The κ-Field Mass as the Physical Hessian of the Commitment Constraint Surface: A Derivation of $m^2 = \lambda_{eff} \xi^{-2}$ from First Principles in the VERSF Framework

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For the General Reader

Every physical field has a mass — a number that sets how quickly its influence fades with distance and how much energy is needed to excite it. In conventional physics, this mass is a free parameter: we measure it in experiment and insert it into the equations. The VERSF programme aims to derive such numbers rather than assume them.

A companion paper (*Derivation of the κ-Field Mass from Minimal Fact Architecture*) showed that the κ-field mass must take the form $m = C_m \xi^{-1}$, where ξ is the coherence scale and $C_m = \sqrt{4/3}$ is a dimensionless spectral invariant of the $K = 7$ Fano-plane constraint structure. That paper derived C_m by computing the lowest eigenvalue of a closure operator built from the commitment architecture. What it did not do — and what is done here — is prove why that eigenvalue equals the squared mass in physical units.

The missing step is an old idea in physics dressed in new clothing. When atoms in a crystal are displaced from their equilibrium positions, they experience a restoring force proportional to the Hessian of the lattice potential. The frequencies at which the crystal vibrates — the phonon frequencies — are the square roots of the Hessian's eigenvalues. The mass of a phonon, in the field-theoretic sense, is set by the lowest such eigenvalue.

The commitment constraint structure of VERSF works the same way. The $K = 7$ architecture defines an equilibrium configuration for the commitment density field at each point in spacetime. The closure operator L_{eff} is precisely the Hessian of the constraint potential — the restoring-force operator for departures from equilibrium. Its lowest eigenvalue, converted to physical units using the only available length scale ξ, gives the squared mass of the κ-field propagating mode. The identification $m^2 = \lambda_{eff} \xi^{-2}$ is not an assumption. It is the statement that the κ-field is the phonon of the commitment constraint lattice.

Abstract

A previous paper in the VERSF programme (*Derivation of the κ-Field Mass from Minimal Fact Architecture* [T4]) derived the dimensionless spectral invariant $C_m = \sqrt{4/3}$ as the square root of the minimum positive eigenvalue $\lambda_{eff} = 4/3$ of the physical closure operator $L_{eff} = PL_1P^T$, and proposed $m = C_m \xi^{-1}$ as the κ-field mass. The identification $m^2 = \lambda_{eff} \xi^{-2}$ was presented as transparent from the Klein–Gordon dispersion relation but was not derived. This paper closes that gap, addressing three vulnerabilities that a sharp referee would raise against any less complete treatment.

We establish three results that together make the identification $m^2 = \lambda_{eff} \xi^{-2}$ logically necessary rather than merely natural.

First, we prove a **Uniqueness Theorem for the Constraint Penalty**: the only penalty functional on the physical state space V_p that is local, quadratic, and invariant under the K = 7 automorphism group PGL(3,2) ≅ PSL(2,7) is proportional to the Gram form $\sigma^T L_{eff} \sigma$, with the proportionality constant fixed to unity by the Gram normalisation. This eliminates the referee objection "why that potential?" — no other potential is admissible.

Second, we establish a **No-Alternative-Scaling Lemma**: in the CCC sector with natural units $\hbar = c = 1$, the relation $\rho = \xi^{-4}$ reduces all available dimensional quantities to a single scale ξ, and the unique dimensionless parametrisation of a commitment density perturbation $\delta s \in [L^{-1}]$ is $\sigma = \delta s \cdot \xi$. This eliminates the objection "why that scaling?" — no alternative exists within the CCC sector without importing external structure.

Third, we prove a **Constraint-to-Mass Proposition**: the constraint penalty V_c is quadratic in the field s (not in its gradients $\partial_\mu s$) and is a Lorentz scalar. By the Lagrangian Uniqueness Theorem [T5], the unique Lorentz-invariant Lagrangian containing such a term is the Klein–Gordon Lagrangian, and V_c therefore necessarily appears as a mass term with $m^2 = \lambda_{eff} \xi^{-2}$. This eliminates the deepest objection: "why does constraint energy propagate as a relativistic scalar field?" — the constraint energy cannot take any other form within an admissible Lagrangian.

Together, the three results reduce the mass identification to a logical consequence of the K = 7 constraint structure and the CCC energy density scale. No free parameters appear, and each step is locked rather than argued.

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1. Introduction

The VERSF programme is built on two foundational results that anchor the κ-field mass derivation. The Causal–Coherence Compatibility (CCC) condition establishes that irreversible fact formation requires a minimum action budget $\rho L^4 \gtrsim \hbar c$ in a region of size L and energy density ρ, defining the coherence scale

$\xi = (\hbar c/\rho)^{(1/4)}$

as the smallest region capable of sustaining a committed fact [T1]. Separately, the K = 7 no-go theorem proves that the minimal internal architecture of a stable fact requires exactly seven mutually consistent constraint relations, realised canonically by the Fano plane PG(2,2) [T2].

From these two results, a companion paper [T4] derived the κ-field mass in the form $m = C\_m\ \xi^{-1}$, with $C\_m = \sqrt{(4/3)}$ obtained as the square root of the minimum positive eigenvalue $\lambda\_eff = 4/3$ of the physical closure operator $L\_eff = PL_1P^T$ acting on the 4-dimensional physical state space V_p. The derivation proceeded by:

(i) establishing $m \sim \xi^{-1}$ by dimensional closure over the CCC variables alone;
(ii) constructing L_eff from the Fano-plane incidence structure;
(iii) computing λ_eff from the projected spectral data;
(iv) identifying $m^2 = \lambda\_eff\ \xi^{-2}$.

Step (iv) was described as "transparent from the Klein–Gordon dispersion relation" but was not formally justified. A careful referee would identify three points where the argument remains vulnerable: the choice of constraint potential, the choice of scaling for the departure parameter, and the assumption that a constraint penalty propagates as a relativistic scalar. The present paper addresses all three.

The central result is:

Theorem (κ-Field Mass). *Under the CCC condition and the K = 7 minimal fact architecture, the squared mass of the κ-field equals the minimum positive eigenvalue of the physical closure operator in physical units:*

$$\mathbf{m^2 = \lambda\_eff / \xi^2 = (4/3) / \xi^2}$$

This is a derived result. Each step in the derivation is the unique admissible choice given the K = 7 and CCC constraints.

The argument proceeds as follows. In §2 we establish $\rho = \xi^{-4}$ as the unique CCC energy density. In §3 we define the constraint surface $\mathcal{C}$. In §4 we prove the Uniqueness Theorem for the constraint potential, showing the Gram form is the only admissible penalty. In §5 we prove the No-Alternative-Scaling Lemma, locking $\sigma = \delta s \cdot \xi$. In §6 we prove the Constraint-to-Mass Proposition, establishing that V_c cannot appear in an admissible Lagrangian as anything other than a KG mass term. In §8 we assemble the main theorem. Sections 8–9 develop the phonon analogy; §12 addresses remaining objections.

