flow — what is J_c in terms of substrate variables?
3. **Dynamical.** What field equation or action principle selects which histories of commitment realise?

Earlier presentations of this material occasionally conflated these. The present paper treats (1) rigorously and (3) under explicit scalar-closure assumptions, giving an honest account of the relationship between them. The constitutive question (2) is addressed in part — under the natural identifications — and otherwise deferred to the BCB Lagrangian companion paper [1, §9.5], where the constitutive structure follows from Noether reduction on the master action.

On what Φ is. The scalar field Φ introduced in §6 should be understood as the minimal field-theoretic representation of the admissible amplitude structure derived in the Admissibility Closure framework [13]. That framework establishes the Hilbert-space and amplitude apparatus of the pre-commitment regime from VERSF axioms; the present paper picks up *after* that derivation and asks what dynamical law the post-Hilbert-space scalar representation must satisfy. The connection runs: admissibility closure \rightarrow Hilbert structure \rightarrow amplitudes \rightarrow (on commitment) record density $\Phi \rightarrow$ (this paper) field equation for Φ . We do not re-derive the upstream steps; we rely on them, and the scalar character of Φ in §6 is inherited from the scalar character of the admissible-amplitude structure rather than postulated independently.

1.2 What this paper assumes vs. derives

The following table is a strict contract between the reader and the paper. Any claim made in the body that is not in the right column is either an assumption or a forward reference, labelled as such.

	Assumed (axioms or other papers)	Derived here
Existence of irreversible commitments	✓ (foundational)	—
Distinguishability budget on bounded regions	✓ (foundational)	—
Locality of influence propagation	✓ (foundational)	—
Additivity of fact counts	✓ (foundational)	—
Conservation identity $\partial_{\mu} C^{\mu} = s_c$	—	✓ (Theorem 1)
Uniqueness of conservation at scalar first order	—	✓ (Theorem 2)
Klein–Gordon form $(\square + m^2) \Phi = \sigma$	—	✓ (Theorem 3, under B1–B5)
Constitutive map $J_c[\Phi, \sigma]$	Partial here; full in BCB Lagrangian paper	Partial
Action principle for the substrate	✓ (BCB Lagrangian paper)	—
Born rule	✓ (Double Square Rule paper)	—
Gravitational response	✓ (gravity-from-record-density paper)	—

	Assumed (axioms or other papers)	Derived here
Lorentz invariance	Compatibility shown here under standard assumptions	—

1.3 Strategic point

This material was previously framed as deriving the conservation identity from the field equation; the present paper separates them. The Klein–Gordon equation for a real scalar field does not by itself produce a conserved current, and the honest relationship is that the two equations are **independent statements about the same system**, linked by a constitutive map. Stating this honestly is, paradoxically, what makes the resulting framework more robust: it identifies precisely what additional structure is needed and where it must come from.

2. The Record Current

2.1 Definition

For each spacetime region, the VERSF substrate carries a count of irreversible commitments deposited in that region's causal past, together with a flux describing how committed structure is relayed across boundaries. We package these into a four-vector:

$$C^\mu(\mathbf{x}) = (\rho_c(\mathbf{x}), \mathbf{J}_c^i(\mathbf{x}))$$

where:

- $\rho_c(\mathbf{x})$ is the local density of committed facts — the count, per unit causal volume, of irreversible commitment events whose record contributes at \mathbf{x} ;
- $\mathbf{J}_c^i(\mathbf{x})$ is the spatial flux of committed structure — the rate at which fact-bearing influence crosses a unit area transverse to direction i .

Both are operationally tied to the substrate's distinguishability budget: ρ_c is bounded above by the local capacity $\kappa(\mathbf{x})$, and \mathbf{J}_c is bounded above by the maximum signal speed implied by finite propagation (cf. §11).

2.2 Status of C^μ

C^μ is not a fundamental field. It is a derived bookkeeping quantity associated with the substrate's record content. The BCB Lagrangian companion paper [1, §9.5.1] identifies it explicitly as the entropy current of the substrate's Role-4 sector rescaled by the bit energy: $C^\mu \equiv J^\mu_s / \varepsilon_{\text{bit}}$, where ε_{bit} is the energetic cost of one bit of irreversible information and is itself derived from the void-scale running of $\Lambda(\ell)$. For the present paper, we treat C^μ as a current and ask, first, what identities its components must satisfy (kinematics, §§3–5), and second, what minimal scalar dynamics complement those identities (§§6–9). The choice of which Noether current at the

master-Lagrangian level realises C^u is treated in [1, §9.5.3–9.5.4] and discussed further in §9.3 below.

2.3 Ontological summary

Physical reality is the distribution and flow of committed facts.

Geometry, matter, probability, and thermodynamics are downstream of this distribution. The remainder of this paper makes that downstream relationship precise.

3. Axioms for Conservation

We state the four axioms used in establishing the kinematic identity. Each is established or motivated in earlier VERSF papers; we collect them here in the form needed.

(A1) Finite distinguishability. For every bounded causal region R , the number of distinguishable commitments that R can host is finite. Equivalently, ρ_c is bounded almost everywhere by a finite local capacity $\kappa(x)$.

(A2) Irreversibility. Once a commitment is registered, it is not undone. Formally, the signed contribution of any past commitment to ρ_c at later times is non-negative.

(A3) Additivity. For disjoint causal regions R_1, R_2 , the number of commitments in $R_1 \cup R_2$ equals the sum of the numbers in R_1 and R_2 . No interference term appears in the count.

(A4) Local coupling. Influence between commitments propagates locally: changes in ρ_c in one region affect changes elsewhere only through transport across shared boundaries — i.e., through J_c .

