Einstein-Type Dynamics from Commitment-Generated Stress–Energy in VERSF

Bianchi-Compatible Geometry from Inherited Conservation, Source-Organized and Geometry-Organized Equivalent Formulations, Non-Markovian Memory Curvature from a Nonlocal Retarded Functional, Anisotropic Transport-Curvature Corrections, and Continuous Recovery of General Relativity in the Weak-Memory / Weak-Anisotropy Limit

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General-Reader Summary

The previous paper in this programme built the source side of VERSF gravity: it identified the effective stress–energy tensor generated by irreversible commitment flow and showed that the structure of this source is heavily constrained by inherited theorems — finite distinguishability, irreversible fact formation, causal propagation, CRE invariance, and K = 7 closure. It deliberately did not write a field equation; the geometric side was left as an open problem.

That problem is the subject of this paper.

In Einstein's theory of gravity, the field equation has the schematic form *geometry on the left, source on the right*: a divergence-free curvature tensor on one side, a divergence-free stress–energy tensor on the other. The two sides are linked by a coupling constant whose numerical value pins the strength of gravity. The miracle that makes this work is that the geometric side, the Einstein tensor, has identically vanishing divergence — a fact known as the contracted Bianchi identity — which exactly matches the conserved character of the source side.

The first move of this paper is to observe that the same logic runs in VERSF, but from the opposite direction. The source paper *already* established that the effective commitment-stress tensor is divergence-free. That conservation is therefore not something to derive — it is something to use. Any admissible geometric tensor on the other side of the field equation must match it: it must also be divergence-free.

The first major result, the *Bianchi-Compatible Geometry Theorem*, exploits this. Among the local second-order symmetric tensors built from the emergent continuum metric, the unique admissible candidate — the unique one that is automatically divergence-free under the inherited geometric structure — is the Einstein tensor (with a cosmological-constant term). Ordinary Einstein curvature therefore is not discarded by VERSF. It re-emerges as the unique continuum-limit geometric structure compatible with conserved irreversible commitment transport.

But that is only the leading admissible piece. The source side built in the previous paper contains two structures absent from standard general relativity: a non-Markovian memory sector and an

anisotropic transport-curvature sector. Both of these carry through to the geometric side as deviations from pure Einstein gravity. The memory sector produces *retarded geometric response* — the geometry does not respond only to the present commitment state but also, weakly, to accumulated commitment history. The anisotropic transport sector produces *directional curvature corrections* analogous to a geometric shear stress.

The paper is careful not to overclaim. A *GR Recovery Theorem* shows that in the regime where memory effects are weak, transport anisotropy is small, and the commitment density varies slowly, the full VERSF equations reduce continuously to Einstein's equations with the matching condition κ_eff = 8πG. Ordinary general relativity therefore appears as the low-memory, weak-anisotropy effective limit of the deeper commitment-based dynamics — not as something replaced.

The broader picture: in standard general relativity, geometry is a fundamental object and matter sources its curvature. In VERSF, irreversible commitment structure is the fundamental object, and what we call geometry is the large-scale continuum response to conserved commitment transport, weighted by memory accumulation and anisotropic distinguishability.

The paper supplies the first dynamical field equation in the VERSF programme. It does not yet derive the effective gravitational coupling κ_eff from substrate dynamics, it does not yet supply a unified covariant action from which all sectors descend, and it does not yet treat quantum fluctuations of the geometry. Each of these is recorded as an open problem.

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Abstract

The preceding paper established the effective stress–energy tensor of the VERSF framework,

$$T^{\text{eff}}_{\mu\nu} = T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(\text{int})}_{\mu\nu},$$

with the sectors encoding propagating commitment-density dynamics (κ), non-Markovian memory accumulation (Ξ), anisotropic transport-curvature stress ($\hat{Q}$), and inter-sector exchange (int). It proved total conservation $\nabla^\mu T^{\text{eff}}_{\mu\nu} = 0$, total CRE invariance, the Admissible Source Uniqueness Theorem, and sector-level uniqueness, but explicitly left open the geometric field equation sourced by T^{eff}.

The present paper closes that gap.

The construction proceeds in five steps.

1. **Inherited conservation as an admissibility filter on geometry (§3).** Total conservation of T^{eff}, combined with locality, parity-evenness, second-order scope, Lorentz covariance, and CRE invariance, restricts the admissible geometric tensor on the left-hand side of any field equation to a narrow class.
2. **Bianchi-Compatible Geometry Theorem (Theorem 4.1).** Within that class, on the (D+1)-dimensional CRE-quotient continuum with $D \geq 2$ spatial dimensions and linear-in-second-derivative scope, the unique divergence-free symmetric rank-2 tensor is the Einstein tensor plus a cosmological-constant term:

$$E_{\mu\nu} = G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu}.$$

For $D + 1 = 4$, this reduces to the Lovelock-classified unique second-order admissible form; for $D + 1 > 4$, higher-order Lovelock terms become topologically nontrivial and are recorded as a subordinate open problem (OP10).

3. **Equivalent source-organized and geometry-organized formulations (§5).** The full field equation admits two equivalent presentations:

(Source-organized) $G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = \kappa_{\text{eff}} T^{\text{eff}}_{\mu\nu}$;

(Geometry-organized) $G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} + M^{(\Xi)}_{\mu\nu} + Q^{(\hat{Q})}_{\mu\nu} + I^{(\text{int})}_{\mu\nu} = \kappa_{\text{eff}} T^{(\kappa)}_{\mu\nu}$,

with the identifications $M^{(\Xi)}_{\mu\nu} := -\kappa_{\mathrm{eff}}\, T^{(\Xi)}_{\mu\nu}$, $Q^{(\hat{Q})}_{\mu\nu} := -\kappa_{\mathrm{eff}}\, T^{(\hat{Q})}_{\mu\nu}$, $I^{(\mathrm{int})}_{\mu\nu} := -\kappa_{\mathrm{eff}}\, T^{(\mathrm{int})}_{\mu\nu}$. The two are algebraically equivalent; the second makes the deviation from ordinary Einstein gravity look like geometric corrections.

4. **Non-Markovian memory curvature (§7).** The memory-curvature tensor $M^{(\Xi)}_{\mu\nu}$ inherits the nonlocal-retarded functional structure of the source paper's §6.6 and is locally representable in the derivative-expansion regime of Proposition 7.4. Its asymptotic envelope is algebraic, $\sim \cos(m\tau + \varphi)/\tau$, inherited from the worldline-projected memory kernel.
5. **GR Recovery Theorem (Theorem 10.1).** In the simultaneous limit of weak memory amplitude, weak transport anisotropy, and slowly varying commitment density (regimes L1–L3 inherited from the source paper, Proposition 6.5), the full VERSF field equation reduces continuously to

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T^{(\mathrm{matter})}_{\mu\nu},$$

with the matching conditions $\kappa_{\mathrm{eff}} = 8\pi G$ (G Newton's constant) and $T^{(\kappa)}_{\mu\nu} \to T^{(\mathrm{matter})}_{\mu\nu}$ in the appropriate effective-matter limit. The effective gravitational coupling itself inherits the substrate-level structural form

$$\kappa_{\mathrm{eff}} = 8\pi C_\lambda \hbar \xi^2 / c^3$$

from the fold-density programme (§2.7), so the matching pins the closure-normalisation factor C_λ rather than introducing an arbitrary empirical parameter.

The paper does **not** derive the closure-normalisation factor C_λ (only its structural form is inherited), does not derive the *numerical values* of the three anisotropic-transport Wilson coefficients ($\alpha_{\hat{Q}}$, $\beta_{\hat{Q}}$, $\gamma_{\hat{Q}}$) (their effective-action origin is supplied in §8.2 / Proposition 8.3), does not derive the full bilocal kernel $K(x, x')$ of the memory-curvature operator from refinement dynamics (only its worldline-asymptotic form is inherited from the κ-field memory programme), does not derive the unified covariant action from substrate dynamics (its effective-action form is supplied in §14.1 / Proposition 14.3, with $T^{(\mathrm{int})}_{\mu\nu}$ appearing as a Lagrange-multiplier-enforced constrained completion), does not derive Standard-Model matter coupling, and does not treat quantum fluctuations of the geometry. Each is explicitly recorded as an open problem.

Its contribution is structural: it supplies the first admissible dynamical geometry equation compatible with the previously fixed source structure, and proves that ordinary general relativity sits within the framework as a continuous effective limit.

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1. Introduction

The Lorentzian completion paper installed the geometric stage; the source-side paper installed what lives on that stage and bends it. Both deliberately stopped short of dynamics. The present paper writes the field equation that couples them.

Why this paper is sharply constrained. A from-scratch derivation of a gravitational field equation faces an enormous moduli space of possibilities — every modified-gravity programme of the past fifty years lives in some slice of it. The present construction does not face that problem. The source paper has already fixed the right-hand side up to a small parameter set, and has proven that total conservation $\nabla^\mu T^{eff}_{\mu\nu} = 0$ holds identically. This conservation forces the left-hand side to be divergence-free, which — combined with locality, parity-evenness, second-order scope, and CRE invariance — collapses the geometric moduli space to a single leading admissible form. The first central result, the Bianchi-Compatible Geometry Theorem, makes this collapse explicit.

Two equivalent presentations. The field equation can be written with all four source sectors on the right-hand side (the source-organized form), or with the memory, anisotropy, and exchange sectors moved over to the left-hand side and renamed as geometric corrections (the geometry-organized form). The two are algebraically identical, but they emphasize different aspects: the first makes manifest that the construction inherits cleanly from the source paper; the second makes manifest the deviations from ordinary Einstein gravity. The paper carries both, with the source-organized form as the primary statement.

Two structurally distinctive sectors. The memory-curvature contribution $M^{(\Xi)}_{\mu\nu}$ inherits the nonlocal-retarded character of $T^{(\Xi)}_{\mu\nu}$ from the source paper. This is the first structurally distinctive feature of VERSF gravity: geometry is not instantaneously sourced by local stress–energy as in standard general relativity. The anisotropic transport-curvature contribution $Q^{(\hat{Q})}_{\mu\nu}$ inherits the directional structure of $T^{(\hat{Q})}_{\mu\nu}$, producing curvature corrections beyond the isotropic Einstein form. Both are subordinate to the leading Einstein structure in the recovery limit, but both are structurally non-removable away from it.

GR is recovered, not replaced. The GR Recovery Theorem of §10 shows that in the weak-memory, weak-anisotropy, slowly-varying regime, the full VERSF equations reduce continuously to Einstein's equations with $\kappa_{eff} = 8\pi G$. This is the correct relationship: general relativity is the effective leading-order continuum-limit geometry compatible with conserved irreversible commitment transport. VERSF gravity adds structurally honest sub-leading corrections without contradicting the recovered limit.

Inheritance, not reinvention. The Lorentzian continuum geometry $g_{\mu\nu}$, the four-sector decomposition of $T^{eff}_{\mu\nu}$, total conservation, total CRE invariance, the inter-sector exchange tensor $T^{(int)}_{\mu\nu}$, the $K = 7$-fixed mass $m^2 = (4/3)\,\xi^{-2}$, the worldline-projected memory kernel $M(\tau) \sim \cos(m\tau + \varphi)/\tau$, the §6.6 nonlocal variational structure, Proposition 6.5 of the source paper on the derivative-expansion regime, and the inherited structural theorems (Facthood Necessity, commitment uniqueness, CRE, CCC, $K = 7$ closure) are all taken as inputs and cited as such. The present paper adds: the admissibility filter on the geometric side, the Bianchi-Compatible Geometry Theorem, the dual-form field equation, the lifting of the §6.6 nonlocal structure into a memory-curvature sector, and the GR recovery argument.

Scope. The paper supplies the first admissible dynamical field equation, proves continuous recovery of ordinary general relativity in the appropriate effective limit, and identifies the residual open problems with explicit cross-references. It does *not* derive κ_{eff}, does not supply a unified covariant action, does not derive Standard-Model matter coupling, and does not address quantum fluctuations of the geometry.

Epistemic discipline. Results are labelled *proven*, *conditional*, or *conjectural*, with conditional results stating the additional assumptions explicitly.

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2. Inherited Structures

We inherit the following without re-derivation, in each case from the indicated source.

2.1 Lorentzian continuum geometry

From the Lorentzian completion paper:

$$g_{\mu\nu} = \mathrm{diag}(-c_\star^2, h_{ij}),\quad h_{ij} = \Omega^2 \delta_{ij} + \lambda \hat{Q}_{ij},$$

with Greek indices $\mu, \nu, \ldots$ running over $(0, 1, \ldots, D)$, $0 = \tau$ the observable commitment-time coordinate, and Latin indices $i, j, \ldots$ over the spatial sector $(1, \ldots, D)$. The signature is mostly-plus throughout. All covariant derivatives ∇_μ are with respect to g, with Levi-Civita connection (torsion-free, metric-compatible). The d'Alembertian is $\Box_g := \nabla_\mu \nabla^\mu$.

2.2 Effective stress–energy tensor

From the source-side paper (Theorem 8.2):

$$T^{\mathrm{eff}}_{\mu\nu} = T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(\mathrm{int})}_{\mu\nu},$$

with the sector contents:

- **κ-sector** — local Klein–Gordon stress with $K = 7$-fixed mass $m^2 = (4/3)\,\xi^{-2}$,

$$T^{(\kappa)}_{\mu\nu} = \nabla_\mu \kappa\, \nabla_\nu \kappa - \tfrac{1}{2} g_{\mu\nu} \nabla_\alpha \kappa\, \nabla^\alpha \kappa - \tfrac{1}{2} m^2 g_{\mu\nu} \kappa^2,$$

with κ governed by $(\Box_g - m^2)\,\kappa = -\rho_{\mathrm{committed}}$ in the mostly-plus signature inherited above.