Notational conventions. Natural units $\hbar = c = 1$ unless otherwise noted; [L] denotes dimension of length; the commitment density field s(x,t) has dimension $[s] = [L^{-1}]$ (consistent with the Klein–Gordon Lagrangian $\frac{1}{2}(\partial s)^2 - \frac{1}{2}m^2 s^2$, which requires $[\mathscr{L}] = [L^{-4}]$ in 3+1 dimensions); ξ has dimension [L]; λ_eff = 4/3 is dimensionless. The physical closure operator $L\_eff = (4/3)I_4$ on $V\_p \cong \mathbb{R}^4$ as derived in [T4].

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2. Dimensional Foundations: The CCC Condition and the Fundamental Energy Density

The CCC condition at threshold reads:

$$\chi(\xi) = \rho\xi^4 / \hbar c = 1$$

In natural units ($\hbar = c = 1$) this simplifies to:

$$\rho\xi^4 = 1 \Longrightarrow \rho = \xi^{-4} \quad (1)$$

This is not an approximation. It is the definition of ξ as the scale at which the action budget is exactly sufficient for one irreversible commitment event. Equation (1) establishes ρ as a derived quantity: given ξ, the void energy density is ξ^{-4} in natural units.

Lemma (Fundamental Energy Density). *The unique energy density constructible from the CCC variables $\{\rho, \hbar, c\}$ alone, evaluated at the coherence threshold, is $\rho = \xi^{-4}$. No independent energy density scale exists within the CCC sector.*

Proof. The CCC variables have dimensions $[\rho] = [L^{-4}]$, $[\hbar] = [L^{-1} \cdot T]$ (natural units), $[c] = [L \cdot T^{-1}]$. The unique combination with dimension $[L^{-4}]$ is $(\hbar c)^n \rho^m$ with $4m - 4 = 0$, $n = 0$, giving ρ itself. At the CCC threshold, $\rho = \xi^{-4}$ in natural units. No dimensionally independent energy density exists. ■

Its role here is to establish that $\rho = \xi^{-4}$ is the *only* energy density the CCC sector provides, and therefore the only admissible scale for measuring constraint violations — a fact that will lock the No-Alternative-Scaling Lemma of §5.

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3. The Commitment Constraint Surface

At each spacetime point x, the local commitment configuration is described by a vector $\varphi(x) \in V\_p \cong \mathbb{R}^4$, where V_p is the 4-dimensional physical state space of intrinsic fact-carrying degrees of freedom [T2, T4].

Definition (Constraint Surface $\mathcal{C}$). The commitment constraint surface operates at two levels that must be kept distinct.

At the *field level*, $\mathcal{C}$ is the set of equilibrium values $s_0 \in \mathbb{R}$ of the commitment density field — those values at which the local fact-formation process is in structural equilibrium with the K = 7 constraint architecture, neither under-committed nor over-committed.

At the *internal state-space level*, perturbations away from any s_0 are described by departure vectors $\sigma \in V\_p \cong \mathbb{R}^4$. The K = 7 architecture assigns a positive penalty to every non-zero $\sigma \in V\_p$ via the Gram matrix $L_0 = BB^T$ (constructed in §4): for all $\sigma \neq 0$, $\sigma^T L_0 \sigma > 0$. The equilibrium s_0 is therefore isolated — it is not part of a flat direction — and every perturbation in V_p incurs a positive energy cost.

Remark. The closure operator $L\_eff = (4/3)I_4$ on V_p is positive definite with no zero modes [T4]. The K = 7 projection theorem guarantees a spectral gap $\lambda\_eff = 4/3 > 0$: a zero mode in V_p would require the Fano-plane representation to be reducible on V_p, which is excluded by the irreducibility of V_p under the automorphism group PGL(3,2) ≅ PSL(2,7) [T4, F3]. Therefore every perturbation off equilibrium costs positive energy and the κ-field is massive throughout the admissible architecture.

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4. The Constraint Potential: Uniqueness Theorem

This section addresses the objection "why that potential?" directly, by proving that no other penalty functional is admissible under the $K = 7$ constraint structure.

Construction of the Gram penalty. The $K = 7$ architecture is encoded in the Fano-plane incidence matrix B, whose columns are the seven constraint vectors $\{c_i\}$ acting on $V_6 \supseteq V\_p$ [T2, T4]. The entries of B are in $\{0,1\}$: each entry records whether a given point lies on a given line of the Fano plane. This integer structure is not a normalisation convention — it is fixed by the combinatorial geometry of PG(2,2). In particular, each constraint line passes through exactly 3 points, so $|c_i|^2 = 3$ for all i, with no freedom to rescale. Any rescaling $c_i \to k\, c_i$ would violate the integer incidence structure of PG(2,2), which fixes $|c_i|^2 = 3$ combinatorially; therefore no continuous normalisation freedom exists. For a departure vector σ, constraint i is violated by $c_i\cdot\sigma$. The total squared violation — the canonical penalty for departing from $\mathcal{C}$ — is:

$$Q(\sigma) = \Sigma_{i=1}^{7} (c_i\cdot\sigma)^2 = \sigma^T(\Sigma_{i=1}^{7} c_i c_i^T)\sigma = \sigma^T BB^T\sigma = \sigma^T L_0 \sigma$$

The coefficient in this expression is $\alpha = 1$, fixed by the integer incidence structure: the norms $|c_i|^2 = 3$ are determined by the Fano plane, not chosen, and no free coupling constant appears in the sum of squared violations. After removing the uniform mode, $L_1 = BB^T - (9/7)J_7$ is formed by subtracting the rank-1 projector onto the all-ones vector (scaled appropriately); this removes the uniform mode corresponding to a global rescaling of commitment density across all seven constraint lines — a bulk shift that represents no internal structural departure and must be projected out before computing the physical Hessian. Projecting to V_p then gives $L\_eff = PL_1P^T$ and the projected penalty:

$$Q\_p(\sigma) = \sigma^T L\_eff\ \sigma$$

Theorem (Uniqueness of Constraint Penalty). *The unique penalty functional on V_p that is (i) local, (ii) at least quadratic at $\mathcal{C}$, (iii) invariant under the $K = 7$ automorphism group PGL(3,2) ≅ PSL(2,7), and (iv) constructible from the $K = 7$ constraint vectors alone without external parameters, is:*

$$V\_c(\sigma) = \tfrac{1}{2}\, \sigma^T L\_eff\ \sigma \cdot \rho \quad (2)$$

with coefficient $\alpha = 1$. No alternative penalty exists within the admissible architecture.

Proof. We verify that each condition uniquely forces the form (2).

(i) Locality. The penalty depends only on the local departure σ(x), not on σ at other points. This excludes non-local penalties (e.g., integral operators).

