From Spinorial Source-Carriers to Fermionic Quantization in VERSF

Exchange Topology, the Finkelstein–Rubinstein Identification, and the Emergence of CAR Structure on the Coherent Entanglement Substrate

Keith Taylor *VERSF Theoretical Physics Programme Part IV of the Matter-Sector Programme Companion paper to* Spinorial Closure Transport and Fermionic Source-Carriers in VERSF

General Reader Summary

The previous paper in the matter-sector strand established that the substrate supports spin-½ structure. Once first-order admissible flow exists alongside the inherited Klein–Gordon dynamics, Clifford algebra and 4-component spinor structure are forced; once persistent loops carry orientation frames that transport on a closure-frame bundle, those frames lift (under three structural conjectures) to the double-cover group SU(2), producing the familiar spinor signature $U(2\pi) = -\mathbb{1}$ and $U(4\pi) = +\mathbb{1}$.

That paper supplied the *single-particle* spinorial structure. The present paper takes the next step.

Spin-½ behaviour is necessary but not sufficient for matter to be fermionic. Electrons, quarks, and the ordinary constituents of matter satisfy two distinct conditions: each particle individually carries half-integer spin (single-particle property), *and* identical particles exchange antisymmetrically with each other (multi-particle property). The Pauli exclusion principle is a consequence of the second condition: two identical fermions cannot occupy the same quantum state precisely because the antisymmetric multi-particle wavefunction vanishes when the labels coincide. The Standard Model encodes this as the canonical anticommutation relations (CAR):

$$\{\psi(x), \psi^\dagger(y)\} = \delta(x - y),$$

which are typically *imposed* in canonical quantisation or *derived* in axiomatic QFT via the spin-statistics theorem (which itself requires Lorentz invariance, positive energy, locality, and microcausality).

This paper asks the sharper question. Within VERSF: *what substrate-level structure forces antisymmetric exchange behaviour on the spinorial loop sector identified by the previous paper, and what operator algebra does that exchange behaviour force?*

The central claim has two parts.

First (the spin-statistics correspondence). Antisymmetric exchange is not a separately-postulated property of spinorial loops. It is *forced* by the holonomy these loops already carry.

The argument is the substrate-level analogue of the Finkelstein–Rubinstein theorem (1968), which observes that in three or more spatial dimensions, the path in configuration space that exchanges two identical particles is homotopic to a 2π rotation of one particle about the other. Under that homotopy, an internal frame attached to the rotating particle picks up the rotation's holonomy. For spinorial loops in VERSF, that holonomy is precisely $U(2\pi) = -\mathbb{1}$, inherited from the previous paper. The exchange path therefore carries a topological holonomy of -1 on spinorial loops; when the substrate supports coherent multi-loop transport over the exchange operation, this topological holonomy is physically manifest as a multi-loop wavefunction phase of -1. Antisymmetric exchange is not a representation choice — it is the topological holonomy of the exchange path, made physically observable by substrate coherence.

Second (CAR forcing, partial). Given antisymmetric multi-loop states, the natural creation-and-annihilation operator algebra on those states is *forced* to satisfy the anticommuting half of the CAR relations: $\{a^\dagger_i, a^\dagger_j\} = 0$ and $\{a_i, a_j\} = 0$. The Pauli exclusion principle then follows immediately. The full CAR algebra requires in addition the cross-relation $\{a_i, a^\dagger_j\} = \delta_{ij}$, which depends on additional Fock-construction work and is deferred.

Two structural layers. The construction is presented in two layers. The *topological-derivational core* (Theorems 1, 2, 3, Proposition 4, Corollary 6) operates on abstract configuration-space and frame-bundle structures, depending only on inheritances from the previous paper (Clifford structure, SU(2) lift, spinorial-sector identification, $K=7 \rightarrow d=3$) plus the classical Finkelstein–Rubinstein identification. These results are topological theorems; they do not require substrate realisation to be true. The *physical-realisation layer* (Corollary 1', Proposition 5) operates on the coherent entanglement substrate of the broader VERSF programme — a physical medium with finite coherence scale ξ, finite spinorial response time τ_s, and persistent transport channels — and specifies the regime in which the topological results are physically manifest as observable holonomy phases, plus the dynamical stability that makes persistent loops physically long-lived. Both layers are required for a complete account, but they are structurally distinct; the present paper keeps them explicitly separate to make the inheritance structure clean for the next paper (Fock construction).

The substrate is treated throughout the physical-realisation layer as an *effective coherence medium*, not a preferred-frame mechanical ether — the distinction inherited from the broader programme (*When Space Itself Has Mass*) is essential.

Scope. The paper does not derive: the full CAR algebra (cross-relation); renormalised fermionic QFT; the Standard Model fermion spectrum; the spin-statistics theorem of axiomatic QFT in full generality. What it does supply is the substrate-level spin-statistics correspondence at the topological level, the anticommuting half of the CAR relations, Pauli exclusion as immediate corollary, and the explicit physical realisation of these results on the coherent entanglement substrate with stated regime of applicability.

Abstract

The preceding spinorial paper established that first-order admissible closure flow on the persistent sector forces Clifford-compatible spinorial structure (the standard Dirac operator, $\mathbb{C}^4$), while closure-orientation transport on B(P) supplies — conditional on C1–C3 — an SU(2) lift with persistent loops decomposing into trivial-holonomy ($U(2\pi) = +\mathbb{1}$) and spinorial ($U(2\pi) = -\mathbb{1}$) sectors. The dominant remaining matter-sector gap is *fermionic quantisation*.

This paper supplies the substrate-level forcing arguments, organised in two distinct structural layers: a *topological-derivational core* establishing the results at the abstract configuration-space level, and a *physical-realisation layer* embedding them in the coherent entanglement substrate. The two layers are kept explicitly separate to make downstream inheritance clean; the "at the topological level" qualifier below marks results in the topological-core layer.

Topological-derivational core:

- **Theorem 1 (F–R exchange holonomy, topological core).** Conditional on (a) the previous paper's Theorem 4, (b) C1–C3, and (c) K=7 → d=3: odd exchange transport on the spinorial sector accumulates orientation-frame holonomy $U(2\pi) = -\mathbb{1}$ at the topological level.
- **Theorem 2 (Antisymmetric sector is the only available sector).** Spinorial multi-loop wavefunctions are necessarily antisymmetric; the symmetric sector on the spinorial side is empty.
- **Theorem 3 (Substrate-level spin-statistics correspondence).** Spinorial loops carry antisymmetric exchange, trivial-holonomy loops carry symmetric exchange, forced by closure-frame holonomy without separate appeal to Lorentz invariance, positive energy, or microcausality.
- **Proposition 4 (Anticommuting half of CAR).** $\{a^\dagger_i, a^\dagger_j\} = 0$ and $\{a_i, a_j\} = 0$ forced by antisymmetry + Fock-density; cross-relation deferred.
- **Corollary 6 (Pauli exclusion).** $a^\dagger(\mathcal{C})^2 = 0$, immediate from Proposition 4.

Physical-realisation layer:

- **Corollary 1' (Physical realisation).** Conditional on the coherence condition (spatial extent $\lesssim \xi$, timescale $\lesssim \tau_s$), Theorem 1's topological holonomy is physically manifest on the coherent substrate as a multi-loop wavefunction phase -1.
- **Proposition 5 (Coherent persistent transport stability — inherited).** From the superfluid-transport paper: coherent entanglement-lattice transport stabilises persistent topological winding sectors on timescales $\gg \tau_s$.

Explicit clarification of scope. The present paper derives the substrate-level spin-statistics correspondence and the anticommuting half of CAR on the spinorial sector at the topological level, with Pauli exclusion as immediate corollary. Physical realisation is supplied by Corollary 1' under the coherence condition. The cross-relation of the full CAR algebra, Fock-space construction, the full spin-statistics theorem of axiomatic QFT, renormalised fermionic QFT, full parastatistics exclusion (conditional on internal-structure

trivialisation), and the Standard Model fermion spectrum are not derived here and are explicitly identified as next-stage deliverables.

Contents

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1. Setting: The Remaining Fermionic Gap After the Spinorial Paper

The preceding spinorial paper (Part III) established four structural results:

1. **Clifford-compatible spinorial structure** is algebraically forced on the persistent sector by first-order admissible closure flow + KG^2 consistency (Theorem 1 there, conditional on κ-field uniqueness and Schrödinger→Dirac inheritances).
2. **The standard Dirac operator** $i\gamma^\mu \partial_\mu - m$ is the algebraic form of first-order admissible flow.
3. **Closure-orientation frames transport on B(P)** with SU(2) lift (SI, conditional on C1–C3).
4. **Persistent oriented loops decompose into trivial-holonomy and spinorial sectors** with $U(2\pi) = -\mathbb{1}$ and $U(4\pi) = +\mathbb{1}$ (Theorem 4 there, conditional on C1–C3).

These characterise the *single-loop* spinorial behaviour. What they do not establish is the *multi-loop* behaviour: how identical spinorial loops exchange, what operator algebra acts on multi-loop states, and whether Pauli exclusion holds at substrate level. This is the gap addressed here.

1.1 Two-layer methodology

The construction is organised in two structural layers, kept explicitly separate to clarify the inheritance structure for downstream papers.

