Effective Stress–Energy from Irreversible Commitment Flow in VERSF

Unified Commitment Ontology, Source Uniqueness from Inherited Structural Theorems, Identification of the κ-Field with the Commitment-Density Field, Non-Markovian Memory Stress, and Anisotropic Transport-Curvature Sourcing of Emergent Lorentzian Geometry

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General-Reader Summary

The previous paper in this programme finished building the *geometric* side of VERSF gravitational physics. It identified the effective Lorentzian continuum — the geometric stage on which any future gravitational field equation must be written. What it deliberately did not do is build the *source* side: it said nothing about what kind of "stuff" lives on that stage and bends it.

That is the gap this paper closes.

In ordinary gravity, the source is the stress–energy tensor T_μν — a single symmetric object packaging energy density, pressure, and stress. Wherever there is mass, motion, or pressure, T_μν is nonzero, and that nonzero value is what curves the geometry. To get gravitational dynamics in VERSF, the same kind of object has to be built — but it has to be built out of *VERSF's* primitives, not standard matter fields. The fundamental stuff in VERSF is not particles; it is irreversible commitment events.

The first move of this paper is to recognize that the apparently distinct things VERSF talks about — commitment density, the propagating κ-field, accumulated memory, anisotropic transport curvature — are not independent ingredients. They are different *manifestations* of the same single underlying quantity: committed distinguishability. The count of irreversible commitment events gives a density; that density sources a propagating field (the κ-field); the field carries memory of its own history; and the geometry it produces inherits anisotropy from how the substrate transports distinguishability. The picture is one of unified commitment flow, not a federation of independent sectors.

Once that ontological unification is made, the paper proves something strong: every admissible source structure for the emergent Lorentzian geometry has to be a functional of these primitives, and nothing else. The proof draws on theorems already established elsewhere in the programme — that physical theories require stable records, that irreversible commitment is the unique route to such records, that observable physics is invariant under commitment reordering, that causal propagation is finite-speed, and that the substrate's internal structure is fixed by K = 7 closure. Together these force the source tensor into a very specific shape with only a small number of free parameters remaining, and one of those parameters — the mass of the κ-field — is fixed by the same K = 7 closure architecture, not phenomenologically inserted.

The paper does not write down Einstein equations. It deliberately stops just before them. What it does is supply the source object — symmetric, conserved, invariant under commitment reordering, causally well-behaved, and with no admissible alternative — that any later field equation will have to use.

Contents

Abstract

The preceding paper completed the kinematic geometry layer of the VERSF programme by deriving an effective Lorentzian continuum metric of the form

$$ds^2 = -c{\star}^2\, d\tau^2 + h_{ij}\, dx^i\, dx^j,\quad h_{ij} = \Omega^2\, \delta_{ij} + \lambda\, \hat{Q}_{ij}.$$

That construction installed the geometric stage on which gravitational dynamics must be formulated, but explicitly left open the *source* side of any field equation. The present paper closes that gap.

The construction proceeds in four steps.

1. **Unified commitment ontology (§2.6).** The apparently distinct VERSF source sectors — committed-record density, κ-field propagation, accumulated memory, anisotropic transport stress — are not independent primitives. They are different manifestations of a single chain: irreversible commitment count → density $\rho_{committed}$ → propagating commitment-density field $s \equiv \kappa$ → accumulated memory field Ξ → effective geometry. The identification $s \equiv \kappa$ under retarded boundary conditions is inherited from the κ-field equivalence theorem.
2. **Source Uniqueness Theorem (Theorem 3.1).** Combining the Facthood Necessity theorem, the commitment uniqueness theorems, CRE invariance, Causal–Coherence Compatibility, finite-speed causal propagation, and $K = 7$ closure, *every* admissible local source tensor on the CRE-quotient continuum reduces to a local covariant functional of $(\rho_{committed}, \kappa, \Xi, \hat{Q}_{\mu\nu})$. No admissible source structure exists outside this family.
3. **Three sector tensors (§§5–7).** The κ-sector stress tensor follows from the unique admissible action, with mass $m^2 = (4/3)\, \xi^{-2}$ structurally fixed by $K = 7$ closure and the CCC coherence scale ξ — no free mass parameter (Theorem 5.2). The memory-field sector contributes a non-Markovian stress built from the accumulated commitment history with kernel $M(\tau) \sim \cos(m\tau + \varphi)/\tau$ inherited from the worldline-memory programme; the local tensor form (Definition 6.1) is the leading derivative-expanded effective representation of a fundamentally nonlocal retarded functional (§6.6, Proposition 6.5, §11). The anisotropic transport-curvature sector is the most general parity-even second-order rank-2 tensor built from $\hat{Q}_{\mu\nu}$ and $\nabla R^{(\infty)}$, carrying three effective couplings (Theorem 7.4).
4. **Total source structure (Theorem 8.2).** The total effective stress–energy tensor decomposes uniquely as

$$T^{eff}_{\mu\nu} = T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(int)}_{\mu\nu},$$

where $T^{(int)}_{\mu\nu}$ is the inter-sector exchange tensor whose total divergence cancels that of the other three sectors. Total conservation, total CRE invariance, and the weak commitment energy condition are recorded as Theorems 8.3, 8.4, and 9.2.

The paper does **not** derive Einstein equations, fix the values of the three transport-curvature couplings, derive the effective gravitational coupling, or close the matter-coupling problem. Its

contribution is structural: it supplies the source tensor — uniquely determined up to a small parameter set, with no admissible alternative outside the unified commitment ontology — that any later dynamical equation must consume.

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1. Introduction

The Lorentzian completion paper closed the *kinematic and causal* layer of the geometry programme. Given the refinement-stable spatial transport metric $h_{ij} = \Omega^2 \delta_{ij} + \lambda \hat{Q}_{ij}$ and the invariant admissible commitment-propagation speed $c\star$, it produced the effective Lorentzian continuum metric

$$g_{\mu\nu} = \mathrm{diag}(-c\star^2, h_{ij}),$$

uniquely up to a constant translation of the observable commitment-time coordinate. That paper deliberately stopped short of dynamics. Its closing line identified two open problems as the natural next questions: the geometric field equation for $g_{\mu\nu}$ (OP4 in that paper) and the effective stress–energy tensor sourcing it (OP5).

OP5 is the subject of the present paper.

Why source structure first. In standard treatments, the source tensor is read off from a matter action that pre-exists the geometric construction. In VERSF, no such matter action is pre-given: the substrate primitives are committed records, transport curvature, and a propagating commitment-density field, none of which start life as a Lagrangian for "matter" in the conventional sense. Before any field equation can be written, the source side must be constructed from substrate inputs the geometry sector already supplies, and it must be shown that the construction is *structurally constrained* — that there is not a vast moduli space of admissible sources, because in that case the dynamical paper would face an under-determined problem.

The unification move. The construction adopts a unified ontology in which all admissible source structure reduces to committed distinguishability and its causal propagation. This is not a new postulate; it is a consequence of the chain of equivalences already established in the programme: irreversible commitment events generate a density ($\rho_{\text{committed}}$) which sources the unique admissible retarded propagating field (the κ-field, identified with the commitment-density field s under retarded boundary conditions by the κ-field equivalence theorem), which in turn generates a non-Markovian accumulated memory (Ξ) and inherits anisotropy from the transport-curvature sector ($\hat{Q}_{\mu\nu}$). The "many independent sectors" picture is replaced by a single chain of manifestations.

Inheritance, not reinvention. The Lorentzian continuum geometry $g_{\mu\nu}$, the committed-record current and density, the κ-field with its structurally-fixed mass, the memory kernel $M(\tau)$, the refinement-stable transport-curvature tensor $R^{(\infty)}_{ij}$ and its quadratic invariant $\hat{Q}_{ij}$, the Facthood Necessity theorem, the commitment uniqueness theorems, Commitment Reordering

Equivalence (CRE), Causal–Coherence Compatibility (CCC), and $K = 7$ closure are all established in companion papers cited in §2. The present paper takes them as inputs.

Scope. This paper supplies the source tensor $T^{\mathrm{eff}}_{\mu\nu}$, proves uniqueness up to a finite parameter set, and identifies the structural constraints on any future dynamical paper. It does *not* derive the geometric field equation, does *not* fix the values of effective couplings beyond those already fixed by inherited theorems, does *not* address quantum fluctuations, and does *not* close the matter-coupling problem. All of these are explicitly listed as open problems in §12.

Epistemic discipline. Results are labelled:

- **Proven** — established here under stated hypotheses, or in cited prior papers.
- **Conditional** — follows from proven results given an explicitly stated additional input.
- **Conjectural** — plausibility argument only, flagged as such.

2. Inherited Structures

We inherit the following without re-derivation.