- **Memory sector** — non-Markovian, derived from the accumulated memory field Ξ. The local effective tensor

$T^{(\Xi)}_{\mu\nu} = \nabla_\mu \Xi\, \nabla_\nu \Xi - \tfrac{1}{2} g_{\mu\nu} \nabla_\alpha \Xi\, \nabla^\alpha \Xi + g_{\mu\nu} V_\Xi(\Xi)$, $V_\Xi(\Xi) = \tfrac{1}{2} \mu^2 \Xi^2$ (leading bilocal order)

is the leading derivative-expanded effective local representation of a fundamentally nonlocal retarded bilocal functional (source paper §6.6, Proposition 6.5).

- **Anisotropic transport-curvature sector** — three-coupling parity-even rank-2 structure,

T^(Q̂)_μν = α_Q̂ Q̂_μν + β_Q̂ G_μν^(R) + γ_Q̂ g_μν V_Q̂,

with G_μν^(R) := ∇_μ R^(∞)_αβ ∇_ν R^(∞)αβ − ½ g_μν ∇^λ R^(∞)_αβ ∇_λ R^(∞)αβ the transport-curvature gradient-quadratic tensor (source paper Definition 7.1, with the notation G changed here to G^(R) to avoid collision with the Einstein tensor) and V_Q̂ := Q̂_αβ Q̂^αβ.

- **Inter-sector exchange tensor** — fixed uniquely by total conservation (source paper Theorem 8.3):

∇^μ T^(int)_μν = − ∇^μ T^(κ)_μν − ∇^μ T^(Ξ)_μν − ∇^μ T^(Q̂)_μν.

2.3 Total conservation, CRE invariance, energy conditions

∇^μ T^eff_μν = 0 (Theorem 8.3 of the source paper).

T^eff_μν is CRE-invariant (source paper Theorem 8.4) and satisfies the weak commitment energy condition under explicit sufficient conditions (source paper Theorem 9.2).

2.4 Memory-kernel asymptotic structure

From the oscillatory-memory programme and the worldline-projected kernel structure of the source paper §2.7:

M(τ) ~ cos(mτ + φ) / τ (worldline regime, late-time asymptotic),

with m the K = 7-fixed κ-field mass.

2.5 Nonlocal variational architecture of the memory sector

From the source paper §6.6:

S_mem = ½ ∫ d^(D+1)x ∫ d^(D+1)x′ √|g(x)| √|g(x′)| Ξ(x) K(x, x′) Ξ(x′),

with K a causal retarded kernel decomposing under (x ↔ x′) into symmetric and antisymmetric parts (source paper Proposition 6.5 proof):

K(x, x′) = K_sym(x, x′) + K_anti(x, x′).

K_sym contributes to T^(Ξ)_μν via the derivative-expansion regime; K_anti is absorbed into T^(int)_μν as the dissipative sector. The same decomposition lifts to the geometric memory operator in §7 below.

2.6 Inherited structural theorems

The Facthood Necessity Theorem, the commitment uniqueness theorems, the Commitment Reordering Equivalence (CRE) construction, the Causal–Coherence Compatibility (CCC) framework, and the K = 7 closure architecture are inherited as established in the corresponding companion papers. They were the load-bearing inputs for the Admissible Source Uniqueness Theorem (source paper Theorem 3.1); they remain load-bearing here for the corresponding admissibility filter on the geometric side.

2.7 Inherited structural form of the effective gravitational coupling

The fold-density programme of the VERSF framework establishes that Newton's gravitational coupling is not arbitrary but admits the constrained substrate-level form

$$G = C_\lambda \hbar \xi^2 / c^3,$$

where ξ is the VERSF coherence scale (CCC-fixed), C_λ is the closure-normalisation factor determined by fold energetics and the K = 7 closure architecture, and the formula is stated in the inherited *substrate-scale-normalised units* of the fold-density papers. In those conventions, ξ is the dimensional reference length of the coherence threshold, and the placement of $\hbar$ and c factors follows from how ξ is normalised at the substrate level. Explicit dimensional verification against SI requires translating ξ to a Planck-related length via the substrate-scale relations of the fold-density papers; the present paper inherits the formula without independent dimensional re-derivation. The derivation rests on finite distinguishability, irreversible fold commitment, the CCC coherence threshold, and the unique admissible fold-density sourcing law.

Combining this with the matching condition of the GR Recovery Theorem (§10), the effective gravitational coupling κ_{eff} of the VERSF dynamical field equation inherits the structural form

$$\kappa_{eff} = 8\pi C_\lambda \hbar \xi^2 / c^3.$$

This identification is *conditional* on the fold-density derivation chain, the collapse ansatz for fold energetics, and the K = 7 closure architecture, and is inherited without independent re-derivation here. Its consequence for the present paper is that κ_{eff} is *not* a fully open parameter as it was for the corresponding open problem OP6 of the source paper. The structural functional form is fixed; only the closure-normalisation factor C_λ remains substrate-derivatively incomplete (OP6 of the present paper, reformulated accordingly in §15).

Conceptually, this means curvature strength, memory propagation, and continuum geometry all inherit their scales from the same finite-distinguishability structure. Gravity is therefore not an independent interaction added to the framework, but the large-scale geometric response of the substrate to irreversible commitment density.

2.8 Inherited tensorial closure: forced rank-2 symmetric response structure

The tensorial-closure programme of the VERSF framework establishes — independently of the source-side conservation argument and independently of the geometric-side admissibility filter

— that the gravitational response sector is *forced* into a symmetric rank-2 tensor structure by the substrate closure constraints themselves.

The programme shows that, under finite distinguishability, irreversible commitment, locality, observer invariance, additivity, and operator-content minimality:

- scalar closure is insufficient to encode directional relational response,
- vector response sectors fail to support universal coupling,
- antisymmetric rank-2 sectors fail to couple universally to the symmetric committed source,
- and higher-rank sectors violate leading-order minimality.

The unique admissible leading-order response representation is therefore

$(1, 1) \oplus (0, 0)$ *(the symmetric rank-2 Lorentz representation plus its scalar trace component)*,

equivalently a symmetric rank-2 tensor field plus a scalar trace component.

This result sharpens the interpretation of the Bianchi-Compatible Geometry Theorem (§4) substantially. The Einstein tensor is not selected merely because it is divergence-free; it appears because (i) the response sector is *already* forced into a symmetric rank-2 structure by the inherited tensorial closure, (ii) universal coupling requires tensorial response, and (iii) Bianchi compatibility then uniquely constrains the admissible local geometry sector to $G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu}$. The (G1) symmetry condition of Definition 3.1 below is therefore not a free admissibility postulate; it is the geometric-side image of the inherited tensorial closure result.

2.9 Convention on observer fields and $\rho_{\text{committed}}$

The observer field u^μ used in defining $\rho_{\text{committed}} := -u_\mu J^\mu$ is the unit timelike congruence intrinsic to the commitment current J^μ (source paper §2.3 observer convention), not an external choice. $\rho_{\text{committed}}$ is therefore a CRE-invariant scalar density on the quotient continuum.

2.10 Notation

Symmetrization: $A_{(\mu\nu)} := \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$. We work in $D \geq 2$ spatial dimensions, i.e. $(D + 1) \geq 3$ total continuum dimensions; the dimensional triviality at $D = 2$ ($\hat{Q} \equiv 0$ by orbit counting) carries over from the source paper. The physically central case is $D + 1 = 4$ (3+1-dimensional total), where the Lovelock specialization of §4 is sharpest and the Bianchi-Compatible Geometry Theorem (Theorem 4.1) admits no higher Lovelock extensions. For $D + 1 > 4$, higher-curvature Lovelock terms (Gauss–Bonnet and its generalisations) become topologically non-trivial and constitute the residual room for extension recorded as OP10 in §15.

3. Admissibility Requirements on the Geometric Side

Conservation of the source tensor immediately imposes admissibility conditions on the geometric tensor on the other side of any candidate field equation. These conditions are *consequences* of the inherited structure, not free postulates.

Definition 3.1 — Admissible geometric tensor

A symmetric rank-2 tensor $E_{\mu\nu}$ is *admissible* on the emergent Lorentzian continuum if it satisfies:

(G1) **Symmetry.** $E_{\mu\nu} = E_{\nu\mu}$.

(G2) **Locality at the geometric level.** $E_{\mu\nu}$ at x depends only on $g_{\mu\nu}$ and its finitely many covariant derivatives at x.

(G3) **Local Lorentz covariance.** $E_{\mu\nu}$ transforms as a rank-2 tensor under local Lorentz frame changes.

(G4) **Parity-evenness.** $E_{\mu\nu}$ is invariant under spatial parity acting on the continuum, in the same convention as the source paper §7.

(G5) **Second-order scope.** $E_{\mu\nu}$ is at most second order in derivatives of $g_{\mu\nu}$, with the additional Lovelock-type restriction of being *linear* in second derivatives (i.e. no curvature-squared or higher-curvature primitive terms appear at this order). Higher-derivative extensions are deferred (OP10).

(G6) **Divergence-freeness.** $\nabla^\mu E_{\mu\nu} = 0$ identically (i.e. as a consequence of the geometric structure, not merely on shell).

(G7) **CRE invariance.** $E_{\mu\nu}[g[H_1]] = E_{\mu\nu}[g[H_2]]$ whenever $H_1 \sim_{\mathrm{CRE}} H_2$, where g[H] is the continuum metric generated by the proto-history H (source paper §2.4).

Remark — symmetry between source-side and geometry-side filters

Definition 3.1 is the geometry-side analogue of the (A1)–(A7) admissibility conditions imposed on $T_{\mu\nu}$ in the source paper (Theorem 3.1 there). The structural parallel is intentional: the same inherited theorems (causal propagation, parity-evenness, CRE, $K = 7$ closure, locality, Lorentz covariance) constrain both sides of the field equation, and they do so in a *matched* way. This matching is what makes the Bianchi-Compatible Geometry Theorem in §4 a sharp uniqueness result rather than a modelling choice.

Why G6 is forced

By the inherited result $\nabla^\mu T^{\mathrm{eff}}_{\mu\nu} = 0$, any candidate field equation of the form $E_{\mu\nu} = \kappa_{\mathrm{eff}} T^{\mathrm{eff}}_{\mu\nu}$ (with $\kappa_{\mathrm{eff}} \neq 0$) requires $\nabla^\mu E_{\mu\nu} = 0$. This was already noted as Theorem 9.3 of the source paper. G6 is therefore not an independent postulate; it is the geometric-side image of inherited source conservation.

Why the linear-in-second-derivative restriction in G5

The (G5) restriction excludes parity-even curvature-squared terms (Gauss–Bonnet, R^2, $R_{\mu\nu} R^{\mu\nu}$, $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$) at the level of the leading admissible field equation. Two justifications:

(i) *Source-paper symmetry.* The source paper restricted $T_{\mu\nu}$ to parity-even, second-order, linear-in-derivative scope (Theorem 8.2 of the source paper, "parity-even second-order scope"). Mirroring this on the geometric side preserves the structural parallel of Definition 3.1.

(ii) *Lovelock matching at $D + 1 = 4$.* For the physically central case, the Lovelock theorem identifies $G_{\mu\nu} + \Lambda g_{\mu\nu}$ as the unique symmetric, divergence-free, second-order, linear-in-second-derivative tensor; the higher-curvature Lovelock terms vanish topologically in $D + 1 = 4$. The restriction (G5) is therefore tight in 3+1 dimensions and only becomes non-trivial in higher D, which is treated as a subordinate open problem (OP10).

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4. Bianchi-Compatible Geometry Theorem

Theorem 4.1 — Bianchi-Compatible Geometry Theorem [proven, conditional on Definition 3.1 and the Lovelock theorem]

Under the admissibility conditions (G1)–(G7) of Definition 3.1, on the (D+1)-dimensional CRE-quotient Lorentzian continuum inherited from §2.1, the unique admissible geometric tensor is

$$E_{\mu\nu} = G_{\mu\nu} + \Lambda_{\mathrm{eff}}\, g_{\mu\nu},$$

where $G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor of $g_{\mu\nu}$ and $\Lambda_{\mathrm{eff}} \in \mathbb{R}$. For $D + 1 = 4$, this is the unique admissible form; for $D + 1 > 4$, higher-order Lovelock terms (Gauss–Bonnet and its generalizations) become topologically non-trivial and constitute a subordinate open problem (OP10).

Proof. The argument proceeds in three steps.

Step 1 — enumeration of candidate parity-even symmetric rank-2 tensors at order ≤ 2 in derivatives. Under (G1)–(G5), the most general candidate is the linear combination

$$E_{\mu\nu} = a\, R_{\mu\nu} + b\, g_{\mu\nu} R + c\, g_{\mu\nu},$$

with $a, b, c \in \mathbb{R}$. The term $c\, g_{\mu\nu}$ is the zero-derivative cosmological-constant contribution; $a\, R_{\mu\nu}$ and $b\, g_{\mu\nu} R$ are the linear-in-second-derivative curvature contributions. Curvature-squared and higher-derivative terms are excluded by (G5). This enumeration is the complete Lovelock-class list at leading admissible order: it is the linear span of all symmetric, parity-even, rank-2 tensors that are second order in metric derivatives and linear in second derivatives, with no further parity-even structure admissible under the inherited continuum architecture at this order.

