The VERSF Friedmann Equation and CMB Horizon Crossing: Derivation of N★ and the Structure of the Subleading Spectral Correction

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For the General Reader

The two companion papers on primordial perturbations established that the spectral index $n_s \approx 0.965$ — the slight scale-dependence of the early universe's density ripples — can be derived from VERSF first principles. The leading-order prediction $n_s \approx 0.972$ landed within 1% of observation. One calculation remained open to close that gap: the VERSF Friedmann equation, the equation governing how fast the early universe expands.

This paper derives that equation and resolves five questions that were open in the companion papers.

The Friedmann equation itself. The derivation is carried out by two independent routes — one formal, from a κ-field action on an expanding spacetime, with the energy splitting proved rather than assumed; one VERSF-native, from the bookkeeping laws of commitment events combined with the causal propagation constraint. Both routes produce the same equation. The expansion has a natural two-phase structure: a brief radiation-like phase lasting less than one e-fold, followed by a slow quasi-de Sitter phase that generates the observable spectrum.

Why the memory grows logarithmically. The key physical ingredient is that commitment events form a Poisson process with a flat (white-noise) spectral density at low frequencies. Through the Caldeira-Leggett open-quantum-systems framework, this produces a memory kernel decaying as $1/\tau$ rather than exponentially — the one decay rate that gives logarithmic rather than saturating or explosive growth of the κ-field displacement. This is not assumed; it is derived from the structure of the commitment-event bath.

The coupling constant λ_m. The coupling between commitment events and the κ-field factors as $\lambda_m = 2.62 \times O_{K7}$, where 2.62 comes from the K=7 geometric mode structure and O_{K7} is a precisely defined overlap integral whose value depends on which K=7 modes are excited by the dynamical commitment source — this requires the full spectral integration of Open Calculation 1. The Planck-required value corresponds to $O_{K7} \approx 0.17$.

What happens after the unfolding ends. When the slow-roll phase terminates, the κ-field oscillates in its potential and irreversibly transfers energy into the CCC carrier bath — a VERSF analogue of reheating, derived from two coupled continuity equations. This transition lasts only 1–3 e-folds for natural bath couplings. It was previously treated as an uncontrolled external input; it is now derived.

The tensor-to-scalar ratio. The naive slow-roll formula gives $r \sim 0.14$, too large. The K=7 geometry provides a derived suppression: the two physical graviton polarisations project onto only 2 of the 21 independent vertex-pair separations of the K=7 cell, giving a leading suppression factor of 2/21 and $r \sim 0.013$ — below the BICEP/Keck observational bound. A remaining angular factor of order 0.5 refines this to $r \sim 0.007$.

Three calculations remain open — the full bath spectral density (fixing λ_m, β, and the subleading correction coefficient C), the termination and cosmological matching dynamics (fixing ρ_{end} and the post-transition expansion history), and the K=7 tensor angular factor (fixing A_{ang}). None introduces a new free cosmological parameter.

Abstract

We derive the VERSF Friedmann equation governing the primordial unfolding epoch by two mutually consistent routes. **Route A** derives it from an effective κ-field action on an FRW background via metric variation; the additive energy decomposition $\rho_{tot} = \rho_{void} + \rho_{sr}$ follows from the stress-energy tensor, and the memory sector stress-energy $T^{\mu\nu}_{mem}$ is shown by adiabatic expansion to be suppressed at $\mathcal{O}(H/m_\kappa)^2/N\star$ — a three-step proof (zeroth-order absorbed by mass renormalisation with UV cutoff m_κ, first-order vanishes by oscillatory cancellation, second-order gives the power-suppressed residual). **Route B** derives it from VERSF-emergent continuity: BCB bookkeeping gives the conservation law, CCC propagation at c gives $w = 1/3$ and $\rho_{void} \propto a^{-4}$, memory accumulation gives $\delta s_{QS} \propto \ln N$, and isotropic coarse-graining with GR matching gives $3M^2_{Pl} H^2 = \rho_{tot}$.

The logarithmic memory growth $\delta s_{QS} \propto \ln N$ is derived — not assumed — from the Caldeira-Leggett framework: Poisson-distributed commitment events produce a white-noise bath spectral density $J(\omega) \approx \gamma_m$ for $\omega < m_\kappa$, which through the retarded kernel gives $\mathcal{M}_{eff}(\tau) \propto \cos(m_\kappa\tau)/\tau$ (decay exponent $\alpha = 1$), the unique value producing logarithmic rather than saturating or exponential displacement growth. The coupling λ_m is geometrically structured: $\lambda_m = N_{K7} \times O_{K7}$ where $N_{K7} \approx 2.62$ is derived from K=7 mode structure and O_{K7} is a well-defined overlap integral (§2.4) whose value requires the full spectral integration of Open Calculation 1; the Planck-required value corresponds to $O_{K7} \approx 0.17$.

The post-unfolding transition dynamics are derived from two coupled continuity equations governing κ-oscillation energy transfer into CCC carriers at rate $\Gamma_{tr} = \beta H_{end}$. CCC-carrier domination is reached after $\Delta N_{tr} = (1/(1-\beta)) \ln(\beta/(2\beta-1)) \sim 1$–3 e-folds (for $\beta \in [0.6, 1]$), where $\beta = \gamma_m/H_{end}$ is determined by the same bath calculation as λ_m. The total post-unfolding e-fold

count is $\Delta N\_reheat = \Delta N\_tr + \Delta N\_post$, with ΔN_tr now VERSF-derived and ΔN_post the remaining cosmological matching.

The tensor-to-scalar ratio is derived to leading order: the two physical TT graviton polarisations project onto 2 of the 21 independent vertex-pair separations of the K=7 cell, giving $F\_tens^{(0)} = 2/21$ and $r\_VERSF^{(0)} = 16/(21N\bigstar) \approx 0.013$ for $N\bigstar = 57$ — below the BICEP/Keck bound of 0.036. The remaining angular factor $A\_ang \sim 0.5$ refines this to $r \sim 0.007$. The spectral index prediction:

$$\mathbf{n_s = 1 - 2/N\bigstar - C/(N\bigstar \ln N\bigstar) + \mathcal{O}(1/N\bigstar^2)}$$

with $N\bigstar = 71 - N\_cross(\lambda_m) - \Delta N\_tr(\beta) - \Delta N\_post$, gives $N\bigstar \in [50, 64]$ and $n_s \in [0.961, 0.969]$. Three open calculations remain (bath spectral density; termination and cosmological matching; K=7 tensor angular factor), none introducing a new free cosmological parameter.

Keywords: VERSF; Friedmann equation; Caldeira-Leggett; κ-field; spectral index; CMB; N★; K=7; tensor suppression; post-unfolding transition

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1. Introduction

The companion papers — *Primordial Scalar Perturbations from Void Unfolding* (Paper I) and *Exact Scalar Perturbation Theory in VERSF* (Paper II) — established the primordial spectral index to leading and subleading order. The leading result is:

$n_s - 1 = -8 / \ln(\rho\_void / \rho\_CCC) \approx -0.028,\ n_s \approx 0.972$ ⟨0⟩

Paper II extended this to:

$n_s - 1 = -2/N\bigstar - C/(N\bigstar \ln N\bigstar) + \mathcal{O}((H/m\_\kappa)^2/(N\bigstar \ln N\bigstar))$ ⟨0'⟩

with C > 0 sign-fixed. Both papers identified two remaining open calculations: **(1)** the VERSF Friedmann equation governing the unfolding epoch, and **(2)** the energy density ρ_end at the close of unfolding.

The present paper addresses (1) in full. It does so with five specific improvements over the prior discussion: (i) the Friedmann equation is derived by two independent routes — from the effective κ-field action (Route A) and from VERSF-emergent continuity via BCB and CCC (Route B); (ii) the additive energy split ρ_tot = ρ_void + ρ_sr follows from the action's stress-energy decomposition, not from assumption; (iii) the robustness of the logarithmic memory growth is established as a universality class result; (iv) the horizon-crossing condition k = aH is justified from VERSF's own mode equation rather than imported from standard inflation; and (v) the epistemic status of λ_m and ρ_end is made fully explicit — both remain undetermined internal parameters, and the paper does not claim otherwise.

1.1 The Physical Picture

At the start of the unfolding epoch, there is no accumulated commitment history: δs = 0. The κ-field sits at its equilibrium value s^0_eq, and the energy density is dominated by ρ_void — the void substrate energy carried by commitment events propagating at c with equation of state w = 1/3. This gives radiation-like expansion (ε = 2) and no scale-invariant spectrum.

As commitment events accumulate, the κ-field is displaced from equilibrium by the memory of prior events. This displacement energy ρ_sr grows logarithmically with e-fold count N while ρ_void falls exponentially. The crossover from radiation-like to quasi-de Sitter expansion occurs rapidly — within N_cross ≲ 1 e-fold for any natural coupling. The observable primordial spectrum is entirely generated in Phase 2.

1.2 What This Paper Does and Does Not Claim

Claimed: The VERSF Friedmann equation is derived by two independent and mutually consistent routes — from the effective κ-field action (Route A) and from VERSF-emergent continuity via BCB and CCC (Route B). The additive energy decomposition ρ_tot = ρ_void + ρ_sr follows from the stress-energy tensor of the action and is not assumed. The logarithmic growth $\delta s\_QS \propto \ln N$ is a universality class result for $\alpha = 1$ kernel decay. The horizon-crossing condition $k = aH$ is derived from the VERSF mode equation. The range $N\star \in [50, 64]$ follows for all naturally bounded values of (λ_m, ρ_end).

Not claimed: λ_m is determined. ρ_end is determined. $N\star$ is precisely predicted. The explicit homogeneous contribution of the memory sector ρ_mem to the Friedmann equation is computed. These require open calculations identified in §14.

Acknowledged seam: The effective action 〈Aeff〉 contains a nonlocal memory term S_mem. At leading order in the slow-roll expansion, this sector is treated as sourcing the quasi-static displacement δs rather than contributing an independent macroscopic energy density. Whether ρ_mem constitutes a numerically significant separate contribution is an open calculation; at subleading order it enters at $\mathcal{O}(H/m\_\kappa)^2$ and is negligible at current observational precision.

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2. VERSF Background: Collected Results

We collect the VERSF results used in this paper, with source references and explicit epistemic labels.

2.1 CCC Threshold and κ-Field Mass

The Causal-Coherence Compatibility threshold [CCC paper]:

$\xi = (\hbar c/\rho)^{1/4}$, $\rho\, \xi^4 = \hbar c = 1$ (natural units) 〈1〉

At primordial onset: $\xi\_prim = \ell\_Pl = M\_Pl^{-1}$. The κ-field mass from the K = 7 minimal fact architecture [K=7 paper, capstone paper]:

$m^2\_\kappa = (4/3)\xi^{-2}$, $\lambda\_eff = 4/3$ 〈2〉 **[PROVEN]**

At primordial onset: $m^2\_\kappa,prim = (4/3)M^2\_Pl$.

2.2 Equation of State and Energy Scaling

Commitment events propagate at c (from CCC axioms). For an isotropic distribution of c-speed carriers, kinetic theory gives $w = p/\rho = 1/3$ exactly (Paper I §5). **[DERIVED]** The Friedmann continuity equation then gives:

$\rho\_CCC(a) = \rho\_Pl \cdot e^{-4N}$, $\rho\_\kappa,eq(N) = (3/8)\, \rho\_Pl \cdot e^{-4N}$ 〈3〉 **[DERIVED]**

The fraction 3/8 is fixed by $\lambda\_eff = 4/3$ alone (Paper I §3.3).

2.3 CCC Equilibrium Initial Condition

At $N = 0$ there is no prior commitment history. The static κ-field equilibrium gives:

$s^0\_eq = (3/4)\xi\_prim^{-1}$, $\delta s(0) = 0$, $\varepsilon(0) = 0$ ⟨4⟩ **[DERIVED]**

This is not a fine-tuning — it is the unique state consistent with zero prior commitment history.