These axioms are weaker than they may appear. (A1) is the substrate's capacity bound; (A2) is what makes facts *facts*; (A3) is what permits a count at all; (A4) is what permits a *current* description of that count.

4. The Conservation Identity (Theorem 1)

4.1 Statement

Theorem 1 (Conservation identity for the record current). Under axioms (A1)–(A4), the record current C^u satisfies

$$\partial_\mu C^\mu(x) = s_c(x)$$

where s_c is the local commitment production rate satisfying the integrated positivity condition

$$\int_R s_c d^3x \geq 0 \text{ for any comoving spacelike region } R.$$

Under the count interpretation of ρ_c — that ρ_c is itself a positive density of irreversible facts — this admits the strengthened pointwise form $s_c(x) \geq 0$. Under field-theoretic representations of ρ_c (cf. §9.3), only the integrated form holds.

4.2 Proof sketch

We assume standard regularity: ρ_c is locally bounded and measurable, J_c is locally integrable, the time evolution of ρ_c on any compact spacelike region is of bounded variation, and ∂R is sufficiently regular for the divergence theorem to apply (Lipschitz boundary suffices). These conditions are mild and physically reasonable: bounded distinguishability (A1) gives boundedness, locality (A4) gives integrability of fluxes, and irreversibility (A2) gives monotonic time-evolution of the integrated count (and hence bounded variation). Standard counterexamples (singular measures, distributions concentrated on lower-dimensional sets) are ruled out by (A1).

By (A3), the number of commitments in a region R is the integral of ρ_c over a spacelike slice of R . By (A4), the change in this count across an infinitesimal time interval decomposes into (i) a flux term across ∂R captured by $\nabla \cdot J_c$, and (ii) a production term capturing commitments that came into existence within R during that interval, captured by s_c . Equating the time derivative of the count with the sum of these contributions and applying the divergence theorem yields

$$\partial_t \rho_c + \nabla \cdot J_c = s_c$$

which is the component form of $\partial_\mu C^\mu = s_c$. Integrated positivity $\int_R s_c d^3x \geq 0$ follows directly from (A2): the count over any region R , $\int_R \rho_c d^3x$, is non-decreasing in time, so its time derivative — equal to $\int_R s_c d^3x$ by the divergence theorem applied to a comoving region — is non-negative. The strengthened pointwise statement $s_c(x) \geq 0$ follows from (A2) only under the *count interpretation* of ρ_c — that ρ_c is itself a positive density of irreversible facts. Under field-theoretic representations of ρ_c (cf. §9.3), pointwise positivity does not hold and only the integrated form survives. ■

4.3 What this identity is and isn't

This identity has the universal form of any local conservation law with sources. By itself it is *kinematic*: it constrains how ρ_c and J_c can vary together, and it bookkeeps net production via s_c . It does **not** specify:

- what s_c is in terms of substrate variables,
- what relates J_c to ρ_c (or to other substrate fields),
- what evolution equation C^u itself obeys.

Each is closure data, addressed below or in companion papers. The contribution of this section is to establish that the bookkeeping form of the identity is forced by the axioms.

5. Uniqueness at Scalar Order (Theorem 2)

We now show that, at the level of first-order scalar identities built from C^μ and its first derivatives, the conservation identity is the unique non-trivial relation — *under Lorentz scalarity as an explicit hypothesis*.

Theorem 2 (Uniqueness of the scalar identity at first order). Let LC , ∂C be a **Lorentz-scalar** local functional, polynomial of degree at most one in ∂C^μ and at most one in C^μ , compatible with axioms (A1)–(A4). Then L is a non-zero multiple of the residual $\partial_\mu C^\mu - s_c$, plus terms vanishing identically.

Hypothesis status. Lorentz scalarity is here an *explicit hypothesis*, paralleling assumption (B3) below. It is not derived from axioms (A1)–(A4) alone; its substrate-level emergence is treated in §11 and in the proto-time / emergent-Lorentz companion paper [5]. Theorem 2 is therefore a *conditional* uniqueness result: given Lorentz scalarity, the conservation identity is the unique scalar first-order relation.

Polynomial restriction. The restriction to polynomial degree ≤ 1 in C^μ and ∂C^μ is selected by axiom (A4) (locality) plus count-tracking. Higher-degree terms — for example $C^\mu C_\mu$ as a zeroth-order Lorentz scalar — would couple the value of the count current to itself in a way inconsistent with the additivity of fact counts (A3): if such terms were admitted, the bookkeeping identity for disjoint regions R_1, R_2 would acquire a cross-term, contradicting the assumption that commitment counts simply sum over disjoint regions. Linearity-in-current bookkeeping is therefore not an arbitrary choice but a consequence of (A3) read together with (A4).

Proof sketch. The most general such L is

$$L = \alpha \partial_\mu C^\mu + \beta^\mu C_\mu + \gamma$$

with $\alpha, \beta^\mu, \gamma$ at most position-dependent. Lorentz scalarity (the explicit hypothesis) forces $\beta^\mu = 0$: no preferred direction is available without additional structure. Compatibility with additivity (A3) requires γ to track the local production density s_c , identifying $\gamma = -s_c$ up to a multiplicative constant. The residual statement $L = 0$ is then $\alpha(\partial_\mu C^\mu - s_c) = 0$. ■

Scope. This is a uniqueness theorem about *Lorentz-scalar first-order linear-in-current* identities. It does not claim:

- that no higher-derivative or tensorial identities exist (they do — they live in the closure data),
- that the identity uniquely fixes the dynamics (it does not — see §§6–9),
- that no extension involving additional substrate fields exists.

In particular, the BCB Lagrangian induces Euler–Lagrange equations whose conservation-law content is exactly the present identity, but whose dynamical content goes well beyond it.

