κ-Field Wave Dynamics, Geometric Memory, and Non-Markovian Decay in VERSF

Second Companion Paper to: *Memory-Modified Decay: How the Past Participates in VERSF*

For the General Reader

The first paper in this series (*Memory-Modified Decay: How the Past Participates in VERSF*, hereafter Paper I) showed that radioactive decay is modified by the κ-field — a physical field sourced by irreversible commitment events — and that this modification produces an oscillatory algebraic tail $\cos(mt)/t$ in the late-time survival curve. Paper I derived this result rigorously from the Volterra equation, using a memory kernel $K(\tau) \sim \cos(m\tau)/\tau$ as input. It attributed the kernel to the κ-field but left its derivation to a companion paper.

This paper provides that derivation — and more. It asks and answers two prior questions simultaneously:

Question 1: Where does the kernel $K(\tau) \sim \cos(m\tau)/\tau$ come from? A single commitment event produces a κ-field disturbance decaying as $\tau^{-3/2}$ — why should many events together produce a $1/\tau$ kernel? We show via exact Fourier analysis that the full spatial integral of the κ-field Green's function over a uniform three-dimensional source is not $1/\tau$ but $\sin(m\tau)/m$ — constant-amplitude oscillation. The $1/\tau$ kernel specifically emerges when sources are effectively distributed along a one-dimensional geometry (such as a single nucleus's causal history), where the dimensional scaling formula $G_{eff}^{(d)} \sim \tau^{(d-3)/2}$ gives τ^{-1} for d=1. This d=1 result is the conjectured physical origin of the $\cos(m\tau)/\tau$ kernel of Paper I; its rigorous derivation including the full transverse spatial integration is identified as an open problem. The two behaviours — $\sin(m\tau)/m$ and $\cos(m\tau)/\tau$ — have fundamentally different mathematical structures (poles versus logarithmic branch cuts in the Laplace domain) with physically different consequences for the decay correction.

Question 2: What is the dynamical mechanism? Rather than treating the κ-field as a fixed background, we project the κ-field wave equation onto the system's location to obtain an explicit second-order ODE for the local geometric field g(t). This ODE has the form of a damped oscillator sourced by prior decay events. Solving it exactly shows that the decay law acquires a correction $\sim e^{-\gamma t} \cos(mt)$ — a damped oscillatory transient that dominates at intermediate times.

The two answers are deeply connected, and a third answer — the selection mechanism — completes the picture. The $\sin(m\tau)/m$ kernel (exact 3D spatial integral) has simple poles in the Laplace domain at $p = \pm im$; these poles, shifted into the left half-plane by explicit damping, produce the $e^{-\delta t} \cos(mt)$ correction derived by the ODE method. The $\cos(m\tau)/\tau$ kernel (governing a single nucleus via causal-coherence selection) has logarithmic branch cuts, producing the algebraic 1/t tail of Paper I. The selection mechanism — the observation that a single nucleus "listens" only to past commitment events that remain phase-coherent with its own commitment geometry — is derived asymptotically under the Gaussian tube ansatz in §4. Crucially, the tube is *transversely narrow* (width $\ell_{cc} \ll m^{-1}$, suppressing sources far from the nucleus's worldline) but *longitudinally unrestricted* (the nucleus's causal history extends indefinitely in time with no temporal cutoff). This transversely narrow, longitudinally unrestricted geometry is what produces $\cos(m\tau)/\tau$ rather than the full 3D kernel $\sin(m\tau)/m$.

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Abstract

We derive the effective memory kernel governing non-Markovian decay within the Void Energy-Regulated Space Framework (VERSF) from the κ-field wave equation, and simultaneously provide the dynamical geometric mechanism through which the resulting memory modifies the decay law. Paper I (*Memory-Modified Decay: How the Past Participates in VERSF*) employed a kernel $K(\tau) \sim \cos(m\tau)/\tau$ in the Volterra equation to derive an algebraic 1/t correction to radioactive decay; this paper provides the derivations that Paper I deferred.

Via exact Fourier analysis we show that the full spatial integral of the retarded κ-field Green's function over a uniform three-dimensional source gives $K_{exact}(\tau) = \sin(m\tau)/m$ — constant-amplitude oscillation, not $1/\tau$ decay. This carries simple poles in the Laplace domain at $p = \pm im$, shifted into the left half-plane by a pole displacement $\delta = \varepsilon/(2(\lambda^2 + m^2))$, implying exponentially damped oscillatory corrections $e^{-\delta t} \cos(mt)$ to the Volterra evolution. The $\cos(m\tau)/\tau$ kernel of Paper I — which carries logarithmic branch cuts and generates algebraic 1/t tails — arises in

physically distinct source configurations: for a d-dimensional source, $G_{eff}^{(d)} \sim \tau^{(d-3)/2} \cos(m\tau)$, giving constant amplitude for d=3 (established), $\tau^{-1/2}$ for d=2 (established via J_0 Bessel asymptotics), and the conjectured $\tau^{-1} = \cos(m\tau)/\tau$ for d=1. The d=1 result is consistent with the scaling pattern but awaits rigorous derivation including the transverse momentum integration.

We then introduce a causal-coherence weighting W_{cc} — motivated by the CCC principle that a given nucleus is influenced only by past commitment events that remain phase-coherent with its local commitment geometry — and show that a Gaussian tube W_{cc} that is transversely narrow ($\ell_{cc} \ll m^{-1}$) and longitudinally unrestricted ($L_{cc} \to \infty$) gives $K_{sel}(\tau) \sim \ell_{cc}^2 \cos(m\tau)/\tau$, proved in the $L_{cc} \to \infty$ limit by explicit stationary phase on both the light-cone (delta function) and interior Bessel contributions. This establishes, conditionally on the Gaussian tube form and the identification $\ell_{cc} \sim \xi$ (CCC coherence scale), the complete derivation chain from the κ-field wave equation to Paper I's Volterra kernel. Deriving $\ell_{cc} \sim \xi$ from fold-interface dynamics is the remaining open task.

We establish a three-level unified hierarchy: single event → $\tau^{-3/2}$; 3D ensemble → $\sin(m\tau)/m$ (poles → damped oscillation, this paper); 1D effective geometry → $\cos(m\tau)/\tau$ (branch cuts → algebraic tail, Paper I). The precise VERSF mechanism selecting the 1D effective regime, and the amplitude A_g of the geometric oscillation, are identified as the two priority open calculations.

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1. Introduction

Exponential decay arises from the Markovian assumption: each unstable system has a constant decay probability per unit time, independent of its history. In the Void Energy-Regulated Space Framework (VERSF), this assumption is relaxed. Physical evolution occurs within a geometry shaped by irreversible commitment events, whose influence propagates through the κ-field and persists in time.

Paper I (*Memory-Modified Decay: How the Past Participates in VERSF*) made this precise at the level of the Volterra equation: if the κ-field memory kernel decays as $K(\tau) \sim \cos(m\tau)/\tau$, then the survival curve acquires an oscillatory algebraic tail $\cos(mt)/t$, established both perturbatively and non-perturbatively via a Laplace branch-cut argument. Paper I attributed the kernel to the κ-field propagator and cited the present work for its derivation.

This paper provides that derivation. We approach the problem from two complementary angles that turn out to be deeply unified:

The kernel approach (§2–§5): We compute the spatial integral of the retarded κ-field Green's function over an ensemble of distributed commitment sources. The exact 3D integral gives $\sin(m\tau)/m$. For a single nucleus, a transversely narrow ($\ell_{cc} \ll m^{-1}$), longitudinally unrestricted worldline tube weighting of the exact Green's function gives $K_{sel} \sim \cos(m\tau)/\tau$, where the $1/\tau$

envelope comes from the light-cone (delta function) contribution and the $\cos(m\tau)$ oscillation comes from the interior Bessel contribution evaluated by stationary phase.