Step 2 — divergence-free condition as an identity in the metric variables. Taking the covariant divergence and using the contracted Bianchi identity $\nabla^\mu R_{\mu\nu} = \tfrac{1}{2} \nabla_\nu R$:

$$\nabla^\mu E_{\mu\nu} = a \nabla^\mu R_{\mu\nu} + b \nabla_\nu R + 0 = (\tfrac{1}{2} a + b) \nabla_\nu R.$$

By (G6), the divergence must vanish *identically as a polynomial in the metric variables and their derivatives* — i.e. for arbitrary $g_{\mu\nu}$ compatible with the inherited Lorentzian structure, not on the solutions of a particular equation. The covariant gradient $\nabla_\nu R$ is a non-trivial local invariant of the connection (it does not vanish identically on the space of admissible metrics: it vanishes on flat backgrounds and on metrics with constant scalar curvature, but those are non-generic). Therefore the coefficient $(\tfrac{1}{2} a + b)$ multiplying it must itself vanish:

$$b = - \tfrac{1}{2} a.$$

Substituting back:

$$E_{\mu\nu} = a (R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R) + c\, g_{\mu\nu} = a\, G_{\mu\nu} + c\, g_{\mu\nu}.$$

Step 3 — normalization and Lovelock specialization. Absorbing a into the coupling constant κ_{eff} of the field equation (§6) gives the displayed form $E_{\mu\nu} = G_{\mu\nu} + \Lambda_{\mathrm{eff}}\, g_{\mu\nu}$, with $\Lambda_{\mathrm{eff}} := c/a$. For $D + 1 = 4$, the Lovelock theorem identifies this as the unique symmetric, divergence-free, second-order, linear-in-second-derivative tensor — independent of the enumeration in Step 1, which is reproduced as a special case. For $D + 1 > 4$, the Lovelock theorem permits additional Gauss–Bonnet-type terms that satisfy (G1)–(G6) but are absent from the second-order linear scope; whether the inherited substrate dynamics select against them, or require their inclusion at higher D, is recorded as OP10.

(G7) — CRE invariance — is automatic: $R_{\mu\nu}$, R, and $g_{\mu\nu}$ are all built from g, and g[H] is CRE-invariant by construction (inherited from the Lorentzian completion paper).

Corollary 4.2 — No higher-derivative primitive term at this order [proven]

Under (G1)–(G7), no parity-even curvature-squared or higher-derivative term enters at the order considered. In particular, terms of the form $R^2 g_{\mu\nu}$, $R_{\mu\nu} R$, $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} g_{\mu\nu}$, or $\Box R\, g_{\mu\nu}$ are excluded from the leading admissible $E_{\mu\nu}$.

Proof. Direct from (G5) and the divergence-free condition: such terms either fail (G5) (being higher than second order in derivatives) or, if formally second-order, fail (G6) by introducing curvature-derivative terms in $\nabla^\mu E_{\mu\nu}$ that cannot be cancelled within the second-order scope.

Structural significance

Theorem 4.1 is the geometric-side dual of the Admissible Source Uniqueness Theorem (source paper Theorem 3.1). The two together fix both sides of the field equation up to a small parameter set: the source side via the $(\kappa, \Xi, \hat{Q}, \text{int})$ decomposition and three inherited or open couplings; the geometry side via Einstein + Λ at leading admissible order. The dynamical paper's structural task — couple them — is now sharply scoped.

Remark — what Theorem 4.1 does and does not say

What it says: the *leading admissible* geometric tensor is $G_{\mu\nu} + \Lambda_{\text{eff}}\, g_{\mu\nu}$. Any departure from this form must come from extensions of the admissibility class — higher-derivative terms (OP10), nonlocal terms (sourced by the memory sector, §7), or anisotropic terms not expressible as ordinary metric-derived curvature (sourced by the transport sector, §8).

What it does not say: it does not fix the numerical value of Λ_{eff}, it does not fix the coupling κ_{eff} of the eventual field equation, and it does not by itself force any nonlocal or anisotropic *geometric* corrections. Those corrections enter via the source-side structure of the previous paper; the geometric side at second-order linear-in-derivative scope contains only the Einstein and cosmological-constant pieces.

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5. Equivalent Source-Organized and Geometry-Organized Formulations

The full field equation admits two equivalent presentations, related by algebraic reorganization. Both are useful; the choice between them is presentational, not physical.

Definition 5.1 — Source-organized field equation

The *source-organized* form of the VERSF dynamical field equation is

$$G_{\mu\nu} + \Lambda_{\text{eff}}\, g_{\mu\nu} = \kappa_{\text{eff}}\, T^{\text{eff}}_{\mu\nu}, \quad T^{\text{eff}}_{\mu\nu} = T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(\text{int})}_{\mu\nu}.$$

All four source sectors appear on the right-hand side; the left-hand side carries only the Einstein-plus-Λ structure of Theorem 4.1.

Definition 5.2 — Geometry-organized field equation

The *geometry-organized* form is

$$G_{\mu\nu} + \Lambda_{\text{eff}}\, g_{\mu\nu} + M^{(\Xi)}_{\mu\nu} + Q^{(\hat{Q})}_{\mu\nu} + I^{(\text{int})}_{\mu\nu} = \kappa_{\text{eff}}\, T^{(\kappa)}_{\mu\nu},$$

with the algebraic identifications

$M^{(\Xi)}_{\mu\nu} := -\kappa_{\text{eff}} T^{(\Xi)}_{\mu\nu}$, $Q^{(\hat{Q})}_{\mu\nu} := -\kappa_{\text{eff}} T^{(\hat{Q})}_{\mu\nu}$, $I^{(\text{int})}_{\mu\nu} := -\kappa_{\text{eff}} T^{(\text{int})}_{\mu\nu}$.

Only the κ-sector source remains on the right-hand side; the memory, anisotropy, and exchange contributions appear as additive geometric corrections to the left-hand side.

Proposition 5.3 — Equivalence of the two formulations [proven]

The source-organized and geometry-organized forms of Definitions 5.1–5.2 are algebraically identical. Both inherit total conservation $\nabla^\mu(\text{LHS}) = \nabla^\mu(\text{RHS})$ automatically: the source-organized form by the inherited $\nabla^\mu T^{\text{eff}}_{\mu\nu} = 0$ of §2.3, the geometry-organized form by the same identity combined with the divergence-free character of $G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu}$ (Theorem 4.1).

Proof. Subtracting $M^{(\Xi)}_{\mu\nu} + Q^{(\hat{Q})}_{\mu\nu} + I^{(\text{int})}_{\mu\nu}$ from both sides of the geometry-organized equation and using the identifications of Definition 5.2 gives the source-organized form. Both inherit divergence-freeness as displayed.

Remark — when each formulation is the natural one

The source-organized form (Definition 5.1) is the cleaner mathematical object: it states the field equation in its most economical form, with the LHS at leading geometric admissibility and all source content collected on the RHS. It is the natural form for the GR Recovery Theorem of §10, where the matching condition $\kappa_{\text{eff}} = 8\pi G$ is read off directly.

The geometry-organized form (Definition 5.2) makes manifest the *deviations* from ordinary Einstein gravity. $M^{(\Xi)}_{\mu\nu}$, $Q^{(\hat{Q})}_{\mu\nu}$, and $I^{(\text{int})}_{\mu\nu}$ appear as geometric corrections analogous to higher-derivative or nonlocal modifications of GR. This is the natural form for §§7–8, where the structural content of those corrections is unpacked.

The paper carries both, presenting the source-organized form as primary and the geometry-organized form as the reorganization that makes the structural distinctiveness of VERSF gravity visible.

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6. VERSF Dynamical Field Equation

We now assemble the full equation.

Definition 6.1 — VERSF dynamical field equation

The emergent Lorentzian continuum metric $g_{\mu\nu}$ satisfies, in source-organized form,

$$G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = \kappa_{\text{eff}} T^{\text{eff}}_{\mu\nu},$$

with T^eff_μν given by the four-sector decomposition of §2.2 and κ_eff ∈ ℝ\{0} the effective gravitational coupling (open, OP6). The equivalent geometry-organized form is Definition 5.2.

Theorem 6.2 — Conservation compatibility [proven]

The field equation of Definition 6.1 is internally consistent in the sense that

∇^μ (LHS) = ∇^μ (RHS)

identically, with both sides divergence-free.

Proof. LHS divergence: ∇^μ G_μν = 0 by the contracted Bianchi identity; ∇^μ (Λ_eff g_μν) = Λ_eff ∇^μ g_μν = 0 by metric-compatibility of the Levi-Civita connection. So ∇^μ (LHS) = 0. RHS divergence: κ_eff ∇^μ T^eff_μν = 0 by the inherited ∇^μ T^eff_μν = 0 of §2.3.

Theorem 6.3 — CRE compatibility [proven]

Definition 6.1 is CRE-invariant: both sides transform identically under proto-history reorderings preserving the committed-record structure.

Proof. LHS: G_μν and g_μν are functionals of g, and g[H] is CRE-invariant by construction. RHS: T^eff_μν is CRE-invariant by source paper Theorem 8.4.

Theorem 6.4 — Admissibility uniqueness at leading order [proven, conditional on Theorem 4.1 and source paper Theorem 8.2]

Under the combined admissibility conditions of Definition 3.1 (geometric side) and the source paper's Theorem 3.1 (source side), and under the parity-even second-order linear-in-derivative scope inherited on both sides, Definition 6.1 is the unique admissible leading-order field equation up to:

- the value of Λ_eff (OP6),
- the value of κ_eff (OP6),
- the values of the three anisotropic-transport couplings (α_Q̂, β_Q̂, γ_Q̂) inherited from source paper Theorem 7.4 (present paper OP4, inherited from source paper OP2),
- the specific functional structure of T^(int)_μν, fixed uniquely by source paper Theorem 8.3 once the other three sectors are specified,
- the kernel structure of the memory sector at its nonlocal level (OP3 of the present paper).

Proof. Theorem 4.1 fixes the leading admissible geometric tensor. Source paper Theorem 8.2 fixes the source-side decomposition. The two together fix the field equation up to the listed parameters and inherited or open structural inputs. No further geometric primitive term is admissible at this order by Corollary 4.2; no further source primitive term is admissible by source paper Theorem 3.1.

Why no separate "I^(int)" term in the source-organized form

In the source-organized form (Definition 6.1), the inter-sector exchange tensor $T^{(\mathrm{int})}_{\mu\nu}$ is *already inside* $T^{\mathrm{eff}}_{\mu\nu}$. Its role is to enforce $\nabla^{\mu} T^{\mathrm{eff}}_{\mu\nu} = 0$ by construction (source paper Theorem 8.3), and no separate geometric term is required to balance it. In the geometry-organized form (Definition 5.2), it appears explicitly as $I^{(\mathrm{int})}_{\mu\nu}$ on the LHS, with the same role on that side. The two presentations are equivalent.

7. Memory-Curvature Sector and the Nonlocal/Effective Hierarchy

The memory-sector source contribution $T^{(\Xi)}_{\mu\nu}$ is the leading derivative-expanded effective local representation of a fundamentally nonlocal retarded bilocal functional (source paper §6.6, Proposition 6.5). When passed through the field equation, this nonlocal structure lifts to the geometric side: in the geometry-organized form (Definition 5.2), the memory-curvature tensor $M^{(\Xi)}_{\mu\nu} := -\kappa_{\mathrm{eff}} T^{(\Xi)}_{\mu\nu}$ inherits the same hierarchy of locality regimes.

Definition 7.1 — Memory-curvature tensor (leading effective local form)

In its leading derivative-expanded form (the regime of Proposition 6.5 of the source paper),

$$M^{(\Xi)}_{\mu\nu} = -\kappa_{\mathrm{eff}} \left[\nabla_{\mu} \Xi \, \nabla_{\nu} \Xi - \tfrac{1}{2} g_{\mu\nu} \nabla_{\alpha} \Xi \, \nabla^{\alpha} \Xi + \tfrac{1}{2} \mu^{2} g_{\mu\nu} \Xi^{2} \right],$$

with Ξ the accumulated memory field of source paper §2.7 and μ^2 the leading bilocal mass parameter (subject to OP4 of the source paper for substrate derivation).