2.1 Lorentzian transport-generated geometry

From the Lorentzian completion paper:

$$g_{\mu\nu} = \mathrm{diag}(-c_\star^2, h_{ij}), \quad h_{ij} = \Omega^2 \delta_{ij} + \lambda \hat{Q}_{ij}.$$

Greek indices $\mu, \nu, \ldots$ run over $(0, 1, \ldots, D)$, with $0 = \tau$ (observable commitment-time coordinate); Latin indices $i, j, \ldots$ run over the spatial sector $(1, \ldots, D)$. All covariant derivatives ∇_μ are with respect to g.

2.2 Refinement-stable transport-curvature tensor

The antisymmetric refinement-stable causal transport-curvature tensor $R^{(\infty)}_{ij}$ and its symmetric traceless quadratic invariant

$$\hat{Q}_{ij} = R^{(\infty)}_{ik} R^{(\infty)}{}_j{}^k - (1/D) \|R^{(\infty)}\|_F^2 \cdot \delta_{ij}$$

are inherited from the spatial-geometry paper. $\hat{Q}_{ij}$ lifts to the continuum as the purely spatial, symmetric, traceless tensor

$$\hat{Q}_{\mu\nu} := \gamma_\mu{}^\alpha \gamma_\nu{}^\beta \hat{Q}_{\alpha\beta}, \quad \gamma_{\mu\nu} := g_{\mu\nu} + c_\star^2 u_\mu u_\nu,$$

where u^μ is a unit future-directed observer field with $g_{\mu\nu} u^\mu u^\nu = -c_\star^{-2}$.

2.3 Committed-record density

The committed-record current J^μ is the continuum field encoding the flow of irreversible distinguishability. It is required to be future-directed and causal. The committed-record density measured by u^μ is

$$\rho_{\text{committed}} := -u_\mu J^\mu \geq 0.$$

$\rho_{\text{committed}}$ is the local count of irreversible commitment events per unit four-volume, in the rest frame of u^μ. It is the *source* of the κ-field (§2.5).

Observer convention. In all couplings to $\rho_{\text{committed}}$ throughout this paper — the κ-field source term, the memory-kernel integrand, and any composite construction — the observer field u^μ is the unit timelike congruence *intrinsic* to the commitment current J^μ. That is, u^μ is the four-velocity selected by the commitment-flow structure itself: $u^\mu := J^\mu / (-J^\nu J_\nu / c_\star^2)^{1/2}$ wherever J^μ is timelike (which is required by §2.3). It is not an external observer choice. Under this convention $\rho_{\text{committed}}$ is a CRE-invariant scalar density on the quotient continuum, and the κ-field equation of §2.5 is unambiguous.

2.4 Commitment Reordering Equivalence

Two proto-histories H_1, H_2 are commitment-reordering equivalent, $H_1 \sim_{\text{CRE}} H_2$, if they generate identical committed records, identical causal relations among them, identical cone fields, and identical transport-generated continuum geometry. Observable physics descends to the quotient $H_{\text{obs}} := H_{\text{proto}} / \sim_{\text{CRE}}$.

2.5 κ-field and structurally-fixed mass

The κ-field is the unique admissible retarded propagating mode of the commitment-density field $s(x, \tau)$. The κ-field equivalence theorem (inherited from the κ-field uniqueness papers) establishes that

$s(x, \tau) \equiv \kappa(x, \tau)$ under retarded physical boundary conditions,

with κ governed by the Klein–Gordon equation in the mostly-plus signature inherited from §2.1 ($g_{\mu\nu} = \mathrm{diag}(-c_\star^2, h_{ij})$) with d'Alembertian $\Box_g = \nabla_\mu \nabla^\mu$:

$$(\Box_g - m^2)\, \kappa = -\rho_{\text{committed}}.$$

This is the convention-consistent form for the kinetic-term sign of §5.1 below; the same equation reads $(\Box + m^2)\kappa = \rho_{\text{committed}}$ in the opposite-signature convention used in some companion papers. **Translation rule.** Throughout this paper, both the κ equation of motion and any inherited result are stated in the mostly-plus signature of §2.1. Companion-paper results stated in the mostly-minus convention (where $(\Box + m^2)\kappa = \rho_{\text{committed}}$) transform by flipping the sign of the operator only: m^2 and $\rho_{\text{committed}}$ are unchanged; the operator combination $(\Box \mp m^2)$

carries the convention dependence. Source-term sign on the right-hand side flips correspondingly to maintain physical content (κ as the retarded response to the source $\rho_{\text{committed}}$).

The κ-field mass is *not* a phenomenological parameter. It is structurally fixed by the $K = 7$ closure architecture and the CCC coherence scale ξ:

$m^2 = (4/3)\,\xi^{-2}$, $m = \sqrt{(4/3)} \cdot \xi^{-1}$.

The κ-sector therefore carries no free structural mass parameter; both m and the equation of motion are inherited.

2.6 Unified commitment ontology

The preceding VERSF papers establish that the apparently distinct sectors of the framework — committed-record density, propagating commitment-density field, accumulated memory, and anisotropic transport stress — are not independent primitives. They are different manifestations of a single chain:

$N_{\text{committed}} \to \rho_{\text{committed}} \to s(x, \tau) \equiv \kappa(x, \tau) \to \Xi(x, \tau) \to g_{\mu\nu}$.

Here:

- $N_{\text{committed}}$ is the count of irreversible commitment events,
- $\rho_{\text{committed}}$ is the local committed-event density,
- $s(x, \tau)$ is the commitment-density field,
- $\kappa(x, \tau)$ is its propagating retarded mode (equivalent to s under retarded boundary conditions; §2.5),
- $\Xi(x, \tau)$ is the accumulated non-Markovian memory field (§2.7),
- $g_{\mu\nu}$ is the emergent Lorentzian geometry sourced by the effective record structure.

The present paper adopts this unified ontology: *all admissible source structures reduce to committed distinguishability and its causal propagation*. The effective stress–energy tensor is therefore not a federation of independent sectors but a coarse-grained accounting of the same irreversible record dynamics in different functional aspects — local (κ), nonlocal-in-time (Ξ), and anisotropic-in-space ($\hat{Q}$).

2.7 Accumulated memory field

The κ-field memory papers establish that irreversible commitment sourcing generates a non-Markovian causal memory field

$\Xi(x, \tau) := \int d^D x' \int_{-\infty}^{\tau} d\tau'\, M(x, \tau; x', \tau')\, \rho_{\text{committed}}(x', \tau')$,

where M is the retarded memory kernel inherited from the oscillatory-memory-corrections programme. In the causal-coherence-selected worldline regime, M possesses algebraic late-time structure

$M(\tau) \sim \cos(m\tau + \varphi) / \tau$,

with m the structurally-fixed κ-field mass of §2.5 and φ a phase inherited from the worldline construction. The memory kernel is *not* the retarded Green function of $(\square_g - m^2)$; it is the worldline-projected memory kernel of the same operator, which carries different observable consequences (the algebraic envelope is the signature of the κ-field's coupling to the commitment-history sector, not of its local propagation).

2.8 K = 7 closure and Facthood Necessity

Two inherited structural theorems play load-bearing roles in §3:

- **Facthood Necessity Theorem.** Any admissible physical theory must produce stable distinguishable records. Theories that fail to generate stable records are not admissible at all.
- **K = 7 closure architecture.** The admissible internal structure of distinguishability transport is fixed by the K = 7 wheel structure. No additional independent internal degrees of freedom are admissible.

Both are inherited; see the K = 7 papers and the Facthood Necessity papers for proofs.

2.9 Notation and conventions

Symmetrization brackets: $A_{(\mu\nu)} := \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$. Frame norms: $\|\cdot\|_F$. We work in $D \geq 2$ spatial dimensions; the dimensional triviality at $D = 2$ ($\hat{Q} \equiv 0$ by orbit counting) carries over from the spatial-geometry paper.

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3. Admissible Source Uniqueness

The preceding sections list six independent structural constraints inherited from the programme: finite distinguishability, irreversible commitment, CRE invariance, CCC, finite-speed causal propagation, and K = 7 closure. We now show that these jointly constrain the admissible source structure of the emergent Lorentzian geometry to a single covariant functional family.

Theorem 3.1 — Admissible Source Uniqueness [proven, conditional on inherited structural theorems]

Let $T_{\mu\nu}$ be an admissible effective source tensor on the CRE-quotient continuum, satisfying:

(A1) locality ($T_{\mu\nu}$ at x depends only on fields and their finitely-many derivatives at x, possibly with retarded causal integration over the past lightcone);

(A2) CRE invariance ($T_{\mu\nu}[H_1] = T_{\mu\nu}[H_2]$ whenever $H_1 \sim_{CRE} H_2$);

(A3) causal propagation (T_μν depends on the substrate state only via fields with finite-speed causal evolution, with c⋆ as the admissible propagation speed);

(A4) finite distinguishability (T_μν is built from fields whose information content per unit volume is bounded by the finite-distinguishability constraint);

(A5) irreversible commitment (the underlying substrate dynamics consistent with T_μν reduces, on the CRE quotient, to irreversible record-generation);

(A6) compatibility with K = 7 closure (no additional independent internal degrees of freedom beyond those fixed by the K = 7 architecture);

(A7) local Lorentz covariance in the committed-record sector.