The projection approach (§5–§8): We project the κ-field wave equation onto the system's location to obtain a second-order ODE for the local geometric field $g(t)$, then solve the coupled decay–geometry system to extract the time-domain correction.

In §9 we show that these two approaches are manifestations of the same underlying physics, distinguished by the Laplace structure of the effective kernel: poles for the exact 3D result (→ damped oscillation), branch cuts for the 1D effective kernel (→ algebraic tail). The three papers — Paper I and the two halves of the present paper — describe three levels of a unified hierarchy.

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2. κ-Field Dynamics and the Single-Source Green's Function

The κ-field satisfies the sourced massive wave equation

$$(\partial_t^2 - \nabla^2 + m^2)\, \kappa(x, t) = \rho_{committed}(x, t) \qquad (1)$$

where m is the κ-field mass scale and $\rho_{committed}$ is the committed-event density sourced by prior decays. The retarded solution is

$$\delta\kappa(x, t) = \int d^3x' \int_{-\infty}^{t} dt'\, G_{ret}(x - x', t - t')\, \rho_{committed}(x', t') \qquad (2)$$

Single-source Green's function. For a point source at the origin, the large-time asymptotic of the retarded Green's function at fixed r is

$$G_{ret}(r, \tau) \sim A_r \cos(m\tau + \varphi_r) / \tau^{3/2} \text{ as } \tau \to \infty \qquad (3)$$

where A_r and φ_r are constants depending on m and r. We sketch the derivation. The exact retarded Green's function in 3+1 dimensions involves the Bessel function J_1 evaluated on the interior of the light cone:

$$G_{ret}(r, \tau) = \delta(\tau - r)/(4\pi r) - \theta(\tau - r) \cdot m\, J_1(m\sqrt{\tau^2 - r^2}) / (4\pi\sqrt{\tau^2 - r^2})$$

For large τ at fixed r, $\sqrt{\tau^2 - r^2} \approx \tau$ and $J_1(m\tau) \sim \sqrt{2/\pi m\tau} \cos(m\tau - 3\pi/4)$ by the standard large-argument Bessel asymptotics. The dominant term therefore behaves as

$$G_{ret}(r, \tau) \sim -m/(4\pi) \cdot \sqrt{2/\pi m\tau} \cos(m\tau - 3\pi/4) / \tau \sim A_r \cos(m\tau + \varphi_r) / \tau^{3/2}$$

confirming equation (3) with explicit constants A_r and phase $\varphi_r = -3\pi/4$. This $\tau^{-3/2}$ decay reflects three-dimensional wave spreading: the κ-field disturbance from a single commitment event distributes over an ever-growing sphere, with amplitude falling as the surface area grows.

The question. Paper I employs $K(\tau) \sim \cos(m\tau)/\tau$ rather than $\tau^{-3/2}$. Since physical memory is sourced by an ensemble of commitment events, not a single isolated decay, the resolution must involve spatial integration over the source distribution. The next two sections compute this integral.

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3. Spatial Integration: Exact Result and Dimensional Scaling

Setup. We consider a spatially homogeneous distribution of commitment sources with number density w_0 and local commitment rate $R(t)$. The κ-field perturbation at the observation point is

$$\delta\kappa(t) = w_0 \int_0^t R(t')\, G_{eff}(t - t')\, dt' \quad (4)$$

where the spatially integrated Green's function is

$$G_{eff}(\tau) = \int d^3r\, G_{ret}(r, \tau) \quad (5)$$

Exact result via Fourier analysis. Taking the spatial Fourier transform of equation (1), the retarded Green's function satisfies

$$(\partial_t^2 + k^2 + m^2)\, \tilde{G}_{ret}(k, \tau) = \delta(\tau), \quad \tilde{G}_{ret} = 0 \text{ for } \tau < 0$$

with solution

$$\tilde{G}_{ret}(k, \tau) = \theta(\tau) \sin(\omega_k \tau) / \omega_k, \quad \omega_k = \sqrt{k^2 + m^2} \quad (6)$$

By the Fourier inversion theorem, $\int d^3r\, G_{ret}(r, \tau) = \tilde{G}_{ret}(k=0, \tau)$. Evaluating (6) at $k = 0$ where $\omega_0 = m$:

$$G_{eff}(\tau) = \theta(\tau) \sin(m\tau) / m \quad (7)$$

This is exact — no asymptotic approximation has been made. The $\tau^{-3/2}$ falloff of the single-source Green's function at fixed r is completely cancelled by the growing volume of source material entering causal contact at each time τ. The exact 3D spatially integrated kernel is constant-amplitude oscillation.

Dimensional scaling law. For a d-dimensional source distribution embedded in 3D space, the effective kernel scales asymptotically as

$$G_{eff}^{(d)}(\tau) \sim C_d \cdot \tau^{(d-3)/2} \cdot \cos(m\tau + \varphi_d) \quad (8)$$

The d=3 case is established exactly above (equation 7, constant amplitude). The d=2 case — sources on a surface — gives $G_{eff}^{(2)}(\tau) = J_0(m\tau)/2$, derived directly from the 2D surface integral in cylindrical coordinates:

$G\_eff^{(2)}(\tau) = \int_0^\infty 2\pi\rho\, d\rho\, G\_ret(\rho, \tau) = (1/2) - (m/2) \int_0^\tau \rho\, J_1(m\sqrt{\tau^2-\rho^2})/\sqrt{\tau^2-\rho^2}\, d\rho$

Substituting $u = m\sqrt{\tau^2-\rho^2}$ in the second integral gives $\int_0^{m\tau} J_1(u)\, du = 1 - J_0(m\tau)$, so the two terms combine to give $J_0(m\tau)/2$ exactly. For large τ, $J_0(m\tau) \sim \sqrt{2/\pi m\tau} \cos(m\tau - \pi/4)$, confirming the $\tau^{-1/2}$ scaling of equation (8) at d=2.

The d=1 case — sources along a worldline — is expected to give $G\_eff^{(1)} \sim \cos(m\tau)/\tau$ by the same pattern. Section §4 derives this asymptotically under the Gaussian tube ansatz without invoking $G\_eff^{(1)}$ as an intermediate step; the worldline integral $\int_{-\infty}^\infty G\_ret(|z|, \tau)\, dz$ is left as an identified open calculation (Priority 3 in §13).

When does the 1/τ kernel govern the dynamics? The exact 3D result (7) is $\sin(m\tau)/m$ — constant-amplitude, no $1/\tau$ decay. The kernel $\cos(m\tau)/\tau$ arises in *physically distinct source configurations*, not as a component of the 3D result. Section §4 identifies the causal-coherence selection mechanism as the physical VERSF origin of the 1D regime. The key point is that $\sin(m\tau)/m$ and $\cos(m\tau)/\tau$ are effective kernels for different source geometries with different Laplace structures (§5), carrying correspondingly different physical consequences.

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4. Causal-Coherence Selection: From 3D Ensemble to Worldline Memory

Section §3 established that the full 3D spatially integrated kernel is $\sin(m\tau)/m$ (constant-amplitude oscillation, poles), while the $\cos(m\tau)/\tau$ kernel of Paper I requires effectively one-dimensional source geometry. This section proposes the VERSF selection mechanism that produces the 1D regime for a single unstable nucleus, and identifies the calculation that would make the proposal a theorem.

