

# Two Kinds of Time: Proto-Time and Physical Time in the VERSF Framework

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## For the General Reader

Everyone knows what time feels like. It moves in one direction, it cannot be reversed, and things that have happened cannot unhappen. A moment ago is gone; a moment from now has not yet arrived. This is so obvious it barely seems worth saying.

And yet physics cannot explain it.

The equations at the heart of quantum mechanics — the theory describing atoms, electrons, and the building blocks of everything — treat time as a simple external backdrop. Time is just there, like a stage the actors perform on, and the equations say nothing about where the stage came from or why it runs in only one direction. If you filmed a quantum system and played the footage backwards, the equations would still be satisfied. The physics works equally well in reverse. But the world clearly does not.

General relativity, Einstein's theory of gravity and spacetime, does something more sophisticated: it describes how time *behaves* — how it slows near massive objects, how it bends and curves alongside space. But it still does not explain where time *comes from*. It describes the stage in exquisite detail without asking who built it.

When physicists try to combine these two great theories into a single description of the universe, time disappears entirely. The fundamental equation of quantum gravity — the Wheeler–DeWitt equation — describes a universe that is completely static. There is no time in it at all. And yet here we are, experiencing relentless forward motion, ageing, remembering the past and anticipating the future.

This paper proposes a resolution. We argue that what we call "time" is actually two different things that happen to coexist, and that physicists have been treating them as the same thing.

The first kind of time — which we call **proto-time** — is the ordering of quantum possibilities. Before anything definite has happened, quantum states exist in a kind of pre-physical sequence. One state follows another. This ordering is real, but nothing has yet been *recorded* about it. No clock has ticked. No fact has formed. Proto-time is the universe before the ink is dry.

The second kind — **physical time** — is what clocks actually measure. It is generated, moment by moment, whenever something irreversible occurs: whenever a definite outcome crystallises

out of quantum possibility and leaves a permanent mark on the world. Every irreversible event — a particle absorbed, a photon detected, a memory formed — is a small act of time-creation. Physical time is not a backdrop. It is a *product*. It accumulates wherever definite facts are being made.

These two kinds of time coexist. Proto-time advances everywhere, always, even in regions of the universe where nothing irreversible is happening. Physical time grows on top of it, locally, at a rate set by how much irreversible activity is occurring in that place.

This distinction — once made precisely — resolves three puzzles at once. The quantum measurement problem (why do quantum possibilities ever become definite outcomes?) dissolves: measurement is simply the moment physical time is generated from proto-time. The mystery of time's direction is explained: physical time can only accumulate forwards, because irreversible facts cannot be un-made. And the frozen universe of quantum gravity is explained: the universe has no *physical* time at the global level because there is nothing outside it to record its state — but proto-time still advances within it, and local physical times emerge wherever subsystems interact irreversibly with each other.

Gravity fits too. Clocks run slower near massive objects because the conditions for forming irreversible facts are suppressed there — physical time accumulates more slowly, and the rate of that accumulation is precisely what the metric of spacetime encodes.

The technical content that follows makes all of this mathematically precise. But the core idea can be stated simply: **time is not something the universe exists inside. It is something the universe produces — one irreversible fact at a time.**

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## Abstract

We argue that the universe contains two distinct but coexisting temporal structures: **proto-time**  $s$ , the global ordering parameter of unitary quantum evolution, and **physical time**  $t$ , which emerges locally from irreversible commitment events. These are not two names for the same thing. Proto-time orders states without generating records and advances universally, including in regions where no physical time accumulates. Physical time is local, irreversible, and tied to the formation of definite facts. Their coexistence —  $s$  running underneath while  $t$  accumulates on top — resolves three long-standing problems simultaneously: the measurement problem in quantum mechanics, the problem of time in quantum gravity, and the origin of temporal directionality. We show that the framework is consistent with both quantum mechanics and general relativity, recovers gravitational time dilation as variation in the local rate  $dt/ds$ , and interprets the Wheeler–DeWitt static universe as the global proto-time substrate from which local physical times emerge. Three falsifiable predictions follow from the two-layer structure. Technical derivations supporting the commitment density  $\lambda(s) = \Sigma/k_B \ln 2$  are collected in appendices.

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# 1. The Problem: One Universe, Two Time Puzzles

Time appears twice in fundamental physics, and both appearances are problematic.

In quantum mechanics, time enters as a fixed external parameter. The Schrödinger equation

$$i\hbar \partial|\psi(t)\rangle/\partial t = \hat{H}|\psi(t)\rangle$$

presupposes a time  $t$  without explaining where it comes from. The equation is reversible — given  $|\psi(t)\rangle$ , one can recover  $|\psi(t_0)\rangle$  exactly — yet physical time is not reversible. Records form, entropy increases, and the past cannot be undone. Standard quantum mechanics contains no mechanism generating this asymmetry. Time is an input, not an output.

In general relativity, time is a component of the spacetime manifold. Proper time along a worldline is

$$d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu$$

This describes how time behaves once events exist, not how events come to exist in the first place. GR operates entirely at the level of realized, definite facts.

The conjunction of the two theories deepens the puzzle. In the semiclassical regime, a quantum system can be in superposition — genuinely indeterminate between outcomes — while the classical spacetime surrounding it presupposes that events are already definite. How does quantum indeterminacy resolve into classical actuality? What role does time play in that resolution? These questions constitute the measurement problem, and they are ultimately questions about the origin of temporal structure.

At the level of quantum gravity, the problem becomes acute. The Wheeler–DeWitt equation

$$\hat{H}|\Psi\rangle = 0$$

describes the wavefunction of the universe as entirely static: there is no external time with respect to which the universe evolves. Time has apparently disappeared from the most fundamental equation we have. And yet here we are, experiencing it.

Both theories describe dynamics *within* time. Neither describes the process by which time is *generated*. That missing layer is the subject of this paper.

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## 2. The Proposal: Two Kinds of Time Coexist

### 2.1 Proto-Time

We introduce a pre-physical ordering parameter  $s \in \mathbb{R}$  with a total order  $<$ . A quantum system evolving under Hamiltonian  $\hat{H}$  without irreversible environmental coupling is parameterized by  $s$ :

$$i\hbar \partial|\psi(s)\rangle/\partial s = \hat{H}|\psi(s)\rangle$$

We call  $s$  **proto-time**. It provides a relational ordering of quantum states —  $|\psi(s_2)\rangle$  follows  $|\psi(s_1)\rangle$  if  $s_1 < s_2$  — but it is formally distinguished from physical time by three criteria:

**(C1) Non-measurability.** Proto-time  $s$  does not appear in the algebra of observables of any physical subsystem. No measurement carried out by any internal clock yields  $s$  as a value. It is a parameter of the global wavefunction, not a locally accessible quantity.

**(C2) Global definition.** Proto-time is defined with respect to the evolution of the entire closed universe wavefunction  $|\Psi(s)\rangle$ . Physical time, by contrast, is always locally defined — different regions accumulate it at different rates.

**(C3) No conjugate observable.** There is no Hermitian operator  $\hat{S}$  satisfying  $[\hat{S}, \hat{H}] = i\hbar$  on the physical Hilbert space — a consequence of Pauli's theorem applied to  $s$ . Physical time  $t$  is not a global canonical parameter either; it is an emergent local quantity.

These are not merely ontological labels. They have formal consequences. In particular, (C1) means that no experiment *internal* to the universe can directly measure the rate at which  $s$  advances. Only the ratio  $dt/ds = \lambda(s)$  is accessible, through physical clocks — and physical clocks work by commitment events, which is where physical time comes from.

Proto-time  $s$  is not an observable parameter but a bookkeeping structure for ordering unitary evolution; all physical predictions are expressed in terms of observable quantities independent of this choice.

### 2.2 Physical Time

**Physical time**  $t$  is not a background. It is generated locally, incrementally, by irreversible physical processes. We call these **commitment events**: the moments at which a quantum possibility becomes a definite physical fact by forming an irreversible record in the environment.

The rate at which physical time accumulates at a given location is the **commitment density**  $\lambda(s)$ , and physical time is:

$$t(s) = \int_0^s \lambda(s') ds'$$

This is the central equation of the framework. Physical time is the accumulated measure of commitment events, integrated over proto-time.  $\lambda(s) = 0$  means no physical time accumulates — the region is pre-temporal, existing only in proto-time.  $\lambda(s) > 0$  means physical time is being generated by irreversible processes.

The commitment density is grounded in thermodynamics (Appendix A):

$$\lambda(s) = \Sigma(s) / (kB \ln 2)$$

where  $\Sigma(s)$  is the local entropy production rate with respect to proto-time. Every quantity on the right is a universal constant ( $kB, \ln 2$ ) or a measurable physical observable ( $\Sigma$ ). No free parameters appear.

### 2.3 What Coexistence Means

The claim is not that proto-time and physical time are two descriptions of the same thing. They are structurally different:

Property	Proto-time $s$	Physical time $t$
Domain	Global, universal	Local, regional
Measurable by internal clocks?	No	Yes
Reversible?	Yes (unitary dynamics)	No ( $\lambda \geq 0$ )
Accumulated by?	Parameterisation of	$\Psi(s)$ — not a separate process
Zero in vacuum?	No	Yes (if $\lambda = 0$ )

Coexistence means that  $s$  advances everywhere and always — including in regions where no commitment events occur and no physical time accumulates. A perfectly isolated quantum system, undergoing unitary evolution with no environmental coupling, exists entirely in proto-time. It has a position in the ordering  $<$  but contributes zero to any local physical time. When it interacts irreversibly with its environment — when a record forms — the commitment event generates a quantum of physical time, and that event is located at a definite proto-time value  $s^*$ .

The two layers are not in competition. Proto-time is the substrate; physical time is what grows on it where conditions are right.

## 3. Consistency with Quantum Mechanics

### 3.1 The Schrödinger Equation Lives in Proto-Time

The Schrödinger equation, as written in standard QM, is parameterized by an external time parameter. In the VERSF framework, that parameter is proto-time  $s$ . For an isolated system:

$$i\hbar \partial|\psi(s)\rangle/\partial s = \hat{H}|\psi(s)\rangle$$

This is not a modification of quantum mechanics. It is the same equation with an explicit identification of what the parameter means: the global ordering of quantum states, prior to and independent of any local physical time.