Then every admissible local source tensor reduces to a local covariant functional of the four primitives (ρ_committed, κ, Ξ, Q̂_μν):

T_μν = **𝓕**_μν[ρ_committed, κ, Ξ, Q̂_αβ],

for some local covariant tensor functional **𝓕**_μν. No admissible source structure exists that is fundamentally independent of irreversible commitment density.

Proof. The argument proceeds in five steps, each citing an inherited theorem.

Step 1 — admissible sources must reduce to stable records. By the Facthood Necessity Theorem (§2.8), any admissible physical theory must produce stable distinguishable records. Any source variable that does not ultimately reduce to such records is therefore inadmissible at the level of physical theory, not merely at the level of measurement. By (A4), every source variable carries bounded information content; combined with Facthood Necessity, every admissible source variable must reduce to a stable record structure.

Step 2 — irreversible commitment is the unique route to stable records. The commitment uniqueness theorems establish that, within the VERSF substrate, irreversible commitment events are the unique admissible mechanism producing stable distinguishable records. By (A5), the substrate dynamics underlying T_μν reduces to irreversible record-generation. Combined with Step 1, every admissible source variable must reduce to committed distinguishability — equivalently, to a functional of ρ_committed and its causal-propagation structure.

Step 3 — CRE removes proto-time-ordering dependence. The proto-time programme establishes that observable physics is defined on the CRE quotient and is invariant under proto-temporal reorderings preserving the committed-record structure. By (A2), T_μν is CRE-invariant; therefore T_μν cannot depend on proto-time ordering directly, only on CRE-invariant combinations of committed structures. This rules out any source variable carrying proto-time content not reducible to the committed-record sector.

Step 4 — causal propagation restricts admissible transport to the κ and Ξ functionals. The Lorentzian completion paper and the CCC framework jointly establish finite-speed propagation, local hyperbolicity, and a causal signal structure. By (A1) and (A3), T_μν may depend on the substrate state only through fields with finite-speed causal evolution. Within the unified ontology of §2.6, the admissible causal-transport fields are exhausted by: (i) the local commitment density ρ_committed; (ii) its causal-propagation mode κ (governed by the K = 7-fixed Klein–Gordon equation, §2.5); and (iii) its non-Markovian memory accumulation Ξ (built from ρ_committed via the worldline-projected kernel M, §2.7). No other causal-transport channel is admissible at the source level.

Step 5 — K = 7 closure excludes additional independent local degrees of freedom. By (A6) and the K = 7 closure architecture, the internal structure of distinguishability transport is fixed and admits no independent additional local sectors. The anisotropic transport-curvature sector Q̂_μν (§2.2) is *not* an independent additional sector — it is the unique parity-even traceless quadratic invariant of R^(∞)_ij admissible at this order (Theorem 3.6 of the spatial-geometry paper) — and it enters T_μν as a structural consequence of the same transport sector that generates the geometry itself. No further primitive local source sectors are admissible.

Combining Steps 1–5: every admissible source variable reduces to one of (ρ_committed, κ, Ξ, Q̂_μν), and T_μν is therefore a local covariant tensor functional of these four primitives.

Remark — what Theorem 3.1 does and does not say

What it says: the *space* of admissible source structures is restricted to functionals of the four named primitives. No new primitive sector can appear at the source level without violating one of (A1)–(A7) and thereby one of the inherited structural theorems.

What it does not say: it does not yet specify the *form* of the functional 𝓕_μν. Specifying the form is the task of §§5–7, which build the three explicit sector tensors T^(κ)_μν, T^(Ξ)_μν, T^(Q̂)_μν, plus the inter-sector exchange tensor T^(int)_μν, and prove a stronger sector-level uniqueness theorem (Theorem 8.2).

Corollary 3.2 — No fluid sector is independent [proven]

There is no admissible source sector independent of (ρ_committed, κ, Ξ, Q̂_μν). In particular, a "commitment fluid" sector — carrying pressure, energy flux, and anisotropic stress as independent fluid variables — would either reduce to a functional of the four primitives (in which case it is not an independent sector) or violate (A6) by introducing additional internal degrees of freedom not fixed by K = 7 closure.

Proof. Direct from Theorem 3.1: any candidate fluid variables (p, q^μ, π_μν) either decompose under the four-primitive structure or contradict K = 7 closure.

Corollary 3.2 resolves the apparent multi-sector freedom present in less constrained treatments. What looks like distinct commitment-density, coherence-pressure, energy-flux, and anisotropic-stress sectors is in fact the single causal flow of $\rho_{\text{committed}}$, propagating through κ, accumulating as Ξ, and inheriting spatial anisotropy from $\hat{Q}$.

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4. Structural Requirements on $T^{\text{eff}}_{\mu\nu}$

The Source Uniqueness Theorem identifies the admissible functional inputs; we now identify the structural requirements on the tensor $T^{\text{eff}}_{\mu\nu}$ built from those inputs. As in the geometry papers, these are *consequences* of the inherited structure, not free postulates.

Requirement R1 — Symmetry

$T^{\text{eff}}_{\mu\nu} = T^{\text{eff}}_{\nu\mu}$.

Why required. Any candidate geometric field equation $E_{\mu\nu}[g] = \kappa_{\text{eff}} T^{\text{eff}}_{\mu\nu}$, with $E_{\mu\nu}$ a symmetric tensor functional of $g_{\mu\nu}$, demands a symmetric source.

Requirement R2 — CRE invariance

If $H_1 \sim_{\text{CRE}} H_2$, then $T^{\text{eff}}_{\mu\nu}[H_1] = T^{\text{eff}}_{\mu\nu}[H_2]$. This is condition (A2) of Theorem 3.1; it is restated here for the assembled total tensor.

Requirement R3 — Causal energy flow (weak commitment energy condition)

For every future-directed causal v^μ,

$T^{\text{eff}}_{\mu\nu} v^\mu v^\nu \geq 0$.

Why required. The source-side image of the cone-compatibility condition (C3) inherited from the Lorentzian completion paper. Without R3, an observer with future-directed causal four-velocity could measure negative commitment-energy density, contradicting the structural meaning of irreversible commitment flow.

Requirement R4 — Conservation or controlled exchange

Either

$\nabla^\mu T^{\text{eff}}_{\mu\nu} = 0$,

or, sector-wise,

$\nabla^\mu T^{(A)}_{\mu\nu} = S^{(A)}_\nu, \quad \sum_A S^{(A)}_\nu = 0$, where the second form expresses sector-by-sector conservation up to inter-sector exchange currents whose total vanishes. The inter-sector exchange tensor $T^{(\mathrm{int})}_{\mu\nu}$ of §8 is the natural place to absorb the $S^{(A)}_{\nu}$ into a single divergence-free total.

Why required. Theorem 9.3 below shows that for any divergence-free geometric tensor $E_{\mu\nu}$, sourcing requires $\nabla^\mu T^{\mathrm{eff}}_{\mu\nu} = 0$.

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5. The κ-Sector

The κ-field is the unique admissible retarded propagating mode of the commitment-density field (§2.5). Its stress tensor follows from the unique admissible action.

Definition 5.1 — κ-field action

The κ-field action on the completed continuum is

$$S_\kappa = \int d^{(D+1)}x \sqrt{|g|}\,[\,-\tfrac{1}{2} \nabla_\mu \kappa \nabla^\mu \kappa - \tfrac{1}{2} m^2 \kappa^2 + \kappa \rho_{\mathrm{committed}}\,].$$

The first two terms are the canonical massive Klein–Gordon kinetic and mass terms with the $K = 7$-fixed mass $m = \sqrt{(4/3)} \cdot \xi^{-1}$. The third term is the linear source coupling of κ to the commitment-density field; it is the unique admissible coupling that produces

$$(\Box_g - m^2)\,\kappa = -\rho_{\mathrm{committed}}$$

on variation with respect to κ, in agreement with §2.5.

Theorem 5.2 — Uniqueness of the κ-action [proven, conditional on locality, Lorentz covariance, finite propagation, linear sourcing, and the K = 7-fixed mass]

S_κ is the unique admissible local Lorentz-covariant action for κ producing the $K = 7$-fixed sourced Klein–Gordon equation.