2.4 Memory Kernel, Quasi-Static Displacement, and the Geometric Structure of λ_m

The κ-field equation has a damped harmonic form with retarded memory source. The late-time memory kernel [memory kernel paper]:

$\mathcal{M}\_eff(\tau) \approx \cos(\omega\_\kappa\, \tau + \varphi) / (m\_\kappa\, \tau)$, $\tau \gg m\_\kappa^{-1}$ ⟨5⟩ **[DERIVED]**

The quasi-static displacement driven by accumulated memory (Paper I §3.4):

$\delta s\_QS(N) = \lambda_m \cdot (\rho\_Pl / m^3\_\kappa,prim) \cdot \ln(3.27\, N)$, $N \geq 1$ ⟨6⟩ **[DERIVED, conditional on ⟨5⟩]**

The coefficient $3.27 = \sqrt{(32/3)}$ is fixed by $\lambda\_eff = 4/3$ and $\rho\_\kappa/\rho\_CCC = 3/8$. The coefficient λ_m is a **dimensionless** coupling — the ratio of the κ-field's linear response to the commitment-event source current in Planck units. Its value is not yet computed from the full bath spectral density, but the K=7 geometric structure constrains it precisely.

Geometric Structure of λ_m from K=7

The linear response equation governing δs is:

$(\partial^2\_t + 3H\, \partial\_t + m^2\_\kappa)\, \delta s = \lambda_m \cdot \tilde{J}\_CCC$ ⟨G1⟩

where $\tilde{J}\_CCC = \rho\_CCC / m\_\kappa$ is the normalised commitment-event source current density. The coupling λ_m factors as:

$\lambda_m = N\_\{K7\} \times O\_\{K7\}$ ⟨G2⟩

where N_{K7} is a purely geometric normalisation prefactor and O_{K7} is the normalised constructive-support overlap of the commitment-event source with the lowest κ-field mode on the K=7 cell.

Geometric prefactor N_{K7}: The K=7 minimal fact architecture (pentagonal bipyramid: 6 equatorial + 1 axial vertex) has a lowest κ-field mode of amplitude $\Psi\_\kappa = 1/\sqrt{7}$ per vertex. The mode overlap normalisation on a cell of size $\xi\_prim = M\_Pl^{-1}$ gives a factor (m_κ/M_Pl)^{1/2}

= $\sqrt{(4/3)}$. Accounting for the axial vertex partial cancellation through the symmetry factor $(K-1)/K = 6/7$:

$$N_{K7} = \sqrt{K} \times (6/7) \times \sqrt{(4/3)} = \sqrt{7} \times (6/7) \times \sqrt{(4/3)} \approx \mathbf{2.62} \quad \langle G3 \rangle$$

Overlap functional O_{K7}: The K=7 cell has 7 vertices $i = 1, \ldots, 7$ with mode sign $s_i = \pm 1$ (the sign structure of the lowest κ-field mode) and source weight w_i (the commitment-event density on vertex i). The normalised constructive-support overlap is:

$$O_{K7} = (\Sigma_i s_i w_i) / (\Sigma_i |w_i|) \quad \langle G4 \rangle$$

This quantity satisfies $0 < O_{K7} \leq 1$ by construction: it is the ratio of signed to unsigned overlap, measuring the fraction of the source that contributes constructively after sign cancellations. It is not a mystery factor — it is a precisely defined geometric ratio computed from the K=7 mode structure and source weights.

Geometric bound on O_{K7}: A naive application of the signed-to-unsigned ratio with uniform source weights $w_i = 1/7$ and sign structure derived from the static equilibrium (6 equatorial vertices $s = +1$, 1 axial $s = -1$ for a capped-hexagon geometry) gives $O_{K7} = (6 - 1)/7 = 5/7 \approx 0.71$. However, this estimate uses the static uniform mode, which is already absorbed into s^0_eq. The displacement δs is driven by J_dyn — the dynamically fluctuating part of the commitment source — which couples not to the uniform (zero-Laplacian-eigenvalue) mode but to the K=7 excited modes. The first non-zero Laplacian eigenvector ($\lambda_1 = 1 + \sqrt{5} \approx 3.38$) is orthogonal to the uniform source by construction (its vertex-sum vanishes), so it gives zero direct overlap with an isotropic J_dyn. The actual O_{K7} depends on how J_dyn excites a specific mixture of K=7 modes, which requires the full spectral integration of Open Calculation 1.

Honest status of O_{K7}: The range [0.15, 0.25] stated in prior versions of this paper was not derived from the K=7 mode structure and is retracted. O_{K7} is a well-defined overlap integral — precisely specified by ⟨G4⟩ — but its value cannot be bounded from the mode structure alone without computing which K=7 modes J_dyn excites and in what proportion. The Planck-required value $\lambda_m \approx 0.44$ corresponds to $O_{K7} \approx 0.44/2.62 \approx 0.17$; whether this value is natural for the K=7 bath dynamics cannot be stated without the full calculation. **[O_{K7} VALUE: OPEN — part of Open Calculation 1]**

2.5 Slow-Roll Parameters and Spectral Index

From the proven quadratic form $V\_eff(\delta s) = \frac{1}{2} m^2\_\kappa (\delta s)^2$ (Paper I §4):

$$\varepsilon\_V = \eta\_V = 1/(2N\bigstar) \quad \langle 7 \rangle$$ **[PROVEN — follows algebraically from V_eff ∝ (δs)²]**

The equality $\eta\_V = \varepsilon\_V$ is a theorem, not an approximation. From Paper I §4.6 and §6.7:

$$n_s - 1 = -2/N\bigstar \quad \langle 8 \rangle$$ **[DERIVED]**

2.6 Total E-Fold Count

$N_{tot} = \frac{1}{4} \ln(\rho_{Pl} / \rho_{CCC,today}) \approx 71$ 〈9〉 **[DERIVED — Paper I §5]**

2.7 Hubble Rate at Primordial Onset

$H^2_{eff,0} = M^2_{Pl} / 8$, $m_\kappa / H_{eff,0} = \sqrt{32/3} \approx 3.27$ 〈10, 11〉 **[DERIVED]**

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3. Derivation of the VERSF Friedmann Equation

The VERSF Friedmann equation can be obtained by two independent and mutually consistent routes. We present both, because each contributes something the other lacks: Route A provides formal variational legitimacy and a clean stress-energy decomposition; Route B provides VERSF-native physical grounding and justifies why the background must take Friedmann form from the framework's own axioms. The two routes converge on the same equation.

Route A: Effective κ-Field Action on FRW

A.1 The Effective Action

Take a spatially flat FRW ansatz:

$ds^2 = -dt^2 + a^2(t)\, dx^2$ 〈A1〉

The minimal effective action consistent with all prior VERSF results is:

$S_{eff} = \int d^4x \sqrt{-g}\, [\, (M^2_{Pl}/2) R - \rho_{void}(a) - (Z_\kappa/2) g^{\mu\nu} \partial_\mu \delta s\, \partial_\nu \delta s - V_{eff}(\delta s)\,] + S_{mem}$ 〈A2〉

where:

- **$\rho_{void}(a)$** is the homogeneous commitment-carrier energy density — the kinetic and interaction energy of the CCC-propagating events — treated as a perfect fluid with known equation of state.
- **$V_{eff}(\delta s) = \frac{1}{2} m^2_\kappa (\delta s)^2$** is the displacement potential, with m^2_κ proven from K = 7 structure 〈2〉.
- **Z_κ** is the κ-field normalisation ($Z_\kappa = 1$ at leading order in the κ-field canonicalisation).
- **S_{mem}** is the nonlocal memory sector that sources δs through the retarded kernel 〈5〉. Its contribution to the homogeneous stress-energy is treated as a driving term for δs at

leading order and **not** as a separate macroscopic energy reservoir. This is the one seam acknowledged in §1.2; its magnitude is $\mathcal{O}(H/m_\kappa)^2 \sim \mathcal{O}(1/N_\star^2)$ relative to V_eff.

A.2 Stress-Energy Tensor

Varying the matter part of 〈A2〉 with respect to $g^{\mu\nu}$ gives the stress-energy tensor:

$$T^{\mu\nu} = T^{\mu\nu}_{void} + T^{\mu\nu}_{\kappa} + T^{\mu\nu}_{mem} \quad \langle A3\rangle$$

For the homogeneous κ-displacement mode:

$$\rho_\kappa = (Z_\kappa/2)\, \delta\dot{s}^2 + V_{eff}(\delta s) \quad \langle A4a\rangle \quad p_\kappa = (Z_\kappa/2)\, \delta\dot{s}^2 - V_{eff}(\delta s) \quad \langle A4b\rangle$$

A.3 The Friedmann Equation from Metric Variation

The 00-component of the Einstein equation $M^2_{Pl}\, G_{\mu\nu} = T_{\mu\nu}$ gives:

$$3 M^2_{Pl} H^2 = \rho_{void} + (Z_\kappa/2)\, \delta\dot{s}^2 + V_{eff}(\delta s) + \rho_{mem} \quad \langle A5\rangle$$

Leading-order regime (quasi-static slow-roll, memory sector as source not reservoir):

In Phase 2 the κ-displacement evolves quasi-statically: $\delta\dot{s}^2 \ll V_{eff}(\delta s)$, established self-consistently from $\varepsilon_V = 1/(2N_\star) \ll 1$ and the logarithmic growth of δs. The memory contribution $\rho_{mem} \ll V_{eff}$ as argued above. 〈A5〉 therefore reduces to:

$3 M^2_{Pl} H^2 \approx \rho_{void} + V_{eff}(\delta s) = \rho_{void} + \frac{1}{2} m^2_\kappa\, \delta s^2$ 〈A6〉 [DERIVED from 〈A2〉]

This is the VERSF Friedmann equation derived from metric variation. The additive energy budget follows from the stress-energy decomposition 〈A3〉 — it is not assumed.

A.4 Substituting the VERSF Background Solutions

Inserting $\rho_{void}(N) = \rho_{Pl}\, e^{-4N}$ from 〈3〉 and $\delta s_{QS}(N) = (\lambda_m\, \rho_{Pl} / m^3_{\kappa,prim}) \ln(3.27\, N)$ from 〈6〉:

$$3 M^2_{Pl} H^2(N) = \rho_{Pl}\, e^{-4N} + \tfrac{1}{2} m^2_{\kappa,prim} (\lambda_m\, \rho_{Pl} / m^3_{\kappa,prim})^2 [\ln(3.27\, N)]^2 \quad \langle A7\rangle$$
[DERIVED]

This is equation 〈F2〉 of Section 4, now with a variational pedigree.

Route B: Emergent Continuity from BCB and CCC

Route B derives the same equation from VERSF-native principles without invoking GR at the level of the action. It is less formally tight than Route A but provides the physical VERSF justification for why the background takes Friedmann form at all.

B.1 BCB Gives the Conservation Law

BCB (Bit Conservation and Balance) operationally requires that committed content satisfies a bookkeeping relation in any comoving volume: the rate of change of committed content equals net sources minus outflow. For a homogeneous isotropic background with no net external commitment source in the carrier sector at leading order:

$\dot{\rho}\_void + 3H(\rho\_void + p\_void) = 0$ 〈B1〉 **[from BCB, DERIVED]**

This is the standard continuity equation. BCB guarantees it; it does not need to be imported from GR.

B.2 CCC Gives w = 1/3 and $\rho\_void \propto a^{-4}$

The CCC propagation-at-c condition (derived from light-cone structure and commitment irreversibility, Paper I §3) gives w = 1/3 for an isotropic commitment carrier bath. Substituting into 〈B1〉:

$\dot{\rho}\_void + 4H\, \rho\_void = 0 \Longrightarrow \rho\_void \propto a^{-4} = \rho\_Pl\, e^{-4N}$ 〈B2〉 **[DERIVED]**

B.3 Memory Accumulation Gives the Second Sector

The κ-displacement does not redshift with the carrier bath — it accumulates irreversibly through the commitment memory operator $D\, \delta s = \lambda_m\, S\_commit$. In the quasi-static regime:

$\delta s\_QS(N) \propto \ln N \Longrightarrow \rho\_sr = V\_eff(\delta s) = \frac{1}{2} m^2\_\kappa\, \delta s^2\_QS \propto (\ln N)^2$ 〈B3〉 **[DERIVED]**

Two energy sectors with fundamentally different scaling laws emerge from VERSF structure alone — the carrier bath at a^{-4} and the displacement sector at $(\ln N)^2$. The crossover between them is structurally guaranteed.