When  $\lambda(s) = \lambda_0$  is constant — when commitment events occur at a steady rate — physical time is simply proportional to proto-time:  $t = \lambda_0 s$ . The Schrödinger equation then reads exactly as in standard QM, with  $t$  replacing  $s$ . All standard predictions are recovered identically.

### 3.2 Observation is Commitment

In standard QM, measurement is treated as an external intervention that "collapses" the wavefunction. This has never been satisfactorily explained. In the VERSF framework, it has a precise meaning: **observation is the process by which a proto-fact undergoes commitment.**

A quantum system prior to measurement is a **proto-fact**: physically real in the sense that it determines the probability distribution over future outcomes and participates in interference phenomena, but not yet a committed fact because no irreversible environmental record has formed.

When the system interacts irreversibly with a measuring apparatus — when the environmental branches corresponding to different outcomes become operationally distinguishable — a commitment event occurs. The proto-fact becomes a committed fact. Physical time has been generated. The "collapse" is not a physical process separate from dynamics; it is the moment at which proto-time translates into physical time.

This reframing has a precise consequence:

**Proposition:** No proto-fact is directly observable. What is observed in any measurement is the result of a commitment event, not the proto-fact itself.

*Reason:* Observation requires a physical record in the observing system. A physical record is a committed fact. Therefore, the immediate object of observation is always a committed fact. The proto-fact prior to commitment is accessible only through its statistical influence on the distribution of committed outcomes — through the Born-rule probability measure it assigns to the space of possible records.

This is not a limitation of our instruments. It is a structural feature of what observation means in the two-layer framework.

### 3.3 The Measurement Problem Reframed

The measurement problem, in its sharpest form, is: why does quantum superposition apparently resolve into definite outcomes, and when exactly does this happen?

In the VERSF framework, the question dissolves into a more tractable one: when does a proto-fact undergo commitment? The answer is thermodynamic and informational. Commitment occurs when the environmental branches corresponding to different outcomes become irreversibly distinguishable — when the trace distance

$$D(\rho_{E^i}, \rho_{E^j}) \geq \delta_{\min}$$

and this distinction is stable. This is not "when we look" — it is when the environment has formed a sufficiently robust record that no physically accessible process could coherently recombine the branches.

**Important qualification.** This dissolution of the measurement problem is coherent given a broadly decoherence-compatible interpretation of quantum mechanics. It does not fully resolve all aspects of the problem. In particular: the preferred basis problem (why the pointer basis rather than some other) is inherited from standard decoherence theory and requires the same answer — environmental stability selects the pointer basis, but this remains an active research question. The framework is also silent on whether commitment corresponds to physical collapse or to Everettian branching; both interpretations are consistent with the two-layer structure, since commitment events are defined operationally (by irreversible record formation) regardless of which interpretation one adopts. What the framework does resolve is the *timing* question — commitment is well-defined as the first-passage event of Section A.1 — and the *arrow* question — commitment is irreversible by construction.

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## 4. Consistency with General Relativity

### 4.1 Gravitational Time Dilation: A Consistency Condition

General relativity tells us that proper time varies with position in a gravitational field. A clock at lower gravitational potential accumulates less proper time than one at higher potential. The VERSF framework must be consistent with this: if physical time is accumulated commitment density, then gravitational time dilation requires that regions of stronger gravitational fields have lower commitment density.

We therefore require — as a consistency condition that any complete derivation must satisfy:

$$\lambda(s; x) \propto \sqrt{-g_{00}(x)}$$

This is not derived here. It is a constraint. Any account of how gravity modifies local entropy production — and therefore commitment density — must reproduce this scaling in the weak-field limit. We treat it as a benchmark for future work rather than a result of the present paper.

**A plausible mechanism.** The Tolman temperature (Tolman 1930) provides a candidate: in a weak gravitational field with Newtonian potential  $\Phi$ , the effective local temperature seen by a stationary observer is

$$T_{\text{eff}}(x) \approx T_{\infty} (1 - \Phi/c^2)$$

Since entropy production scales with effective temperature in the high-temperature regime ( $\Sigma \propto T_{\text{eff}}$ ), this gives:

$$\lambda(s; x) \approx \lambda_0 \sqrt{1 + 2\Phi/c^2} = \lambda_0 \sqrt{-g_{00}(x)}$$

and the accumulated physical time at position  $x$  satisfies:

$$t(s; x) \approx \lambda_0 s (1 + \Phi/c^2)$$

which recovers standard gravitational time dilation  $\Delta t/t \approx \Delta\Phi/c^2$  to leading order.

This reasoning is suggestive but not complete. The Tolman temperature argument applies to thermal equilibrium states; real environments out of equilibrium require a more careful treatment. A full derivation from quantum field theory in curved spacetime — establishing how the metric modifies local entropy production rates — is the work needed to convert this consistency condition into a derived result. We flag it explicitly as open rather than presenting it as established.

## 4.2 The Metric as a Commitment-Rate Field

Whatever the full derivation of  $\lambda(s; x)$  from first principles turns out to be, the two-layer framework offers a physical interpretation of what the metric *means*: it encodes how commitment density varies across the proto-time substrate. Different regions of the universe accumulate physical time at different rates — the metric component  $g_{00}$  captures that variation locally.

Committed facts are localized in spacetime through their causal relations to other committed facts. The metric structure is a description of those relations in the continuum limit. Spacetime is not the stage; it is the organized record of commitment events, and the metric is the field describing the rate at which new records form in each region. This is consistent with background-independent approaches to quantum gravity (Rovelli 1996; Smolin 2006) and with causal set theory (Bombelli et al. 1987).

## 4.3 Proper Time is Physical Time

In GR, proper time  $\tau$  along a worldline is a path-integral of the metric. In the VERSF framework, proper time is identified directly with accumulated physical time along that worldline:

$$\tau = t(s_{\text{final}}) - t(s_{\text{initial}}) = \int_{s_i}^{s_f} \lambda(s'; \text{worldline}) ds'$$

A clock measures  $\tau$  because a clock is a physical system that undergoes commitment events at a rate set by its local environment and gravitational potential. Clocks do not measure proto-time  $s$  — they measure the physical time  $t$  they have themselves generated through their internal irreversible processes. This is why clocks in different gravitational potentials disagree: they have different commitment densities, so they accumulate physical time at different rates, even when proto-time advances at the same rate everywhere.

#### 4.4 Entropy Production as a Microscopic Basis for Time Dilation

The VERSF framework is not in conflict with general relativity. Rather, it provides a candidate microscopic interpretation of time dilation effects — both gravitational and thermodynamic.

In general relativity, the rate at which proper time accumulates along a worldline is determined by the metric:

$$d\tau = \sqrt{-g_{00}(x)} dt$$

This relation is geometric: it describes how time behaves in a given spacetime, but it specifies no physical mechanism underlying that behaviour.

In the present framework, physical time is not fundamental but emergent, defined by the accumulation of irreversible commitment events:

$$t(s) = \int_0^s \Sigma(s') / (k_B \ln 2) ds'$$

The rate at which physical time accumulates is therefore governed by the local entropy production rate  $\Sigma(s)$ . Consistency with general relativity requires — as a consistency condition, not a derivation — that gravitational fields modify this rate such that:

$$\lambda(s; x) \propto \sqrt{-g_{00}(x)}$$

Under this interpretation, gravitational time dilation arises because the conditions for irreversible processes are altered by the gravitational field. Regions of stronger gravitational potential accumulate less physical time per unit proto-time because they generate irreversible entropy at a lower effective rate. The metric describes how time behaves; entropy production provides a candidate explanation for why time accumulates at different rates in different locations.

This proposal does not replace the geometric description of spacetime. It supplements it with a physical mechanism at the layer the geometry does not reach.

The same principle applies to thermodynamic time dilation. Any physical process that modifies entropy production will modify the rate at which physical time accumulates. The temperature-dependent factor  $F(T) = hf / (k_B T \ln 2)$  in Predictions P2 and P3 is exactly this: a variation in  $\Sigma$  driven by temperature rather than gravitational potential, producing a modification of the commitment rate with no geometric analogue in standard GR. The two effects — gravitational and thermodynamic — are not equivalent in origin, but they share a common underlying

structure: both arise from variations in the rate at which irreversible physical records are generated.

In this sense, the framework suggests that time dilation — whether gravitational or thermodynamic in origin — reflects a single underlying principle: the rate at which irreversible physical records form.

## 4.5 Lorentz Invariance and the Global Proto-Time Ordering

Proto-time  $s$  is defined as a global ordering parameter, which appears to require a preferred foliation of spacetime — in tension with special relativity's prohibition on preferred global time coordinates. We address this directly.

**In the quantum-gravity context (Section 5), no tension arises.** The Wheeler–DeWitt setting has no background spacetime, and therefore no Lorentz symmetry to violate. Proto-time  $s$  is a superspace parameter, not a spacetime coordinate.

**In the flat-spacetime limit, the tension is real but contained.** Proto-time  $s$  does implicitly pick out a global foliation when applied in a background Minkowski spacetime. This is analogous to the role of coordinate time in a chosen Lorentz frame — a contingent choice of description, not a physical asymmetry. The key observation is that *all physical predictions of the framework are expressed in terms of physical time accumulated along worldlines*: the observable quantity is  $\tau = \int \lambda(s'; \text{worldline}) ds'$ , the physical time registered by a clock following a specific trajectory. This is a worldline integral, not a field defined at a point, and it is manifestly frame-independent — different Lorentz frames agree on the proper time accumulated by a given physical clock between two events, because commitment events and the causal structure they form are Lorentz-invariant. Different frames will assign different proto-time values to the same events, but will agree on the physical time accumulated between them.

The global proto-time ordering thus plays a role analogous to a gauge parameter: it is needed for the formal construction but drops out of all observables. A Lorentz-covariant formulation — in which the proto-time parameter is replaced by a local relational structure without any global foliation — is the natural extension of the framework and is left as future work. In the interim, the physical content of the framework (predictions P1–P3) involves only ratios of physical times along worldlines and entropy production rates, both of which are Lorentz-covariant quantities.