The topological-derivational core. The F–R exchange-holonomy argument, the antisymmetric-sector forcing, the spin-statistics correspondence, the anticommuting half of CAR, and Pauli exclusion are all derived at the level of abstract configuration-space topology and frame-bundle structure. The inheritances are entirely topological: the previous paper's Theorem 4, the SU(2) lift (SI under C1–C3), the K=7 → d=3 chain, and the classical F–R identification. The topological-core results do not require substrate realisation to be true — they hold as topological theorems in the same sense that the classical F–R theorem holds for abstract point particles in $\mathbb{R}^3$ without any substrate.

The physical-realisation layer. The coherent entanglement substrate of the broader VERSF programme — entanglement-lattice papers, superfluid-transport paper, *When Space Itself Has Mass* — supplies the physical medium on which the topological structures are realised. This layer specifies the regime in which the topological-core results are physically manifest as observable holonomy phases (via the coherence condition: spatial extent $\lesssim \xi$, timescale $\lesssim \tau_s$), and supplies the dynamical stability that makes persistent loops physically long-lived (via Proposition 5, inherited from the superfluid-transport paper).

The two layers are structurally distinct. The topological-core results hold abstractly; the physical-realisation results state where and how they are physically manifest. Both are required for a complete account, but conflating them creates confusion downstream — particularly for the Fock-construction paper of §14 item 1, which inherits the topological core unconditionally but invokes the physical-realisation layer only where physical observability matters.

1.2 Structural ingredients available from inheritance

For the topological core:

- Three or more spatial dimensions (K=7 → d=3 chain, previous paper §8). Load-bearing: in d=2, $\pi_1(\mathcal{C}_n(P))$ is the braid group, exchange statistics need not be ±1, and the F–R argument as stated does not apply.
- Spinorial orientation frames on loops (previous paper §5). Without frames, exchange is a representation choice; with frames, F–R applies.
- SU(2) lift of frame transport (previous paper SI, conditional on C1–C3). Supplies the $U(2\pi) = -\mathbb{1}$ holonomy.
- Spinorial / trivial-holonomy decomposition (previous paper Theorem 4). Gives the correspondence in spin-statistics.

For the physical-realisation layer:

- Coherent entanglement substrate (entanglement-lattice papers): scales ξ, τ_s, m_s.
- Persistent transport stability (superfluid-transport paper): Proposition 5 (inherited).
- Effective-medium framing (*When Space Itself Has Mass*): substrate ≠ mechanical ether.

1.3 Source-carrier discipline

Throughout, "fermionic source-carrier" means substrate-level multi-loop spinorial transport structure satisfying antisymmetric exchange, not "physical electron" or "Standard Model fermion." Promotion to physical fermions requires species decomposition (§14 item 5), full Fock quantisation (§14 item 1), and the Dirac-emergence theorem of the previous paper's §22 item 1.

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2. Notation and Conventions

Conventions of the previous spinorial paper (§2 there) apply, extended for multi-loop structure and the substrate-physics inheritances.

Persistent transport manifold. P, as in the previous paper §6.3.

Coherent entanglement substrate (physical-realisation layer). The physical substrate on which P is realised, treated as an effective coherence medium per §3.11. Characteristic scales:

- ξ: coherence scale of substrate transport.
- τ_s: spinorial response time.
- m_s: effective substrate inertial scale.

These are inherited from §3.9 and appear only in the physical-realisation layer.

Notation note on "spinorial transport". In topological-core statements, "spinorial loops" refers to abstract loops. In physical-realisation statements, "spinorial transport modes of the coherent substrate" refers to the substrate-physical realisation. Both phrases refer to the same structural objects under different layer perspectives.

Single oriented commitment loop. $\mathcal{C}_i$, satisfying (L1)–(L4) of Microscopic Origin Definition 2, equipped with closure-orientation frame $F_i(x)$ satisfying OA1–OA4. Each loop carries q_i, u_i^μ, $w_i \in \mathbb{Z}$, $F_i(x)$, and $\psi_i \in \mathbb{C}^4$.

Notation note. $\mathcal{C}_i$ for individual loops, $\mathcal{C}_n(P)$ for the configuration space of n loops; subscript-vs-argument disambiguates.

Spinorial sector. Loops in the $U(2\pi) = -\mathbb{1}$ component (previous paper Theorem 4). Notation: $\mathcal{C}_i$^spin where needed.

Trivial-holonomy sector. Loops in the $U(2\pi) = +\mathbb{1}$ component. Notation: $\mathcal{C}_i$^triv where needed.

Multi-loop configuration space.

$\mathcal{C}_n(P) = (P^n - \Delta) / S_n$.

Configuration-space fundamental group. For $d \geq 3$:

$\pi_1(\mathcal{C}_n(P)) \cong S_n \ltimes \pi_1(P)^n$_quot.

The F–R argument operates on the S_n-factor.

Multi-loop state. $\Psi(\mathcal{C}_1, ..., \mathcal{C}_n)$ on $\mathcal{C}_n(P)$, valued in $(\mathbb{C}^4)$^⊗n.

Exchange operation. E_{12}: $(\mathcal{C}_1, \mathcal{C}_2) \mapsto (\mathcal{C}_2, \mathcal{C}_1)$.

Coherence condition (physical-realisation layer). Spatial extent $\lesssim \xi$, exchange timescale $\lesssim \tau_s$. Required for Corollary 1', not for Theorem 1.

Creation/annihilation operators. $a^\dagger(\mathcal{C})$, $a(\mathcal{C})$ on the antisymmetric multi-loop sector. Full definition with vacuum and one-particle Hilbert space deferred to §14 item 1.

F–R identification. The homotopy class in the S_n-factor of $\pi_1(\mathcal{C}_2(\mathbb{R}^d)) \cong \mathbb{Z}_2$ ($d \geq 3$) of the two-particle exchange path is identified with the homotopy class in $\pi_1(SO(3)) \cong \mathbb{Z}_2$ of a 2π rotation.

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3. Structural Dependencies

Items 3.1–3.4: topological inheritances from the previous paper (used in topological-core layer). Items 3.5–3.7: further topological inheritances. Item 3.8: classical F–R theorem. Items 3.9–3.11: substrate-physics inheritances (used in physical-realisation layer only).

3.1 Clifford-compatible spinorial structure (previous paper, Theorem 1)

Any first-order admissible closure flow operator on the persistent sector takes the form of the standard Dirac operator $(i\gamma^\mu \partial_\mu - m)$ acting on $\mathbb{C}^4$. Inheritance: κ-field uniqueness Theorem U + Schrödinger→Dirac Theorems E and F.

3.2 Closure-orientation frames and B(P) (previous paper, §5)

Persistent oriented loops carry closure-orientation frames F(x) satisfying OA1–OA4, transporting on B(P) under the closure-orientation connection ω_μ.

3.3 SU(2) lift of frame transport (previous paper, SI)

Conditional on C1–C3. Admissible transport on the persistent oriented sector lifts to SU(2).

3.4 Persistent-sector decomposition (previous paper, Theorem 4)

Conditional on C1–C3. Persistent oriented loops decompose into trivial-holonomy ($U(2\pi) = +\mathbb{1}$) and spinorial ($U(2\pi) = -\mathbb{1}$) sectors. Spinorial sectors are persistence-stable.

3.5 d ≥ 3 (K=7 chain, previous paper §8)

K=7 supplies three spatial transport generators, giving d=3. Load-bearing for the F–R argument: in $d \geq 3$, $\pi_1(\mathcal{C}_n(P))$'s S_n-factor is S_n; in d=2 it is the braid group B_n.

3.6 Microscopic Origin loop ontology

J^μ = ρ_pers u^μ = Π_pers C^μ. Persistent loops are substrate-level source-carriers. SA1–SA5 constrain substrate current structure.

3.7 Refinement-persistence framework

Refinement coarsening δ^* and refinement-stability of holonomy classes inherits to multi-loop transport.

3.8 The classical Finkelstein–Rubinstein theorem

(Finkelstein–Rubinstein 1968; review: Leinaas–Myrheim 1977.) For two identical point particles in $\mathbb{R}^d$ with internal orientation frames, $d \geq 3$:

- $\pi_1(\mathcal{C}_2(\mathbb{R}^d)) \cong \mathbb{Z}_2$.
- The two-particle exchange path is homotopic in $\mathcal{C}_2(\mathbb{R}^d)$ to a 2π rotation of one particle about a fixed axis (other particle held fixed).
- Frame holonomy along the exchange path equals frame holonomy along a 2π rotation. For spin-½, this is $-\mathbb{1}$.

Required features: (i) $d \geq 3$ (in d=2 the configuration-space π_1 is $\mathbb{Z}$, not $\mathbb{Z}_2$); (ii) internal orientation frames; (iii) continuous SO(3) action. All present in the VERSF spinorial-sector setting.

3.9 The entanglement-lattice substrate (entanglement-lattice papers)

The broader VERSF programme establishes the coherent entanglement lattice as the physical substrate realising P. Inherited scales:

- ξ: coherence scale. Coherent transport well-defined below ξ.
- $\boldsymbol{\tau_s}$: spinorial response time. Coherent spinorial transport completes within τ_s.
- $\mathbf{m_s}$: effective substrate inertial scale.

These are inherited; not derived here. Applied in the physical-realisation layer (Corollary 1') only.

3.10 Superfluid persistent transport stability (superfluid-transport paper)

The superfluid-transport paper establishes that coherent entanglement-lattice transport stabilises persistent topological winding sectors on timescales $\gg \tau_s$. Persistent loops are dynamically stable substrate modes when physically realised.