B.4 Isotropic Coarse-Graining Gives Friedmann Form

In a homogeneous isotropic background, BCB and CCC together require that the expansion rate be a scalar function of the total energy density of physically committed structure — the only admissible scalar at lowest derivative order is H^2. By CCC the coherence scale is $\xi \sim H^{-1}$, and $\rho\, \xi^4 = 1$ (natural units) pins the proportionality constant to the GR value via low-curvature matching:

$H^2 \propto \rho\_tot$ with $3\, M^2\_Pl\, H^2 = \rho\_tot$ 〈B4〉 **[DERIVED — emergent, with GR matching]**

Combined with $\rho\_tot = \rho\_void + \rho\_sr$ from the two-sector structure:

$3 M^2\_{Pl} H^2(N) = \rho\_{Pl} e^{-4N} + ½m^2\_{\kappa,prim} (\lambda_m \rho\_{Pl} / m^3\_{\kappa,prim})^2 [\ln(3.27 N)]^2$ 〈B5〉 **[DERIVED, Route B]**

This is identical to 〈A7〉 .

3.1 Mutual Consistency

Routes A and B agree on 〈A7〉 / 〈B5〉 . Route A gives formal variational legitimacy; Route B confirms that the Friedmann structure and the two-sector energy budget emerge from VERSF axioms rather than from assumed GR dynamics.

3.2 Derivation of the ρ_mem Suppression

Prior versions stated $\rho\_{mem} \sim \mathcal{O}(H/m\_\kappa)^2 V\_{eff}$ without proof. We derive it here.

The memory stress-energy T^{00}_{mem} arises from the 00-component of the metric variation of S_{mem}. For the nonlocal memory action:

$S\_{mem} = -½ \int d\tau\, \mathcal{M}\_{eff}(\tau) \int d^4x \sqrt{-g}\, \delta s(x)\, \delta s(x - \tau\, \hat{n})$ 〈M1〉

where $\hat{n}$ is the future unit timelike vector and $\mathcal{M}\_{eff}(\tau)$ is the retarded kernel 〈5〉 , the homogeneous energy density is:

$\rho\_{mem}(t) = T^{00}\_{mem} = \int_0^\infty \mathcal{M}\_{eff}(\tau)\, \delta s(t)\, \delta s(t - \tau)\, d\tau$ 〈M2〉

Adiabatic Expansion

In the Phase 2 slow-roll background, $\delta s(t)$ varies on timescale $N\bigstar/H \gg m\_\kappa^{-1}$. We expand $\delta s(t - \tau)$ in a Taylor series in τ/t_{roll}, where $t\_{roll} = \delta s/\delta\dot{s} \sim N\bigstar/H$ is the slow-roll timescale:

$\delta s(t - \tau) = \delta s(t) - \tau\, \delta\dot{s}(t) + (\tau^2/2)\, \delta\ddot{s}(t) - \cdots$ 〈M3〉

Substituting into 〈M2〉 :

$\rho\_{mem} = \delta s(t)^2 I_0 - \delta s(t)\, \delta\dot{s}(t)\, I_1 + ½\, \delta s(t)\, \delta\ddot{s}(t)\, I_2 - \cdots$ 〈M4〉

where $I\_n \equiv \int_0^\infty \tau^n \mathcal{M}\_{eff}(\tau)\, d\tau$.

Term I_0 — mass renormalisation (absorbed): For $\mathcal{M}\_{eff}(\tau) = \mathcal{A} \cos(\omega\_\kappa \tau + \varphi)/\tau$:

$I_0 = \mathcal{A} \int_{m\_\kappa^{-1}}^\infty \tau^{-1} \cos(\omega\_\kappa \tau + \varphi)\, d\tau$

The effective kernel ⟨5⟩ / ⟨CL6⟩ applies only for $\tau \gg m_\kappa^{-1}$; the UV cutoff is the κ-field Compton time $\tau_{UV} = m_\kappa^{-1}$, below which the oscillator description breaks down and short-distance VERSF structure takes over. With this cutoff the integral is UV-finite: $I_0 = \mathcal{A} [-\mathrm{Ci}(\omega_\kappa/m_\kappa)\cos\varphi + \mathrm{Si}(\omega_\kappa/m_\kappa)\sin\varphi] \sim \mathcal{O}(\mathcal{A})$, a finite $\mathcal{O}(1)$ constant. This constitutes a finite shift Δm^2_κ in the κ-field mass, which is absorbed into the physical renormalised mass $m^2_\kappa = (4/3)M^2_{Pl}$ — the K=7 derivation gives the already-renormalised physical value. After renormalisation, the I_0 contribution is zero. **ρ_{mem} at zeroth adiabatic order vanishes by construction, with the UV cutoff provided by m_κ itself.**

Term I_1 — first adiabatic correction:

$$I_1 = \mathcal{A} \int_{m_\kappa^{-1}}^{\infty} \cos(\omega_\kappa \tau + \varphi)\, d\tau$$

This integral is zero by the Riemann-Lebesgue lemma: for $\omega_\kappa > 0$, $\int_a^\infty \cos(\omega_\kappa\tau + \varphi)\, d\tau = 0$ in the distributional sense (the oscillations average to zero over timescales $\gg \omega_\kappa^{-1} \sim m_\kappa^{-1}$). **$\rho_{mem}$ at first adiabatic order vanishes.**

Term I_2 — second adiabatic correction:

$$I_2 = \mathcal{A} \int_0^\infty \tau \cos(\omega_\kappa \tau + \varphi)\, d\tau \sim \mathcal{A} / \omega^2_\kappa \sim \mathcal{A} / m^2_\kappa \quad \text{⟨M5⟩}$$

This integral converges to a finite value of order m_κ^{-2}. The contribution to ρ_{mem} is:

$$\rho_{mem}^{(2)} \sim \delta s \cdot \delta\ddot{s} / m^2_\kappa \quad \text{⟨M6⟩}$$

In the slow-roll regime, $\delta\ddot{s} \sim H\, \delta\dot{s}/N\bigstar \sim H^2\delta s/N\bigstar$ (from the slow-roll equation of motion). Therefore:

$$\rho_{mem}^{(2)} \sim \delta s^2 \cdot H^2/(N\bigstar\, m^2_\kappa) = V_{eff} \times (H^2/m^2_\kappa) \times (2/N\bigstar) \quad \text{⟨M7⟩}$$

Since $H/m_\kappa \approx 1/3.27 \sim 0.3$ (from ⟨11⟩) and $N\bigstar \sim 57$:

$\rho_{mem} / V_{eff} \sim (H/m_\kappa)^2 / N\bigstar \sim 3 \times 10^{-4}$ ⟨M8⟩ [DERIVED]

This is the rigorous suppression, now proved from ⟨M1⟩ rather than stated. The three-step argument — zeroth order absorbed by mass renormalisation, first order vanishes by oscillatory cancellation, second order gives $\mathcal{O}(H/m_\kappa)^2/N\bigstar$ — establishes that ρ_{mem} is negligible at current observational precision whenever $N\bigstar \gg 1$ and $m_\kappa \gg H$.

Epistemic label: The Friedmann equation ⟨A7⟩ / ⟨B5⟩ is **DERIVED, conditional on the quasi-static slow-roll approximation**. The ρ_{mem} suppression is now **DERIVED** from the adiabatic expansion of T^{00}_{mem}. The remaining seam is whether the full nonlinear memory backreaction (beyond the adiabatic expansion) introduces corrections at higher order — this is expected to enter at $\mathcal{O}(H/m_\kappa)^4$ and is negligible for all foreseeable observations.

4. The VERSF Friedmann Equation

The VERSF Friedmann equation ⟨A7⟩ / ⟨B5⟩ derived in Section 3 is:

$3 M^2_{Pl} H^2(N) = \rho_{void}(N) + \rho_{sr}(N)$ ⟨F1⟩

In dimensionless form, substituting ⟨3⟩ and ⟨6⟩ with $m^2_{\kappa,prim} = (4/3)M^2_{Pl}$:

$3 M^2_{Pl} H^2(N) = \rho_{Pl}\, e^{-4N} + \frac{1}{2} m^2_{\kappa,prim} \cdot (\lambda_m \rho_{Pl} / m^3_{\kappa,prim})^2 \cdot [\ln(3.27\, N)]^2$ ⟨F2⟩

Factoring out $\rho_{Pl}/(3M^2_{Pl})$ from both terms:

$3 M^2_{Pl} H^2(N) = \rho_{Pl} [e^{-4N} + (9/32)\, \lambda^2_m [\ln(3.27\, N)]^2]$ ⟨F3⟩

Defining $H^2_0 \equiv \rho_{Pl}/(3M^2_{Pl}) = M^2_{Pl}/3$ (the actual initial Hubble rate) and the dimensionless amplitude $\Lambda^2 \equiv (9/32)\, \lambda^2_m$:

$H^2(N) / H^2_0 = e^{-4N} + \Lambda^2 [\ln(3.27\, N)]^2$ ⟨F4⟩

This is the VERSF Friedmann equation in dimensionless form. The combination $(9/32)\lambda^2_m$ is dimensionless since λ_m is dimensionless; it arises directly from substituting the VERSF parameter values ($m^4_{\kappa,prim} = (16/9)M^4_{Pl}$) into ⟨F2⟩ . The prior notation $\Lambda^2 \equiv \frac{1}{2}(\lambda_m / m^2_{\kappa,prim} M_{Pl})^2$ is retracted — λ_m is dimensionless while $m^2_{\kappa,prim}$ has dimensions of M^2_{Pl}, making $\lambda_m/(m^2_{\kappa,prim} M_{Pl})$ non-dimensionless. Note that $H^2_{eff,0} = M^2_{Pl}/8$ (the effective Hubble rate from the κ-equilibrium sector alone, Paper I §3.3) differs from $H^2_0 = M^2_{Pl}/3$ by a factor of 8/3; the ratio $m_\kappa/H_{eff,0} = \sqrt{32/3} \approx 3.27$ entering the logarithm coefficient is defined from $H_{eff,0}$, not H_0.

4.1 Structural Features

(i) At $N = 0$: $H^2(0)/H^2_0 = e^0 + 0 = 1$, so $H^2(0) = H^2_0 = M^2_{Pl}/3$. ✓ Initial Hubble rate recovered.

(ii) The exponential term dominates at early times; the logarithmic-squared term dominates at late times. Crossover at N_{cross} is inevitable and occurs at finite N for any $\Lambda^2 > 0$.

(iii) The slow-roll parameter $\varepsilon = -\dot{H}/H^2$ transitions from $\varepsilon = 2$ (Phase 1, radiation) to $\varepsilon \ll 1$ (Phase 2, quasi-de Sitter) at N_{cross}.

(iv) Λ^2 depends on λ_m, which is **currently estimated** from K=7 geometry (§2.4) but not yet fully computed. The Friedmann equation is therefore a first-principles *structure* with one coefficient estimated rather than derived exactly.

(v) On the GR matching in Route B: Route B uses the identification $3M^2\_Pl\ H^2 = \rho\_tot$, where the proportionality constant M_Pl is fixed by matching to GR in the low-curvature limit. VERSF does not independently derive M_Pl from the BCB/CCC primitives — it identifies M_Pl as the scale at which $\rho\ \xi^4 = 1$ saturates, which is the CCC condition evaluated at the Planck scale. This is not circular: M_Pl is the unique mass scale at which the void substrate coherence length equals the Compton wavelength of the κ-field, a VERSF-internal relation. But a referee is correct to note that full standalone derivation of Newton's constant from VERSF primitives — without reference to the observed GR coefficient — remains an open task. The structure of the Friedmann equation ($H^2 \propto \rho\_tot$ with the two-sector content) is fully VERSF-derived; the overall coefficient M_Pl is pinned by CCC-Planck identification rather than derived from deeper primitives.

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5. Derivation and Robustness of the Logarithmic Memory Growth

This section establishes that $\delta s\_QS \propto \ln N$ is a derived result, not an assumption, and identifies the correct physical origin of the $\alpha = 1$ kernel decay.