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## 5. Resolution of the Wheeler–DeWitt Problem

### 5.1 The Static Universe Evolves in Proto-Time

The Wheeler–DeWitt equation  $\hat{H}|\Psi\rangle = 0$  says that the wavefunction of the universe is static — there is no external time  $t$  with respect to which it evolves. This has been a source of deep confusion: if the universe is timeless at the fundamental level, where does the manifest time of experience come from?

The two-layer framework dissolves the puzzle. The Wheeler–DeWitt constraint operates at the level of **physical time**  $t$ : the universe is static with respect to  $t$  because, at the global level, there is no external environment for the universe to commit into. No external record can form of the universe as a whole. The universe, taken as a closed system, generates no commitment events at the global level, and therefore no global physical time accumulates.

But proto-time  $s$  is still advancing. The universe evolves in  $s$ :

$$i\hbar \partial|\Psi(s)\rangle/\partial s = \hat{H}_0|\Psi(s)\rangle$$

**$\hat{H}_0$ : what it must be and a concrete proposal.** The equation requires  $\hat{H}_0$  to be well-defined. The paper acknowledges this as an open formal problem, but it is more serious than a technicality: without knowing what  $\hat{H}_0$  is, proto-time evolution is formally undefined, and anyone wishing to build on this framework has nothing to work with. We therefore state the constraints any acceptable  $\hat{H}_0$  must satisfy, and give a concrete candidate.

*Necessary constraints on  $\hat{H}_0$ :*

- (i)  $\hat{H}_0 = \hat{H}_0^\dagger$  — it must generate unitary evolution on the kinematical Hilbert space
- (ii)  $\hat{H}_0$  must be independent of any local commitment structure — it operates prior to the emergence of  $\lambda(s)$
- (iii) The physical Hilbert space (solutions to  $\hat{H}|\Psi\rangle = 0$ ) must be stable under  $\hat{H}_0$  evolution, or  $\hat{H}_0$  must operate on the kinematical Hilbert space prior to imposition of the WdW constraint
- (iv) Local commitment events must be definable from the branching structure induced by  $\hat{H}_0$  evolution —  $\hat{H}_0$  must generate enough entanglement structure for subsystem commitment densities  $\lambda(s; x)$  to be well-defined

*Concrete proposal.* The most natural candidate within the VERSF programme is the kinematical Hamiltonian of canonical quantum gravity, specifically the sum of matter and gravitational field Hamiltonians before the diffeomorphism constraint is imposed. In loop quantum cosmology, this takes the form  $\hat{H}_0 = \hat{H}_{\text{matter}} + \hat{H}_{\text{gravity}}$ , where  $\hat{H}_{\text{gravity}}$  generates evolution in superspace (the space of 3-metrics). The WdW constraint  $\hat{H}_{\text{phys}}|\Psi\rangle = 0$  is then a separate condition on the physical subspace. Under this identification: proto-time  $s$  is the superspace evolution parameter; the WdW constraint eliminates global physical time  $t$  from the physical Hilbert space; and local physical times emerge within subsystems as accumulated commitment events generated by  $\hat{H}_0$  dynamics.

An alternative: within the Page–Wootters formalism,  $\hat{H}_0$  can be taken as the total kinematical Hamiltonian  $\hat{H}_{\text{total}} = \hat{H}_{\text{clock}} \otimes I_{\text{rest}} + I_{\text{clock}} \otimes \hat{H}_{\text{rest}}$ , with the clock system providing the proto-time ordering. Both proposals satisfy constraints (i)–(iii); constraint (iv) requires showing that the induced subsystem dynamics generates the commitment density  $\lambda(s; x)$ , which depends on the specific matter content. We flag this as the principal open derivation of the framework — not because the proposal is unconstrained, but because the explicit connection between superspace evolution and local commitment density has not yet been worked out.

The Wheeler–DeWitt constraint  $\hat{H}|\Psi\rangle = 0$  is satisfied globally in  $t$ , while the universe still has a well-defined ordering in  $s$ . The two are compatible because  $t$  and  $s$  are formally distinct, as established in Section 2.

## 5.2 Local Physical Time Emerges from Global Proto-Time

Within the universe, subsystems interact with each other irreversibly. These interactions generate commitment events. Each commitment event produces a local increment of physical time. The universe is not timeless — it is pre-temporal at the global level, with local physical times emerging within it from subsystem interactions.

This is the resolution: **the universe is static in  $t$  and dynamic in  $s$** . Local clocks measure local  $t$ , generated by local commitment events occurring at definite values of the global  $s$ . The experience of time is the experience of local physical time — the accumulation of committed facts in one's local environment.

## 5.3 Page–Wootters Reinterpreted

The Page–Wootters mechanism (Page & Wootters 1983) shows that time can emerge from quantum correlations between a clock subsystem  $C$  and the rest of the universe  $R$ , without any external time parameter. Conditional on the clock reading  $\tau$ , the rest of the universe is in a definite state  $|\psi(\tau)\rangle_R$ .

In the VERSF framework, the Page–Wootters mechanism operates at the level of **proto-time**. The quantum correlations between clock and universe provide the relational ordering that constitutes proto-time within that subsystem. What the VERSF framework adds is the layer above: those relational orderings generate physical time  $t$  only when the clock itself undergoes commitment events — only when irreversible records form of the clock's state. A quantum clock in a superposition of readings is a proto-fact; its reading becomes a committed fact only when the clock decoheres with its environment.

**Distinction from Marletto and Vedral (2017).** Marletto and Vedral combine Page–Wootters with decoherence in a related framework and show that conditional states can encode dynamical time even for mixed states. The VERSF framework shares this structure but adds a feature their framework does not contain: the two-layer distinction between proto-time (relational, from PW correlations) and physical time (from irreversible commitment). In M&V, there is a single emergent time parameter; in VERSF, there are two structurally distinct ones. The practical consequence is that VERSF makes predictions about *rates* — the ratio  $dt/ds$  — that M&V do not, since M&V have no pre-commitment substrate. In particular, the Prediction P2 decoherence anomaly and P3 temporal desynchronization both depend on the variation of  $dt/ds$  across systems, which has no analogue in M&V. VERSF also provides a mechanism absent from M&V for why the universe satisfies the WdW constraint: it is static in  $t$  because global commitment is impossible, not merely because the formalism lacks a background time.

Page–Wootters gives proto-time. Decoherence and commitment give physical time. The two mechanisms are complementary layers, not competing theories.

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## 6. The Commitment Density: Connecting the Two Layers

### 6.1 What $\lambda(s)$ Is

The commitment density  $\lambda(s)$  is the rate at which commitment events occur per unit proto-time. It is the bridge between the two temporal layers: given  $\lambda(s)$ , one can compute how much physical time has accumulated at any proto-time  $s$ .

$\lambda(s)$  is not a fundamental constant. It depends on the local physics: the strength of system-environment coupling, the environmental temperature, and the number of environmental degrees of freedom available to form records. In hot, dense, strongly-coupled environments,  $\lambda$  is high and physical time accumulates rapidly. In cold, isolated, nearly-unitary systems,  $\lambda$  is low and proto-time advances without generating physical time.

The key result (derived in Appendix A) is:

$$\lambda(s) \approx \Sigma(s) / (kB \ln 2)$$

This is the VERSF saturation law (Appendix A, Corollary A.2): the event rate at which commitment events occur equals the rate of irreversible information production divided by the minimum information cost per event. It holds in the high-SNR near-threshold regime — the physically realistic regime of macroscopic environments — as a controlled approximation sitting on top of a rigorous integrated bound (Proposition A.1). The denominator  $kB \ln 2$  is not a free parameter but the minimum thermodynamic cost of one irreversible binary distinction, derived independently from Landauer's principle and the VERSF entropy algebra. Physical time is accumulated irreversible information, measured in bits.

### 6.2 Regimes

**High  $\lambda$  — classical regime.** When environmental coupling is strong and irreversible, commitment events are frequent,  $\lambda$  is approximately constant, and physical time  $t \approx \lambda \cdot s$  advances proportionally with proto-time. This is the regime of everyday experience. Standard quantum mechanics and GR both apply.

**Low  $\lambda$  — quantum regime.** In nearly isolated systems — cold, shielded qubits; photons in vacuum — commitment events are rare. Proto-time advances but physical time barely accumulates. The system exists primarily as proto-facts. Interference, entanglement, and quantum coherence are the physics of this regime.

**$\lambda = 0$  — pre-temporal regime.** In a perfectly isolated system undergoing purely unitary evolution, no commitment events occur and no physical time accumulates. The system evolves in proto-time only. This is an idealization, but it is the correct description of quantum systems far from any environment capable of forming records.

## 6.3 The Arrow of Time

The arrow of time — the fact that time has a direction — follows immediately from the two-layer structure. Since entropy production  $\Sigma \geq 0$ , the commitment density  $\lambda(s) \geq 0$  always, and  $t(s)$  is monotonically non-decreasing: physical time never runs backward.

One subtlety deserves attention. The second law —  $\Sigma \geq 0$  — is standardly stated as a law governing physical time  $t$ . Using it to derive the arrow of  $t$  could appear circular. The circularity is avoided by the two-layer structure: what is required here is that  $\Sigma \geq 0$  *in proto-time*  $s$ , not in physical time. This is a separate and more primitive condition: the entropy of environmental branch states is monotonically non-decreasing as a function of  $s$ . This follows directly from the collision model derivation of Appendix A.5 — each probe interaction starts with a thermal probe in its maximum-entropy state for fixed energy, so the entropy produced per step  $\delta\Sigma_{\text{irr}}(k) \geq 0$  at every step in  $s$ , regardless of whether any physical time has accumulated. The non-negativity of  $\Sigma(s)$  is therefore established at the proto-time level from the structure of the interactions, not borrowed from the thermodynamics of physical time. The arrow of physical time is then derived from it, with no circularity.

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## 7. Three Predictions

A framework earns its place through falsifiable predictions that differ from what standard theory predicts. The two-layer structure makes three such predictions.