The causal-coherence principle. A decay event at time t is not influenced equally by all prior commitment events in the surrounding 3D medium. It is influenced by the subset of prior events that remain *causally phase-coherent* with that specific nucleus's local commitment geometry. This is not an independent postulate — it follows from the CCC framework: the threshold condition $\chi(L) = \rho L^4/\hbar c \sim 1$ distinguishes events near the commitment boundary from those far above it, and only coherent past events contribute meaningfully to future fact formation at a given locus.

The key observation is that this selected subset is effectively one-dimensional. The full κ-field propagates in 3D, but the memory channel relevant to a given nucleus is a filtered projection of that field — a projection that selects a narrow causal tube aligned with the nucleus's own commitment history. This is not dimensional reduction of space itself but dimensional reduction of the relevant memory channel.

Mathematical formulation. The exact retarded solution at the target nucleus's location x_0 is

$\delta\kappa(x_0, t) = \int d^3x' \int_{-\infty}^{t} dt'\, G_{ret}(x_0 - x', t - t')\, \rho_{committed}(x', t')$

Replace the raw source density with a causally-selected source:

$$\delta\kappa_{sel}(x_0, t) = \int d^3x' \int_{-\infty}^{t} dt'\, G_{ret}(x_0 - x', t - t')\, \rho_{committed}(x', t')\, W_{cc}(x', t'; x_0, t) \qquad (4a)$$

where $W_{cc}(x', t'; x_0, t)$ is a causal-coherence weighting centred on the target nucleus at (x_0, t). This weighting encodes the CCC principle: only past commitment events that remain geometrically and phase coherent with the local commitment structure at (x_0, t) contribute to the selected source.

The tube ansatz. The simplest model for W_{cc} is a Gaussian tube with transverse width ℓ_{cc}:

$$W_{cc}(\rho, z; \tau) = \exp[-\rho^2 / (2\ell_{cc}^2)] \cdot \exp[-(z - \tau)^2 / (2L_{cc}^2)] \qquad (4b)$$

The derivation of $K_{sel}(\tau)$ separates into two contributions — K_{LC} (light-cone delta function) and K_{int} (interior Bessel) — which require different treatment of the longitudinal width L_{cc}. For K_{LC}, the longitudinal Gaussian centred at $z \approx \tau$ is used to select the forward ($z > 0$) light-cone branch and suppress the backward branch; it evaluates to approximately 1 near $z \approx \tau$ (where $\rho \ll \tau$), so K_{LC} does not depend sensitively on L_{cc}. For K_{int}, the stationary phase evaluation is at $\theta = 0$ (corresponding to $z = 0$ in the substitution $z = \tau \sin\theta$), which lies at the interior of the causal region. The Gaussian contribution at $z = 0$ is $\exp(-\tau^2/2L_{cc}^2)$, which is exponentially small for finite L_{cc} with $\tau \gg L_{cc}$. For K_{int} to give the full $\int_0^{\pi/2} J_1(m\tau \cos\theta)\, d\theta$ and hence the $\cos(m\tau)/\tau$ result, one must take $\mathbf{L_{cc} \to \infty}$ — a longitudinally unrestricted tube — so the Gaussian is effectively unity over $z \in [0, \tau]$.

The tube is therefore *transversely narrow* ($\ell_{cc} \ll m^{-1}$) but *longitudinally unrestricted* ($L_{cc} \to \infty$). This is the physically natural description of a single nucleus's causal history: the worldline extends indefinitely in the temporal direction with no longitudinal cutoff, while transverse coherence is lost at scale $\ell_{cc} \sim \xi$. We therefore describe W_{cc} as a *transversely narrow worldline tube*, not a "causal filament near the light cone." The factorisation of W_{cc} into transverse and temporal factors is valid because ρ is a spatial variable and $\tau = t - t'$ is temporal, so a temporal coherence envelope $e^{-\tau/\tau_{cc}}$ factors outside the spatial integral without affecting the spatial calculation. The inequality $\ell_{cc} \sim \xi \ll m^{-1}$ (placing individual nuclei in the narrow-tube regime) is expected on physical grounds but requires the κ-field coupling calculation of Paper I's §10.1 for verification.

The selected kernel is:

$$K_{sel}(\tau) = 2\pi \int_0^{\infty} \rho\, d\rho \int_{-\infty}^{\infty} dz\, G_{ret}(\sqrt{\rho^2 + z^2}, \tau)\, W_{cc}(\rho, z; \tau) \qquad (4c)$$

Asymptotic derivation of $K_{sel}(\tau)$. We now compute $K_{sel}(\tau)$ for $\tau \gg \ell_{cc}, L_{cc}$, $m\tau \gg 1$, using the exact Green's function

$$G_{ret}(r, \tau) = \theta(\tau)\, \delta(\tau - r)/(4\pi r) - \theta(\tau - r) \cdot m\, J_1(m\sqrt{\tau^2 - r^2}) / (4\pi\sqrt{\tau^2 - r^2})$$

The two terms contribute separately as K_LC (light-cone delta function) and K_int (interior Bessel).

K_LC: the light-cone term. Insert $G_{LC} = \delta(\tau - r)/(4\pi r)$ into (4c). The delta function $\delta(\tau - \sqrt{\rho^2 + z^2})$ collapses the integral to the surface $r = \tau$. Using $\delta(\tau - \sqrt{\rho^2 + z^2}) = (\tau/|z|)[\delta(z - z^*(\rho)) + \delta(z + z^*(\rho))]$, where $z^*(\rho) = \sqrt{\tau^2 - \rho^2}$, and discarding the $z < 0$ branch (suppressed by the longitudinal Gaussian centred at $z \approx +\tau$):

$$K_{LC}(\tau) = (1/2) \int_0^\tau \rho \, d\rho / z^*(\rho) \cdot W_{cc}(\rho, z^*(\rho); \tau)$$

For $\rho \ll \tau$: $z^*(\rho) \approx \tau - \rho^2/(2\tau)$, so $z^*(\rho) - \tau \approx -\rho^2/(2\tau)$ and $1/z^*(\rho) \approx 1/\tau$. The longitudinal Gaussian evaluates to $\exp(-(z^*(\rho)-\tau)^2/2L_{cc}^2) \approx \exp(-\rho^4/8\tau^2 L_{cc}^2) \approx 1$, which is subleading relative to the transverse Gaussian for $\rho \sim \ell_{cc}$. Therefore:

$$K_{LC}(\tau) \approx (1/(2\tau)) \int_0^\infty \rho \exp(-\rho^2/2\ell_{cc}^2) \, d\rho = \boldsymbol{\ell_{cc}^2/(2\tau)} \qquad (4d)$$

This is a real, non-oscillatory $1/\tau$ term — it carries the amplitude scaling but no oscillatory m-dependence. Note that this term does *not* vanish: the integral $\int \rho \, e^{-\rho^2/2\ell_{cc}^2} \, d\rho = \ell_{cc}^2$ is finite and the τ^{-1} factor is explicit. The Laplace transform of K_LC is $\int_0^\infty e^{-p\tau} (\ell_{cc}^2/2\tau) \, d\tau \sim (\ell_{cc}^2/2) \log(1/p)$ near $p = 0$ — a logarithmic branch point at $p = 0$, distinct from the $p = \pm im$ branch cuts of the oscillatory kernel. Inserted into the Volterra equation, K_LC generates an additional non-oscillatory algebraic correction to $N(t)$ alongside the oscillatory $\cos(mt)/t$ term. This correction is sub-leading to the oscillatory term for most observation windows but should be accounted for in precision fits (see Laplace table in §5).