Definition 7.2 — Memory-curvature tensor (nonlocal retarded form)

In its full nonlocal form, $M^{(\Xi)}_{\mu\nu}$ is the metric variation of the geometric memory action

$$S^{(\mathrm{geom})}_{\mathrm{mem}} = -\tfrac{1}{2} \kappa_{\mathrm{eff}} \int d^{D+1}x \int d^{D+1}x' \sqrt{|g(x)|} \sqrt{|g(x')|} \, \Xi(x) \, K(x, x') \, \Xi(x'),$$

via

$$M^{(\Xi)}_{\mu\nu}(x) = \left(2 / \sqrt{|g(x)|}\right) \cdot \delta S^{(\mathrm{geom})}_{\mathrm{mem}} / \delta g^{\mu\nu}(x),$$

with $K(x, x')$ the inherited causal retarded kernel of source paper §6.6, decomposing under $(x \leftrightarrow x')$ into

$$K(x, x') = K_{\mathrm{sym}}(x, x') + K_{\mathrm{anti}}(x, x')$$

as in source paper Proposition 6.5. The metric variation contains two structurally distinct contributions: the variation of $\sqrt{|g(x)|}$ at the local evaluation point (giving the algebraic $g_{\mu\nu}$ multiplier terms) and the variation through the implicit g-dependence of Ξ via the inherited definition $\Xi(x) = \int M(x; x') \rho_{committed}(x')$ on the curved background (giving the gradient kinetic terms). The (K_{sym}, K_{anti}) decomposition organises the variation as follows:

$$\delta S^{(geom)}_{mem} / \delta g^{\mu\nu} = \delta S^{(geom,sym)}_{mem} / \delta g^{\mu\nu} + \delta S^{(geom,anti)}_{mem} / \delta g^{\mu\nu}.$$

The symmetric piece contributes the entire $M^{(\Xi)}_{\mu\nu}$ as displayed. The antisymmetric piece is *absorbed into the inter-sector exchange tensor* $I^{(int)}_{\mu\nu}$ of the geometry-organized field equation:

$$I^{(int)}_{\mu\nu} \supseteq (2 / \sqrt{|g|}) \cdot \delta S^{(geom,anti)}_{mem} / \delta g^{\mu\nu},$$

in lockstep with the source-side treatment (source paper Proposition 6.5, where the antisymmetric kernel piece is absorbed into $T^{(int)}_{\mu\nu}$). This routing is the geometric-side image of the source-side dissipative/effective split and is the structural reason that $M^{(\Xi)}_{\mu\nu}$ is well-defined as a symmetric rank-2 metric-variation tensor despite K being not manifestly symmetric in (x, x').

Clarification on locality

Definition 7.1 is the *leading-order effective local representation* of Definition 7.2, valid in the derivative-expansion regime (L1)–(L3) of source paper Proposition 6.5. The geometric memory operator is fundamentally nonlocal and retarded; the local tensor displayed in Definition 7.1 is its long-wavelength truncation. The same approximation hierarchy that governs $T^{(\Xi)}_{\mu\nu}$ in the source paper governs $M^{(\Xi)}_{\mu\nu}$ here.

Theorem 7.3 — Algebraic asymptotic envelope of the memory-curvature contribution [proven, conditional on the inherited worldline-projected kernel and a Tauberian-type regularity hypothesis on the Ξ-sector source data]

The memory-curvature contribution to the geometric response inherits the asymptotic envelope of the worldline-projected memory kernel:

$$\| M^{(\Xi)}_{\mu\nu}(x, \tau) \| \sim \cos(m\tau + \varphi) / \tau \text{ as } \tau \to \infty,$$

in the worldline regime of source paper §2.7, where $\| \cdot \|$ denotes a frame-independent magnitude.

Hypotheses on the Ξ-sector source data. Let $\chi(x, \tau)$ denote the Ξ-sector source data along the worldline. We assume:

(H1) *Compact past support* — there exists τ_0 such that $\chi(x, \tau') = 0$ for $\tau' < \tau_0$ (or, equivalently, that χ has sufficient decay as $\tau' \to -\infty$ that the convolution integral is absolutely convergent for all finite τ);

(H2) *Bounded variation at the recall time* — χ is bounded and of bounded variation in the relevant past lightcone region;

(H3) *Mellin regularity* — the symmetric part K_{sym} of the inherited memory kernel has Mellin transform regular on the strip $\mathrm{Re}(s) \in (0, 1)$, with the Mellin pole structure dominated by the contribution at the κ-field frequency m (this is the appropriate regularity condition for an oscillatory algebraic kernel; standard Tauberian theorems for monotonic kernels do not apply).

Proof. From source paper §2.7, the worldline-projected kernel has late-time asymptotic $M(\tau) \sim \cos(m\tau + \varphi) / \tau$ as $\tau \to \infty$. The retarded geometric memory contribution at worldline parameter τ is the convolution

$$(M * \chi)(\tau) = \int_{\tau_0}^{\tau} M(\tau - \tau')\, \chi(\tau')\, d\tau'.$$

Take the Mellin transform of both sides under (H3). The Mellin transform of the oscillatory algebraic kernel $\cos(m\tau + \varphi) / \tau$ admits a meromorphic continuation with poles at the relevant strip determined by the frequency m and phase φ; under (H1)–(H2), the Mellin transform of χ is holomorphic on a suitable strip containing the kernel's pole structure. The leading late-time asymptotic of the convolution is then read off from the residue at the dominant Mellin pole (a *complex Tauberian* or *Mellin-contour* argument appropriate for oscillatory kernels, distinct from the Hardy–Littlewood–Karamata theorem for monotonic kernels):

$$(M * \chi)(\tau) \sim \bar{\chi} \cdot \cos(m\tau + \varphi) / \tau \text{ as } \tau \to \infty,$$

where $\bar{\chi}$ is a τ-independent constant determined by the finite Mellin moments of χ against K_{sym} at the dominant pole. The factor κ_{eff} multiplying $T^{(\Xi)}_{\mu\nu}$ to produce $M^{(\Xi)}_{\mu\nu}$ is τ-independent and does not affect the envelope. The displayed result follows.

The Mellin regularity hypothesis (H3) is the load-bearing analytic input; (H1)–(H2) are mild and are satisfied by any physically reasonable source-data class. The structural conclusion — that the memory-curvature contribution inherits the kernel's algebraic decay rather than acquiring exponential decay through convolution — depends on (H3); a pathological kernel Mellin structure could in principle break the envelope inheritance, but no such pathology arises in the inherited worldline-projected kernel.

Proposition 7.4 — Local effective limit of the memory-curvature sector [proven, conditional on regime (L1)–(L3) of source paper Proposition 6.5]

Under the inherited approximation regime (L1)–(L3), Definition 7.2 reduces to Definition 7.1 via the same derivative-expansion construction as source paper Proposition 6.5. The leading bilocal kernel moments a_0, a_1 of the source paper translate directly to the corresponding coefficients in $M^{(\Xi)}_{\mu\nu}$:

$$\tfrac{1}{2} \mu^2 g_{\mu\nu} \Xi^2 \leftrightarrow a_0 \text{ contribution}, \quad \nabla_\mu \Xi\, \nabla_\nu \Xi - \tfrac{1}{2} g_{\mu\nu} \nabla_\alpha \Xi\, \nabla^\alpha \Xi \leftrightarrow a_1 \text{ contribution},$$

with the relative factor $-\kappa_{\mathrm{eff}}$ in the identification $M^{(\Xi)}_{\mu\nu} = -\kappa_{\mathrm{eff}} T^{(\Xi)}_{\mu\nu}$ translating the source-side stress structure into a geometric-side correction.

Proof. Apply source paper Proposition 6.5 to T^(Ξ)_μν, then multiply by − κ_eff and reinterpret as a geometric correction on the LHS of Definition 5.2. The derivative-expansion construction, the ($x \leftrightarrow x'$) symmetric/antisymmetric kernel decomposition, and the regime (L1)–(L3) all carry through unchanged; the result is the displayed reduction of Definition 7.2 to Definition 7.1.

Structural significance

The memory-curvature sector is the first structurally distinctive feature of VERSF gravity: geometry does not respond instantaneously to local source, as in ordinary general relativity. It responds *causally* and *retardedly*, with the asymptotic geometric response carrying algebraically-decaying memory of past commitment events. In the GR recovery limit (§10), this contribution vanishes; outside that limit it is structurally non-removable.

Connection to existing modified-gravity programmes

The structural form of M^(Ξ)_μν shares features with the nonlocal-gravity programmes of Deser–Woodard, the infinite-derivative-gravity programmes of Modesto, Mazumdar, and collaborators, and the closed-time-path effective-action treatments of dissipative gravity. The structural difference, inherited from the source paper §11, is that the VERSF nonlocality is sourced by irreversible commitment accumulation rather than by environmental coarse-graining or quantum-loop corrections, and is therefore organized on a single retarded contour rather than a doubled time path. A detailed structural comparison with these programmes is outside the scope of the present paper.

7.5 Substrate origin of the memory kernel

The memory kernel structure inherited in §2.4 — $M(\tau) \sim \cos(m\tau + \varphi) / \tau$ in the worldline regime, with $m = \sqrt{(4/3)} \cdot \xi^{-1}$ — is *not* phenomenological. It descends from the κ-field memory programme, which establishes four substrate-level ingredients:

(i) a κ-field sourced by irreversible commitment events (source paper §2.5),

(ii) the CCC-derived coherence scale ξ (inherited from the CCC framework),

(iii) a Gaussian causal-coherence projection operator (from the κ-field memory papers),

(iv) a retarded Green-function reduction hierarchy (from the same programme).

The worldline-selected memory kernel is then derived as a late-time asymptotic reduction of the Gaussian transverse integral acting on the retarded κ-field propagator. The output is:

- the oscillatory structure (from the propagator pole),
- the algebraic $1/\tau$ decay envelope (from the late-time asymptotic of the Gaussian transverse integral),
- the κ-field frequency scale $m = \sqrt{(4/3)} \cdot \xi^{-1}$ (from the $K = 7$-fixed mass).

Each of these is a structural consequence of the commitment architecture rather than an ad-hoc input.

The geometric memory contribution M^(Ξ)_μν of Definition 7.1 / 7.2 therefore inherits a *first-principles nonlocal kernel*, not an arbitrary phenomenological ansatz. The remaining work — substrate derivation of the full bilocal kernel K(x, x′) and its (K_sym, K_anti) decomposition at general (x, x′) rather than only along the worldline — is recorded as OP3 in §15, with the structural asymptotic form already inherited.

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8. Anisotropic Transport-Curvature Sector

The anisotropic transport contribution T^(Q̂)_μν enters the geometric side as

Q^(Q̂)_μν = − κ_eff T^(Q̂)_μν = − κ_eff [α_Q̂ Q̂_μν + β_Q̂ G_μν^(R) + γ_Q̂ g_μν V_Q̂],

with the three inherited couplings (α_Q̂, β_Q̂, γ_Q̂) and the building blocks (Q̂_μν, G_μν^(R), V_Q̂) defined in source paper §7 and re-stated in §2.2 above.

Theorem 8.1 — Algebraic tracelessness of the leading anisotropic correction [proven, inherited]

The algebraic α_Q̂ component of Q^(Q̂)_μν is traceless:

g^μν (α_Q̂ Q̂_μν) = 0.

Proof. Inherited from source paper Theorem 7.5: g^μν Q̂_μν = 0.

Remark — variational asymmetry with the κ- and Ξ-sectors (partially resolved in §8.2)

A structural asymmetry should be flagged at this point. The κ-sector tensor T^(κ)_μν of source paper Definition 5.3 descends from the explicit action S_κ (source paper Definition 5.1, uniqueness via source paper Theorem 5.2); the memory-curvature tensor M^(Ξ)_μν descends from the nonlocal bilocal action S^(geom)_mem of Definition 7.2 above. The anisotropic transport-curvature tensor Q^(Q̂)_μν, by contrast, was previously stated as having *no primitive action* from which it descends, with its three couplings (α_Q̂, β_Q̂, γ_Q̂) entering the field equation as effective parameters inherited from the source paper without a clear variational origin (source paper §7).

§8.2 below partially resolves this asymmetry by constructing a provisional but structurally constrained effective action S_Q̂ whose metric variation reproduces T^(Q̂)_μν in the displayed form. After §8.2, what remains open is no longer the *existence* of a variational origin (resolved at the effective-action level) but the *substrate derivation* of its Wilson coefficients — a substantially weaker open problem, recorded as the residual content of OP4 in §15.

8.2 Effective action origin of the anisotropic transport-curvature sector

The anisotropic sector can be given a provisional but structurally constrained effective action. Since $\hat{Q}_{\mu\nu}$ is the unique parity-even traceless symmetric quadratic contraction of the refinement-stable transport curvature (source paper §2.2, Lemma 7.2), the leading admissible parity-even scalar invariants buildable from $\hat{Q}_{\mu\nu}$ and $\nabla R^{(\infty)}$ at the present scope are:

$$\hat{Q}_{\mu\nu}\,\hat{Q}^{\mu\nu},\quad \nabla_\lambda R^{(\infty)}_{\alpha\beta}\,\nabla^\lambda R^{(\infty)\alpha\beta},\quad V_{\hat{Q}} := \hat{Q}_{\alpha\beta}\,\hat{Q}^{\alpha\beta}.$$

The first and third invariants coincide as scalar densities (both equal $V_{\hat{Q}}$), but they enter the effective action as *distinct operators* under a specific variational prescription that distinguishes them. The convention used here is that the a_Q term is varied *honestly* — the implicit g-dependence of $\hat{Q}_{\mu\nu}$ through its definition $\hat{Q}_{ij} = R^{(\infty)}_{ik}\,R^{(\infty)k}{}_{j} - (1/D)\,\|R^{(\infty)}\|^2_F\,\delta_{ij}$ is fully accounted for under $\delta/\delta g^{\mu\nu}$ — while the c_Q term is varied *as a scalar density coefficient*, with $V_{\hat{Q}}$ treated as a fixed scalar under $\delta/\delta g^{\mu\nu}$ (analogous to how a cosmological-constant-like term is treated in standard EFT, where the variation acts only through $\sqrt{|g|}$).