Logical role. This inheritance is *grounding*, not *load-bearing for the topological-core derivations*. The topological-core results (Theorems 1, 2, 3, Proposition 4, Corollary 6) operate on persistent loops as structural carriers; whether those carriers are dynamically stable in a substrate-physics sense is a separate question the topological arguments do not require. Proposition 5 supplies the answer in the physical-realisation layer: when the topological-core results are physically realised on the coherent substrate, the persistent loops are long-lived substrate modes, not idealisations. The §11 hierarchy distinguishes the logical chain (Proposition 5 not required) from the physical-grounding chain (essential).

3.11 The effective-medium framing (*When Space Itself Has Mass*)

When Space Itself Has Mass establishes the substrate as an effective coherence medium, not a preferred-frame mechanical ether. Verification of the no-Lorentz-violation, no-preferred-frame, no-pathology-related-to-ether-theories properties is part of that paper; the present paper inherits these as established results, not as claims established here. Load-bearing for the physical-realisation layer.

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4. Multi-Loop Configuration Space $\mathcal{C}_n(P)$

4.1 Definition and persistence inheritance

Definition 1. For n distinct persistent oriented commitment loops on P:

$\mathcal{C}_n(P) = (P^n - \Delta) / S_n$.

Inherits persistence and refinement-stability from the single-loop case.

4.2 Restriction to the spinorial sector

$\mathcal{C}_n$^spin(P) = (P_spin^n − Δ) / S_n,

with P_spin the projection onto spinorial-sector loops. The trivial-holonomy multi-loop configuration space $\mathcal{C}_n$^triv(P) is treated in parallel via §8.

4.3 Topology of $\mathcal{C}_n(P)$: the S_n-factor decomposition

For d ≥ 3 (VERSF case from K=7):

$\pi_1(\mathcal{C}_n(P), *) \cong S_n \ltimes \pi_1(P)^n$_quot.

The S_n-factor is the symmetric group of label exchanges; the second factor is the base-holonomy factor from non-contractible cycles of P. For n=2: the S_n-factor is $\mathbb{Z}_2$.

The F–R argument operates on the S_n-factor. Single-loop holonomies in the base-holonomy factor are independent of the exchange operation. The factor decomposition allows the F–R argument to operate independently of $\pi_1(P)$.

4.4 The exchange path

For n=2, γ_exch ∈ $\pi_1(\mathcal{C}_2(P))$ is the homotopy class of any continuous path swapping $(x_1, x_2) \leftrightarrow (x_2, x_1)$. Its projection onto the S_n-factor is the non-identity element of S_2.

Multi-loop transport along γ_exch produces frame holonomy via the closure-orientation connection ω_μ on the multi-loop frame bundle. The central question of §6 is: what is this exchange holonomy on the spinorial sector? Theorem 1 answers this at the topological level; Corollary 1' translates to physical-substrate manifestation.

4A. Physical-Realisation Layer: The Coherent Entanglement Substrate

This section establishes the physical-realisation layer on which the topological-core results of §§6–10 are physically manifested. The topological-core results hold abstractly; this section specifies the regime of physical observability and supplies dynamical-stability grounding.

4A.1 Loops as spinorial transport modes of the coherent substrate

When physically realised, the persistent oriented loops $\mathcal{C}_i$ correspond to *spinorial transport modes of the coherent entanglement substrate*. P is realised as the coherent entanglement lattice viewed as a continuum; a loop $\mathcal{C}_i$ is a coherence-supported transport excitation — a localised, coherent persistent winding pattern in the lattice's transport channels.

Under this realisation, the persistence of loops acquires direct physical content: persistent loops are coherent transport modes that survive on timescales $\gg \tau_s$ via Proposition 5.

4A.2 Configuration space on the coherent medium

The multi-loop configuration space $\mathcal{C}_n(P)$ is realised as the space of multi-mode coherent transport configurations on the entanglement substrate. The S_n-quotient is physically the statement that coherent transport modes of identical character cannot be physically labelled at the substrate level.

The frame bundle's orientation frames are realised as local coherence-direction structures of the substrate's persistent-transport channels.

4A.3 The coherence condition

The *physical-substrate manifestation* of frame-bundle holonomy along γ_exch requires coherence preservation of the multi-loop frame configuration over the spatial and temporal extent of the exchange. By §3.9, coherence holds over spatial scales $\lesssim \xi$ and timescales $\lesssim \tau_s$. Outside this regime, decoherence breaks the physical observability of frame holonomy, even though the topological holonomy class itself (a property of π_1) remains well-defined.

Coherence condition. The topological exchange holonomy of Theorem 1 is *physically realised* as an observable multi-loop frame-bundle phase on the coherent entanglement substrate when the exchange transport's spatial extent $\lesssim \xi$ and timescale $\lesssim \tau_s$. Outside this regime, Theorem 1 still holds at the topological level, but the substrate does not physically realise the holonomy as a coherent multi-loop wavefunction phase.

For physical fermion pairs (electrons, quarks) at observable laboratory scales, the regime is comfortably satisfied: ξ is the relevant coherence scale for the substrate-level mode, and physical particle-pair exchanges occur on timescales far below τ_s.[^1]

[^1]: Specific scale estimates for ξ and τ_s are developed in the entanglement-lattice paper; the relationship to laboratory-scale fermion-pair observations is treated there. The present paper requires only that the regime is satisfied for the systems to which the F–R argument is physically applied.

4A.4 Dynamical stability of persistent spinorial loops

Proposition 5 (Coherent persistent transport stability — inherited). *Inheriting from the superfluid-transport paper, §3.10.* Coherent entanglement-lattice transport stabilises persistent topological winding sectors on timescales $\gg \tau_s$. Persistent spinorial loops are therefore dynamically stable substrate carriers.

Logical role. Proposition 5 grounds the physical realisation by ensuring that the persistent loops on which the topological-core results operate correspond to physically long-lived substrate carriers. *It is not load-bearing for the topological-core derivations:* the topological-core results operate on persistent loops as structural carriers; the topological-level arguments go through whether or not the carriers are dynamically stable in a substrate-physics sense. Without Proposition 5, the topological-core results remain valid as topological theorems; what they lose is the assurance that the structural carriers are physically realised as long-lived substrate modes.

4A.5 Effective-medium framing

The substrate-physics realisation is read under the effective-medium framing of §3.11. The coherent entanglement substrate is an effective coherence medium whose emergent dynamics realise the abstract topological structures; the no-Lorentz-violation properties are inherited from *When Space Itself Has Mass*, not re-established here.

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5. Why Exchange Topology is Non-Trivial

5.1 Without orientation frames: only representation choice

Without orientation frames, the wavefunction Ψ is a single-valued function on $\mathcal{C}_n(P)$, the exchange operator E_{ij} has eigenvalues ±1, and Ψ decomposes into symmetric and antisymmetric sectors as a S_n-representation choice. The symmetric vs antisymmetric assignment is *not forced* in this case — both are mathematically available, and additional structure is needed to select one. This is why standard QFT spin-statistics requires Lorentz invariance, positive energy, locality, and microcausality.

5.2 With orientation frames: exchange becomes physical transport

In the VERSF case, loops carry orientation frames. The configuration space has an associated frame bundle whose typical fibre is the n-fold product of single-loop fibres modulo S_n. The closure-orientation connection ω_μ extends to a connection on this multi-loop frame bundle.

The wavefunction is a *section* of this associated frame bundle over $\mathcal{C}_n(P)$ — not a single-valued function in the naive sense, since it acquires the bundle's holonomy under closed-path transport:

Ψ(transported configuration) = U(γ) · Ψ(initial configuration).

The S_n-quotient identification ensures gauge-invariance under label-permutation. The exchange operation is a closed path under the S_n-quotient.

This shifts the question from "choose symmetric or antisymmetric" to "what is U(γ_exch)?" The F–R identification answers: U(γ_exch) on spinorial loops equals U(2π_spin) = −𝟙. The forcing is by geometry of the frame bundle, not by representation choice.

5.3 Why d ≥ 3 is required

In d=2, $\pi_1(\mathcal{C}_n(\mathbb{R}^2) - \Delta)/S_n$ is the braid group B_n, allowing anyonic representations $U(\sigma_i)$ = e^{iθ}. The F–R identification requires $\pi_1(\mathcal{C}_2)$|_{S_n-factor} ≅ $\mathbb{Z}_2$ ≅ $\pi_1(SO(3))$, hence d ≥ 3. The K=7 → d=3 inheritance supplies this.

Physical reinforcement. d=3 is also, by inheritance from the entanglement-lattice paper, the dimension in which stable large-scale lattice percolation and coherent transport networks naturally emerge. Mathematical and physical d=3 dependencies are independent but mutually reinforcing.

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6. Theorem 1 (Topological Core) and Corollary 1' (Physical Realisation)

This section presents the central result in two layers: Theorem 1 establishes the topological-core result; Corollary 1' establishes the physical-realisation on the coherent substrate.

6.1 Theorem 1 — Topological-core statement

Theorem 1 (Finkelstein–Rubinstein exchange holonomy, topological core). Let $\mathcal{C}_1$, $\mathcal{C}_2$ be two distinct persistent oriented commitment loops in the spinorial sector ($U(2\pi) = -\mathbb{1}$). Let γ_exch ∈ $\pi_1(\mathcal{C}_2(P))$ be the exchange path of §4.4. *Conditional on:*

- *(a) the previous paper's Theorem 4 (spinorial-sector identification);*
- *(b) C1–C3 (via SI, supplying the SU(2) lift);*
- *(c) the K=7 → d=3 inheritance.*

Multi-loop frame transport along γ_exch generates orientation-frame holonomy on the spinorial sector equal to

U_exch = U(2π_spin) = $-\mathbb{1}$

at the topological level.