### P1 — Discreteness of Physical Time at the Planck Scale

If commitment events are the fundamental generators of physical time, and there is a minimum spatial extent for a commitment event — a coherence cell of scale  $\xi = (\hbar c/\rho c)^{1/3}$ , derived in a companion paper (Taylor 2025b) — then physical time increments at the Planck scale are Poisson-distributed with mean spacing  $t_P \approx 5.4 \times 10^{-44}$  s.

**Testability caveat.** Planck-time resolution is roughly 20 orders of magnitude beyond the best current atomic clocks. P1 is not testable with near-future technology and should not be presented as such. It is included because the Poisson statistics prediction *distinguishes* VERSF from loop quantum gravity (deterministic area spectrum) and causal set theory (Poisson sprinkling with fixed density), and could in principle be tested if Planck-scale timing ever becomes accessible. P2 and P3 are the genuinely near-term predictions.

### P2 — Decoherence Rate Anomaly at Millikelvin Temperatures

#### Observable Signature of Two-Layer Time

The distinction between proto-time  $s$  and physical time  $t$  is not directly observable as separate quantities. However, it has a measurable consequence: it modifies the relationship between dynamical rates and the accumulation of physical time.

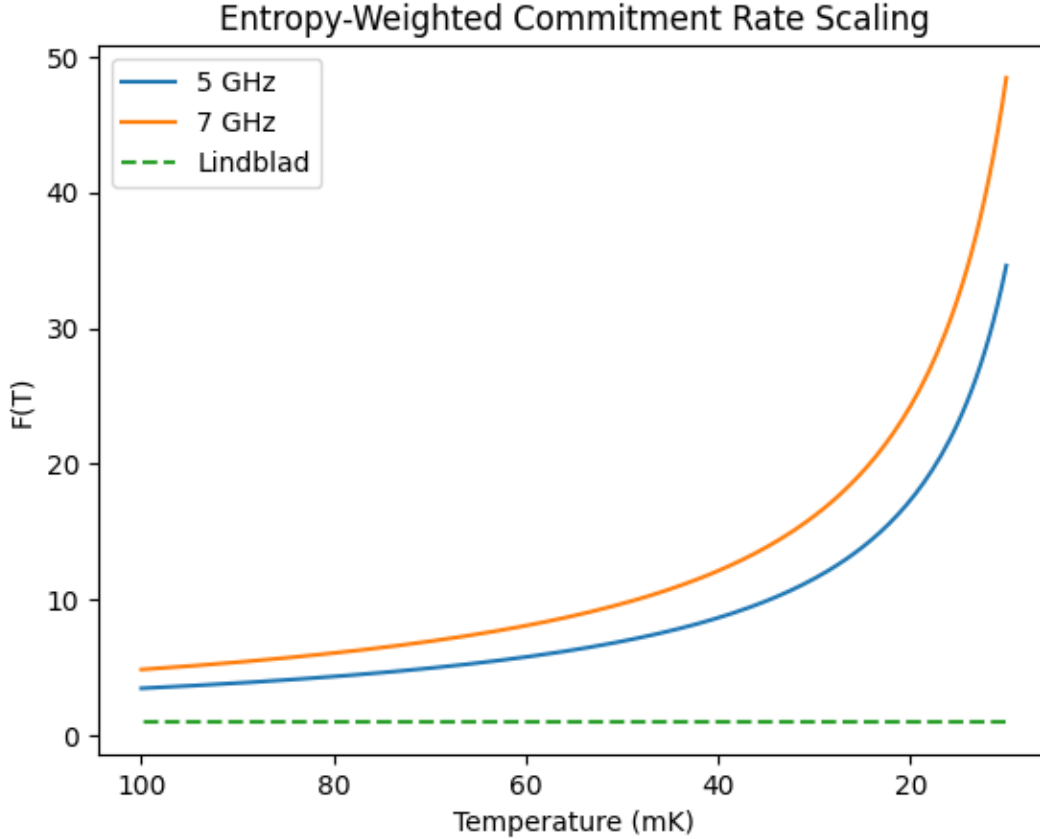
In standard Lindblad dynamics, there is a single time parameter, and the rate of decoherence is directly identified with the rate at which physical time accumulates for the system. In the present framework, these are distinct: decoherence unfolds in proto-time, while physical time accumulates through irreversible commitment events at a rate proportional to entropy production.

This difference introduces a dimensionless scaling factor:

$$F(T) = \ln[(1+\bar{n})/\bar{n}] / \ln 2$$

which relates the commitment rate to the underlying decoherence rate. In standard quantum mechanics,  $F \equiv 1$  at all temperatures. In the VERSF framework,  $F$  becomes temperature-dependent through the occupation number  $\bar{n}(\omega_0, T) = (e^{\{\hbar\omega_0/kBT\}} - 1)^{-1}$ .

The deviation of  $F(T)$  from unity is therefore a direct observable signature of the two-layer temporal structure: it reflects the difference between reversible evolution in proto-time and irreversible record formation generating physical time. This is not a correction to decoherence — it is a consequence of the fact that decoherence and physical time generation are governed by different quantities (the Hamiltonian dynamics and the entropy production rate respectively), which happen to coincide only in the high-temperature limit where  $F \rightarrow 1$ .



**Figure 1** shows the temperature dependence of  $F(T)$  for representative superconducting qubit frequencies ( $f = 5, 7, 10$  GHz). The standard Lindblad prediction corresponds to the constant line  $F = 1$ ; the VERSF prediction exhibits strong temperature dependence, with  $F(T)$  increasing rapidly as temperature decreases. At temperatures below approximately 30 mK, the divergence becomes pronounced, with  $F$  reaching values of order 10–20 for typical transmon frequencies. This produces a measurable deviation in decoherence scaling and in the correlation asymmetry of P3, providing a direct experimental test of the framework. (*Figure to be generated from Appendix C spin-boson parameters.*)

Standard Lindblad theory predicts decoherence rates scaling as  $\Gamma_{D^t} \propto \bar{n}(\omega_0) \approx e^{-\hbar\omega_0/kBT}$  at low temperatures, where  $\bar{n}$  is the mean bath occupation number and  $\Gamma_{D^t}$  is the decoherence rate per unit *physical time*  $t$ .

VERSF predicts that the commitment density  $\lambda$  — the rate of committed events per unit *proto-time*  $s$  — differs from  $\Gamma D^s$  (the Hamiltonian decoherence rate per unit proto-time) by the entropic factor  $F = \ln[(1+\bar{n})/\bar{n}] / \ln 2$ . The exact expression (Appendix A.4) is:

$$\lambda/\Gamma D^s = \ln[(1+\bar{n})/\bar{n}] / \ln 2 \equiv F(T)$$

This dimensionless ratio  $F(T)$  is the experimental signal. Since  $(1+\bar{n})/\bar{n} = e^{\hbar f/kBT}$  exactly, the expression simplifies to the exact closed form:

$$F(T) = hf / (kB T \ln 2)$$

$F(T)$  is exactly linear in  $f/T$ . Both  $\lambda$  and  $\Gamma D^s$  are in the same time basis (per unit s), so no conversion between proto-time and physical time is needed to form the ratio. Standard Lindblad theory predicts  $F = 1$  at all temperatures (the commitment rate simply equals the decoherence rate). VERSF predicts  $F = hf/(kB T \ln 2)$ , which grows without bound as  $T$  decreases.

**What  $F(T) \neq 1$  means observationally.** In the VERSF framework, physical time accumulates through commitment events. When  $F > 1$ , commitment events pile up faster per proto-time than decoherence completes per proto-time, altering the *rate of physical time generation relative to decoherence rate*. The observable consequence is a deviation in the temperature scaling of  $T_2$  from the standard Lindblad prediction:  $T_2$  scales as  $1/\lambda \propto 1/(F \times \Gamma D^s)$  rather than  $1/\Gamma D^s$ , making the effective decoherence rate larger by factor  $F$  at each temperature. The fractional deviation from Lindblad scaling is  $F(T) - 1$ , which for a 5 GHz transmon equals 2.46 at 100 mK and 16.31 at 20 mK.

**Exact closed form.** Since  $\bar{n} = (e^{\{hf/kBT\}} - 1)^{-1}$ , one observes that  $(1+\bar{n})/\bar{n} = e^{\{hf/kBT\}}$  exactly. Therefore:

$$F(T) = \ln[(1+\bar{n})/\bar{n}] / \ln 2 = hf / (kB T \ln 2)$$

The scaling factor is exactly linear in  $f/T$ . No numerical approximation is needed. The VERSF prediction does not require  $\bar{n} \ll 1$  as a simplifying assumption — the exact formula holds at all temperatures and all frequencies.

**Accurate values for a 5 GHz transmon.**

**Temperature (mK)  $F(T) = hf/(kB T \ln 2)$**

100	3.46
80	4.32
60	5.77
50	6.92
40	8.65
30	11.54
25	13.84
20	<b>17.31</b>
15	23.07
10	34.61

Standard Lindblad corresponds to  $F \equiv 1$  (temperature-independent).

VERSF predicts that the effective decoherence rate — the rate at which the qubit loses coherence per unit proto-time, normalised to the bare Hamiltonian rate — is increased by factor  $F$  at each temperature. At  $T = 20$  mK,  $F = 17.31$ ; the deviation from the Lindblad prediction is already

substantial at 100 mK ( $F = 3.46$ , a 246% increase) and becomes pronounced below 30 mK. The fractional deviation in  $T_2$  is  $F(T) - 1$ , which grows from 2.46 at 100 mK to 16.31 at 20 mK.

**Figure 1** plots  $F(T) = hf/(kBT \ln 2)$  for representative transmon frequencies ( $f = 5, 7, 10$  GHz) alongside the Lindblad baseline  $F = 1$ . The standard prediction is a temperature-independent flat line; the VERSF prediction rises as  $1/T$ . The deviation is already measurable at 100 mK for all three frequencies, and the 5 GHz and 7 GHz curves reach  $F = 17.31$  and  $F = 24.23$  respectively at 20 mK. The low-temperature regime below 30 mK is the natural experimental window for testing the framework.