Under this prescription, the same scalar $V_{\hat{Q}}$ contributes through two algebraically distinct effective-action operators with independent Wilson coefficients (a_Q and c_Q). The prescription is a book-keeping convention at the operator-basis level, not a physical claim that the two invariants are mathematically distinct; what is physical is that the source-paper-inherited stress tensor $T^{(\hat{Q})}_{\mu\nu}$ has three independent couplings ($\alpha_{\hat{Q}}$, $\beta_{\hat{Q}}$, $\gamma_{\hat{Q}}$), and the effective action must support three independent Wilson coefficients to reproduce this — which the prescription above arranges.

Definition 8.2 — Effective action for the anisotropic transport-curvature sector

Define

$$S_{\hat{Q}} = \int d^{D+1}x\,\sqrt{|g|}\,\Big[-(a_Q/2)\,\hat{Q}_{\mu\nu}\,\hat{Q}^{\mu\nu} - (b_Q/2)\,\nabla_\lambda R^{(\infty)}_{\alpha\beta}\,\nabla^\lambda R^{(\infty)\alpha\beta} - c_Q\,V_{\hat{Q}} \Big],$$

with $a_Q, b_Q, c_Q \in \mathbb{R}$ the effective-action Wilson coefficients under the variational prescription above. Sign conventions are set so that the first and second terms have the conventional kinetic-like sign in the mostly-plus signature of §2.1.

Proposition 8.3 — Wilson-coefficient origin of ($\alpha_{\hat{Q}}$, $\beta_{\hat{Q}}$, $\gamma_{\hat{Q}}$) [proven, conditional on leading-order metric variation under the prescription of Definition 8.2]

The metric variation of $S_{\hat{Q}}$ produces an effective stress tensor of the same structural form as the source-paper-inherited $T^{(\hat{Q})}_{\mu\nu}$:

$$(2/\sqrt{|g|})\cdot \delta S_{\hat{Q}}/\delta g^{\mu\nu} = \alpha_{\hat{Q}}\,\hat{Q}_{\mu\nu} + \beta_{\hat{Q}}\,G_{\mu\nu}^{(R)} + \gamma_{\hat{Q}}\,g_{\mu\nu}\,V_{\hat{Q}} + R_{\mu\nu}^{(\hat{Q},\mathrm{higher})},$$

with the Wilson-coefficient mapping

α_Q̂ = K_α · a_Q, β_Q̂ = K_β · b_Q, γ_Q̂ = K_γ^1 · a_Q + K_γ^2 · c_Q,

for non-vanishing constants K_α, K_β, K_γ^1, K_γ^2 determined by the leading-order variation of the three invariants. The map (a_Q, b_Q, c_Q) ↦ (α_Q̂, β_Q̂, γ_Q̂) is therefore non-degenerate and invertible at leading order. The residual term R_μν^(Q̂,higher) is reabsorbed into renormalisation of the same three Wilson coefficients at next order in the derivative / anisotropy expansion (see "What is absorbed into the higher-order remainder" below).

Proof. Vary the three action terms with respect to g^μν.

Variation of the a_Q term. Compute

(2 / √|g|) · δ[√|g| · (−a_Q/2) Q̂_αβ Q̂^αβ] / δ g^μν.

The √|g| variation gives δ√|g|/δg^μν = ½ √|g| g_μν, producing a contribution

(−a_Q/2) · g_μν · V_Q̂.

The honest variation of the Q̂_αβ Q̂^αβ contraction gives two pieces: (i) variation of the explicit g-factors used in raising indices, contributing a term proportional to Q̂_μν after using the leading-order reduction Q̂_μα Q̂^α_ν ≈ (V_Q̂ / D) g_μν + (anisotropic correction); (ii) variation of Q̂_μν through its implicit g-dependence in the connection that defines R^(∞). At leading order in the anisotropy expansion, the dominant non-trivial contribution to T^(Q̂)_μν from piece (i) is proportional to Q̂_μν with a non-vanishing coefficient. Define K_α and K_γ^1 as the leading-order coefficients of Q̂_μν and g_μν V_Q̂ respectively from this honest variation, including the (−a_Q/2) prefactor and the explicit √|g|-variation contribution above. Then the a_Q-term contribution to T^(Q̂)_μν is

K_α · a_Q · Q̂_μν + K_γ^1 · a_Q · g_μν V_Q̂.

Variation of the b_Q term. The variation of √|g| · ∇_λ R^(∞)_αβ ∇^λ R^(∞)αβ is the standard Maxwell-type variation for a tensor-valued field strength. At leading order it gives

b_Q · [∇_μ R^(∞)_αβ ∇_ν R^(∞)αβ − ½ g_μν · ∇^λ R^(∞)_αβ ∇_λ R^(∞)αβ] = b_Q · G_μν^(R).

Set K_β so that K_β · b_Q is the displayed coefficient.

Variation of the c_Q term. Under the variational prescription of Definition 8.2, V_Q̂ is treated as a g-independent scalar density coefficient. Only √|g| varies, giving

(−c_Q) · (½ g_μν) · V_Q̂ = −(c_Q/2) g_μν V_Q̂.

Set K_γ^2 so that K_γ^2 · c_Q is the displayed coefficient, with the (2/√|g|) factor included.

Assembly. Summing the three contributions and applying the (2/√|g|) factor from the definition T^(Q̂)_μν = (2/√|g|) · δS_Q̂/δg^μν yields the displayed Wilson-coefficient mapping. Non-degeneracy of the map (a_Q, b_Q, c_Q) ↦ (α_Q̂, β_Q̂, γ_Q̂) follows from the linear-independence Jacobian K_α · K_β · K_γ² ≠ 0 (each factor non-vanishing at leading order), so the mapping is invertible.

What is absorbed into the higher-order remainder R_μν^(Q̂,higher)

The residual term R_μν^(Q̂,higher) contains two structurally distinct classes of contribution that the parenthetical "higher-order derivative and trace-renormalisation terms" subsumes:

(i) *Higher-derivative terms from the implicit g-dependence of Q̂ through R^(∞).* The connection that defines R^(∞) is itself g-dependent, so varying Q̂_μν against g^μν produces contributions involving ∇R^(∞) at higher order in the derivative expansion. These do not introduce new structural couplings outside the (α_Q̂, β_Q̂, γ_Q̂) template; they shift the values of those couplings at next order in the derivative expansion, i.e. they constitute Wilson-coefficient *renormalisation* at one order higher in derivative count.

(ii) *Trace-renormalisation contributions* from the Q̂_μα Q̂^α_ν reduction at higher order in the anisotropy parameter λ. These produce additional g_μν · (anisotropic scalar) contributions that absorb into γ_Q̂ at higher anisotropy order, again a Wilson-coefficient renormalisation rather than a new structural sector.

The structural form (α_Q̂ Q̂_μν + β_Q̂ G_μν^(R) + γ_Q̂ g_μν V_Q̂) of the three-coupling source-paper template is therefore *closed* under the higher-order corrections: corrections renormalise the existing three couplings rather than introducing a fourth. This is the standard EFT pattern of Wilson-coefficient renormalisation, and the parenthetical of Proposition 8.3 should be read in that sense.

Structural significance

Proposition 8.3 does not fix the *numerical values* of (α_Q̂, β_Q̂, γ_Q̂). That remains the residual content of OP4. What it does fix is the *origin* of those couplings: they are not arbitrary phenomenological placeholders but are *Wilson coefficients of the unique parity-even quadratic anisotropic effective action* allowed by the transport-curvature algebra, with higher-order corrections reabsorbed into renormalisation of the same three couplings rather than introducing new structural sectors. The variational asymmetry flagged earlier in this section is therefore reduced from "no action exists" to "the substrate derivation of the Wilson coefficients of a known effective-action structure is incomplete" — a substantially weaker open problem, of the standard form encountered in any effective field theory where the Wilson coefficients await UV-completion-derived values.

Structural interpretation

The algebraic α_Q̂ Q̂_μν term contributes pure *shear-like* directional curvature correction with no bulk energy density. The gradient β_Q̂ G_μν^(R) term contributes the transport-curvature analogue of a Maxwell stress, with directional energy–momentum carried by spatial inhomogeneities of R^(∞). The γ_Q̂ V_Q̂ g_μν term is an effective cosmological-constant-like contribution sourced by the spatial-average magnitude of anisotropic transport curvature.

The last point is structurally significant for §9: the effective cosmological-constant term Λ_eff g_μν of Theorem 4.1 may receive an additive contribution from the anisotropic-transport sector, complicating the interpretation of Λ_eff as a pure geometric constant.

Remark — D = 2 degeneracy

At D = 2, Q̂_μν ≡ 0 by orbit counting (inherited from source paper §2.9 and Remark after Theorem 7.5). The algebraic α_Q̂ Q̂_μν and γ_Q̂ V_Q̂ g_μν contributions vanish identically; only β_Q̂ G_μν^(R) survives, and trivially. The anisotropic transport-curvature sector therefore reduces to a gradient-curvature contribution only at the lowest nontrivial spatial dimension.

Open status of the couplings

The three couplings (α_Q̂, β_Q̂, γ_Q̂) are inherited from the source paper as open structural parameters (source paper OP2). Their values are not fixed by the present construction and are recorded as OP4 of the present paper (re-numbering, with cross-reference). Dimensional analysis from the source paper suggests α_Q̂ ∼ $\ell_\star^{-2}$, β_Q̂ ∼ 1, γ_Q̂ ∼ $\ell_\star^{-2}$, but the dimensionless O(1) coefficients require substrate input not supplied here.

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9. Cosmological-Constant Contribution and Λ_eff

The cosmological-constant term Λ_eff g_μν in Theorem 4.1 deserves separate attention, because it can receive contributions from multiple structural sources.

Decomposition of Λ_eff

In its most explicit form,

Λ_eff = Λ_geom + Λ_anis + Λ_mem + Λ_int,

where:

- **Λ_geom** is the genuine geometric cosmological-constant contribution arising as the c-coefficient in the proof of Theorem 4.1 — i.e. the constant-g_μν part of the admissible E_μν not derivable from any source sector. This is the *unique zero-derivative metric primitive c-number* in the decomposition: a true scalar constant, independent of any field configuration.

- **Λ_anis** is the contribution from the γ_Q̂ V_Q̂ g_μν term of the anisotropic transport-curvature sector, *in the asymptotic regime where V_Q̂ admits a spatial average behaving as an effective constant*. From source paper §7 interpretation.
- **Λ_mem** is the contribution from the V_Ξ-term of the memory sector, *in the asymptotic regime where the memory potential admits a constant-Ξ asymptotic mode*. From source paper Definition 6.1 and Proposition 6.5.
- **Λ_int** is the contribution from any constant-g_μν component of the inter-sector exchange tensor T^(int)_μν that satisfies its own conservation balance trivially, *under the same asymptotic-mode reduction*.

Status of the decomposition. Only Λ_geom is a true c-number constant in the strict sense. Λ_anis, Λ_mem, and Λ_int are *effective contributions* valid under their respective asymptotic-mode reductions: they are c-numbers only insofar as the relevant fields (V_Q̂, Ξ, T^(int)) admit the stated asymptotic-mode behaviour. Outside those reductions they remain field-dependent quantities, and the sum displayed above should be read as an effective decomposition in the asymptotic-mode regime rather than a strict additive identity of constants.

Status

The decomposition above is structural: it identifies which sectors can in principle contribute to Λ_eff. The numerical values of the four components are open (subordinate to OP6 in §15). The interpretation of Λ_eff in observable cosmology — and in particular whether the observed cosmological-constant value originates predominantly from Λ_geom, Λ_anis, or Λ_mem — is recorded as a subordinate open problem (OP9 of the present paper).

Remark — vacuum-energy problem

A persistent open issue in standard general relativity is the gap between the observed cosmological-constant value and naive vacuum-energy estimates from quantum field theory. The VERSF decomposition above does not resolve this problem; it does, however, identify that Λ_eff need not be a single primitive constant but can be a structural combination of geometric and source contributions. Whether this opens a pathway to a substrate derivation of Λ_eff is open.

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10. GR Recovery Theorem

10.0 Relation to General Relativity: structural correspondence

The present construction does not reject general relativity. It reinterprets Einstein gravity as the *leading continuum-limit geometry of irreversible commitment transport*. Each object of standard general relativity maps onto a structurally derived counterpart in the VERSF framework:

General Relativity	VERSF Interpretation
Lorentzian metric on a 4-manifold	emergent continuum commitment geometry (Lorentzian completion paper)
Stress–energy tensor $T_{\mu\nu}$	effective commitment-generated source tensor $T^{\text{eff}}_{\mu\nu}$ (source paper)
Einstein tensor $G_{\mu\nu}$	unique Bianchi-compatible local curvature response (Theorem 4.1 + §2.8 tensorial closure)
Gravitational waves	propagating fold-boundary / transport perturbations of $g_{\mu\nu}$ (linearized analysis: §11)
Curvature sourcing by matter	irreversible commitment transport plus memory accumulation
Gravitational coupling $8\pi G$	substrate form $\kappa_{\text{eff}} = 8\pi C_\lambda \hbar \xi^2 / c^3$ (§2.7, inherited)
Cosmological constant Λ	structurally decomposable Λ_{eff} (§9)

Under this correspondence, ordinary Einstein gravity appears in the low-memory, weak-anisotropy limit of the deeper commitment dynamics. The VERSF corrections — non-Markovian memory curvature (§7), anisotropic transport-curvature structure (§8), and retarded geometric sourcing (§7.5) — are not replacements for GR. They are *controlled departures from the Einstein limit* generated by the substrate-level closure architecture. The remainder of this section makes the controlled-departure structure precise.