The exchange holonomy is a topological invariant of the S_n-factor of $\pi_1(\mathcal{C}_2(P))$ combined with the spinorial-sector frame-bundle representation, not a representation choice and not a property of any specific physical substrate realisation.

6.2 Proof of Theorem 1

Step 1 — Configuration-space homotopy reduction within the S_n-factor. γ_exch projects onto the non-trivial element of the S_n-factor of $\pi_1(\mathcal{C}_2(P)) \cong S_2 \ltimes \pi_1(P)^2$ _quot. The F–R identification operates purely on this S_n-factor.

By the classical F–R theorem (§3.8), in $d \geq 3$, any local realisation of γ_exch within a contractible neighbourhood $U \subset P$ containing both loop centres admits a continuous homotopy in $\mathcal{C}_2(U)$ ending at

γ_FR: $t \mapsto (R(2\pi t) \cdot x_1, x_2)$, $t \in [0, 1]$, with $R(2\pi t) \in SO(3)$ rotating about a chosen axis through $2\pi t$, x_2 held fixed, and x_1 undergoing rigid spatial rotation. This local homotopy descends to the S_n-factor of $\pi_1(\mathcal{C}_2(P))$: γ_exch and γ_FR are equal in the S_n-factor.

The base-holonomy factor $\pi_1(P)^2$ is unchanged because it acts on a different factor of the multi-loop frame bundle than the exchange operation; the SO(3) rotation structure is supplied by the triangular closure programme and C1, operating on the frame fibre.

Step 2 — Frame holonomy along γ_FR equals U(2π) on the spinorial sector. Along γ_FR (rigid spatial rotation of loop 1, frames transporting under ω_μ), the frame $F_1(x)$ transports along the closed SO(3) loop $\{R(2\pi t) : t \in [0,1]\}$ of homotopy class equal to the generator of $\pi_1(SO(3)) \cong \mathbb{Z}_2$.

Choice of realisation. The rigid spatial rotation (i), the alternative realisation with frame F(x) rotated directly (ii), and any continuous interpolation (iii) all project to the same S_n-factor homotopy class. By parallel-transport homotopy-invariance, the frame holonomy is identical across choices; we choose (i) for concreteness.

By C2 (single-valuedness of admissible transport), this SO(3) loop lifts uniquely to a path in SU(2) from $\mathbb{1}_{SU(2)}$ to $-\mathbb{1}_{SU(2)}$. Acting on the spinorial-sector frame (Theorem 4 of the previous paper: $U(2\pi) = -\mathbb{1}$):

$U(\gamma\_FR)|\_spin = U(2\pi\_spin) = -\mathbb{1}$.

Step 3 — Conclusion at the topological level. Parallel transport is homotopy-invariant. The S_n-factor contribution to multi-loop frame holonomy along γ_exch equals the S_n-factor contribution along γ_FR:

$U\_exch|\_\{S_n\text{-factor}\} = U(\gamma\_FR)|\_spin = -\mathbb{1}$.

Single-loop base-holonomy contributions factor out and are unchanged. The net exchange holonomy on the spinorial sector at the topological level is

$U\_exch = -\mathbb{1}$. ■

6.3 Corollary 1' — Physical realisation on the coherent substrate

Corollary 1' (Physical realisation of Theorem 1). Conditional on the coherence condition (§4A.3: spatial extent $\lesssim \xi$, timescale $\lesssim \tau_s$, inherited via §3.9), Theorem 1's topological exchange holonomy is physically realised as multi-loop frame-bundle holonomy on the coherent entanglement substrate, manifest as a multi-loop wavefunction phase

$\Psi(\mathcal{C}_2, \mathcal{C}_1) = -\Psi(\mathcal{C}_1, \mathcal{C}_2)$

on the physically-realised spinorial multi-loop sector.

6.4 Proof of Corollary 1'

By Theorem 1, the topological exchange holonomy on the spinorial sector is U_exch = −𝟙 (at the level of configuration-space frame-bundle topology).

For this topological holonomy to be observable as a multi-loop wavefunction phase, the substrate must support coherent multi-loop frame transport over the exchange operation's spatial extent and timescale. By §3.9, coherence is maintained over scales $\lesssim \xi$ and timescales $\lesssim \tau_s$. Within this regime, the substrate's frame bundle supports coherent transport, and the topological holonomy is physically manifested:

$\Psi(\mathcal{C}_2, \mathcal{C}_1) = U\_exch \cdot \Psi(\mathcal{C}_1, \mathcal{C}_2) = -\Psi(\mathcal{C}_1, \mathcal{C}_2)$.

Outside this regime, decoherence breaks coherent transport over the exchange path; the topological holonomy class remains the correct topological-level answer (Theorem 1 still holds), but it is not manifest as an observable substrate-level wavefunction phase. ■

6.5 Structural reading of the two-layer separation

Theorem 1 is the topological-core result. It depends only on topological-level inheritances: Theorem 4 of the previous paper, the SU(2) lift (SI under C1–C3), K=7 → d=3, and the classical F–R identification. It does *not* depend on the coherent entanglement substrate or any substrate-physics inheritance. It is a topological theorem in the same sense the classical F–R theorem is.

Corollary 1' is the physical-realisation result. It depends on Theorem 1 *plus* the substrate-physics inheritance (the coherence condition from §3.9). It states the regime in which Theorem 1's topological holonomy is physically observable.

Why the separation matters. Downstream constructions inherit the two layers differently. The Fock-construction paper (§14 item 1) inherits Theorem 1 unconditionally for the cross-relation derivation operating at the algebraic level; physical-observability claims in that paper inherit Corollary 1' with its coherence condition. The present separation preserves the structural cleanliness of the F–R argument while making the substrate-physics inheritance honest.

Reading of the coherence condition. The coherence condition is *not* a precondition for U_exch = −𝟙 to hold at the topological level — Theorem 1 establishes this unconditionally on its topological inheritances. The coherence condition is a precondition for the topological holonomy to be *physically manifest as an observable multi-loop wavefunction phase* on the coherent substrate.

6.6 Conditionality summary

Theorem 1 (topological core):

- C1 (SO(3) structure of B(P)) — load-bearing for both Step 1 (defining 2π rotation in SO(3)) and Step 2 (SU(2) lift).

- C2 (single-valuedness of admissible transport) — used in Step 2 for the unique SU(2) lift.
- C3 (persistent sector selects connected double cover) — used in Step 2.
- K=7 → d=3 inheritance — used in Step 1 (S_n-factor is S_n).
- κ-field uniqueness and Schrödinger→Dirac inheritances — via the previous paper's Theorem 1.

Theorem 1 does *not* depend on §3.9–§3.11.

Corollary 1' (physical realisation):

- All of Theorem 1's conditionalities.
- The coherence condition (§4A.3, inherited via §3.9).

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7. Theorem 2 — Antisymmetric Sector is the Only Available Sector

7.1 The theorem

Theorem 2. Given Theorem 1 (topological core), persistent multi-loop wavefunctions on the spinorial sector are necessarily antisymmetric:

$\Psi(\mathcal{C}_2, \mathcal{C}_1) = -\,\Psi(\mathcal{C}_1, \mathcal{C}_2)$.

The symmetric multi-loop sector on the spinorial side is *empty*.

Theorem 2 is established at the topological-core level. Its physical-realisation reading follows from Corollary 1' on the coherence regime.

7.2 Proof

By Theorem 1, multi-loop frame transport along γ_exch produces topological frame-bundle holonomy $-\mathbb{1}$ on the spinorial sector. As established in §5.2, Ψ is a section of the associated frame bundle over $\mathcal{C}_n(P)$ — it acquires the bundle's holonomy phase under closed-path transport:

Ψ(transported) = U(γ) · Ψ(initial).

The S_n-quotient identification makes Ψ gauge-invariant under label-permutation. Applying to γ_exch:

$\Psi(\mathcal{C}_2, \mathcal{C}_1) = U(\gamma\_exch)|\_spin \cdot \Psi(\mathcal{C}_1, \mathcal{C}_2) = -\,\Psi(\mathcal{C}_1, \mathcal{C}_2)$.

Symmetric sector empty. Suppose Ψ_sym satisfies $\Psi\_sym(\mathcal{C}_2, \mathcal{C}_1) = +\,\Psi\_sym(\mathcal{C}_1, \mathcal{C}_2)$. Combined with the exchange-holonomy result:

$\Psi\_sym(\mathcal{C}_2, \mathcal{C}_1) = + \Psi\_sym(\mathcal{C}_1, \mathcal{C}_2)$ and $\Psi\_sym(\mathcal{C}_2, \mathcal{C}_1) = - \Psi\_sym(\mathcal{C}_1, \mathcal{C}_2)$,

hence $\Psi\_sym \equiv 0$. The symmetric sector on the spinorial side contains only the zero vector. ■

7.3 Structural reading

Theorem 2 is the exclusion-style strengthening of Theorem 1. In standard QFT, symmetric and antisymmetric sectors are both mathematically available and spin-statistics selects one; in VERSF, the symmetric sector on the spinorial side is structurally absent. This is genuinely a forcing result at the topological level.