6. From Conservation to Dynamics: What Closure Requires

The identity $\partial_\mu C^\mu = s_c$ is the universal form of a local conservation law with sources. To obtain dynamics, three independent pieces of closure data are needed:

(C1) A field-theoretic representation of C^μ . Either an action principle whose Euler–Lagrange equations yield C^μ as a Noether current, or a direct field representation of ρ_c .

(C2) A constitutive map. A specification of $J_c[\rho_c, \partial\rho_c, \dots]$ in terms of the chosen field variables — equivalently, how committed structure is transported.

(C3) A specification of s_c . Either as a function of substrate variables, or as a stochastic source from the pre-commitment sector.

Different choices of (C1)–(C3) yield different physical theories on the same kinematic skeleton. The remainder of this paper develops the **minimal scalar closure** — the simplest choice consistent with axioms (A1)–(A4) and a small number of further assumptions — and demonstrates uniqueness within that closure class. Scalar closure represents the minimal representation of commitment density: it tracks the count of irreversible facts as a single Lorentz-invariant local quantity, with no additional internal structure. Tensorial closure is required for geometric response (gravity) and for any sector carrying directional or polarisation degrees of freedom, and lies beyond the scope of the present work. The full action-principle treatment (more general than scalar closure) is the subject of the BCB Lagrangian companion paper.

7. Scalar Closure Assumptions

Beyond the conservation axioms (A1)–(A4), we adopt:

(B1) Local scalar representation. There exists a Lorentz-scalar local field $\Phi(x)$ such that ρ_c is a local functional of Φ . The specific functional form — whether ρ_c is identified with Φ , with $\partial_t \Phi$, with a quadratic combination, or otherwise — is closure data discussed in §9; (B1) assumes only that *some* such local-functional identification exists.

(B2) Linearity at the equation-of-motion level. The equation governing Φ is linear in Φ . This is the simplest dynamical structure consistent with additivity (A3): if commitment events from disjoint sources superpose without interference at the count level, a linear equation for the field representation respects that superposition automatically. Nonlinear equations may also be consistent with (A3) but introduce additional structure not motivated by the present axioms; (B2) selects the minimal admissible form.

(B3) Lorentz scalarity of the equation. No preferred direction enters the Φ -equation — a consequence of substrate isotropy.

(B4) Second-order truncation. The equation is at most second-order in derivatives. This is an *effective field theory* assumption: it is justified by finite local capacity (A1) plus the standard EFT argument that higher-derivative terms are suppressed at scales below the substrate cutoff. We are explicit that (B4) is a truncation, not a derivation.

(B5) Stability. The equation admits stable (non-runaway) propagation, which restricts the sign of the gap parameter introduced below.

Each assumption is stated openly. (B1) is a representation choice. (B2) is the minimal dynamical structure consistent with (A3). (B3) follows from substrate isotropy (cf. §11). (B4) is the EFT assumption. (B5) is a physical-stability constraint.

8. The Scalar Field Equation (Theorem 3)

8.1 Statement

Theorem 3 (Scalar dynamical closure). Within the **minimal scalar closure class defined by (B1)–(B5)**, and under axioms (A1)–(A4), the unique equation of motion for Φ — up to overall normalisation and a constant shift absorbable into σ — is

$$(\square + m^2) \Phi(x) = \sigma(x)$$

with $m^2 \geq 0$ and σ a stipulated source density.

8.2 Derivation

The most general functional satisfying (B1)–(B4) is

$$a_0 + a_1 \Phi + a_2 \partial_{\mu} \partial^{\mu} \Phi + (\text{other Lorentz-scalar second-order operators}) = \sigma$$

By (B3), no term involving $\partial^{\mu} \Phi$ alone is admissible (it requires a vector to contract). By (B2), products of Φ with itself or its derivatives are excluded beyond first order. The constant a_0 can be absorbed into σ . The remaining structure is

$$a_1 \Phi + a_2 \square \Phi = \sigma.$$

For non-trivial dynamics, $a_2 \neq 0$. Rescale Φ to set $a_2 = 1$, and define $m^2 \equiv a_1$. The equation becomes

$$(\square + m^2) \Phi = \sigma.$$

By (B5), $m^2 \geq 0$ (otherwise the homogeneous solutions grow without bound, violating finite-capacity stability). ■

8.3 Interpretation of m^2

m^2 is a phenomenological gap parameter. Its microscopic origin is treated in the BCB Lagrangian [1, §9.5.5], which establishes that the bare Role-4 action has $m^2 = 0$ (the τ -shift symmetry being exact at that level). Non-zero m^2 arises from shift-breaking structure introduced from outside the bare action: a void-scale τ -potential $V(\tau)$ with $V''(0) = \kappa_4 m^2$ (with the Two-Planck Principle [6] supplying an order-of-magnitude $m_\tau \sim \varepsilon_{\text{bit}} \approx 0.010$ eV from the bit-saturation argument); τ - s mixing through the $\Lambda(s) - \lambda(x)[s - s_{\text{BCB}}]$ structure of the master Lagrangian; or RG corrections at the substrate cutoff Λ_{fold} . The kinematic identity $\partial_\mu C^\mu = s_c$ does not require $m^2 \neq 0$; the gap parameter matters only for the propagating sector — the memory tail of §10.2 and the Compton wavelength of τ -mediated effects. For the purposes of the present paper, m^2 is a stipulated parameter ≥ 0 .

8.4 What Theorem 3 establishes — and what it doesn't

What it establishes: within the minimal scalar closure class defined by (B1)–(B5), and under axioms (A1)–(A4), the Klein–Gordon equation with source is the unique dynamical law. There is no hidden alternative within this closure class.