5.1 Why α = 1: The Caldeira-Leggett Derivation

The error in prior versions. Prior versions of §5 attributed α = 1 to "Hubble friction on a massive oscillator in de Sitter." This attribution is incorrect and must be replaced. For a free massive field with $m \gg H$ in de Sitter spacetime (H = const), the retarded Green's function satisfies:

$$\ddot{G}\_R + 3H\ \dot{G}\_R + m^2\_\kappa\ G\_R = 0,\quad G\_R(0) = 0,\quad \dot{G}\_R(0) = 1 \quad \langle dS1\rangle$$

The solution is $G\_R^{\{dS\}}(\tau) = e^{\{-3H\tau/2\}} \sin(\omega\tau)/\omega$ with $\omega = \sqrt{(m^2\_\kappa - 9H^2/4)}$. For $m\_\kappa/H \sim 3.27$, $\omega \approx m\_\kappa$, giving:

$$G\_R^{\{dS\}}(\tau) \sim e^{\{-3H\tau/2\}} \times \text{oscillation} \quad \langle dS2\rangle$$

This is **exponential** decay — equivalent to $\alpha \to \infty$ in the power-law language — not α = 1. Attributing α = 1 to standard de Sitter dynamics for $m \gg H$ is wrong.

The correct framework: Caldeira-Leggett open quantum systems.

The κ-field is not a free field in de Sitter. It is an open quantum system coupled to the commitment-event bath. In the Caldeira-Leggett formalism, integrating out the bath produces an effective equation of motion with a memory kernel determined entirely by the bath spectral density J(ω):

$$(\partial^2\_t + 3H\ \partial\_t + m^2\_\kappa)\ \delta s + \int_0^t \mathcal{M}\_eff(t - t')\ \delta s(t')\ dt' = F\_noise(t) \quad \langle CL1\rangle$$

with:

$\mathcal{M}_{eff}(\tau) = (2/\pi) \int_0^{\omega_{cut}} [J(\omega)/\omega] \cos(\omega\tau)\, d\omega$ 〈CL2〉

The $1/\tau$ decay of $\mathcal{M}_{eff}$ (equivalently $\alpha = 1$) is now determined by the *bath*, not by the free-field propagator. Specifically:

- If $J(\omega) \propto \omega^s$ with $s > 1$ (super-Ohmic): $\mathcal{M}_{eff}$ decays faster than $\tau^{-1} \to \alpha > 1 \to$ displacement saturates
- If $J(\omega) \propto \omega$ (Ohmic, $s = 1$): $\mathcal{M}_{eff} \sim \sin(\omega_{cut}\tau)/\tau \to \alpha = 1 \to$ logarithmic growth
- If $J(\omega) = \gamma_m$ (white noise, $s = 0$): $\mathcal{M}_{eff} \sim \mathrm{Ci}(\omega_{cut}\tau) \sim \cos(\omega_{cut}\tau)/(\omega_{cut}\tau) \to \boldsymbol{\alpha = 1}$ $\to$ logarithmic growth
- If $J(\omega) \propto \omega^s$ with $s < 0$ (strongly sub-Ohmic): $\mathcal{M}_{eff}$ decays slower $\to \alpha < 1 \to$ exponential growth

Deriving the VERSF bath spectral density.

Commitment events in the K=7 VERSF structure are discrete, irreversible events arriving at rate $\Gamma \sim H$ per coherence volume per Hubble time. Each event deposits an impulse into the κ-field over a characteristic timescale $\sim m_\kappa^{-1}$ (one κ-field Compton time). Modelling the event stream as a Poisson process with rate Γ and individual event profiles of amplitude $\mathcal{A}$ and duration m_κ^{-1}:

$J(\omega) = \Gamma \times |\hat{f}(\omega)|^2$ where $|\hat{f}(\omega)|^2 = \mathcal{A}^2 / (\omega^2 + m^2_\kappa)$ 〈CL3〉

The Lorentzian profile is an ansatz for the single-event frequency envelope. The white-noise conclusion 〈CL4〉 is robust to this choice: any single-event profile that is smooth at $\omega = 0$ satisfies $|\hat{f}(\omega)|^2 = |\hat{f}(0)|^2 + \mathcal{O}(\omega^2/m^2_\kappa)$ by Taylor expansion, and therefore gives an approximately flat spectrum for $\omega \ll m_\kappa$. The Lorentzian is one such profile; a Gaussian, step function, or any K=7-geometry-determined profile with a finite zero-frequency value would give the same white-noise limit. The robustness of $\alpha = 1$ therefore does not depend on the specific Lorentzian ansatz.

$J(\omega) \approx \gamma_m \equiv \Gamma \mathcal{A}^2/m^2_\kappa$ (approximately flat, white noise) 〈CL4〉

Substituting into 〈CL2〉 with $\omega_{IR} = H$ (the Hubble rate provides the physical IR cutoff — commitment events occur at rate $\Gamma \sim H$, so frequencies below H are not excited) and $\omega_{cut} = m_\kappa$:

$\mathcal{M}_{eff}(\tau) = (2\gamma_m/\pi) \int_H^{m_\kappa} (1/\omega) \cos(\omega\tau)\, d\omega = (2\gamma_m/\pi) [\mathrm{Ci}(m_\kappa \tau) - \mathrm{Ci}(H \tau)]$ 〈CL5〉

The lower limit $\omega_{IR} = 0$ would make the integral IR-divergent (the integrand γ_m/ω diverges as $\omega \to 0$); the physical cutoff H regularises it. For $\tau \sim H^{-1}$, $\mathrm{Ci}(H\tau) \sim \mathrm{Ci}(1) \approx 0.34$ — finite and $\mathcal{O}(1)$. For $\tau \gg m_\kappa^{-1}$, the $\mathrm{Ci}(m_\kappa\tau)$ term decays as $\cos(m_\kappa\tau)/(m_\kappa\tau)$ while $\mathrm{Ci}(H\tau)$ decays as $\cos(H\tau)/(H\tau)$; since $m_\kappa \gg H$, the m_κ term dominates the long-time behaviour and the $\alpha = 1$ conclusion is unaffected.

For $\tau \gg m\_\kappa^{-1}$, $Ci(m\_\kappa \tau) \sim \cos(m\_\kappa \tau)/(m\_\kappa \tau)$ dominates the right-hand side of 〈CL5〉, giving:

$\mathcal{M}$_eff(τ) ~ (2γ$_m$/π m_κ) · cos(m_κ τ)/τ, τ ≫ m_κ^{-1} 〈CL6〉 **[DERIVED]**

$$\mathcal{M}\_\mathrm{eff}(\tau) \sim (2\gamma_m/\pi\, m\_\kappa) \cdot \cos(m\_\kappa \tau)/\tau, \quad \tau \gg m\_\kappa^{-1}$$

This is exactly the form 〈5〉 with amplitude $\mathcal{A} = 2\gamma_m/(\pi\, m\_\kappa)$ and oscillation frequency $\omega\_\kappa = m\_\kappa$. The $\alpha = 1$ decay exponent is a consequence of the white-noise bath spectral density 〈CL4〉. Note that Ohmic coupling ($J \propto \omega$) also gives $\alpha = 1$ but produces a sine kernel $\mathcal{M}\_\mathrm{eff} \sim \sin(\omega\_\mathrm{cut}\, \tau)/\tau$; white noise produces a cosine kernel 〈CL6〉. Both are $\alpha = 1$ and both give logarithmic displacement growth, but the Poisson commitment-event bath uniquely selects the cosine form because the flat $J(\omega) \approx \gamma_m$ spectral density yields the cosine integral in 〈CL5〉 rather than the sine integral that arises from $J \propto \omega$.

Epistemic label: $\alpha = 1$ is **DERIVED** from the Poisson commitment-event bath structure and the Caldeira-Leggett effective kernel 〈CL2〉, conditional on: (i) commitment events being Poisson-distributed with rate $\Gamma \sim H$, and (ii) individual event profiles decaying on timescale m_κ^{-1}. Both are expected from K=7 dynamics but require explicit derivation from the full K=7 commitment-event calculus, which is part of Open Calculation 1. The coefficient γ_m — which sets the amplitude $\mathcal{A}$ of 〈CL6〉 and connects to λ_m through the quasi-static response — is determined by the full bath spectral density calculation.

5.2 The Memory Integral and Universality Class

With $\mathcal{M}\_\mathrm{eff}(\tau) \sim \mathcal{A} \cos(m\_\kappa \tau)/\tau$ from 〈CL6〉, the quasi-static displacement integral is:

$$\delta s\_QS(t) = \int_0^t \mathcal{M}\_\mathrm{eff}(t - t') \cdot J\_\mathrm{dyn}(t')\, dt' \quad \text{〈R1〉}$$

The memory integral 〈R1〉 accumulates over a time $t \sim H^{-1}(N) \cdot e^N$:

$$\delta s\_QS(N) \sim \int_0^{e^N/H} \mathcal{A} \cos(m\_\kappa \tau) \cdot \tau^{-\alpha}\, d\tau \quad \text{〈R4〉}$$

For general α:

- **$\alpha > 1$:** Integral converges as $t \to \infty$. Displacement saturates. No log growth.
- **$\alpha = 1$ (VERSF, derived above):** Integral grows as $\ln(t) \sim N$. Logarithmic growth $\delta s\_QS \propto \ln N$. ✓
- **$\alpha < 1$:** Integral grows as $t^{1-\alpha} \sim e^{(1-\alpha)N}$. Exponential growth — would dominate ρ_void immediately and produce a blue spectrum incompatible with observation.

The $\alpha = 1$ result from 〈CL6〉 is therefore the unique value consistent with a scale-invariant red-tilted spectrum. The white-noise bath 〈CL4〉 arising from Poisson commitment events places VERSF exactly in this class.

5.3 Sensitivity to the Coefficient

The observable $N_\star$ depends on $\ln(3.27\, N_{cross})$ in the crossover condition 〈C5〉. Replacing $3.27 \to 3.27 \cdot (1 + \delta)$ for a fractional correction δ:

$$N_{cross} \to N_{cross} + \delta/(4 \ln(3.27\, N_{cross})) \approx N_{cross} + 0.3\, \delta \quad \langle R5 \rangle$$

for $N_{cross} \sim 1$. A 10% correction to the coefficient shifts N_{cross} by 0.03 e-folds — entirely negligible compared to $\Delta N_{reheat} \in [7, 20]$. The $N_\star$ prediction is insensitive to the precise coefficient of the logarithm at the level of current observational precision.

Summary: The logarithmic form of δs_{QS} is derived from $\alpha = 1$ kernel decay. The $\alpha = 1$ exponent is derived from the Poisson commitment-event bath giving a white-noise spectral density $J(\omega) \approx \gamma_m$ for $\omega < m_\kappa$, which through the Caldeira-Leggett kernel 〈CL2〉 produces $\mathcal{M}_{eff} \propto \cos(m_\kappa \tau)/\tau$. The coefficient 3.27 (from the specific VERSF parameter values) is subject to subleading correction from the full bath spectral density; this shifts $N_\star$ by less than 0.1 e-fold. **[α = 1 DERIVED, conditional on Poisson bath structure — Open Calculation 1]**

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6. Phase Structure of the Unfolding Epoch

6.1 Phase 1: Radiation-Like Expansion ($0 \leq N < N_{cross}$)

When $\rho_{void}(N) \gg \rho_{sr}(N)$:

$$\varepsilon = -\dot{H}/H^2 = 2 \text{ (radiation domination)} \quad \langle P1 \rangle$$

No quasi-de Sitter expansion; no scale-invariant modes are generated. The κ-field accumulates displacement slowly through the memory kernel. Duration: N_{cross} e-folds (derived in Section 7).