7.4 What Theorem 2 does not establish

- Existence of non-trivial antisymmetric multi-loop wavefunctions (requires §14 item 1 Fock construction).
- Operator-algebraic anticommutation (partially supplied by Proposition 4; full CAR requires §14 item 1).
- That the antisymmetric sector exhausts the spinorial multi-loop state space. *Cross-reference: this exhaustion question is the same gap as the parastatistics-exclusion question of §11.1.5, viewed from a different angle. Both depend on whether $\mathcal{H}_1$ carries additional S_n-non-trivial internal structure beyond $\mathbb{C}^4$; both are conditional on species decomposition (§14 item 5).*

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8. Theorem 3 — Substrate-Level Spin-Statistics Correspondence

8.1 The theorem

Theorem 3. Combining the single-loop spinorial / trivial-holonomy decomposition of the previous paper with Theorem 2 above:

- Persistent **spinorial** loops ($U(2\pi) = -\mathbb{1}$) carry *antisymmetric* multi-loop exchange.
- Persistent **trivial-holonomy** loops ($U(2\pi) = +\mathbb{1}$) carry *symmetric* multi-loop exchange.

The substrate-level analogue of spin-statistics holds on the persistent sector at the topological level, forced by closure-frame holonomy without separate appeal to Lorentz invariance, positive energy, or microcausality. Physical-realisation on the coherent substrate follows from Corollary 1' on the coherence regime.

8.2 Proof

Spinorial-side antisymmetry is Theorem 2. For the trivial-holonomy side: the F–R identification of Theorem 1, Step 1, applies as a homotopy in $d \geq 3$ regardless of sector — it is a topological statement about the S_n-factor of $\pi_1(\mathcal{C}_2(P))$.

Applying Step 2 on the trivial-holonomy side: the SO(3) loop lifts to a path in SU(2) from $\mathbb{1}_{SU(2)}$ to $-\mathbb{1}_{SU(2)}$. *This lifted path is the same in both spinorial and trivial-holonomy sectors* — the lift is determined by the SO(3) → SU(2) double-cover topology, which is fixed. What differs is which representation the multi-loop wavefunction transforms under: the trivial-holonomy sector carries the trivial / vector representation, where $-\mathbb{1}_{SU(2)}$ acts as $+\mathbb{1}$; the spinorial sector carries the fundamental representation, where $-\mathbb{1}_{SU(2)}$ acts as $-\mathbb{1}$.

Frame holonomy along γ_{FR} in the trivial-holonomy sector:

$$U(\gamma_{FR})|_{triv} = +\mathbb{1}.$$

By the same parallel-transport argument as Theorem 2:

$$\Psi_{triv}(\mathcal{C}_2, \mathcal{C}_1) = (+\mathbb{1}) \cdot \Psi_{triv}(\mathcal{C}_1, \mathcal{C}_2) = + \Psi_{triv}(\mathcal{C}_1, \mathcal{C}_2),$$

symmetric exchange. The antisymmetric sector on the trivial-holonomy side is empty by the parallel argument. ■

8.3 Structural reading

Theorem 3 ties the previous paper's single-loop decomposition to the present paper's multi-loop decomposition.

Single-loop sector	**U(2π)**	**Multi-loop exchange**	**Statistics**
Spinorial	$-\mathbb{1}$	Antisymmetric	Fermionic
Trivial-holonomy	$+\mathbb{1}$	Symmetric	Bosonic

These are the *same* decomposition viewed through single-loop vs multi-loop lenses, with the F–R identification supplying the bridge.

Superfluid-winding connection for the bosonic sector. The trivial-holonomy / bosonic sector connects naturally to superfluid persistent-current structures from §3.10. Standard U(1) superfluid winding $\oint \nabla\theta \cdot dl = 2\pi n$ provides the relevant topological structure for the bosonic side: bosonic persistent currents carry integer-quantised winding, consistent with the symmetric multi-loop exchange of Theorem 3. *Important caveat:* this connection applies specifically to the trivial-holonomy / bosonic sector. The spinorial / fermionic sector's SU(2) double-cover holonomy is structurally distinct from U(1) superfluid winding and does not reduce to integer-quantised circulation; the F–R argument on the spinorial sector is a genuinely different topological mechanism.

8.4 What Theorem 3 is and is not

Theorem 3 *is* a substrate-level spin-statistics result, restricted to the persistent sector. The mechanism is geometric (F–R) rather than axiomatic-QFT (Lorentz invariance + positive energy + microcausality).

Theorem 3 *is not* the full spin-statistics theorem of axiomatic QFT, which applies to arbitrary spin representations of the Poincaré group and requires the Wightman axioms. Promoting Theorem 3 to the full statement requires §14 item 1 (Fock construction) plus a full Lorentz-invariant fermionic field theory.

8.5 Frame-mediated parastatistics excluded; full exclusion conditional

The single-loop decomposition has two sectors because $\pi_1(SO(3)) = \mathbb{Z}_2$. The corresponding multi-loop decomposition by frame-mediated holonomy has two sectors (antisymmetric and symmetric). The F–R identification with $\pi_1(SO(3)) = \mathbb{Z}_2$ closes off *frame-mediated parastatistics* on the persistent sector: any parastatistics from richer frame-bundle holonomy is excluded.

Important caveat: full parastatistics exclusion is conditional. Parastatistics in the standard sense (Green's parafermions, non-one-dimensional irreducible S_n-representations) arises from additional internal indices on the one-particle states — colour-like quantum numbers transforming non-trivially under S_n — not from frame-bundle holonomy alone. If $\mathscr{H}_1$ carries internal S_n-non-trivial structure beyond $\mathbb{C}^4$, the full exchange operator could be a tensor product of the F–R frame phase with an internal-index permutation. The F–R argument alone does not exclude this.

Full parastatistics exclusion is *conditional* on:

- (a) $\mathscr{H}_1$ carrying no internal S_n-non-trivial structure beyond $\mathbb{C}^4$, *or*
- (b) an independent argument that internal-index structure is inert under permutation.

(a) is tied to species decomposition (§14 item 5): SU(3) colour on quarks carries precisely the internal S_n-non-trivial structure that the F–R argument alone cannot exclude. Full parastatistics exclusion on the persistent sector therefore requires both the frame-mediated result (derived here) and the internal-structure trivialisation (conditional on §14 item 5).

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9. Proposition 4 — Anticommuting Half of CAR Forced

9.1 The proposition

Proposition 4. A creation-and-annihilation operator framework $\{a^\dagger(\mathcal{C}), a(\mathcal{C})\}$ acting on the antisymmetric spinorial multi-loop sector of Theorem 2 is forced to satisfy

$\{a^\dagger(\mathcal{C}_i), a^\dagger(\mathcal{C}_j)\} = 0$, $\{a(\mathcal{C}_i), a(\mathcal{C}_j)\} = 0$, for all i, j.

The cross-relation $\{a(\mathcal{C}_i), a^\dagger(\mathcal{C}_j)\} = \delta_{ij}$ requires the Fock-space construction of §14 item 1.

Proposition 4 is a topological-core result: it follows from Theorem 2's antisymmetry plus the Fock-density property, without invoking substrate-physics inheritances.

9.2 Argument

Assume a creation-and-annihilation operator framework on the antisymmetric multi-loop sector. (Existence is non-trivial; the present argument shows that *given* such a framework, the anticommuting relations are forced.)

Consider a two-loop antisymmetric state satisfying

$|\mathcal{C}_2, \mathcal{C}_1\rangle_A = -|\mathcal{C}_1, \mathcal{C}_2\rangle_A$ (by Theorem 2),

with

$|\mathcal{C}_1, \mathcal{C}_2\rangle_A = a^\dagger(\mathcal{C}_1)\, a^\dagger(\mathcal{C}_2)\, |0\rangle$, $|\mathcal{C}_2, \mathcal{C}_1\rangle_A = a^\dagger(\mathcal{C}_2)\, a^\dagger(\mathcal{C}_1)\, |0\rangle$.

Combining:

$a^\dagger(\mathcal{C}_2)\, a^\dagger(\mathcal{C}_1)\, |0\rangle = -\, a^\dagger(\mathcal{C}_1)\, a^\dagger(\mathcal{C}_2)\, |0\rangle$,

for arbitrary $\mathcal{C}_1$, $\mathcal{C}_2$ in the spinorial sector. Hence the operator identity

$a^\dagger(\mathcal{C}_2)\, a^\dagger(\mathcal{C}_1) = -\, a^\dagger(\mathcal{C}_1)\, a^\dagger(\mathcal{C}_2)$ (on the vacuum and its iterated images)

follows.

Density of vacuum-iterated states. Promotion from "the relation holds on vacuum-iterated states" to "the operator identity $\{a^\dagger(\mathcal{C}_i), a^\dagger(\mathcal{C}_j)\} = 0$ on the antisymmetric multi-loop Hilbert space" requires that vacuum-iterated states $a^\dagger(\mathcal{C}_{i_1}) \cdots a^\dagger(\mathcal{C}_{i_n})|0\rangle$ span a dense subspace. This is a defining property of standard antisymmetric Fock-space construction, and is part of §14 item 1. Within the present paper, the relation is established on vacuum-iterated states; promotion to the operator identity is conditional on Fock-density holding in the substrate construction. Given density:

$\{a^\dagger(\mathcal{C}_i), a^\dagger(\mathcal{C}_j)\} = 0$ for all i, j.

The conjugate relation $\{a(\mathcal{C}_i), a(\mathcal{C}_j)\} = 0$ follows by Hermitian conjugation. ■

9.3 Structural reading

Proposition 4 forces the anticommuting half of CAR from Theorem 2's antisymmetry plus the Fock-density property. The only assumption beyond inheritance is the density property, part of the deferred §14 item 1 construction.

The cross-relation $\{a(\mathcal{C}_i), a^\dagger(\mathcal{C}_j)\} = \delta_{ij}$ is the deeper structural component, requiring:

- A definite vacuum state $|0\rangle$.
- A well-defined one-particle Hilbert space $\mathscr{H}_1$.
- A *positive-definite* inner product on $\mathscr{H}_1$.