(ii) Quadratic. At the equilibrium point $\mathcal{C}$, the gradient of any penalty vanishes by definition of equilibrium. Therefore the Taylor expansion of any admissible penalty begins at quadratic order. Sub-quadratic (linear) terms are zero; the leading non-trivial contribution is the Hessian term. The Nonlinear Exclusion Theorem of [T5] further shows that super-quadratic (anharmonic)

terms are excluded at the fundamental level. Even absent [T5], higher-order terms would introduce additional dimensionless couplings not constructible within the CCC sector — where $\rho = \xi^{-4}$ provides the sole energy scale — thereby violating the no-free-parameter condition of the programme. Therefore the penalty is exactly quadratic on both grounds.

(iii) Invariance under PGL(3,2). The automorphism group of the Fano plane acts on V_6, preserving the K = 7 constraint structure. The restriction of this action to V_p is an irreducible representation of PGL(3,2) [T4, F3]. By Schur's Lemma, the only bilinear form on V_p invariant under an irreducible group action is proportional to the identity: any invariant quadratic form Q on V_p satisfies $Q(\sigma) = c\cdot|\sigma|^2$. Since $L\_eff = (4/3)I_4$ on V_p, this forces $Q\_p(\sigma) = c\cdot\sigma^T L\_eff\ \sigma$ for some scalar c. Invariance under PGL(3,2) fixes the *structure* of the penalty uniquely — only the scalar c remains free.

(iv) Constructible from K = 7 constraint vectors alone; α = 1. The Gram matrix $L_0 = BB^T$ is constructed entirely from the integer incidence matrix B with entries in {0,1}. The norms $|c_i|^2 = 3$ are fixed by the combinatorial structure of the Fano plane (each line through 3 points), not by any free parameter. Therefore the scalar c in condition (iii) is fixed without external input: c = 1 follows from the Gram normalisation $Q(\sigma) = \Sigma_i(c_i\cdot\sigma)^2$ with $|c_i|^2 = 3$ as given by [T2]. No external parameter is available to modify c within the CCC sector: $\rho = \xi^{-4}$ is the only energy scale (Fundamental Energy Density Lemma, §2), and it enters multiplicatively as the conversion to physical units, not as a modification of c. Therefore c = 1 and α = 1 exactly.

Combining (i)–(iv): the penalty is local, quadratic, group-invariant, and Gram-normalised. These four conditions together force $V\_c = ½\ \sigma^T L\_eff\ \sigma \cdot \rho$ uniquely. The scalar c is therefore uniquely determined and equals 1; $\rho = \xi^{-4}$ enters equation (2) separately as the dimensional prefactor converting constraint-space energy to physical energy density, and has no freedom to modify c.
■

Corollary (L_eff as Hessian). *The Hessian of V_c with respect to σ is:*

$\partial^2 V\_c / \partial\sigma_i\partial\sigma_j = \rho\cdot(L\_eff)_{ij} = \xi^{-4}(L\_eff)_{ij}$ (3)

This follows immediately from the quadratic form structure and is not an additional assumption.

Remark on the role of irreducibility. The Schur's Lemma step is essential. If V_p carried a reducible representation of PGL(3,2), invariant quadratic forms could mix different irreducible subspaces with independent coefficients, leaving a free parameter. The irreducibility of V_p — proved in [T4] — eliminates this freedom and makes the penalty structurally unique. The K = 7 architecture is not merely sufficient for the Uniqueness Theorem; it is necessary.

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5. The Dimensionless Departure Parameter: No-Alternative-Scaling Lemma

The commitment density field $s(x,t)$ has dimension $[s] = [L^{-1}]$. Physical perturbations are $\delta s(x,t) = s(x,t) - s_0$, also of dimension $[L^{-1}]$. To enter the dimensionless constraint penalty V_c (equation (2)), δs must be made dimensionless.

Lemma (No Alternative Scaling). *Within the CCC sector, the unique dimensionless parametrisation of a commitment density perturbation $\delta s \in [L^{-1}]$ is:*

$\sigma = \delta s \cdot \xi$ (4)

No alternative scaling exists without importing structure external to the CCC sector.

Proof. We must form a dimensionless quantity from δs and CCC-sector variables. In natural units ($\hbar = c = 1$), the CCC sector is characterised by the single constraint $\rho\xi^4 = 1$, reducing all available dimensional quantities to a single independent scale. Specifically:

— the CCC variables are $\{\rho, \xi\}$ subject to $\rho = \xi^{-4}$;
— in natural units, $\rho^{(1/4)} = \xi^{-1}$, so ρ^n for any n is a power of ξ^{-1};
— therefore the only independent CCC-sector length scale is ξ.

The most general dimensionless combination of δs with CCC-sector quantities is $\delta s \cdot \xi^a \cdot \rho^b$ for real a, b. Dimensionlessness requires $[L^{-1}][L]^a[L^{-4}]^b = 1$, giving $a - 4b = 1$. Using $\rho = \xi^{-4}$ in natural units, $\rho^b = \xi^{-4b}$, so the combination becomes $\delta s \cdot \xi^{(a-4b)} = \delta s \cdot \xi^1$ for all admissible (a, b). The combination is $\sigma = \delta s \cdot \xi$ regardless of the specific choice of a and b satisfying the constraint. Every CCC-sector-constructible dimensionless combination of δs is equal to $\delta s \cdot \xi$ in natural units.

The alternative $\delta s/s_0$ would require knowledge of the equilibrium commitment density s_0 — a background quantity not determined by the K = 7 spectral data or the CCC threshold condition. It constitutes external input and is therefore inadmissible within the CCC sector.

Therefore $\sigma = \delta s \cdot \xi$ is unique. ■

Remark (physical content of σ). The amplitude $|\sigma| = 1$ corresponds to $\delta s = \xi^{-1}$ — one quantum of commitment above equilibrium per coherence length, the minimum resolvable perturbation of the commitment density field. In the CCC framework, $\rho\xi^4 = 1$ sets one commitment event per coherence volume; a perturbation $\delta s = \xi^{-1}$ adds exactly one event per coherence length, so σ counts commitment quanta above equilibrium in natural units. This physical interpretation is consistent with and confirms the uniqueness argument: σ is both the unique dimensionless parametrisation and the natural quantum of commitment perturbation.

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6. The Constraint-to-Mass Proposition

The two preceding sections establish what the constraint potential is (Uniqueness Theorem, §4) and how to parametrise it (No-Alternative-Scaling Lemma, §5). Before proving the Main

Theorem, two clarifications are needed: first, the connection between the vector $\sigma \in V_p$ used in §§3–4 and the scalar $\sigma = \delta s \cdot \xi$ introduced in §5; second, why the constraint potential contributes to the *mass* of the κ-field rather than to some other coupling.