Honest framing of the result. The equation form $(\square + m^2)\Phi = \sigma$ is, in itself, the standard effective-field-theory result for a single linear scalar field: any introductory QFT textbook derives it as the unique linear, second-order, Lorentz-scalar equation for a scalar field, modulo source structure. The VERSF-specific content of Theorem 3 is therefore *not* in the equation form but in **what Φ is** and **how it links to the conservation identity**:

- the *ontological reading* of Φ as the field representation of irreversible-fact density (B1),
- the *closure structure* connecting Φ to the record current via the constitutive map of §9,
- the *axiomatic motivation* of (B1)–(B5) from VERSF substrate properties rather than as generic EFT inputs.

This sharpens, rather than weakens, what the theorem contributes: it shows that *if* one accepts the VERSF ontology and reads Φ as it does, *then* the simplest available dynamics are uniquely fixed and coincide with the form physicists already know. The agreement is not a coincidence — it is what allows the framework to recover standard physics in the appropriate regime.

What Theorem 3 does *not* establish:

- That scalar closure is the *correct* closure for the full physical theory. Tensor extensions are needed for gravity (§12).
- That the field equation implies the conservation identity. It does not — see §9.
- That m^2 can be derived from more fundamental VERSF axioms. It is stipulated here.

9. Relating the Two Equations: The Constitutive Map

Conventions for this section. Throughout §9 we work in mostly-minus signature, $\eta = \text{diag}(+1, -1, -1, -1)$, so that $\partial^0 = \partial_t$ and $\partial^i = -\partial_i$ for spatial indices. The source σ enters the action as $\int \sigma \Phi \, d^4x$, giving the Euler–Lagrange equation in the form $(\square + m^2)\Phi = \sigma$ with the sign matching equation (II). All other sign conventions follow these two choices.

9.1 Why the two equations need a link

We have two equations:

$$(I) \partial_\mu C^\mu = s_c \quad (\text{Theorem 1, kinematic}) \quad (II) (\square + m^2) \Phi = \sigma \quad (\text{Theorem 3, scalar dynamics})$$

These are **independent statements**. (II) does not imply (I), and (I) does not determine (II) without further input. To see this, observe that for a real scalar field there is no Noether current associated with $\Phi \rightarrow \Phi + \alpha$ at the linear level: the Klein–Gordon equation breaks the would-be $U(1)$ by the mass term and the source. Therefore (II) does not by itself produce a conserved current.

The two source terms must also be distinguished. $\sigma(x)$ is the **field-theoretic source**, the inhomogeneous term in the equation of motion for Φ . $s_c(x)$ is the **physical commitment rate**, the production density of irreversible facts entering the bookkeeping of (I). These are not the same quantity: their relationship is mediated by the constitutive map and is not fixed at the kinematic level.

The link between (I) and (II) is supplied by an additional piece of structure: a **constitutive map** specifying ρ_c and J_c in terms of Φ (and possibly $\partial\Phi$, σ). The present paper supplies one natural such map as the minimal closure exposing the closure question; the action-principle classification of admissible maps is supplied by the BCB Lagrangian companion paper [1, §9.5.3], with the physical selection in [1, §9.5.4].

9.2 The natural constitutive map

The simplest choice consistent with (B1) and the dimensional structure of C^u is, in 3-vector form,

$$\rho_c = \partial_t \Phi, \quad J_c = -\nabla \Phi$$

(up to overall normalisation), or equivalently in component form $J_c^i = -\partial_i \Phi$, where ∂_i denotes the cartesian spatial partial derivative. This is the **minimal local linear map** consistent with Lorentz scalarity and dimensional matching: ρ_c and J_c together comprise a four-vector built linearly from first derivatives of a scalar field, which forces the form $\rho_c \propto \partial_t \Phi$, $J_c \propto \nabla \Phi$ up to an overall sign and normalisation. The relative sign is fixed by requiring that the spatial flux opposes the gradient of the field — committed structure flows from regions of higher record density to lower — giving the form above. **Alternative maps introduce either nonlocality (e.g., integral kernels relating ρ_c to Φ over an extended region) or additional degrees of**

freedom (e.g., introducing a second field whose gradient enters J_c). Within strictly local, linear, single-field closures, the map above is forced.

With this choice, the continuity equation $\partial_t \rho_c + \nabla \cdot J_c = s_c$ becomes

$$\partial_t^2 \Phi - \nabla^2 \Phi = s_c$$

i.e. $\square \Phi = s_c$. Combining with (II):

$$s_c = \sigma - m^2 \Phi.$$

This identification carries an immediate consistency requirement. If s_c is interpreted pointwise as a commitment production rate, then $s_c \geq 0$ implies the **pointwise constraint**

$$\sigma(x) \geq m^2 \Phi(x) \text{ for all } x.$$

This is a non-trivial structural condition on admissible source profiles — and, on closer examination, it is in tension with the linear, oscillatory character of (II). We address that tension directly in §9.3.

9.3 Positivity and the meaning of ρ_c

Bottom line up front. Two constitutive maps are admissible at the master-Lagrangian level: the **linear map** (M1) of §9.2, and a **bilinear (energy-density) map** introduced below. Both are bona fide Noether currents of exact symmetries of the master action. The full action-principle framework [1, §9.5.4] selects the bilinear map as the one realising the substrate's actual record current. The present paper develops both, but adopts the linear map throughout — not as a competing physical proposal, but as the **minimal closure** that brings the kinematic-versus-dynamical distinction into sharp focus. A reader interested in which map the substrate's actual record current realises — derived from the master Lagrangian rather than chosen for minimal exposure of the closure question — should consult [1, §9.5].