Proof. Within the class of local Lorentz-covariant single-scalar actions of second order in derivatives, the most general Lagrangian is

$$\mathscr{L} = a \nabla_\mu \kappa \nabla^\mu \kappa + b\, \kappa \Box_g \kappa + c\, m^2 \kappa^2 + W(\kappa) + \boldsymbol{\mathcal{J}}(\kappa, \rho_{\mathrm{committed}}),$$

with $a, b, c \in \mathbb{R}$, W an arbitrary function of κ, and $\boldsymbol{\mathcal{J}}$ an arbitrary coupling functional. Integration by parts reduces $\kappa \Box_g \kappa$ to $-\nabla_\mu \kappa \nabla^\mu \kappa$ up to boundary terms, folding b into a. The standard convention $a = -\tfrac{1}{2}$ ensures positive kinetic energy for timelike gradients. By Step 2 of Theorem 3.1, the sourcing must be by $\rho_{\mathrm{committed}}$; by additivity of sourcing (committed-record contributions add), $\boldsymbol{\mathcal{J}}$ must be linear in $\rho_{\mathrm{committed}}$; by Lorentz covariance and finite propagation, $\mathcal{J}$ must be local in κ — leaving only $\mathcal{J} = \kappa\, \rho_{\text{committed}}$ (up to a coupling constant, normalized to unity by absorption into the definition of $\rho_{\text{committed}}$). The mass coefficient $c\, m^2$ is fixed by the $K = 7$-fixed m^2 of §2.5. Nonlinear self-interaction $W(\kappa)$ beyond the $m^2 \kappa^2$ term is excluded by the *nonlinear exclusion* result of the κ-field uniqueness papers (no admissible higher-order self-interaction is consistent with the $K = 7$ closure architecture and CCC coherence-scale fixing). Hence S_κ is unique.

Definition 5.3 — κ-field stress tensor

Variation of S_κ with respect to $g^{\mu\nu}$, holding $\rho_{\text{committed}}$ fixed as a non-dynamical external source under the metric variation, gives

$$T^{(\kappa)}_{\mu\nu} := \nabla_\mu \kappa\, \nabla_\nu \kappa - \tfrac{1}{2} g_{\mu\nu} \nabla_\alpha \kappa\, \nabla^\alpha \kappa - \tfrac{1}{2} m^2 g_{\mu\nu} \kappa^2.$$

The source-coupling term $\kappa\, \rho_{\text{committed}}$ contributes to the κ-field equation (§2.5) but does not contribute to $T^{(\kappa)}_{\mu\nu}$, since $\rho_{\text{committed}}$ is treated as fixed under the metric variation. The energy–momentum exchange between the κ-sector and the commitment-current sector is carried entirely by the inter-sector exchange tensor $T^{(\text{int})}_{\mu\nu}$ of §8 (Theorem 8.3).

Theorem 5.4 — On-shell divergence balance [proven]

The κ-sector stress tensor is *not* separately conserved; its divergence is sourced by the coupling to $\rho_{\text{committed}}$:

$$\nabla^\mu T^{(\kappa)}_{\mu\nu} = -\rho_{\text{committed}}\, \nabla_\nu \kappa.$$

The non-conservation is the expected signature of a sourced scalar field; the divergence is absorbed into the inter-sector exchange tensor $T^{(\text{int})}_{\mu\nu}$ of §8 (Theorem 8.3).

Proof. Compute

$$\nabla^\mu T^{(\kappa)}_{\mu\nu} = (\nabla^\mu \nabla_\mu \kappa)\, \nabla_\nu \kappa + \nabla^\mu \kappa\, \nabla_\mu \nabla_\nu \kappa - \tfrac{1}{2} \nabla_\nu(\nabla_\alpha \kappa\, \nabla^\alpha \kappa) - m^2 \kappa\, \nabla_\nu \kappa.$$

On a torsion-free Levi-Civita connection, $\nabla_\mu \nabla_\nu \kappa = \nabla_\nu \nabla_\mu \kappa$ for the scalar κ, so

$$\nabla^\mu \kappa\, \nabla_\mu \nabla_\nu \kappa = \nabla^\mu \kappa\, \nabla_\nu \nabla_\mu \kappa = \tfrac{1}{2} \nabla_\nu(\nabla_\alpha \kappa\, \nabla^\alpha \kappa),$$

and the middle two terms cancel identically. The remainder is

$$\nabla^\mu T^{(\kappa)}_{\mu\nu} = (\Box_g \kappa - m^2 \kappa)\, \nabla_\nu \kappa.$$

Substituting the κ-field equation $(\Box_g - m^2)\kappa = -\rho_{\text{committed}}$ from §2.5 gives

$$\nabla^\mu T^{(\kappa)}_{\mu\nu} = -\rho_{\text{committed}}\, \nabla_\nu \kappa.$$

Theorem 5.5 — κ-sector weak commitment energy condition [proven, conditional on $m^2 \geq 0$]

For every future-directed causal v^μ, T^(κ)_μν v^μ v^ν ≥ 0.

Proof. Standard scalar-field WEC computation. The kinetic contribution (∇_μ κ v^μ)^2 is nonnegative; the kinetic mixed contribution −½ (g_μν v^μ v^ν)(∇^α κ ∇_α κ) has the sign of (∇^α κ ∇_α κ) since g_μν v^μ v^ν ≤ 0 for causal v; case-splitting on the sign of ∇^α κ ∇_α κ and using $m^2 \geq 0$ verifies the inequality. $m^2 \geq 0$ is automatic from $m^2 = (4/3)\,\xi^{-2} > 0$ (§2.5).

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6. The Memory Sector

The accumulated memory field Ξ(x, τ) inherited from §2.7 contributes a stress tensor structurally distinct from the local κ-sector: it is non-Markovian, history-dependent, and carries delayed sourcing.

Definition 6.1 — Memory stress tensor (leading effective local form)

Define

T^(Ξ)_μν := ∇_μ Ξ ∇_ν Ξ − ½ g_μν ∇_α Ξ ∇^α Ξ + g_μν V_Ξ(Ξ),

where V_Ξ is the effective memory potential inherited from the κ-field memory papers. At the leading bilocal order derived in Proposition 6.5, V_Ξ is quadratic: V_Ξ(Ξ) = ½ μ^2 Ξ^2. The general-function form V_Ξ(Ξ) displayed above admits higher polynomial powers as a generalisation beyond the bilocal sector; characterising those higher-power contributions is part of OP4.

Clarification on locality [structural]

Definition 6.1 should not be read as a claim that the memory sector is fundamentally local. The underlying dynamics are retarded and nonlocal (§2.7, Theorem 6.3); the tensor T^(Ξ)_μν displayed above is the leading-order *effective local representation* valid in the derivative-expansion regime made explicit by Proposition 6.5 below. The distinction matters: the memory sector is structurally non-Markovian even when its long-wavelength effective description appears approximately local. A fully covariant nonlocal formulation is sketched in §11.

Theorem 6.2 — Symmetry and CRE invariance [proven, conditional on inherited memory-sector inputs]

T^(Ξ)_μν is symmetric and CRE-invariant.

Proof. Symmetry is manifest from each term in Definition 6.1. CRE invariance: Ξ is built from ρ_committed via the kernel M (§2.7); both ρ_committed and M are CRE-invariant by inheritance (M depends only on the commitment-history sector and the K = 7-fixed mass m, both CRE-invariant). Therefore Ξ is CRE-invariant, and so is any tensor built from Ξ and its covariant derivatives.

Theorem 6.3 — Non-Markovian functional dependence [structural]

T^(Ξ)_μν is not a local functional of ρ_committed at (x, τ). It depends on ρ_committed throughout the entire past lightcone of (x, τ), weighted by the kernel M.

Proof. Direct from Definition 2.7: Ξ(x, τ) = ∬ M(x, τ; x′, τ′) ρ_committed(x′, τ′) d^Dx′ dτ′, integrated over the causal past. The gradient ∇_μ Ξ then carries history dependence through the support of M.

Interpretation

Unlike T^(κ)_μν, which is local in κ and depends on ρ_committed only at the instant the field equation is evaluated, T^(Ξ)_μν carries *persistent causal residue*: the geometry retains a memory of prior commitment events with algebraically-decaying envelope M(τ) ~ cos(mτ + φ)/τ in the worldline regime (§2.7). This is structurally distinct from standard general relativity, where geometry is instantaneously sourced by local stress–energy without persistent history dependence.

Theorem 6.4 — Memory-sector divergence balance [conditional on inherited memory-kernel dynamics]

The memory sector is not separately conserved: ∇^μ T^(Ξ)_μν is sourced by the inherited memory-kernel dynamics from the κ-field memory papers, with the resulting divergence absorbed into the inter-sector exchange tensor T^(int)_μν of §8. Closure of this balance is OP3 (subsidiary to OP9 per §11 — substrate-derivative closure at the leading-derivative level can proceed without waiting on the full nonlocal microscopic closure, but the two open problems share their fundamental resolution).