From vector to scalar: the role of degeneracy. In §§3–4, σ denotes a departure vector in $V_p \cong \mathbb{R}^4$, and $Q_p(\sigma) = \sigma^T L_{eff}\, \sigma$ is a quadratic form on that 4-dimensional space. In §5, $\sigma = \delta s \cdot \xi$ is introduced as a scalar — the dimensionless version of the scalar field perturbation δs(x,t). These are connected as follows. Because $L_{eff} = (4/3)I_4$ is proportional to the identity on V_p, every unit direction $\hat{e} \in V_p$ is an eigenvector with the same eigenvalue $\lambda_{eff} = 4/3$. The scalar projection of the departure vector onto any unit direction ê yields $Q_p = \lambda_{eff}(\delta s \cdot \xi)^2$, independently of which ê is chosen. Because L_{eff} is proportional to the identity, the quadratic form defines a unique scalar amplitude independent of basis choice, allowing consistent identification with a scalar field degree of freedom; no preferred direction in V_p is selected and no ambiguity of "choosing a direction" arises. The κ-field is this scalar amplitude, and its mass is well-defined precisely because the degeneracy of L_{eff} makes the choice of direction irrelevant. The transition from vector to scalar is exact, not approximate, and follows from the isotropy established in §4 via Schur's Lemma.

Proposition (Constraint Energy as Mass Term). *The constraint potential V_c is quadratic in the field s (not in $\partial_\mu s$) and is a Lorentz scalar. By the Lagrangian Uniqueness Theorem [T5], the unique Lorentz-invariant Lagrangian for s containing V_c is the Klein–Gordon Lagrangian, and V_c necessarily appears as the mass term with $m^2 = \lambda_{eff}\, \xi^{-2}$.*

Proof. We establish each claim in turn.

V_c is quadratic in s, not in $\partial_\mu s$. From equation (2) and the conversion $\sigma = \delta s \cdot \xi$ (equation (4)), $V_c = \frac{1}{2} \lambda_{eff} (\delta s)^2 \xi^{-2}$. This is a function of the field value δs(x,t) at a point — not of its spacetime derivatives $\partial_\mu(\delta s)$. Constraint violations are local: they penalise the *amplitude* of departure from $\mathcal{C}$, not the *rate of change* of that departure. Derivative terms in a potential would penalise configurations that vary rapidly in spacetime; constraint violations penalise configurations that deviate from equilibrium regardless of how quickly they vary.

V_c is a Lorentz scalar. The field s(x,t) is a scalar field [T5]. λ_{eff} is dimensionless and group-theoretically invariant (established in §4). ξ is a scalar. Therefore $V_c = \frac{1}{2} \lambda_{eff} (\delta s)^2 \xi^{-2}$ is a product of scalars — a Lorentz scalar. ■

Why this forces the mass term. In a Lorentz-invariant Lagrangian $\mathcal{L}$ for a scalar field s, terms can be organised by their derivative content:

— zero derivatives: functions of s only, forming the potential V(s);
— one derivative: terms linear in $\partial_\mu s$ cannot form Lorentz scalars without an additional four-vector, and no such vector is available in the scalar field sector; such terms are therefore absent;
— two derivatives: $\partial_\mu s\, \partial^\mu s$, the kinetic term;
— higher: excluded by the Nonlinear Exclusion Theorem [T5] at the fundamental level.

V_c is a zero-derivative term quadratic in s. It therefore contributes to the potential sector $V(s) = \frac{1}{2}m^2s^2$. By the Lagrangian Uniqueness Theorem [T5], the unique admissible Lagrangian for s is the Klein–Gordon form $\frac{1}{2}(\partial s)^2 - \frac{1}{2}m^2s^2$, and the potential sector is entirely the mass term. Therefore $V_c = \frac{1}{2}m^2(\delta s)^2$ with m^2 identified by matching. There is no other term in the admissible Lagrangian into which V_c could flow.

The propagation question resolved. The referee objection "why does constraint energy propagate as a relativistic scalar field?" rests on an implicit assumption that the constraint and the propagation are independent choices. They are not. The Lagrangian Uniqueness Theorem [T5] establishes that any admissible Lagrangian for the commitment density field already has the Klein–Gordon form — including its kinetic term — independently of V_c. The constraint potential V_c does not *cause* s to propagate relativistically; rather, s is already a relativistic scalar (by Lorentz covariance and [T5]), and V_c is the specific quadratic potential that fixes the mass parameter of that propagation. The two facts are logically independent: [T5] fixes the form; the present paper fixes the mass parameter within that form.

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7. Uniqueness of the κ-Field Equation

The three preceding sections established: (i) the constraint potential V_c is uniquely fixed (§4); (ii) the dimensionless parametrisation $\sigma = \delta s \cdot \xi$ is unique (§5); (iii) V_c necessarily appears as a Klein–Gordon mass term (§6). We now close the chain by proving that the full field equation for s(x,t) is itself uniquely determined within the admissibility class — so that not only the mass parameter but the entire dynamical law is forced.

Theorem (Uniqueness of the κ-Field Equation). *Let s(x,t) be the commitment density field associated with irreversible fact formation. Under the admissibility conditions: (i) locality, (ii) Lorentz invariance, (iii) single scalar degree of freedom (minimal fact architecture, §§3–6), (iv) second-order dynamics, and (v) quadratic constraint penalty (Uniqueness Theorem, §4); the unique admissible equation of motion is:*

$(\square + m^2)s = n_{committed}$ (10)

with $m^2 = \lambda_{eff}\, \xi^{-2}$. No alternative local, Lorentz-invariant field equation satisfies all five conditions.

Proof.

Step 1 — General form of admissible field equations. A local field equation for a scalar field must be built from s and its derivatives $\partial_\mu s$, $\partial_\mu \partial_\nu s$, … Under Lorentz invariance, the only admissible scalar differential operators are s itself and $\square s = \partial_\mu \partial^\mu s$. The most general candidate equation therefore takes the form:

$A \cdot \square s + B \cdot s + C \cdot F(s) = J(x)$

where $F(s)$ represents possible nonlinear terms and $J(x)$ is a source.

Step 2 — Exclude nonlinear terms. The constraint potential is exactly quadratic (§4, Uniqueness Theorem); higher-order terms introduce dimensionless couplings not constructible within the CCC sector, violating the no-free-parameter condition. Therefore $F(s) = 0$ at the fundamental level. This is the same exclusion used in §4 condition (ii) and in §13.3.

Step 3 — Exclude higher-derivative dynamics. Adding operators such as $\Box^2 s$ or $\partial^4 s$ introduces additional propagating modes, Ostrogradsky ghost instabilities, and — critically within the CCC framework — new energy scales beyond ξ. Such scales are inadmissible: ξ is the unique length scale of the CCC sector (Fundamental Energy Density Lemma, §2), and any additional scale would break dimensional closure and violate the no-free-parameter condition. Therefore only second-order derivatives are admissible, and the equation reduces to:

$$A \cdot \Box s + B \cdot s = J(x)$$

Step 4 — Fix kinetic normalisation. The coefficient A can be absorbed by a field rescaling $s \to s' = \sqrt{A} \cdot s$, which leaves the physical content unchanged ($n\_committed$ rescales accordingly as $n\_committed \to n\_committed/\sqrt{A}$, preserving its dimension $[L^{-3}]$ and physical interpretation as committed event density) and yields $\Box s + \tilde{B} \cdot s = J(x)$ with $\tilde{B} = B/A$. The canonical normalisation of the kinetic term $\frac{1}{2}(\partial s)^2$ is fixed by Lorentz covariance and the Lagrangian Uniqueness Theorem [T5]; no freedom remains in A.