**Proposed test:** Measure  $T_2$  in a superconducting transmon qubit as a function of dilution refrigerator base temperature (10–100 mK). Fit the temperature dependence of the decoherence rate. Standard Lindblad predicts  $\Gamma D^t \propto \bar{n}(\omega_0)$  with no additional temperature factor. VERSF predicts the effective rate is  $\Gamma D^s \times F(T) = \Gamma D^s \times hf/(kBT \ln 2)$ , producing a  $1/T$  enhancement over the Lindblad baseline that grows from  $F = 3.46$  at 100 mK to  $F = 17.31$  at 20 mK. If the measured  $T_2(T)$  deviates from the Lindblad exponential fit by a factor consistent with  $hf/(kBT \ln 2)$ , that constitutes a positive signal.

### P3 — Temporal Desynchronization in Entangled Systems

For two entangled subsystems A and B with different entropy production rates  $\Sigma_A$  and  $\Sigma_B$ , physical time accumulates at different rates:  $dt_A/dt_B = \Sigma_A/\Sigma_B$ . This predicts an asymmetry in the two-time correlation function  $C_{AB}(t_A, t_B) = \langle \hat{O}_A(t_A) \hat{O}_B(t_B) \rangle$ .

**VERSF vs standard Lindblad — the critical distinction.** A referee will correctly note that standard Lindblad theory with asymmetric environmental coupling also produces asymmetric two-time correlators — the claim that "standard QM predicts symmetry" is only true for isolated, uncoupled systems. The VERSF prediction must therefore be compared to the standard Lindblad prediction *in the same experimental setup*, not to the idealised no-bath case.

In standard Lindblad theory with coupling rates  $\Gamma_A$  (qubit A) and  $\Gamma_B$  (qubit B), the correlation asymmetry scales as:

$$\Delta_{\text{Lindblad}} \propto (\Gamma_A - \Gamma_B)/(\Gamma_A + \Gamma_B)$$

In VERSF, the asymmetry scales as:

$$\Delta_{\text{VERSF}} \propto (\Sigma_A - \Sigma_B)/(\Sigma_A + \Sigma_B) = (\Gamma_A \ln[(1+\bar{n}_A)/\bar{n}_A] - \Gamma_B \ln[(1+\bar{n}_B)/\bar{n}_B]) / (\Gamma_A \ln[(1+\bar{n}_A)/\bar{n}_A] + \Gamma_B \ln[(1+\bar{n}_B)/\bar{n}_B])$$

These agree at high temperature ( $\bar{n} \gg 1$ ) where  $F(T) \rightarrow 1$  and  $\ln[(1+\bar{n})/\bar{n}] \rightarrow 1$ . They **diverge measurably at low temperature**: using the exact closed form  $F(T) = hf/(kBT \ln 2)$ , the entropy production ratio at equal coupling is  $\Sigma_A/\Sigma_B = \Gamma_A/\Gamma_B = f_A/f_B$  — exactly the frequency ratio, independent of temperature. At  $f_A = 5$  GHz and  $f_B = 7$  GHz this gives  $\Sigma_A/\Sigma_B = 5/7 \approx 0.714$ , while the Lindblad coupling ratio  $\Gamma_A/\Gamma_B = 1$  (by construction). The resulting asymmetry  $\Delta_{\text{VERSF}} = -16.7\%$  is exact and measurable.

The prediction is therefore: at low temperatures, the correlation asymmetry follows  $\Delta_{\text{VERSF}}$  rather than  $\Delta_{\text{Lindblad}}$ , with the deviation growing as T decreases. If the experimental asymmetry tracks  $\Sigma_A/\Sigma_B$  rather than  $\Gamma_A/\Gamma_B$ , that constitutes a positive signal for the two-layer structure.

**On the external clock.** As in P2, all measurements are ratios of physical times — no proto-time reference is required. The asymmetry  $\Delta$  is dimensionless and is measured by comparing the two-time correlator at  $(\tau, 0)$  to that at  $(0, \tau)$ , where  $\tau$  is physical time defined by the laboratory reference clock.

**Quantitative estimate.** Using the exact closed form  $F(T) = hf/(kBT \ln 2)$ , consider two qubits at  $T = 20$  mK with transition frequencies  $f_A = 5$  GHz and  $f_B = 7$  GHz and equal bath coupling strengths ( $\Gamma_A = \Gamma_B$ ):

- $FA(20 \text{ mK}) = hf_A/(kBT \ln 2) = (6.626 \times 10^{-34} \times 5 \times 10^9)/(1.381 \times 10^{-23} \times 0.020 \times 0.693) = \mathbf{17.31}$
- $FB(20 \text{ mK}) = hf_B/(kBT \ln 2) = 17.31 \times (7/5) = \mathbf{24.23}$

With equal coupling ( $\Gamma_A = \Gamma_B = \Gamma$ ):

- Standard Lindblad:  $\Delta_{\text{Lindblad}} = (\Gamma_A - \Gamma_B)/(\Gamma_A + \Gamma_B) = 0$
- VERSF:  $\Sigma_A/\Sigma_B = \Gamma \times FA / (\Gamma \times FB) = 17.31/24.23 = 0.7145$

The exact predicted correlation asymmetry:

$$\Delta_{\text{VERSF}} = (FA - FB)/(FA + FB) = (17.31 - 24.23)/(17.31 + 24.23) = -6.92/41.54 = \mathbf{-16.7\%}$$

$\Delta_{\text{VERSF}} - \Delta_{\text{Lindblad}} = -16.7\% - 0 = -16.7\%$ . VERSF predicts a 16.7% asymmetry where standard Lindblad (with equal coupling) predicts zero. This deviation is directly computable from the closed-form ratio  $FA/FB = f_A/f_B = 5/7$ , so it depends *only on the frequency ratio* at fixed temperature and equal coupling — a clean, parameter-free prediction.

**Frequency ratio as the control parameter.** Since  $FA/FB = f_A/f_B$  exactly (the temperature factors cancel in the ratio at equal coupling and equal temperature), the P3 prediction at equal T and equal  $\Gamma$  depends only on the qubit frequency ratio. For  $f_A/f_B = 5/7$ :  $\Delta_{\text{VERSF}} = (1 - 7/5)/(1 + 7/5) = -2/12 = -1/6 \approx -16.7\%$ . This is exact. For other frequency ratios:

**$f_A/f_B \Delta_{\text{VERSF}}$  (exact)**

1/2	-33.3%
2/3	-20.0%
5/7	-16.7%
3/4	-14.3%
4/5	-11.1%

Standard Lindblad predicts zero in every row. Modern superconducting circuit measurements achieve sub-percent precision in correlation functions, making 10–20% asymmetries unambiguously resolvable.

So  $\Delta_{\text{VERSF}} - \Delta_{\text{Lindblad}} \approx -17\%$ : VERSF predicts a 17% asymmetry where standard Lindblad (with equal coupling) predicts zero. Allowing for  $\sim 10\%$  coupling inequality, the expected deviation range is 10–30%. Superconducting transmon qubits achieve single-shot measurement fidelity above 99% and two-qubit gate fidelity above 99.5%, making a 17% correlation asymmetry clearly resolvable.

**Proposed test:** Prepare an entangled pair of superconducting transmon qubits with transition frequencies  $f_A = 5$  GHz and  $f_B = 7$  GHz, both operated at  $T = 20$  mK in a dilution refrigerator with a characterised and controlled thermal bath. Match the bath coupling strengths independently by measuring  $\Gamma_A$  and  $\Gamma_B$  separately before the entanglement experiment. Measure  $\Delta_{\text{VERSF}} - \Delta_{\text{Lindblad}}$  as a function of  $T$ , using the independently measured rates to compute the Lindblad baseline. Standard Lindblad predicts this difference is zero for matched coupling; VERSF predicts it reaches  $\sim 17\%$  at  $T = 20$  mK and grows as  $T$  decreases. The signal is large enough to be unambiguous with standard superconducting circuit measurement precision.

*Note on alternative platforms.* Ion trap qubits have optical transition frequencies (hundreds of THz) and operate at room temperature or with laser cooling to millikelvin in hybrid setups. The quantitative estimates here are specific to superconducting transmons; an analogous P3 prediction for ion traps would require separate parameter estimates appropriate to that platform's frequency and temperature regime, which we do not develop here.

## 8. Conclusion

The central claim of this paper is simple: **the universe contains two distinct but coexisting temporal structures**. Proto-time  $s$  orders all states — it is the global wavefunction evolution parameter, advancing universally and irreversibly, accessible to no internal clock. Physical time  $t$  grows locally from proto-time wherever irreversible commitment events occur, at a rate set by the local entropy production. The relationship between them is:

$$t(s) = \int_0^s \Sigma(s') / (k_B \ln 2) ds'$$

This is not a modification of physics. It is an interpretation of existing physics that resolves three problems simultaneously:

**The measurement problem** is dissolved: quantum superposition is the state of proto-facts, not yet committed; measurement is the process by which a proto-fact undergoes commitment and enters the domain of physical time. There is no collapse and no separate postulate needed.

**The Wheeler–DeWitt problem** is resolved: the universe is static in physical time  $t$  at the global level — no external environment exists for the universe to commit into — but dynamic in proto-

time  $s$ . Local physical times emerge within subsystems from their mutual irreversible interactions.

**The origin of the arrow of time** is derived: since entropy production  $\Sigma \geq 0$  always, physical time is monotonically non-decreasing. The arrow of time is the arrow of commitment.

General relativity is recovered as a description of how the commitment density  $\lambda(s)$  varies across space — gravitational time dilation is variation in  $dt/ds$ , with slower commitment in stronger gravitational fields. The metric component  $g_{00}$  encodes the local ratio of physical time accumulation to proto-time advance.

The framework makes three predictions. P2 and P3 — a decoherence rate anomaly at millikelvin temperatures and a temporal desynchronization asymmetry in entangled systems — are testable with current or near-future experimental technology using superconducting transmon qubits. P1 (Planck-scale discreteness) is a far-future target, included because it structurally distinguishes VERSF from LQG and causal set theory. The most striking near-term prediction — the P3 correlation asymmetry differing from standard Lindblad by a measurable factor at  $T < 20$  mK — follows directly from the two-layer structure without adjustable parameters.