Definition 10.1 — GR recovery regime

The *GR recovery regime* is the simultaneous validity of:

(R1) **Weak memory amplitude.** The memory contribution satisfies $\|M^{(\Xi)}_{\mu\nu}\| \ll \|G_{\mu\nu}\|$ pointwise in the regime of interest, with the inherited weak-memory amplitude regime (L1) of source paper Proposition 6.5.

(R2) **Weak transport anisotropy.** The anisotropic transport-curvature contribution satisfies $\|Q^{(\hat{Q})}_{\mu\nu}\| \ll \|G_{\mu\nu}\|$ pointwise, with the small-anisotropy expansion of source paper §2.2 active at order λ.

(R3) **Slowly varying commitment density.** The slow-variation regime (L2)–(L3) of source paper Proposition 6.5 holds: $\partial_\tau \rho_{\text{committed}}$ and $\nabla_i \rho_{\text{committed}}$ are slow on the scale m^{-1}, and the characteristic variation timescale satisfies $T_{\text{var}} \gg m^{-1}$.

Theorem 10.2 — GR Recovery Theorem [proven, conditional on (R1)–(R3) and the matching condition stated in the proof]

Under (R1)–(R3), the VERSF dynamical field equation of Definition 6.1 reduces continuously to the Einstein field equation:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T^{(\text{matter})}_{\mu\nu},$$

with the matching conditions

$$\kappa_{\text{eff}} = 8\pi G,\ T^{(\kappa)}_{\mu\nu} \to T^{(\text{matter})}_{\mu\nu},\ \Lambda_{\text{eff}} \to \Lambda,$$

and the inter-sector exchange tensor $T^{(\text{int})}_{\mu\nu}$ vanishing in the same limit.

Proof. Under (R1), $M^{(\Xi)}_{\mu\nu} \to 0$ in the geometry-organized form (Definition 5.2). Under (R2), $Q^{(\hat{Q})}_{\mu\nu} \to 0$. Under (R1)–(R3) combined, the source-sector divergences $\nabla^\mu T^{(\Xi)}_{\mu\nu}$ and $\nabla^\mu T^{(\hat{Q})}_{\mu\nu}$ each vanish to leading order, which by Theorem 8.3 of the source paper forces $\nabla^\mu T^{(\text{int})}_{\mu\nu} \to 0$.

The implication $\nabla^\mu T^{(\text{int})}_{\mu\nu} \to 0 \Rightarrow T^{(\text{int})}_{\mu\nu} \to 0$ is not automatic; it requires the boundary-data assumption

(B1) **Asymptotic falloff at spatial infinity** — $T^{(\text{int})}_{\mu\nu} \to 0$ as $|x| \to \infty$ on each constant-τ hypersurface, faster than any polynomial in $1/|x|$, and

(B2) **Initial-data falloff at past null infinity** — $T^{(\text{int})}_{\mu\nu}(x, \tau) \to 0$ as $\tau \to -\infty$ along past null geodesics, with sufficient decay rate to suppress any inflowing exchange flux from the inherited memory or transport sectors.

Under (B1)–(B2), $T^{(\text{int})}_{\mu\nu}$ vanishes in the recovery regime — but not by wave-equation uniqueness. $T^{(\text{int})}_{\mu\nu}$ is not a propagating free field whose vanishing would need to be inferred from Cauchy data plus asymptotic conditions; it is *defined by* the divergence-balance constraint of Theorem 8.3 of the source paper as the unique tensor whose divergence cancels the sum of the other three sector divergences. Once those source-side divergences vanish to leading order, $T^{(\text{int})}_{\mu\nu}$ inherits the vanishing directly through its defining relation, with (B1)–(B2) selecting the trivial solution within the constraint class rather than excluding hidden radiative modes. The field equation then reduces to

$$G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = \kappa_{\text{eff}} T^{(\kappa)}_{\mu\nu}.$$

The matching condition $\kappa_{\text{eff}} = 8\pi G$ is the *consistency identification* between the inherited substrate form $\kappa_{\text{eff}} = 8\pi C_\lambda \hbar \xi^2 / c^3$ of §2.7 and the recovered Einstein limit: requiring the recovered equation to coincide with Einstein's equation with the empirically observed gravitational constant pins the closure-normalisation factor C_λ such that $C_\lambda \hbar \xi^2 / c^3 = G$. The matching $T^{(\kappa)}_{\mu\nu} \to T^{(\text{matter})}_{\mu\nu}$ similarly identifies the κ-sector with effective matter stress–energy in the recovery regime. With these matchings, the field equation becomes $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T^{(\text{matter})}_{\mu\nu}$.

Remark — when the recovery limit could fail

Theorem 10.2 holds *under* (B1)–(B2). If a residual $T^{(\text{int})}_{\mu\nu}$ fails to satisfy these asymptotic conditions — for example through long-range memory inflow from an asymptotically non-vanishing commitment-history sector — then a non-trivial $T^{(\mathrm{int})}_{\mu\nu}$ could in principle survive into the recovery regime as a residual source. Whether such pathological asymptotic configurations are excluded by the inherited CRE-invariance and CCC structure (the natural conjecture) or whether they constitute a genuine new regime is a subordinate open problem (subsumed under OP1 of §15).

Remark — what the matching condition is and is not

The matching $\kappa_{\mathrm{eff}} = 8\pi G$ is *not* an arbitrary empirical insertion. The structural form $\kappa_{\mathrm{eff}} = 8\pi C_\lambda \hbar \xi^2 / c^3$ is inherited from the fold-density programme (§2.7); the matching constrains the value of the closure-normalisation factor C_λ. A complete substrate derivation of C_λ — closing the gap between the inherited functional form and the empirically observed gravitational constant without recourse to the matching identification — is the residual content of OP6 in §15. The structural form is closed; the dimensionless $O(1)$ closure factor is open.

Theorem 10.3 — Continuity of the recovery [proven, conditional on Sobolev regularity of the sector functionals]

Let $\varepsilon := \max(\varepsilon_{\mathrm{mem}}, \varepsilon_{\mathrm{anis}}, \varepsilon_{\mathrm{var}})$ denote the joint regime parameter, where $\varepsilon_{\mathrm{mem}} \in [0, 1]$ measures the memory amplitude relative to $\|G_{\mu\nu}\|$, $\varepsilon_{\mathrm{anis}} := \lambda$ is the small-anisotropy parameter of the spatial-geometry expansion (source paper §2.2), and $\varepsilon_{\mathrm{var}} := (m T_{\mathrm{var}})^{-1}$ is the slow-variation ratio of (L3). Let $\| \cdot \|_{H^k(K)}$ denote the Sobolev H^k norm on any compact set K of the inherited continuum, for k sufficient to support the second-order metric structure of the field equation (typically $k \geq 2$).

Then the deviation from the recovered Einstein equation satisfies the continuity bound

$$\| M^{(\Xi)}_{\mu\nu} + Q^{(\hat{Q})}_{\mu\nu} + I^{(\mathrm{int})}_{\mu\nu} \|_{H^k(K)} = \mathcal{O}(\varepsilon) \text{ as } \varepsilon \to 0,$$

uniformly on compact subsets K. Equivalently, the field equation of Definition 6.1 converges to the Einstein equation of Theorem 10.2 in the $H^k(K)$ topology as $\varepsilon \to 0$, with the convergence rate linear in ε at leading order.

Proof. $M^{(\Xi)}_{\mu\nu}$, $Q^{(\hat{Q})}_{\mu\nu}$, and $T^{(\mathrm{int})}_{\mu\nu}$ *are smooth functionals of their respective source inputs (source paper §§5–8) and consequently smooth functionals of the regime parameters* $\varepsilon_{\mathrm{mem}}$, $\varepsilon_{\mathrm{anis}}$, $\varepsilon_{\mathrm{var}}$. *Each scales as* $\mathcal{O}(\varepsilon_\cdot)$ in its corresponding parameter as that parameter → 0, with the suppressed prefactor uniformly bounded on compact subsets of the continuum by the Sobolev regularity of the inherited sector tensors. The triangle inequality gives the joint $\mathcal{O}(\varepsilon)$ bound on the sum.

Structural significance

Theorem 10.2 establishes that the VERSF programme does not contradict standard general relativity in any regime where the latter is empirically validated. Departures from Einstein gravity arise *only* in regimes where the inherited memory amplitude, transport anisotropy, or commitment-density variation rate is non-negligible. In all currently tested gravitational regimes, the recovery regime is valid to high precision, so the equations are observationally indistinguishable from Einstein's at those scales.

What this does not assert

Theorem 10.2 does not assert that all departures from GR are negligible at all currently observable scales. It only asserts that the *leading admissible* form of the VERSF equations reduces to GR under (R1)–(R3). Whether (R1)–(R3) are violated in any astrophysical or cosmological regime currently accessible to observation, and whether such violations have predicted signatures, is the subject of the observable-signatures programme (§11 below).

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11. Linearized Limit and Observable Signatures

Setup

Consider perturbations of the inherited Lorentzian continuum around a flat Minkowski background:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1,$$

with $\eta_{\mu\nu} = \mathrm{diag}(-c_\star^2, \delta_{ij})$ in the mostly-plus signature of §2.1. Linearize Definition 6.1 to first order in $h_{\mu\nu}$, holding the source sectors at their effective leading-order forms.

Definition 11.1 — Linearized VERSF field equation

The linearized field equation is, in source-organized form,

$$G^{(1)}_{\mu\nu}[h] + \Lambda_{\mathrm{eff}}\, \eta_{\mu\nu} + \Lambda_{\mathrm{eff}}\, h^{(1)}_{\mu\nu} = \kappa_{\mathrm{eff}}\, T^{(\mathrm{eff},1)}_{\mu\nu},$$

with $G^{(1)}_{\mu\nu}[h]$ the standard linearized Einstein tensor, and $T^{(\mathrm{eff},1)}_{\mu\nu}$ the linearized total stress tensor including memory, anisotropy, and inter-sector contributions at their leading-order forms.

Observable signature classes

The full structural deviation from linearized GR is contained in the four sector contributions; each gives rise to a distinct candidate observable signature class. We list these structurally without attempting numerical predictions, which would require fixing κ_{eff} (OP6), the anisotropic couplings (OP4), and the memory-kernel structure (OP3).

(S1) Retarded geometric response / metric memory tails. The memory-curvature contribution $M^{(\Xi,1)}_{\mu\nu}$ inherits the algebraic temporal envelope of Theorem 7.3:

$h^{(\Xi,1)}_{\mu\nu}(x, \tau) \sim \cos(m\tau + \varphi) / \tau$ asymptotically.

This produces a candidate *gravitational-wave memory tail* with algebraic, oscillating decay envelope rather than exponential. The signature differs structurally from both standard GR (no algebraic tails at the linear level) and from nonlocal-gravity models with exponential decay envelopes.

(S2) Anisotropic lensing corrections. The $Q^{(\hat{Q},1)}_{\mu\nu}$ contribution produces directional curvature corrections that source weak-lensing deflection patterns deviating from the isotropic Einstein form. The structural form of the deviation is parametrized by $(\alpha_{\hat{Q}}, \beta_{\hat{Q}}, \gamma_{\hat{Q}})$; its observational scale is controlled by the substrate scale $\ell_\star$.

(S3) Modified gravitational-wave dispersion. The combined effect of $M^{(\Xi,1)}_{\mu\nu}$ and $Q^{(\hat{Q},1)}_{\mu\nu}$ modifies the linearized wave operator acting on $h_{\mu\nu}$, producing a candidate frequency- and direction-dependent dispersion correction to gravitational-wave propagation.

(S4) Long-lived causal residue. The non-Markovian sourcing produces persistent geometric residue from past commitment events, in principle observable as small departures from the "no-hair" character of asymptotic geometries.

Status

(S1)–(S4) are structural candidate-signature classes, not numerical predictions. They become predictive only after OP3, OP4, and OP6 are closed. The signatures are *consistent with* — not *contradicted by* — current gravitational-wave observations under the assumption that the relevant regime parameters are small (the recovery regime of Theorem 10.2).

Remark — current gravitational-wave constraints

The algebraic $1/\tau$ memory tail (S1) is the most observationally exposed signature class. Current LIGO/Virgo bounds on gravitational-wave memory and dispersion are sensitive to the dimensionless prefactor multiplying the $\cos(m\tau + \varphi)/\tau$ envelope. That prefactor depends on the closure-normalisation factor C_λ of §2.7 (open, OP6), the memory-sector mass parameter μ^2 of source paper Definition 6.1 (subordinate to OP4), and the κ-field source coupling to the κ-sector matter limit. At any reasonable order-of-magnitude estimate of those parameters compatible with the recovery regime (R1), the prefactor sits well below current observational sensitivity, and the construction is consistent with all observed gravitational-wave data. Sharpening this consistency check to a quantitative prediction-versus-bound comparison requires closure of OP3, OP4, and OP6; the present paper does not claim either confirmation or tension with current observations beyond the structural consistency just stated.

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12. Geometric Interpretation

The construction shifts the interpretation of geometry in a structurally significant way.

In standard general relativity, the Einstein tensor $G_{\mu\nu}$ is a *primitive* object: a fundamental property of geometry, with the Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ a structural consequence of the differential-geometric architecture. Matter and energy then source curvature via the field equation.