Step 5 — Identify the mass coefficient. From §6, the only admissible zero-derivative quadratic term in the Lagrangian is $\frac{1}{2}m^2 s^2$, giving the variation $\delta V/\delta s = m^2 s$. Therefore $\tilde{B} = m^2$. The specific value $m^2 = \lambda\_eff\, \xi^{-2} = (4/3)\xi^{-2}$ is derived in §8 below from the CCC energy density and the $K = 7$ spectral data; that derivation is logically independent of the present step, which establishes only that $\tilde{B}$ must equal some positive m^2 fixed by the constraint potential. The numerical value is not assumed here.

Step 6 — Source term. The only admissible source is $n\_committed(x)$: local fact production, with dimension $[L^{-3}]$ (committed events per unit volume), constructed from within the CCC architecture. No other source exists within the admissibility class without introducing external structure.

Combining Steps 1–6: every free element of the general form is eliminated. The equation $(\Box + m^2)s = n\_committed$ is the unique survivor. ■

Failure table for the field equation.

Alternative	Failure mode
Nonlinear terms $F(s)$	Introduce new couplings; CCC closure broken
Higher derivatives $\Box^2 s$	Introduce new scales and ghost instabilities
Multiple scalar fields	Violate minimal fact architecture (§§3–6)
Non-Lorentz-invariant terms	Violate admissibility condition (ii)

Alternative	Failure mode
Different potential	Excluded by Uniqueness Theorem (§4)
Different mass	Excluded by Main Theorem (§8)

Corollary (Full Dynamical Closure). *Within the VERSF admissibility class, the commitment density field is uniquely described by the Klein–Gordon equation with mass $m^2 = \lambda_{eff}\, \xi^{-2}$. No alternative field dynamics exist without violating at least one of: CCC scaling, K = 7 minimal fact architecture, Lorentz admissibility, or the no-free-parameter condition.*

The chain is now complete. The locked sequence is: facthood → CCC → ξ; K = 7 → λ_eff; symmetry → unique penalty; CCC scaling → unique σ; Lagrangian admissibility → unique form; second-order dynamics → unique equation. Zero free parameters remain at any stage.

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8. Main Theorem: $m^2 = \lambda_{eff}\, \xi^{-2}$

With the three preparatory results in place — Uniqueness of V_c (§4), uniqueness of σ (§5), and V_c as mass term (§6) — the Main Theorem follows by direct computation.

Theorem (κ-Field Mass Identification). *Under the CCC condition and the K = 7 minimal fact architecture, with $\lambda_{eff} = 4/3$ the minimum positive eigenvalue of L_{eff} and ξ the coherence scale:*

$m^2 = \lambda_{eff}\, \xi^{-2} = (4/3)\, \xi^{-2}$ (5)

Each step is the unique admissible choice. No free parameters appear.

Proof.

Step 1 — Constraint potential in terms of σ. By the Uniqueness Theorem (§4) and $L_{eff} = (4/3)I_4$:

$V_c = \frac{1}{2} \lambda_{eff} |\sigma|^2 \cdot \rho$ (6)

Step 2 — Convert to physical field variable. By the No-Alternative-Scaling Lemma (§5), $\sigma = \delta s \cdot \xi$; by equation (1), $\rho = \xi^{-4}$:

$V_c = \frac{1}{2} \lambda_{eff} (\delta s \cdot \xi)^2 \cdot \xi^{-4} = \frac{1}{2} \lambda_{eff} (\delta s)^2 \cdot \xi^{-2}$ (7)

Step 3 — Dimensional check. $[V_c] = [\lambda_{eff}][\delta s]^2[\xi]^{-2} = 1 \cdot [L^{-1}]^2 \cdot [L]^{-2} = [L^{-4}]$ = energy density in 3+1D natural units. ✓

Step 4 — Identify mass term. By the Constraint-to-Mass Proposition (§6), V_c contributes to the Lagrangian as the unique zero-derivative quadratic term in s:

$V\_KG = ½ m^2 (\delta s)^2$ (8)

Step 5 — Match. Equating (7) and (8) and cancelling $½(\delta s)^2$:

$m^2 = \lambda\_eff\ \xi^{-2} = (4/3)\ \xi^{-2}$ (9)

Therefore $m = \sqrt{(4/3)} \cdot \xi^{-1}$, consistent with [T4]. ■

The inputs and their locking mechanisms.

Input	Source	Locked by
$\lambda\_eff = 4/3$	K = 7 spectral computation [T4]	Fano-plane structure; no free eigenvalue
$\rho = \xi^{-4}$	CCC threshold [T1]	Fundamental Energy Density Lemma (§2)
$\sigma = \delta s \cdot \xi$	CCC sector scaling	No-Alternative-Scaling Lemma (§5)
$V\_c = ½\lambda\_eff\|\sigma\|^2\rho$	Gram-matrix construction + integer incidence structure of Fano plane (§4, [T2])	Uniqueness Theorem (§4)
V_c → mass term	Admissible Lagrangian [T5]	Constraint-to-Mass Proposition (§6)

Each row is a theorem or established result, not a modelling choice. The first four rows are independent inputs; the fifth (V_c → mass term) follows from §6 given the first four. The identification $m^2 = \lambda\_eff\ \xi^{-2}$ is a logical consequence of all five.

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9. The κ-Field as a Propagating Constraint Mode

The Main Theorem establishes m at zero momentum. We confirm that this mass governs propagation.

The field equation from the constraint Lagrangian. The full Lagrangian for the commitment density field [T3, T5]:

$\mathcal{L} = ½(\partial\_\mu s)(\partial^\wedge\mu s) - ½m^2s^2 + s \cdot n\_committed$

The kinetic term $½(\partial s)^2$ is fixed by Lorentz covariance [T5], independent of V_c. The potential term $½m^2s^2$ is V_c from §8. The source n_committed has dimension $[L^{-3}]$ (committed events per unit volume), distinct from the void energy density $\rho = \xi^{-4}$; the notation is chosen to prevent confusion. The Euler–Lagrange equation is:

$(\square + m^2)s = n\_committed$ (10)

Mode structure. Because $L\_eff = (4/3)I_4$ is proportional to the identity on V_p, all directions in physical state space carry the same mass $m = \sqrt{(4/3)}\ \xi^{-1}$. The κ-field is not the lowest eigenmode of a spectrum — it is the unique propagating excitation of the commitment constraint structure. This is consistent with the minimal field content condition (A6) of [T5].

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10. The Commitment-Lattice Phonon Analogy: Precise Statement

Phonon analogy table.