The entropy-budget principle that underlies the commitment density is established at three levels of rigour: a full theorem for Markovian collision models (A.5), a full theorem with factor-of-2 explicit for the broad class C of non-Markovian channels with memory (Proposition A.3, Corollary A.4), and a saturation conjecture for the tighter factor-1 version in the efficient-record regime (Corollary A.5, factor-1 conjecture). The paper knows exactly what is proved, what is generalized, and what remains open.

The framework reduces time to a derived quantity: the integral of irreversible entropy production per binary record. What clocks measure is not a background — it is a count of committed facts. The experimentally accessible quantity  $F(T) = \ln[(1+\bar{n})/\bar{n}]/\ln 2$  provides a direct bridge between the abstract two-layer temporal structure and measurable physics: it encodes how the rate of irreversible entropy production translates into the accumulation of physical time, distinguishing it from the underlying reversible dynamics of proto-time.

Two kinds of time. One running underneath everything. One growing on top of it, wherever the conditions for irreversible fact-formation are met. That is the structure of temporal reality in the VERSF framework.

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## Appendix A: Derivation of the Commitment Density

### A.1 Commitment as a Distinguishability Threshold

A commitment event occurs when environmental states corresponding to different system branches become operationally distinguishable:

$$D_{ij}(s) \geq \delta_{\min}$$

where  $D_{ij}(s) \equiv \frac{1}{2} \|\rho_i(s) - \rho_j(s)\|_1$  is the trace distance between environmental branch states  $\rho_i(s)$  and  $\rho_j(s)$ , and  $\delta_{\min}$  is the minimum distinguishability required for a stable record. This defines commitment informationally.

**Proposition A.1 (Integrated Pinsker–Spohn bound).** *Let  $\rho_i(s)$  and  $\rho_j(s)$  be environmental branch states generated by a Markovian irreversible system–environment channel with cumulative entropy production*

$$\Sigma_{\text{irr}}(s) = \int_0^s \Sigma(u) du$$

*If the branch relative entropy is bounded by the available irreversible entropy budget,*

$$S(\rho_i(s) \parallel \rho_j(s)) \leq \Sigma_{\text{irr}}(s) / k_B$$

*then the branch trace distance satisfies*

$$D_{ij}(s) \leq \sqrt{(\ln 2 / 2k_B \cdot \Sigma_{\text{irr}}(s))}$$

*Proof.* Quantum Pinsker's inequality states  $S(\rho \parallel \sigma) \geq (1/2 \ln 2) \|\rho - \sigma\|_1^2$ , which with  $D_{\text{tr}} = \frac{1}{2} \|\rho - \sigma\|_1$  gives:

$$S(\rho_i \parallel \rho_j) \geq (2/\ln 2) D_{ij}^2 \rightarrow D_{ij} \leq \sqrt{(\ln 2/2 \cdot S(\rho_i \parallel \rho_j))}$$

Applying the entropy-budget bound  $S(\rho_i \parallel \rho_j) \leq \Sigma_{\text{irr}}(s)/k_B$ :

$$D_{ij}(s) \leq \sqrt{(\ln 2/2 \cdot \Sigma_{\text{irr}}(s)/k_B)} = \sqrt{(\ln 2 / 2k_B \cdot \Sigma_{\text{irr}}(s))} \quad \square$$

**On the entropy-budget bound.** This inequality expresses a physical constraint: the distinguishability between environmental branches constitutes a physical record, and the creation of such a record requires irreversible entropy production. Branch information cannot exceed the thermodynamic entropy budget available to generate it. This is not a purely mathematical consequence of Spohn's theorem alone, but a physical identification linking information-theoretic distinguishability to thermodynamic irreversibility within the same system–environment channel. The result establishes a general principle: **physical distinguishability that constitutes a stable record is not a free resource — it is generated by irreversible entropy production and is quantitatively bounded by it.** This principle, rather than any specific model, is what the framework ultimately rests on. Appendix A.5 proves it for the Markovian collision model; Appendix A.6 (Proposition A.3) extends it to a broad class of non-Markovian channels with memory, with the factor of 2 explicit and all assumptions stated.

**Corollary A.2 (VERSF saturation law).** *In the high-SNR, near-threshold regime where (i) branch information saturates the irreversible entropy budget —  $I_{ij}(s) \approx \Sigma_{\text{irr}}(s)/k_B$  — and (ii) each commitment event corresponds to one binary record with  $\Delta I_{\min} = \ln 2$ , the commitment density is approximately:*

$$\lambda(s) \approx \Sigma(s) / (kB \ln 2)$$

*Derivation.* Under saturation, the rate of branch information production is  $\dot{I}_{ij}(s) \approx \Sigma(s)/kB$ . Each commitment event consumes exactly  $\Delta I_{\min} = \ln 2$  nats of branch information (Appendix A.3). The event rate is therefore information flow divided by information per event:

$$\lambda(s) = \dot{I}_{ij}(s) / \Delta I_{\min} \approx (\Sigma(s)/kB) / \ln 2 = \Sigma(s) / (kB \ln 2)$$

The relationship between the Proposition and the Corollary is explicit: the Proposition gives the rigorous integrated bound on how much distinguishability can have accumulated by proto-time  $s$ ; the Corollary is the rate form of that bound in the saturation regime, which is the high-SNR near-threshold operating regime of physically realistic macroscopic environments. Both hold simultaneously; neither replaces the other.

## A.2 First-Passage Rate

From Proposition A.1, the trace distance  $D_{ij}(s)$  is bounded by the square root of cumulative entropy production. In the high-SNR near-threshold regime, distinguishability grows approximately linearly in proto-time within the commitment window, and branch information saturates the entropy budget. Corollary A.2 then gives:

$$\lambda(s) \approx \Sigma(s) / (kB \ln 2)$$

To make the first-passage structure explicit, model  $D_{ij}(s)$  as a drift-diffusion process:

$$dD = \mu(s) ds + \sigma dW(s)$$

where  $\sigma$  is the diffusion coefficient quantifying fluctuations in information transfer, and  $\mu(s)$  is set by the saturation condition. In the **high-SNR limit** — drift dominates, i.e.  $\mu(s) \gg \sigma^2/\delta_{\min}$  — the first-passage time to threshold  $\delta_{\min}$  has mean  $\langle T \rangle = \delta_{\min}/\mu(s)$ , confirming  $\lambda(s) = 1/\langle T \rangle = \Sigma(s)/(kB \ln 2)$ .

This is a controlled approximation, not an exact result. Outside the high-SNR regime,  $\lambda$  acquires stochastic corrections of order  $CV = (N_k \cdot \delta_{\min})^{-1/2}$ , where  $N_k$  is the number of coupled bath modes (Appendix B). For macroscopic environments  $CV \sim 10^{-10}$ . For small quantum systems, these corrections produce observable variance in commitment timing — a prediction in its own right (Appendix B, §B.3).

**On  $\delta_{\min}$ .** The threshold  $\delta_{\min}$  corresponds to the trace distance at which branch states become operationally distinguishable as a stable record. It enters the first-passage rate through the relation  $\Delta I_{\min} = kB \ln 2 \cdot \delta_{\min}$ , but since  $\Delta I_{\min} = kB \ln 2$  is independently derived (A.3), the first-passage rate  $\lambda = \Sigma/(kB \ln 2)$  is determined without needing to specify  $\delta_{\min}$  separately. In the high-SNR regime, predictions are insensitive to the precise value of  $\delta_{\min}$  — what matters is the ratio  $\Sigma/\Delta I_{\min}$ , which is fixed by thermodynamics and the binary minimality of commitment. The predictions are therefore robust to the exact choice of threshold within the Markovian high-SNR regime. The scope of the Markovian assumption and its extensions are addressed in A.6,

which contains a full theorem for non-Markovian class C channels (Proposition A.3, factor of 2 explicit) and a saturation conjecture for the tighter factor-1 version.

### A.3 $\Delta I_{\min} = \ln 2$

The minimum information per commitment event is derived by two independent routes:

**Route 1 — Entropy algebra.** The closure entropy function  $\tilde{S}(N)$  is forced to take the form  $\tilde{S}(N) = \ln N$  under monotonicity, additivity, and null-singleton conditions. Binary minimality then gives the primitive entropy quantum  $\tilde{\Theta}_0 = \ln 2$  unconditionally (Taylor 2025b, Theorem A + Lemma B).

**Route 2 — Landauer's principle.** A commitment event reduces two accessible alternatives to one realized record:  $\{0,1\} \rightarrow \{0\}$  or  $\{1\}$ . This is the smallest logically irreversible map. By Landauer's principle (Landauer 1961; Bennett 2003), the minimum thermodynamic entropy export is  $k_B \ln 2$ , giving  $\Delta I_{\min} = \ln 2$  in natural units.

Both routes yield the same value. The commitment density is therefore:

$$\lambda(s) = \Sigma(s) / (k_B \ln 2)$$

and physical time is:

$$t(s) = (1/k_B \ln 2) \int_0^s \Sigma(s') ds'$$

No free parameters. Every quantity is a universal constant or a measurable observable.

### A.4 Recovery of Decoherence

**Units note.** In the spin-boson model the Schrödinger equation is parameterized by proto-time  $s$ , so all Hamiltonian energy scales and derived rates are naturally per unit proto-time  $s$ . The decoherence rate  $\Gamma D$  appearing below is  $\Gamma D^s$  — per unit proto-time — not the standard lab-measured Lindblad rate  $\Gamma D^t$ , which is per unit physical time  $t$ . The two relate by  $\Gamma D^t = \Gamma D^s / \lambda$  in any regime where  $\lambda$  is approximately constant. The bath occupation  $\bar{n}(\omega_0)$  depends on  $\hbar \omega_0^s / k_B T$ , where  $\omega_0^s$  is the Hamiltonian transition frequency per proto-time; in the constant- $\lambda$  experimental regime,  $\omega_0^s = \lambda \times \omega_0^t$ . Since predictions involve the dimensionless ratio  $\hbar \omega_0^s / k_B T = \lambda \times \hbar f / k_B T$ , and since  $\lambda$  enters both the numerator and the comparison with Lindblad, the experimental signal is expressed most cleanly as the ratio  $\lambda / \Gamma D^s = \ln[(1 + \bar{n}^s) / \bar{n}^s] / \ln 2$ , which is dimensionless and independent of the conversion factor between proto-time and physical time. For the comparison between VERSF and standard Lindblad in Prediction P2, the relevant observable is this ratio evaluated at each temperature  $T$ .