Proposition 6.5 — Local effective limit of the memory sector [proven, conditional on stated approximation regime]

Suppose the memory kernel satisfies:

(L1) weak-memory amplitude (the kernel M is treated to leading order in the small-amplitude expansion of the worldline-projected response),

(L2) slowly varying commitment density (∂_τ ρ_committed and ∇_i ρ_committed are slow on the scale m^{-1}), (L3) characteristic variation timescale T_var ≫ m^{-1}.

Then the nonlocal memory action of §6.6 admits a covariant derivative expansion of its *symmetric (time-reversal-even) effective sector*

S_mem^(sym) = ∫ d^(D+1)x √|g| [a_0 Ξ^2 + a_1 ∇_μ Ξ ∇^μ Ξ + $\mathcal{O}(\partial^4)$],

where a_0, a_1 ∈ ℝ are effective coefficients determined by moments of the symmetric part of the retarded kernel, and higher-order derivative terms are suppressed by powers of $(m\ T\_var)^{-1}$. The antisymmetric (dissipative, reactive) part of the kernel is absorbed into the inter-sector exchange tensor T^(int)_μν of §8 and does not contribute to T^(Ξ)_μν at this order. The local stress tensor T^(Ξ)_μν of Definition 6.1 is the leading-order metric variation of this symmetric expanded action and is therefore the leading-order effective local representation of the nonlocal memory sector.

Proof sketch. Decompose the bilocal kernel under (x ↔ x′) into symmetric and antisymmetric parts:

K(x, x′) = K_sym(x, x′) + K_anti(x, x′), K_sym(x, x′) := ½ (K(x, x′) + K(x′, x)), K_anti := ½ (K(x, x′) − K(x′, x)).

This is the standard Schwinger–Keldysh decomposition (with the convention that K_sym encodes the time-reversal-even effective response and K_anti the dissipative/reactive response); the retarded support of K means K is not manifestly symmetric in (x, x′), and the decomposition makes the two contributions explicit. The K_anti part contributes only odd derivative terms in the gradient expansion (∇_μ Ξ ∇^μ Ξ-style structures with an explicit antisymmetric weight); under standard Schwinger–Keldysh organisation these constitute the dissipative sector and enter T^(int)_μν of §8 rather than T^(Ξ)_μν.

For the K_sym part: expand the retarded convolution in powers of continuum gradients around (x, τ):

Ξ(x′) = Ξ(x) + (x′ − x)^μ ∇_μ Ξ(x) + ½ (x′ − x)^μ (x′ − x)^ν ∇_μ ∇_ν Ξ(x) + ⋯.

Substitute into S_mem^(sym) = ½ ∬ Ξ(x) K_sym(x, x′) Ξ(x′) d^(D+1)x d^(D+1)x′ and integrate term-by-term against the symmetric kernel moments

a_n := ∫ d^(D+1)x′ K_sym(x, x′) (x′ − x)^{⊗n}.

Odd-order moments a_(2k+1) vanish identically by the (x ↔ x′) symmetry of K_sym (this is now a structural property of the symmetric part, not a separate assumption about isotropy). Even-order moments survive: a_0 supplies the displayed potential term; a_2 is a rank-2 tensor moment whose trace part supplies the kinetic coefficient a_1 := −½ Tr(a_2)/(D + 1) after integration by parts of Ξ ∇_μ ∇_ν Ξ against the trace, with the traceless part of a_2 contributing higher-order anisotropic structure absorbed into $\mathcal{O}(\partial^4)$; a_4 onwards contribute $\mathcal{O}(\partial^4)$ suppressed by $(m\ T\_var)^{-1}$ under (L3).

The leading two terms reproduce the form of Definition 6.1 under the identifications $a_0 \leftrightarrow \frac{1}{2} \mu^2$ (with sign convention $-\frac{1}{2} \mu^2 \Xi^2 \to V_\Xi$; see the V_Ξ scope remark below) and the standard kinetic normalization on a_1.

Sign convention for V_Ξ. Under the action conventions of §5.1 (kinetic term $-\frac{1}{2} \nabla_\mu \kappa \nabla^\mu \kappa$ with mass term $-\frac{1}{2} m^2 \kappa^2$), the memory-sector mass term enters the bilocal action as $-\frac{1}{2} \mu^2 \Xi^2$. The potential V_Ξ in Definition 6.1 picks up a sign relative to the action coefficient: $V_\Xi(\Xi) = +\frac{1}{2} \mu^2 \Xi^2$ corresponds to bilocal action coefficient $a_0 = -\frac{1}{2} \mu^2$. This is the only sign-convention relation in the memory sector and is made explicit here to remove a residual ambiguity between §6.1 and §6.6.

Remark — scope of V_Ξ. Proposition 6.5 derives a quadratic potential $V_\Xi = \frac{1}{2} \mu^2 \Xi^2$ from the leading kernel moment a_0; this is the form consistent with the bilocal action of §6.6 at the order considered. The general function $V_\Xi(\Xi)$ appearing in Definition 6.1 admits higher powers in Ξ as a generalisation beyond the bilocal sector — such terms would arise from trilocal or higher polylocal extensions of S_{mem} not present in the present construction. Characterising the higher-power structure of V_Ξ is part of OP4.

6.6 Nonlocal variational structure of the memory sector

The memory field $\Xi(x, \tau)$ is fundamentally non-Markovian and therefore cannot, in general, be represented by a strictly local second-order scalar action of ordinary Klein–Gordon type. The local form of $T^{(\Xi)}_{\mu\nu}$ in Definition 6.1 is the long-wavelength truncation made explicit by Proposition 6.5; its fundamental origin is a nonlocal retarded functional.

The underlying memory field is

$$\Xi(x, \tau) = \int d^D x' \int_{-\infty}^{\tau} d\tau' \, M(x, \tau; x', \tau') \, \rho_{committed}(x', \tau'),$$

with retarded support $M(x, \tau; x', \tau') = 0$ for $\tau' > \tau$. The memory sector is therefore intrinsically history-dependent and cannot be reduced to a local field value at (x, τ) alone.

Accordingly, the correct fundamental object is not a local action $S[\Xi] = \int d^{D+1}x \, \mathscr{L}(\Xi, \partial\Xi)$, but a *nonlocal retarded bilocal functional*

$$S_{mem} = \frac{1}{2} \int d^{D+1}x \int d^{D+1}x' \, \sqrt{|g(x)|} \, \sqrt{|g(x')|} \, \Xi(x) \, K(x, x') \, \Xi(x'),$$

where K is a causal retarded kernel operator vanishing outside the causal future/past structure selected by the memory kernel. The local stress tensor of Definition 6.1 is then the leading-order effective stress tensor obtained after long-wavelength truncation, weak-memory approximation, and local derivative expansion of the retarded sector (Proposition 6.5).

This interpretation is consistent with the inherited ODE–Volterra equivalence hierarchy, the memory-kernel reduction programme, and the worldline-projected kernel structure of §2.7. A fully covariant microscopic derivation of $K(x, x')$ from refinement dynamics is open (§11).

7. The Anisotropic Transport-Curvature Sector

The anisotropic transport-curvature tensor Q̂_μν enters the source as anisotropic stress. We construct the most general parity-even, second-order, symmetric rank-2 tensor built from Q̂_μν and from gradients of R^(∞), and show it carries exactly three effective couplings.

Definition 7.1 — Building blocks at order λ

At order λ in the small-anisotropy expansion of the spatial-geometry paper, the available parity-even rank-2 tensors built from Q̂_μν, R^(∞)_αβ, and their derivatives are:

(B1) Q̂_μν itself (algebraic, traceless, symmetric, purely spatial),

(B2) the gradient-quadratic tensor

G_μν := ∇_μ R^(∞)_αβ ∇_ν R^(∞)αβ − ½ g_μν ∇^λ R^(∞)_αβ ∇_λ R^(∞)αβ,

symmetric by construction and parity-even,

(B3) the scalar invariant V_Q̂ := Q̂_αβ Q̂^αβ, entering the source only as the coefficient of a g_μν term.

Lemma 7.2 — Completeness of (B1)–(B3) at order λ [proven, conditional on parity-even, second-order scope]

The most general symmetric, parity-even, second-order rank-2 tensor built from g_μν, Q̂_μν, R^(∞)_αβ, ∇R^(∞), and contractions thereof — with no additional inputs — is a linear combination of (B1), (B2), and (B3) up to terms that vanish identically by tracelessness of Q̂.

Proof. We enumerate candidate parity-even symmetric rank-2 tensors at order λ and show each reduces to a combination of (B1)–(B3) or lies outside scope.

Algebraic rank-2 tensors at order λ in Q̂. These are exhausted by Q̂_μν itself (the traceless part) and a g_μν · ‖R^(∞)‖² term (the trace part, absorbed into the conformal background Ω² in the spatial-geometry paper).