Crystal concept	VERSF concept	Mathematical object
Lattice equilibrium	Constraint surface $\mathcal{C}$	Equilibrium s_0
Atomic displacement	Commitment perturbation	δs(x,t)
Lattice constant	Coherence scale	ξ
Dynamical matrix	Closure operator	$L\_eff = (4/3)I_4$
Dynamical matrix eigenvalue	Spectral invariant	λ_eff = 4/3
Energy density unit	Void energy density	$\rho = \xi^{-4}$
Phonon frequency²	κ-field mass²	$m^2 = \lambda\_eff\ \xi^{-2}$
Phonon field	Commitment perturbation field	κ(x,t) = δs(x,t)

Where the analogy is exact. The dynamical matrix $D_{ij} = \partial^2V/\partial u_i\partial u_j$ determines phonon frequencies ω^2 = eigenvalue(D)/M; the phonon equation is $\partial^2u/\partial t^2 + (D/M)u = 0$. In VERSF: $L\_eff = \partial^2V\_c/\partial\sigma^2|\_\{\sigma=0\}$ determines $m^2 = \lambda\_eff\ \xi^{-2}$; the field equation is $(\square + m^2)\delta s = 0$. The analogy is exact at quadratic order, which is the only level the K = 7 architecture determines.

Where the analogy breaks down — a testable prediction. Crystal phonons exhibit band structure because the dynamical matrix varies across the Brillouin zone. In VERSF, $L\_eff = (4/3)I_4$ is k-independent and isotropic: the κ-field has a single dispersion branch $\omega^2 = k^2 + m^2$ with no anisotropy. This is a direct, falsifiable consequence of the K = 7 constraint structure and the Schur's Lemma isotropy result of §4. Any experimental detection of dispersion anisotropy — for instance, through direction-dependent oscillation frequencies in the decay correction of §11.6 — would falsify the K = 7 architecture. The isotropy of L_eff is not an aesthetic feature of the analogy; it is a prediction.

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11. Consistency Checks

10.1 Dimensional consistency. $[m^2] = [\lambda\_eff][\xi]^{-2} = 1\cdot[L]^{-2} = [E]^2$ in natural units. Correct for a mass squared. ✓

10.2 Recovery of CCC scaling. $m \sim \xi^{-1} = (\rho/\hbar c)^{(1/4)}$. This is the scaling derived in [T4] by dimensional closure. The prefactor $\sqrt{(4/3)}$ is now derived from the spectral invariant, not assumed. ✓

10.3 Absence of massless limit. $m = 0$ would require $\lambda_{eff} = 0$, i.e., a zero mode of L_{eff} in V_p. This would require the Fano-plane representation to be reducible on V_p, which is excluded by the irreducibility under $PGL(3,2) \cong PSL(2,7)$ [T4, F3]. The κ-field cannot be tuned to zero mass without abandoning the $K = 7$ constraint structure. ✓

10.4 Consistency with Lagrangian Uniqueness [T5]. [T5] proves the *form* of the Lagrangian is uniquely Klein–Gordon. The present paper derives the *mass parameter* within that form. Together they establish a completely determined Lagrangian with no free parameters. ✓

10.5 Consistency with Field Identification [T5]. The identification $s \equiv \kappa$ under retarded boundary conditions [T5] holds for $m > 0$ (the massive case proven in [T5]), a condition now guaranteed by $\lambda_{eff} = 4/3 > 0$. The value $m = \sqrt{(4/3)}\xi^{-1}$ fixes the Green's function G_{ret} and the memory kernel $K(\tau) \sim \cos(m\tau)/\tau$ [T6]. The field identification is unaffected; the specific dynamical structure is now fully determined. ✓

10.6 Consistency with observable predictions. The oscillatory decay correction

$$N(t) \sim e^{(-\lambda t)} + \varepsilon \cdot \cos(mt + \varphi) / [(\lambda^2 + m^2)t]$$

with $m = \sqrt{(4/3)}\xi^{-1}$ is now fully derived from first principles. The frequency $f = \sqrt{(4/3)}\, c/(2\pi\xi)$, the amplitude suppression $(\lambda^2 + m^2)^{-1}$, and the absence of directional variation in f (isotropy prediction, §10) are all structural consequences of the $K = 7$ architecture. ✓

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12. Anticipated Objections and Clarifications

11.1 "The Uniqueness Theorem assumes locality and quadratic order — are these not additional inputs?"

Locality is treated here as a structural consequence of the CCC architecture rather than an independent modelling assumption. The CCC condition defines commitment events as local facts at spacetime points: a fact is an irreversible commitment event at a specific location, and the action budget $\rho L^4 \gtrsim \hbar c$ is evaluated within a region of size L. A non-local penalty — one depending on the commitment configuration at spatially separated points — would require causal information to be available instantaneously across such separations, violating the facthood condition that each commitment event is localised within its coherence volume ξ^4. The derivation of locality from the facthood structure is developed in [T1] and [T7]; the present paper takes it as an established structural consequence of the CCC programme. Quadratic order follows from the equilibrium condition (gradient vanishes at $\mathcal{C}$) and the Nonlinear Exclusion Theorem of [T5], both of which are theorems of the VERSF architecture.

11.2 "Schur's Lemma requires irreducibility of the PGL(3,2) action on V_p — is this established?"

Yes. The irreducibility of the 4-dimensional representation of PGL(3,2) ≅ PSL(2,7) on V_p is established in [T4] via the character theory of PGL(3,2) and the projection analysis that defines V_p as the complement of the uniform mode in V_6. The argument is outlined in the §4 Remark above; the full group-theoretic details are in [T4].

11.3 "The Constraint-to-Mass Proposition uses the Lagrangian Uniqueness Theorem — isn't that circular?"

No. The Lagrangian Uniqueness Theorem [T5] establishes the *form* of the admissible Lagrangian — Klein–Gordon — independently of any specific mass value. The Constraint-to-Mass Proposition uses this established form to show that V_c (a zero-derivative quadratic in s) must enter that Lagrangian as the mass term, ruling out other roles. The mass value $m^2 = \lambda_{eff}\, \xi^{-2}$ is then derived from V_c. The logical order is: form first (via [T5]), mass second (via the present paper). There is no circularity.

11.4 "Why does constraint energy propagate at all — why is there a kinetic term?"

The kinetic term $\frac{1}{2}(\partial s)^2$ is not derived from V_c. It is fixed independently by Lorentz covariance: any local Lagrangian for a scalar field in 3+1D Minkowski space must include a term quadratic in $\partial_\mu s$ for the field to propagate at finite speed. This is established in [T5] via the Lagrangian Uniqueness Theorem (admissibility condition on causal propagation). The constraint potential V_c sets the mass; the kinetic term sets the propagation. The two are logically independent, sourced by different admissibility conditions.

11.5 "The CCC condition is a threshold inequality — why use equality?"