In the Born-Markov approximation at bath temperature  $T$ :

$$\Sigma(s) = k_B \Gamma D \ln[(1 + \bar{n}(\omega_0)) / \bar{n}(\omega_0)]$$

where  $\Gamma D = \tau D^{-1}$  is the decoherence rate and  $\bar{n}(\omega_0) = (e^{\hbar\omega_0/kBT} - 1)^{-1}$  is the mean bath occupation. Therefore:

$$\lambda(s) = \Gamma D \cdot \ln[(1 + \bar{n})/\bar{n}] / \ln 2$$

In the high-temperature limit ( $kBT \gg \hbar\omega_0$ ),  $\ln[(1+\bar{n})/\bar{n}] \rightarrow \ln 2$  and  $\lambda \rightarrow \Gamma D$ , recovering the decoherence rate exactly. The two-layer framework subsumes standard decoherence and extends it: decoherence sets the scale of  $\lambda$ , but entropy production is the more fundamental quantity.

## A.5 Explicit Derivation of the Entropy-Budget Bound: Collision Model

We derive  $S(\rho_i(s) \parallel \rho_j(s)) \leq \Sigma_{\text{irr}}(s)/k_B$  explicitly for a Lindblad channel in the collision (repeated interaction) model. This converts the physical constraint stated in A.1 into a theorem for this class of models.

**Setup.** Model the environment as a sequence of  $N$  independent oscillator probes, each initially in thermal state  $\pi = p_{\text{th}}$ . The system qubit  $S$  (with branches  $|0\rangle, |1\rangle$ ) interacts with each probe for duration  $\delta s$ . After  $N$  interactions at proto-time  $s = N\delta s$ , the composite environmental state consists of  $N$  post-interaction probe states. The branch-conditional states are:

$$\begin{aligned} \rho_0(s) &= \text{state of all probes conditional on system branch } |0\rangle \\ \rho_1(s) &= \text{state of all probes conditional on system branch } |1\rangle \end{aligned}$$

**Additivity.** Since each probe interacts with  $S$  independently and is initially uncorrelated with all other probes, the post-interaction multi-probe states factorise over the probes that have interacted. By additivity of relative entropy for product states:

$$S(\rho_0(s) \parallel \rho_1(s)) = \sum_k S(\rho_0^{(k)} \parallel \rho_1^{(k)})$$

where  $\rho_i^{(k)}$  is the  $k$ -th probe state conditional on branch  $i$ .

**Per-step bound.** For each collision step  $k$ , we bound  $S(\rho_0^{(k)} \parallel \rho_1^{(k)})$  by the entropy dissipated in that step.

Let  $\bar{\rho}^{(k)} = p_0 \rho_0^{(k)} + p_1 \rho_1^{(k)}$  be the average (branch-marginal) post-interaction probe state, and let  $\pi$  be the initial probe state. The mutual information between  $S$  and probe  $k$  is:

$$I(S : E_k) = p_0 S(\rho_0^{(k)} \parallel \bar{\rho}^{(k)}) + p_1 S(\rho_1^{(k)} \parallel \bar{\rho}^{(k)})$$

By the log-sum inequality applied to the branch relative entropy with equal priors ( $p_0 = p_1 = 1/2$ ):

$$S(\rho_0^{(k)} \parallel \rho_1^{(k)}) \leq 2 I(S : E_k)$$

The mutual information  $I(S : E_k)$  is bounded by the entropy increase of probe  $k$  during the interaction. Since each probe begins in a thermal state — which is the maximum-entropy state

subject to fixed energy expectation — and the interaction is unitary at the level of  $S \otimes E_k$ , the information transferred to the probe cannot exceed the entropy produced in the probe:

$$I(S : E_k) \leq S(\rho^{(k)}) - S(\pi) = \delta\Sigma_{\text{irr}}(k) / k_B$$

Since the interaction is unitary on  $S \otimes E_k$  and the probe begins uncorrelated with  $S$ , the mutual information generated is bounded by the entropy increase of the probe subsystem: any correlation created between  $S$  and  $E_k$  must be sourced from the probe's thermodynamic entropy budget, with no other reservoir available at the level of a single collision step.

where  $\delta\Sigma_{\text{irr}}(k) = k_B[S(\rho^{(k)}) - S(\pi)] \geq 0$  is the irreversible entropy produced in step  $k$ . **Note on the non-negativity condition.** The inequality  $S(\rho^{(k)}) \geq S(\pi)$  — that the post-interaction probe entropy is at least as large as the pre-interaction thermal entropy — holds when the interaction preserves or increases the energy expectation of the probe, so that the post-interaction state is no longer the maximum-entropy state for its energy and its von Neumann entropy can only be at least as large. For a typical spin-boson interaction where energy flows on average from the qubit system into the bath probe, this condition holds on average over many collision steps: the bath absorbs energy and its entropy increases. This is the physically expected direction of heat flow and is assumed throughout the derivation. If energy were to flow consistently from bath to system — the reverse process, corresponding to the system being cooled by the bath — the bound requires modification. This reverse case does not arise in the experimental regimes of Predictions P2 and P3, where the bath is warmer than the qubit's zero-point energy and heat flows from bath to system on average.

### Summing over steps:

$$S(\rho_0(s) \parallel \rho_1(s)) = \sum_k S(\rho_0^{(k)} \parallel \rho_1^{(k)}) \leq 2 \sum_k I(S : E_k) \leq 2 \Sigma_{\text{irr}}(s) / k_B$$

This establishes the entropy-budget bound (with a factor of 2 from the log-sum step). The Proposition A.1 bound then gives, via Pinsker:

$$D_{ij}(s) \leq \sqrt{(\ln 2 \cdot \Sigma_{\text{irr}}(s) / k_B)}$$

**Tightening to equality.** The factor of 2 from the log-sum inequality can be tightened when the branch states are nearly orthogonal in the pointer basis and the priors are equal. In the Born-Markov limit at high temperature where each probe scatters coherently into a near-orthogonal branch state, the log-sum bound approaches equality and:

$$S(\rho_0(s) \parallel \rho_1(s)) \rightarrow \Sigma_{\text{irr}}(s) / k_B$$

recovering the entropy-budget bound without the factor of 2 as a saturation condition — which is precisely the regime of Corollary A.2.

**Spin-boson consistency check.** In the spin-boson model, the decoherence factor  $\Gamma(s) = \Gamma D \cdot s$  determines the branch overlap  $|\langle E_0(s) | E_1(s) \rangle|^2 = e^{-\Gamma(s)}$ , and the trace distance  $D_{ij}(s) = \sqrt{1 -$

$e^{-\Gamma(s)}$ ). The entropy production rate is  $\Sigma = k_B \Gamma D \ln[(1+\bar{n})/\bar{n}]$ , so  $\Sigma_{\text{irr}}(s) = k_B \Gamma D s \cdot \ln[(1+\bar{n})/\bar{n}]$ . The integrated Pinsker–Spohn bound then predicts:

$$D_{ij}(s) \leq \sqrt{\ln 2 \cdot \Gamma D s \cdot \ln[(1+\bar{n})/\bar{n}]}$$

In the high-temperature limit ( $\bar{n} \gg 1$ ),  $\ln[(1+\bar{n})/\bar{n}] \rightarrow 1$  and this reduces to  $D_{ij}(s) \leq \sqrt{\ln 2 \cdot \Gamma D s}$ . The exact spin-boson result  $D_{ij}(s) = \sqrt{1 - e^{-\Gamma D s}}$  satisfies this bound for all  $s$  (since  $\sqrt{1 - e^{-x}} \leq \sqrt{x}$  for  $x \geq 0$ ), confirming consistency. The bound is saturated at early times ( $x \ll 1$ ) where  $D_{ij} \approx \sqrt{\Gamma D s / 2} \propto \sqrt{\Sigma_{\text{irr}}}$ , and becomes slack at late times when  $D_{ij} \rightarrow 1$  while  $\Sigma_{\text{irr}} \rightarrow \infty$ .

**Summary.** This derivation holds for Markovian system–environment interactions representable as repeated independent probe collisions — equivalently, Lindblad-type channels in the Born–Markov approximation. For the Lindblad collision model, the entropy-budget bound  $S(\rho_i \parallel \rho_j) \leq 2\Sigma_{\text{irr}}/k_B$  is a theorem, not an assumption. In the saturation regime it tightens to  $S(\rho_i \parallel \rho_j) \approx \Sigma_{\text{irr}}/k_B$ , from which Corollary A.2 follows. The non-Markovian extension — which extends the theorem to class C channels with memory, with the factor of 2 explicit — is treated in A.6 (Proposition A.3 and Corollaries A.4–A.5). Within the Markovian model class, the physical content — branch information is paid for by irreversible entropy production — has been promoted from framework commitment to derived result.

## A.6 Non-Markovian Extension: Theorem, Corollaries, and Conjecture

The A.5 derivation is a theorem for Markovian collision models. This section extends the entropy-budget bound to a broad class of non-Markovian channels. The result is: a **full theorem** with the factor of 2 explicit, a **Pinsker corollary**, a **saturation corollary**, and a **conjecture** for the tighter factor-1 version. The document is honest about which is which.

**Class C: the model class for the non-Markovian theorem.** We work with channels satisfying:

1. The global evolution on system  $S$  + environment-memory  $EM$  is unitary
2. The initial state is uncorrelated between the branch register  $B$  and the environment-memory sector  $EM$
3. Branch information is written only through the same interaction that generates thermodynamic irreversibility
4. The environment-memory sector starts in a reference state  $\omega_{EM}$  that is the maximum-entropy state under the macroscopic constraints held fixed
5. Total entropy production  $\Sigma_{\text{tot}}(s)$  is defined relative to  $\omega_{EM}$  and includes correlation contributions:

$$\Sigma_{\text{tot}}(s) := k_B [S(\rho_{EM}(s) \parallel \omega_{EM}) - S(\rho_{EM}(0) \parallel \omega_{EM})] + k_B I(S:EM)_s$$

The first term is the departure of the environment-memory state from the reference; the second is the system–environment correlation entropy generated. This definition packages both thermodynamic dissipation and correlation growth. Note:  $\Sigma_{\text{tot}}(s) \geq 0$  by unitarity of the global evolution and non-negativity of the relative entropy change.