Order-λ² candidates. The composite Q̂_μα Q̂^α_ν is quadratic in Q̂ and therefore of order λ², outside the order-λ scope of the present construction. Such terms re-enter at next order and are not admissible here.

Order-λ candidates from ∇∇V_Q̂. The double-derivative tensor ∇_μ ∇_ν V_Q̂ reduces to a g_μν multiple of V_Q̂ up to total-divergence terms at order λ: ∇_μ ∇_ν V_Q̂ = ½ g_μν □*g V_Q̂* + *∇*(μ ∇_ν) V_Q̂ traceless-part, where the traceless part vanishes at the order considered under the slow-variation regime assumed for the substrate inputs, and the trace part folds into γ_Q̂ g_μν V_Q̂ of (B3) up to a total divergence absorbed into the variational boundary.

Gradient-quadratic contractions. At second order in ∇R^(∞), the only parity-even symmetric rank-2 contraction is G_μν of (B2). Alternative contractions either (i) reduce to G_μν up to a total divergence by Bianchi-type identities on R^(∞), or (ii) vanish by antisymmetry of R^(∞)_αβ in (α, β).

Mixed contractions of ∇R^(∞) with Q̂. Candidate cross-terms such as ∇_μ R^(∞)_αβ Q̂^αβ are scalar (one index too few) and so contribute only as g_μν multipliers; the only such admissible structure is g_μν V_Q̂ of (B3) up to total divergence. Cross-terms with the right index count, such as ∇^α R^(∞)_αμ Q̂^β_ν, vanish either by antisymmetry of R^(∞) under the contraction with the symmetric Q̂ or reduce to G_μν.

g_μν multiplier. The g_μν · V_Q̂ multiplier of (B3) is the unique scalar-times-metric term at quadratic order in Q̂ admissible at this order.

No further parity-even second-order rank-2 tensor is admissible.

Definition 7.3 — Anisotropic transport-curvature stress tensor

Define

T^(Q̂)_μν := α_Q̂ Q̂_μν + β_Q̂ G_μν + γ_Q̂ g_μν V_Q̂,

with effective couplings α_Q̂, β_Q̂, γ_Q̂ ∈ ℝ inherited from the substrate dynamics. Their values are open (§12, OP2).

Theorem 7.4 — Uniqueness up to three couplings [proven, conditional on parity-even second-order scope]

Under the conditions of Lemma 7.2, T^(Q̂)_μν of Definition 7.3 is the unique parity-even, second-order, symmetric rank-2 tensor built algebraically from Q̂_μν and from gradients of R^(∞) at this order. Three real couplings (α_Q̂, β_Q̂, γ_Q̂) parametrize the construction.

Proof. Symmetry: each of Q̂_μν, G_μν, g_μν V_Q̂ is symmetric. Uniqueness: by Lemma 7.2, any other parity-even second-order symmetric rank-2 tensor built from the same ingredients is a linear combination of (B1)–(B3); reparametrizing gives the three-coupling form.

Theorem 7.5 — Algebraic tracelessness [proven]

The algebraic component α_Q̂ Q̂_μν is traceless: g^μν (α_Q̂ Q̂_μν) = 0.

Proof. g^μν Q̂_μν = 0 by §2.2.

Interpretation

The algebraic $\hat{Q}$-sector contributes *pure shear-like anisotropic stress* — no bulk energy density. The gradient sector $G_{\mu\nu}$ is the analogue of the Maxwell stress tensor for the transport-curvature field, capturing the energy and momentum carried by spatial inhomogeneities of $R^{(\infty)}$. The $\gamma_{\hat{Q}} V_{\hat{Q}} g_{\mu\nu}$ term is an effective cosmological-constant-like contribution sourced by the spatial-average magnitude of anisotropic transport curvature.

Remark — D = 2 degeneracy

At $D = 2$, $\hat{Q}_{\mu\nu} \equiv 0$ by orbit counting (inherited from the spatial-geometry paper). The algebraic and $\gamma_{\hat{Q}} V_{\hat{Q}}$ sectors vanish; only the gradient sector $G_{\mu\nu}$ survives, trivially. $T^{(\hat{Q})}_{\mu\nu}$ reduces to zero at this order, and the source structure at $D = 2$ is κ-plus-memory only.

8. Total Effective Stress–Energy and Sector Uniqueness

We now assemble the total source tensor and prove the sector-level uniqueness theorem that strengthens Theorem 3.1.

Definition 8.1 — Total effective stress–energy tensor

Define

$$T^{\mathrm{eff}}_{\mu\nu} := T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(\mathrm{int})}_{\mu\nu},$$

where $T^{(\mathrm{int})}_{\mu\nu}$ is the inter-sector exchange tensor whose divergence $\nabla^{\mu} T^{(\mathrm{int})}_{\mu\nu}$ exactly cancels the sum of the individual sector divergences (Theorem 8.3).

Theorem 8.2 — Sector-level uniqueness [proven, conditional on Theorem 3.1 and the parity/order scope of §§5–7]

Under Theorem 3.1 (admissible source primitives restricted to $\rho_{\mathrm{committed}}$, κ, Ξ, $\hat{Q}_{\mu\nu}$) and the parity-even second-order scope inherited from the geometry programme, the most general admissible source tensor on the completed Lorentzian continuum decomposes uniquely as

$$T^{\mathrm{eff}}_{\mu\nu} = T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(\mathrm{int})}_{\mu\nu},$$

with $T^{(\kappa)}_{\mu\nu}$ from Definition 5.3, $T^{(\Xi)}_{\mu\nu}$ from Definition 6.1, $T^{(\hat{Q})}_{\mu\nu}$ from Definition 7.3, and $T^{(\mathrm{int})}_{\mu\nu}$ fixed by Theorem 8.3. Parameter content:

- $T^{(\kappa)}$: no free structural parameters (m fixed by $K = 7$ closure; source coupling fixed by additivity).
- $T^{(\Xi)}$: inherited memory-kernel structure (no free structural parameters at the level of the stress tensor; V_{Ξ} inherited).
- $T^{(\hat{Q})}$: three effective couplings $\alpha_{\hat{Q}}$, $\beta_{\hat{Q}}$, $\gamma_{\hat{Q}}$ (open, OP2).

- $T^{(int)}$: no free structural parameters (fixed by divergence balance, Theorem 8.3).

Proof. Theorem 3.1 restricts admissible source variables to ($\rho_{committed}$, κ, Ξ, $\hat{Q}_{\mu\nu}$). The κ-sector is fixed by Theorem 5.2 (action uniqueness); the memory sector by §6 (functional form fixed by the inherited kernel M); the transport-curvature sector by Theorem 7.4 (uniqueness up to three couplings); the inter-sector exchange tensor by the conservation requirement R4 (Theorem 8.3 below). No further independent sector arises within the parity-even second-order scope, since by Theorem 3.1 any candidate sector outside the listed primitives is inadmissible.

Theorem 8.3 — Total conservation by construction [proven]

The inter-sector exchange tensor $T^{(int)}_{\mu\nu}$ is uniquely fixed by the requirement that $\nabla^\mu T^{eff}_{\mu\nu} = 0$:

$$\nabla^\mu T^{(int)}_{\mu\nu} = -\nabla^\mu T^{(\kappa)}_{\mu\nu} - \nabla^\mu T^{(\Xi)}_{\mu\nu} - \nabla^\mu T^{(\hat{Q})}_{\mu\nu}.$$

With $T^{(int)}_{\mu\nu}$ so defined, total conservation $\nabla^\mu T^{eff}_{\mu\nu} = 0$ holds identically.

Proof. By linearity of the divergence,

$$\nabla^\mu T^{eff}_{\mu\nu} = \nabla^\mu T^{(\kappa)}_{\mu\nu} + \nabla^\mu T^{(\Xi)}_{\mu\nu} + \nabla^\mu T^{(\hat{Q})}_{\mu\nu} + \nabla^\mu T^{(int)}_{\mu\nu}.$$

Setting the right-hand side to zero and solving for $\nabla^\mu T^{(int)}_{\mu\nu}$ gives the stated balance.

Remark — what $T^{(int)}$ encodes

$T^{(int)}_{\mu\nu}$ packages all inter-sector energy–momentum exchange: the source-coupling contribution from $\kappa\ \rho_{committed}$ (Theorem 5.4), the memory-kernel dynamics from the Ξ-sector (Theorem 6.4), and any divergence contributions from the gradient-curvature sector. By construction, the sum is divergence-free. Closure of $T^{(int)}_{\mu\nu}$ as a derivable rather than imposed structure — i.e., showing that its divergence balance follows from a unified action rather than being assembled by hand — is OP1.

Theorem 8.4 — Total CRE invariance [proven, conditional on CRE invariance of inputs]

$T^{eff}_{\mu\nu}$ is CRE-invariant.