ξ is defined as the scale at which the threshold is exactly met. The κ-field mass governs dynamics at the coherence scale, where the constraint is marginally satisfied. Sub-threshold regions do not support committed facts; the field equation does not apply there. Using $\rho = \xi^{-4}$ is therefore correct and not an approximation.

11.6 "The phonon analogy — the κ-field is relativistic, phonons are not."

The analogy holds at the mass term, where both reduce to the rest energy from the quadratic potential. The relativistic kinetic term $\frac{1}{2}(\partial s)^2$ and the non-relativistic phonon kinetic term $\frac{1}{2}\dot{u}^2$ differ in structure, but the mass identification m^2 = eigenvalue(D)/M (phonon) and $m^2 = \lambda_{eff}\, \xi^{-2}$ (κ-field) are structurally identical at zero momentum. The two kinetic sectors are independently fixed by their respective symmetry requirements and do not affect the mass comparison.

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13. Failure Modes of Alternative Architectures

A strong derivation does not merely exhibit one successful route to a result; it shows why nearby alternatives fail. Having established in §§4–8 that the κ-field mass $m^2 = \lambda\_eff\ \xi^{-2}$ is uniquely fixed within the admissible K = 7 / CCC architecture, we now make explicit the corresponding failure modes of alternative constructions. The point of this section is not rhetorical. It is to show that the derived mass formula is not simply one option among many, but the surviving outcome after the admissibility conditions eliminate the obvious competitors.

12.1 What counts as an alternative?

Within the present context, there are only four broad ways the mass derivation could have come out differently:

(1) the internal constraint architecture could differ from K = 7;
(2) the constraint penalty could differ from the Gram quadratic form;
(3) the dimensionless scaling of the field perturbation could differ from $\sigma = \delta s \cdot \xi$;
(4) the quadratic constraint penalty could enter the Lagrangian in some role other than the Klein–Gordon mass term.

Sections 4–6 showed that options (2)–(4) are excluded within the admissible architecture. Here we show that option (1) also fails in the precise sense that it destroys one or more of the structural properties on which the derivation depends: irreducibility, isotropy, uniqueness, or freedom from external parameters.

12.2 Failure of non-K = 7 architectures

The K = 7 architecture is not merely a convenient combinatorial choice. It is the minimal architecture for a stable fact established in [T2], and the present derivation relies on three specific consequences of that structure: the existence of a canonical incidence matrix B; the projected closure operator L_eff acting irreducibly on V_p; and the isotropic result $L\_eff = (4/3)I_4$.

If K < 7, the constraint system is underdetermined. Either the projected physical space contains flat directions, giving zero modes in the penalty Hessian, or the architecture fails to support a stable committed fact at all. In either case the spectral gap $\lambda\_eff > 0$ is lost, and the κ-field either becomes massless or ceases to be well-defined as a propagating constraint mode. This is incompatible with the existence of a robust local fact and therefore excluded by the K = 7 no-go theorem.

If K > 7, the problem is different. The system becomes overstructured. Additional constraints beyond the minimal admissible set generally introduce one of two failures:

— *reducibility*: the physical state space decomposes into multiple invariant sectors, allowing more than one independent quadratic invariant and hence reintroducing free coefficients into the penalty functional;
— *anisotropy*: the projected Hessian is no longer proportional to the identity, so λ_eff splits into several eigenvalues.

In either case the result is fatal to the uniqueness argument. Reducibility breaks the Schur's Lemma step used in §4, while anisotropy destroys the single-branch dispersion relation of §10. Instead of one κ-field mass, one obtains either a multiplet of masses or a direction-dependent stiffness. In all known overstructured constructions, additional constraints beyond the minimal admissible set either introduce reducibility or anisotropy on the projected physical space; no counterexample preserving both irreducibility and isotropy is known, and any such case would require explicit verification against the $K = 7$ no-go result of [T2]. The $K = 7$ case is therefore structurally distinguished: it is the unique case simultaneously sufficient for stable facthood and minimal enough to retain irreducibility and isotropy on V_p.

12.3 Failure of alternative penalty functionals

A second class of alternatives keeps $K = 7$ fixed but modifies the constraint penalty away from the Gram form $Q\_p(\sigma) = \sigma^T L\_eff\ \sigma$. There are only three genuine possibilities.

Different quadratic form. On V_p, invariance under the irreducible action of PGL(3,2) ≅ PSL(2,7) forces every invariant quadratic form to be proportional to the identity, and hence proportional to L_eff. Any non-Gram quadratic form is therefore either equivalent to the Gram form up to an overall factor — already absorbed by the Gram construction — or breaks the symmetry and is inadmissible.

Continuous rescaling of the Gram form. The constraint vectors arise from the integer incidence structure of PG(2,2) with entries in {0,1}; each line contains exactly 3 points, so $|c_i|^2 = 3$ is combinatorially fixed. Any rescaling $c_i \rightarrow k\ c_i$ would destroy the incidence interpretation. No continuous normalisation freedom exists, as established in §4.

Higher-order terms. Terms such as $|\sigma|^4$ or $|\sigma|^6$ fail twice: at the fundamental level they are excluded by the Nonlinear Exclusion Theorem of [T5]; more basically, they introduce additional dimensionless couplings not constructible from the CCC sector alone, violating the no-free-parameter condition. No alternative penalty survives: non-Gram quadratics violate symmetry, rescaled Gram forms violate the combinatorial incidence structure, and higher-order terms violate admissibility and parameter closure.

12.4 Failure of alternative scalings

A third class of alternatives preserves the penalty form while changing the dimensionless parametrisation of the field perturbation. §5 proved that within the CCC sector the unique dimensionless combination of $\delta s \in [L^{-1}]$ is $\sigma = \delta s \cdot \xi$.

A scaling such as $\delta s/s_0$ requires the equilibrium value s_0, which is not fixed by the CCC threshold or the $K = 7$ spectral data — it is a background choice imported from outside the derivation.

A scaling using ρ rather than ξ, for example δs·ρ^(−1/4), is not genuinely different: because $\rho = \xi^{-4}$, it reduces identically to $\delta s \cdot \xi$. Likewise any expression of the form $\delta s \cdot \xi^a \rho^b$ with total dimension zero collapses to $\delta s \cdot \xi$ after using the CCC relation. There is no true family of alternatives inside the CCC sector — all admissible scalings are equivalent to the same one.

The only remaining alternatives require a new independent scale ℓ^*. But such a scale is neither part of the K = 7 spectral data nor the CCC threshold relation. Introducing it breaks the closure of the derivation and converts the mass formula into a parametrised ansatz rather than a theorem.

12.5 Failure of alternative Lagrangian roles

A fourth possibility accepts the quadratic scalar potential $V_c = \frac{1}{2}\lambda_{eff}(\delta s)^2\xi^{-2}$ but denies it should be interpreted as a mass term. This also fails.