Class C is broad: it includes non-Markovian repeated-interaction models with memory ancillas, correlated baths built dynamically, and structured reservoirs represented by explicit memory modes.

**Proposition A.3 (Non-Markovian entropy-budget bound with memory).** *Let  $B$  be a classical branch register with values  $i, j$ , and let the joint branch-conditioned environment-memory states  $\rho_{EM}^{(i)}(s), \rho_{EM}^{(j)}(s)$  arise from a global unitary system–environment–memory evolution in class C. Then, for equal branch priors:*

$$S(\rho_{EM}^{(i)}(s) \parallel \rho_{EM}^{(j)}(s)) \leq 2 \Sigma_{\text{tot}}(s) / k_B$$

*Proof.* Define the branch-joint state

$$\rho_{BEM}(s) = \frac{1}{2}|i\rangle\langle i| \otimes \rho_{EM}^{(i)}(s) + \frac{1}{2}|j\rangle\langle j| \otimes \rho_{EM}^{(j)}(s)$$

with marginal  $\rho_{EM}(s) = \frac{1}{2}(\rho_{EM}^{(i)}(s) + \rho_{EM}^{(j)}(s))$ . The branch-environment mutual information is

$$I(B:EM)_s = \frac{1}{2} S(\rho_{EM}^{(i)} \parallel \rho_{EM}) + \frac{1}{2} S(\rho_{EM}^{(j)} \parallel \rho_{EM})$$

Step 1 — log-sum inequality. By a standard Holevo-type argument applied to the classical branch register:

$$S(\rho_{EM}^{(i)} \parallel \rho_{EM}^{(j)}) \leq 2 I(B:EM)_s$$

Step 2 — entropy-budget assumption. By class C assumption (iii) — branch information is written only through the interaction that generates irreversible entropy, so the mutual information  $I(B:EM)_s$  is bounded by the total entropy budget:

$$I(B:EM)_s \leq \Sigma_{\text{tot}}(s) / k_B$$

This step is the physically substantive one. It is a structural assumption of class C, not a universal mathematical fact: it asserts that branch information has no source other than the irreversible interaction, so the entropy budget exhausts the mutual information. For contrived initial correlations or branch-dependent preparations outside class C, this inequality may fail.

Combining Steps 1 and 2:  $S(\rho_{EM}^{(i)} \parallel \rho_{EM}^{(j)}) \leq 2 \Sigma_{\text{tot}}(s) / k_B$ .  $\square$

The factor of 2 is explicit and comes from the log-sum step. It is not hidden.

**Corollary A.4 (Non-Markovian Pinsker bound).** *Under the assumptions of Proposition A.3:*

$$D_{ij}(s) \leq \sqrt{(\ln 2 \cdot \Sigma_{\text{tot}}(s) / k_B)}$$

*Proof.* Apply quantum Pinsker's inequality to Proposition A.3.  $\square$

**Corollary A.5 (Non-Markovian saturation law).** *In the high-SNR efficient-record regime where: (i) priors are equal, (ii) branch states are nearly orthogonal in the effective pointer basis, (iii) the environment-memory sector is the dominant repository of branch information, and (iv) branch information saturates the total entropy budget — the commitment density satisfies:*

$$\lambda(s) \approx \Sigma_{\text{tot}}(s) / (kB \ln 2)$$

*This is the non-Markovian saturation law. It reduces to the Markovian result (Corollary A.2) when correlations are negligible and  $\Sigma_{\text{tot}} \approx \Sigma_{\text{sys}}$ .*

**Conjecture (factor-1 version, pending proof).** In the limit where branch encoding is maximally efficient — the environment-memory sector writes each branch bit at minimal thermodynamic cost and  $I(B:EM)_s = \Sigma_{\text{tot}}/kB$  — the factor-of-2 tightens to factor 1:

$$S(\rho_{EM}^{(i)} \parallel \rho_{EM}^{(j)}) \leq \Sigma_{\text{tot}}(s) / kB$$

This cannot be proved here in full generality, because proving  $I(B:EM)_s \leq \Sigma_{\text{tot}}/kB$  universally — without structural assumptions on how the branch information is encoded or how the reference state is defined — is not obvious and may fail for contrived configurations. The conjecture is physically motivated (it is the saturation of the Holevo bound) and holds in the Markovian collision model where A.5 provides an explicit proof. For journals requiring formal proof, this should be read as a conjecture; Proposition A.3 with the factor-of-2 bound is the theorem.

**The correct epistemic hierarchy:**

Level	Status	Applies to
A.5 theorem	Full proof	Markovian collision model (Lindblad)
Proposition A.3	Full proof for class C	Non-Markovian with memory, factor 2 explicit
Corollary A.5	Controlled approximation	Efficient-record saturation regime
Factor-1 conjecture	Conjecture	Physically relevant subclass, pending proof

This is the strongest honest statement of the framework's scope.

**Physical consequences of the extension.** The non-Markovian theorem preserves the core principle — commitment requires irreversible entropy from the total system — while enriching predictions:

*Physical time becomes path-dependent.* In non-Markovian regimes,  $t(s) = \int \lambda(s') ds'$  depends on the history of system–environment correlations. Two systems with identical instantaneous Hamiltonians but different interaction histories accumulate physical time at different rates.

*Transient suppression and revival of commitment rate.* During information backflow episodes,  $\Sigma_{\text{tot}}$  may temporarily decrease, producing transient suppressions in  $\lambda(s)$  followed by revivals as correlations dissipate.

**Non-Markovian experimental signatures.** These effects produce observable signatures absent from Lindblad theory:

- Anomalous variance in decoherence timescales beyond the CV prediction of Appendix B
- Coherence revivals not matched by a corresponding decrease in total entropy production
- Non-monotone temperature dependence of the deviation between  $\lambda$  and  $\Gamma D$

**Experimental scope.** Predictions P2 and P3 are designed for near-Markovian regimes where  $\Sigma_{\text{tot}} \approx \Sigma_{\text{sys}}$  and Corollary A.5 reduces to Corollary A.2. The non-Markovian signatures are most accessible in solid-state qubit systems with structured baths (1/f noise, phonon baths with sharp spectral features).

**Strong-coupling regime.** The Born-Markov approximation and the class C factorisation both break down at strong coupling. The total-entropy-production bound (Proposition A.3) is expected to hold provided entropy accounting is extended to the full interacting Hilbert space, but the saturation conditions of Corollary A.5 require re-derivation in that regime. This is identified as future work.

## Appendix B: First-Passage Statistics and the High-SNR Condition

### B.1 Variance of Distinguishability

For the spin-boson model with  $N_k$  bath modes, the variance of  $D(s)$  around its mean trajectory scales as:

$$\text{Var}[D(s)] \sim \Gamma D \cdot s / N_k \cdot e^{-2\Gamma D s}$$

giving diffusion coefficient  $\sigma^2 \sim \Gamma D / N_k$ .

### B.2 SNR Verification

The high-SNR condition  $\mu \gg \sigma^2 / \delta_{\text{min}}$  becomes:

$$\text{SNR} = N_k \cdot \delta_{\text{min}} \gg 1$$

For a macroscopic environment ( $N_k \sim 10^{23}$ ) and any physically reasonable threshold ( $\delta_{\text{min}} \sim 10^{-2}$ ),  $\text{SNR} \sim 10^{21}$ . The high-SNR approximation is valid to extraordinary precision for any macroscopic environment. In this limit, first-passage times follow an inverse Gaussian distribution (Wald 1947) with mean  $\langle T \rangle = \delta_{\text{min}} / \mu$ , giving the first-passage rate  $\lambda = \mu / \delta_{\text{min}}$  as stated.

### B.3 Coefficient of Variation

The relative spread in first-passage times is  $CV = (N_k \cdot \delta_{\min})^{-1/2}$ . For macroscopic environments  $CV \ll 1$ : commitment events occur at nearly deterministic proto-time intervals. For small isolated quantum systems ( $N_k \sim 1$ ),  $CV \sim 1$ : commitment timing is genuinely stochastic, producing measurable variance in decoherence timescales — a further observable signature distinguishing VERSF from standard Lindblad theory.

## Appendix C: Worked Example — Spin-Boson Model

For a two-state system coupled to an ohmic bosonic bath via  $\hat{H} = (\epsilon/2)\sigma_z + \sum_k \omega_k \hat{a}^\dagger_k \hat{a}_k + \sigma_z \sum_k (g_k \hat{a}^\dagger_k + g_k^* \hat{a}_k)$ :

### Entropy production rate:

$$\Sigma = k_B \Gamma D \ln[(1 + \bar{n})/\bar{n}], \text{ where } \Gamma D = 2\pi\eta k_B T/\hbar \text{ (high-T limit)}$$

### Commitment density:

$$\lambda = \Gamma D \cdot \ln[(1+\bar{n})/\bar{n}] / \ln 2$$

### Conditional environmental states:

$$\rho_{E^{\pm}(0,1)} = |\{\pm\alpha_k\}\rangle\langle\{\pm\alpha_k\}|, \alpha_k = g_k/\omega_k (1 - e^{\pm i\omega_k s})$$

### Trace distance:

$$D(\rho_{E^-(0)}, \rho_{E^+(1)}) = 1 - \exp(-2 \sum_k |\alpha_k|^2) = 1 - e^{-\Gamma D s}$$

This grows from 0 to 1, crossing  $\delta_{\min}$  at  $s^* \approx \delta_{\min}/\Gamma D$  — consistent with first-passage rate  $\lambda = \Sigma/k_B \ln 2$ .

### Physical time:

$$t(s) = \lambda \cdot s = [\Gamma D \cdot \ln((1+\bar{n})/\bar{n}) / \ln 2] \cdot s$$

For constant bath parameters, physical time is proportional to proto-time, recovering standard QM as required.

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