K_int: the interior Bessel term. In the limit $L_{cc} \to \infty$, the longitudinal weighting is effectively unity over $z \in [0, \tau]$. Applying the narrow-tube approximation $G_{ret}(\sqrt{\rho^2+z^2}, \tau) \approx G_{ret}(|z|, \tau)$ (valid with relative error $O(\ell_{cc}^2/z^2)$ integrated against $e^{-\rho^2/2\ell_{cc}^2}$ over ρ) and performing the transverse Gaussian integral:

$$K_{int}(\tau) \approx -m \, \ell_{cc}^2 \int_0^\tau J_1(m\sqrt{\tau^2 - z^2}) / \sqrt{\tau^2 - z^2} \, dz$$

Substituting $z = \tau \sin\theta$ (so $\sqrt{\tau^2 - z^2} = \tau \cos\theta$):

$$K_{int}(\tau) \approx -m \, \ell_{cc}^2 \int_0^{\pi/2} J_1(m\tau \cos\theta) \, d\theta \qquad (4e)$$

For $m\tau \gg 1$, the large-argument asymptotic $J_1(x) \sim \sqrt{2/\pi x} \cos(x - 3\pi/4)$ and an endpoint stationary-phase evaluation at $\theta = 0$ (where the phase $h(\theta) = \cos\theta$ has $h'(0) = 0$) give:

$$\int_0^{\pi/2} J_1(m\tau \cos\theta) \, d\theta \sim -\cos(m\tau)/(m\tau)$$

Therefore:

$$K_{int}(\tau) \sim -m \, \ell_{cc}^2 \cdot (-\cos(m\tau)/(m\tau)) = \ell_{cc}^2 \cos(m\tau)/\tau \qquad (4f)$$

Full result. Combining K_LC and K_int:

$K\_sel(\tau) \sim \ell\_cc^2/(2\tau) + \ell\_cc^2 \cos(m\tau)/\tau = (\ell\_cc^2/\tau)[1/2 + \cos(m\tau)]$

The oscillatory term $\ell\_cc^2 \cos(m\tau)/\tau$ dominates the time-varying part of $K\_sel(\tau)$. The constant offset 1/2 is non-oscillatory and is already accounted for in K_LC (§5, Laplace table). The dominant oscillatory component is:

$K\_sel(\tau) \sim \ell\_cc^2 \cos(m\tau)/\tau$, with sub-leading non-oscillatory correction $\ell\_cc^2/(2\tau)$

Setting $A\_cc = \ell\_cc^2$ and absorbing any initial-condition phase into φ:

$K\_sel(\tau) \sim A\_cc \cos(m\tau + \varphi) / \tau, A\_cc \propto \ell\_cc^2$ (4g)

This is precisely the kernel $K(\tau) \sim A \cos(m\tau + \varphi)/\tau$ of Paper I. The $1/\tau$ amplitude comes from the light-cone (delta function) term K_LC; the oscillatory $\cos(m\tau)$ phase comes from the interior Bessel term K_int. The amplitude $A\_cc \propto \ell\_cc^2$ makes the result observable at smaller scales for larger coherence tubes.

Proposition (Causal-Coherence Tube Selection). *Let $G\_ret(r,\tau)$ be the retarded Green's function of the massive κ-field in 3+1 dimensions, and let $K\_sel(\tau) = \int d^3x\, G\_ret(|x|,\tau)\, W\_cc(x,\tau)$ with W_cc a Gaussian tube of transverse width ℓ_cc and $L\_cc \to \infty$ (transversely narrow, longitudinally unrestricted). Then for $\tau \gg \ell\_cc$ and $m\tau \gg 1$:*

$K\_sel(\tau) \sim A\_cc \cos(m\tau + \varphi)/\tau, A\_cc \propto \ell\_cc^2$

This establishes that the $1/\tau$ Volterra kernel arises as the asymptotic memory law of a transversely narrow worldline tube. The physical identification $\ell\_cc \sim \xi$ (CCC coherence scale) — required to apply this result to individual nuclei — is structurally motivated but not yet derived from fold-interface dynamics (Priority 3, §13).

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5. Laplace Structure: Poles versus Branch Cuts

The distinction between $G\_eff = \sin(m\tau)/m$ and $K(\tau) = \cos(m\tau)/\tau$ is not merely quantitative. Their Laplace transforms have qualitatively different structures with qualitatively different physical consequences.

Exact 3D kernel: $\hat{K}(p) = 1/(p^2 + m^2)$, simple poles.

$\hat{K}(p) = \int_0^\infty e^{-p\tau} \sin(m\tau)/m \, d\tau = 1/(p^2 + m^2)$ (9)

This has simple poles at $p = \pm im$. In the Volterra equation $\hat{N}(p) = N_0/[p + \lambda - \varepsilon \hat{K}(p)]$, the poles of $\hat{N}(p)$ satisfy $(p + \lambda)(p^2 + m^2) = \varepsilon$. Near $p = im$ for small ε, writing $p = im + \delta$:

$(im + \delta + \lambda)(2im\delta + O(\delta^2)) = \varepsilon \to \delta \approx \varepsilon / [2im(\lambda + im)]$

Computing explicitly: $2im(\lambda + im) = 2im\lambda - 2m^2$, so

$\delta = \varepsilon / (-2m^2 + 2im\lambda) = \varepsilon(-2m^2 - 2im\lambda) / (4m^2(\lambda^2 + m^2))$

Taking the real part: $\mathrm{Re}(\delta) = -2m^2\varepsilon / (4m^2(\lambda^2 + m^2)) = -\varepsilon/(2(\lambda^2 + m^2)) < 0$. The poles are displaced into the left half-plane by

$$\boldsymbol{\delta_{real} = \varepsilon / (2(\lambda^2 + m^2))} \quad (9a)$$

Their Bromwich contribution gives the exponentially damped oscillatory correction

$\sim e^{-\delta_{real} \cdot t} \cos(mt + \psi)$ with $\delta_{real} = \varepsilon/(2(\lambda^2 + m^2))$.

Near-light-cone kernel: $\hat{K}(p) \sim C \log(p - im)$, logarithmic branch cut.

$$\boldsymbol{\hat{K}(p) \sim C \log(p - im) + \text{analytic as } p \to im} \quad (10)$$

This is the logarithmic branch cut identified in Paper I (§6). It cannot be removed by perturbation theory. The Bromwich inversion generates a contribution

$\sim \cos(mt)/t$ at large t.

Summary:

Kernel	Laplace structure	N(t) correction	Source geometry
$\sin(m\tau)/m$ (exact 3D)	Poles at $p = \pm im$	$e^{-\delta t} \cos(mt + \psi)$	3D uniform ensemble
$K_{LC} \sim \ell_{cc}^2/(2\tau)$ (non-oscillatory)	Log branch point at $p = 0$	non-oscillatory $1/t$	Worldline tube, sub-leading
$\cos(m\tau)/\tau$ (worldline tube, $L_{cc} \to \infty$)	Log branch cut at $p = im$	$\cos(mt)/t$ algebraic (Paper I)	Worldline tube, dominant oscillatory

The K_{LC} contribution has a branch point at $p = 0$ and generates a non-oscillatory algebraic N(t) correction sub-leading to $\cos(mt)/t$ for most observation windows. In precision fits, both contributions from the worldline tube — the non-oscillatory K_{LC} and the oscillatory K_{int} — should be retained.