Proof. Each sector is CRE-invariant: $T^{(\kappa)}$ inherits CRE invariance of κ (built from CRE-invariant $\rho_{committed}$ via the $K = 7$-fixed Klein–Gordon equation); $T^{(\Xi)}$ by Theorem 6.2; $T^{(\hat{Q})}$ by CRE invariance of $\hat{Q}_{\mu\nu}$ and $R^{(\infty)}$ (inherited); $T^{(int)}$ by construction from CRE-invariant divergences. Therefore the total is CRE-invariant.

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9. Effective Energy Conditions and Field-Equation Compatibility

Definition 9.1 — Weak commitment energy condition

$T^{eff}_{\mu\nu}$ satisfies the *weak commitment energy condition* (WCEC) if $T^{eff}_{\mu\nu} v^\mu v^\nu \geq 0$ for every future-directed causal v^μ.

Theorem 9.2 — Sufficient WCEC conditions [proven]

$T^{eff}_{\mu\nu}$ satisfies the WCEC if all of the following hold:

(W1) $m^2 > 0$ (automatic from §2.5: $m^2 = (4/3)\, \xi^{-2}$),

(W2) $V_\Xi(\Xi) \geq 0$ (inherited admissibility constraint on the memory-sector potential),

(W3) the kinetic terms of κ and Ξ enter with the conventional sign,

(W4) the anisotropic transport-curvature contribution satisfies

$$|T^{(\hat{Q})}_{\mu\nu} v^\mu v^\nu| \leq T^{(\kappa)}_{\mu\nu} v^\mu v^\nu + T^{(\Xi)}_{\mu\nu} v^\mu v^\nu,$$

for every future-directed causal v^μ,

(W5) the inter-sector exchange tensor satisfies $T^{(int)}_{\mu\nu} v^\mu v^\nu \geq -(T^{(\kappa)} + T^{(\Xi)} + T^{(\hat{Q})})_{\mu\nu} v^\mu v^\nu$.

Proof. Conditions (W1)–(W3) give $T^{(\kappa)}_{\mu\nu} v^\mu v^\nu \geq 0$ (Theorem 5.5) and $T^{(\Xi)}_{\mu\nu} v^\mu v^\nu \geq 0$ (analogous WEC argument for the memory scalar). Condition (W4) bounds the transport-curvature contribution by the positive κ- and Ξ-contributions. Condition (W5) ensures that the inter-sector exchange does not over-cancel the positive sectors. Combining all five gives nonnegative total.

Remark — sufficiency only

(W4) and (W5) are sufficient but not necessary. The precise admissibility domain for $T^{(\hat{Q})}$ and $T^{(int)}$ — in particular, whether the three couplings ($\alpha_{\hat{Q}}$, $\beta_{\hat{Q}}$, $\gamma_{\hat{Q}}$) admit a substrate derivation guaranteeing (W4) without ancillary assumption — is open (§12, OP2).

Theorem 9.3 — Conservation is necessary for any divergence-free geometric tensor [proven]

Suppose a candidate gravitational field equation has the form

$$E_{\mu\nu}[g] = \kappa_{eff} T^{eff}_{\mu\nu},$$

with $E_{\mu\nu}$ a symmetric tensor functional of $g_{\mu\nu}$ satisfying $\nabla^\mu E_{\mu\nu} = 0$. Then $\nabla^\mu T^{eff}_{\mu\nu} = 0$.

Proof. Taking the covariant divergence of both sides: $0 = \nabla^\mu E_{\mu\nu} = \kappa_{eff} \nabla^\mu T^{eff}_{\mu\nu}$. For $\kappa_{eff} \neq 0$, the result follows.

Corollary 9.4 — Sector decomposition fixes $T^{(int)}$ uniquely [proven]

Under Theorem 9.3 and the sector decomposition of Theorem 8.2, $T^{(int)}_{\mu\nu}$ is the unique tensor satisfying the divergence balance of Theorem 8.3. Any future Einstein-type field equation therefore *requires* $T^{(int)}_{\mu\nu}$ as defined; no admissible field equation can avoid the inter-sector exchange tensor.

Structural consequence for the next paper

The dynamical problem is now sharply scoped:

- The *space* of admissible sources is fixed by Theorem 3.1.
- The *form* of the sectors is fixed by Theorem 8.2.
- The *exchange* tensor $T^{(int)}$ is fixed by Theorem 8.3.
- The *conservation* requirement is mandatory (Theorem 9.3).
- The *CRE-invariance* is automatic (Theorem 8.4).
- The *causal-energy* requirement is sufficient under explicit conditions (Theorem 9.2).

What remains for the dynamical paper: (i) determine which divergence-free geometric tensor $E_{\mu\nu}[g]$ sources $T^{eff}_{\mu\nu}$; (ii) fix the value of κ_{eff} (the effective gravitational coupling); (iii) supply or derive the values of $\alpha_{\hat{Q}}$, $\beta_{\hat{Q}}$, $\gamma_{\hat{Q}}$; (iv) close $T^{(int)}$ as a derivable rather than imposed structure (OP1); (v) close the memory-kernel dynamics into a derivable balance (OP3).

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10. Epistemic Status

Result	Status
Lorentzian metric $g_{\mu\nu}$	inherited from the Lorentzian completion paper
Committed-record density $\rho_{committed}$	inherited from the commitment-sector programme
$s \equiv \kappa$ identification under retarded boundary conditions	inherited from the κ-field equivalence theorem
$K = 7$-fixed mass $m^2 = (4/3)\, \xi^{-2}$	inherited from the κ-field uniqueness papers
Unified commitment ontology (§2.6)	inherited from the programme as a whole
Memory kernel $M(\tau) \sim \cos(m\tau+\varphi)/\tau$	inherited from the oscillatory-memory programme
Facthood Necessity Theorem	inherited

Result	Status
$K = 7$ closure architecture	inherited
CRE quotient $\sim_{CRE}$	inherited
Admissible Source Uniqueness (Thm 3.1)	proven, conditional on inherited structural theorems
No independent fluid sector (Cor 3.2)	proven
Structural requirements R1–R4 (§4)	derived from inherited structure
κ-action uniqueness (Thm 5.2)	proven, conditional on locality + Lorentz covariance + linear sourcing + nonlinear exclusion
κ-sector on-shell divergence balance (Thm 5.4)	proven
κ-sector WEC (Thm 5.5)	proven
Memory-sector symmetry and CRE invariance (Thm 6.2)	proven
Non-Markovian functional dependence (Thm 6.3)	proven (structural)
Memory-sector divergence balance (Thm 6.4)	conditional on inherited memory-kernel dynamics
Local effective limit of memory sector (Prop 6.5)	proven, conditional on weak-memory / slow-variation regime (L1)–(L3)
Nonlocal retarded variational structure (§6.6)	structural; covariant closure open (OP9)
Approximation-hierarchy framing of memory sector (§11)	structural; substrate-level closure open (OP9)
Completeness of (B1)–(B3) at order λ (Lem 7.2)	proven, conditional on parity-even/second-order scope
Transport-curvature sector uniqueness (Thm 7.4)	proven within the same scope
Algebraic tracelessness of $\hat{Q}$-sector (Thm 7.5)	proven
Sector-level uniqueness of T^{eff} (Thm 8.2)	proven, conditional on Thm 3.1 + parity/order scope
Total conservation by construction (Thm 8.3)	proven
Total CRE invariance (Thm 8.4)	proven, conditional on CRE-invariant inputs
Sufficient WCEC conditions (Thm 9.2)	proven
Conservation necessary for divergence-free $E_{\mu\nu}$ (Thm 9.3)	proven
Values of $\alpha_{\hat{Q}}$, $\beta_{\hat{Q}}$, $\gamma_{\hat{Q}}$	open (OP2)
Closure of $T^{(int)}$ from a unified action	open (OP1)

Result	Status
Memory-kernel divergence balance closure	open (OP3)
Form of V_Ξ	open (OP4)
Geometric field equation E_μν[g]	open (OP5)
Effective gravitational coupling κ_eff	open (OP6)
Matter coupling	open (OP7)
Quantum fluctuations of T^eff_μν	open (OP8)

11. Toward a Covariant Nonlocal Action for the Memory Sector

The preceding sections establish the existence of a nonlocal retarded memory sector (§6.6) and identify its leading-order effective local representation (Proposition 6.5). They do *not* supply a fully covariant microscopic derivation of the kernel operator K(x, x′) from the underlying refinement dynamics. The current status of the memory sector is therefore analogous to:

- effective influence-functionals in nonequilibrium field theory,
- Schwinger–Keldysh closed-time-path effective actions,
- and Volterra-type causal response systems.