Because V_c contains no derivatives, it belongs to the zero-derivative sector of the Lagrangian. Because it is quadratic in the scalar field and Lorentz invariant, it cannot contribute to the kinetic sector, to a derivative coupling, or to a tensor interaction. Within the admissible scalar-field Lagrangian classified in [T5], the only available home for such a term is the Klein–Gordon potential $V_{KG} = \frac{1}{2}m^2s^2$. Any attempt to assign V_c a different dynamical role would require enlarging the admissibility class — for example by allowing extra fields, nonlocality, higher derivatives, or Lorentz-violating structures. But once those are admitted, one is no longer discussing the VERSF commitment field derived from minimal fact architecture. One has changed the theory.

12.6 Summary of exclusions

Alternative	**Failure mode**
K < 7	Flat directions, loss of spectral gap, failure of stable facthood
K > 7	Reducibility or anisotropy; loss of uniqueness and single-branch dispersion
Non-Gram quadratic penalty	Violates PGL(3,2) invariance or is equivalent to Gram form
Rescaled Gram penalty	Violates fixed combinatorial incidence structure of PG(2,2)
Higher-order penalty	Introduces external couplings; excluded by admissibility
Alternative scaling of δs	Collapses to $\delta s\cdot\xi$ or imports external scale
Alternative role for V_c in $\mathscr{L}$	Impossible within local Lorentz-invariant scalar-field admissibility

The exclusion is not generic or aesthetic. Each alternative breaks one of the specific structures on which the derivation rests: facthood, irreducibility, isotropy, closure, or Lorentz admissibility.

12.7 Consequence for the mass formula

The mass formula $m^2 = \lambda_{eff}\,\xi^{-2}$ should therefore be read not merely as the output of one successful construction, but as the surviving element of a constrained architecture after the nearby alternatives have been removed. Any alternative mass assignment would require violating at least one of the following: the CCC scaling relation; the K = 7 minimal fact architecture; the irreducible PGL(3,2) symmetry of V_p; or the admissible Klein–Gordon form of the commitment-field Lagrangian. This is the sense in which the κ-field mass is structurally locked.

The formula is not simply compatible with the architecture; it is what remains when the admissibility conditions have done their full eliminative work. This is not merely uniqueness within a model, but uniqueness under admissibility: the mass is fixed not by choice, but by the requirement that a local, causal, irreducible fact-forming structure exists at all.

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14. Scope and Open Questions

What this paper derives:

- Uniqueness of the constraint penalty V_c under locality, quadratic order, PGL(3,2) invariance, and Gram normalisation (§4)
- Uniqueness of the dimensionless departure parameter $\sigma = \delta s \cdot \xi$ within the CCC sector (§5)
- That V_c necessarily appears as a KG mass term in any admissible Lagrangian (§6)
- That $m^2 = \lambda_{eff} \xi^{-2}$ follows from these three locked results (§8)
- Precise statement of the phonon analogy and its isotropy prediction (§10)

What is taken from prior work:

- $\lambda_{eff} = 4/3$ from the K = 7 spectral computation [T4]
- $\rho = \xi^{-4}$ from the CCC condition [T1]
- The Lagrangian Uniqueness Theorem [T5]
- The field identification $s \equiv \kappa$ [T5]
- Irreducibility of V_p under PGL(3,2) [T4]

What remains open:

Anharmonic corrections to V_c are excluded at the fundamental level by [T5] but may arise in effective field theory at scales above ξ. Their structure is not determined by the present analysis.

The relation between V_c and the fold-sector Lagrangian of the full BCB framework [T3] requires a companion derivation.

The value of ξ depends on ρ (void energy density), identified with the cosmological vacuum energy. An internal derivation of ρ, rather than identification with the observed cosmological constant, remains open [T1, T2].

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15. Conclusion

We have derived $m^2 = \lambda_{eff} \xi^{-2} = (4/3)\xi^{-2}$ from first principles, and shown that no alternative architecture, penalty, scaling, or Lagrangian assignment is consistent with the admissibility conditions — closing both the derivation gap in [T4] and the eliminative gap implicit in any uniqueness claim.

The derivation is structured around three locking results:

1. Uniqueness of Constraint Penalty (§4). The only penalty functional on V_p that is local, quadratic, invariant under PGL(3,2), and constructible from the K = 7 constraint vectors without external parameters is $V_c = \frac{1}{2} \sigma^T L_{eff} \sigma \cdot \rho$, with coefficient $\alpha = 1$ forced by the Gram normalisation and the constant of proportionality fixed to unity by Schur's Lemma. No alternative potential is admissible.

2. No-Alternative-Scaling Lemma (§5). In the CCC sector with $\rho = \xi^{-4}$, all dimensional quantities reduce to a single scale ξ. Every CCC-sector-constructible dimensionless combination of δs collapses to $\delta s \cdot \xi$ in natural units. No alternative scaling exists without external input.

3. Constraint-to-Mass Proposition (§6). V_c is quadratic in s (not $\partial_\mu s$) and is a Lorentz scalar. By the Lagrangian Uniqueness Theorem [T5], the unique admissible Lagrangian for s has Klein–Gordon form, and V_c can only enter as the mass term. The constraint energy cannot propagate as anything other than the rest mass of the κ-field.

Given these three results, the Main Theorem (§8) is a four-line calculation. The derivation proceeds by successive eliminations: admissibility fixes the Lagrangian form, CCC fixes the scale, K = 7 fixes the spectrum, and symmetry fixes the penalty — leaving the mass as the only surviving parameter. The mass $m^2 = (4/3)\xi^{-2}$ is a logical consequence of the K = 7 constraint structure and the CCC energy density scale. It is not fitted, not assumed, and not left to analogy. Any alternative mass assignment would require violating at least one of CCC scaling, K = 7 minimal fact structure, irreducible PGL(3,2) symmetry, or admissible Klein–Gordon dynamics; the κ-field mass is therefore not one possible choice within the framework, but the only surviving one.

The phonon picture that motivates the paper remains apt: L_eff is the dynamical matrix, ξ is the lattice constant, and the κ-field mass is the phonon frequency of the commitment constraint lattice. But the present version of that picture is a theorem, not a metaphor. The isotropy of $L_{eff} = (4/3)I_4$ on V_p means the κ-field has a single degenerate dispersion branch — a falsifiable prediction that distinguishes the K = 7 architecture from any anisotropic alternative.

Every quantitative prediction downstream of m — the oscillation frequency $f = \sqrt{(4/3)}\, c/(2\pi\xi)$, the memory kernel $K(\tau) \sim \cos(m\tau)/\tau$, the decay correction $N(t) \sim e^{(-\lambda t)} + \varepsilon \cos(mt+\varphi)/[(\lambda^2+m^2)t]$, the gravitational memory signature $\delta g_{\mu\nu} \sim \cos(mt)/t$ — is now grounded without remainder. The κ-field equation is not postulated but forced: once locality, Lorentz invariance, minimal fact architecture, and CCC scaling are imposed, the Klein–Gordon equation with mass $m^2 = \lambda_{eff}\, \xi^{-2}$ is the only admissible dynamical law. The κ-field mass is a theorem of the commitment architecture.

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