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6. Geometric Formulation of Decay Dynamics

We now turn from the global kernel analysis to the local projection approach, which produces the same physical content in the time domain and provides the explicit dynamical mechanism.

Let $N(t)$ denote the number of undecayed systems at time t. In VERSF, the effective decay rate depends on the local κ-field environment. We expand the effective rate around the background κ-field value κ_0:

$$\lambda_{eff}(\kappa) = \lambda_0 + (d\lambda_{eff}/d\kappa)|_{\kappa_0} \cdot (\kappa - \kappa_0) + O((\kappa - \kappa_0)^2)$$

Defining the dimensionless geometric field $g(t) \equiv (\kappa(x_{sys}, t) - \kappa_0)/\kappa_{ref}$ and the coupling $\alpha = (\kappa_{ref}/\lambda_0)(d\lambda_{eff}/d\kappa)|_{\kappa_0}$, this gives to leading order:

$$\lambda(t) = \lambda_0 [1 + \alpha g(t)] + O(\alpha^2 g^2) \quad (11)$$

Equation (11) is the leading term in the Taylor expansion of $\lambda_{eff}(\kappa)$ around κ_0 — not an independent postulate. The coupling α is precisely the quantity $\varepsilon \sim d\lambda_{eff}/d\kappa|_{\kappa_0}$ of Paper I's §10.3 (with κ_{ref} absorbed). The decay equation becomes

$$dN/dt = -\lambda_0 [1 + \alpha g(t)] N \quad (12)$$

The standard exponential law is recovered when $g(t) \equiv 0$. All deviations from exponential decay are encoded in the past history of $g(t)$.

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7. Local Projection: The Geometric Field ODE

Why the field equation is second-order. The simplest relaxation model would be a first-order equation $\dot{g} + \gamma g$ = source, producing purely exponential memory decay with no oscillatory corrections. The κ-field requires a second-order equation because it supports propagating modes: the dispersion relation $\omega^2 = k^2 + m^2$ (from §3) requires two time derivatives. A first-order (diffusive) field would have dispersion $\omega \sim -ik^2$, with no oscillatory solutions. The second-order ODE for $g(t)$ is therefore a consequence of the κ-field's wave character, not an additional postulate.

Projection. The κ-field satisfies $(\Box + m^2)\kappa = \rho_{committed}$ (equation 1). Projecting onto the dominant mode at the system's location x_{sys} and adding phenomenological damping at rate γ (arising from energy loss to spatial dispersion — the same process responsible for the interior/boundary decomposition in §3):

$$\ddot{\kappa} + 2\gamma \dot{\kappa} + (m^2 + \gamma^2) \kappa = \rho_{source}(t)$$

Defining $\omega_g^2 = m^2 + \gamma^2$ from the dispersion relation, and dividing by κ_{ref}:

$$\ddot{g}(t) + 2\gamma \dot{g}(t) + \omega_g^2 g(t) = \beta \cdot [-(1/N) dN/dt]^* \quad (13)$$

where:

- $\gamma > 0$ is the geometric damping rate (spatial dispersion of κ-field coherence),

- $\omega_g^2 = m^2 + \gamma^2$ from the dispersion relation, giving oscillation frequency $\Omega = \sqrt{\omega_g^2 - \gamma^2} = m$,
- β is a source coupling with dimensions time^{-1} (it absorbs the dimensional mismatch between the κ-field normalisation κ_{ref} and the commitment event rate dN/dt, which has dimensions time^{-1}; β is not dimensionless),
- N^* is the number of systems at the CCC commitment threshold — i.e., the N for which $\chi(L) = \rho L^4/\hbar c \sim 1$ — set by the system's commitment density, not a free parameter.

Connection to the Fourier result. The projection approach and the spatial integration approach are two routes to closely related objects. The Green's function of the *undamped* homogeneous ODE $\ddot{g} + \omega_g^2 g = 0$ with $g(0) = 0$, $\dot{g}(0) = 1$ is $\sin(m\tau)/m$, which is exactly the spatially integrated zero-mode $\tilde{G}_{ret}(k=0, \tau) = \sin(m\tau)/m$ of §3. For $\gamma > 0$, the ODE Green's function is the damped version $(1/m)e^{-\gamma\tau} \sin(m\tau)$, which is the physically relevant intermediate-time object and reduces to $\sin(m\tau)/m$ exactly only in the limit $\gamma \to 0$. The identification between the two approaches is therefore exact in the undamped limit and holds as a controlled approximation for small γ/m. This correspondence confirms that the exact 3D kernel $\sin(m\tau)/m$ (poles) governs the ODE description at intermediate times, while the $1/\tau$ boundary contribution (branch cuts) governs the late-time algebraic tail of Paper I.

Amplitude scaling. Since the source term is proportional to $\lambda_0 N_0/N^*$, the amplitude A_g of the geometric oscillation scales as N_0/N^*. For systems near or above the CCC threshold ($N_0 \sim N^*$), the correction is unsuppressed and most readily observable. For systems far below threshold ($N_0 \ll N^*$), the correction is suppressed, consistent with the expectation that $\varepsilon \ll 1$ in ordinary macroscopic conditions.

Three postulates made explicit:

1. Commitment events source the geometric field (RHS of equation 13, proportional to $-dN/dt$).
2. The field relaxes with memory — not instantaneously — through the damping term $2\gamma \dot{g}$.
3. The field supports propagating oscillatory modes via $\omega_g^2 g$, inherited from the dispersion relation $\omega^2 = k^2 + m^2$.

The underdamped case $\gamma < \omega_g$ (equivalently $\gamma < m/\sqrt{2}$ roughly) produces oscillatory corrections; the overdamped case produces purely decaying corrections.

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8. Coupled Decay System and Perturbative Solution

Equations (12) and (13) form a coupled system. For $|\alpha g| \ll 1$ — justified by operating far above the CCC commitment threshold, established in Paper I's §10.3 — we solve perturbatively.

Exact integral form. Separating variables in equation (12): $dN/N = -\lambda_0[1 + \alpha g(t)]dt$, integrating from 0 to t:

$N(t) = N_0 \exp(-\lambda_0 t - \alpha \lambda_0 \int_0^t g(s)\, ds)$ (14)

This is exact given equation (12). All deviations from exponential decay are encoded in $\int_0^t g(s)\, ds$.

Zeroth-order source. At zeroth order in α, $N(t) \approx N_0 e^{-\lambda_0 t}$, giving source term

$$-(1/N^*)\, dN/dt \approx (\lambda_0 N_0 / N^*)\, e^{-\lambda_0 t}$$

which decays exponentially with rate λ_0.

Driven and free regimes. For $t \sim 1/\lambda_0$ (driven regime): $g(t)$ is sourced by the decaying exponential and builds amplitude A_g. For $t \gg 1/\lambda_0$ (free regime): the source has decayed away and $g(t)$ satisfies the homogeneous equation with initial conditions set at $t_0 \sim \text{few}/\lambda_0$.

The open amplitude A_g. The amplitude and phase of the free oscillation are set by matching at $t \sim t_0$, requiring the driven Green's function problem to be solved. This is deferred to future work and is the priority open calculation (see §12). Throughout the remainder, A_g appears as a parameter; the qualitative form of the correction is established independently.

In the underdamped case, the free solution for $t > t_0$ is:

$g(t) = A_g e^{-\gamma(t-t_0)} \cos(\Omega(t-t_0) + \varphi),\ \Omega = \sqrt{\omega_g^2 - \gamma^2} = m$ (15)

The equality $\Omega = m$ follows directly from $\omega_g^2 = m^2 + \gamma^2$.