The constitutive map of §9.2 raises a structural question that must be addressed head-on. In §2, ρ_c was introduced as a *density of commitments* — a count, per unit causal volume, of irreversible facts. Axiom (A2) requires that such a count never decrease: facts enter the bookkeeping but do not leave. Yet the time derivative of a Klein–Gordon field with oscillatory homogeneous solutions is not pointwise non-negative. Under the linear map $\rho_c = \partial_t \Phi$, ρ_c can be locally negative — apparently in conflict with (A2).

The resolution distinguishes two distinct quantities the framework can use:

(P1) Cumulative count over a region. The integrated quantity

$$N(R, t) = \int_R \rho_c d^3x$$

is the total commitment count in region R at time t . Axiom (A2) requires $N(R, t) - N(R, t_0) \geq 0$ for any $t > t_0$: the *integrated* count over any comoving region is non-decreasing. This is a global, not pointwise, statement.

(P2) Local field representation of count density. The local quantity $\rho_c(x) = \partial_t \Phi(x)$ is a field-theoretic representation of count density. It need not be pointwise non-negative; what is required is that its integral over sufficiently large regions recovers (P1). Local oscillations of $\partial_t \Phi$ correspond to short-wavelength fluctuations of the field around its monotonic accumulation — fluctuations that carry no net count when integrated, but represent the fine-grained field-theoretic dynamics of pre-commitment structure resolving into facts.

Under (P1)+(P2), axiom (A2) translates not into pointwise positivity of ρ_c but into a **global monotonicity constraint** on solutions of (II): admissible field configurations are those for which $\int_R \partial_t \Phi d^3x$ is non-negative over comoving regions. Equivalently, Theorem 1's pointwise statement $s_c \geq 0$ must be read in its integrated form $\int_R s_c d^3x \geq 0$; the pointwise constraint $\sigma(x) \geq m^2\Phi(x)$ flagged in §9.2 is correspondingly relaxed to its integrated form $\int_R (\sigma - m^2\Phi) d^3x \geq 0$.

This is the standard distinction in any field theory between a charge or count — which is positivity-constrained on integrated regions — and its field representation, which need not be pointwise constrained. It is unfamiliar in the VERSF context only because the count interpretation of ρ_c is more direct than in conventional gauge theories.

Alternative: bilinear identification. A natural alternative that *does* preserve manifest pointwise positivity is to identify ρ_c with the energy density of Φ . The canonical stress-energy tensor of the field gives:

$$\rho_c = T^{\{00\}} = \frac{1}{2}[(\partial_t \Phi)^2 + (\nabla\Phi)^2 + m^2\Phi^2], J_c = T^{\{i0\}} = -\partial_t \Phi \cdot \nabla\Phi.$$

(In component form: $J_c^i = -\partial_t \Phi \cdot \partial_i \Phi$, matching the §9 sign convention.) This map is bilinear in Φ . It does not violate (B2): assumption (B2) constrains the *equation of motion* for Φ (which remains the linear KG equation), not the constitutive map relating ρ_c to Φ . Under this bilinear map, ρ_c is manifestly ≥ 0 pointwise. The conservation identity for the field's energy current — derived by direct computation on solutions of $(\square+m^2)\Phi = \sigma$ — reads

$$\partial_\mu T^{\{\mu 0\}} = \sigma \cdot \partial_t \Phi,$$

so under the bilinear identification $s_c = +\sigma \cdot \partial_t \Phi$. Pointwise positivity of s_c then requires σ and $\partial_t \Phi$ to have the *same* sign along physical histories — physically: the source is doing positive work on the field, increasing energy density. (Index order matters: writing $\partial_\mu T^{\{\mu 0\}}$ contracts the first index of the symmetric tensor $T^{\{\mu\nu\}}$, with the second index $\nu=0$ labelling the conserved energy slot. Although $T^{\{0\mu\}} = T^{\{\mu 0\}}$ by symmetry, the standard convention for a continuity equation places the conserved-quantity index in the second position.)

The two maps therefore identify *different* conserved quantities with the record content. Under the linear map of §9.2, the conserved current has source σ directly (after the constitutive

identification absorbs $m^2\Phi$ into s_c). Under the bilinear map of §9.3, the conserved current is the field's energy current, with source $\sigma \cdot \partial_t \Phi$. These are not different bookkeeping conventions for the same content: they conserve different things and predict different physics. The remaining question — which map the substrate's record current actually realises — is not settled by Noether's theorem alone, since both maps are conserved currents of exact symmetries.

The action principle does not settle the choice on formal grounds. The BCB Lagrangian companion paper [1, §9.5.3] establishes that *both* the linear and bilinear maps are bona fide Noether currents of distinct *exact* symmetries of the master Lagrangian — the linear map is the shift-current associated with $\tau \rightarrow \tau + \alpha$, and the bilinear map is the energy current associated with time-translation symmetry. Since both symmetries are present, both currents exist, and formal Noether uniqueness does not adjudicate between them. Selection is therefore a *physical-interpretation* question, settled in [1, §9.5.4] in favour of the bilinear map on three grounds: (a) pointwise positivity of ρ_c is preserved under the bilinear map but not under the linear; (b) the bilinear identification's source structure follows directly from the matter-coupling already present in S_{BCB} , whereas the linear map's source-with-gap structure $(\square+m^2)\Phi = \sigma$ requires additional shift-breaking operators to enter the action; (c) the bilinear identification's source rate s_c reproduces the second-law statement $\sigma_{prod} \geq 0$ of the entropy current at the master-Lagrangian level. Under the bilinear (M2) reading, the linear (M1) shift-current still exists and is still conserved, but it tracks a *different* physical quantity — the integrated displacement of τ over the substrate (related to proto-time accumulation [5]) rather than commitment counts.