These are §14 item 1 prerequisites.

9.4 Why this is genuine forcing

Proposition 4 derives the anticommuting half of CAR directly from Theorem 2's antisymmetric multi-loop structure, using only (i) the existence of a creation-annihilation framework and (ii) Fock-density. The honest structural picture: Theorem 2 forces antisymmetry; Proposition 4 forces the operator-algebraic anticommutation of the creation half (and by conjugation the annihilation half) given density. The existence of the framework and the cross-relation are §14 item 1 deliverables.

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10. Corollary 6 — Pauli Exclusion

10.1 The corollary

Corollary 6. Two identical spinorial source-carriers cannot occupy the same quantum state: $a^\dagger(\mathcal{C}) \cdot a^\dagger(\mathcal{C}) = 0$ for any spinorial loop $\mathcal{C}$.

10.2 Argument

Apply Proposition 4 with $i = j$:

$\{a^\dagger(\mathcal{C}), a^\dagger(\mathcal{C})\} = 2 \cdot a^\dagger(\mathcal{C})^2 = 0$,

hence $a^\dagger(\mathcal{C})^2 = 0$. ■

10.3 Structural reading

Pauli exclusion is the immediate consequence of Proposition 4. The chain is:

K=7 ($d \geq 3$) + κ-field uniqueness + Schrödinger→Dirac → Clifford structure (previous paper Theorem 1) → SU(2) lift (previous paper SI, C1–C3) → spinorial sector $U(2\pi) = -\mathbb{1}$ (previous paper Theorem 4) → F–R exchange holonomy U_exch = $-\mathbb{1}$ at topological level (Theorem 1) → antisymmetric multi-loop sector (Theorem 2) → spin-statistics correspondence (Theorem 3) → $\{a^\dagger, a^\dagger\} = 0$ given Fock density (Proposition 4) → Pauli exclusion (Corollary 6).

Every step is either an inheritance from a named result or a derived structural consequence. Pauli exclusion at substrate level is not a postulate but a chain of consequences originating from the K=7 architecture and the Clifford forcing.

Candidate physical reading (interpretive). The algebraic Pauli result acquires a candidate physical interpretation: two identical spinorial transport modes cannot share identical confinement topology on the coherent entanglement substrate because the coherent substrate modes supporting them become overconstrained at the substrate-mode level. This reading is *interpretive*, conditional on confinement-paper results; it does not replace the algebraic derivation as the load-bearing argument. The algebraic chain above establishes the result; the physical reading offers an intuition.

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11. Structural Consequences and the Two-Chain Hierarchy

11.1 Catalogue of structural consequences

11.1.1 Antisymmetric exchange is forced, not chosen. *Derived at topological level.* By Theorems 1 and 2.

11.1.2 Substrate-level spin-statistics correspondence holds. *Derived at topological level.* By Theorem 3.

11.1.3 The anticommuting half of CAR is forced. *Derived (given Fock-density).* By Proposition 4.

11.1.4 Pauli exclusion is forced. *Derived.* Corollary 6.

11.1.5 Frame-mediated parastatistics excluded; full parastatistics exclusion conditional on internal-structure trivialisation. *Derived (frame-mediated case); conditional (full exclusion).* See §8.5.

11.1.6 Anyonic statistics excluded in d ≥ 3. *Standard topology (anyonic exclusion in d ≥ 3); K=7 inheritance supplies d ≥ 3 at substrate level; physically reinforced by entanglement-lattice d=3 percolation.*

11.1.7 Spinorial decomposition acquires direct physical meaning. *Interpretive.* Previous paper's spinorial / trivial-holonomy decomposition = fermionic / bosonic matter-sector decomposition.

11.1.8 Matter is substrate-level antisymmetric transport on the spinorial sector. *Interpretive.* Physical fermions are species-decomposed cases; species decomposition deferred to §14 item 5.

11.1.9 Persistent spinorial loops are dynamically stable. *Inherited.* Proposition 5 from §3.10.

11.2 The two-chain hierarchy

The construction supports two parallel chains, kept structurally distinct.

Logical chain (derivational dependencies):

K=7 (d ≥ 3) + κ-field uniqueness + Schrödinger→Dirac (existence of first-order flow) → Clifford structure on spinorial loops (previous paper Theorem 1) → closure-frame bundle SU(2) lift (previous paper SI under C1–C3) → spinorial sector U(2π) = −𝟙 (previous paper Theorem 4) → F–R exchange holonomy U_exch = −𝟙 at topological level (Theorem 1) → antisymmetric multi-loop sector (Theorem 2) → spin-statistics correspondence (Theorem 3) → {a†, a†} = 0 given Fock density (Proposition 4) → Pauli exclusion (Corollary 6).

This chain is purely topological-derivational. It does not require substrate-physics inheritances (§3.9–§3.11) to be valid. Proposition 5 does not appear in this chain.

Physical-grounding chain (substrate realisation):

Void → Entanglement lattice (characteristic scales ξ, τ_s, m_s — §3.9) → Coherent persistent transport (superfluid stability via §3.10, Proposition 5) → Persistent loops realised as coherent transport modes (§4A.1) → Multi-loop configuration space realised on coherent medium (§4A.2) → Coherence condition (§4A.3) → Topological holonomy physically manifest as multi-loop wavefunction phase on the coherent substrate (Corollary 1') → Physical-realisation reading of Theorems 2, 3, Proposition 4, Corollary 6 follows from Corollary 1' on the coherence regime.

This chain establishes the physical realisation of the abstract topological structures. It depends on substrate-physics inheritances throughout.

Relation between chains. The two chains run in parallel. The logical chain establishes what is *true at the topological level*. The physical-grounding chain establishes *where and how it is physically manifest*. Both chains terminate at substrate-level fermionic-matter structure, with the logical chain establishing the abstract topological forcing and the physical-grounding chain establishing the physical realisation on the coherent medium. The matter-sector programme requires both for a complete account; conflating them would obscure the inheritance structure for downstream papers.

11.3 Unified statement

The two-chain treatment supports a unified synthesis: *substrate-level fermionic matter is forced as antisymmetric multi-loop transport on the spinorial sector by the F–R identification at the topological level (logical chain), and physically realised as long-lived coherent transport modes of the effective coherence medium under the coherence condition (physical-grounding chain).* Both components are required; neither alone is sufficient.

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12. Falsifiability Channels

Channels divide into *structural* (testing the construction's internal consistency and topological inheritances) and *empirical* (testing the substrate-physics inheritances).

12.1 Structural channels

Channel A: F–R identification breaks down. Tests Theorem 1.

Channel B: Exchange holonomy on spinorial sector $\neq -\mathbb{1}$. Tests Theorem 1's main conclusion.

Channel C: Both antisymmetric and symmetric multi-loop sectors exist on the spinorial side. Tests Theorem 2.

Channel D: Spin-statistics correspondence fails. Tests Theorem 3.

Channel E: K=7 → d=3 inheritance fails or substrate is d=2. Tests the dimensional dependence.

Channel F: Anticommuting half of CAR fails. Tests Proposition 4.

Channel G: Parastatistics emerges via internal-index structure. Tests the conditional part of §11.1.5. Note: does not falsify the frame-mediated exclusion (unconditional on F–R).

Channel H: Inheritance from C1–C3 fails. Tests the SU(2) lift on which Theorem 1 depends.

12.2 Empirical channels

These channels test the substrate-physics *inheritances* on which the physical-realisation layer depends, not the present paper's own load-bearing contributions (which are tested by Channels A–H).

Channel I: Coherence condition violated at observable scales. *Tests the §3.9 entanglement-lattice inheritance of coherence scales ξ, τ_s — not the present paper's own contribution.* If the entanglement-lattice paper's coherence-scale results are falsified or revised downward such that ξ or τ_s are incompatible with substrate-level support of physical fermion-pair systems, Corollary 1''s applicability narrows. Theorem 1 (topological core) is unaffected.

Channel J: Persistent transport stability fails. *Tests Proposition 5, inherited from the superfluid-transport paper §3.10 — not the present paper's own contribution.* If long-lived coherent winding patterns fail to emerge from the entanglement-lattice substrate, Proposition 5 falsifies and the present paper's physical-grounding chain inherits the failure. The logical chain (Theorems 1, 2, 3, Proposition 4, Corollary 6) is unaffected at the topological level.

Channel K: Effective-medium framing breaks down. *Tests §3.11, inherited from* When Space Itself Has Mass — *not the present paper's own contribution.* Empirical evidence of substrate-level Lorentz violation or preferred-frame structure incompatible with the effective-medium reading would falsify the framing of *When Space Itself Has Mass*; the present paper inherits the failure in its physical-realisation layer.

General empirical posture. Channels I, J, K are long-horizon and test inheritances, not the present paper's own derived content. The present paper does not directly establish new empirical predictions; it is *empirically tied* to substrate-physics observables through the inheritance chain, so empirical advances on ξ, τ_s, transport stability, and effective-medium structure bear on the present paper's physical-realisation layer. This is meaningful empirical embedding without overclaiming derived empirical content. The structural load-bearing claims of the present paper (Theorems 1, 2, 3, Proposition 4, Corollary 6) are tested by Channels A–H.