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9. Exact Integral Evaluation: Steady Renormalisation and Oscillatory Transient

Substituting (15) into (14), shifting the time origin to t_0:

$$\int_{t_0}^t g(s)\, ds = A_g \operatorname{Re}[\, e^{i\varphi} \cdot (1 - e^{-(\gamma+im)(t-t_0)}) / (\gamma + im)\,]$$

Separating into steady and transient parts:

$$= A_g \operatorname{Re}[\, e^{i\varphi} / (\gamma + im)\,] - A_g \operatorname{Re}[\, e^{i\varphi} e^{-(\gamma+im)(t-t_0)} / (\gamma + im)\,]$$

Steady part. The first term converges to the constant

$$C_\infty = A_g (\gamma \cos \varphi + m \sin \varphi) / (\gamma^2 + m^2)$$

This renormalises the initial amplitude: $N_0 \to N_0' = N_0 \exp(-\alpha\lambda_0 C_\infty)$. It produces no oscillatory behaviour.

Oscillatory transient. The second term gives

$$-[A_g / (\gamma^2 + m^2)]\, e^{-\gamma(t-t_0)}\, [\gamma \cos(m(t-t_0) + \varphi) + m \sin(m(t-t_0) + \varphi)]$$

This is the physically observable correction: exponentially damped at rate γ, oscillating at frequency m. Writing $\gamma \cos\theta + m \sin\theta = \sqrt{\gamma^2 + m^2}\, \sin(\theta + \arctan(\gamma/m))$, the full late-time result is:

$$\mathbf{N(t) \approx N_0' e^{-\lambda_0 t} \{ 1 + [\alpha\lambda_0 A_g / \sqrt{\gamma^2 + m^2}]\, e^{-\gamma(t-t_0)} \sin(m(t-t_0) + \varphi + \arctan(\gamma/m)) \}} \quad (16)$$

This is the main result of the geometric approach. The observable modification to exponential decay has:

- Frequency: m (the κ-field mass scale, same as Paper I),
- Damping rate: γ (spatial dispersion rate),
- Amplitude: $\alpha\lambda_0 A_g / \sqrt{\gamma^2 + m^2}$, suppressed by $\alpha \ll 1$.

Phase consistency with Paper I. Paper I's correction carries phase $\varphi_c - 2 \arctan(m/\lambda)$ (companion eq. 9). In the limit $\gamma \to 0$, equation (16) carries phase $\varphi + \pi/2$ (since $\arctan(\gamma/m) \to 0$ and sin → cos). These are consistent provided $\varphi_c = \varphi + \pi/2 + 2 \arctan(m/\lambda)$, which is a relationship between the κ-field state at $t = 0$ (Paper I's reference) and at $t = t_0$ (this paper's reference). Since φ is a free parameter in both — set by the κ-field vacuum configuration at experiment initiation — this is not a contradiction but a phase-shift relation between two conventions. The two expressions are phase-consistent in the overlap limit.

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10. Unified Hierarchy and Consistency with Paper I

We now assemble the full picture. The three papers in this series describe the same physical phenomenon at three levels of description.

The three-level hierarchy:

Level	Object	τ behaviour	Laplace structure	Paper
Microscopic	Single-source $G_{ret}(r, \tau)$ at fixed r	$A_r \cos(m\tau) / \tau^{3/2}$	—	—
Mesoscopic	3D spatially integrated $G_{eff}(\tau)$; ODE Green's function ($\gamma \to 0$)	$\sin(m\tau) / m$	Poles at $p = \pm im$	This paper, §3 and §7
Selected	Causally-coherence-selected kernel $K_{sel}(\tau)$, $\ell_{cc} \ll m^{-1}$	$\cos(m\tau) / \tau$ (derived §4)	Log branch cut at $p = im$	This paper §4 (kernel derived); Paper I (results conditional on kernel)

Transitions:

- *Microscopic → Mesoscopic:* Spatial integration over the full 3D source distribution. The growing causal volume cancels the $\tau^{-3/2}$ single-source decay, giving constant $\sin(m\tau)/m$ (§3, exact).
- *Mesoscopic → Selected:* Causal-coherence filtering with a transversely narrow ($\ell_{cc} \ll m^{-1}$), longitudinally unrestricted tube gives $K_{sel} \sim \ell_{cc}^2 \cos(m\tau)/\tau$ by the stationary phase calculation of §4, completing the derivation chain without requiring $G_{eff}^{(1)}$ as an intermediate step. The worldline integral $G_{eff}^{(1)} = \int_{-\infty}^{\infty} G_{ret}(|z|,\tau)\, dz$ remains an independent open calculation (Priority 3 in §13) that would provide an alternative derivation route but is not a prerequisite for the main result.

Single-nucleus versus bulk-ensemble:

Which level governs depends on the measurement:

- *Single-nucleus decay:* $\ell_{cc} \sim \xi \ll m^{-1}$ (CCC coherence scale); selected kernel $\cos(m\tau)/\tau$; branch cuts; Paper I algebraic tail applies.
- *Bulk ensemble:* full 3D source volume; mesoscopic kernel $\sin(m\tau)/m$; poles; damped oscillatory correction from this paper applies.
- *Real experiment:* mixture, crossover near $\ell_{cc} \sim m^{-1}$.

The open problem. Whether the ODE (mesoscopic) and Volterra (macroscopic) treatments are strictly complementary or partially overlapping is an identified open problem. In particular: the free-field oscillation $g(t) \sim e^{-\gamma t} \cos(mt)$, inserted as a source into Paper I's Volterra integral, generates contributions to $\int K(t-s)N(s)ds$ that may partially reproduce or modify the branch-cut $1/t$ tail. Resolving this requires a unified full-wave treatment without driven/free regime separation — the second priority open calculation (§12).

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11. Physical Interpretation

The mechanism in this paper has five components, ordered from microscopic to macroscopic:

1. **Commitment events source the κ-field.** Each decay is an irreversible commitment event that perturbs the κ-field at the system's location through the RHS of equation (13).
2. **The κ-field propagates and accumulates.** As established in §3, the contributions from all prior events integrate over the causal volume. For a 3D ensemble, the accumulated perturbation is constant-amplitude oscillation $\sin(m\tau)/m$; for effectively 1D source geometry, it is the decaying $\cos(m\tau)/\tau$.
3. **The local geometry oscillates.** Projected onto the system's location, the accumulated κ-field perturbation drives the geometric field $g(t)$ into underdamped oscillation at frequency m, decaying at rate γ.

4. **The oscillating geometry feeds back into the decay rate.** Through equation (11), the effective decay rate at time t depends on g(t), which encodes a structured imprint of all prior decays.
5. **Exponential decay is the limit of static geometry.** Setting $\alpha = 0$ (no coupling) or $\gamma \to \infty$ (instantaneous relaxation) recovers the standard law. In the limit $\gamma \to \infty$, equation (13) collapses to an algebraic instantaneous relation $g \approx \beta(-dN/dt)/(N^* \omega g^2)$, removing all dynamics and with it all memory.

The geometric meaning of the two kernels. The $\sin(m\tau)/m$ kernel represents the κ-field memory accumulated from a spatially extended 3D ensemble — constant-amplitude oscillation whose persistence reflects the exact cancellation between the per-source amplitude decay and the growing volume of contributing sources. The $\cos(m\tau)/\tau$ kernel arises in physically distinct source configurations (effectively 1D geometry or causal-boundary dominance), where this cancellation is incomplete and algebraic decay results. These are not two components of a single kernel but two different effective kernels for two different physical situations. Which governs a given experiment is determined by the source geometry — the open problem of §12.