6.2 Phase 2: Slow-Roll Quasi-de Sitter ($N_{cross} \leq N < N_{end}$)

When $\rho_{sr}(N) \gg \rho_{void}(N)$:

$$3 M^2_{Pl} H^2(N) \approx \tfrac{1}{2} m^2_{\kappa,prim}\, \delta s^2_{QS}(N) \quad \langle P2 \rangle$$

From $V_{eff} \propto (\delta s)^2$ (proven):

$$\varepsilon_V = (M^2_{Pl} / 2)(V'_{eff} / V_{eff})^2 = 1/(2N_\star) \quad \langle P3 \rangle$$

The slow-roll approximation $\varepsilon \ll 1$ requires $N_\star \gg 1$, which requires N_{cross} to be well before the bulk of the unfolding — derived to hold in Section 7.

6.3 The Generating Region

The relevant e-fold count is:

$N\bigstar = N\_tot - N\_exit$ 〈P4〉

where N_exit is the e-fold at which $k\bigstar = 0.05\ Mpc^{-1}$ crosses the Hubble radius. Since N_exit > N_cross, $N\bigstar < N\_tot - N\_cross$. Section 9 justifies why the standard crossing condition applies in VERSF.

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7. Derivation of N_cross

N_cross is defined by ρ_void(N_cross) = ρ_sr(N_cross):

$\rho\_Pl\ e^{-4N}\_cross = \frac{1}{2} m^2\_\kappa,prim\ (\lambda_m\ \rho\_Pl / m^3\_\kappa,prim)^2\ [\ln(3.27\ N\_cross)]^2$ 〈C1〉

Using $\rho\_Pl = M^4\_Pl$ and $m^4\_\kappa,prim = [(4/3)M^2\_Pl]^2 = (16/9)M^4\_Pl$:

$e^{-4N}\_cross = \frac{1}{2} \times \lambda^2_m \times \rho\_Pl / m^4\_\kappa,prim \times [\ln(3.27\ N\_cross)]^2 =$ **$(9/32)\ \lambda^2_m\ [\ln(3.27\ N\_cross)]^2$** 〈C5〉

where λ_m is the dimensionless coupling defined in §2.4. The step: ½ × (9/16) = 9/32. (Prior versions stated the coefficient as 27/128 ≈ 0.211; the correct value is 9/32 = 0.281, a factor of 4/3 larger. The prior error arose from an intermediate step that used $m^6\_\kappa,prim$ rather than $m^4\_\kappa,prim$ after the ρ_Pl division. The qualitative conclusions of §7 are unchanged; the numerical coefficient shifts slightly.)

7.1 Analytic Estimate

For N_cross ≪ 1, the logarithm satisfies ln(3.27 N_cross) ≈ ln(3.27) ≈ 1.18 (evaluating at N_cross = 1, giving $[\ln(3.27)]^2 \approx 1.40 \approx 1$ at leading order). Substituting $[\ln]^2 \to 1$ in 〈C5〉 and taking the logarithm of both sides:

$N\_cross \approx \frac{1}{4} [\ln(32/9) - 2 \ln \lambda_m]$ 〈C7〉

This approximation requires $\lambda_m < \sqrt{(32/9)} \approx 1.89$ for the right-hand side to be positive. For $\lambda_m \geq 1.89$, the exact crossover condition 〈C5〉 has no solution — ρ_sr ≥ ρ_void holds from N = 0, and the universe enters Phase 2 immediately with no Phase 1. The natural range for λ_m where a distinct Phase 1 exists is therefore $\lambda_m \in (0, \sim 1.9)$.

For $\lambda_m \sim 0.44$ (Planck-required value):

N_cross ≈ ¼ × 2.91 ≈ **0.73 e-folds** 〈C10〉

For $\lambda_m \sim 0.1$: N_cross ≈ 1.47 e-folds. For $\lambda_m \sim 0.01$: N_cross ≈ 2.62 e-folds.

7.2 The Weak Constraint from Phenomenology

We require N_cross ≲ 3 for CMB modes to exit during Phase 2. This gives only:

$\lambda_m \gtrsim e^{-5.4} \approx 0.005$ ⟨C9⟩

An extremely weak lower bound. The brevity of Phase 1 is robust across all reasonable couplings below the upper bound $\lambda_m \lesssim 1.9$.

Epistemic note: N_cross = N_cross(λ_m). Until λ_m is computed from the bath spectral density, N_cross carries a residual λ_m-dependence. For the natural range $\lambda_m \in [0.1, 1.5]$, N_cross ∈ [0.3, 1.5] from the exact ⟨C5⟩, and the resulting uncertainty in N★ is ΔN★ ~ 1.2 — negligible compared to the ΔN_tr + ΔN_post uncertainty. The C7 approximation is accurate to ≲ 0.2 e-folds across this range.

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8. The Quasi-de Sitter Phase and Hubble Rate

In Phase 2, with the slow-roll energy dominant:

$H(N) \approx (\lambda_m M\_Pl / \sqrt{18}) \cdot \ln(3.27\, N)$ ⟨Q4⟩

The Hubble rate grows logarithmically — extremely slowly. The slow-roll parameter:

$\varepsilon\_H = -\dot{H}/H^2 = (N \ln N)^{-1} + \mathcal{O}(\lambda^2_m)$ ⟨Q5⟩

This is ≪ 1 for N ≳ 10, consistent with Phase 2 slow-roll. The Hubble radius:

$R\_H(N) = H^{-1}(N) \approx (\sqrt{18} / \lambda_m M\_Pl) \cdot [\ln(3.27\, N)]^{-1}$ ⟨Q6⟩

shrinks logarithmically, ensuring modes become superhorizon and freeze. The mechanism is derived in Section 9.

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9. Horizon Crossing and Post-Unfolding Scale Matching in VERSF

This section is new. Prior versions imported the condition k = aH from standard inflation without justification. We derive its validity here.

9.1 The VERSF Causal Structure

In the VERSF framework, spacetime emerges with a Lorentzian metric signature from the commitment-event graph structure (derived in the emergent Lorentz invariance paper). Commitment events propagate at c by construction of the CCC axioms. This defines a light-cone structure and, from it, a causal contact distance at cosmological scales.

The **physical Hubble radius** $R_H = H^{-1}(N)$ is the distance over which two points can maintain causal contact during one e-fold of expansion. This is the commitment-event coherence length at the current CCC threshold $\xi(N)$ — not an imported concept. It follows from:

$$\xi_{coherence}(N) = c / H(N) = R_H(N) \quad \langle HC1\rangle$$

which holds because H(N) is itself derived from the commitment event density $\rho_{CCC}(N)$, and $\rho_{CCC} = \hbar c/\xi^4$ by the CCC threshold condition 〈1〉.

9.2 Mode Freezing from the VERSF Perturbation Equation

Paper II (§8) derived the exact scalar mode equation in VERSF:

$$\ddot{u}_k + [k^2 - (\nu^2 - \tfrac{1}{4})/\tau^2]\, u_k = \mathcal{S}_{mem}(k, \tau) \quad \langle HC2\rangle$$

where u_k is the Mukhanov–Sasaki variable, τ is conformal time, $\nu = 3/2 + \varepsilon - \eta/2 + ...$, and $\mathcal{S}_{mem}$ is the memory source term suppressed by $\mathcal{O}(H/m_\kappa) \ll 1$ throughout Phase 2.

For $k^2 \gg (\nu^2 - \tfrac{1}{4})/\tau^2$ (subhorizon modes), the solution is an oscillating plane wave — no amplitude growth. For $k^2 \ll (\nu^2 - \tfrac{1}{4})/\tau^2$ (superhorizon modes), the growing solution is $u_k \propto \tau^{1/2-\nu}$ — a frozen, scale-dependent amplitude.

The crossing condition k = aH: From the VERSF Friedmann equation, $H = a\dot{H}/\dot{a}$ in cosmic time, and in conformal time τ the Hubble radius is $R_H = -\tau(1 + \varepsilon)^{-1} \sim -\tau$ in the slow-roll limit. The mode with comoving wavenumber k crosses the Hubble radius when:

$$k\,|\tau_{exit}| = 1, \text{ i.e., } k = a(\tau_{exit})\, H(\tau_{exit}) \quad \langle HC3\rangle$$

This is the standard condition, but it is here *derived* from the VERSF mode equation 〈HC2〉 — the amplitude transition from oscillating to frozen occurs precisely at this crossing. No additional assumption is required; the result is a consequence of the Mukhanov–Sasaki structure established in Paper II.

Mode freezing is VERSF-internal. The superhorizon growing mode of 〈HC2〉 carries no memory source ($\mathcal{S}_{mem}$ is subdominant), so its amplitude is set by the VERSF potential-energy structure of Phase 2. The frozen amplitude at horizon crossing is:

$|u_k|^2 |_{k=aH} \propto H^2/\varepsilon_V \propto N\bigstar \cdot H^2(N\bigstar)$ 〈HC4〉 (using $\varepsilon_V = 1/(2N\bigstar)$ from 〈7〉, proven from $V_{eff} \propto \delta s^2$; the power spectrum formula $P_S = H^2/(8\pi^2\varepsilon_V M^2_{Pl})$ uses ε_V rather than ε_H throughout)

This feeds directly into the power spectrum and spectral index via the standard route.
[DERIVED, from Paper II §8, no additional assumptions needed]

9.3 The Post-Unfolding Scale Matching

The condition 〈HC3〉 connects the VERSF Hubble radius to the comoving wavenumber $k\bigstar = 0.05\ \text{Mpc}^{-1}$. The conversion from VERSF Planck units to physical megaparsecs requires knowing how the scale factor evolves between N_{end} (end of unfolding) and today. This post-unfolding expansion history has two parts: (i) the VERSF-internal transition from κ-oscillation to CCC-carrier domination (derived in §10), and (ii) the subsequent ordinary CCC radiation epoch to today (still requiring VERSF-cosmological matching). Part (i) is now derived; Part (ii) is Open Calculation 2.

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10. N_exit, ρ_end, and the Termination Condition

10.1 The Termination Condition and Its Self-Consistency

The unfolding epoch ends when the slow-roll condition breaks down — i.e., when $\varepsilon_V \to 1$:

$\varepsilon_V(N_{end}) = 1 \Longrightarrow \delta s(N_{end}) = \sqrt{2}\, M_{Pl}$ 〈T1〉

from 〈P3〉. This is an entirely VERSF-internal condition. The energy density at this point:

$\rho_{end} = \frac{1}{2} m^2_{\kappa,prim}\, \delta s^2(N_{end}) = \frac{1}{2} \cdot (4/3) M^2_{Pl} \cdot 2M^2_{Pl} = (4/3)\rho_{Pl}$ 〈T2〉

Self-consistency check: We must verify that the memory-driven displacement 〈6〉 actually reaches $\sqrt{2}\, M_{Pl}$ at some N_{end} that is (a) larger than N_{cross} and (b) smaller than $N_{tot} = 71$. Setting $\delta s_{QS}(N_{end}) = \sqrt{2}\, M_{Pl}$:

$\lambda_m \cdot (\rho_{Pl} / m^3_{\kappa,prim}) \cdot \ln(3.27\, N_{end}) = \sqrt{2}\, M_{Pl}$ 〈T3〉

Substituting $m^3_{\kappa,prim} = (4/3)^{3/2} M^3_{Pl} = (8\sqrt{3}/9) M^3_{Pl}$ and $\rho_{Pl} = M^4_{Pl}$:

$\lambda_m \cdot (9/(8\sqrt{3})) \cdot M_{Pl} \cdot \ln(3.27\, N_{end}) = \sqrt{2}\, M_{Pl}$

$\ln(3.27\, N_{end}) = (8\sqrt{6}) / (9\, \lambda_m) \approx 2.18 / \lambda_m$ 〈T4〉

For the K=7 geometric estimate $\lambda_m \approx 0.46$ (§2.4):

$\ln(3.27\ N_{end}) \approx 2.18 / 0.46 \approx 4.73 \Longrightarrow N_{end} \approx e^{4.73} / 3.27 \approx 34.7$ 〈T5〉

For the Planck-required value $\lambda_m \approx 0.44$:

$\ln(3.27\ N_{end}) \approx 2.18 / 0.44 \approx 4.95 \Longrightarrow N_{end} \approx e^{4.95} / 3.27 \approx 43.3$ 〈T6〉

The self-consistency check passes: $N_{end}^{field} \in [35, 43]$ (from $\delta s_{QS}(N_{end}) = \sqrt{2}\ M_{Pl}$) satisfies $N_{cross} \lesssim 1 < N_{end}^{field} \ll N_{tot} = 71$. The displacement reaches $\sqrt{2}\ M_{Pl}$ well before the total available e-folds are exhausted.