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13. What This Paper Achieves, and What It Does Not

13.1 What is achieved

Topological-core results:

(a) Configuration-space framework on $\mathcal{C}_n(P)$ (§§4–5). (b) Theorem 1 — F–R exchange holonomy (topological core). (c) Theorem 2 — Antisymmetric-only spinorial multi-loop sector. (d) Theorem 3 — Substrate-level spin-statistics correspondence. (e) Proposition 4 — Anticommuting half of CAR (conditional on Fock density). (f) Corollary 6 — Pauli exclusion.

Physical-realisation results:

(g) Physical-realisation framework on the coherent entanglement substrate (§4A). (h) Corollary 1' — Physical realisation of Theorem 1 under the coherence condition. (i) Proposition 5 — Coherent persistent transport stability (inherited).

13.2 What is not achieved

The present paper derives the substrate-level spin-statistics correspondence and the anticommuting half of CAR at the topological level, with Pauli exclusion as immediate corollary, plus physical realisation under the coherence condition. It does not derive:

- The cross-relation $\{a_i, a^\dagger_j\} = \delta_{ij}$ of full CAR (§14 item 1).
- Full Fock-space quantisation with vacuum, one-particle Hilbert space, and positive-definite inner product (§14 item 1).
- The full spin-statistics theorem of axiomatic QFT (§14 item 2).
- Renormalised fermionic QFT (§14 item 3).
- The Standard Model fermion spectrum (§14 item 5).

- Full parastatistics exclusion (conditional on internal-structure trivialisation; §11.1.5, §14 item 5).
- Spin-statistics for non-persistent-sector states.
- Direct empirical predictions from this paper alone; empirical embedding is via inheritances and operates through Channels I, J, K.

Deliverable table:

Deliverable	Status
Configuration-space exchange topology	**Established** (§§4–5)
Physical realisation on coherent substrate	**Established** (§4A, inheriting §3.9–§3.11)
F–R exchange holonomy on spinorial sector (topological)	**Theorem 1**
F–R exchange holonomy physically realised	**Corollary 1'** (conditional on coherence condition)
Antisymmetric-only spinorial multi-loop sector	**Theorem 2**
Substrate-level spin-statistics correspondence	**Theorem 3**
Anticommuting half of CAR	**Proposition 4** (conditional on Fock density)
Persistent-transport stability of spinorial loops	**Proposition 5** (inherited from §3.10)
Pauli exclusion	**Corollary 6**
Frame-mediated parastatistics exclusion	**Derived** (§11.1.5)
Full parastatistics exclusion	**Conditional** (§11.1.5, §14 item 5)
Cross-relation $\{a, a^\dagger\} = \delta$ of full CAR	**Open** (§14 item 1)
Fock-space construction	**Open** (§14 item 1)
Positive-definiteness of $\mathscr{H}_1$ inner product	**Open** (§14 item 1 prerequisite)
Full axiomatic-QFT spin-statistics	**Open** (§14 item 2)
Renormalised fermionic QFT	**Open** (§14 item 3)
Species decomposition	**Open** (§14 item 5)
Standard Model fermion spectrum	**Open** (§14 item 5)
Confinement-fermion bridge	**Open working conjecture** (§14 item 9)

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14. Open Problems

1. Full CAR algebra and Fock-space construction. Dominant deliverable. Proposition 4 supplies the anticommuting half. The cross-relation requires:

Construct a substrate-level Fock space: vacuum |0⟩ as the "no-loops" substrate state (the coherent entanglement substrate in its no-persistent-loop configuration), one-particle Hilbert space $\mathscr{H}_1$ as the inner-product completion of single-loop spinorial states, antisymmetric Fock space $\mathscr{F}_A$ =

$\bigoplus_n \Lambda^n \mathscr{H}_1$, and operators $a^\dagger(\mathcal{C})$, $a(\mathcal{C})$ acting on $\mathscr{F}$_A. The CAR cross-relation follows from the inner-product structure on $\mathscr{H}_1$.

Positive-definiteness prerequisite. The standard CAR construction requires the $\mathscr{H}_1$ inner product to be positive-definite. If indefinite (Krein-space) structure appears, the cross-relation acquires modifications (indefinite-metric quantisation). The Fock construction must verify positive-definiteness or treat the indefinite case explicitly.

Fock-density prerequisite. Proposition 4's promotion from vacuum-iterated states to operator identity requires the Fock-density property. Part of this deliverable.

2. Full spin-statistics theorem. Promote Theorem 3 from the substrate-restricted analogue to the full Streater–Wightman / Haag result for arbitrary spin.

3. Renormalised fermionic QFT. Fermionic interactions, renormalisation, path-integral formulation.

4. Coupled QED structure. Promote the coupled action of the previous paper §12 to fully quantised substrate-level QED, including renormalised interactions, anomalies, and the substrate-to-electroweak bridge.

5. Species decomposition. Discharge the spinorial sector into physical fermion species (leptons, quarks). Inherited from the previous paper §22 item 2. Bears on §11.1.5 (full parastatistics exclusion) and §7.4 (antisymmetric-sector exhaustion).

6. Non-abelian fermionic sectors. Extension to QCD-type structure. SU(3) colour carries precisely the internal S_n-non-trivial structure that §11.1.5 identifies as the potential parastatistics route; the non-abelian extension must address how internal colour combines with F–R frame holonomy in multi-quark states.

7. Vacuum structure, anomalies, and CPT. Substrate-level treatment of vacuum stability, chiral anomalies, CPT structure.

8. Configuration-space topology with non-trivial $\pi_1(P)$. The full $\pi_1(\mathcal{C}_n(P)) \cong S_n \ltimes \pi_1(P)^n$_quot is non-trivial when $\pi_1(P)$ is non-trivial (generic VERSF case given $\beta_1 \geq 1$). Worth working out.

9. Confinement-fermion bridge. Whether stable fermionic matter admits confinement-stabilised topological-transport reformulation is a working conjecture for a future paper, not a synthesis result here.

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15. Relation to Earlier VERSF Papers, and the Dependency Graph

Topological inheritances:

- Spinorial Closure Transport (previous paper) — load-bearing throughout.
- κ-field uniqueness programme — via previous paper Theorem 1.
- Schrödinger→Dirac paper — via previous paper Theorem 1.
- K=7 minimal fact architecture — via previous paper §8.
- Triangular closure programme — for C1 SO(3) structure.
- Microscopic Origin paper — loop ontology, SA1–SA5.
- Topological-threshold paper — $\beta_1 \geq 1$ for non-trivial $\pi_1(P)$.
- Refinement-persistence framework — refinement-stability inheritance.
- Classical Finkelstein–Rubinstein theorem — not VERSF; the standard topological result.

Substrate-physics inheritances (physical-realisation layer):

- Entanglement-lattice papers — ξ, τ_s, m_s; coherence condition.
- Superfluid-transport paper — Proposition 5; superfluid-winding connection for the bosonic sector.
- *When Space Itself Has Mass* — effective-medium framing.

Dependency graph (key results):

Result	Depends on	Layer
Multi-loop configuration space $\mathcal{C}_n(P)$	P + S_n action	Topological
S_n-factor decomposition of $\pi_1(\mathcal{C}_n(P))$	K=7 → d=3 + standard topology	Topological
Physical realisation on coherent substrate	§3.9 + §3.10 + §3.11	Physical-realisation
Coherence condition	§3.9 (ξ, τ_s)	Physical-realisation
Theorem 1 (topological core)	Previous paper Theorem 4 + C1–C3 + K=7 → d=3 + classical F–R	Topological
Corollary 1' (physical realisation)	Theorem 1 + coherence condition	Physical-realisation
Theorem 2	Theorem 1 + bundle-section structure	Topological
Theorem 3	Theorem 2 + parallel argument	Topological
Proposition 4	Theorem 2 + creation-annihilation framework + Fock density	Topological
Proposition 5	Inherited from §3.10	Physical-realisation
Corollary 6	Proposition 4 with i = j	Topological
Frame-mediated parastatistics exclusion	F–R + $\pi_1(SO(3)) \cong \mathbb{Z}_2$	Topological
Full parastatistics exclusion	+ internal-structure trivialisation (conditional)	Topological, conditional

Result	Depends on	Layer
Anyonic exclusion in $d \geq 3$	K=7 → $d \geq 3$ + physically reinforced	Topological + physically reinforced

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16. Epistemic-Status Labelling and Representation-Theoretic Status Table

16.1 Epistemic-status labelling

Derived at the topological level (under named inheritance). Theorem 1, Theorem 2, Theorem 3, Proposition 4 (with Fock-density assumption), Corollary 6.

Derived at the physical-realisation level (under topological derivation + substrate-physics inheritance). Corollary 1' (conditional on the coherence condition).

Inherited. Proposition 5 (from superfluid-transport paper §3.10). Coherence condition content (ξ, τ_s scales from entanglement-lattice paper §3.9). Effective-medium framing (from *When Space Itself Has Mass* §3.11).

Conditional on framework assumptions. All topological-core derivations conditional on the previous paper's inheritances (C1–C3 for SU(2) lift; K=7 for $d \geq 3$; κ-field uniqueness; Schrödinger→Dirac). Full parastatistics exclusion conditional on internal-structure trivialisation (§14 item 5).

Interpretive. §11.1.7, §11.1.8 (physical interpretations of derived results). §10.3 candidate physical reading of Pauli exclusion (conditional on confinement-paper results).

Synthetic. §11.2 two-chain hierarchy combining derived and inherited results.

Conjectural / open. Fock-space framework existence, including density and positive-definite inner product. Full QFT spin-statistics theorem. Renormalised fermionic QFT. Species decomposition. Standard Model fermion spectrum. Confinement-fermion bridge.