Non-negativity. As in Paper I, the perturbative correction in (16) can drive N(t) negative at asymptotically late times. The physically meaningful regime requires $\alpha\lambda_0 A_g / \sqrt{\gamma^2 + m^2} \ll 1$, guaranteed by $\alpha \ll 1$ under the weak coupling assumption. The experimentally relevant regime is the onset of the correction — where the damped oscillatory term is a small fractional modification of $N_0' e^{-\lambda_0 t}$.

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12. Experimental Implications

Equation (16) and the hierarchy of §9 together yield the following testable structure.

Two-regime signature. The modification to exponential decay has two observationally distinct regimes:

- *Intermediate times* ($t \sim$ few/λ_0 to $\sim 1/\gamma$): damped oscillatory modulation of the survival curve with envelope $e^{-\gamma t}$ and frequency m. In a log-survival plot, this appears as a sinusoidal ripple of slowly decreasing amplitude superimposed on the linear decay trend.
- *Late times* ($t \gg 1/\gamma$): persistent algebraic oscillatory tail $\cos(mt)/t$ (Paper I). In a log-survival plot, the curve departs from linearity with a $1/t$ envelope rather than exponential approach to zero.

Three independent observables. A complete dataset spanning both regimes would yield three independent constraints on κ-field parameters: the oscillation frequency m (common to both regimes), the intermediate-time envelope decay rate γ, and the crossover time $1/\gamma$ (which must be consistent across both regimes). These over-constrain the two-parameter (m, γ) system, providing a non-trivial internal consistency check.

Why the intermediate-time regime may be more accessible. The intermediate-time correction has absolute amplitude ~ $\alpha\lambda_0 A_g / \sqrt{\gamma^2 + m^2}$, which for $t \ll 1/\gamma$ is approximately constant. The algebraic tail has amplitude ~ $\varepsilon A N_0/((\lambda^2 + m^2)t)$, decreasing as $1/t$. The intermediate-time correction is therefore generically larger in absolute amplitude for $t \ll 1/\gamma$. However, the experimental signal-to-noise also depends on $\lambda_0 t$: earlier times have higher count rates but potentially smaller fractional corrections. A complete signal-to-noise comparison requires numerical input on α, A_g, γ, and m; the qualitative statement is that both regimes are accessible to sufficiently precise measurements and the intermediate-time regime is the natural initial experimental target.

Detection requirements:

- High-precision long-time decay measurements spanning $t \sim t_0$ to several × $1/\gamma$ for the intermediate regime, and $t \gg 1/\gamma$ for the algebraic tail.
- Count statistics sufficient to resolve oscillatory modulation at fractional amplitude ~ $\alpha\lambda_0 A_g / \sqrt{\gamma^2 + m^2}$.
- Sensitivity to the three independent parameters m, γ, and the crossover time as separate fit quantities.
- Low external perturbation to prevent masking of the geometric signal.

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13. Epistemic Status and Open Problems

Established rigorously in this paper:

- $G_{ret}(r, \tau) \sim A_r \cos(m\tau + \varphi_r)/\tau^{3/2}$ for a single point source, via J_1 Bessel asymptotics, with $\varphi_r = -3\pi/4$ identified explicitly (§2),
- $G_{eff}(\tau) = \sin(m\tau)/m$ for a 3D uniform source, exact via Fourier (§3),
- $G_{eff}^{(2)} = J_0(m\tau)/2$ (exact) for a 2D surface source, via cylindrical coordinate integral; asymptotically $\sim \tau^{-1/2} \cos(m\tau - \pi/4)$ (§3),
- $K_{sel}(\tau) \sim A_{cc} \cos(m\tau + \varphi)/\tau$ with $A_{cc} \propto \ell_{cc}^2$, derived asymptotically under the Gaussian tube ansatz ($\ell_{cc} \ll m^{-1}$, $L_{cc} \to \infty$, $m\tau \gg 1$): the $1/\tau$ amplitude from the light-cone delta function term $K_{LC} \sim \ell_{cc}^2/(2\tau)$, the $\cos(m\tau)$ phase from the interior Bessel term $K_{int} \sim \ell_{cc}^2 \cos(m\tau)/\tau$ (§4, Proposition and equations 4d–4g),
- $\hat{K}(p) = 1/(p^2 + m^2)$ with simple poles; pole displacement $\delta_{real} = \varepsilon/(2(\lambda^2 + m^2))$ computed by explicit perturbative expansion (§5),
- Log branch cut structure of $\cos(m\tau)/\tau$ kernel (§5, following Paper I),
- $N(t)$ correction (16) from the ODE approach, with all integral steps exact (§8–§9),
- Phase consistency of (16) with Paper I eq. (9) in the $\gamma \to 0$ limit (§9).

Established conditionally (proof complete given the ansatz):

- The full derivation chain: κ-field wave equation → Gaussian tube selection → $K_{sel} \sim \cos(m\tau)/\tau$ → Paper I Volterra kernel (§2–§4). This holds asymptotically under the

Gaussian tube ansatz ($\ell\_cc \ll m^{-1}$, $L\_cc \to \infty$); the condition requiring further derivation is $\ell\_cc \leftrightarrow \xi$ from fold-interface dynamics.

Conjectured but not yet derived:

- $\ell\_cc \sim \xi$: the identification of the tube coherence width with the CCC coherence scale from fold-interface dynamics,
- Whether a non-Gaussian W_cc (e.g. derived directly from the CCC threshold condition) gives the same asymptotic (4e) or modifies the amplitude and phase,
- Whether the ODE (3D, poles) and Volterra (1D selected, branch cuts) corrections can be simultaneously active in the same experiment, or are strictly applicable to different measurement configurations.

Priority open problems, ordered:

Priority 1: Derive the Volterra kernel self-consistently. Obtain $K(\tau)$ from equation (1) with $\rho\_committed = \lambda(t)\, N(t)/V$ — the full self-consistent backreaction — without approximating spatial and temporal integration. This determines whether the operative Volterra kernel is $\sin(m\tau)/m$ or $\cos(m\tau)/\tau$, resolving the open problem of §3 and §9.

Priority 2: Compute A_g from the driven Green's function. The amplitude of the geometric oscillation requires the driven-regime solution of equation (13) with source $(\lambda_0 N_0/N^*)\, e^{-\lambda_0 t}$. Until A_g is known, equation (16) is qualitatively correct in form but quantitatively incomplete.

Priority 3: Derive ℓ_cc ↔ ξ from fold-interface dynamics. Equations (4d–4g) establish $K\_sel \sim \ell\_cc^2 \cos(m\tau)/\tau$ asymptotically under the Gaussian tube ansatz with $\ell\_cc \ll m^{-1}$ and $L\_cc \to \infty$. The remaining task is to derive from VERSF fold-interface dynamics that the physical causal-coherence weighting W_cc takes this Gaussian form with ℓ_cc set by the CCC coherence scale ξ. This is the final step that would close the derivation chain from first principles to Paper I's kernel completely.

Priority 3(b): Extend to spatial fields g(x,t). The scalar g(t) is a point-source projection; the full treatment uses g(x,t) with coupling modified by the geometric weighting W(x) of Paper I's §10.2.

Priority 4: Derive γ from fold-interface dynamics. The damping rate γ is here identified with the spatial dispersion rate of κ-field coherence; its microscopic derivation from VERSF fold-interface dynamics is an open problem shared with Paper I.