The key structural difference is that the VERSF memory kernel is sourced by irreversible commitment accumulation rather than by ordinary thermal or quantum environmental coarse-graining. In particular, there is no "doubled" time contour in the VERSF construction as there is in standard Schwinger–Keldysh: irreversibility is *intrinsic* to commitment, not emergent from tracing out an environment, so the closed-time-path structure simplifies to a single retarded contour with the symmetric/antisymmetric kernel decomposition organising the effective/dissipative split (Proposition 6.5). A complete covariant formulation would still require: (i) a single-contour retarded version of the commitment field on the CRE-quotient continuum compatible with the lack of environmental tracing; (ii) a causal-retarded variational structure compatible with the inherited Lorentzian geometry and CRE invariance; and (iii) explicit derivation of K(x, x′) from refinement dynamics. All three remain open.

The present paper therefore operates within the following controlled approximation hierarchy:

Level	Description
Fundamental	Nonlocal retarded commitment-memory functional
Intermediate	ODE–Volterra reduction hierarchy
Effective	Local derivative-expanded stress tensor
Present paper	Leading-order effective local form (Definition 6.1, justified by Proposition 6.5)

This hierarchy is sufficient for source construction, conservation analysis, energy-condition analysis, and preparation for the dynamical field-equation paper, but it is *not* yet sufficient for a complete microscopic quantum-gravitational closure of the memory sector. The closure of the memory sector at the fundamental level — derivation of K from refinement, and the corresponding closed-time-path action — is recorded as OP9 in §12.

12. Limitations and Open Problems

The paper does **not** derive:

- the geometric field equation E_μν[g],
- the value of the effective gravitational coupling κ_eff,
- the values of α_Q̂, β_Q̂, γ_Q̂,
- the closure of T^(int) as a derivable rather than imposed structure,
- the form of the memory potential V_Ξ,
- the substrate derivation of the retarded memory kernel K(x, x′) and the corresponding closed-time-path action for the memory sector,
- the matter coupling — how Standard-Model matter eventually emerges as a downstream sector of commitment flow,
- quantum fluctuations of the source tensor.

Specific open problems:

OP1 — Unified variational source. Can all source sectors and the inter-sector exchange tensor T^(int) be derived from a single effective action

S_source = S_κ + S_Ξ + S_Q̂ + S_matter,

with T^(int) arising automatically from the equations of motion rather than being assembled by divergence balance? A unified action would close R4 substrate-derivatively rather than as an imposed condition.

OP2 — Transport-curvature couplings. Can α_Q̂, β_Q̂, γ_Q̂ be fixed from refinement dynamics, dimensional analysis of the substrate scale ℓ⋆, or a renormalization-group fixed-point condition? Dimensional analysis suggests α_Q̂ ∼ $\ell\star^{-2}$, β_Q̂ ∼ 1, γ_Q̂ ∼ $\ell\star^{-2}$, but pinning the dimensionless O(1) coefficients requires substrate input not supplied here.

OP3 — Memory-kernel divergence balance. The memory-sector divergence ∇^μ T^(Ξ)_μν is presently absorbed into T^(int)_μν via Theorem 8.3. Closing it as a derivable balance from the inherited memory-kernel dynamics — and from the nonlocal retarded functional of §6.6 rather than from the local effective truncation of Definition 6.1 — requires further work in the κ-field memory programme. OP3 is logically subsidiary to OP9 (covariant nonlocal closure) but operationally usable at the leading-derivative level even before OP9 is closed.

OP4 — Memory potential. What is $V_\Xi(\Xi)$? Substrate-derived from the worldline-projected kernel structure, or independent? At the effective level of Proposition 6.5, V_Ξ corresponds (up to sign convention) to the zero-derivative moment a_0 of the retarded kernel; deriving a_0 substrate-derivatively closes V_Ξ. Connection to the commitment-barrier dynamics is the natural avenue.

OP5 — Geometric field equation. Which divergence-free geometric tensor $E_{\mu\nu}[g]$ sources $T^{eff}_{\mu\nu}$? Candidates include the Einstein tensor $G_{\mu\nu}$, the Lovelock family, and modified-gravity tensors built from $\hat{Q}_{\mu\nu}$ and $R^{(\infty)}$. The CRE-invariance constraint on the source side (Theorem 8.4) suggests a corresponding CRE-invariance requirement on $E_{\mu\nu}$, which may narrow the family.

OP6 — Effective gravitational coupling. κ_{eff} is presently undetermined. Its relationship to Newton's G_N, to the substrate scale $\ell\star$, and to the CCC coherence scale ξ is the natural next problem after OP5.

OP7 — Matter coupling. How does standard matter stress–energy emerge? The most likely route is via a refinement-induced coarse-graining of high-density commitment-history configurations into effective particle-like excitations, but the construction is open and lies outside the present scope.

OP8 — Quantum fluctuations. $T^{eff}_{\mu\nu}$ is presently treated as a deterministic functional of refinement-stable inputs. Quantum fluctuations of the underlying substrate translate into fluctuations of $T^{eff}_{\mu\nu}$; characterizing them belongs to the quantum-substrate programme.

OP9 — Covariant nonlocal closure of the memory sector. §11 records the memory sector as a leading-order local truncation of a fundamentally nonlocal retarded functional. Closing the memory sector at the fundamental level — deriving $K(x, x')$ from refinement dynamics, constructing the closed-time-path action on the CRE-quotient continuum, and verifying that the leading-derivative expansion of Proposition 6.5 reproduces Definition 6.1 with substrate-derived coefficients a_0, a_1 — is the natural next step after OP1 and is logically prior to a fully microscopic source theory.

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13. Conclusion

The Lorentzian completion paper installed the geometric stage for VERSF gravitational physics. The present paper builds the source tensor on that stage.

The central result is the *Admissible Source Uniqueness Theorem* (Theorem 3.1): every admissible local source tensor on the CRE-quotient continuum reduces to a local covariant functional of the four primitives ($\rho_{committed}$, κ, Ξ, $\hat{Q}_{\mu\nu}$). The uniqueness is grounded not in a parity or derivative scope chosen by hand but in the inherited structural theorems of the programme — Facthood Necessity, the commitment uniqueness theorems, CRE invariance, the

Lorentzian completion's causal structure, and K = 7 closure. No admissible source structure exists outside this family.

Within the family, the sector-level decomposition is

T^eff_μν = T^(κ)_μν + T^(Ξ)_μν + T^(Q̂)_μν + T^(int)_μν,

with:

- **κ-sector (T^(κ)_μν)** — the local propagating-mode stress with K = 7-fixed mass $m^2 = (4/3)\,\xi^{-2}$. No free structural parameters.
- **Memory sector (T^(Ξ)_μν)** — the non-Markovian, history-dependent contribution. Definition 6.1 is the *leading-order effective local representation* of a fundamentally nonlocal retarded functional with kernel M(τ) ~ cos(mτ + φ)/τ in the worldline regime (§§6.6, 11; Proposition 6.5). Inherited functional structure; covariant nonlocal closure open (OP9).
- **Anisotropic transport-curvature sector (T^(Q̂)_μν)** — the unique parity-even second-order rank-2 tensor built from Q̂_μν and ∇R^(∞), carrying three effective couplings (α_Q̂, β_Q̂, γ_Q̂).
- **Inter-sector exchange tensor (T^(int)_μν)** — fixed uniquely by total conservation (Theorem 8.3); no free structural parameters.

Five structural results secure the construction:

- **Source Uniqueness from inherited theorems (Thm 3.1)** — the *space* of admissible sources is fixed.
- **No independent fluid sector (Cor 3.2)** — what looks like fluid structure (density, flux, pressure) is recovered through the κ, Ξ, Q̂ chain without an independent fluid primitive.
- **κ-action uniqueness with K = 7-fixed mass (Thm 5.2)** — no free mass parameter in the κ-sector.
- **Sector-level uniqueness of T^eff (Thm 8.2)** — the *form* of each sector is fixed up to inherited or open structural inputs.
- **Total conservation by construction (Thm 8.3)** — T^(int) is uniquely determined by divergence balance; total conservation is automatic.

The spine of the construction is the inversion that runs through the geometry papers and now continues into the source side: the structural requirements on T^eff are *consequences* of the inherited substrate structure, not free postulates. The four-sector decomposition is not a modelling choice; it is what remains after the Admissible Source Uniqueness Theorem cuts away every inadmissible alternative. The memory sector remains fundamentally nonlocal and retarded; the local tensor formulation used here is the leading derivative-expanded effective representation of a deeper causal-memory functional whose fully covariant microscopic form remains open (OP9).

The structural question at this layer is therefore no longer

"What sources VERSF gravity?"

— that is settled here — but

"What divergence-free geometric tensor E_μν[g] does T^eff_μν source, what is the value of the effective gravitational coupling κ_eff, and can the inter-sector exchange tensor T^(int) be closed as a derivable rather than imposed structure?" That is the subject of the subsequent papers.