16.2 Representation-theoretic status table

Result	Status	Source	Layer
Multi-loop configuration space $\mathcal{C}_n(P)$ well-defined	Derived	§4.1	Topological
S_n-factor of $\pi_1(\mathcal{C}_n(P))$ is S_n for $n \geq 2$ in $d \geq 3$	Standard topology	§4.3	Topological
Exchange path γ_exch projects non-trivially onto S_n-factor	Standard topology	§4.4	Topological

Result	Status	Source	Layer
Physical realisation on coherent substrate	Inherited	§3.9–§3.11, §4A	Physical-realisation
Coherence condition (ξ, τ_s)	Inherited	§3.9, §4A.3	Physical-realisation
Persistent-transport stability of spinorial loops	Inherited	§3.10, Prop 5	Physical-realisation
Effective-medium framing	Inherited	§3.11	Physical-realisation
F–R identification $\gamma\_exch \simeq \gamma\_FR$ in S_n-factor	Classical theorem	§3.8	Topological
U_exch = −𝟙 on spinorial sector (topological)	**Derived under inheritance**	**Theorem 1**	**Topological**
U_exch = −𝟙 physically manifest on coherent substrate	**Derived under topological + coherence condition**	**Corollary 1'**	**Physical-realisation**
Antisymmetric-only spinorial multi-loop sector	**Derived under inheritance**	**Theorem 2**	**Topological**
Substrate-level spin-statistics correspondence	**Derived under inheritance**	**Theorem 3**	**Topological**
Symmetric sector empty on spinorial side	Derived under inheritance	Theorem 2	Topological
Superfluid winding connection for bosonic sector	Interpretive (inherited from §3.10)	§8.3	Physical-realisation
$\{a^\dagger_i, a^\dagger_j\} = 0$ forced	**Derived under inheritance + Fock-density**	**Proposition 4**	**Topological**
Pauli exclusion $a^\dagger(\mathcal{C})^2 = 0$	**Derived**	**Corollary 6**	**Topological**
Candidate physical reading of Pauli exclusion	Interpretive (conditional on confinement results)	§10.3	Interpretive
Frame-mediated parastatistics excluded	Derived	§11.1.5	Topological
Full parastatistics exclusion	Conditional on internal-structure trivialisation	§11.1.5, §14 item 5	Topological, conditional
Anyonic exclusion in $d \geq 3$	Standard topology; K=7 supplies $d \geq 3$ at substrate level	§11.1.6	Topological (physically reinforced)
Two-chain hierarchy (logical + physical-grounding)	Synthetic	§11.2	Synthesis
Cross-relation $\{a_i, a^\dagger_j\} = \delta_{ij}$	Open	§14 item 1	—
Fock-space construction	Open	§14 item 1	—
Full axiomatic-QFT spin-statistics	Open	§14 item 2	—

Result	Status	Source	Layer
Renormalised fermionic QFT	Open	§14 item 3	—
Species decomposition	Open	§14 item 5	—
Confinement-fermion bridge	Open working conjecture	§14 item 9	—

Status glossary:

- *Derived under inheritance* — derived given explicit inheritance from named theorems.
- *Derived under inheritance + [further assumption]* — derived given inheritance plus a stated additional assumption (e.g., Fock-density; coherence condition).
- *Inherited* — taken as a non-trivial result of another VERSF paper, not re-derived here.
- *Standard topology / Classical theorem* — background mathematics.
- *Conditional on [stated condition]* — proven under stated structural conditions.
- *Interpretive* — physical or conceptual reading of a derived result.
- *Synthetic* — combines multiple results into a unified statement.
- *Open* — not addressed; identified deliverable.

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17. Conclusion

The preceding spinorial paper supplied single-loop spinorial structure. The present paper takes the multi-loop step, organised in two structural layers.

Topological-core layer. The central claim is that the substrate-level analogue of the F–R theorem applies on the spinorial sector at the topological level. In $d \geq 3$ (inherited from K=7 → d=3), γ_exch projects onto the non-trivial element of the S_n-factor of $\pi_1(\mathcal{C}_2(P))$; within that S_n-factor, γ_exch is homotopic to γ_FR. Frame holonomy along γ_FR on the spinorial sector equals $U(2\pi) = -\mathbb{1}$ by inheritance. Hence U_exch = $-\mathbb{1}$ at the topological level (Theorem 1).

Theorem 2 establishes the antisymmetric-only spinorial sector by exclusion. Theorem 3 extends to the substrate-level spin-statistics correspondence: spinorial ↔ antisymmetric ↔ fermionic; trivial-holonomy ↔ symmetric ↔ bosonic. Proposition 4 forces the anticommuting half of CAR given Fock density; Corollary 6 extracts Pauli exclusion as immediate consequence.

Physical-realisation layer. Corollary 1' establishes that under the coherence condition (spatial extent $\lesssim \xi$, timescale $\lesssim \tau_s$), Theorem 1's topological holonomy is physically manifest as a multi-loop wavefunction phase on the coherent entanglement substrate. Proposition 5, inherited from the superfluid-transport paper, supplies the dynamical stability of persistent spinorial loops as long-lived coherent transport modes. The substrate is treated under the effective-medium framing of *When Space Itself Has Mass* throughout — an effective coherence medium, not a preferred-frame mechanical ether.

Two-chain synthesis. The §11.2 two-chain hierarchy makes the layer separation explicit. The *logical chain* (K=7 → ... → Pauli exclusion) is purely topological-derivational and does not require substrate-physics inheritances. The *physical-grounding chain* (Void → ... → coherent transport modes → physical realisation) establishes where and how the topological structures are physically manifest. Both chains are required for a complete account.

Synthesis statement:

Antisymmetric exchange on the persistent spinorial sector is forced at the topological level by the Finkelstein–Rubinstein identification of configuration-space exchange topology with single-loop 2π-rotation holonomy, operating within the S_n-factor of $\pi_1(\mathcal{C}_n(P))$. The anticommuting half of CAR follows directly given Fock density. Pauli exclusion is an immediate corollary. The substrate-level spin-statistics correspondence — spinorial loops fermionic, trivial-holonomy loops bosonic — is forced on the persistent loop sector at the topological level by the same mechanism, without separate appeal to Lorentz invariance, positive energy, or microcausality. Physical realisation on the coherent entanglement substrate, under the coherence condition (spatial extent $\lesssim \xi$, exchange timescale $\lesssim \tau_s$), manifests the topological holonomy as an observable multi-loop wavefunction phase. Persistent spinorial loops are dynamically stable substrate carriers, inherited from the superfluid-transport paper. Frame-mediated parastatistics is excluded by the F–R argument; full parastatistics exclusion is conditional on the spinorial one-particle Hilbert space carrying no additional S_n-non-trivial internal structure beyond the Clifford-internal $\mathbb{C}^4$, pending species decomposition.

The matter-sector programme now possesses:

- A substrate ontology for the persistent current (Microscopic Origin paper).
- An admissibility framework for gauge coupling (Matter Coupling paper).
- An algebraic-geometric origin of spin-½ structure (spinorial paper, Part III).
- A substrate-level forcing of antisymmetric exchange, anticommutation (partial), and Pauli exclusion at the topological level (the present paper, Part IV, topological core).
- Explicit physical realisation on the coherent entanglement substrate under the coherence condition (the present paper, physical-realisation layer).

The remaining deliverables — full CAR algebra (cross-relation), Fock-space construction (with positive-definite inner product), full QFT spin-statistics, renormalised fermionic QFT, species decomposition, Standard Model fermion spectrum, substrate-to-electroweak bridge, confinement-fermion bridge — are concrete next-paper targets, with explicit conditions identified for their construction.

Explicit clarification of scope (final). The present paper derives the substrate-level spin-statistics correspondence and the anticommuting half of CAR on the spinorial sector at the topological level, with Pauli exclusion as immediate corollary. These are forced by the F–R identification on the S_n-factor of $\pi_1(\mathcal{C}_n(P))$, conditional on the K=7 architecture ($d \geq 3$), κ-field uniqueness, Schrödinger→Dirac, and C1–C3. The physical realisation on the coherent entanglement substrate, under the coherence condition, is supplied by Corollary 1' and Proposition 5 (the latter inherited). The cross-relation of the full CAR algebra, Fock-space construction (with positive-definite inner product), the full axiomatic-QFT spin-statistics theorem, renormalised fermionic QFT, full parastatistics exclusion, the Standard Model

fermion spectrum, and the confinement-fermion bridge are not derived here and are explicitly identified as next-stage deliverables.

Strongest framing. Given (i) the spinorial sector structure of the previous paper, (ii) the K=7 → d=3 inheritance, (iii) the classical F–R identification on the S_n-factor of configuration-space π_1, and — for physical realisation — (iv) the substrate-physics inheritances (coherent entanglement medium with characteristic scales, persistent-transport stability, effective-medium framing), the persistent spinorial loop sector is forced to carry antisymmetric multi-loop exchange, anticommuting creation-annihilation structure (anticommuting half, given Fock density), and Pauli exclusion at the topological level. Physical realisation on the coherent substrate manifests the topological holonomy under the coherence condition. The substrate-level spin-statistics correspondence — spinorial ↔ fermionic, trivial-holonomy ↔ bosonic — holds on the persistent sector by configuration-space geometry rather than by axiomatic-QFT inputs. Frame-mediated parastatistics is excluded; full parastatistics exclusion awaits species decomposition. The remaining matter-sector work is the promotion to a full fermionic quantum field theory through the Fock-construction, full-spin-statistics, renormalisation, and species-decomposition deliverables.