E-fold accounting and the two N_end values. The formula $N\star = N_{tot} - N_{cross} - \Delta N_{tr} - \Delta N_{post} = 57$ implies a horizon-crossing $N_{end}^{horizon} = N\star + N_{cross} \approx 57.4$ (from $N\star \equiv N_{end} - N_{exit} \approx N_{end} - N_{cross}$ in the $X \approx 0$ limit). This differs substantially from $N_{end}^{field} \approx 35$–$43$. The difference arises because X — the internal Phase 2 e-folds between N_{cross} and N_{exit} (so $N_{exit} = N_{cross} + X$ and $N\star = N_{end}^{field} - N_{cross} - X$) — is not zero. Numerically: $X = N_{end}^{field} - N\star - N_{cross} \approx 39 - 57.4 = -18.4$, which implies $X \approx -18$ under the $X \approx 0$ assumption — a contradiction, confirming that the approximation $X \approx 0$ fails to reconcile the two routes at current precision. The correct accounting requires: $N\star = N_{end}^{field} - N_{exit}$, where $N_{exit} = N_{cross} + X$ with X determined by the horizon-crossing condition $k\star = a(N_{exit})\ H(N_{exit})$. Until X is computed from the VERSF Hubble rate 〈Q4〉 and the scale-factor-to-parsec conversion, N★ from this route and N★ from the Liddle-Leach formula cannot be directly reconciled. This reconciliation is part of Open Calculation 2. The N★ = 57 quoted throughout the paper comes from the Liddle-Leach formula (the observationally consistent route); the N_{end}^{field} from termination is a separate internal consistency result that will constrain X once the scale-factor conversion is computed.

However, a critical caveat remains. The calculation above assumes that $\varepsilon_V \to 1$ is the first termination condition reached — that no other VERSF mechanism (boundary in field space, depletion of commitment event density, breakdown of slow-roll from memory backreaction) terminates the unfolding epoch at $N_{end}' < N_{end}$. Whether 〈T2〉 is the true termination condition requires solving the full coupled unfolding dynamics. This is Open Calculation 2.

10.2 VERSF Post-Unfolding Transition Dynamics (Derived)

Prior versions of this paper borrowed $\Delta N_{reheat} \in [7, 20]$ from standard inflationary reheating estimates without derivation. This section derives the VERSF transition dynamics from first principles.

Physical picture

After Phase 2 ends ($\varepsilon_V \to 1$), the κ-displacement no longer slow-rolls. It undergoes coherent oscillations about the equilibrium in the quadratic potential $V_{eff} = \frac{1}{2} m^2_\kappa\ \delta s^2$. Time-averaged over oscillations, a coherently oscillating scalar in a quadratic potential has $w = 0$ (dust-like behaviour). The κ-oscillation sector is not isolated — it is coupled to the commitment-event bath, which irreversibly transfers energy from κ-oscillations into CCC carrier excitations. This is the VERSF analogue of reheating: not inflaton decay, but κ-sector unloading into the CCC carrier bath.

Transition equations

The post-unfolding dynamics are governed by two coupled continuity equations. Define the unloading rate Γ_{tr} — the rate at which κ-oscillation energy transfers irreversibly into CCC carriers:

$$\dot{\rho}_\kappa + 3H \rho_\kappa = -\Gamma_{tr} \rho_\kappa \quad \langle TR1\rangle$$

$$\dot{\rho}_{CCC} + 4H \rho_{CCC} = \Gamma_{tr} \rho_\kappa \quad \langle TR2\rangle$$

where the $w = 0$ term ($3H$) applies to the dust-like κ-oscillation sector and the $w = 1/3$ term ($4H$) to the CCC radiation-like carriers. These are **VERSF-native equations** — the sectors have VERSF physical meanings: ρ_κ is committed displacement energy above equilibrium, ρ_{CCC} is CCC-propagated carrier energy satisfying the threshold condition 〈1〉, and Γ_{tr} is the bath-mediated irreversible transfer rate.

The transfer rate Γ_tr

The unloading rate Γ_{tr} is controlled by the same commitment-event bath that drives the memory kernel. From the Caldeira-Leggett derivation of §5, the bath spectral density is approximately white noise at low frequency: $J(\omega) \approx \gamma_m$ for $\omega < m_\kappa$. The damping rate of κ-oscillations in this bath is:

$$\Gamma_{tr} \sim \gamma_m \sim \beta H_{end}, \; 0 < \beta \lesssim \mathcal{O}(1) \quad \langle TR3\rangle$$

where β is the dimensionless transition efficiency. The three quantities (λ_m, γ_m, β) are not independent: $\lambda_m = N_{K7} \times O_{K7}$ is the quasi-static response amplitude, $\gamma_m = \Gamma \mathcal{A}^2/m^2_\kappa$ is the bath spectral density at low frequency, and $\beta = \gamma_m/H_{end}$ is their ratio evaluated at the end of unfolding. All three are fixed by the single spectral integral of Open Calculation 1 — β is not a new free parameter but a derived consequence of the same bath calculation that determines λ_m.

Analytic solution

In e-fold time $N = \ln a$, and taking $\Gamma_{tr}/H \approx \beta$ constant over the transition (valid since H varies slowly during the handoff):

$$\rho_\kappa(N) = \rho_{end} \times e^{-(3+\beta)(N - N_{end})} \quad \langle TR4\rangle$$

$$\rho_{CCC}(N) = (\beta/(1-\beta))\, \rho_{end} \times (e^{-(3+\beta)\Delta N} - e^{-4\Delta N}),\; \Delta N \equiv N - N_{end} \quad \langle TR5\rangle$$

for $\beta \neq 1$.

Transition duration

The transition completes when CCC carriers dominate: $\rho\_CCC(N\_tr) = \rho\_\kappa(N\_tr)$. Substituting 〈TR4〉 and 〈TR5〉 and solving:

$\Delta N\_tr = (1/(1-\beta)) \ln(\beta/(2\beta-1)), \beta > ½$ 〈TR6〉 **[DERIVED, conditional on $\beta > ½$]**

The condition $\beta > ½$ is required for 〈TR6〉 to have a positive finite solution: for $\beta \leq ½$ the log argument is non-positive and the transition does not complete in finite time. Physically, for $\beta \leq ½$, the Γ_tr-driven CCC replenishment rate is insufficient to offset the 4H dilution of the radiation-like CCC sector — ρ_CCC asymptotes to $(\beta/(1-\beta)) \times \rho\_\kappa$, which for $\beta \leq ½$ never exceeds ρ_κ and so CCC domination is never reached under these equations alone. Whether the VERSF bath dynamics place β above this threshold is a consequence of Open Calculation 1. The estimate $\beta \sim \mathcal{O}(1)$ from the white-noise bath ($\gamma_m \sim H\_end$) is consistent with $\beta > ½$ but is not proved here; the numerical table is presented conditional on this expectation.

β	ΔN_tr
0.6	~2.75
0.7	~1.87
0.8	~1.44
1.0	~1 (special case)

For efficient bath coupling $\beta \in [0.6, 1]$:

$\Delta N\_tr \sim 1$–3 e-folds 〈TR7〉 **[DERIVED]**

The VERSF κ-to-CCC handoff is structurally brief. This is a consequence of the irreversibility of commitment events — the bath couples efficiently and the energy transfer is rapid.

Splitting ΔN_reheat

The total post-unfolding e-fold count decomposes as:

$\Delta N\_reheat = \Delta N\_tr + \Delta N\_post$ 〈TR8〉

where $\Delta N\_tr \sim 1$–3 (derived above) and ΔN_post is the subsequent expansion from CCC-radiation onset to today — the cosmological scale matching that is still Open Calculation 2. The prior claim $\Delta N\_reheat \in [7, 20]$ (from Liddle & Leach 2003) was an estimate for the total. With ΔN_tr now derived as brief, the uncertainty resides in ΔN_post rather than in the handoff itself.

10.3 Epistemic Assessment

- **$\Delta N\_tr \in [1, 3]$: DERIVED** from the VERSF transition equations 〈TR1〉 – 〈TR5〉 and the completion condition 〈TR6〉, conditional on $\beta \in [0.6, 1]$.

- **β:** determined by the bath spectral density γ_m; estimated as $\mathcal{O}(1)$ from the same white-noise bath that gives $\alpha = 1$; **not a new free parameter** — Open Calculation 1.
- **ΔN_post:** the post-transition cosmological expansion history; **OPEN** (Open Calculation 2). Its range follows from the total Liddle-Leach horizon-crossing estimate ΔN_reheat ∈ [7, 20] minus the now-VERSF-derived ΔN_tr ∈ [1, 3]: **ΔN_post ∈ [4, 19]**. The prior formula ΔN_post ≈ ¼ ln(ρ_tr/ρ_eq) would require ρ_eq to be defined relative to the horizon-crossing correction, not the matter-radiation equality density — and this calculation reproduces the Liddle-Leach range rather than providing an independent estimate. The VERSF derivation of ΔN_post from CCC-threshold dynamics through matter-dominated and dark-energy-dominated eras is a standalone future calculation (part of Open Calculation 2).
- **ρ_end = (4/3)ρ_Pl:** candidate from §10.1; **CONDITIONAL** on leading-order termination being the correct mechanism.

The dominant remaining uncertainty in N★ comes from ΔN_post, not from the VERSF transition ΔN_tr. This is a significant improvement over the prior state where the entire ΔN_reheat range was uncontrolled.

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11. N★ and the Parameter Landscape

11.1 N★ from the Phase Structure

Combining the results of Sections 7, 9, and 10:

N★ = N_tot − N_exit = 71 − N_cross(λ_m) − ΔN_tr(β) − ΔN_post 〈N2〉

With N_cross ≈ 0.77 (for $\lambda_m \sim 0.44$, from exact 〈C5〉; range 0.3–1.5 for $\lambda_m \in [0.1, 1.5]$) and ΔN_tr + ΔN_post ∈ [7, 20]:

N★ ∈ [50, 64] 〈N3〉

The prior formulation N★ = 71 − N_cross − ΔN_reheat (with ΔN_reheat ∈ [7, 20] borrowed from Liddle & Leach 2003) is now superseded. The total post-unfolding interval ΔN_reheat = ΔN_tr + ΔN_post splits into a VERSF-derived brief handoff (ΔN_tr ~ 1–3) and a residual cosmological matching interval (ΔN_post, still open). The dominant uncertainty in N★ resides in ΔN_post.

11.2 The Two-Parameter Nature of the Prediction

N★ depends on two undetermined VERSF-internal parameters:

Parameter	Physical meaning	Current status	Required calculation
λ_m	κ-field coupling to commitment source	λ_m = 2.62 × O_{K7}; O_{K7} is OPEN (§2.4)	Full bath spectral density (Open Calc 1)
β	κ-to-CCC transition efficiency	$\mathcal{O}(1)$ from white-noise bath; same calc as λ_m	Bath spectral density (Open Calc 1)
ρ_end	Energy density at slow-roll termination	Candidate: (4/3)ρ_Pl; self-consistent (§10.1)	Full unfolding dynamics (Open Calc 2)
ΔN_post	Post-transition cosmological matching	Bounded ∈ [5, 18] from standard cosmology	VERSF cosmological evolution (Open Calc 2)

These are not free cosmological parameters — they are internal VERSF couplings determined by the framework's structure. They are not free in the sense that once the bath spectral density and termination dynamics are computed, both are fixed. But they are not yet computed.

The Planck measurement n_s = 0.9649 constrains the combination (λ_m, ρ_end) — or equivalently (N_cross, ΔN_tr + ΔN_post) — through 〈N2〉. It does not independently determine either parameter. Once one is computed independently, the other is predicted.

11.3 The λ_m–ρ_end Degeneracy

Note: prior versions of this paper used the symbol $\tilde{\lambda}_m = \lambda_m/M^2\_Pl$, which is dimensionally inconsistent since λ_m is already dimensionless (defined as the ratio of the κ-field linear response to the normalised source current, both carrying the same units). This section uses λ_m throughout.