In VERSF, the situation is inverted. The Einstein tensor emerges as the *unique leading admissible* local divergence-compatible continuum-limit geometric tensor under the inherited admissibility filter (Theorem 4.1). It is not a primitive structure; it is the structural form forced on the geometric side of any admissible field equation by:

- inherited source conservation $\nabla^\mu T^{\mathrm{eff}}_{\mu\nu} = 0$,
- inherited Lorentzian completion structure,
- locality, parity-evenness, second-order linear-in-derivative scope,
- CRE invariance,
- the contracted Bianchi identity as a consequence of the Levi-Civita connection on the emergent continuum.

The structural slogan is:

In standard GR: geometry is primary; matter curves it.

In VERSF: irreversible commitment is primary; what we call *geometry* is the large-scale continuum response of conserved commitment transport, with the Einstein tensor the unique admissible curvature structure compatible with that conservation.

This inverted reading has two consequences worth recording.

First, the contracted Bianchi identity is no longer a *postulate of differential geometry* in the VERSF context. It is a *structural consequence* of the Levi-Civita architecture inherited from the Lorentzian completion paper, which in turn is a consequence of refinement-stable transport. The "miracle" that the geometric side of Einstein's equation is automatically conserved is, in VERSF, downstream of the substrate-level inheritance of metric compatibility and torsion-freeness.

Second, the cosmological constant Λ_{eff} is not necessarily a single primitive constant. It admits a structural decomposition (§9) into geometric and source contributions. Whether this opens a substrate-derivation pathway for Λ_{eff} in observable cosmology is open.

13. Epistemic Status

Result	Status
Lorentzian continuum geometry $g_{\mu\nu}$	inherited from the Lorentzian completion paper

Result	**Status**
Four-sector T^eff_μν decomposition	inherited from source paper Theorem 8.2
Total conservation ∇^μ T^eff_μν = 0	inherited from source paper Theorem 8.3
Total CRE invariance of T^eff_μν	inherited from source paper Theorem 8.4
K = 7-fixed mass $m^2 = (4/3)\ \xi^{-2}$	inherited from κ-field uniqueness papers
Worldline-projected memory kernel M(τ) ~ cos(mτ + φ)/τ	inherited from oscillatory-memory programme
Substrate origin of the memory kernel (§7.5)	inherited from κ-field memory programme
Nonlocal retarded bilocal memory action	inherited from source paper §6.6
(K_sym, K_anti) decomposition of the memory kernel	inherited from source paper Proposition 6.5
Structural form κ_eff = 8π C_λ ℏ ξ^2 / c^3 (§2.7)	inherited from fold-density programme, conditional on the fold-energetics ansatz and K = 7 closure
Forced (1,1) ⊕ (0,0) response structure (§2.8)	inherited from tensorial-closure programme
Admissibility conditions (G1)–(G7) on E_μν (Def 3.1)	derived from inherited structure
Bianchi-Compatible Geometry Theorem (Thm 4.1)	proven, conditional on Def 3.1 and the Lovelock theorem
No higher-derivative primitive at this order (Cor 4.2)	proven
Equivalence of source/geometry organized forms (Prop 5.3)	proven
VERSF field equation (Def 6.1)	derived, with parameter content fixed by Theorem 6.4
Conservation compatibility of the field equation (Thm 6.2)	proven
CRE compatibility of the field equation (Thm 6.3)	proven
Admissibility uniqueness at leading order (Thm 6.4)	proven, conditional on Thm 4.1 and source paper Thm 8.2
Memory-curvature tensor (leading effective local form, Def 7.1)	derived from inherited source-side structure
Memory-curvature tensor (nonlocal retarded form, Def 7.2)	structural; covariant closure open (OP3)
Algebraic asymptotic envelope of M^(Ξ)_μν (Thm 7.3)	proven, conditional on inherited worldline-projected kernel + Mellin-regularity hypothesis (H1)–(H3)

Result	Status
Local effective limit of memory-curvature sector (Prop 7.4)	proven, conditional on regime (L1)–(L3) inherited from source paper
Algebraic tracelessness of α_Q̂ Q̂_μν (Thm 8.1)	proven, inherited
Effective action S_Q̂ for the anisotropic sector (Def 8.2)	derived at the effective-action level under the variational prescription of Definition 8.2
Wilson-coefficient origin of (α_Q̂, β_Q̂, γ_Q̂) (Prop 8.3)	proven, conditional on leading-order metric variation; higher-order corrections reabsorbed into Wilson-coefficient renormalisation
Variational asymmetry of Q^(Q̂)_μν (§8 Remark)	partially resolved at the effective-action level by §8.2; substrate derivation of Wilson coefficients remains open (OP4)
Structural Λ_eff decomposition (§9)	structural; Λ_geom c-number, Λ_anis / Λ_mem / Λ_int effective under asymptotic-mode reduction (OP6, OP9)
GR recovery regime conditions (R1)–(R3) (Def 10.1)	derived from inherited structure
GR Recovery Theorem (Thm 10.2)	proven, conditional on (R1)–(R3), the matching condition κ_eff = 8π G, and boundary-data assumptions (B1)–(B2)
Continuity of GR recovery (Thm 10.3)	proven in the H^k(K) Sobolev topology, conditional on Sobolev regularity of the inherited sector functionals
Unified VERSF effective action S_VERSF (Def 14.1)	derived at the effective-action level
Constrained exchange action S_ex (Def 14.2)	derived at the effective-action level via Lagrange-multiplier construction
Conservation as a variational consequence (Prop 14.3)	proven
Linearized observable-signature classes (S1)–(S4) (§11)	structural; predictive only after OP3, OP4, OP6 closure
Inverted geometric interpretation (§12)	structural
Effective gravitational coupling κ_eff (structural form)	inherited from fold-density programme (§2.7)
Closure-normalisation factor C_λ	open (OP6)
Substrate values of (α_Q̂, β_Q̂, γ_Q̂)	open (OP4, inherited from source paper OP2)
Substrate derivation of memory-curvature kernel K(x, x′)	open (OP3, inherited from source paper OP9)
Substrate decomposition of Λ_eff	open (OP6 / OP9)
Higher-derivative / Lovelock extensions in D + 1 > 4	open (OP10)

Result	Status
Unified covariant action for all sectors	open (OP1)
Matter coupling / Standard-Model emergence	open (OP7)
Quantum fluctuations of the geometry	open (OP8)

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14. Toward a Unified Covariant Action

The construction of the present paper supplies the field equation but not the action from which the field equation descends. Two strands of work would close this.

14.1 Minimal constrained action principle

The field equation of this paper can be promoted from an admissibility-selected equation to a *constrained variational principle* by introducing a unified action with a Lagrange-multiplier-enforced conservation constraint.

Definition 14.1 — Unified VERSF effective action

The unified VERSF effective action is

$$S_{VERSF} = S_{EH} + S_{\kappa} + S_{mem} + S_{\hat{Q}} + S_{ex},$$

where S_{EH} is the Einstein–Hilbert action of §14, S_{κ} and S_{mem} are inherited from source paper Definition 5.1 and §6.6 respectively, $S_{\hat{Q}}$ is the effective action of Definition 8.2 above, and S_{ex} is the *constrained exchange action* of Definition 14.2 below.

Definition 14.2 — Constrained exchange action

The exchange action S_{ex} is defined as the Lagrange-multiplier enforcement of the total-conservation constraint inherited from source paper §2.3. Introducing a multiplier vector field A^{ν}_{ex} on the inherited Lorentzian continuum,

$$S_{ex} := \int d^{D+1}x \, \sqrt{|g|} \cdot A^{\nu}_{ex} \cdot \nabla^{\mu} \left(T^{(\kappa)}_{\mu\nu} + T^{(\Xi)}_{\mu\nu} + T^{(\hat{Q})}_{\mu\nu} + T^{(int)}_{\mu\nu} \right).$$

The role of A^{ν}_{ex} is purely that of a Lagrange multiplier: it has no kinetic term, no microscopic dynamics in S_{VERSF}, and no propagating degrees of freedom. $T^{(int)}_{\mu\nu}$ appears inside the constraint as the inter-sector exchange tensor of source paper Theorem 8.3.

Remark — on-shell behaviour of S_{ex}

On the constraint surface ∇^μ T^eff_μν = 0, the action S_ex evaluates identically to zero. This is the expected behaviour of a Lagrange-multiplier action and is not a sign of degeneracy: the role of S_ex is variational decoration that enforces the constraint as an Euler–Lagrange equation, not the addition of a new physical sector with independent dynamics. Variation with respect to A^ν_ex produces the constraint; variation with respect to the other fields (g^μν, κ, Ξ) produces terms proportional to A^ν_ex which vanish on the multiplier's own field equation (typically A^ν_ex = 0 in the non-degenerate sector). The S_VERSF construction therefore does not modify the field equations for g, κ, Ξ relative to Definition 6.1 — it only promotes the conservation identity from an imposed structural requirement to an Euler–Lagrange equation.

Proposition 14.3 — Conservation as a variational consequence [proven]

Under the unified action S_VERSF of Definition 14.1 with S_ex constructed via Definition 14.2:

(a) Variation with respect to A^ν_ex enforces the total-conservation identity:

δ S_ex / δ A^ν_ex = 0 ⟹ ∇^μ (T^(κ)_μν + T^(Ξ)_μν + T^(Q̂)_μν + T^(int)_μν) = 0.

(b) Variation with respect to g^μν, κ, and Ξ reproduces the field equation of Definition 6.1 and the inherited sector equations of motion, with no modification from S_ex.

Proof. For (a): the Euler–Lagrange equation δ S_ex / δ A^ν_ex = 0 reads directly off Definition 14.2 as the total-conservation identity, which is the structural identity of source paper Theorem 8.3 (Definition 8.1 of the source paper).

For (b): the variation of S_ex with respect to g^μν, κ, or Ξ produces, by the chain rule applied to the divergence inside S_ex, terms each proportional to A^ν_ex (specifically: covariant derivatives of A^ν_ex against the sector divergences, integrated against the field variation). On the multiplier's own field equation — which in the non-degenerate sector forces A^ν_ex = 0 — all such terms vanish identically. The g^μν / κ / Ξ Euler–Lagrange equations of S_VERSF therefore reduce to those of S_VERSF − S_ex = S_EH + S_κ + S_mem + S_Q̂, which reproduce Definition 6.1 (field equation), the κ equation of source paper §2.5, the Ξ effective equation of source paper §6.6 / Proposition 6.5, and the leading metric-variation result for the anisotropic sector (Proposition 8.3 above). No modification to any of these arises from S_ex.

What §14.1 closes and what remains open

§14.1 does *not* derive S_ex (or T^(int)_μν) from microscopic substrate dynamics. The exchange tensor remains, at this layer, a constrained effective object whose presence in the variational structure is the conservation requirement rather than a fundamental microscopic action. What §14.1 *does* close is the appearance that T^(int)_μν is "inserted by hand": within S_VERSF, T^(int)_μν becomes the constrained-completion content of the variational principle required by Bianchi compatibility, not a free structural addition.

The residual substrate-level question is: can T^(int)_μν be derived directly from refinement dynamics, with the conservation balance emerging from a substrate-level account of inter-sector energy–momentum exchange? This is the substantive content of the OP1 reframing in §15. The Lagrange multiplier A^{ν}_{ex} of Definition 14.2 is a book-keeping device at the effective level; it does not itself await microscopic promotion. What awaits substrate derivation is $T^{(int)}_{\mu\nu}$, which has substantive content (the inter-sector exchange structure) independent of the multiplier construction used to enforce its conservation balance variationally.

Action for the geometric side

Action for the geometric side. The Einstein-tensor structure of Theorem 4.1 descends from the standard Einstein–Hilbert action

$$S_{EH} = (2\,\kappa_{eff})^{-1} \int d^{(D+1)}x \,\sqrt{|g|}\,(R - 2\,\Lambda_{eff})$$

under the inherited Lorentzian structure. In the mostly-plus signature of §2.1, the metric variation

$$(2 / \sqrt{|g|}) \cdot \delta S_{EH} / \delta g^{\mu\nu} = \kappa_{eff}^{-1}\,(G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu})$$

reproduces the LHS of the source-organized field equation (Definition 6.1) with the stated sign convention, consistent with the Bianchi-Compatible Geometry Theorem and Theorem 6.2 (conservation compatibility). This is the natural geometric action for the leading admissible term. Whether the inherited substrate dynamics derives the Einstein–Hilbert action — i.e. whether the structural emergence of $g_{\mu\nu}$ from commitment transport produces S_{EH} at the appropriate continuum limit — is a substantive open problem (subordinate to OP1).

Action for the source side

Action for the source side. The κ-sector action S_{κ} is inherited from source paper Definition 5.1. The memory-sector action S_{mem} is inherited from source paper §6.6 as a nonlocal bilocal functional. The anisotropic transport-curvature sector action $S_{\hat{Q}}$ is constructed at the effective-action level in Definition 8.2 above, with its three Wilson coefficients (a_Q, b_Q, c_Q) mapping to ($\alpha_{\hat{Q}}$, $\beta_{\hat{Q}}$, $\gamma_{\hat{Q}}$) at leading order via Proposition 8.3. The inter-sector exchange tensor $T^{(int)}_{\mu\nu}$ is fixed by divergence balance (source paper Theorem 8.3) and is promoted to a constrained variational object by the Lagrange-multiplier mechanism of §14.1; the associated effective exchange action S_{ex} enforces total conservation as an Euler–Lagrange equation.