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14. Summary

This paper provides both the kernel derivation that Paper I deferred and the dynamical geometric mechanism through which the κ-field memory modifies decay. The argument has two parallel strands that converge in §9.

Strand 1 (kernel emergence and selection, §2–§5):

1. The single-source κ-field Green's function decays as $\tau^{-3/2}$, derived via J_1 Bessel asymptotics (§2).
2. Spatial integration over a 3D uniform source gives exactly $\sin(m\tau)/m$ — constant-amplitude oscillation (§3).
3. The causal-coherence selection mechanism (§4): under the Gaussian tube ansatz with $\ell_{cc} \ll m^{-1}$ and $L_{cc} \to \infty$, $K_{sel}(\tau) \sim \ell_{cc}^2 \cos(m\tau)/\tau$ is derived asymptotically by stationary phase (equations 4d–4g). This is the derivation chain Paper I cited. The remaining open step is $\ell_{cc} \leftrightarrow \xi$ from fold-interface dynamics.
4. The two kernels have different Laplace structures: poles with explicit shift $\delta = \varepsilon/(2(\lambda^2 + m^2))$ for $\sin(m\tau)/m$; log branch cut for $\cos(m\tau)/\tau$ (§5).

Strand 2 (geometric dynamics, §6–§9):

1. The effective decay rate is a Taylor expansion in the local κ-field perturbation, giving equation (11) (§6).
2. Projecting the κ-field wave equation onto the system's location gives an underdamped ODE (13) for the geometric field $g(t)$, with $\Omega = m$ from the dispersion relation (§7).
3. The coupled decay–geometry system is solved perturbatively; all deviations from exponential decay are encoded in the integral of $g(s)$ (§8).
4. Evaluating this integral exactly gives the damped oscillatory correction (16) with frequency m and damping rate γ (§9).

Unified conclusion:

The κ-field correction to radioactive decay has two regimes distinguished by measurement scale. For a bulk ensemble (3D source geometry, mesoscopic kernel $\sin(m\tau)/m$, poles), the operative correction is the damped oscillatory transient $\sim e^{-\delta t} \cos(mt + \psi)$ derived here. For a single nucleus (causal-coherence selected asymptotically under the Gaussian tube ansatz, selected kernel $K_{sel} \sim \ell_{cc}^2 \cos(m\tau)/\tau$, branch cuts), the operative correction is the algebraic 1/t tail of Paper I. The derivation chain from the κ-field wave equation to Paper I's Volterra kernel is now complete, asymptotically under the tube ansatz and conditionally on $\ell_{cc} \sim \xi$ from fold-interface dynamics. The amplitude A_g of the geometric oscillation is the second priority open calculation.

References

VERSF Programme Papers

[P1] Taylor, K. *Memory-Modified Decay: How the Past Participates in VERSF.* VERSF Theoretical Physics Programme. (Hereafter Paper I. The primary companion to this work; derives the algebraic 1/t correction to radioactive decay from the Volterra equation with kernel

$K(\tau) \sim \cos(m\tau)/\tau$, establishes it non-perturbatively via a Laplace branch-cut argument, and grounds the model parameters in established VERSF sectors in §10.)

[P2] Taylor, K. *Causal-Coherence Compatibility in the VERSF Framework (CCC Paper).* VERSF Theoretical Physics Programme. (Establishes the commitment threshold condition $\chi(L) = \rho L^4/\hbar c \gtrsim 1$, defines the coherence scale ξ, and introduces the CCC principle that governs which prior events contribute to local fact formation. The causal-coherence weighting W_{cc} of §4 is motivated directly by the CCC framework.)

[P3] Taylor, K. *BCB Lagrangian Unification and the κ-Field Sector.* VERSF Theoretical Physics Programme. (Derives the κ-field wave equation $(\Box + m^2)\kappa = \rho_{committed}$ from BCB action principles; provides the κ-field mass scale m, source coupling α_κ, and the κ-field coupling calculation referenced as Paper I's §10.1 and §10.3.)

[P4] Taylor, K. *The Fold Interface Law and Emergent Geometric Structure.* VERSF Theoretical Physics Programme. (Derives the fold-interface dynamics that govern the microscopic origin of the damping rate γ and the coherence scale ξ; the derivation of $\ell_{cc} \leftrightarrow \xi$ (Priority 3 of this paper) requires results from this work.)

[P5] Taylor, K. *From Necessary Facts to Physical Structure: Synthesis.* VERSF Theoretical Physics Programme. (Programme synthesis paper; provides the broader context for the VERSF hierarchy of which the κ-field memory kernel is one sector.)

Standard Mathematical and Physical References

[M1] Abramowitz, M. & Stegun, I.A. (eds.) *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables.* Dover Publications, 1964. (Standard reference for Bessel function identities and asymptotics used throughout: $J_1(x) \sim \sqrt{2/\pi x}\cos(x - 3\pi/4)$ for large x (§9.2); $J_0(m\tau)/2$ as the 2D surface kernel (§9.1); integral representations of J_0 and J_1 (§9.1.18–9.1.21); the exponential integral E_1 and its logarithmic behaviour near the origin (§5.1).)

[M2] NIST Digital Library of Mathematical Functions. *dlmf.nist.gov.* F.W.J. Olver et al. (eds.), 2010–. (Online successor to [M1]; used for Bessel asymptotics (§10.17), the exponential integral E_1 (§6.2), and Laplace transform tables (§1.14).)

[M3] Bender, C.M. & Orszag, S.A. *Advanced Mathematical Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory.* Springer, 1999. (Standard reference for the stationary phase and endpoint stationary phase methods used in §4 to evaluate $\int_0^{\pi/2} J_1(m\tau\cos\theta)\,d\theta$; see Chapter 6.)

[M4] Gripenberg, G., Londen, S.-O. & Staffans, O. *Volterra Integral and Functional Equations.* Cambridge University Press, 1990. (Comprehensive treatment of Volterra integro-differential equations of the type used in Paper I [P1] and referenced throughout this paper; covers the

Laplace transform theory of convolution equations, pole and branch-cut structure, and Tauberian theorems for large-time behaviour.)

[M5] Gel'fand, I.M. & Shilov, G.E. *Generalized Functions, Volume 1: Properties and Operations.* Academic Press, 1964. (Reference for the retarded Green's function of the massive Klein-Gordon equation in 3+1 dimensions (§2 of this paper), including the exact form involving J_1 and the delta function on the light cone. See Vol. 1, §2.4.)

[M6] Peskin, M.E. & Schroeder, D.V. *An Introduction to Quantum Field Theory.* Addison-Wesley, 1995. (Standard field theory reference for retarded propagators of massive scalar fields; the Fourier representation $\tilde{G}\_ret(k, \tau) = \theta(\tau) \sin(\omega\_k \tau)/\omega\_k$ used in §3 is derived in §2.4. Also provides background on Feynman propagators and the spectral decomposition of two-point functions relevant to the Laplace analysis of §5.)

[M7] Itzykson, C. & Zuber, J.-B. *Quantum Field Theory.* McGraw-Hill, 1980. (Alternative reference for the massive retarded Green's function and its Bessel function form; see §3-1.)

[M8] Titchmarsh, E.C. *Introduction to the Theory of Fourier Integrals.* Oxford University Press, 2nd ed., 1948. (Reference for the Fourier transform identity $\int d^3r\, G\_ret(r,\tau) = \tilde{G}\_ret(k=0,\tau)$ used in §3, and for the Bromwich inversion formula and contour deformation methods used in §5 to extract 1/t tails from logarithmic branch cuts.)