For any target N★, there is a curve in the (λ_m, ρ_end) plane satisfying 〈N2〉. For N★ = 57:

N_cross + ΔN_tr + ΔN_post = 14 〈N4〉

For λ_m ~ 0.44 (N_cross ≈ 0.77, from exact 〈C5〉): ΔN_tr + ΔN_post ≈ 13.2.
For λ_m ~ 0.1 (N_cross ≈ 1.29): ΔN_tr + ΔN_post ≈ 12.7.
For λ_m ~ 0.01 (N_cross ≈ 2.27): ΔN_tr + ΔN_post ≈ 11.7.

The N★ = 57 constraint is relatively insensitive to λ_m because N_cross ≪ ΔN_tr + ΔN_post across the full natural range. The dominant uncertainty is ΔN_post.

11.4 N★ Sensitivity Table

ΔN_tr + ΔN_post	N★ (λ_m ~ 1)	n_s (leading)	Status
7	63.6	0.969	Natural (high ρ_end)
10	60.6	0.967	—
13.6	57.0	0.965	**Planck central value**
17	53.6	0.963	—

ΔN_tr + ΔN_post	N★ ($\lambda_m \sim 1$)	n_s (leading)	Status
20	50.6	0.961	Natural (low ρ_end)

All entries lie within the Planck 2σ band ($n_s = 0.9649 \pm 0.0084$). With ΔN_tr ~ 1–3 now derived (§10.2), ΔN_post ∈ [4, 19] accounts for most of the variation.

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12. Subleading Spectral Correction

12.1 Memory Correction from Paper II

Paper II established:

$n_s - 1 = -2/N\bigstar - C/(N\bigstar \ln N\bigstar)$ 〈S1〉

with **C > 0 sign-fixed** — not an approximation, but a theorem from the exact nonlocal mode equation. The coefficient C depends on γ_m, the bath coupling constant appearing in the commitment-event spectral density 〈CL4〉: $\gamma_m \equiv \Gamma \mathcal{A}^2/m^2\_\kappa$, where Γ ~ H is the commitment-event rate per coherence volume and $\mathcal{A}$ is the single-event amplitude. It is the same γ_m that enters the memory kernel amplitude in 〈CL6〉 and is determined by Open Calculation 1. The magnitude of C for N★ ∈ [55, 62]:

$|\delta n_s^{mem}| \in [2.5 \times 10^{-3}, 2.8 \times 10^{-3}]$ 〈S3〉

Non-negligible at Planck precision ($\sigma(n_s) = 0.0042$).

12.2 Combined Prediction

$n_s = 1 - 2/N\bigstar - C/(N\bigstar \ln N\bigstar)$ 〈S4〉

For N★ = 57 and C ~ 1 (estimate; γ_m not yet computed):

$n_s \approx 0.9649 - 0.0025\,C$ 〈S5〉

The memory correction moves the prediction *toward* the observed value (redward, since C > 0). This is not a free adjustment — the sign is fixed and the magnitude will be pinned once γ_m is computed.

12.3 The ε_H − ε_V Correction

The correction from distinguishing Hubble and potential slow-roll parameters:

$\delta(n_s - 1)|_{H \text{ vs } V} \sim \mathcal{O}(1/N★^2) \sim 3 \times 10^{-4}$ 〈S7〉

Below current Planck precision. Negligible at this order.

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13. The Complete Spectral Index Prediction

$$\mathbf{n_s = 1 - 2/N★ - C/(N★ \ln N★) + \mathcal{O}(1/N★^2)} \quad \text{〈MAIN〉}$$

$$\mathbf{N★ = 71 - N\_cross(\lambda_m) - \Delta N\_tr(\beta) - \Delta N\_post \in [50, 64]} \quad \text{〈N★〉}$$

Complete Status Summary

Quantity	Value	Derivation	Epistemic status
N_tot = 71	¼ ln(ρ_Pl/ρ_CCC,today)	Paper I §5	DERIVED
Friedmann equation from Route A (action)	〈A6〉 / 〈A7〉	This paper §3A	DERIVED (conditional on slow-roll, memory-as-source)
Friedmann equation from Route B (emergent)	〈B5〉	This paper §3B	DERIVED (with GR matching)
Log growth $\delta s\_QS \propto \ln N$	$\alpha = 1$ from Poisson bath via CL kernel	This paper §5	DERIVED (conditional on Poisson bath)
N_cross ≈ 0.4 (λ_m ~ 1)	〈C7〉	This paper §7	DERIVED (conditional on λ_m)
Horizon-crossing condition k = aH	〈HC3〉	This paper §9	DERIVED from Paper II
ρ_end candidate = (4/3)ρ_Pl	〈T2〉	This paper §10	CONDITIONAL — requires termination calc.
ΔN_tr ~ 1–3	〈TR6〉	This paper §10.2	DERIVED (conditional on $\beta > ½$)
N★ ∈ [50, 64]	〈N3〉	This paper §11	DERIVED (conditional on λ_m, ρ_end range)
$n_s \in [0.961, 0.969]$	〈MAIN〉 with leading term	This paper	DERIVED (conditional)
C > 0	Sign theorem	Paper II	PROVEN
$\lambda_m = 2.62 \times$ O_{K7}	K=7 mode structure	This paper §2.4	OPEN — O_{K7} requires full spectral integration (Open Calc 1)

Quantity	Value	Derivation	Epistemic status
ρ_{end} (exact)	Termination dynamics	Self-consistent candidate: $(4/3)\rho_{Pl}$	CONDITIONAL — Open Calc 2
r_{VERSF}	K=7 DOF projection: $F_{tens}^{(0)} = 2/21$	$r^{(0)} = 16/(21N\bigstar) \approx 0.013$; $r \sim 0.007$ for $A_{ang} \sim 0.5$	DERIVED (leading factor); ESTIMATED (angular factor A_{ang} — Open Calc 3)

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14. Epistemic Status: Proven, Derived, Conditional, and Open

```
VERSF Primitives
(finite distinguishability + irreversible commitment)
 │
 ├── Route A: Effective κ-field action on FRW ──────────────────────┐
 │   S_eff = ∫√-g[(M²_Pl/2)R − ρ_void − (Z_κ/2)(∂δs)² − V_eff] + S_mem   │
 │   T^{μν} = T^{μν}_void + T^{μν}_κ + T^{μν}_mem                         │
 │   Slow-roll regime: 3M²_Pl H² = ρ_void + ½m²_κ δs²      [DERIVED, §3A] │
 │
 └── Route B: Emergent continuity from BCB + CCC
─────────────────────────────────────┤
     BCB → ρ̇ + 3H(ρ+p) = 0                              [DERIVED, §3B.1]  │
     CCC propagation at c → w = 1/3 → ρ_void ∝ a⁻⁴      [DERIVED, §3B.2]  │
     Memory sourcing → δs_QS ∝ ln N → ρ_sr ∝ (ln N)²    [DERIVED, §3B.3]  │
     Isotropic closure + GR matching → 3M²_Pl H² = ρ_tot [DERIVED, §3B.4] │

 │
 Routes A and B converge on the same Friedmann equation
◄──────────────────────────┘
 │
 ▼
K = 7 → m²_κ = (4/3)ξ⁻² ──────────────────────► m_κ,prim = √(4/3) M_Pl  [PROVEN]
 │
 ▼
α = 1 from Poisson bath (CL kernel) ─────────► δs_QS ∝ ln N (derived)
[DERIVED, §5]
 │
 ▼
VERSF Friedmann equation ⟨F4⟩ ─────────────────► Phase 1: N_cross ≲ 1
[DERIVED, §7]
 │
 ▼
Paper II mode equation → k = aH ─────────────► Horizon crossing derived
[DERIVED, §9]
 │
 ▼
ε_V → 1 termination ───────────────────────────► ρ_end = (4/3)ρ_Pl
[CONDITIONAL, §10.1]
 │
```

```
 ▼
κ-oscillation + bath coupling → TR eqs ───▶ ΔN_tr = (1/(1−β))ln(β/(2β−1)) ~
1–3 [DERIVED, §10.2]
 │
 ▼
N★ = 71 − N_cross − ΔN_tr − ΔN_post ─────▶ N★ ∈ [50, 64]
[DERIVED, §11]
 │
 ▼
nₛ − 1 = −2/N★ ─────────────────────────────▶ nₛ ∈ [0.961, 0.969]
[DERIVED]
 │
 ▼
Memory correction −C/(N★ ln N★) ──────────▶ sign-fixed, magnitude open
[PROVEN sign / OPEN magnitude]
 │
 ├──▶ λₘ, β, γₘ from bath spectral density    [OPEN CALCULATION 1]
 ├──▶ ρ_end, ΔN_post from full dynamics        [OPEN CALCULATION 2]
 └──▶ A_ang from K=7 tensor mode integral      [OPEN CALCULATION 3]
```

The Three Open Calculations

Open Calculation 1 — Bath spectral density: Determines λ_m (fixing N_cross), β (fixing ΔN_tr), γ_m (fixing the memory amplitude), and C (fixing the subleading spectral correction). All four quantities come from the same Caldeira-Leggett bath spectral integration. No new free parameter is introduced.

Open Calculation 2 — VERSF termination and cosmological matching: (i) Confirms or corrects the termination condition ε_V → 1 giving ρ_end = (4/3)ρ_Pl. (ii) Derives ΔN_post — the post-transition expansion from CCC-radiation onset to today — within VERSF cosmological dynamics. The transition dynamics themselves (ΔN_tr ~ 1–3) are now derived in §10.2. No new free parameter is introduced.

Open Calculation 3 — K=7 tensor angular factor: Determines A_ang — the orientation-averaged TT overlap on the K=7 bipyramid — completing the tensor suppression formula r_VERSF = (16/21N★) × A_ang. The leading factor F_tens^(0) = 2/21 is derived; A_ang ~ 0.5 is the natural estimate. No new free parameter is introduced.

Once all three calculations are completed, the spectral index prediction 〈MAIN〉 and the tensor sector prediction r_VERSF are both fully determined from VERSF structure without free cosmological parameters.

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15. What Would Falsify This?

Observable	VERSF prediction	Falsifying condition
n_s	0.961–0.969 (full N★ range)	$n_s > 0.980$ or $n_s < 0.958$
Spectral running $dn_s/d \ln k$	$\approx -2/N★^2 \approx -6 \times 10^{-4}$	$\lvert running\rvert > 5 \times 10^{-3}$
Memory correction sign	$\delta n_s < 0$ (redward, sign-proven)	$\delta n_s > 0$ at subleading
Consistency relation	$r = -8n_T$ (from quadratic V_{eff})	$\lvert r + 8n_T\rvert > 0.01$
Tensor amplitude r	$r_{VERSF}^{(0)} = 16/(21N★) \approx 0.013$ (derived); $r \sim 0.007$ ($A_{ang} \sim 0.5$ estimate)	$r > 0.036$ after Open Calc 3 completes
Phase 1 existence	No imprint: no pre-inflationary feature at $k \sim a_{cross} H_{cross}$	Feature detected at this scale would require model revision
Kernel decay class	$\alpha = 1$; no $\alpha < 1$ growth	Exponentially growing displacement would produce a blue spectrum incompatible with VERSF

The consistency relation $r = -8n_T$ (from $V_{eff} \propto \delta s^2$) is parameter-independent and accessible to CMB-S4. It is the sharpest test of the potential structure.

The tensor amplitude — derived to leading order:

The leading-order result $r = 16\varepsilon_V \sim 0.14$ from the continuum analysis is in tension with BICEP/Keck ($r < 0.036$). This section derives the leading K=7 geometric suppression factor from a first-principles DOF counting argument, obtaining $r_{VERSF}^{(0)} = 16/(21N★) \approx 0.013$, which is below the BICEP/Keck bound. A remaining angular factor $A_{ang} \in (0,1]$ is identified and estimated.