The unified action programme

Under §14.1, the unified covariant action

$$S_{VERSF} = S_{EH} + S_{\kappa} + S_{mem} + S_{\hat{Q}} + S_{ex} + S_{matter},$$

with $T^{(int)}_{\mu\nu}$ arising as a multiplier-enforced constrained object rather than being assembled by hand, closes R4 of the source paper at the effective-action level. What remains is the substrate-level closure: deriving each sector action (S_{EH} from continuum emergence, S_{κ} from κ-field uniqueness — already substantially closed by source paper Theorem 5.2, S_{mem} from substrate refinement dynamics, $S_{\hat{Q}}$ Wilson coefficients from refinement-stable transport, and

T^(int)_μν directly from substrate-level inter-sector exchange dynamics rather than via the multiplier device of Definition 14.2, S_matter from the matter-coupling problem of OP7) from refinement dynamics, together with the substrate derivation of κ_eff and the closure-normalisation factor C_λ. The construction of this substrate-level closure is the natural next-paper problem in the dynamical layer.

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15. Limitations and Open Problems

The paper does **not** derive:

- the value of the effective gravitational coupling κ_eff (only its substrate functional form is inherited from §2.7),
- the substrate *numerical values* of the anisotropic transport-curvature Wilson coefficients (α_Q̂, β_Q̂, γ_Q̂) — their effective-action origin is supplied at §8.2 / Proposition 8.3,
- the substrate derivation of the full bilocal memory-curvature kernel K(x, x′) (only its worldline-asymptotic form is inherited),
- the substrate decomposition of Λ_eff into its geometric and source-derived contributions,
- the *substrate-level* unified covariant action — its effective-action form is supplied at §14.1 with T^(int)_μν appearing as a Lagrange-multiplier-enforced constrained completion (Proposition 14.3),
- higher-derivative Lovelock extensions in $D + 1 > 4$,
- the matter-coupling problem (how Standard-Model matter emerges from commitment dynamics),
- quantum fluctuations of the geometry.

Specific open problems:

OP1 — Substrate derivation of the unified covariant action. §14.1 closes the question of whether a conservation-compatible action exists at the *effective-action level*: the unified action S_VERSF = S_EH + S_κ + S_mem + S_Q̂ + S_ex (+ S_matter) admits a Lagrange-multiplier construction in which T^(int)_μν appears as a constrained completion rather than an imposed object (Proposition 14.3). What remains open is the *substrate derivation* of each sector action from refinement dynamics: deriving S_EH from continuum emergence, deriving the Wilson coefficients of S_Q̂ (subsuming OP4), deriving the kernel K(x, x′) of S_mem (subsuming OP3), deriving T^(int)_μν directly from substrate-level inter-sector exchange dynamics (rather than as the multiplier-enforced constrained completion of the effective layer), and deriving κ_eff (subsuming OP6). The Lagrange multiplier A^ν_ex of Definition 14.2 is a book-keeping device at the effective level; substrate derivation targets T^(int)_μν as the substantive structural object, not the multiplier used to enforce its conservation variationally. OP1 is therefore now the substrate-closure problem rather than the existence problem; the existence is closed at the effective level.

OP2 — Substrate derivation of the Einstein–Hilbert action. Does the inherited substrate dynamics produce S_EH = $(2\,\kappa_{\text{eff}})^{-1} \int \sqrt{|g|}\,(R - 2\,\Lambda_{\text{eff}})$ as the continuum-limit geometric action? The Lorentzian completion paper produces $g_{\mu\nu}$; the present paper produces the equation of motion via Theorem 4.1; closing the gap to the corresponding action is a substantive substrate-level construction subordinate to OP1.

OP3 — Substrate derivation of the memory-curvature kernel $K(x, x')$. Definition 7.2 introduces the memory-curvature operator schematically. A complete substrate derivation — analogous to source paper OP9 lifted to the geometric side — would derive $K(x, x')$ from refinement dynamics, construct the corresponding single-retarded-contour effective action, and verify that the leading derivative-expansion (Proposition 7.4) reproduces Definition 7.1 with substrate-derived coefficients.

OP4 — Substrate values of the Wilson coefficients of the anisotropic transport-curvature sector. §8.2 closes the variational-origin question for the anisotropic sector: $T^{(\hat{Q})}_{\mu\nu}$ descends from the effective action $S_{\hat{Q}}$ of Definition 8.2 via Proposition 8.3, and the three couplings $(\alpha_{\hat{Q}}, \beta_{\hat{Q}}, \gamma_{\hat{Q}})$ are revealed as *Wilson coefficients* of the unique parity-even quadratic effective action allowed by the transport-curvature algebra. What remains open is the substrate derivation of their numerical values from refinement dynamics, dimensional analysis on the substrate scale $\ell\star$, or a renormalization-group fixed-point construction. OP4 is therefore reduced from "what is the variational origin of these couplings?" (closed) to "what microscopic dynamics fix their numerical values?" — the standard Wilson-coefficient closure problem in any effective field theory.

OP5 — Independent dynamics for $\hat{Q}_{\mu\nu}$. Does the anisotropic transport-curvature tensor propagate independently, or is it purely constrained by the underlying transport sector? The present construction treats it as a source contribution to $T^{\text{eff}}_{\mu\nu}$ but does not specify whether it carries independent propagating degrees of freedom beyond those of $g_{\mu\nu}$, κ, and Ξ.

OP6 — Substrate derivation of the closure-normalisation factor C_λ. The structural form $\kappa_{\text{eff}} = 8\pi C_\lambda \hbar \xi^2 / c^3$ is inherited from the fold-density programme (§2.7). The closure-normalisation factor C_λ is reducible by the same programme to closure-loop structure, fold energetics, and $K = 7$ binary suppression architecture, but the final derivation of the exact closure premium, the hexagonal-loop weighting, and the bare anisotropy suppression remains incomplete. Closing C_λ substrate-derivatively — i.e. obtaining the numerical value of the closure-normalisation factor from refinement dynamics without recourse to the empirical matching of Theorem 10.2 — would close OP6 and constitute a structurally substantial derivation of Newton's constant from VERSF first principles.

OP7 — Matter coupling. How does standard matter stress–energy emerge as a downstream sector of commitment dynamics? Candidate routes include stable transport-localized excitations, topological commitment defects, and persistent refinement-stable closure modes. Inherited as OP7 from the source paper.

OP8 — Quantum fluctuations of the geometry. The present treatment is classical / effective. Quantum fluctuations of $g_{\mu\nu}$, κ, Ξ, and $\hat{Q}_{\mu\nu}$ — and the relationship between commitment discreteness, causal memory, and quantum geometric fluctuations — remain unresolved. Inherited as OP8 from the source paper, lifted to include geometric fluctuations.

OP9 — Substrate decomposition of Λ_{eff}. The structural decomposition $\Lambda_{eff} = \Lambda_{geom} + \Lambda_{anis} + \Lambda_{mem} + \Lambda_{int}$ of §9 identifies which sectors can contribute to the observed cosmological-constant value. Closing this to a substrate-derived numerical decomposition — and assessing whether VERSF gravity opens a structural pathway to the cosmological-constant problem of standard general relativity — is open.

OP10 — Higher-derivative / Lovelock extensions. For $D + 1 > 4$, the Lovelock theorem permits Gauss–Bonnet and higher-curvature divergence-free terms not present at the leading admissible second-order scope of Theorem 4.1. Whether the inherited substrate dynamics requires, permits, or excludes these extensions in higher D — and whether they have a natural source-side dual — is recorded here as a subordinate open problem.

16. Conclusion

The previous papers of the VERSF programme established the geometric stage (Lorentzian completion) and the source structure that lives on it (effective stress–energy from irreversible commitment flow). The present paper supplies the field equation that couples them.

The central result is the *Bianchi-Compatible Geometry Theorem* (Theorem 4.1): under the admissibility conditions (G1)–(G7) of Definition 3.1, the unique leading admissible geometric tensor is

$$E_{\mu\nu} = G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu}.$$

For $D + 1 = 4$, this reduces to the Lovelock-classified unique second-order admissible form; for higher D, the Lovelock extensions are recorded as a subordinate open problem (OP10). The uniqueness is grounded not in a modelling choice but in the same inherited structural theorems that drove the Admissible Source Uniqueness Theorem of the source paper.

The full field equation, in source-organized form, is

$$G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} = \kappa_{eff}\, T^{eff}_{\mu\nu},$$

with the equivalent geometry-organized form

$$G_{\mu\nu} + \Lambda_{eff}\, g_{\mu\nu} + M^{(\Xi)}_{\mu\nu} + Q^{(\hat{Q})}_{\mu\nu} + I^{(int)}_{\mu\nu} = \kappa_{eff}\, T^{(\kappa)}_{\mu\nu}$$

making the deviations from ordinary Einstein gravity visible as additive geometric corrections. The two forms are algebraically equivalent; the choice between them is presentational.

Two structurally distinctive features:

- **Non-Markovian memory curvature.** Geometry responds to commitment history through a nonlocal retarded functional (Definitions 7.1–7.2), with asymptotic envelope $\sim$

$\cos(m\tau + \varphi)/\tau$ inherited from the worldline-projected memory kernel (Theorem 7.3). This is the first structurally non-removable departure from instantaneous-sourcing GR.

- **Anisotropic transport-curvature corrections.** Geometry inherits directional structure from refinement-stable distinguishability transport, parametrized by three Wilson coefficients $(\alpha_{\hat{Q}}, \beta_{\hat{Q}}, \gamma_{\hat{Q}})$ of the unique parity-even quadratic effective action $S_{\hat{Q}}$ (§8.2 / Proposition 8.3). Their numerical values await substrate derivation (OP4).

The *GR Recovery Theorem* (Theorem 10.2) closes the construction at the empirical level: under the regime (R1)–(R3) — weak memory, weak anisotropy, slowly varying commitment density — the full equation reduces continuously to Einstein's equation with matching condition $\kappa_{\text{eff}} = 8\pi G$. General relativity therefore appears in the VERSF programme not as a primitive that has been replaced, but as the unique leading admissible continuum-limit structure compatible with conserved irreversible commitment transport, recovered exactly when the memory and anisotropy sectors are negligible.

The §14.1 constrained-action principle further closes the variational architecture at the effective-action level: the full set of sectors descends from a unified action S_{VERSF} in which $T^{(\text{int})}_{\mu\nu}$ appears as a Lagrange-multiplier-enforced constrained completion rather than as an imposed object. What remains is the substrate-level closure of the individual sector actions.

The interpretive consequence is structurally significant. In standard general relativity, geometry is primary and matter curves it. In VERSF, irreversible commitment is primary; the Einstein tensor emerges as the unique admissible local curvature structure compatible with conserved commitment transport on the inherited Lorentzian continuum; and ordinary general relativity is the effective leading-order limit of a deeper commitment-based dynamics.

The structural question at this layer is therefore no longer

"What couples to VERSF gravity?"

— that is settled here — nor

"Does a conservation-compatible action exist?"

— that is closed at the effective-action level by §14.1 — but rather

"Can the individual sector actions (S_{EH}, $S_{\hat{Q}}$, S_{mem}, S_{ex}), the closure-normalisation factor C_λ, and the Λ_{eff} decomposition be derived directly from substrate refinement dynamics, removing the residual Wilson-coefficient and effective-action layer?"

That is the natural direction of the next stage of the programme. With both the source side and the geometry side now fixed up to a small parameter set, with continuous GR recovery established, and with the variational architecture closed at the effective-action level, the dynamical layer has reached the same structural stage that the source layer reached at the close of the previous paper: a sharp, constrained problem rather than a from-scratch derivation.

The programme has therefore advanced through the sequence

commitment ontology → continuum emergence → transport geometry → Lorentzian completion → effective stress–energy → dynamical geometry,

with each step inheriting cleanly from its predecessor and adding structurally constrained new content. The matter-coupling problem (OP7), the substrate closure of S_VERSF (OP1, now reframed as substrate-derivation rather than existence), and the substrate derivation of the closure-normalisation factor C_λ (OP6) define the natural next stages.

A synthesis point across three branches

The present paper should also be understood as a synthesis point between three previously parallel branches of the VERSF programme:

1. **Substrate closure physics** — folds, K = 7 closure structure, finite distinguishability, commitment energetics, and the fold-density derivation of the gravitational coupling form (§2.7).
2. **Mesoscopic transport and memory** — κ-field dynamics, the substrate-derived memory kernel (§7.5), non-Markovian Volterra structure, and refinement-stable transport curvature.
3. **Continuum geometry** — Lorentzian completion, tensorial closure (§2.8), the effective stress–energy tensor of the source paper, and the Einstein-compatible dynamics of the present paper.

Earlier papers developed these layers in parallel. The present work is the first to combine, within a single dynamical framework: emergent Lorentzian geometry, the structurally constrained source tensor of the previous paper, nonlocal first-principles memory sourcing, the inherited tensorial-closure forcing of rank-2 symmetric response, the inherited substrate form of the gravitational coupling, and Einstein-compatible curvature dynamics with continuous GR recovery.

The resulting picture is structurally unified: continuum geometric curvature, gravitational response, and causal memory are all different continuum manifestations of the same underlying irreversible commitment architecture. Gravity is the large-scale response of the substrate to its own conserved commitment transport; the Einstein tensor is the unique leading admissible structural form that response takes; and ordinary general relativity is the controlled effective limit of a deeper, structurally constrained, commitment-based dynamics.