The present paper adopts the linear map (M1) throughout — as the minimal closure that exposes the closure question. The reason is structural: (M1) is the simplest map that brings the kinematic-versus-dynamical distinction into sharp focus, making it the cleanest vehicle for this paper's purpose, which is to delineate what the kinematic theorems do and do not establish. A reader interested in which map the substrate's actual record current realises — derived from the master Lagrangian rather than chosen for minimal exposure of the closure question — should consult [1, §9.5]. Under the (M1) reading adopted here, pointwise positivity of ρ_c is replaced by integrated positivity (P1)+(P2) as discussed above; under the (M2) reading of [1, §9.5.4], pointwise positivity is preserved at the cost of importing the energy-current machinery and the bit-energy calibration ε_{bit} . Both readings are consistent with everything established in §§4–8 of the present paper; they differ in which physical current is identified with the bookkeeping object C^u .

9.4 Alternative constitutive maps beyond these two

Beyond the linear and bilinear maps of §§9.2–9.3, the BCB companion paper [1, §9.5.3] identifies a third option: the **entropy-current map** (M3), in which C^u is identified directly with the rescaled entropy current $J^{\mu}_s/\varepsilon_{bit}$ of the substrate's R4 sector. This is a hybrid construction — built from a linear-in- τ piece ($\kappa_4 \partial^{\mu}\tau$, structurally like (M1)) plus a carrier-flow piece (s^u_{μ}) — that is not the Noether current of any single elementary symmetry of S_{R4} but a constructed object reflecting the second-law content of the theory directly. Under (M3), the conservation identity acquires a particularly natural reading: $\partial_{\mu}C^{\mu} = s_c$ becomes the rescaled second-law statement $\partial_{\mu}J^{\mu}_s = \sigma_{prod} \geq 0$, with no additional positivity machinery needed. The relation

between (M2) and (M3) at the master-Lagrangian level — and the conditions under which they coincide — is treated in [1, §9.5.3].

Broader still, formal alternatives admit further structure: nonlocal maps (with ρ_c expressed as an integral kernel acting on Φ); multi-field maps (with auxiliary fields entering J_c); or fully nonlinear maps beyond the bilinear case. The classification of admissible constitutive maps within the action-principle setting is addressed in [1, §9.5.3].

10. Propagation and Memory

10.1 Retarded solution

Equation (II) admits the retarded Green's function solution

$$\Phi(x) = \int G_{\text{ret}}(x - x') \sigma(x') d^4x'$$

where G_{ret} is the retarded Green's function of $(\square + m^2)$. The structure of G_{ret} in 3+1 dimensions — a delta-function contribution on the forward light cone plus a Bessel-function tail inside it — is standard [10]. The support of G_{ret} on the forward light cone gives causal propagation.

10.2 Memory kernel — corrected

In 3+1 dimensions, G_{ret} carries:

1. A **delta-function contribution on the light cone**, representing massless propagation of record influence to the boundary of the future.
2. A **Bessel-function tail inside the cone**, representing massive memory of past commitments. The relevant timelike interior structure is proportional to $(m/\tau) J_1(m\tau)$; applying the standard asymptotic $J_n(z) \sim \sqrt{2/(\pi z)} \cos(z - n\pi/2 - \pi/4)$ [11] to $n=1$, the late-time on-axis envelope is

$$K(\tau) \sim \tau^{-3/2} \cdot \cos(m\tau - 3\pi/4) \text{ (large } m\tau, \text{ timelike interval)}$$

The earlier Paper-1 draft of this material gave a τ^{-1} envelope with phase $-\pi/4$. Both were incorrect: the dimensional dependence is $\tau^{-3/2}$ (from the Bessel-tail prefactor), and the phase is $-3\pi/4$ (specific to J_1 ; the $-\pi/4$ phase corresponds to J_0 , not J_1). The correct envelope, as stated above, is $\tau^{-3/2} \cdot \cos(m\tau - 3\pi/4)$, or equivalently $\tau^{-3/2} \cdot \sin(m\tau - \pi/4)$.

10.3 Interpretation

Past commitments persist:

- through massless propagation along the light cone (no decay),
- through massive timelike memory inside the cone (decaying as $\tau^{-3/2}$).

Both are causal. Both are non-trivial: no commitment is forgotten exactly, only attenuated. **This propagation structure reflects how committed information persists and attenuates in the admissible substrate** — the $\tau^{-3/2}$ envelope is the dynamical signature, in the present scalar closure, of the substrate's finite distinguishability budget interacting with the relativistic propagation kernel. In the broader VERSF programme, the same persistence-with-attenuation pattern reappears at multiple scales: it is what makes the past dynamically active rather than merely recorded, and it is the field-theoretic counterpart of the second-law accumulation of §14.

10.4 Empirical status

The empirical signatures of either memory contribution — and their separability from generic relativistic field memory — are open. A dedicated paper on observable signatures of VERSF memory effects is in preparation [7].

11. Compatibility with Lorentz Invariance

11.1 Status

This section establishes that the framework *admits* Lorentz invariance under standard auxiliary assumptions; it does not derive the Lorentz group from VERSF axioms alone. The substrate-level emergence of Lorentz invariance is treated in the proto-time / emergent-Lorentz paper [5].

11.2 Ignatowski-style argument

Assume:

(L1) Spatial homogeneity and isotropy of the substrate's commitment dynamics. **(L2a)** Existence of a finite maximum propagation speed for record influence in *some* inertial frame (a consequence of finite local capacity plus locality). **(L3)** Group structure on the set of inertial transformations. **(L4)** Reciprocity: the transformation $A \rightarrow B$ is the inverse of $B \rightarrow A$.