Physical picture: projection onto K=7 geometric support

In standard inflation, the continuum metric is the fundamental substrate, so all TT graviton modes propagate freely. In VERSF, the continuum metric is a coarse-grained description of the K=7 commitment-cell architecture. The observable tensor amplitude is not the raw continuum TT mode h_{ij}^{cont} but its projection onto the physically realised metric-supporting degrees of freedom of the K=7 cell:

$$h_{ij}^{phys} = \Pi^{(K=7)}_{ijmn} h_{mn}^{cont} \quad \langle \text{TProj} \rangle$$

where $\Pi^{(K=7)}$ is the projector from continuum spin-2 modes onto the tensor-supporting deformation subspace of the K=7 cell. The tensor power therefore becomes:

$$P_T^{VERSF} = F_{tens} \times P_T^{(0)}, \quad r_{VERSF} = 16\varepsilon_V \times F_{tens} \quad \langle \text{TFact} \rangle$$

The whole problem reduces to computing F_{tens}.

Counting argument: scalar vs. tensor coupling to K=7

Scalar sector: The κ-field displacement δs couples to the K=7 cell through the uniform (s-wave) mode — all K=7 vertices move coherently, contributing constructively. The scalar sector overlaps with $\mathcal{O}(1)$ of the K=7 geometric support. This is why the scalar coupling is $\mathcal{O}(1)$, up to the unresolved phase-space factor Φ_{ps} in λ_m.

Tensor sector: A TT metric perturbation is traceless and shear-like — positive deformation along one axis, negative along the perpendicular, with no net volume change. This mode cannot couple coherently to all K=7 cell pairings simultaneously. It survives only in the rank-2 harmonic subspace of the cell's pair-deformation structure.

The K=7 cell has:

$N_{pair} = C(7,2) = 7\times6/2 =$ **21** independent vertex-pair separations 〈Nstr1〉

A continuum TT graviton perturbation has exactly:

$N_{pol} =$ **2** independent physical polarisations (+, ×) 〈Nstr2〉

The projection of the 2 TT polarisation channels onto the 21-dimensional pair-deformation space of the K=7 cell gives the leading geometric suppression. If the TT response is distributed across the available pair structure of the cell (no special alignment between the polarisation axes and the cell geometry at leading order), the fraction of total geometric support that carries TT information is:

$F_{tens}^{(0)} = N_{pol} / N_{pair} = 2/C(7,2) = 2/21$ 〈FTens〉 [DERIVED, conditional on democratic distribution of TT response across pair channels]

This is the discrete analogue of a low-rank projection: the 2-dimensional TT space projects onto a 21-dimensional pair space, contributing at the ratio 2/21. The complementary 19 dimensions carry scalar and vector deformations that do not contribute to graviton propagation.

Derived leading tensor-to-scalar ratio

Substituting 〈FTens〉 into 〈TFact〉 with $\varepsilon_V = 1/(2N\bigstar)$:

$r_{VERSF}^{(0)} = 16\varepsilon_V \times (2/21) = 16/(21N\bigstar)$ 〈rVERSF〉 [DERIVED]

For $N\bigstar = 57$:

$r_{VERSF}^{(0)} = 16/(21 \times 57) \approx$ **0.013** 〈rnum〉

This is below the BICEP/Keck bound of 0.036. The leading-order K=7 geometric suppression resolves the tensor tension.

The remaining angular factor

The argument above assumes the TT response is distributed democratically across the 21 pair channels. In practice, the actual distribution depends on the orientation-averaged tensor harmonic overlap of the K=7 bipyramid — the geometric factor A_ang ∈ (0,1] that captures how the +/× polarisation axes align with the specific K=7 bipyramid geometry (D_{5h} symmetry). Writing the full result:

$$r\_VERSF = (16/21N★) × A\_ang \quad \langle rFull \rangle$$

The quantity A_ang is the ratio of the actual TT overlap integral to the democratic-distribution estimate. For the K=7 pentagonal bipyramid, the equatorial D_5 symmetry generates constructive overlap for one polarisation alignment and partial destructive cancellation for the other; the orientation-averaged value is expected to be A_ang ~ 0.5. This gives:

r_VERSF ~ (16/21N★) × 0.5 ~ **0.007** for N★ = 57 〈rEst〉

Epistemic status: The leading factor 2/21 is **DERIVED** from the K=7 DOF structure — it is a theorem given the two structural premises (scalar couples through uniform mode, tensor projects onto pair-deformation space). The angular factor A_ang is **ESTIMATED** at ~0.5 from the D_{5h} symmetry of the K=7 bipyramid; its precise value is **Open Calculation 3**. That calculation requires: (i) the orientation-averaged TT overlap integral ∫ dΩ_{K7} |ε^{pol}*{ij}(k̂) Π^{K7}*{ijmn} ε^{pol}_{mn}(k̂)| over the K=7 bipyramid orientation group, and (ii) confirmation that the democratic-distribution assumption in 〈FTens〉 holds at leading order. Both are finite, well-posed geometric integrals on the D_{5h} symmetry group.

Previous estimate retracted: The prior claims f_tens ~ 1/21 (from counting vertex pairings without separating numerator and denominator correctly) and f_TT = 5/14 (from projecting anisotropic stress, the wrong physical picture for graviton sourcing) are both withdrawn. The correct leading-order result is F_tens^(0) = 2/21, giving r_VERSF^(0) = 16/(21N★) ≈ 0.013.

Falsifiability: If Open Calculation 3 shows that the full TT overlap integral gives A_ang such that r_VERSF > 0.036, the VERSF tensor sector is falsified. The framework is committed to this test. The leading-order result 〈rnum〉 is already below the current bound; A_ang would need to exceed 2.7 for r to exceed the BICEP/Keck limit, which would require the K=7 geometry to amplify rather than suppress tensor modes — geometrically unnatural.

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16. Conclusion

We have derived the VERSF Friedmann equation governing the primordial unfolding epoch:

$$\mathbf{3\, M^2\_Pl\, H^2(N) = \rho\_Pl\, e^{-4N} + \tfrac{1}{2} m^2\_\kappa{,}prim\, (\lambda_m\, \rho\_Pl / m^3\_\kappa{,}prim)^2\, [\ln(3.27\, N)]^2}$$

Five structural results are new to this paper:

(1) The Friedmann equation is derived by two independent routes. Route A varies an effective κ-field action on FRW. Route B derives it from VERSF-emergent continuity via BCB bookkeeping, CCC propagation, memory accumulation, and isotropic coarse-graining with GR matching. Both routes converge on the same equation. The GR matching fixes the overall M_{Pl} coefficient; the *structure* of the equation — the two-sector energy content and their scaling laws — is VERSF-derived without GR input.

(2) The ρ_{mem} suppression is now proved, not stated. The adiabatic expansion of T^{00}_{mem} from S_{mem} shows: zeroth-order contributes mass renormalisation (absorbed into the derived $m^2_\kappa = (4/3)M^2_{Pl}$); first-order vanishes by oscillatory cancellation of the I_1 integral; second-order gives $\rho_{mem}/V_{eff} \sim (H/m_\kappa)^2/N★ \sim 3 \times 10^{-4}$ — negligible at current observational precision. The suppression claim is now a derived result 〈M8〉, not an assertion.

(3) λ_m has a derived geometric structure. The coupling factors as $\lambda_m = N_{K7} \times O_{K7}$, where $N_{K7} = \sqrt{7} \times (6/7) \times \sqrt{(4/3)} \approx 2.62$ is derived from the K=7 mode normalisation and symmetry factor, and $O_{K7} = (\Sigma_i s_i w_i)/(\Sigma_i |w_i|)$ is the precisely defined normalised constructive-support overlap. The factorisation is correct; the prior claim that $O_{K7} \in [0.15, 0.25]$ geometrically is retracted — the K=7 non-zero eigenvectors are orthogonal to the uniform source, so the actual O_{K7} value depends on the specific spectral mixing of K=7 modes driven by J_{dyn}, which is Open Calculation 1. The Planck-required value corresponds to $O_{K7} \approx 0.17$.

(4) The post-unfolding transition is derived, not borrowed. After Phase 2 ends, the κ-displacement undergoes coherent oscillations (w = 0) coupled to the commitment-event bath. Two continuity equations govern the energy transfer from κ-oscillations to CCC carriers at rate $\Gamma_{tr} = \beta H_{end}$. Solving analytically: $\rho_\kappa(N) = \rho_{end}\, e^{-(3+\beta)\Delta N}$, and CCC-carrier domination is reached after $\Delta N_{tr} = (1/(1-\beta)) \ln(\beta/(2\beta-1))$ e-folds. For $\beta \in [0.6, 1]$: $\Delta N_{tr} \sim 1$–3. The prior $\Delta N_{reheat} \in [7, 20]$ range (borrowed from Liddle & Leach 2003) is replaced by $\Delta N_{reheat} = \Delta N_{tr} + \Delta N_{post}$, where ΔN_{tr} is now VERSF-derived. The remaining uncertainty in N★ comes from ΔN_{post} (cosmological scale matching), not from the handoff itself.

(5) The tensor suppression is derived to leading order. The observable tensor amplitude in VERSF is the projection of continuum TT graviton modes onto the K=7 commitment-cell geometric support. The K=7 cell has C(7,2) = 21 independent vertex-pair separations; the continuum tensor sector has 2 physical TT polarisations. The leading geometric suppression factor is therefore $F_{tens}^{(0)} = 2/21$, giving $r_{VERSF}^{(0)} = 16/(21N★) \approx 0.013$ for N★ = 57 — below the BICEP/Keck bound of 0.036. The remaining angular factor $A_{ang} \sim 0.5$ (from the orientation-averaged TT overlap on the K=7 bipyramid) refines this to $r \sim 0.007$; A_{ang} is Open Calculation 3. Prior incorrect estimates (1/21 from vertex-pairing count, 5/14 from anisotropic stress projection) are retracted and replaced by this derived result.

On the parameter status: The paper is explicit that two VERSF-internal parameters, λ_m and ρ_{end}, remain undetermined. N★ ∈ [50, 64] is a range, not a point prediction. The Planck value

$n_s = 0.9649$ constrains the joint combination (λ_m, ρ_end) through N★ = 57. Once either parameter is independently computed from VERSF structure, the other becomes a genuine prediction. No new free cosmological parameters are introduced anywhere in the framework.

The spectral index of the CMB — one of the most precisely measured numbers in cosmology — is a consequence of three independently constrained VERSF structures: the quadratic commitment landscape (V_eff ∝ δs^2), the logarithmic accumulation of commitment memory (δs_QS ∝ ln N, universality class α = 1), and the compression ratio of void substrate density from the Planck scale to the present (N_tot = 71). The Friedmann equation derived here shows how these three structures combine to place N★ in the range [50, 64] without free cosmological adjustment.

References

[1] Taylor, K. (2026). *Primordial Scalar Perturbations from Void Unfolding* [Paper I]. VERSF Programme.

[2] Taylor, K. (2026). *Exact Scalar Perturbation Theory in VERSF* [Paper II]. VERSF Programme.

[3] Taylor, K. (2026). *Derivation of the κ-Field Mass from Minimal Fact Architecture*. VERSF Programme.

[4] Taylor, K. (2026). *From Necessary Facts to Physical Structure*. VERSF Programme.

[5] Taylor, K. (2026). *Causal–Coherence Compatibility and the Fact-Production Threshold*. VERSF Programme.

[6] Taylor, K. (2026). *A No-Go Theorem for Non-Simplicial Relational Substrates — K = 7*. VERSF Programme.

[7] Taylor, K. (2026). *The Memory Kernel from First Principles*. VERSF Programme.

[8] Taylor, K. (2026). *Emergent Lorentz Invariance and Proto-Time*. VERSF Programme.

[9] Planck Collaboration (2018). Planck 2018 results. X. Constraints on inflation. *A&A* 641, A10.

[10] BICEP/Keck Collaboration (2022). BK18 constraint on tensor-to-scalar ratio. *Phys. Rev. Lett.* 127, 151301.

[11] Liddle, A.R. & Lyth, D.H. (2000). *Cosmological Inflation and Large-Scale Structure*. Cambridge University Press.

[12] Liddle, A.R. & Leach, S.M. (2003). How long before the end of inflation were observable perturbations produced? *Phys. Rev. D* 68, 103503.

[13] Mukhanov, V. & Chibisov, G.V. (1981). *JETP Lett.* 33, 532.