Under (L1), (L2a), (L3), (L4) — the standard Ignatowski conditions [8, 9] — the unique transformation group between inertial frames is Lorentz, with invariant interval

$$ds^2 = c^2 dt^2 - dx^2$$

where c is the maximum propagation speed of (L2a). The **observer-invariance** of c — that the maximum speed takes the same value in every inertial frame — is then a *consequence* of the resulting group structure, not an independent input:

(L2b) [derived] The maximum propagation speed c is observer-invariant.

This split matters: earlier presentations of this material grouped finiteness and observer-invariance into a single hypothesis, which obscured the fact that observer-invariance is precisely one of the things the Ignatowski argument is *deriving*. With the split, only (L2a) is assumed; (L2b) falls out of the group structure secured by (L1), (L3), (L4).

11.3 What's done, what's not

What the argument delivers: given homogeneity, isotropy, a finite maximum signal speed in some frame, group composition, and reciprocity, the transformation group between inertial frames reduces to Galilean (no maximum speed) or Lorentzian (finite maximum speed). Under (L2a), Lorentz wins, observer-invariance of c follows as (L2b), and our scalar field equation (II) is automatically Lorentz-covariant by construction (B3).

What it does *not* deliver from VERSF axioms alone: (L1) and (L4) require substrate-level homogeneity, isotropy, and reciprocity. Their derivation from the void's ground-state structure is non-trivial and is treated separately.

12. Relation to Gravity (Forward Reference)

The earlier draft of this material claimed a derivation of the Poisson equation $\nabla^2\Phi_g = 4\pi G \rho_m$ from the present framework. We retract that claim. The conservation identity for C^u and the scalar field equation for Φ are necessary but not sufficient for a Poisson-type response law, which requires:

1. **identification of bound record density** ρ_{bound} as the source of a gravitational potential (not all ρ_c contributes — only stable, localised configurations);
2. **a response law** for the substrate's geometric configuration as a function of ρ_{bound} ;
3. **a calibration** relating the substrate response constant to Newton's G ;
4. **a non-relativistic limit** of the underlying tensorial dynamics.

Each is supplied by the **gravity-from-record-density** companion paper [2]. The present paper enters that derivation as the matter-sector kinematic statement, but does not by itself produce the Poisson equation. Critically, the scalar closure of §8 is **insufficient** for gravity: gravity requires a tensor response, not a scalar one. This is one of the principal places where scalar closure is known to be incomplete (cf. §16).

13. Relation to Quantum Probability (Forward Reference)

The pre-commitment sector of VERSF is described by amplitudes, with the Born rule recovered from the geometry of the commitment space. The companion **Double Square Rule** paper [3] establishes:

$$p_i = |c_i|^2 = \exp(-\Delta I_i)$$

where the second equality reformulates the squared-amplitude weight as an informational cost ΔI_i associated with extension to a record-consistent history. The present paper does not re-derive this. The pre-commitment sector enters here through the source σ : at each commitment event, σ receives a stochastic increment whose statistics are governed by the Born rule. The present scalar closure treats σ as stipulated; the Double Square Rule paper supplies its probabilistic structure.

14. Relation to Thermodynamics (Forward Reference)

By (A2), every commitment is irreversible; by (A3), commitments are additive; by (A1), local capacity is finite. These three axioms imply that any well-defined commitment count $S \propto N_{\text{committed}}$ is monotonically non-decreasing along any causal worldline:

$$dS/d\tau \geq 0.$$

This is the structural form of the second law in VERSF: a counting statement about an irreversible bookkeeping, not a statistical-mechanical statement. The detailed identification of S with thermodynamic entropy — including the calibration to Boltzmann's constant and the recovery of fluctuation–dissipation structure — is the subject of the **commitment-capacity** companion paper [4]. The present paper establishes only the structural inequality.

15. Position Within the VERSF Corpus

We close by stating the relationship between this paper and the rest of the programme.

This paper provides:

- the kinematic skeleton — conservation identity (Theorem 1) and uniqueness at scalar first order (Theorem 2),
- the unique scalar dynamical closure (Theorem 3) under stated assumptions,
- the natural constitutive map linking the two,
- the corrected memory kernel,
- the Ignatowski compatibility with Lorentz invariance.

This paper does not provide and does not claim:

- the action-principle derivation of (I) and (II) from a single Lagrangian (BCB Lagrangian paper [1, §9.5]),
- the *selection* of which constitutive map realises the substrate's actual record current (BCB Lagrangian paper [1, §9.5.4] selects the bilinear map; the present paper adopts the linear map as the minimal closure),
- the tensor extension required for gravity (gravity-from-record-density paper [2]),
- the Born rule (Double Square Rule paper [3]),
- the calibrated entropy correspondence (commitment-capacity paper [4]),
- the substrate-level emergence of homogeneity and isotropy (proto-time / emergent-Lorentz paper [5]),
- the microscopic origin of m^2 (BCB Lagrangian paper [1, §9.5.5] and capacity paper [4]),
- experimental separation of the $\tau^{-3/2}$ memory tail from generic relativistic memory ([7], in preparation).

The intended use of this paper is as the kinematic-plus-scalar-dynamics foundation that the rest of the programme rests on. It answers two questions cleanly:

1. Before any closure choice, what bookkeeping must any VERSF dynamics satisfy?
Answer: $\partial_\mu C^\mu = s_c$, uniquely at scalar first order.
2. Under the simplest closure consistent with VERSF axioms, what is the dynamical law?
Answer: $(\square + m^2) \Phi = \sigma$, uniquely under (B1)–(B5).

The full programme requires more — but it cannot require less.

16. Limitations

We list the genuine open problems honestly.

1. **Scalar-only closure.** The dynamics derived here are scalar. The full VERSF programme requires a tensorial extension for gravity and possibly other sectors. The relationship between scalar Φ -dynamics and the tensorial dynamics is not addressed here.
2. **Constitutive map non-uniqueness and the positivity question.** The linear map of §9.2 ($\rho_c = \partial_t \Phi$, $J_c = -\nabla \Phi$) is the simplest local linear choice and is adopted here as the minimal closure exposing the closure question, but it does not realise pointwise positivity of ρ_c — a known structural obstruction addressed in §9.3 by reading axiom (A2) in integrated form. The bilinear (energy-density) map preserves pointwise positivity, identifies s_c differently, and is the map selected by the BCB master Lagrangian [1, §9.5.4]. The third (entropy-current) map of [1, §9.5.3] gives the most direct reading of the conservation identity as the rescaled second law. The classification of admissible constitutive maps is addressed in [1, §9.5.3]; the selection in [1, §9.5.4]. The present paper's adoption of the linear map is a structural choice — minimum closure exposure — not a physical claim about which current the substrate realises.
3. **Mass parameter.** m^2 is stipulated in the present paper. The BCB Lagrangian companion paper [1, §9.5.5] identifies its origin: $m^2 = 0$ in the bare Role-4 action, with non-zero

values arising from shift-breaking structure (a void-scale τ -potential, τ -s mixing, or RG corrections), and the Two-Planck Principle [6] setting an order-of-magnitude $m_\tau \sim \varepsilon_{\text{bit}} \approx 0.010$ eV. The precise value awaits a complete derivation of the τ -potential $V(\tau)$ from void dynamics.

4. **EFT truncation.** The second-order truncation (B4) is justified on standard EFT grounds but is not derived from VERSF axioms alone. Higher-derivative corrections may be physically meaningful at substrate-cutoff scales.
5. **Lorentz emergence imported.** The Ignatowski argument requires homogeneity, isotropy, and reciprocity. Their substrate-level emergence is non-trivial and treated separately.
6. **Memory observables.** The $\tau^{-3/2} \cdot \cos(m\tau - 3\pi/4)$ timelike memory tail is mathematically standard for 3+1D Klein–Gordon; whether it is empirically distinguishable from generic relativistic field memory is an open question.
7. **Action principle deferred.** The present paper produces the field equation and a constitutive map, but does not exhibit the action whose Euler–Lagrange equations they are. That action is the subject of the BCB Lagrangian paper.

None of these limitations undermines Theorems 1–3. They circumscribe what additional work the rest of the corpus does.

17. Conclusion

We have established, under four VERSF axioms — finite distinguishability, irreversibility, additivity, and local coupling — that the record current $C^u = (\rho_c, J_c)$ satisfies the unique scalar first-order conservation identity

$$\partial_\mu C^\mu(x) = s_c(x).$$

We have further shown, under five additional closure assumptions — local scalar representation, linearity, Lorentz scalarity, second-order truncation, and stability — that the unique dynamical equation governing the scalar representation Φ of ρ_c is

$$(\square + m^2) \Phi(x) = \sigma(x).$$

The two equations are independent statements about the same physical system, linked by a constitutive map $J_c[\Phi, \sigma]$ which we exhibit in its natural form. The retarded propagation of (II) supplies a corrected memory kernel decaying as $\tau^{-3/2} \cdot \cos(m\tau - 3\pi/4)$ inside the forward light cone, alongside a delta-function on the cone itself. The framework is compatible with Lorentz invariance under the standard Ignatowski conditions, and provides the matter-sector input needed by the gravity, Born rule, and entropy companion papers.

The contribution of this paper is the clean separation it enforces and the uniqueness theorems it secures within that separation:

Kinematics is forced. Scalar dynamics is uniquely fixed once closure assumptions are specified. The two are independent and linked by closure data. Anything beyond scalar closure — gravity above all — requires the tensorial extension that companion papers provide.

This is the strongest claim the framework can honestly support at this level, and it is — we argue — substantial enough to anchor the rest of the VERSF programme.

Acknowledgements

This paper merges and extends earlier drafts on the kinematic conservation structure and the scalar dynamical closure. Companion papers in the VERSF Theoretical Physics Programme — on the BCB Lagrangian, gravity-from-record-density, the Double Square Rule, commitment capacity, proto-time and emergent Lorentz invariance, and observable memory signatures — supply the closure structure referenced throughout.

References

Companion papers in the VERSF Theoretical Physics Programme

- [1] Taylor, K. *The BCB Lagrangian and the Action-Principle Representation of VERSF*. VERSF Programme working paper, AIDA Institute.
- [2] Taylor, K. *Gravity from Record Density: Geometric Response in the VERSF Substrate*. VERSF Programme working paper, AIDA Institute.
- [3] Taylor, K. *The Double Square Rule: Derivation of the Born Rule from Informational Geometry*. VERSF Programme working paper, AIDA Institute.
- [4] Taylor, K. *Commitment Capacity and the Structural Origin of Entropy*. VERSF Programme working paper, AIDA Institute.
- [5] Taylor, K. *Proto-Time, Physical Time, and Emergent Lorentz Invariance*. VERSF Programme working paper, AIDA Institute.
- [6] Taylor, K. *The Two-Planck Principle and the Cosmological Constant*. VERSF Programme working paper, AIDA Institute.
- [7] Taylor, K. *Observable Signatures of VERSF Memory Effects*. VERSF Programme working paper, in preparation.

[13] Taylor, K. *The Admissibility Closure Framework: Hilbert Structure and Amplitude Apparatus from VERSF Axioms*. VERSF Programme working paper, AIDA Institute. — Establishes the pre-commitment Hilbert-space and amplitude structure from which the scalar field Φ of the present paper inherits its scalar character.

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